Research article

Wavelet-based systematic risk estimation: application on GCC stock markets: the Saudi Arabia case

Anouar Ben Mabrouk[1,2,3,*]

[1] Department of Mathematics, Higher Institute of Applied Mathematics and Computer Science, University of Kairouan, Tunisia

[2] Laboratory of Algebra, Number Theory and Nonlinear Analysis, Department of Mathematics, Faculty of Sciences, University of Monastir, Tunisia

[3] Department of Mathematics, Faculty of Science, University of Tabuk, Saudi Arabia

* **Correspondence:** Email: anouar.benmabrouk@fsm.rnu.tn, amabrouk@ut.edu.sa; Tel: +21677238217; Fax: +21677226575.

Abstract: Systematic risk estimation is widely applied by investors and managers in order to predict risks in the market. One of the most applied measures of risk is the so-called Capital Asset Pricing Model, shortly CAPM. It has been studied empirically focusing on the impact of return interval on the betas. This paper lies in this topic and attempts to estimate the CAPM at different time scales for GCC markets by adapting a wavelet method to examine the relationship between the return of the stock and its systematic risk at different time scales. The main novelty is by applying non-uniform intervals of time. Differently from existing literature, we use random ones. The proposed procedure is acted empirically on a sample corresponding to Saudi Tadawul market as the most important GCC representative market actively traded over the period January 01, 2013 to September 20, 2018, which is characterized by many political, economic and financial movements such as Qatar embargo, Yemen war, NEOM project, 2030 KSA vision and the Arab spring effects. The findings in the present work may be good basis for understanding current and future GCC markets situation and may be thus a basis for investors' decisions in such markets.

Abbreviations: GCC: Gulf Cooperation Council; CAPM: Capital Asset Pricing Model; KSA: Kingdom of Saudi Arabia; NEOM: New Future (A planned cross-border city in the Tabuk province of northwestern Saudi Arabia); FOMC: Federal Open Market Committee; COVID-19: Coronavirus Disease of 2019; Cap Go: Capital Goods; CoPr Svc: Commercial and Professional

Services; Media Ent: Media and Entertainment; Div Fin: Diversified Financials; R Estate: Real Estate Mgmt & Dev't

1. Introduction and motivations

In this work, we focus on developing a best mathematical approach for the so-called systematic risk or capital asset pricing model using wavelet theory. Recall that such a theory has been proved to be a powerful tool in the economic/financial field. Financial indicators are subject to volatility and high fluctuations in all markets, which makes their study and their understanding using classical methods un-sufficient. One of the main power characteristic of wavelets is their ability to detect and/or localize fluctuations and volatility.

Financial markets are the essential component of countries' economies. All industrial countries have at least one purse and most developing countries have created one or have to create. Financial markets have enabled economic agents to reconcile the antagonistic objectives of their clientele. These are objectives of profitability, security and liquidity. To obtain portfolios that combine high levels of profitability with high levels of security and thus being less risky, financial market participants have several financial asset management instruments. Earlier, in the 50th of the last century, Markowitz (1952) has put the basics of the theory of financial asset management and the functioning of the financial markets leading to a rigorous link between risk and return on securities. Next, the studies have been growing up in both theory and empirically. See Sharpe (1964), Lintner (1965a), Lintner (1965b), Mossin (1966) and Black (1972), Black et al. (1972). Henceforth, the CAPM become a central model in financial theory that makes it possible to describe in a simple way the link of profitability of financial assets and their risks. The validity of the CAPM has been by the next investigated by Roll (1977) based on USA market and stating that the omission of some low capitalization securities could affect the CAPM. In addition, all assets such as bonds, gold and real estate should be taken into account.

More recently, the CAPM has been improved by several empirical studies. In particular, Fama and French (1996) announced the death of systematic risk beta, using annual data of the well known indexes NYSE, AMEX, and NASDAQ returns during the period time 1963 to 1990. They stated that the systematic risk beta may not provide a strong indication for explaining the average change in the market index. Re-considered next by Kothari and Shanken (1998), results of Fama and French (1993) have been proved to be more significant by using monthly data on equity returns rather than annual data. Even though, Kothari and Shanken (1998) noticed that the use of annual returns to estimate the beta may be a cause of measurement problems. This is essentially due to the seasonal effect of returns, the non-synchronization of actions, etc. Based on the annual returns from 1927 to 1990, the authors concluded that the beta is statistically significant and that the regular contribution of size to explain the difference in sample yielded beyond the beta is minimal. For a deep backgrounds on CAPM the readers may refer to Aydogan (1989), Banz (1981), Basu (1977), Black (1972), Black et al. (1972), Breeden (1979), Chae and Yang (2008), Chan and Lakonishok (1993), Fama and MacBeth (1973), Fama and French (2004), Fama and French (2006), Galagedera (2007), Gibbons (1982), Gursoy and Rejepova (2007), Handa et al. (1989), Handa et al. (1993), Ho et al. (2000), Karan and Karadagli (2001), Merton (1973), Perold (2004).

Some empirical extensions of the CAPM taking into account the time variations of beta and time variations of the risk premium or both of them have been developed. The so-called CAPM Conditional Testing has been established joining an old study on the same subject due to Levhari and Levy (1977) where the use of a shorter time than real time has been proved to provide biased beta estimator. Handa et al. (1993) showed that if the difference in the time interval of the returns is taken into account, for the same action, different estimators can be obtained. Furthermore, Handa et al. (1993) rejected the beta estimator that uses monthly returns and accepted instead the use of annual returns. Cohen et al. (1986) showed that the beta estimator is sensitive to the time intervals used for equity returns. As a result, the problem of the validity of the CAPM arises, and more particularly the validity of the relationship between the systematic risk and the return of securities listed on the financial markets taking into account the time factor in the returns of the shares. This key issue of the CAPM audit, which has already been the subject of extensive research in industrialized countries, is still relevant.

As mentioned by Marfatia (2017a) and Marfatia (2017b), one of the limitations of conventional approach is the lack of distinction between the time domain and the frequency ones which is a crucial task in both econometric and economic rationale.

The present study lies in the whole scope of the integration and/or the study and evaluation of risks in stock markets according to time changes. Many studies have been developed to do such task. In Marfatia (2017a) a wavelet-based study has been developed to investigate the impact of risks in international stock markets. The approach proposed in Marfatia (2017a) consists in combining wavelet techniques with time-varying conditional volatility to investigate essentially the co-movement of risks at both the country level and regional level. The authors concluded that co-movement of risks between the US market and European markets are strong mostly at lower frequencies. However, the co-movement of risks between the country and its local region are strong at higher frequencies. Moreover, at high frequencies the relationship between the country in hand and the US or other global markets is also weak. The study concluded also that the spill-over of risks are mostly limited at lower frequencies contrarily to existing studies which claim that especially during the recent financial crisis, the spill-over of risks are largely a global phenomenon.

In Marfatia (2017b), linkages of the housing market and macroeconomy has been investigated using wavelet approach. Cross-wavelet coherence has shown that the relationship varies significantly relatively to countries, time, frequencies, and the direction of causation. The author claimed that house price movements are related to interest rates at short-run, however per capita income growth is related to the interest rates at the long-run. Moreover, the author studied the role of industrial production and income growth and concluded that the direction of causation between the housing market and macroeconomic variables depends strongly on time-frequency domain.

Our main aim here is to understand the nature of the relationship linking systematic risk and the return on equities for GCC stock markets by using wavelet theory as a mathematical tool recently introduced in the field of finance and proved its performance compared to classical tools. A new method to forecast systematic risk based on wavelets for missing data and further exploring the relationship between the return of the stock and its systematic risk at different time scales is proposed.

Wavelets offer efficient algorithms for practical problems where classical techniques have shown limitations. Moreover, they provide an attractive mathematical formalism in the reformulation of several problems and in different scientific fields, especially in time series analysis. Wavelets form a mathematical tool that transforms time domain data into different frequency horizons. They represent

the advantage of being localized in both the time and frequency domains. They allow to observe and to analyse data at different time scales, which in turns makes it possible to overcome the inadequacies of the classical analysis of the CAPM. For more backgrounds on wavelets and their applications the authors may refer to Arfaoui et al. (2017), Arfaoui et al. (2020), Arfaoui et al. (2020), Mabrouk et al. (2008), Mabrouk et al. (2008), Mabrouk et al. (2010), Mabrouk et al. (2011), Mabrouk and Zaafrane (2013), Mabrouk et al. (2015), Mahmoud et al. (2016), Percival and Walden (2000), Selcuk (2005), Zemni et al. (2019a), Zemni et al. (2019a).

From an empirical point of view, we aim to apply the wavelet technique to estimate the systematic risk of an action relatively to the market return. This validity could also be a real foundation for effective financial decisions. Indeed, the effectiveness of a financial decision depends to a large extent on the accuracy of the valuation of the securities as well as the most precise knowledge possible of their subsequent evolutions and their risks. We therefore try, in this work to provide a better comprehension of the GCC financial markets in the face of a modern financial theory such as the CAPM.

To resume, one aim in the present research is to find a mathematical approach to the GCC market indices by applying wavelet methods. The new idea consists in applying random and/or different time supports allowing the subdivision of the whole support into random parts instead of using the classical sampling (weeks, months, years). We recall in particular, that the Kingdom of Saudi Arabia has established 2030 projects that have a direct impact on the national market as well as the rest of the Arab Gulf and international markets such as and particularly NEOM project which will have a profound impact on the market. This makes it of interest to study such market and understand its complexities.

The proposed procedure is acted on samples composed of stocks in GCC markets actively traded over a critic period strongly and directly related to the last political changes especially in Arab countries, the GCC embargo against Qatar country. The period may also be considered as a pre-corona stage as we now see that the current pandemic COVID-19 has been spread in quietly all the world. GCC continent is one of the places that are related to all the world because of their geographical position and their strong relation to the worldwide economies as biggest petroleum countries. Moreover, the largest workers community is focusing in these countries. Saudi Arabia is also characterized by the saint cities of Muslims which consequently bring a big number of pilgrims and visitors each year. This may be a strong cause of dispersion of viruses which may be next transferred with pilgrims to other continents. Saudi Arabia has also made a worldwide program known as NEOM international project which is the basic point in the 2030-vision of the kingdom. The sample of study will be based on Tadawul index during the period from January 01, 2013 to September 20, 2018. Notice that this period is also characterized by the direct link to Syrian and Yemen movements and the present Arab war against some parties in Yemen which is somehow leaded by Saudi Arabia. We will discuss the effect of such links on the stability of GCC markets. We may also recall many financial movements such as Aramco subscription and generally the 2030 KSA vision. The findings in the present work may be a good basis for understanding GCC markets situation, behavior and future and thus a basis for investors' decisions in such markets.

We recall that in the data basis applied, the main problem confronted is the lack of data where the samples present missing values for many cases. Consequently, our work becomes a twofold study. In a first step, we developed a wavelet-based method to reconstruct missing data which by the next leads to complete and adjusted basis on the whole period above. The complete basis is applied next for CAPM and thus the comprehension of the Saudi Tadawul market.

The rest of the present work is organized as follows. In section 3, a literature review on forecasting systematics risk is briefly presented. Section 4 is devoted to the development of our methodology. We especially re-develop the wavelet analysis of time series briefly to apply it by the next for missing data reconstruction which in turns will be applied for completing the data basis used later. The mathematical formulation of the CAPM is provided by the next. Finally, this section is achieved by the development of the wavelet CAPM. Section 5 is subject of our empirical results and their discussions on the KSA Tadawul stock market. Section 6 is a conclusion. Finally, section 7 is an appendix in which we provided empirical results of the 3-scale law as a supplementary part to section 5 to improve the idea of time changes in stock market prediction.

2. Systematic risk forecasting literature review

We propose in this section to conduct a literature review on the systematic risk forecasting. However, we will not be addressing mathematical formulations in this section and include them instead in the methodology section to provide readers a more comprehensible mathematical methodology.

The CAPM was firstly discovered by Sharpe (1964) and next Lintner (1965a), Lintner (1965b). It has been applied next for half a century in estimating the capital cost for companies and evaluating the performance of managed portfolios. However, its empirical evaluation has been always affected by the availability or not of the data. The empirical problems of the CAPM might reflect theoretical failures arising from many superficial assumptions.

The CAPM is based indeed on several assumptions, such as mono-periodicity, the market perfection aspect without taxes nor transaction costs, homogeneity in anticipations, unlimited short selling, loans and borrowing at the risk-free and limitless rate, strictly increasing and strictly concave Von-Neumann-Morgenstern utility functions, mean-variance preferences based on restrictions relating to the return or the utility function, investor aversion to risk, competition and market efficiency. However, such assumptions might not be generally accepted simultaneously, which yields limitations for the CAPM. As a consequence, some solutions have been proposed such as the CAPM with taxes (Brennan (1973), Litzenberger and Ramaswamy (1979)), the CAPM with transaction costs (Lévy (1978)), the CAPM in continuous time (Merton (1973)) and the CAPM with non-homogeneous anticipations (Sharpe (1970a), Sharpe (1970c), Sharpe (1970b)).

The choice of a portfolio is generally made after a financial analysis of a set of more or less independent actions. Robust measurement of the crucial variables in the investment decision should be ensured. Numerous studies have investigated the financial analysis of individual actions allowing to make a good decision. Markowitz (1952) has shown that the investor seeks to optimize his choices by taking into account the expected return on his investments and the risk of his portfolio. His model suggests selecting several stocks based on statistical criteria such as the profitability of a stock in order to obtain optimal portfolios. However, Vasichek et al. (1972) asserted that the profitability of a stock is not sufficient to characterize an investment opportunity. The dispersion of returns around the average return reflects the uncertainty of the investment (the risk). The total risk of a portfolio can always be measured by the variance or the standard deviation of its profitability which constitutes a convenient measure of dispersion. This allows investors to minimize the risk of their actions.

According to Markowitz, the CAPM is used to solve the problem of the portfolio structure which incorporates the quantified treatment of risk by estimating the demand function of assets. This makes it

possible to study the equilibrium of the market. Such a method has a main drawback as it does not take into consideration the time factor, which causes a strong limitation.

The CAPM is a diagram that analyses the return and risk of an investment. Sharpe (1964) and Lintner (1965b) suggested to estimate the prices of transferable securities allowing supply and demand to be balanced and allowing a general equilibrium of the market. Therefore, the CAPM can determine the prices of equilibrium securities through supply and demand according to a linear relationship between portfolio profitability and total risk. Indeed, the level of risk is obtained by varying the loan or the borrowing which can be made up of all the securities listed on a market. Thus, any combination of an equity portfolio and an investment in the risk-free asset will be indicated by a straight line in the risk-return space. Therefore, any portfolio, by definition, is perfectly diversified, since it involves all stocks in the proposals of their market capitalizations.

Mathematically, for a market in equilibrium, for any portfolio or asset, the CAPM is expressed by means of a linear relationship between the expected return on the share or portfolio i and the market premium where the linearity coefficient is often denoted β_i and is known as systematic risk or beta. This coefficient is mainly used for two purposes. The first involves the ranking of assets and portfolios against systematic risks by practitioners. The second aims to test the mean-variance efficiency. Beta is usually estimated using the standard market model, which is expressed by the linear regression model. It also represents the coefficient of elasticity of the price of the security with respect to the stock index representing the market.

Sharpe (1964) proved that for each share i relating to a given portfolio the systematic risk β_i is expressed by the quotient of the covariance between the rate of return of asset i and the rate of return of the portfolio by the square of market risk. More precisely, the expected return on security i is expressed by the risk-free interest rate increased by a risk premium composed of two elements. The first element is the excess of the expected market rate of return over the return on the risk-free asset, measuring the risk premium that the investor should perceive by agreeing to bear a risk equal to that of the market. The second element β_i is the measure of the importance of the risk of security i in relation to the risk of the market. Sharpe (1970b) and Vasichek et al. (1972) concluded firstly that the CAPM is purely normative, subject to empirical validation and secondly that the rate of return of each share in excess of the risk-free interest rate depends only on β_i.

The CAPM gives a coherent answer, as for the evaluation of the expected profitability of an asset according to the risk. This profitability can be used as the discount rate in the valuation of the asset. Indeed, the relationship between the risk and the profitability of a financial asset is linear, provided that this risk is measured by its variance with the market taken as a whole and not its variance or its standard deviation. Each investor has the choice between obtaining a certain but low profitability or taking a risk offset by a higher expected profitability. Markowitz was the first to formalize and quantify the diversification effect according to which a combination of many assets in a portfolio reduces the total risk for a given expected rate of return.

Sharpe W. F. considered a model which results in the equilibrium relationship between the expected profitability of a financial asset and its risk. This model makes it possible to determine the choice of investment and to compare alternatives to different random gains by using the Esperance-Variance criterion in order to choose the best alternative.

In the same context, Von Neuman and Morgestern in the 1944-th formally proved that any individual yielding to a few intuitive axioms of rationality aims to maximize the expectation of the utility of their

wealth. Indeed, the utility function reflects the preference of each individual, hence each individual is specific, and depends in particular on his initial wealth at the time of the decision and his aversion to risk. The conclusion lies in the fact that the individual faced with alternatives with random consequences aims to maximize the expectation of utility.

Diversification is also an important factor in estimating the CAPM. It is strongly related to the behaviour of the expectation-variance of the portfolio. The profitability of the portfolio is the weighted average of the expectations. The contribution of each security to the expected profitability of the portfolio is therefore proportional to its expected profitability. In terms of risk, it is possible to measure that of the portfolio by the variance of its profitability.

According to Markowitz, an efficient portfolio is characterized by a maximum expectation of return or by a minimum variance for a given expectation of return. The efficient frontier is the set of all efficient portfolios. The efficient frontier takes two different forms depending on the absence or presence of a risk-free return asset. The minimum variance frontier determines portfolios of risky assets and shows the lowest risk for a given level of profitability. As for the portfolios with the highest levels of profitability for a given level of risk, they are represented by the efficient frontier corresponding to the upper part of the frontier at minimum variance.

However, several criticisms have been pointed out for the CAPM (Desmoulins-Lebeault (2003), Magni (2007a), Magni (2007b)). Some are linked to the portfolio and some are linked to market. Roll (1977) introduced an empirical criticism due to the representative portfolio of the market. This portfolio groups together assets held by a set of investments, a set of stocks, bonds and other types of securities that are not in practice negotiable and liquid such as real estate assets of investments and their human capital. This puts into question the results of the econometric tests and the practical utility in terms of management and performance analysis. Roll (1977) claimed that according to CAPM, the ideal portfolio is able to positively manage the different variables. Roll (1977) discussed the problem of the right of the market. It is a conceptual order that challenges the theoretical foundations of investment performance indicators over another period of time. If the market portfolio is efficient, securities are on average to the right of the market. It is therefore sometimes impossible to observe deviations from this line over a long period. In reality, these deviations do not indicate whether a security is over or under-priced.

The estimated betas are relative for the entire period studied and assumed to be stable relatively to a certain scale. While in reality the variance with the market varies over time and therefore there may be some statistical issues related to errors in the beta estimates of individual securities and their instabilities which are not taken into account.

Overall, the CAPM formulation showed remarkable empirical robustness in determining excess return, taking systematic risk into account. CAPM is characterized by a special case which forms several types of risk into one defined by systematic risk (non-diversifiable risk or market risk). In this sense, two securities with the same level of risk can lead to different profits. Investors tend to choose the security with the highest profit. This choice will lead to a drop in profit. However, if there are two portfolios A and B of the same risk, where B is the optimal portfolio according to the CAPM, portfolio A generates a higher return than that of B. In such a case, investors who seek new opportunities will be willing to invest in A although it is undervalued by the CAPM, and to sell the optimal portfolio B which has the same systematic risk.

Besides, other studies have focused on empirical extensions of the CAPM taking into account the time variations of the risk beta or variations in the time of the risk premium or both of them. This area

of research is often referred to as the CAPM conditional test. Studies on this topic show the major impact of time intervals on the estimate of beta, understanding of the market and the economic policies in general (See for instance Levhari and Levy (1977), Marfatia (2014), Marfatia (2015), Marfatia (2017a), Marfatia (2017b), Marfatia (2020)). Handa et al. (1993) showed that if the difference in the time interval of returns is taken into account, for the same action, we may obtain different estimators. Handa et al. (1993) rejected the CAPM beta estimator which uses monthly returns, but accepted the one based on annual returns. Cohen et al. (1986) showed that the beta estimator is sensitive to the time intervals used for stock returns.

Consequently, the problem of the validity of the CAPM according to time variation arises. More precisely, the validity of the relation between the systematic risk and the return of the securities quoted on the financial markets taking into account the time factor in the returns of the equities seems to be necessary. This essential issue of CAPM verification is still relevant today. To our knowledge, few works have looked specifically at this question, in particular by using a method as powerful as that of wavelets. The time-scale division induced by the wavelet transform overcomes the shortcomings of the classical CAPM analysis.

Our objective here is to test the validity of the CAPM on the Saudi Arabia Tadawul market. We aim firstly to analyze the systematic risk and to point out its limitations taking into account the time factor. The second is to identify the relationship between equity returns and their systematic risks in the Saudi Arabia Tadawul market using the wavelet approach taking already the time factor into account.

In our knowledge, the application of wavelets for GCC stock markets is still recent and the literature on it needs more developments. However, in other markets such a tool is now developed and has induced more understanding for the markets analyzed such as SP500 (USA), CAC40 (France) and ISE (Istanbul). Gençay et al. (2003) applied wavelets for the stock markets of the US, UK and Germany to estimate the best time scale for measuring systemic risk. The authors concluded that the relationship between risk and return is a multi-scale phenomenon. Fernandez (2006) analyzed the Santiago stock market in Chile using time-scaling methodology. Rhaiem et al. (2007a), Rhaiem et al. (2007b) studied the French CAC40 index as market portfolio and the daily EURIBOR as the risk-free rate. The predictions of the CAPM are claimed to be more relevant in the short term than in the long term, which makes the French market different from those of the US, UK and Germany. Aktan et al. (2009) applied the wavelet multi-scaling method for the Istanbul Stock Exchange during the period from January 2003 to October 2007. It is shown that a positive relationship between risk and returns is most significant at the medium levels, concluding that the effect of market returns on an asset is stronger in this time horizon.

3. Methodology

Before introducing the methodology applied in our work it is necessary to recall some basic facts. Indeed, one of the important factors that may affect the model described here may be the US policy actions and the risk perceptions which significantly impact international stock markets. However, we did not include these factors in the present model as our aim is not to change the CAPM model which already exists but to improve it firstly by using the wavelet time-frequency action. This fact may be considered as a limitation for the present work and opens instead a good idea for an eventual extension. Besides, some recent studies have discussed the role of these factors and their estimations in other models which confirms their importance and the motivation to include them in any eventual

extension of the present model. Indeed, in Kishor and Marfatia (2013) time-varying response of foreign stock markets to US monetary policy shocks has been estimated. The authors noticed a significant time-variation in the response of the global equity markets to US monetary policy surprises especially during the crisis periods. The model applied looks like the present one as it aims to estimate a risk-like parameter β^i as follows

$$R_t^i = \alpha^i + \beta^i \Delta r_t^u + e_t^i \tag{1}$$

where R_t^i is the abnormal return of country i at the event date t and Δr_t^u is the monetary policy surprise. The coefficient β^i reflects the response of the abnormal return of a country i's stock market to unanticipated interest rate increase in the US.

Now our methodology consists in a first step to apply wavelet decomposition of time series to overcome the problem of missing data confronted when gathering the data applied in our work. Next, as the data is completed the whole basis will be subject of both CAPM and wavelet CAPM processing.

3.1. Wavelets for time series

Wavelet analysis allows the representation of time series into species relative to the time and frequency information known as time-frequency decomposition. It consists in decomposing a series in different frequency components with a scale adapted resolution and thus permits to observe and to analyze data at different scales. Wavelet analysis starts from one source function ψ known as the mother wavelet and next composes dilation-translation copies to get a complete system for finite energy time series. Each wavelet basis element is defined for $j, k \in \mathbb{Z}$ as a copy of ψ at the scale j and the position k by $\psi_{j,k}(t) = 2^{-j/2}\psi(2^j t - k)$. The quantity 2^j corresponds to the frequency of the series while the index k localizes volatility or fluctuations. Let for $j \in \mathbb{Z}$ fixed, $W_j = span(\psi_{j,k}, k)$ known as the j-level detail space. A time series $X(t)$ is projected onto W_j yielding a component $DX_j(t)$ given by

$$DX_j(t) = \sum_k d_{j,k}\psi_{j,k}(t) \tag{2}$$

The $d_{j,k}$ are the detail coefficients of the series $X(t)$ expressed by means of the ordinary inner product in the functional space $L^2(\mathbb{R})$ as

$$d_{j,k} = <X, \psi_{j,k}> = \int_{\mathbb{R}} X(t)\psi_{j,k}(t)dt \tag{3}$$

The spaces W_j's form an orthogonal decomposition covering the space of finite energy series $L^2(\mathbb{R})$. This means that the series $X(t)$ can be completely reconstructed as a sum of its projections on the detail spaces and that these projections are mutually uncorrelated. In wavelet theory, the mother wavelet yields a second function called father wavelet or scaling function denoted here by φ. (See Daubechies (1992)). Similarly to ψ, the function φ yields dilation-translation copies $\varphi_{j,k}(t) = 2^{-j/2}\varphi(2^j t - k)$ generating subspaces V_j. The sequence $(V_j)_j$ is called a multi-resolution analysis (multi-scale analysis) on $\mathbb{R}$ and V_j is called the j-level approximation space. It is well known in wavelet theory that $V_j \subset V_{j+1}$, $j \in \mathbb{Z}$, which means that the approximation of the time series at the level j and $j + 1$ can be viewed from each other and so from any horizon $p \geq j + 1$. In physics-mathematics this is called the zooming rule. It holds also that for all $j \in \mathbb{Z}$, $f(t) \in V_j$ iff $f(2t) \in V_{j+1}$, which reflects the fact that, not only the signal f from horizon j can be seen in the horizon $j + 1$ but also his contracted or dilated copies. As for the detail subspaces, the approximation subspaces V_j's satisfy also a completeness relation meaning that

no information is lost when considering all approximations and a second property meaning that all the information is lost at finer scales. Finally the Vj's satisfy a shift-invariance property in the sense that $f(t) \in V_j$ iff $f(t - k) \in V_j$, $j, k \in \mathbb{Z}$, which means that the multi-resolution analysis permits to detect the properties of the signal along the whole time support. Combining all the properties above we deduce that the approximation space is decomposed into a low-level approximation part supplemented with a detail one. Under these properties, the following decomposition is proved for $j \in \mathbb{Z}$,

$$X(t) = \sum_j DX_j(t) = \sum_{j \leq J} DX_j(t) + \sum_{j \geq J+1} DX_j(t) \tag{4}$$

The component $AX_J(t) = \sum_{j \leq J} DX_j(t)$ is called the approximation of $X(t)$ at the level J and it reflects the trend or the global shape of $X(t)$. It also belongs to the space V_J. Thus, using the definition of the V_J's, the component $AX_J(t)$ may be expressed using the basis $(\varphi_{J,k})_k$ as

$$AX_J(t) = \sum_k a_{J,k} \varphi_{J,k}(t) \tag{5}$$

where the $a_{J,k}$ are the approximation coefficients of the series $X(t)$ expressed by $a_{J,k} = <X, \varphi_{J,k}>$. As a result, we obtain the following relation known as the wavelet decomposition of $X(t)$

$$X(t) = AX_J(t) + \sum_{j \geq J+1} DX_j(t) \tag{6}$$

It is composed of one part reflecting the global behavior of the series and a second part reflecting the higher frequency oscillations or the fine scale deviations of the series near its trend. In practice we cannot obviously compute the complete set of coefficients. We thus fix a maximal level of decomposition J and consider the decomposition for any $J_0 < J$,

$$X_J(t) = AX_{J_0}(t) + \sum_{J_0 < j \leq J} DX_j(t). \tag{7}$$

There is no theoretical method for the exact choice of the parameters J_0 and J. However, the minimal parameter J_0 does not have an important effect on the total decomposition and usually chosen to be 0. But, the choice of J is always critical. One selects J related to the error estimates.

In finance, economics, management and generally actuarial sciences, compared to classical theories wavelet analysis is still less used although it proved good results and needs to be more developed. Recently the literature starts growing rapidly. See Arfaoui et al. (2017), Arfaoui et al. (2020), Arfaoui et al. (2020), Mabrouk et al. (2008), Mabrouk et al. (2008), Mabrouk et al. (2010), Mabrouk et al. (2011), Mabrouk and Zaafrane (2013), Mabrouk et al. (2015), Conlon et al. (2008), Cifter and Ozun (2007), Cifter and Ozun (2008), DiSario et al. (2008), Fernandez (2006), Gençay et al. (2002), Gençay et al. (2003), Gençay et al. (2005), Mahmoud et al. (2016), In and Kim (2006), In and Kim (2007), In et al. (2008), Percival and Walden (2000), Selcuk (2005), Sharkasi et al. (2006), Xiong et al. (2005), Yamada (2005), Zemni et al. (2019a), Zemni et al. (2019a).

3.2. Wavelets for missing data in markets

As mentioned in the introduction, the main problem in studying markets' movements and/or situations is the lack of data and sometimes its uncertainty. This leads researchers to develop prior procedures to complete the data used in the market study. In our situation here this problem is present strongly in the data used. We noticed that the origin of our data results in some missing values in many actions shared in the market index used here and which is the well-known Tadawul index of the Saudi stock market. To overcome this problem and instead of searching left and right for the data and confronting may be with the same problem in other origins, we proposed to act a wavelet method to reconstruct missing values. The data basis is essentially extracted from the web site www.investing.com

The present section is devoted to present the wavelet-based method to reconstruct such missing data. The method consists in providing a prediction procedure able to predict a short time interval series on an arbitrarily set of backwards and/or forwards (past and/or future, prior and/or post) values. Recall that generally to conduct a best prediction or reconstruction of time series we need usually a long-time interval for training. This fact may not be satisfied in general situations. For some situations such as ownership-structure and diversification variable the samples are usually short. One has one main value on a year. Furthermore, when applying wavelet analysis to approximate and/or to forecast time series, the majority of the existing studies assume the presence of some seasonality, periodicity and/or autoregressive aspect in the series. See for example Mabrouk et al. (2010), Soltani (2002), Soltani et al. (2007) and the references therein.

In the present paper, we act a simple method already tested in Mabrouk et al. (2010) leading to good prediction. The method is principally characterized by the non-necessity to test it on the detail parts components of the series nor its wavelet coefficients. This is essentially due to the fact that we use few values of series leading to short sub-samples and next act the prediction on the sub-samples. In such parts the dynamic behavior is not important. However, the most positive point in the method is the fact that it necessitates only to compute the values of the source scaling function and the associated wavelet on a suitable grid, the dyadic or the integer grid in the supports of the mother and father wavelets.

Let $X(t)$, $t = 1, 2, \ldots, N$, be a time series. The procedure to be applied is a dynamic recursive scheme consisting in applying firstly a partial estimator at short horizons to all the observations (t_i, X_i), $i = 1, 2, \ldots, N$, to yield firstly the predicted value of X_{N+1}. This last is then included as new observation to predict X_{N+2}. We then follow the same steps until reaching the desired horizon. During the procedure, we apply the J-level wavelet decomposition (4). This necessitates to know the values of the mother wavelet ψ on the dyadic grid $\{2^j(N + 1) - k, k\}$ and the scaling function φ on the integer grid $\{N + 1 - k, k\}$ in the supports. Assume we have got $X_J(N)$. The next value will be estimated by ($J_0 = 0$ in (7))

$$X_J(N + 1) = \sum_k a_{0,k}\varphi(N + 1 - k) + \sum_{j=1}^{J} \sum_k d_{j,k}\psi(2^j(N + 1) - k) \tag{8}$$

This means that for evaluating the predicted value of X_{N+1}, it suffices to do this for AX_J and the DX_j's. This motivates the use of Daubechies compactly supported wavelets which are well evaluated on the integer grid.

3.3. The mathematical formulation of the CAPM

The selection of a portfolio is usually preceded by a financial analysis of a set of stocks. A good analysis provided with a successful quantification of the important variables of the investment decision on the financial markets will be a central factor for investors success. One of the variables is the profitability, defined as the sum of capital gains and dividends reported at the beginning of the period. The expected profitability of a portfolio is equal to the weighted average of the expected returns on the different securities that make it up. A second factor is the risk of an action. Recall that the dispersion of returns near the average profitability reflects the uncertainty or the risk of the investment. The standard deviation and/or the variance are convenient measures of this dispersion. To avoid risk, investors prefer investing with a low variance.

In its original and/or simple variant, the CAPM is mathematically expressed as

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + u_{i,t} \tag{9}$$

where $R_{i,t}$ is the return rate of an action i at the period time t, $R_{m,t}$ is the return of the market measured by means of a general index at the same period of time t. The parameter β_i is a specific factor to each action i, indicating the relation between the fluctuations of the action i return rate and the fluctuations of the general index of the market, called often the beta coefficient or the systematic risk. The factor $u_{i,t}$ is a random factor representing the hidden fluctuations of $R_{i,t}$ that are not explained by the market, or generally an error term. Finally, the parameter α_i is added to guarantee a null expectation of $u_{i,t}$. Therefore, CAPM breaks down the total variability of an action into two parts. A first part, due to the influence of the market and which corresponds to the systematic risk and a second due to the specific characteristics of the action and which corresponds to the variations of the specific prices of such action, called sometimes the diversified or the specific risk. The CAPM allows to study the behavior of all operators in a market and to build a theory of equilibrium. The market is a global reference framework defining the conditions that prevail in all the transactions that economic agents carry out. It allows the confrontation of supply and demand. The equilibrium is reached under many hypotheses such as the null transaction cost, the perfect divisibility of assets, free-tax dividends, and capital gains, intervention of both buyers and sellers in the market without influencing prices. We may also consider the possibility that investors can lend or borrow required amount at a pure interest rate, without influencing the level, and the borrowing rate is equal to the loan rate. Some hypothesis are also based on investors behavior such as risk aversion behavior, equality of expectations for operators as to profitability expectations associated with the different assets and their risks, the rationality and safety of operators over a given horizon, to maximize their wealth.

Many variants of the mathematical CAPM formulation have been developed in the literature. Sharpe (1964) proposed a variant of CAPM based on expected returns stating that

$$E(R_i) = r_f + \frac{\sigma_{im}}{\sigma_m^2}(E(R_m) - r_f) \tag{10}$$

which by setting $\beta_i = \dfrac{\sigma_{im}}{\sigma_m^2}$ becomes

$$E(R_i) = r_f + \beta_i(E(R_m) - r_f) \tag{11}$$

It means that the expected return of the action i is estimated by the risk-free interest added with a risk prime. This later is composed of the excess of the expected rate of return of the market minus the

remuneration of the risk-free asset multiplied by the factor β_i. It thus depends on the risk premium that the investor should receive when agreeing to bear a risk equal to that of the market and the importance of the risk of the action i against the risk of the market. In literature, there are other categories of the CAPM such that CAPM without a risk-free asset and CAPM with transaction costs. See for example Black (1972), Black et al. (1972). Assume that a number K of investors are present. The k-th investor will allocate his wealth T_k in a number M_k of actions in the market. To be in an equilibrium situation, we should have

$$E_i = r_f + \frac{\sigma_{ik}}{\sigma_k^2}(E_k - r_f) \tag{12}$$

where E_i is the expectation of the return of action i, E_k is the expectation of the return of the optimal portfolio detained by the investor, σ_k^2 is the variance of the optimal portfolio and σ_{ik} is the covariance between the return of action i and the portfolio. Denoting similarly to previous variants $\beta_{ik} = \frac{\sigma_{ik}}{\sigma_k^2}$ the last equation becomes

$$E_i = r_f + \beta_{ik}(E_k - r_f) \tag{13}$$

β_{ik} is the volatility of the action i relatively to the optimal portfolio. Taking into account all investors wealth, Lévy (1978) proposed the form

$$E_i = r_f + \frac{1}{TK}\sum_{k=1}^{K} T_k(E_k - r_f) \tag{14}$$

where $TK = \sum_{k=1}^{K} T_k$. This formula means that the required risk premium of the action i is evaluated as the average of the premiums demanded by the investors, weighted by their respective wealth.

CAPM may also depend on inflation. Indeed, assuming the existence of an asset with certain nominal return, Friend et al. (1976) proposed that the relation between the risk and the return of the different assets could be written as

$$E(R_i) = r_f + \sigma_{i\pi} + \frac{E_m - r_f - \sigma_{m\pi}}{\alpha\sigma_{im}^2 - \sigma_{m\pi}}(\alpha\sigma_{im} - \sigma_{i\pi}) \tag{15}$$

where $\sigma_{i\pi}$ is the covariance between the rate of return on assets i and the rate of inflation, $\sigma_{m\pi}$ is the covariance between the market rate of return and the rate of inflation and α is the ratio between the nominal value of risky assets and that of all assets.

Finally, taking into account the taxation, the CAPM may be modified to reflect the presence of taxation based on the relationship between the expected return and the risk of the security as

$$E_i(1 - T_0) = r_f(1 - T_0) + \beta_i(E_m - r_f)(1 - T_0) \tag{16}$$

where E_i is the expectation of the yield of action i, T_0 is the tax rate, β_i is the systematic risk and E_m corresponds to the expectation of the market return.

Different forms of the CAPM may be also found in the literature that take into account other factors such as heterogeneity of anticipations relatively to future performances of actions, etc. See for example

Aktan et al. (2009). In the present paper, we assume that the CAPM defines the required return on an investment according to the equation

$$E(R_i) = r_f + \beta_i(E(R_m) - r_f) \tag{17}$$

where $E(R_i)$ is the asset's expected return, r_f is the risk-free rate, $E(R_m)$ is the expected return of the market portfolio, and β_i is the measure of risk for asset i evaluated as

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} = \frac{\sigma_{im}}{\sigma_m^2} \tag{18}$$

In empirical finance, β_i is estimated usually via the ordinary least square estimate from the linear regression

$$R_{it} - r_{ft} = \alpha_{it} + \beta_i(R_{mt} - r_{ft}) + \varepsilon_{it} \tag{19}$$

where ε_{it} is the error term while α_{it} is a constant. Consequently, the risk beta allows to decompose the variance of an asset i as

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2 \tag{20}$$

Hence, the variance σ_i^2 can be decomposed into a first component $\beta_i^2 \sigma_m^2$ corresponding to the firm's systematic risk and a second one $\sigma_{\varepsilon_i}^2$ corresponding to the firm's unsystematic risk.

3.4. The wavelet CAPM

To estimate the CAPM with wavelets the first step is to pass by the variance and the covariance of the statistics (time) series and introduce the analogues for the components due to the wavelet decomposition. Therefore, the wavelet analysis consists here also in splitting the variance of the series into sub-variances relative to scales or the levels j which will be called the variance of the series at the scale j (Percival and Walden (2000)). Let v_X^2 be the variance of $X(t)$ and $v_X^2(j)$ be the variance of the projection at the scale or the level j, we have

$$v_X^2 = \sum_{j=1}^{\infty} v_x^2(j) \tag{21}$$

This permits to focus on the sub-variances of the components at a level j instead of considering the whole series. This in turns facilities the analysis of the fluctuations and the dynamics of the series. Denote L the length of the wavelet support, $N_j = [N/2^j]$ the number of wavelet coefficients at the level j and $L_j = [2^{-j}(L - 2)(2^j - 1)]$ the number of boundary wavelet coefficients at the level j. The variance at the level j is estimated as

$$\widehat{v}_X^2(j) = \frac{1}{2^j(N_j - L_j)} \sum_{k=L_j-1}^{N_j-1} d_{j,k}^2 \tag{22}$$

Similarly, we have an analogue formulation for the covariance at the level j for a couple of series (X, Y) as

$$\widehat{v}_{XY}^2(j) = \frac{1}{2^j(N_j - L_j)} \sum_{k=L_j-1}^{N_j-1} d_{j,k}^X d_{j,k}^Y \tag{23}$$

To apply wavelets for evaluating the CAPM we firstly decompose returns into their wavelet decompositions. This leads to component-wise returns according to the levels. These new time series will be considered as the returns to be applied. See for example Gençay et al. (2003). The risk beta will be estimated by the wavelet risk beta at the level or the scale j as

$$\widehat{\beta}_i(j) = \frac{\widehat{\upsilon}_{R_i R_m}(j)}{\widehat{\upsilon}_{R_m}^2(j)} \tag{24}$$

where $\widehat{\upsilon}_{R_i R_m}(j)$ is the wavelet covariance of the component i of the portfolio with the market at the scale j. $\widehat{\upsilon}_{R_m}^2(j)$ is the wavelet variance of the market at the scale j.

Next, in order to illustrate the explanatory power of market returns on the determination of portfolio returns, we estimate the coefficient $R_i^2(j)$ for each time scale used as follows

$$R_i^2(j) = \beta_i(j)^2 \frac{\sigma_{R_m}^2(j)}{\sigma_{R_i}^2(j)} \tag{25}$$

4. Empirical results and discussions

One of the main facts in analyzing stock markets is the uncertainty phenomenon related to many factors such as data collection and recording, policy makers, etc. In Marfatia (2014) the impact of monetary policy on the real economy has been investigated based on policy makers and market participants. A novel approach has been developed for evidence supporting the Lucas island model predictions. Based on the estimation of the time varying response of the S&P stock returns to monetary policy surprises derived from the high frequency Federal funds futures market, the study concluded that at higher level of uncertainty, the impact of FOMC policy surprise on the time varying S&P returns decreases. Besides, the volatility in the short-term bond market has been shown to offer highest explanatory power in explaining the impact of uncertainty on the effectiveness of Fed's policy surprises. This study interconnects with the present one in the common point of taking into consideration of the uncertain character of financial data. However, in the present work we considered these uncertain values to be part of missing data and thus a post reconstruction step based on wavelet theory has been conducted to fill the gaps of missing and uncertain data. The study of the affects of such missing and/or uncertain data on the risk may be a good eventual future direction. Moreover, taking into consideration the US policy is of great interest in the model as US policy is strongly affecting the one in GCC countries especially KSA and vice-versa. We may recall here the last phenomenon of floating the oil market which affected immediately the US oil market and thus the US policy makers. We have noticed enormous politicians from both republic and democratic parties that strongly criticizing and threatening to change the way of collaboration with KSA despite their strategic relations.

In Marfatia (2015) the role of financial stress and risks in the monetary policy's time-varying impact on the US bond markets has been studied. The author investigated precisely the link between the treasury and monetary policy shocks and the impact of the financial market volatility. Besides the author concluded that a time-varying coefficient is more adequate to express the impact of the monetary policy on the market and to express adequately the uncertainty in economic data and its role in estimating interest rates as well as the level of financial risks. These facts join in some sense the present work as we suggest here also that time variation is an essential character in estimating the coefficient of correlation

between the whole market index and the firms and did not assume that such coefficient remains constant over time. However, our present model did not include neither the US monetary policy nor the GCC monetary policy as explicit factors, despite their importance and their hidden role and hidden affect on the market studied. These factors, which constitute some limitations of the present model, may be interesting extending studies in the future for both the theoretical mathematical model and the decision makers in the market as well as policy makers.

The main object of our work is to examine the effect of time scales on the systematics risk of the stock components of the market. We test the following hypotheses:

- Hypothesis 0: The risk beta is stable according to time scales.

- Hypothesis 1: There is a linear dependence between action's returns and their systematic risks.

This paper empirically tests a wavelet methodology of beta estimation proposed previously on daily data for GCC markets collected on the period of January 01, 2013 to September 20, 2018 resulting of a sample size $N = 1427$ due to the well-known Tadawul index for Saudi Arabia stock market. We focused on a portfolio composed of 148 actions as listed in Table 1 with corresponding sectors. Table 1 provides the components of Tadawul market applied in our study and their classification according to the global classification standards.

In fact, Tadawul stock market is already classified into industrial sectors since 2008. Next, with the continuous developments in KSA economy, new companies as well as industries have emerged into the market. Consequently, necessary changes have been conducted in the classification of companies listed on Tadawul to reflect the emergence of the new elements. However, Tadawul's previous sector classification was not based on global classification standards which induces some limitations in the classification.

In the new Tadawul market structure there are 20 sectors presented as follows: Energy, Materials, Capital Goods, Commercial & Professional Svc, Transportation, Consumer Durables & Apparel, Consumer Services, Media and Entertainment, Retailing, Food & Staples Retailing, Food & Beverages, Health Care Equipment & Svc, Pharma Biotech & Life Science, Banks, Diversified Financials, Insurance, Telecommunication Services, Utilities, REITs, Real Estate Mgmt & Dev't. Table 1 provides the re-organization of the Tadawul stock market components according to the global classification standards reminiscent of a bit modification where we merged some sectors into one according to their closeness. The Consumer Durables & Apparel Components is merged with Consumer Services to constitute the new sector Consumer. The sectors Foods & Bevarage and Food & Staples Retailing are merged to compose the new sector Foods. Finally we merged the sector Health Care Equipment & Svc with the sector Pharma Biotech & Life Science into the new sector Health and Life Science. Consequently we obtain 17 sectors as shown in Table 1, which resembles to the classical segmentation of Saudi market.

The choice of this market is motivated by the fact that Tadawul is the best representative index of the Saudi market supervised by the Capital Market Authority. It is also considered as the largest capital market in the Middle East and North Africa. Recently at the end of 2019, Tadawul has taken place in the first 10 largest stock markets in the world. It lists more than 150 publicly traded companies. We applied in our study a number of 148 ones because of the non-availability of sufficient data and or the very weak effect of some companies in the applied period of study. For example, a first test has yielded for ABIDC action a zero beta for both scale laws of the periods of time applied.

Table 1. Tadawul companies and sectors.

Sector	Company Name	Abbreviation
	Alujain Corporation	ACAlujain
	Arabian Cement Co	ACCAC
	Al Jouf Cement Company	AJCC
	Advanced Petrochemical Company	APC1
	Arabian Pipes Company	APC2
	Basic Chemical Industries Co	BCIC
	Eastern Province Cement Co	EPCC
	Filing & Packing Materials Co	FPMC
	Hail Cement Company	HCC
	Methanol Chemicals Company	MCC
	Najran Cement Company	NCC
	Nama Chemicals Co	NCC2
	National Gypsum Company	NGC
	National Industrialization Co	NIC
	National Metal Manufacturing Co	NMMC
	National Petrochemical Company	NPC
	Rabigh Refining & Petrochemical Co	RRPC
	Saudi Arabia Fertilizers Co	SAFC
	Sahara	SAHARA
Materials	Saudi Arabian Mining Company	SAMC
	Saudi Basic Industries Corp	SBIC
	Saudi Cement Company	SCC2
	Saudi Chemical Company	SCC4
	Saudi Industrial Investment Group	SIIG
	Saudi Int Petrochemical Co	SIPC
	Saudi Kayan Petrochemical Company	SKPC
	Southern Province Cement Co	SPCC
	Saudi Paper Manufacturing Co	SPMC
	Saudi Steel Pipe Company	SSPC
	Takween Advanced Industries	TAI
	Tabuk Cement Co	TCC
	The National Co for Glass Industry	TNCGI
	The Qassim Cement Co	TQCC
	United Wire Factories Company	UWFC
	Yanbu Cement Co	YCC
	Yanbu National Petrochemical Co	YNPC
	Yamamah Saudi Cement Co	YSCC
	Zamil Industrial Investment Co	ZIIK

Sector	Company Name	Abbreviation
Energy	Aldrees Petroleum & Transport Svcs	APTS
	Saudi Arabia Refineries Co	SARC
Capital Goods	Al-Ahsa Development Co	AADC
	Abdullah A. M. Al-Khodari Sons Co	AKSC
	Al-Babatain Power & Telecom Co	ABPTC
	Astra Industrial Group	AIG
	Middle East Specialized Cables Co	MESCC
	Saudi Arabian Amiantit Co	SAAC
	Saudi Cable Company	SCC
	Saudi Ceramic Co	SCC3
	Saudi Industrial Export Co	SIEC
	Saudi vitrified clay pipes co	SVCPC
	The National Shipping Co	TNSC
Commercial & Professional Svc	Allianz SE VNA O. N	ASVON
	Saudi Printing & Packaging Company	SPPC
Transportation Components	Saudi Industrial Services Co	SISC
	Saudi Public Transport Co	SPTC
	Saudi Transport & Investment Co	STIC
	United Int Transportation Company	UITC
Consumer	Al-Abdullatif Co	ACAbdullatif
	Al Sorayai Trading & Industrial Group	ASTIG
	Saudi Industrial Development Co	SIDC
	Tourism Enterprise Co	TEC
	Al Khaleej Training & Education Co	ATEC
	Dur Hospitality	DH
	Herfy Food Services Co	HFSC
Retailing	United Electronics Company	UEC
	Al Hassan Ghazi Ibrahim Shaker	AKGIS
	Jarir Marketing Co	JMC
	Fawaz Abdulaziz AlHokair Company	FAAC
	Saudi Automotive Services Co	SASC
Foods	Almarai Company	ACAlmarai
	Al-Jouf Agriculture Development Co	AJADC
	Ash-Sharqiyah Development Company	ASDC
	Food Products Co	FPC
	Halwani Bros	HB
	Jazan Development Co	JDC
	National Agriculture Development Co	NADC
	Qassim Agriculture Co	QAC
	Saudia Dairy and Foodstuff Co	SDFC
	Saudi Fisheries Co	SFC
	Savola Group	SG
	Tabuk Agriculture Development Co	TADC

Sector	Company Name	Abbreviation
Foods	Anaam International Holding Group	AIHG
	National Agriculture Marketing Co	NAMC
	Abdullah Al Othaim Markets	AAOMC
Media and Entertainment	Saudi Research and Marketing Group	SRMG
	Tihama Advertising &Public Relations	TAPR
Health & Life Science	Mouwasat Medical Services Company	MMSC
	Saudi Pharmaceutical Appliances	SPA
Banks	Alawwal Bank	ABALAwal
	Alinma Bank	ABAINma
	Arab National Bank	ANB
	Al Rajhi Bank	ARB
	Bank AL Bilad	BA
	Bank Al Jazira	BA2
	Banque Saudi Fransi	BSF2
	Riyad Bank	RB
	Samba Financial Group	SFG
	The Saudi British Bank	TSBB
	The Saudi Investment Bank	TSIB
Diversified Financials	Al-Baha Investment and Development Company	ABIDC
	Al Ahli Takaful Company	ATC
	Aseer Trading Tourism &Manufacturing	ATTM
	Falcom Financial Services	FFS
	Kingdom Holding Company	KHC
	Saudi Advanced Industries Co	SAIC
	Solidarity Saudi Takaful Co	SSTC
	SABB Takaful	ST
Insurance	Al-Ahlia Insurance	AAIC
	Al Alamiya Cooperative Insurance	AACI
	Arabia Cooperative Insurance	ACI
	Amana Cooperative Insurance	ACIC
	Allied Cooperative Insurance Group	ACIG
	Arabia Insurance Cooperative Co	AICC
	Al-Rajhi Cooperative Insurance	ARCI
	AS-AXA SA	ASAXASA
	Al Sagr Co-operative Insurance Co	ASCIC
	Arabian Shield Coop Insurance Co	ASCIC2
	Bupa Arabia for Coop. Insurance	BACI
	Buruj Cooperative Insutance Co	BCIC2
	Gulf General Cooperative Insurance	GGCI
	Gulf Union Cooperative Insurance	GUCI
	Malath Cooperative Insurance Co	MCIC
	Rade Union Cooperative Insurance	RUCI
	Saudi Arabian Cooperative Insurance	SACI

Sector	Company Name	Abbreviation
Insurance	Salama Cooperative Insurance Co	SCIC
	Saudi Enaya Cooperative Insurance	SECI
	Saudi Indian Company Insurance	SICI
	Saudi RE Cooperative Reinsurance	SRCR
	Saudi United Cooperative Insurance	SUCI
	The Company for Coop. Insurance	TCCI
	The Mediterranean &Gulf Insurance Co	TMGIC
	United Cooperative Assurance Co	UCAC
	Wataniya Insurance Company	WIC
Telecommunication Services	Etihad Atheeb Telecommunication	EAT
	Etihad Etisalat Co	EEC
	Mobile Telecommunications Company	MTC
	Saudi Telecom	ST2
Utilities	National Gas & Industrialization Co	NGIC
	Saudi Electricity Company	SEC
REITs	Taiba Holding Co	THC
Real Estate Mgmt & Dev't.	Arriyadh Development Co	ADC
	Dar Alarkan Real Estate Development	DARED
	Emaar The Economic City	ETEC
	Jabal Omar Development Company	JODC
	Knowledge Economic City	KEC
	Makkah Construction & Development Co	MCDC
	Red Sea Housing Services Company	RSHSC
	Saudi Real Estate Co	SREC

4.1. Wavelet completion of the data basis

Recall as mentioned in the introduction that when collecting the data about financial GCC markets such as KSA Tadawul applied here, the main problem confronted is the lack of data where the samples present for many cases missing values. Consequently, we started by acting a wavelet method to reconstruct missing data which by the next leads to complete and adjusted basis on the period of study. On the total basis used we noticed 231 missing values dispersed on the whole market and on different time dates (daily) on 14 actions in the market. The complete basis is applied next for CAPM and thus the comprehension of the market. The missing values are distributed as in Table 2. We applied as in Mabrouk et al. (2010) a training set of 5 prior values for different time dates (daily) on 14 actions in the market.

In fact several wavelet functions have been tested to provide a best filtering. The tests results in the well-known Daubechies wavelet Db6. The authors may refer to Daubechies (1992) where a mathematical method has been developed to compute numerically the values of the mother/father wavelet on dyadic points in the compact supports. The essential idea there was based on the fact that the vector of values of the wavelet at these points is an eigenvector for a special matrix associated also to dyadic eigenvalues. In fact, in Mabrouk et al. (2010) we applied a newer version of Daubechies wavelets developed differently and applied also in Mabrouk et al. (2010) based on connection coefficients of wavelets and leading to the fact that the matrices obtained in Daubechies (1992) may be transformed

into circulant and hollow ones. This permits a great gain in machine memory, algorithms speed and error estimates.

Table 2. Missing data.

Stock	Number of missing values
AADC	12
ACAbdullatif	18
AJCC	21
AKGIS	6
ASAXASA	23
ATEC	21
DH	17
FFS	19
JDC	20
MCDC	21
QAC	19
SAHARA	15
YSCC	10
ZIIK	8
Total	231

4.2. The CAPM processing

Next as our basis of data has been completed to obtain 100% trading days we are able to develop the CAPM analysis. We propose to study the relationship between excess return on each individual stock and the time scales of market portfolio using the usual OLS estimator for β_i issued from the regression (19). The daily return of each stock is calculated as the log-price difference

$$R_{it} = \log P_{i,t} - \log P_{i,t-1} \tag{26}$$

where $P_{i,t}$ is the price of asset i at day t. The market return R_{mt} is taken as the log-difference of the index due to the action i as

$$R_{mt} = \log C_t - \log C_{t-1} \tag{27}$$

where C_t is the index value at day t.

Table 3 below shows the descriptive statistics of the market. The statistics corresponds precisely to the return excess for each company relatively to the risk-free. Table 3 presents descriptive statistics of excess return on the stocks in the sample and on the proxy for the market portfolio the Tadawul. The median value of all assets in the present sample is approximately zero except for the one of Tadawul being equal to $0.662.10^{-3}$. The flatness and distortion features of all stocks' returns are different from each other. Moreover, the Jarque-Bera test leads to $JB = 1$ which rejected the null hypothesis at the 5% significance level, and 0 otherwise.

Table 3. Descriptive statistics of excess returns.

Stocks	Mean	Maximum	Minimum	SD.	Skewness	Kurtosis
Tadawul	0.0001	0.0855	−0.0755	0.0111	−0.6594	13.1546
AADC	−0.0003	0.0976	−0.1042	0.0214	−0.4020	7.8649
AAIC	−0.0009	0.6868	−0.7257	0.0410	−0.5965	126.3768
AKSC	−0.0010	0.1211	−0.1079	0.0246	−0.2688	8.1129
AAOMC	0.0009	0.1088	−0.1056	0.0174	0.1619	8.2193
ABALAwal	−0.0004	0.3614	−0.7054	0.0283	−11.0296	310.8295
ABAINma	0.0003	0.0950	−0.1077	0.0168	−0.0143	10.4981
ABIDC	−0.0002	0.0953	−0.1102	0.0172	0.4497	19.5665
ABPTC	−0.0002	0.0953	−0.1126	0.0218	−0.6165	8.6596
AACI	−0.0004	0.1166	−0.5887	0.0365	−2.9527	51.2243
ACAbdullatif	−0.0005	0.0917	−0.1089	0.0188	−0.9383	9.3029
ACAlmarai	0.0003	0.0948	−0.2262	0.0164	−1.6522	33.7767
ACAlujain	−0.0000	0.0970	−0.1102	0.0249	−0.1402	6.5993
ACCAC	−0.0002	0.0949	−0.0827	0.0161	−0.0995	8.0130
ACI	−0.0010	0.1060	−0.6712	0.0346	−5.0857	101.9765
ACIC	−0.0002	0.8330	−0.1121	0.0460	4.1390	78.3266
ACIG	−0.0001	0.1007	−0.1181	0.0315	−0.3735	5.9002
ADC	0.0001	0.1010	−0.1141	0.0175	−0.1528	11.3075
AICC	−0.0004	0.4090	−0.2500	0.0322	1.0842	25.2380
AIG	−0.0006	0.0744	−0.1141	0.0196	−0.7648	8.1017
AIHG	−0.0009	0.1010	−0.1056	0.0248	−0.5832	7.6882
AJADC	−0.0000	0.0932	−0.1041	0.0185	−0.2863	8.7256
AJCC	−0.0003	0.1028	−0.1069	0.0167	−0.6405	13.4413
AKGIS	−0.0013	0.1218	−0.6005	0.0256	−9.1548	215.2429
ANB	0.0000	0.0741	−0.0914	0.0142	−0.0283	7.0484
APC1	0.0005	0.0947	−0.0794	0.0174	0.2549	8.2650
APC2	−0.0005	0.0950	−0.1136	0.0229	−0.3841	8.1570
APTS	−0.0000	0.0992	−0.2109	0.0200	−0.9530	15.0490
ARB	0.0000	0.0916	−0.0694	0.0132	0.1622	9.5371
ARCI	0.0006	0.0950	−0.1044	0.0252	−0.1468	6.4127
ASAXASA	−0.0002	0.1092	−0.5073	0.0314	−3.0477	51.8985
ASCIC	−0.0001	0.0987	−0.2144	0.0278	−0.6780	9.3565
ASCIC2	0.0003	0.0984	−0.1153	0.0298	−0.0923	6.2951
ASDC	−0.0001	0.1152	−0.1104	0.0282	−0.1505	6.4707
ASTIG	−0.0004	0.4990	−0.1088	0.0255	4.9823	107.8562
ASVON	−0.0008	0.1001	−0.1097	0.0297	−0.2984	6.5046
ATC	−0.0004	0.0950	−0.1040	0.0231	−0.2889	7.2039
ATEC	−0.0002	0.0983	−0.1363	0.0235	−0.2669	7.3756
ATTM	−0.0003	0.0953	−0.1132	0.0205	−0.7734	8.7852
BA	0.0003	0.0984	−0.0943	0.0189	0.1910	8.3323

Stocks	Mean	Maximum	Minimum	SD.	Skewness	Kurtosis
BA2	−0.0000	0.0945	−0.1067	0.0186	−0.1549	7.6460
BACI	0.0009	0.1159	−0.6777	0.0310	−7.1585	164.7862
BCIC	−0.0002	0.0953	−0.1286	0.0213	−0.9222	9.6679
BCIC2	−0.0002	0.0984	−0.4638	0.0318	−2.3196	36.2178
BSF2	0.0001	0.0835	−0.1040	0.0168	0.0432	6.6258
DARED	−0.0001	0.1021	−0.1086	0.0257	−0.1485	6.9724
DH	−0.0003	0.1094	−0.1131	0.0199	−0.2447	9.6004
EAT	−0.0008	0.9017	−0.1158	0.0351	11.9202	307.3399
EEC	−0.0010	0.0953	−0.1214	0.0200	−0.7863	11.2386
EPCC	−0.0005	0.0906	−0.1001	0.0137	−0.7080	14.8314
ETEC	0.0002	0.1041	−0.1082	0.0239	0.0271	7.2838
FAAC	−0.0004	0.0984	−0.6727	0.0297	−8.0928	186.2733
FFS	0.0003	0.0925	−0.1027	0.0136	−0.4737	14.0483
FPC	−0.0003	0.1007	−0.1054	0.0266	−0.3694	7.2488
FPMC	−0.0002	0.1078	−0.1105	0.0228	−0.6224	7.8977
GGCI	−0.0005	0.0975	−0.1085	0.0285	−0.3174	6.3612
GUCI	−0.0001	0.4143	−0.1087	0.0295	1.5882	32.0246
HB	0.0002	0.1220	−0.1033	0.0205	0.1787	7.4574
HCC	−0.0005	0.0963	−0.1102	0.0160	−0.4340	10.2036
HFSC	0.0001	0.0925	−0.1049	0.0175	−0.3081	7.8099
JDC	−0.0000	0.0968	−0.1126	0.0221	−0.9225	9.1503
JMC	0.0003	0.0945	−0.0975	0.0147	0.0394	10.2247
JODC	0.0008	0.0988	−0.1138	0.0200	0.1267	8.7210
KEC	−0.0001	0.1004	−0.1138	0.0241	−0.2723	7.7153
KHC	−0.0005	0.1021	−0.1101	0.0197	0.0212	11.1638
MCC	−0.0004	0.0927	−0.1037	0.0195	−0.5431	9.3482
MCDC	0.0005	0.0953	−0.1262	0.0188	0.5072	11.2096
MCIC	−0.0001	0.9192	−0.4867	0.0403	6.9176	206.2428
MESCC	−0.0005	0.1082	−0.1084	0.0244	−0.2775	7.8676
MMSC	0.0009	0.0949	−0.0996	0.0182	0.1273	6.3078
MTC	−0.0007	0.6035	−0.1069	0.0276	7.2330	164.4192
NADC	0.0000	0.0978	−0.2533	0.0221	−1.2536	19.7337
NAMC	0.0000	0.0990	−0.1124	0.0270	−0.3427	8.1218
NCC	−0.0006	0.0950	−0.0996	0.0178	−0.1085	8.1755
NCC2	−0.0001	1.7154	−0.1201	0.0521	24.9600	824.2936
NGC	−0.0006	0.1004	−0.1063	0.0210	−0.2816	9.3964
NGIC	0.0002	0.0930	−0.1035	0.0147	0.3508	13.0524
NIC	−0.0005	0.0997	−0.1082	0.0195	−0.2293	8.5197
NMMC	−0.0002	0.1023	−0.1085	0.0224	−0.6541	8.0132
NPC	−0.0001	0.0990	−0.1054	0.0220	−0.1101	7.9040
QAC	−0.0004	0.5358	−0.1086	0.0273	4.8470	109.9920

Stocks	Mean	Maximum	Minimum	SD.	Skewness	Kurtosis
RB	−0.0004	0.0970	−0.6845	0.0216	−22.1634	706.8519
RRPC	0.0000	0.0971	−0.1066	0.0231	0.0671	7.8090
RSHSC	−0.0004	0.0977	−0.3537	0.0244	−2.4811	37.1443
RUCI	−0.0001	0.0970	−0.1094	0.0250	−0.5178	7.6858
SAAC	−0.0007	0.1032	−0.1071	0.0183	−0.4872	11.1093
SACI	−0.0006	0.1036	−0.6155	0.0330	−4.5685	88.1574
SAFC	−0.0007	0.0942	−0.2485	0.0160	−4.5416	73.6872
SAHARA	0.0001	0.0950	−0.1077	0.0180	−0.2328	8.4039
SAIC	−0.0002	0.0956	−0.1059	0.0223	−0.4760	7.7089
SAMC	0.0004	0.1037	−0.1057	0.0205	−0.0872	8.5748
SARC	−0.0005	0.0989	−0.1087	0.0236	−0.4868	7.9098
SASC	−0.0001	0.1086	−0.2045	0.0239	−1.0248	11.6036
SBIC	0.0001	0.0958	−0.1048	0.0156	0.0344	11.9477
SCC	−0.0001	0.0980	−0.1128	0.0183	−0.3730	10.0518
SCC2	−0.0001	0.6317	−0.1022	0.0275	8.4612	198.6055
SCC3	−0.0004	0.0868	−0.1058	0.0150	−0.8061	13.2266
SCC4	−0.0009	0.0936	−0.2701	0.0187	−2.2048	36.5825
SCIC	−0.0002	0.1021	−0.5216	0.0331	−2.8175	46.8998
SDFC	0.0005	0.0948	−0.1025	0.0181	0.1999	6.5451
SECI	−0.0005	0.7122	−0.1132	0.0354	5.5794	119.1722
SEC	0.0003	0.0941	−0.1058	0.0157	0.2316	12.4987
SFC	−0.0001	0.9771	−0.1107	0.0365	13.3332	361.5026
SFG	−0.0005	0.0976	−0.7988	0.0266	−18.9221	571.6588
SG	0.0012	1.5817	−0.1038	0.0463	27.8451	951.0885
SICI	−0.0007	0.5166	−0.5062	0.0437	0.0462	29.0768
SIDC	−0.0005	0.0970	−0.1091	0.0234	−0.7096	8.0909
SIEC	0.0009	2.2792	−0.1200	0.0669	27.6153	942.0233
SIIG	−0.0001	0.1020	−0.1139	0.0212	−0.1086	6.6162
SIPC	0.0000	0.0915	−0.1030	0.0196	−0.3179	6.8247
SISC	−0.0002	0.0930	−0.1129	0.0194	−0.7895	8.7647
SKPC	−0.0002	0.1047	−0.1141	0.0219	0.0579	8.5149
SPA	0.0000	0.0926	−0.1038	0.0173	−0.2850	7.9182
SPCC	−0.0005	0.1044	−0.0993	0.0152	0.2098	11.5496
SPMC	−0.0006	0.6261	−0.6241	0.0380	2.5707	147.2114
SPPC	−0.0004	0.1014	−0.1360	0.0289	0.1974	7.5492
SPTC	0.0000	0.1018	−0.1030	0.0222	−0.5027	8.8293
SRCR	−0.0003	0.1766	−0.1103	0.0222	0.0748	10.2966
SREC	−0.0001	0.0953	−0.1077	0.0211	−0.3305	8.2718
SRMG	0.0007	0.1058	−0.1091	0.0295	0.5594	7.1668
SSPC	−0.0003	0.1014	−0.1109	0.0199	−0.2914	7.8505
SSTC	−0.0005	0.0946	−0.1051	0.0270	−0.3907	6.9996
ST	0.0004	0.0975	−0.0804	0.0147	0.2952	10.6360

Stocks	Mean	Maximum	Minimum	SD.	Skewness	Kurtosis
STIC	0.0002	0.0953	−0.2765	0.0275	−0.8529	13.4305
ST2	−0.0003	0.0986	−0.1054	0.0266	−0.3334	7.1394
SUCI	0.0004	0.1017	−0.2443	0.0291	−0.4557	9.1944
SVCPC	−0.0003	0.1004	−0.1241	0.0186	0.0852	9.3941
TADC	−0.0003	0.0939	−0.1054	0.0235	−0.5464	7.5586
TAI	−0.0011	0.1121	−0.3038	0.0254	−1.6922	20.8887
TAPR	−0.0003	0.6959	−0.1077	0.0372	4.6034	89.7945
TCC	−0.0003	0.0957	−0.1060	0.0165	0.1651	13.5109
TCCI	0.0003	0.0987	−0.3808	0.0263	−2.7115	40.6446
TEC	−0.0001	0.1054	−0.1181	0.0272	−0.1333	7.1379
THC	0.0003	0.0979	−0.1050	0.0177	0.1665	11.2939
TMGIC	−0.0002	0.9370	−0.2231	0.0397	8.8970	219.8138
TNCGI	−0.0003	0.0950	−0.1132	0.0184	−0.6498	9.6728
TNSC	0.0004	0.1280	−0.1258	0.0208	−0.3959	8.7984
TQCC	−0.0004	0.0656	−0.1018	0.0114	−0.7792	12.6196
TSBB	−0.0001	0.0901	−0.4053	0.0194	−6.3189	137.0194
TSIB	−0.0001	0.0822	−0.1619	0.0145	−0.6161	17.7199
UCAC	−0.0007	0.1007	−0.6010	0.0299	−5.8647	118.2236
UEC	−0.0000	0.0982	−0.1156	0.0220	−0.0306	8.2912
UITC	0.0005	0.0960	−0.1019	0.0207	−0.0130	7.0139
UWFC	−0.0007	0.1010	−0.2848	0.0218	−1.9298	27.3173
WIC	−0.0007	0.0984	−0.4847	0.0312	−2.4851	45.1754
YCC	−0.0003	0.0950	−0.0969	0.0158	0.1400	10.5054
YNPC	0.0002	0.1014	−0.1030	0.0195	0.0737	7.4320
YSCC	−0.0007	0.0899	−0.1045	0.0136	−0.6175	12.2179
ZIIK	−0.0000	0.0922	−0.1051	0.0183	−0.5785	7.9267

Notice from Table 3 that as in the majority of studies of financial markets, return excess of actions relatively to the risk-free as well as the return excess of the market relatively to its risk-free have always low skewness and high kurtosis. Our analysis consists of projecting Equation (19) relatively to time scales to test the effect of time on the systematic risk beta. This will be conducted by splitting the market returns into crystals or horizons relative to different time scales instead of using the classical periods such as weeks, moths, years. The coefficients of the linear regressions will be estimated by the usual OLS of the returns $(R_{it} - r_f)^j$ on the one of the market $(R_{mt} - r_f)^j$ for each level j. This leads to a j-level mathematical formulation as

$$(R_i - r_f)^j = \alpha_i^j + \beta_i^j (R_m - r_f)^j + \varepsilon_i^j = \alpha_i^j + \alpha_i D_m^j + \varepsilon_i^j \tag{28}$$

In the empirical study a set of 6 levels $j = 1, 2, 3, 4, 5, 6$ will be applied. We will proceed differently to classical methods by testing random periods of time instead of classical ones using weeks, months and years. The idea is an extension of our papers Rhaiem et al. (2007a), Rhaiem et al. (2007b) and Aktan et al. (2009). The correspondence scale and dynamic days applied here is resumed in Table 4 where we considered the first two prime numbers as scale laws. For the two scale laws we stopped at the

higher-level $J = 6$ as the next level $J = 7$ corresponds to 128–256 dynamic days and 192–384 dynamic days for the 2-scale and the 3-scale laws respectively and this corresponds to approximately one year.

Table 4. Time scales.

Scale law		Number of dynamic days
	J=1	2–4 dynamic days
	J=2	4–8 dynamic days
	J=3	8–16 dynamic days
2-scale law	J=4	16–32 dynamic days
	J=5	32–64 dynamic days
	J=6	64–132 dynamic days
	J=1	3–6 dynamic days
	J=2	6–12 dynamic days
	J=3	12–24 dynamic days
3-scale law	J=4	24–48 dynamic days
	J=5	48–96 dynamic days
	J=6	96–192 dynamic days

4.3. Main hypothesis validity tests

We propose in this section to discuss the validity of the main hypotheses raised above about the time-scale stability of the risk beta and the linear dependence between action's returns and their systematic risks.

Table 5 shows the different regressions of the return excess of actions relatively to the one of the market at different scales $j = 1, 2, 3, 4, 5, 6$ by the OLS estimates methods. It represents the estimations of the Betas of each stock component at the scales $j = 1, 2, 3, 4, 5, 6$ relatively to the 2-scale law.

Table 5 reflects a quite positive relationship between multi-scale return and systematic risk coefficients. Moreover, the beta changes according to the scale. The looking at the individual results indicated that this relationship becomes quietly stronger at the fourth and fifth scales (16–32 and 32–64 days): The effect of the market return on an individual asset's return will be greater at these scales than the others ones. This means that Tadawul is more efficient at scales 4 and 5. Therefore, the CAPM is a multi-scale phenomenon, and quite longer periods are more relevant in explaining the relationship between stock return and its beta. However, this relevance returns to be perturbed for long periods which means the non-resistance of the market according to time indicator. These facts lead us to think again about other factors that may be included in the model to understand more the market movement. Factors such as US policy actions, local economic policy may improve the studies on such market.

Table 5 shows that the linear dependence is always justified even-though being negative and very weak in many cases and on all the 6 scales. This means that some crisis is always present in the market explained by an opposite variation of the actions and the market. The table shows also that no law may be expected simultaneously for all the contribution of the D_m^i of all actions relatively to the increasing of time scale.

Table 5. Estimations of the return excess of actions on the market for 2-scale law.

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
AADC	2.0454	1.2592	4.7203	−0.1344	0.2696	−0.1017
AAIC	−0.8136	−0.5988	0.7101	3.6282	2.7476	−0.2771
AKSC	0.2910	−0.6088	0.2336	3.6697	0.3298	−0.4860
AAOMC	−2.0076	−1.6669	2.4864	−0.6651	0.5414	−0.0907
ABALAwal	2.2558	1.2156	0.7113	2.1137	0.2595	0.0006
ABAINma	0.2077	−0.9598	−0.1373	2.9759	0.3678	−0.1966
ABIDC	0	0	0	0	0	0
ABPTC	2.0344	−3.1069	0.8582	0.2941	0.2484	0.0735
AACI	−4.9193	−6.9013	−0.3877	0.8579	2.9001	0.4560
ACAbdullatif	1.3031	1.8143	0.0771	1.3672	−1.1050	0.6233
ACAlmaraI	0.1346	0.4174	0.3182	1.6072	−0.4814	−0.0170
ACAlujain	−2.2558	−5.5011	0.2792	−5.3093	1.9286	0.4987
ACCAC	1.5871	−1.0429	0.4471	3.2303	−0.2090	−0.1456
ACI	−1.1459	−0.6471	1.8266	−3.3696	0.6706	−0.4927
ACIC	1.3312	−0.0810	0.6685	−3.1870	−4.6848	0.6405
ACIG	1.3704	−1.8459	−0.9784	0.2556	3.8544	−0.5202
ADC	−1.6338	−1.2348	−0.3794	−1.2973	0.8679	−0.2050
AICC	1.7936	−0.8655	−0.7359	1.4150	0.8197	0.2657
AIG	0.1469	−0.7044	−0.7536	1.0553	−0.6053	−0.1658
AIHG	3.9209	3.6844	−0.4308	3.9178	2.3019	−0.6122
AJADC	0.1802	−0.6197	0.7412	0.8990	−0.3237	0.0060
AJCC	0.5851	0.2148	0.2926	1.1417	0.5260	−0.2301
AKGIS	−0.3550	−3.7764	0.2050	0.8631	0.2353	0.0068
ANB	0.1510	−1.1475	1.5114	−0.0872	−0.6937	−0.1717
APC1	1.2912	0.5540	2.0165	1.2257	−0.2257	−0.1791
APC2	0.6019	−0.2898	0.5744	3.9534	0.8820	−0.3703
APTS	−0.3484	0.5055	0.0923	2.0748	0.2792	−0.1150
ARB	1.7859	0.5454	−0.0968	1.9171	−0.1537	−0.0129
ARCI	−0.1832	0.4277	0.2845	0.2703	−0.4891	−0.1190
ASAXASA	−1.5885	−1.8101	−0.9836	2.6300	−1.2146	−0.2040
ASCIC	−0.9028	0.5524	−0.1869	−1.3193	−2.8156	−0.3546
ASCIC2	−1.1030	−0.7915	2.0132	1.0996	0.7185	−0.2994
ASDC	2.1719	0.8584	1.0935	3.1043	−0.3542	−0.7126
ASTIG	0.0798	−0.6475	2.5086	1.6947	0.0239	−0.5799
ASVON	−0.9754	−10.8147	−1.4902	0.4733	−1.0793	0.1823
ATC	0.2324	−0.3313	−0.7571	1.0422	−1.3387	−0.0540
ATEC	−0.5190	−1.0538	0.2905	3.3250	−0.2892	−0.4030
ATTM	1.5430	−0.0771	−0.1022	4.6420	−0.2408	−0.3051
BA	0.2042	−0.3765	−1.6786	0.8267	0.0514	−0.0353
BA2	0.6170	0.0255	−0.0960	3.1852	1.1053	0.0912

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
BACI	−1.7953	6.4710	−1.9693	0.6657	−1.2850	−0.9257
BCIC	1.6692	1.3176	0.0553	5.2654	−1.6853	−0.0222
BCIC2	−0.3402	−0.8038	3.1566	3.1345	0.9370	−0.3572
BSF2	0.9532	0.8619	0.0967	0.1826	−0.2565	0.0949
DARED	0.0819	−1.6201	−1.3135	5.0073	−0.0942	−0.4284
DH	−0.1791	−0.1964	−0.0991	0.5983	0.8619	−0.3453
EAT	3.5175	1.8236	−0.1227	7.9564	−0.1084	−0.1180
EEC	0.7504	1.3335	−1.3436	−2.8282	−0.5041	0.2862
EPCC	−1.7460	−1.8496	−0.8482	−1.0594	0.6062	0.0826
ETEC	−0.1419	−1.1883	−0.9012	4.7261	−0.1504	−0.2432
FAAC	3.4837	2.7514	0.5599	1.5045	1.1037	0.1344
FFS	−1.3546	−2.0219	−0.7033	0.0681	0.2875	−0.2461
FPC	0.0702	0.1569	0.0989	2.6568	−0.2760	−0.3351
FPMC	1.0234	−0.0070	0.4789	2.1010	−0.5076	−1.1162
GGCI	−1.6632	0.5200	0.6573	3.6334	1.2126	−0.3616
GUCI	3.0394	2.2841	4.2901	1.6468	−0.2954	0.2031
HB	−0.8458	−1.5923	3.7476	0.9129	0.5309	−0.1082
HCC	1.0593	0.3255	−0.1368	1.0735	−0.2680	−0.4327
HFSC	−0.3633	−0.9863	−0.8563	−0.2454	−1.5168	0.2498
JDC	0.0592	−0.1913	0.3649	3.5112	−1.2266	−0.4731
JMC	0.4594	0.6729	0.3395	0.2675	0.4964	−0.1651
JODC	1.4972	0.9974	−0.1874	2.5024	−0.6180	−0.2581
KEC	0.4610	5.0965	3.4134	0.7002	0.4510	−0.5444
KHC	−3.0494	−3.7322	−2.0498	0.7790	−0.4837	0.0575
MCC	−0.9612	−0.6497	−0.4138	1.7948	−0.3047	−0.2888
MCDC	0.2186	0.6632	−0.2014	0.4837	0.6552	−0.1054
MCIC	3.1832	1.1088	2.0118	0.5240	−1.0701	0.1967
MESCC	−2.3770	−2.8424	−2.5796	−1.7246	0.3690	−0.2658
MMSC	−0.1815	0.8594	0.1113	−1.6686	−0.1210	−0.0233
MTC	−3.4109	−1.9792	−0.5610	1.2879	1.7197	−0.1438
NADC	−0.6231	−0.3392	0.9297	0.8857	0.2977	−0.1760
NAMC	−1.9429	−0.7066	0.0368	2.8776	1.3819	−0.3872
NCC	1.1487	0.8169	0.1470	0.3079	−0.3792	−0.5390
NCC2	9.1468	1.2602	−5.2381	−10.6298	0.5239	−0.1011
NGC	2.1524	1.1171	0.0046	2.2828	−0.6344	−0.3704
NGIC	1.7953	1.5640	0.5471	0.1670	0.1474	−0.0615
NIC	1.4052	0.8410	−0.5249	1.6480	−0.6017	−0.2061
NMMC	2.6260	0.1592	0.0231	3.3927	−0.2717	−0.5809
NPC	1.1125	0.7752	−0.4892	2.8176	0.0757	−0.3156
QAC	1.2273	−0.1425	−0.2098	4.4579	0.1034	−0.1594
RB	1.1363	0.8378	0.1758	1.3161	0.2652	−0.0258
RRPC	0.6483	−0.1850	0.4136	0.3193	−0.0602	0.0378
RSHSC	−0.3698	−0.7114	0.4449	3.1914	0.2107	−0.1487

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
RUCI	−0.2733	−1.0438	−0.0889	1.4671	1.2418	−0.3125
SAAC	1.0423	2.1967	0.7674	0.2750	−0.4898	−0.0547
SACI	−1.0528	0.1017	0.2823	4.2541	−2.4115	−0.0486
SAFC	1.1512	0.6889	0.5044	1.1865	0.1779	−0.0260
SAHARA	−0.0427	−0.0528	0.0045	0.7286	−0.3619	−0.2953
SAIC	−1.4498	0.7550	−2.3830	3.1324	−0.1545	−0.9078
SAMC	3.4219	2.5712	0.2028	0.6863	1.4786	−0.2563
SARC	1.0320	0.1089	−0.2897	0.3245	−0.1400	−0.9298
SASC	−0.5931	0.0990	−0.8007	5.8086	−1.0631	−0.6298
SBIC	0.9065	0.4762	−0.4144	1.8419	−0.4755	−0.1427
SCC	0.6076	0.4292	0.5561	0.4767	0.6291	−0.0205
SCC2	−14.4916	−1.9066	−4.0903	1.1632	0.1599	−0.5467
SCC3	2.7830	2.1234	0.5116	1.1399	0.8658	−0.0871
SCC4	−0.6128	0.7771	0.1343	2.2970	1.0891	−0.2841
SCIC	−0.2775	0.4804	−0.4168	2.0828	1.4853	−0.0950
SDFC	−1.4744	0.1362	0.6110	−0.0223	−1.0306	0.0109
SECI	−2.1329	−9.3723	−0.5578	−4.1358	0.5618	−0.1817
SEC	−0.1276	−0.1430	−0.0693	0.9053	0.2668	−0.0806
SFC	0.5515	0.0295	0.3074	2.8764	0.1732	−0.5060
SFG	2.3863	1.0691	−0.4912	1.0404	−0.1379	0.1002
SG	0.0942	−1.1869	0.2493	1.0254	0.1091	0.0541
SICI	3.3619	0.3848	−0.6766	3.9122	0.3650	−0.1877
SIDC	−1.3162	−0.4311	−1.0642	3.2360	−0.0909	−0.4698
SIEC	2.7579	0.7454	4.4298	6.5821	1.9027	−0.3040
SIIG	0.7388	0.4496	0.3281	1.3524	0.1179	−0.2102
SIPC	0.0949	1.1080	0.2261	−0.9483	−0.5055	−0.0899
SISC	−1.0568	0.5250	−0.3734	4.1423	0.4105	−0.3802
SKPC	0.6351	0.2604	−0.3634	1.3533	−0.1275	−0.3112
SPA	1.6959	0.4021	−0.1265	2.3174	0.5124	−0.1977
SPCC	0.1245	0.4730	−1.0012	1.2827	−0.5697	−0.0033
SPMC	−0.0953	0.1594	0.2309	−0.5832	1.4201	0.0667
SPPC	3.0029	2.0906	1.7543	1.0952	−0.2096	−0.5743
SPTC	0.1662	1.4353	−0.0313	4.3419	0.4840	−0.0461
SRCR	2.4947	0.9071	5.0771	−0.8209	−0.0912	−0.4913
SREC	−0.3585	0.0772	−0.4963	2.4197	0.1505	−0.1294
SRMG	4.7696	−0.7324	2.4767	2.4454	−0.5290	0.0696
SSPC	1.9187	1.6177	0.2899	1.2718	0.0358	−0.0169
SSTC	0.4567	1.4386	−0.4690	1.2473	0.7117	1.6682
ST	0.4615	0.7855	0.1862	0.1500	−0.8559	−0.0837
STIC	−0.2576	−0.7365	1.6165	5.3658	−1.0183	−0.0929
ST2	−0.0839	−0.9605	−0.2524	2.7731	−2.3866	−0.2827

STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
SUCI	−1.4984	0.3416	−0.2582	8.6798	1.2857	−0.4694
SVCPC	−0.9153	−0.8431	−0.9134	0.9804	−0.1793	0.1746
TADC	1.3985	0.2504	0.7664	0.3929	−0.5896	−0.2982
TAI	0.8440	0.5883	0.4978	3.9641	0.5515	−0.6518
TAPR	0.1290	2.4773	−0.9535	−0.4211	−1.9360	0.3635
TCC	1.0496	1.4458	−0.7647	−1.0836	0.2322	−0.2075
TCCI	0.9107	−0.1210	0.8547	0.5367	1.4837	−0.2641
TEC	−0.3004	−0.2483	0.1187	5.7231	0.6804	−0.8181
THC	−1.2204	−0.6896	0.1092	2.2283	−0.0053	−0.0980
TMGIC	−1.4057	−2.6089	−0.7206	−0.1546	0.4604	−0.1210
TNCGI	0.7529	−1.1795	1.2706	2.3802	−0.6989	−0.1479
TNSC	2.5977	2.3481	0.1663	2.2420	−0.9197	−0.4980
TQCC	2.4470	1.7056	0.0552	0.4815	−0.4503	−0.1157
TSBB	2.2765	1.7355	0.1684	0.5770	0.4966	−0.1079
TSIB	−0.4280	−0.8878	0.7389	−0.3927	0.4265	−0.2391
UCAC	1.9711	0.3970	−1.8811	−0.7071	0.6535	0.1020
UEC	−0.6223	−0.3789	0.3593	0.6947	1.2473	0.3361
UITC	0.6165	−0.1991	0.4392	1.6566	−0.7038	−0.2740
UWFC	−0.1181	−1.8048	0.3520	4.1857	0.6254	−0.2395
WIC	−2.2706	−3.5767	−0.0353	−0.6691	0.4868	0.1453
YCC	0.5451	0.5711	1.0556	1.5989	−0.0201	0.0016
YNPC	1.7776	0.5990	0.3653	2.2086	0.6763	−0.4501
YSCC	0.3955	0.4720	0.2649	1.2843	0.5708	−0.1295
ZIIK	0.4253	1.6005	−0.2111	−0.5111	−0.7499	−0.0979
Mean	0.3514	−0.0826	0.1707	1.4273	0.0707	−0.1693

To explain more such a contribution, we computed the determination coefficient R^2 in Table 6. It represents the estimations of the determination coefficient R^2 relative to the Betas of each stock component estimated in Table 5 at the scales $j = 1, 2, 3, 4, 5, 6$ relatively to the 2-scale law. Notice from Table 6 that the coefficient R^2 is decreasing as the time scale increases with some perturbed cases where no monotony is conserved along all time horizons. This means among the negative movement of the market already observed that the major influencing parts of the market portfolio on the actions returns is localized in high frequencies. This concluded that at low horizons the market is going down although the linearity is strong at high levels (5 and 6). Economically speaking this a bad information for small companies and/or short investments.

To test more the link between the market return and the actions we plotted for the 2-scale law of time the recomposed crystal of the excess return on the stock versus the corresponding crystal on the market portfolio in Figure 1. Figure 1 plots daily stock returns versus corresponding stock beta at different time scales. The inspection of the figure confirms the relation between the average betas of stocks and average returns at every scale which enforces our earlier conclusions. Furthermore, as the scale increase from low (scale 1) to high (scale 6), the relation between the beta and the return becomes more and more clear. This evidence supports the proposition that the major part of the market's influence on individual asset prices is at higher horizons.
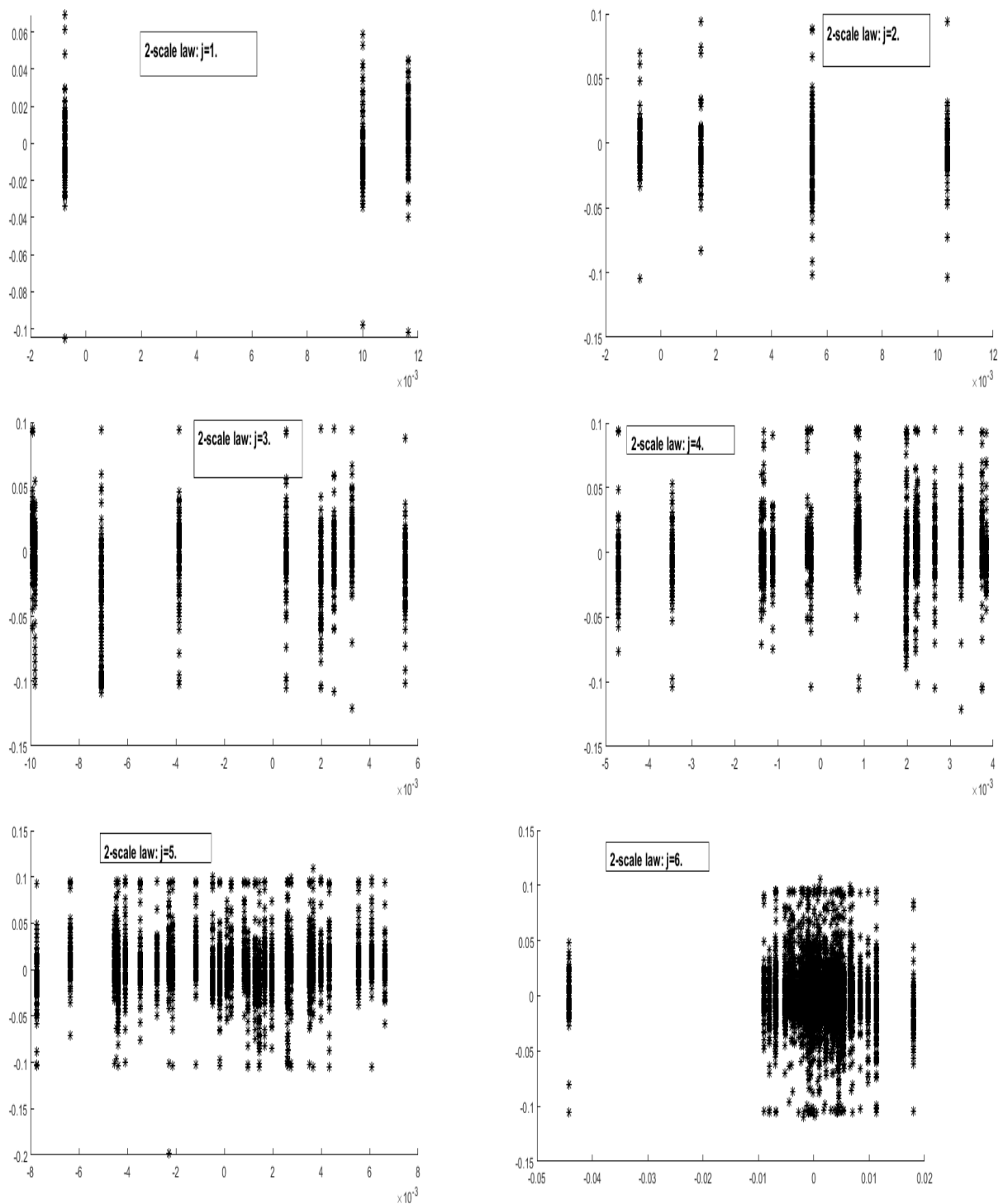
Figure 1. Excess market return (horizontal axis) versus excess return of the action (vertical axis) for different time scales with the 2-scale law.

Table 6. The determination coefficient R^2 relative to Table 5.

| | R^2 for each scale | | | | | |
STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
AADC	0.5513	0.1359	0.6325	0.0002	0.0033	0.0017
AAIC	0.1450	0.0281	0.0120	0.0563	0.0375	0.0034
AKSC	0.3626	0.0286	0.0026	0.3691	0.0031	0.0392
AAOMC	0.8296	0.1393	0.3241	0.0180	0.0075	0.0038
ABALAwal	0.9866	0.2454	0.0666	0.1087	0.0069	0.0000
ABAINma	0.2077	−0.9598	−0.1373	2.9759	0.3678	−0.1966
ABIDC	*NaN*	*NaN*	*NaN*	*NaN*	*NaN*	*NaN*
ABPTC	0.5584	0.2241	0.3029	0.0035	0.0065	0.0005
AACI	0.9971	0.5625	0.0016	0.0066	0.0374	0.0095
ACAbdullatif	0.6384	0.3413	0.0008	0.0812	0.0748	0.0350
ACAlmaraI	0.0150	0.0519	0.0118	0.1490	0.0065	0.0001
ACAlujain	0.7835	0.5221	0.0074	0.1169	0.0562	0.0271
ACCAC	0.1562	0.0155	0.0114	0.2168	0.0014	0.0122
ACI	0.3076	0.5430	0.0597	0.0385	0.0033	0.0047
ACIC	0.3465	0.0007	0.0034	0.0256	0.0517	0.0038
ACIG	0.4663	0.0483	0.0118	0.0003	0.0473	0.0089
ADC	0.7977	0.1493	0.0056	0.0196	0.0527	0.0173
AICC	0.1696	0.0508	0.0137	0.0152	0.0076	0.0045
AIG	0.0051	0.1327	0.0430	0.0911	0.0580	0.0054
AIHG	0.6664	0.8132	0.0083	0.1172	0.1047	0.0187
AJADC	0.0220	0.1731	0.0668	0.0179	0.0123	0.0000
AJCC	0.0522	0.0037	0.0041	0.0825	0.0458	0.0292
AKGIS	0.3382	0.6773	0.0033	0.0260	0.0068	0.0000
ANB	0.0201	0.5334	0.3266	0.0008	0.0216	0.0177
APC1	0.2548	0.0774	0.1005	0.0187	0.0114	0.0117
APC2	0.1998	0.0040	0.0072	0.1661	0.0431	0.0143
APTS	0.1252	0.0125	0.0011	0.2254	0.0128	0.0038
ARB	0.3688	0.0939	0.0014	0.0917	0.0030	0.0001
ARCI	0.0168	0.0087	0.0017	0.0004	0.0047	0.0015
ASAXASA	0.7668	0.1040	0.0275	0.0314	0.0146	0.0021
ASCIC	0.1493	0.0121	0.0007	0.0118	0.0859	0.0085
ASCIC2	0.2443	0.0782	0.1228	0.0135	0.0024	0.0023
ASDC	0.2244	0.0378	0.0218	0.1038	0.0031	0.0578
ASTIG	0.2014	0.1077	0.4786	0.1102	0.0001	0.0208
ASVON	0.5520	0.6574	0.1278	0.0004	0.0112	0.0007
ATC	0.0162	0.0075	0.0142	0.0081	0.0475	0.0003
ATEC	0.6875	0.1231	0.0041	0.1438	0.0038	0.0301
ATTM	0.2088	0.0007	0.0003	0.2669	0.0031	0.0227
BA	0.0152	0.0326	0.2408	0.0096	0.0001	0.0005

			R^2 for each scale			
STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
BA2	0.0468	0.0000	0.0004	0.0741	0.0283	0.0014
BACI	0.9822	0.2175	0.0504	0.0053	0.0405	0.1130
BCIC	0.6013	0.2346	0.0001	0.1289	0.0764	0.0001
BCIC2	0.0943	0.0259	0.0887	0.0702	0.0056	0.0082
BSF2	0.5051	0.1925	0.0041	0.0031	0.0077	0.0042
DARED	0.0064	0.0844	0.0304	0.1732	0.0001	0.0321
DH	0.0251	0.0121	0.0042	0.0226	0.0406	0.0681
EAT	0.3689	0.0500	0.0006	0.1937	0.0002	0.0014
EEC	0.6922	0.4296	0.0290	0.0477	0.0317	0.0512
EPCC	0.6784	0.2600	0.0692	0.1606	0.0575	0.0092
ETEC	0.0047	0.0365	0.0191	0.1689	0.0005	0.0158
FAAC	0.9998	0.3874	0.0314	0.0878	0.0612	0.0060
FFS	0.5238	0.4407	0.1663	0.0002	0.0171	0.0080
FPC	0.0181	0.0210	0.0003	0.0537	0.0024	0.0177
FPMC	0.2145	0.0000	0.0052	0.0200	0.0074	0.1441
GGCI	0.8051	0.0050	0.0098	0.0761	0.0132	0.0034
GUCI	0.9193	0.6496	0.3417	0.0154	0.0005	0.0026
HB	0.6265	0.3599	0.5291	0.0107	0.0056	0.0026
HCC	0.0958	0.0322	0.0006	0.0266	0.0038	0.0462
HFSC	0.0323	0.1624	0.0494	0.0062	0.0778	0.0184
JDC	0.0019	0.0032	0.0024	0.1340	0.0410	0.0338
JMC	0.0951	0.1530	0.0504	0.0087	0.0792	0.0350
JODC	0.5510	0.1758	0.0009	0.1263	0.0180	0.0165
KEC	0.6406	0.2771	0.3365	0.0093	0.0174	0.0332
KHC	0.9947	0.2511	0.6098	0.0295	0.0079	0.0004
MCC	0.7362	0.0374	0.0039	0.0509	0.0062	0.0202
MCDC	0.9850	0.2134	0.0208	0.0576	0.0279	0.0197
MCIC	0.6028	0.2023	0.1153	0.0012	0.0134	0.0070
MESCC	0.4411	0.9385	0.1787	0.0273	0.0030	0.0065
MMSC	0.0146	0.1928	0.0042	0.1552	0.0007	0.0001
MTC	0.4446	0.2618	0.0046	0.0098	0.0306	0.0029
NADC	0.3605	0.0585	0.1084	0.0223	0.0047	0.0106
NAMC	0.9296	0.0180	0.0000	0.1074	0.0408	0.0152
NCC	0.0442	0.1038	0.0019	0.0026	0.0095	0.0667
NCC2	0.9996	0.0044	0.3526	0.2712	0.0052	0.0015
NGC	0.3648	0.1052	0.0000	0.1620	0.0090	0.0400
NGIC	0.3314	0.6376	0.0132	0.0025	0.0017	0.0022
NIC	0.3651	0.2964	0.0154	0.1665	0.0448	0.0157
NMMC	0.9193	0.0013	0.0000	0.0888	0.0023	0.0736
NPC	0.5774	0.2832	0.0173	0.1008	0.0003	0.0128
QAC	0.1159	0.0008	0.0005	0.1427	0.0003	0.0020
RB	0.2963	0.5422	0.0229	0.0588	0.0062	0.0024

| | | | R^2 for each scale | | | |
STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
RRPC	0.8476	0.0153	0.0284	0.0083	0.0003	0.0002
RSHSC	0.0595	0.0982	0.0080	0.1678	0.0019	0.0089
RUCI	0.1524	0.0873	0.0002	0.0260	0.0266	0.0146
SAAC	0.4508	0.5554	0.2000	0.0031	0.0148	0.0026
SACI	0.2828	0.0031	0.0013	0.1097	0.0307	0.0001
SAFC	0.8476	0.2866	0.0772	0.2063	0.0013	0.0005
SAHARA	0.0005	0.0007	0.0000	0.0283	0.0107	0.0396
SAIC	0.9068	0.0147	0.0875	0.1527	0.0005	0.0942
SAMC	0.6533	0.5972	0.0056	0.0262	0.0857	0.0190
SARC	0.9850	0.0025	0.0022	0.0010	0.0003	0.0548
SASC	0.9570	0.0004	0.0177	0.4427	0.0500	0.0714
SBIC	0.5176	0.0946	0.0223	0.0879	0.0185	0.0111
SCC	0.6040	0.1211	0.0512	0.0118	0.0566	0.0002
SCC2	0.9896	0.0610	0.3424	0.0179	0.0027	0.0447
SCC3	0.4674	0.5461	0.0502	0.1523	0.0972	0.0078
SCC4	0.9570	0.3738	0.0022	0.3199	0.0783	0.0326
SCIC	0.0564	0.0085	0.0045	0.0249	0.0158	0.0007
SDFC	0.9850	0.0011	0.0465	0.0000	0.0666	0.0000
SECI	0.0975	0.6284	0.0018	0.0223	0.0013	0.0027
SEC	0.1524	0.0113	0.0028	0.3588	0.0774	0.0095
SFC	0.0259	0.0002	0.0020	0.1340	0.0007	0.0301
SFG	0.8971	0.1041	0.0214	0.0376	0.0014	0.0039
SG	0.1524	0.6770	0.0066	0.0652	0.0006	0.0009
SICI	0.9912	0.0022	0.0091	0.0741	0.0014	0.0022
SIDC	0.8273	0.0084	0.0207	0.0905	0.0001	0.0343
SIEC	0.9820	0.0345	0.5092	0.1512	0.0257	0.0061
SIIG	0.9850	0.0268	0.0079	0.0850	0.0015	0.0166
SIPC	0.0150	0.8035	0.0151	0.0958	0.0781	0.0040
SISC	0.6464	0.0238	0.0032	0.1814	0.0105	0.0396
SKPC	0.3652	0.0863	0.0065	0.1093	0.0006	0.0463
SPA	0.4194	0.6744	0.0029	0.1834	0.0108	0.0150
SPCC	0.0053	0.0451	0.0657	0.2596	0.0458	0.0000
SPMC	0.0038	0.0086	0.0447	0.0354	0.0220	0.0005
SPPC	0.9989	0.1971	0.2367	0.0292	0.0013	0.0184
SPTC	0.3653	0.2404	0.0000	0.2722	0.0095	0.0003
SRCR	0.7167	0.1152	0.5949	0.0079	0.0004	0.0273
SREC	0.0255	0.0030	0.0229	0.0995	0.0029	0.0071
SRMG	0.9915	0.0864	0.1634	0.0293	0.0110	0.0013
SSPC	0.9849	0.8655	0.0077	0.1149	0.0000	0.0001
SSTC	0.0935	0.8642	0.0045	0.0164	0.0045	0.1762
ST	0.3597	0.4065	0.0098	0.0028	0.0349	0.0029
STIC	0.3626	0.2447	0.0870	0.1124	0.0104	0.0008

| | | | R^2 for each scale | | | |
STOCKS	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
ST2	0.0040	0.0174	0.0013	0.0362	0.0550	0.0050
SUCI	0.9995	0.0053	0.0012	0.2316	0.0085	0.0117
SVCPC	0.8539	0.1033	0.1238	0.0525	0.0075	0.0038
TADC	0.5053	0.0126	0.0136	0.0021	0.0102	0.0071
TAI	0.3679	0.0196	0.0045	0.1628	0.0119	0.0337
TAPR	0.1524	0.2582	0.2177	0.0095	0.0441	0.0145
TCC	0.9851	0.2749	0.1572	0.1094	0.0052	0.0313
TCCI	0.9574	0.0068	0.1052	0.0098	0.0672	0.0251
TEC	0.0253	0.0044	0.0006	0.2845	0.0111	0.0694
THC	0.5221	0.0451	0.0011	0.1061	0.0000	0.0021
TMGIC	0.8498	0.4677	0.0494	0.0010	0.0048	0.0040
TNCGI	0.2239	0.1142	0.0503	0.3044	0.0348	0.0067
TNSC	0.8762	0.6472	0.0018	0.0412	0.0419	0.0557
TQCC	0.5566	0.4555	0.0009	0.0309	0.0266	0.0194
TSBB	0.9958	0.2533	0.0045	0.0109	0.0180	0.0043
TSIB	0.4233	0.1382	0.1285	0.0229	0.0218	0.0261
UCAC	0.6874	0.0102	0.0670	0.0047	0.0067	0.0016
UEC	0.0893	0.1027	0.0171	0.0208	0.1551	0.0590
UITC	0.1772	0.0096	0.1136	0.0707	0.0563	0.0137
UWFC	0.0047	0.3556	0.0074	0.2828	0.0376	0.0207
WIC	0.9397	0.1827	0.0000	0.0025	0.0019	0.0010
YCC	0.1972	0.2168	0.0772	0.1497	0.0001	0.0000
YNPC	0.4871	0.0711	0.0056	0.1923	0.0290	0.0611
YSCC	0.1287	0.0558	0.0147	0.0563	0.0865	0.0200
ZIIK	0.6492	0.2813	0.0057	0.0327	0.0809	0.0036

Figure 1 shows some linearity between the individual stock and the market portfolio where on the horizontal axis the excess of the market return is assigned versus the excess of the stock return assigned to the vertical axis. We notice that the linear relationship is particularly strong at the high scales especially scale 5 and 6. Medium and low scales show some perturbation in the movement of the market which confirms our conclusion previously about the market being non encouraging for small companies and short investments.

Next, we acted in a similar way the second 3-scale law on the market in order to study and understand more the movement of the market and its influence on actions and vice-versa. We obtained the results of Table a. For reasons of readability and to help readers focusing we provided the results of the 3-scale law in the Appendix later. Table a represents the estimations of the Betas of each stock component at the scales $j = 1, 2, 3, 4, 5, 6$ relatively to the 3-scale law.

Table a confirms the linear dependence noticed previously and confirms the perturbations in the market with very weak dependence in many cases and on all the 6 scales. Table a shows also that no law may be expected simultaneously for all the contribution of the D_m^j of all actions relatively to the increasing of time scale. To explain more such a contribution, we computed as previously the determination coefficient R^2 in Table b below. It represents the estimations of the determination

coefficient R^2 relative to the Betas of each stock component estimated in Table a at the scales $j = 1, 2, 3, 4, 5, 6$ relatively to the 3-scale law. In fact, the changing in the time scale permits to quantify more the link between the return excess for each action and the time dimension of the market portfolio.

Notice here also from Table b that the coefficient R^2 is decreasing as the time scale increases with some perturbed cases. This explains here also the negative movement of the market already observed and that the major influencing parts of the market portfolio on the actions returns is localized in high frequencies. This confirms that at low horizons the market is going down but no law is conserved for all companies and all levels.

As for the case of the 2-scale law of time intervals we proposed here to test the linearity between the market return and the actions. This is illustrated by Figure A which corresponds to the 3-scale law and which is provided also in Appendix. Figure A shows some linearity between the individual stock and the market portfolio where on the horizontal axis the excess of the market return is assigned versus the excess of the stock return assigned to the vertical axis. We notice that the linear relationship is particularly strong at the high scales especially scale 5 and 6. Medium and low scales show some perturbation in the movement of the market.

However, we noticed from both figures (Figure 1 and Figure A) that the market is somehow perturbed and a high-level $J = 6$ is not somehow compatible strongly in level 5. The same remark may be also noticed for the medium-high level $J = 4$. This means that the application of random periods even-though it confirms the nature and the structure of the market did not permit to discover the hidden aspects that induced the perturbation observed from some short periods.

To resume, the results make possible to quantify the correlation between the return of the stock and its beta at different time scales. The empirical results show according to Figure 1 and Figure A that the relationship between the return of a stock and its beta shows an important ambiguity although being linear. Recall that the CAPM implies that the excess return from assets (in excess of the risk-free asset return) should be proportional to the market premium (market return in excess of the risk–free asset return). Tables 6 and b show some positivity and significance for the relationship between the endogenous and exogenous variables at all scales although being very weak in some cases. When looking at individual excess return, the mean contribution of D_m^j is not clear enough to decide except quietly and sometimes at higher scales $J \geq 4$. This is always explained by the weak determination coefficient R^2. This implies that it is unclear to conclude about the major part of the market portfolio's influence on individual stocks; is it at the medium or higher frequencies.

This allows us to adapt the wavelet methods to discover more aspects in the Saudi Tadawul market. It is well known that wavelets are nowadays the most powerful tools in detecting hidden phenomena in financial markets. To show the volatility and the hidden dynamics of the market index Tadawul we applied firstly a wavelet filtering that consists in decomposing the time series associated into wavelet decomposition at the level 6. The result is illustrated by Figure 2.

The wavelet approach in estimating the systematic risk of an asset in the CAPM is based on a wavelet multi-scaling by decomposing the corresponding financial time series due to the stock return and the market one on a scale-by-scale wavelet basis. At each scale, the wavelet variance of the market return and the wavelet covariance between the market return and the stock return is computed to obtain a scale-by-scale estimate of stock's beta. The empirical results are gathered in Table 7.

It represents the estimations of the Betas of each stock component at the wavelet levels $J = 1, 2, 3, 4, 5, 6$. In the first column we provided also the estimation of the Beta relative to the wavelet
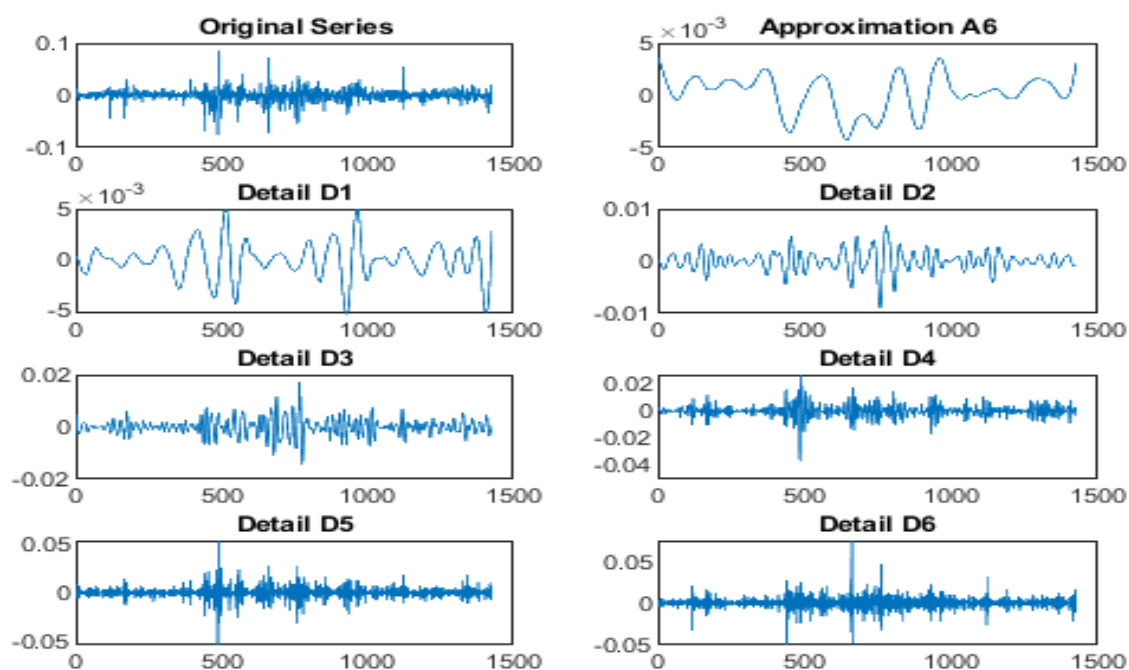
Figure 2. Wavelet decomposition of Tadawul return series at level 6.

approximation at the level $J = 6$ to get a global idea on the systematic risk beta. Recall that in wavelet theory the approximation component A_J provides a global description of the series estimated. This column is no longer provided in all the previous studies.

Notice from Table 7 that more dependence between the market and actions is detected. For example, the ABIDC which has resulted in a zero beta for previous classical methods even by changing the time scale law has yielded by using the wavelet processing a non-zero beta although being negative and small (in absolute value) for all wavelet levels. This means that in such type of markets one should apply always macroscopic and microscopic tools able to detect the hidden facts.

Next to confirm the linear dependence and the efficiency of the wavelet tool we established as previously a computation of the R^2 coefficient of determination in Table 8. It represents the estimations of the determination coefficient R^2 relative to the Betas of each stock component estimated in Table 7 at the wavelet levels $J = 1, 2, 3, 4, 5, 6$. It shows a coherence with results on wavelet estimation of beta.

We notice that the market is going to be efficient at high scales and the dependence grow up in fact as the level increases. This let investors to conclude that investing in such a market may need to come over the microscopic and macroscopic scales of the market and study may be different panels in it to conclude in what sector investments will be of good returns.

In the present case, it appears that food sectors seems to be the most important. This is somehow natural as KSA is an importing country more than a self-producing in this sector. Otherwise, sectors such as energy, transportation and security and safety are somehow encouraging. This is also a good task as KSA is planning to lunch the international project NEOM which will be from the financial and economic points of view an international market where different sectors will intersect and interchange.

Table 7. Wavelet Beta for each level.

			Beta for wavelet levels $J = 1$ to $J = 6$.			
STOCKS	A_6	D_1	D_2	D_3	D_4	D_5	D_6
---	---	---	---	---	---	---	---
AADC	0.0456	0.0542	−0.0179	0.1145	0.2421	−0.0009	0.0273
AAIC	0.0147	−0.0081	0.1691	−0.1342	−0.0624	0.7116	0.1077
AKSC	−0.0146	−0.0049	−0.0910	−0.1801	0.0312	0.4310	0.7640
AAOMC	−0.0022	−0.0936	−0.0098	0.0616	0.2043	0.3210	0.0341
ABALAwal	0.0673	−0.0284	0.1195	−0.0003	0.1863	0.2360	0.8566
ABAINma	−0.0415	−0.0424	−0.0475	−0.1100	0.1312	−0.2777	−0.0323
ABIDC	−0.0242	−0.0569	0.0617	−0.0920	−0.0203	−0.0006	−0.1124
ABPTC	0.0266	0.0268	−0.0001	−0.0281	0.1581	0.1581	0.2604
AACI	0.0281	0.0679	0.1736	−0.3365	−0.2494	1.0131	0.1229
ACAbdullatif	0.0411	0.0297	0.0013	0.0275	0.0980	0.2472	0.2536
ACAlmaraI	−0.0461	−0.0893	−0.1196	−0.0724	0.3399	−0.1337	0.3615
ACAlujain	−0.0968	0.0039	−0.1484	−0.3781	−0.1976	−0.1978	0.0943
ACCAC	0.0686	0.0669	0.0045	0.0525	0.2210	0.1984	0.0702
ACI	−0.0697	−0.1585	−0.0762	0.0461	0.2419	0.0863	−0.6547
ACIC	0.0375	0.0327	−0.1805	0.3293	0.2982	0.7254	−0.3424
ACIG	−0.0507	−0.0400	−0.0705	0.1302	−0.3809	0.3830	0.2661
ADC	0.0190	0.0311	−0.0623	0.1224	0.0979	−0.0768	0.0702
AICC	−0.0112	−0.0615	0.0712	−0.0048	−0.1443	0.3192	0.1213
AIG	−0.0676	−0.0908	−0.0320	−0.1739	0.0568	0.1343	−0.1885
AIHG	−0.0580	−0.0233	−0.2209	0.0852	0.0832	−0.0607	−0.3612
AJADC	−0.0060	0.0003	−0.0215	−0.0347	0.0678	−0.2266	0.2843
AJCC	−0.0060	0.0033	−0.0218	−0.1552	0.1513	0.0814	0.2852
AKGIS	−0.0546	0.0453	−0.2659	−0.2000	0.1226	0.1847	0.1372
ANB	0.0268	0.0506	0.0283	−0.0674	0.0760	−0.0548	−0.2062
APC1	0.0374	0.0536	−0.0135	−0.1091	0.2764	−0.2207	0.7767
APC2	0.0123	0.0329	−0.0178	−0.0574	0.1145	0.1385	0.1497
APTS	−0.0470	−0.0911	−0.0024	0.0020	−0.0018	−0.1974	0.0723
ARB	0.0223	−0.0224	0.0373	0.0208	0.1418	0.2026	0.0611
ARCI	−0.1525	−0.2613	−0.1309	0.2157	−0.1349	−0.0898	−0.2411
ASAXASA	−0.0972	−0.1594	−0.0708	−0.0277	−0.0418	0.5093	0.1081
ASCIC	−0.0530	−0.0495	−0.2044	0.1794	−0.0412	0.1838	0.2804
ASCIC2	−0.1415	−0.2969	−0.1227	0.1461	−0.0200	0.4611	−0.1952
ASDC	0.0770	0.1209	−0.1090	0.3411	0.1059	−0.1814	0.0541
ASTIG	−0.0199	−0.0211	−0.1032	0.1846	0.0056	0.1017	0.2861
ASVON	−0.0351	−0.0789	−0.0799	0.2179	−0.0809	0.2327	0.3148
ATC	−0.1226	−0.1647	−0.1447	0.0564	−0.0239	−0.1422	−0.0088
ATEC	0.0136	0.0351	−0.0492	0.0251	−0.0541	0.0419	0.3493
ATTM	−0.0115	0.0402	−0.1117	0.0373	0.0256	0.0809	−0.1285
BA	−0.0241	−0.0742	−0.0410	−0.1302	0.3286	−0.0013	0.2471

Beta for wavelet levels $J = 1$ to $J = 6$.						
BA2	0.0245	0.0005	−0.0289	0.0601	0.1783	0.1511	0.0892
BACI	−0.0338	−0.1625	−0.1457	0.0285	0.5634	0.3169	0.0614
BCIC	−0.0612	−0.0732	−0.1072	0.0494	0.0063	0.0639	−0.2716
BCIC2	0.0598	−0.0354	0.0842	0.3015	0.0639	0.6607	0.2274
BSF2	0.0299	−0.0036	0.0418	−0.0750	0.3108	0.0878	−0.0789
DARED	0.0332	0.0541	−0.1733	−0.0101	0.3716	0.3324	−0.0017
DH	−0.0157	−0.0057	−0.0923	−0.0019	0.2259	−0.1083	−0.0520
EAT	−0.0633	−0.1789	−0.1411	0.3081	0.0860	0.5561	0.5497
EEC	0.0900	−0.0058	0.1621	0.0578	0.1009	0.2936	0.9460
EPCC	0.0027	−0.0005	0.0537	−0.0043	−0.0976	0.2280	−0.3686
ETEC	−0.0780	−0.0424	−0.1387	−0.2074	0.2058	−0.2062	−0.0935
FAAC	−0.0364	−0.0923	0.0080	−0.1250	0.0767	−0.0097	0.2983
FFS	−0.0376	−0.0093	−0.1266	−0.0069	−0.0209	0.0271	−0.0250
FPC	−0.0072	0.0486	−0.2269	0.2946	0.0863	−0.1109	−0.1828
FPMC	−0.1131	−0.1256	−0.1212	−0.0607	−0.0738	−0.1013	−0.5218
GGCI	−0.0141	−0.0012	−0.1053	0.1598	−0.1022	0.2126	0.1017
GUCI	0.0346	−0.0494	0.1068	0.1781	0.0482	0.3336	0.6681
HB	−0.0031	−0.0287	0.0521	0.0272	0.1787	−0.3348	−0.6415
HCC	−0.0184	−0.0067	−0.0145	−0.0269	0.0276	−0.1004	−0.1397
HFSC	−0.0289	0.0374	−0.1611	−0.0272	−0.0048	0.1469	−0.1942
JDC	−0.0413	−0.0328	−0.1512	−0.0011	0.1987	0.0130	0.0080
JMC	−0.0396	−0.0267	−0.1830	−0.0113	0.1340	0.1954	0.1677
JODC	−0.0396	0.0117	−0.0927	−0.1693	−0.0684	0.2397	0.3081
KEC	0.0228	0.0221	0.0901	−0.7417	0.0984	0.5667	1.4308
KHC	−0.0315	0.0111	0.0801	−0.4839	−0.2192	−0.2052	1.1686
MCC	−0.0131	0.0437	−0.1318	−0.0062	0.0266	−0.0479	0.2546
MCDC	0.0192	0.0079	−0.0850	0.2193	0.1774	−0.1346	0.0068
MCIC	−0.0856	−0.0649	0.0001	0.0876	−0.1942	−0.5787	0.9745
MESCC	0.0624	−0.0036	0.1909	0.0119	0.1185	0.1184	−0.3774
MMSC	−0.0165	0.0246	−0.0466	−0.0205	−0.0512	−0.2765	0.1979
MTC	0.0621	0.0193	−0.0586	0.1656	0.2843	0.5829	−0.0508
NADC	−0.0055	0.0106	−0.0288	−0.1748	0.3539	−0.2887	0.3566
NAMC	0.0176	−0.0139	0.0025	0.0920	0.1666	0.2011	−0.5989
NCC	−0.0208	−0.0568	−0.0378	0.0056	0.1093	0.1240	0.1683
NCC2	−0.0486	−0.1032	0.3798	0.0441	−0.5495	−0.6516	1.4675
NGC	−0.0328	−0.0027	−0.1242	−0.0748	0.1352	0.1267	0.1416
NGIC	−0.0325	−0.0249	−0.1076	−0.0713	0.0575	0.2202	−0.0242
NIC	0.0056	−0.0294	−0.0704	0.1314	0.0778	0.1146	0.0940
NMMC	−0.0580	0.0075	−0.2024	−0.0421	0.0416	0.0608	−0.3702
NPC	−0.0802	−0.0287	−0.1793	0.0420	0.0791	0.1705	0.2557
QAC	0.0070	0.1227	−0.1421	−0.1661	0.2926	0.0481	−0.0265
RB	0.0417	0.0822	0.1209	−0.1099	−0.0373	−0.0026	−0.3406

Beta for wavelet levels $J = 1$ to $J = 6$.							
RRPC	−0.0057	0.0226	−0.0950	−0.1592	0.0971	0.2582	1.4673
RSHSC	−0.1031	−0.1218	−0.1347	−0.0627	−0.1571	0.2579	0.7422
RUCI	−0.0746	−0.1569	−0.1086	0.1516	0.0528	−0.0863	0.6127
SAAC	−0.0211	−0.0015	0.0017	−0.6231	0.0361	0.4772	1.3337
SACI	0.0465	−0.0492	0.3521	0.0392	0.0144	−0.0173	−0.7094
SAFC	−0.0392	−0.0753	−0.0291	0.0116	0.0703	−0.0545	−0.0540
SAHARA	−0.0001	0.0100	−0.0833	−0.0273	0.0647	−0.0915	0.7992
SAIC	−0.0435	−0.0287	−0.1793	0.0420	0.0791	0.1705	0.2557
SAMC	−0.0554	−0.0288	−0.0690	−0.0728	−0.0460	−0.1104	0.1703
SARC	−0.0556	−0.0074	−0.2230	0.1207	−0.0827	0.3700	−0.7227
SASC	−0.1013	−0.2558	0.1039	−0.0344	0.1160	−0.4093	0.1326
SBIC	−0.0063	−0.0007	−0.0411	−0.0692	0.1383	−0.1662	0.4092
SCC	−0.0235	0.0223	−0.1505	0.0182	−0.0065	0.3886	−0.0727
SCC2	0.0233	−0.0196	0.0826	−0.0464	0.0946	0.2569	−0.6639
SCC3	0.0264	0.0216	0.0130	0.0811	0.0034	0.0011	0.1613
SCC4	0.0137	−0.0368	−0.1199	0.0677	0.3075	0.4753	0.2284
SCIC	−0.0645	−0.1311	−0.2697	0.2887	0.1641	0.2769	−0.0454
SDFC	−0.0181	0.0428	−0.2401	0.1097	0.1630	−0.1407	−0.4301
SECI	0.0403	0.0403	0.1164	0.0695	−0.3204	0.4207	0.7303
SEC	0.0524	0.0577	−0.0225	0.1559	−0.0170	0.1915	0.4767
SFC	0.0708	0.1561	0.0207	0.1582	−0.1650	0.3554	0.3235
SFG	0.0239	0.0406	0.0093	−0.0620	0.1675	−0.4243	0.7986
SG	0.0682	−0.0119	0.0034	−0.1310	0.5997	0.4311	0.3788
SICI	0.0808	0.1858	−0.1974	0.5537	−0.2980	0.3184	−0.4706
SIDC	0.0005	0.0041	0.0166	−0.0081	0.0289	0.0964	0.2958
SIEC	0.1569	0.1548	0.3015	0.1853	0.3910	−0.3421	−0.6848
SIIG	−0.0193	−0.0115	−0.0666	−0.2546	0.1769	0.0918	1.0752
SIPC	0.1220	0.0570	0.0461	−0.5714	0.6840	0.7317	1.5726
SISC	−0.0284	0.0242	−0.1223	−0.0651	0.0169	0.1226	0.1384
SKPC	−0.0116	−0.0139	−0.0799	−0.1090	0.2074	0.1536	0.1100
SPA	−0.0043	0.0342	−0.1003	0.0008	0.0688	0.1896	−0.3133
SPCC	0.0039	0.0659	−0.1328	0.0018	0.0911	−0.0435	−0.1378
SPMC	0.0466	0.0674	0.1489	−0.1305	−0.5855	0.0198	2.1554
SPPC	−0.1349	−0.0717	−0.3389	−0.0615	−0.0492	−0.1068	0.2281
SPTC	0.0015	−0.0258	0.0063	−0.1330	0.2367	0.2255	0.5886
SRCR	0.0795	−0.0163	0.0922	0.1830	0.2519	0.6456	0.6302
SREC	−0.0435	−0.0113	−0.1283	0.0082	0.0159	0.1174	−0.2488
SRMG	−0.0822	−0.0163	−0.2175	−0.0583	−0.0043	−0.0628	0.0482
SSPC	−0.0021	−0.0076	−0.1187	0.0045	0.1936	0.3004	0.0021
SSTC	0.1072	0.1209	−0.0049	0.1628	0.2037	0.1308	0.0637
ST	−0.0142	−0.0234	−0.0529	−0.0817	0.1553	−0.1020	0.2956

Beta for wavelet levels $J = 1$ to $J = 6$.						
STIC	−0.0300	−0.0376	0.0693	0.0188	0.0581	−0.1068	−0.4726
ST2	−0.0877	−0.1114	−0.1397	0.1198	−0.1517	0.3265	−0.1733
SUCI	−0.1081	−0.1897	−0.0812	−0.0051	0.0272	−0.1135	0.0635
SVCPC	−0.0933	−0.0127	−0.1554	−0.0354	−0.7366	0.3252	0.7706
TADC	0.0218	0.0159	0.0245	0.0616	0.0529	−0.0515	0.0022
TAI	−0.0085	0.0656	−0.0769	−0.1610	0.1383	0.0659	0.9873
TAPR	0.0865	−0.0391	0.2627	0.0648	−0.0169	0.1629	−0.1250
TCC	0.0662	0.0291	0.1601	−0.0269	0.0820	0.4433	0.0911
TCCI	0.0048	−0.0362	−0.0630	−0.0027	0.3080	0.7444	−0.4341
TEC	0.0098	0.0413	−0.1368	0.1357	0.0449	0.0228	−0.3267
THC	−0.0064	−0.0257	−0.0740	0.1256	0.0902	−0.0468	−0.1454
TMGIC	0.0893	−0.0840	0.3027	−0.0005	0.1539	1.4404	0.1353
TNCGI	0.0131	0.0284	−0.0248	−0.0269	0.1447	0.1147	0.1534
TNSC	−0.0507	−0.1198	−0.0972	0.0384	0.2365	0.3383	0.0835
TQCC	−0.0131	−0.0264	−0.0498	0.0454	−0.0007	0.1612	0.0034
TSBB	0.0299	0.0060	−0.0291	0.0442	0.2327	−0.1622	0.3065
TSIB	0.0455	0.1177	−0.0475	−0.0747	0.0499	−0.0638	0.2830
UCAC	−0.1449	−0.2060	−0.0364	−0.1649	−0.2076	−0.0638	0.9574
UEC	0.0038	−0.0059	−0.1177	0.0701	0.1616	0.3211	0.3109
UITC	−0.0424	−0.0231	−0.1561	−0.0926	0.0089	0.3628	0.2038
UWFC	0.0001	0.0216	−0.0804	0.1801	−0.0675	−0.2087	0.0294
WIC	−0.1450	−0.2493	−0.0908	−0.0261	−0.0027	0.3320	−0.0153
YCC	−0.0224	0.0166	−0.0026	−0.0658	−0.1775	0.2109	−0.3489
YNPC	−0.0060	−0.0205	−0.0110	−0.1010	0.2394	−0.5251	0.7140
YSCC	0.0184	0.0849	−0.0399	0.0296	−0.1814	0.0618	0.0721
ZIIK	−0.0277	0.0439	−0.1857	−0.0914	0.1403	0.1091	0.1613
Mean	−0.0111	−0.0178	−0.0450	−0.0036	0.0649	0.1208	0.1682

Table 8. Wavelet R^2 relative to Table 7 for each level.

STOCKS	$D1$	$D2$	$D3$	$D4$	$D5$	$D6$
AADC	0.0007	0.0001	0.0036	0.0229	0.0000	0.0002
AAIC	0.0000	0.0021	0.0014	0.0005	0.0402	0.0006
AKSC	0.0000	0.0014	0.0056	0.0005	0.0254	0.0626
AAOMC	0.0033	0.0000	0.0017	0.0252	0.0264	0.0004
ABALAwal	0.0001	0.0027	0.0000	0.0108	0.0118	0.1438
ABAINma	0.0007	0.0009	0.0054	0.0167	0.0524	0.0008
ABIDC	0.0013	0.0012	0.0070	0.0002	0.0000	0.0028
ABPTC	0.0002	0.0000	0.0003	0.0092	0.0064	0.0108
AACI	0.0004	0.0030	0.0074	0.0102	0.0881	0.0010
ACAbdullatif	0.0003	0.0000	0.0003	0.0058	0.0285	0.0175
ACAlmaraI	0.0030	0.0068	0.0030	0.0755	0.0075	0.0985
ACAlujain	0.0000	0.0044	0.0308	0.0112	0.0099	0.0012
ACCAC	0.0020	0.0000	0.0013	0.0299	0.0130	0.0034
ACI	0.0025	0.0005	0.0002	0.0165	0.0008	0.0318
ACIC	0.0001	0.0017	0.0060	0.0066	0.0308	0.0027
ACIG	0.0002	0.0006	0.0016	0.0198	0.0139	0.0042
ADC	0.0004	0.0012	0.0072	0.0066	0.0024	0.0029
AICC	0.0004	0.0005	0.0000	0.0071	0.0114	0.0021
AIG	0.0025	0.0003	0.0083	0.0020	0.0066	0.0105
AIHG	0.0001	0.0080	0.0016	0.0027	0.0005	0.0204
AJADC	0.0000	0.0001	0.0005	0.0031	0.0126	0.0165
AJCC	0.0000	0.0002	0.0076	0.0212	0.0030	0.0324
AKGIS	0.0004	0.0114	0.0068	0.0045	0.0054	0.0035
ANB	0.0015	0.0005	0.0026	0.0051	0.0022	0.0186
APC1	0.0012	0.0001	0.0048	0.0436	0.0265	0.2068
APC2	0.0002	0.0001	0.0007	0.0061	0.0041	0.0029
APTS	0.0025	0.0000	0.0000	0.0000	0.0101	0.0019
ARB	0.0003	0.0009	0.0003	0.0216	0.0304	0.0040
ARCI	0.0122	0.0027	0.0097	0.0093	0.0024	0.0169
ASAXASA	0.0031	0.0006	0.0001	0.0005	0.0278	0.0013
ASCIC	0.0004	0.0058	0.0056	0.0005	0.0041	0.0092
ASCIC2	0.0128	0.0016	0.0035	0.0001	0.0190	0.0045
ASDC	0.0024	0.0016	0.0200	0.0023	0.0033	0.0003
ASTIG	0.0001	0.0019	0.0064	0.0000	0.0018	0.0084
ASVON	0.0010	0.0008	0.0063	0.0017	0.0028	0.0066
ATC	0.0058	0.0040	0.0008	0.0003	0.0054	0.0000
ATEC	0.0003	0.0005	0.0001	0.0009	0.0006	0.0260
ATTM	0.0005	0.0035	0.0003	0.0004	0.0018	0.0057
BA	0.0019	0.0005	0.0051	0.0565	0.0000	0.0419

			R^2			
BA2	0.0000	0.0003	0.0011	0.0212	0.0074	0.0025
BACI	0.0029	0.0027	0.0001	0.0666	0.0194	0.0008
BCIC	0.0013	0.0033	0.0006	0.0000	0.0009	0.0317
BCIC2	0.0002	0.0007	0.0095	0.0008	0.0554	0.0076
BSF2	0.0000	0.0008	0.0023	0.0631	0.0041	0.0040
DARED	0.0005	0.0054	0.0000	0.0485	0.0161	0.0000
DH	0.0000	0.0022	0.0000	0.0432	0.0047	0.0012
EAT	0.0026	0.0021	0.0111	0.0010	0.0426	0.0344
EEC	0.0000	0.0074	0.0012	0.0029	0.0151	0.1693
EPCC	0.0000	0.0019	0.0000	0.0093	0.0216	0.0693
ETEC	0.0003	0.0042	0.0094	0.0155	0.0087	0.0022
FAAC	0.0011	0.0000	0.0023	0.0015	0.0000	0.0103
FFS	0.0001	0.0094	0.0000	0.0006	0.0004	0.0003
FPC	0.0004	0.0075	0.0149	0.0022	0.0020	0.0051
FPMC	0.0041	0.0025	0.0009	0.0024	0.0028	0.0498
GGCI	0.0000	0.0014	0.0034	0.0030	0.0073	0.0011
GUCI	0.0003	0.0014	0.0044	0.0005	0.0112	0.0464
HB	0.0002	0.0008	0.0002	0.0146	0.0273	0.1334
HCC	0.0000	0.0001	0.0003	0.0007	0.0036	0.0078
HFSC	0.0006	0.0087	0.0002	0.0000	0.0090	0.0157
JDC	0.0003	0.0048	0.0000	0.0194	0.0001	0.0000
JMC	0.0004	0.0170	0.0001	0.0141	0.0153	0.0203
JODC	0.0000	0.0027	0.0082	0.0027	0.0172	0.0279
KEC	0.0001	0.0018	0.1057	0.0046	0.0618	0.4610
KHC	0.0000	0.0017	0.0561	0.0328	0.0150	0.2636
MCC	0.0006	0.0048	0.0000	0.0005	0.0007	0.0197
MCDC	0.0000	0.0021	0.0138	0.0142	0.0061	0.0000
MCIC	0.0003	0.0000	0.0006	0.0043	0.0254	0.1106
MESCC	0.0000	0.0061	0.0000	0.0049	0.0024	0.0214
MMSC	0.0002	0.0008	0.0001	0.0016	0.0509	0.0260
MTC	0.0001	0.0005	0.0062	0.0230	0.0439	0.0002
NADC	0.0000	0.0002	0.0073	0.0558	0.0217	0.0140
NAMC	0.0000	0.0000	0.0017	0.0078	0.0037	0.0337
NCC	0.0012	0.0005	0.0000	0.0069	0.0047	0.0089
NCC2	0.0004	0.0063	0.0001	0.0225	0.0127	0.1011
NGC	0.0000	0.0041	0.0016	0.0109	0.0051	0.0067
NGIC	0.0003	0.0074	0.0026	0.0028	0.0231	0.0002
NIC	0.0003	0.0014	0.0055	0.0032	0.0051	0.0031
NMMC	0.0000	0.0077	0.0004	0.0008	0.0009	0.0259
NPC	0.0002	0.0069	0.0004	0.0025	0.0069	0.0143
QAC	0.0025	0.0029	0.0046	0.0229	0.0003	0.0001
RB	0.0016	0.0038	0.0033	0.0006	0.0000	0.0307

			R^2			
RRPC	0.0001	0.0017	0.0062	0.0030	0.0165	0.3417
RSHSC	0.0028	0.0032	0.0009	0.0108	0.0156	0.1135
RUCI	0.0048	0.0021	0.0045	0.0011	0.0011	0.0349
SAAC	0.0000	0.0000	0.1209	0.0007	0.0950	0.3936
SACI	0.0002	0.0133	0.0002	0.0000	0.0000	0.0333
SAFC	0.0023	0.0004	0.0001	0.0043	0.0017	0.0015
SAHARA	0.0000	0.0023	0.0003	0.0042	0.0035	0.1846
SAIC	0.0002	0.0069	0.0004	0.0025	0.0069	0.0143
SAMC	0.0002	0.0013	0.0016	0.0012	0.0047	0.0142
SARC	0.0000	0.0092	0.0029	0.0031	0.0224	0.1287
SASC	0.0136	0.0021	0.0003	0.0059	0.0261	0.0023
SBIC	0.0000	0.0008	0.0025	0.0169	0.0111	0.0952
SCC	0.0002	0.0082	0.0001	0.0000	0.0578	0.0024
SCC2	0.0001	0.0011	0.0004	0.0026	0.0117	0.0850
SCC3	0.0003	0.0001	0.0031	0.0000	0.0000	0.0148
SCC4	0.0005	0.0050	0.0015	0.0549	0.0698	0.0149
SCIC	0.0019	0.0076	0.0087	0.0052	0.0067	0.0002
SDFC	0.0006	0.0196	0.0044	0.0226	0.0072	0.0832
SECI	0.0002	0.0013	0.0004	0.0185	0.0155	0.0382
SEC	0.0015	0.0003	0.0119	0.0002	0.0156	0.1007
SFC	0.0021	0.0000	0.0021	0.0054	0.0139	0.0117
SFG	0.0003	0.0000	0.0006	0.0073	0.0377	0.0955
SG	0.0000	0.0000	0.0011	0.0335	0.0113	0.0090
SICI	0.0023	0.0025	0.0170	0.0080	0.0043	0.0071
SIDC	0.0000	0.0001	0.0000	0.0004	0.0018	0.0138
SIEC	0.0006	0.0025	0.0010	0.0081	0.0030	0.0107
SIIG	0.0000	0.0011	0.0174	0.0107	0.0022	0.2091
SIPC	0.0010	0.0007	0.1038	0.1837	0.1835	0.3832
SISC	0.0002	0.0047	0.0011	0.0002	0.0050	0.0045
SKPC	0.0000	0.0016	0.0030	0.0234	0.0073	0.0031
SPA	0.0005	0.0031	0.0000	0.0037	0.0134	0.0411
SPCC	0.0020	0.0101	0.0000	0.0068	0.0006	0.0132
SPMC	0.0004	0.0012	0.0028	0.0766	0.0001	0.3092
SPPC	0.0008	0.0134	0.0005	0.0007	0.0011	0.0070
SPTC	0.0002	0.0000	0.0035	0.0260	0.0112	0.0756
SRCR	0.0001	0.0022	0.0104	0.0225	0.0672	0.0786
SREC	0.0000	0.0041	0.0000	0.0002	0.0020	0.0144
SRMG	0.0000	0.0071	0.0004	0.0000	0.0003	0.0002
SSPC	0.0000	0.0036	0.0000	0.0283	0.0258	0.0000
SSTC	0.0024	0.0000	0.0038	0.0107	0.0028	0.0006
ST	0.0003	0.0012	0.0038	0.0237	0.0070	0.0852
STIC	0.0002	0.0008	0.0000	0.0011	0.0016	0.0268

			R^2			
ST2	0.0022	0.0026	0.0023	0.0091	0.0288	0.0039
SUCI	0.0051	0.0008	0.0000	0.0002	0.0017	0.0008
SVCPC	0.0000	0.0085	0.0006	0.3510	0.0345	0.2701
TADC	0.0001	0.0001	0.0008	0.0011	0.0004	0.0000
TAI	0.0008	0.0011	0.0042	0.0068	0.0004	0.0964
TAPR	0.0002	0.0068	0.0003	0.0000	0.0008	0.0008
TCC	0.0003	0.0103	0.0004	0.0047	0.0940	0.0061
TCCI	0.0002	0.0006	0.0000	0.0324	0.0895	0.0249
TEC	0.0003	0.0024	0.0033	0.0006	0.0001	0.0155
THC	0.0002	0.0020	0.0055	0.0065	0.0008	0.0138
TMGIC	0.0005	0.0063	0.0000	0.0033	0.0930	0.0009
TNCGI	0.0003	0.0002	0.0002	0.0173	0.0051	0.0055
TNSC	0.0038	0.0022	0.0005	0.0281	0.0389	0.0020
TQCC	0.0006	0.0021	0.0020	0.0000	0.0144	0.0000
TSBB	0.0000	0.0003	0.0006	0.0196	0.0108	0.0433
TSIB	0.0080	0.0011	0.0031	0.0023	0.0027	0.0439
UCAC	0.0057	0.0002	0.0032	0.0101	0.0006	0.0683
UEC	0.0000	0.0035	0.0010	0.0120	0.0275	0.0203
UITC	0.0001	0.0054	0.0029	0.0000	0.0325	0.0137
UWFC	0.0001	0.0014	0.0072	0.0021	0.0111	0.0002
WIC	0.0080	0.0008	0.0001	0.0000	0.0138	0.0000
YCC	0.0001	0.0000	0.0020	0.0238	0.0149	0.0649
YNPC	0.0001	0.0000	0.0042	0.0232	0.0913	0.1429
YSCC	0.0048	0.0010	0.0006	0.0266	0.0015	0.0046
ZIIK	0.0007	0.0115	0.0029	0.0150	0.0049	0.0112

Besides, some sectors present a small and sometimes negative relationship between the growth in the market and the one of the company and thus should be corrected to firstly pass to positive influence of the company in the whole market and increase the return. Some steps should be taken into consideration such as the Yemen war at the south and Syrian one where a great part of KSA money is lost.

The finings in Tables 7 and 8 are confirmed in Figure 3 hereafter which illustrates the overage of wavelet excess return of the actions against the wavelet excess return of the market at the different wavelet levels $j = 1, 2, 3, 4, 5, 6$.

According to the classification into sectors we notice from Table 9 that by grouping the components of the market according to the sectors this allows us to carry out a global view of the market. Yet the disruption is very clear in the market in hand. Most sectors show a positive relationship in the medium level $J = 4$ according to the 2-scale law except for a few cases such as health. However, this sector itself is based mainly on imported equipment and a workforce which suffers from qualification except a great part of foreign one. By applying the wavelet model which generally allows to go further into the microscopic state of the market, we notice that the positive relationship between market and sectors becomes more and more important for large scales, $J = 4$ and mostly $J = 5$ and $J = 6$. These conclusions join previous ones which require to introduce other factors such as those due to Marfatia H.A. and may be political and geopolitical factors to better understand this market.

5. Conclusion

In this paper a wavelet study of the largest GCC market Saudi Tadawul has been developed in order to understand such market on a critic period which post follows many important movements infusing directly and strongly on the market. Saudi Tadawul is chosen based on its value as the most representative market for GCC continent and thus its influence and relation to worldwide markets.

In this paper we proposed essentially to test the impact of the time scale on the estimation of the systematic risk Beta in the presence of missing data by using essentially wavelet tools. We essentially focused on the stability of the systematic risk beta according to time scales and to the eventual linear linkage between the assets returns and their systematic risks.

At a first step the empirical results conducted showed that the relationship between the return of a stock and its beta becomes stronger quietly at the medium-higher, but the test of the linearity between the return and its betas showed that there is an important ambiguity. This led us to think about wavelets as more adoptable and relevant methods to enlighten the ambiguity. Wavelets are suitable adopted mathematical tools that have been world-wise developed until the 80's. They have been shown successful applications in physics, mathematics, finance, statistics, etc.

The application of wavelets has proved that the relationship between the return of a stock and its beta is more robust at higher scales 4, 5 and 6. This evidence means that Saudi stock market is more efficient in the high scales. Therefore, the predictions of the CAPM are more relevant at the higher-horizons in a multi-scale framework as compared to other horizons.

Nevertheless the market presents sometimes some perturbations which remain clear even when splitting it into sectors. We think that such perturbations are due to other factors that should be included for any future study to understand more the movement and the situation of the market. Factors such as US policy, local political factors as well as geopolitical ones may be good extensions.
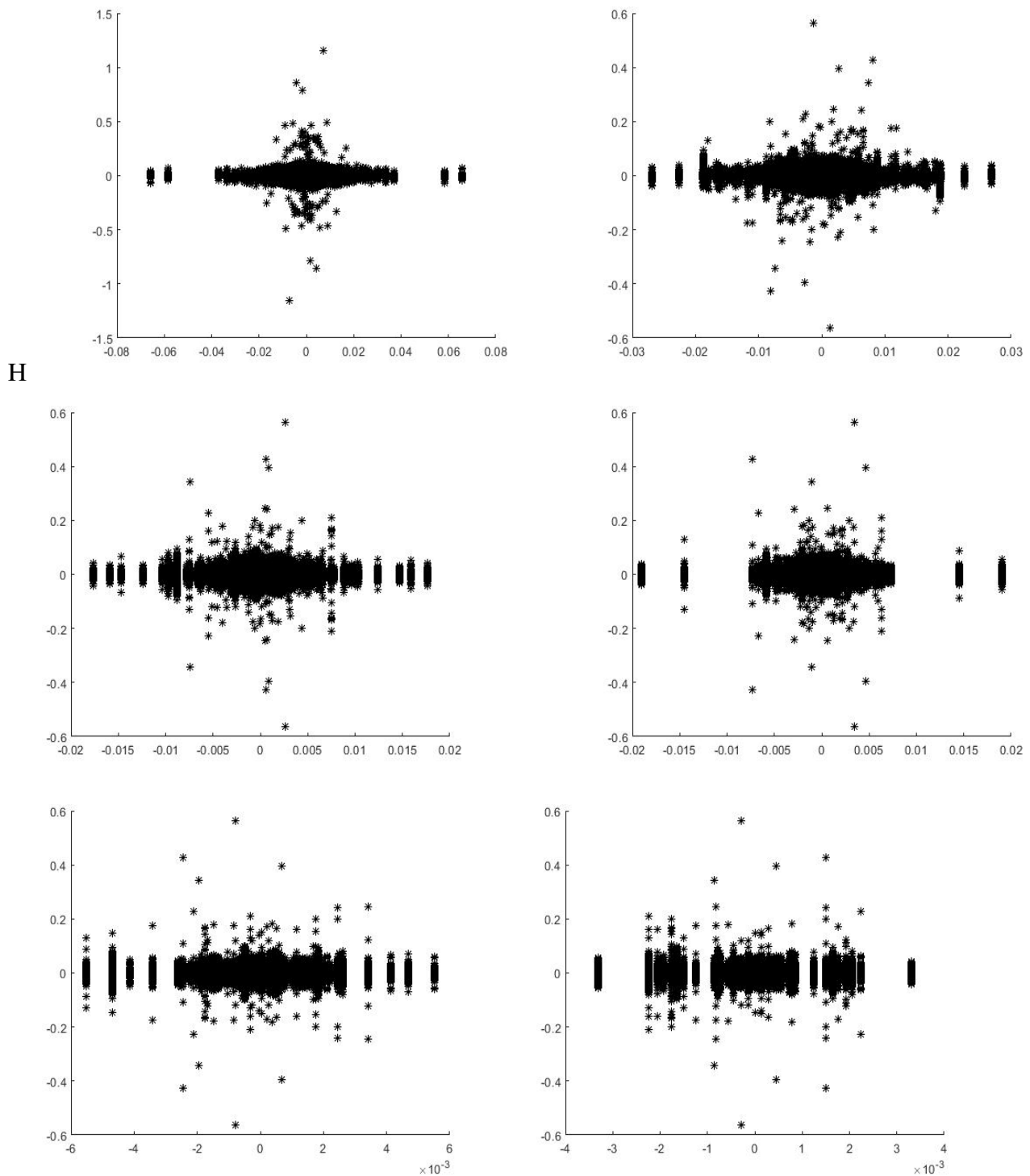
Figure 3. Wavelet excess market return (horizontal axis) versus wavelet excess return of the action (vertical axis) at levels j=1,2,3,4,5,6.

Table 9. The mean Beta for each sector.

Sectors	2-scale law Beta						Wavelet Beta					
	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Energy	0.3418	0.3072	-0.0987	1.1997	0.0696	-0.5224	0.0493	-0.1127	0.0614	-0.0422	0.0863	-0.3252
Materials	0.6043	0.2231	-0.1048	1.0446	0.0677	-0.2219	-0.0017	-0.0470	-0.0443	0.0598	0.0605	0.2630
Cap. Go.	1.1992	0.1656	1.1567	1.5932	0.3301	-0.1080	0.0166	0.0060	-0.0630	0.0294	0.1691	0.1994
CoPr SvC	1.5417	-2.1253	0.1435	1.2702	-0.7362	-0.2967	-0.0901	-0.1720	0.0649	0.0355	0.1547	0.2088
Transport	-0.1329	0.2562	0.4128	3.8766	-0.2069	-0.1983	-0.0156	-0.0507	-0.0680	0.0801	0.1510	0.1145
Consumer	-0.1850	-0.2499	0.1393	2.2427	-0.2051	-0.2490	0.0173	-0.0750	0.0480	0.0492	0.0784	0.0874
Media Ent.	2.4493	0.8724	0.7616	1.0121	-1.2325	0.2165	-0.0277	0.0226	0.0032	-0.0106	0.0500	-0.0384
Retailing	0.4745	-0.1264	0.1326	1.8277	0.4039	-0.0635	-0.0671	-0.0909	-0.0601	0.1222	0.0564	0.2093
Foods	0.1943	-0.0608	0.7407	1.8958	0.0772	-0.2536	0.0150	-0.0778	0.0434	0.1819	-0.0106	-0.0292
Health	0.7572	0.6308	-0.0076	0.3244	0.1957	-0.1105	0.0294	-0.0735	-0.0099	0.0088	-0.0435	-0.0577
Banks	1.0496	0.2654	0.0821	1.2413	0.1573	-0.0457	0.0115	0.0148	-0.0459	0.1605	-0.0281	0.1804
Div. Fin.	-0.3950	-0.3979	-0.7848	1.3826	-0.2593	0.0161	-0.0139	-0.0598	-0.0457	0.0224	-0.0051	0.1886
Insurance	-0.1504	-0.6573	0.4598	1.0127	0.2895	-0.1567	-0.0823	-0.0187	0.0917	-0.0005	0.3518	0.1177
TelecoM	0.1933	0.0543	-0.5699	2.2973	-0.3199	-0.0646	-0.0692	-0.0443	0.1628	0.0799	0.4398	0.3179
Utilities	0.8338	0.7105	0.2389	0.5362	0.2071	-0.0711	0.0164	-0.0650	0.0423	0.0203	0.2059	0.2263
REITs	-1.2204	-0.6896	0.1092	2.2283	-0.0053	-0.0980	-0.0257	-0.0740	0.1256	0.0902	-0.0468	-0.1454
R Estate	-0.0307	0.2600	0.0474	2.2167	0.1841	-0.2578	-0.0061	-0.0906	-0.1052	0.0927	0.1371	0.2768

The study shows that such a market may be encouraging for investments on long time horizons reminiscent of some prudence that should be taken into account for short investments and small companies. The study may be a good example for both investors and researchers to understand other GCC markets that will be future extending studies of the present case.

Acknowledgments

Conflict of interest

The author declares here that no conflicts of interest in this paper.

References

Aktan B, Ozturk M, Rhaeim N, et al. (2009) Wavelet-Based Systematic Risk Estimation An Application on Istanbul Stock Exchange. *Int Res J Financ Econ* 23: 34–45.

Arfaoui S, Rezgui I, Mabrouk AB (2017) Wavelet Analysis On The Sphere, Spheroidal Wavelets, Degryuter, Degruyter, 2017, ISBN 978-3-11-048188-4.

Arfaoui S, Mabrouk AB, Cattani C (2020) New type of Gegenbauer-Hermite monogenic polynomials and associated Clifford wavelets. *J Math Imaging Vision* 62: 73–97.

Arfaoui S, Mabrouk AB, Cattani C (2020) New type of Gegenbauer-Jacobi-Hermite monogenic polynomials and associated continuous Clifford wavelet transform. *Acta Applicandea Math.* https://doi.org/10.1007/s10440-020-00322-0.

Aydogan K (1989) Portfolio Theory, Capital Market Board of Turkey Research Report: Ankara.

Banz RW (1981) The relationship between return and market value of common stock. *J Financ Econ* 9: 3–18.

Basu S (1977) The relationship between earnings' yield, market value and return for NYSE common stocks-further evidence. *J Financ* 32: 663–681.

Mabrouk AB, Mohamed MLB, Omrani K (2008) Numerical solutions for PDEs modeling binary alloy-solidification dynamics, In: Proceedings of 2007 International Symposium on Nonlinear Dynamics, *J Phys*, conference series 96 (2008) 012067.

Mabrouk AB, Kortass H, Ammou SB (2008) Wavelet Estimators for Long Memory in Stock Markets. *Int J Theor Appl Financ* 12: 297–317.

Mabrouk AB, Kahloul I, Hallara SE (2010) Wavelet-Based Prediction for Governance, Diversification and Value Creation Variables. *Int Res J Financ Econ* 60: 15–28.

Mabrouk AB, Abdallah NB, Hamrita ME (2011) A wavelet method coupled with quasi self similar stochastic processes for time series approximation. *Int J Wavelets Multiresolution Inf Process* 9: 685–711.

Mabrouk AB, Zaafrane O (2013) Wavelet Fuzzy Hybrid Model For Physico Financial Signals. *J Appl Stat* 40: 1453–1463.

Mabrouk AB, Rabbouch B, Saadaoui F (2015) A wavelet based methodology for predicting transmembrane segments, In: Poster Session, The International Conference of Engineering Sciences for Biology and Medecine, 1–3 May 2015, Monastir, Tunisie.

Black F, Jensen MC, Scholes M(1972) The capital asset pricing model: Some empirical tests, in Jensen MC (ed), *Studies in the theory of Capital,* New York: Praeger, 1–54.

Black F (1972) Capital Market Equilibrium with Restricted Borrowing. *J Bus* 45: 444–455.

Breeden D (1979) An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities. *J Financ Econ* 73: 265–296.

Brennan MJ (1973) Taxes, market valuation and corporate financial policy. *Natl tax J* 23: 417–427.

Chae J, Yang C (2008) Which idiosyncratic factors can explain the pricing errors from asset pricing models in the Korean stock market? *Asia-Pasific J Financ Stud* 37: 297–342.

Chan LK, Lakonishok J (1993) Are the reports of beta's death premature? *J Portf Manage* 19: 51–62.

Cifter A, Ozun A (2007) Multiscale systematic risk: An application on ISE 30, MPRA Paper 2484, University Library of Munich: Germany.

Cifter A, Ozun A (2008) A signal processing model for time series analysis: The effect of international F/X markets on domestic currencies using wavelet networks. *Int Rev Electr Eng* 3: 580–591.

Cohen K, Hawawin G, Mayer S, et al. (1986) *The Microstructure of Securities Markets,* Prentice-Hall: Sydney.

Conlon T, Crane M, Ruskin HJ (2008) Wavelet multiscale analysis for hedge funds: Scaling and strategies. *Phys A* 387: 5197–5204.

Daubechies I (1992) Ten Lectures on Wavelets, Society for Industrial and Applied Mathematics, Philadelphia.

Desmoulins-Lebeault F (2003) Distribution of Returns and the CAPM Empirical Problems. Post-Print halshs-00165099, HAL.

DiSario R, Saroglu H, McCarthy J, et al. (2008) Long memory in the volatility of an emerging equity market: The case of Turkey. *Int Mark Inst Money* 18: 305–312.

Fama EF (1970) Efficient capital markets: A review of theory and empirical work. *J Financ* 25: 383–417.

Fama E, French K (1992) The Cross-section of Expected Stock Returns. *J Financ* 47: 427–465.

Fama E, French K (1993) Common risk factors in returns on stocks and bonds. *J Financ Econ* 33: 3–56.

Fama E, French KR (1996) The CAPM is Wanted, Dead or Alive. *J Financ* 51: 1947–1958.

Fama E, French KR (2004) The Capital Asset Pricing Model: Theory and Evidence. *J Econo Perspect* 18: 25–46.

Fama E, French KR (2006) The Value Premium and the CAPM. *J Financ* 61: 2163–2185.

Fama E, MacBeth J (1973) Risk, return and equilibrium: Empirical tests. *J Polit Econ* 81: 607–636.

Fernandez V (2006) The CAPM and value at risk at different time-scales. *Int Rev Financ Anal* 15: 203–219.

Friend L, Landskroner Y, Losq E (1976) The demand for risky assets and uncertain inflation. *J Financ* 31: 1287–1297.

Galagedera DUA (2007) A review of capital asset pricing models. *Managerial Financ* 33: 821–832.

Gençay R, Selçuk F, Whitcher B (2002) *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics,* Academic Press, San Diego.

Gençay R, Whitcher B, Selçuk F (2003) Systematic Risk and Time Scales. *Quant Financ* 3: 108–116.

Gençay R, Whitcher B, Selçuk F (2005) Multiscale systematic risk. *J Inte Money Financ* 24: 55–70.

Gibbons MR (1982) Multivariate tests of financial models: A new approach. *J Financ Econ* 10: 3–27.

Gursoy CT, Rejepova G (2007) Test of capital asset pricing model in Turkey. *J Dogus Univ* 8: 47–58.

Handa P, Kothari SP, Wasley C (1989) The relation between the return interval and beta: Implications for size-effect. *J Financ Econ* 23: 79–100.

Handa P, Kothari SP, Wasley C (1993) Sensitivity of multivariate tests of the CAPM to the return measurement interval. *J Financ* 48: 1543–1551.

Ho YW, Strange R, Piesse J (2000) CAPM anomalies and the pricing of equity: Evidence from the Hong Kong market. *Appl Econ* 32: 1629–1636.

Hubbard BB (1998) *The world according to wavelets: The story of a mathematical technique in the making,* 2e, Ak Peters Ltd., MA.

Mahmoud IMM, Ben Mabrouk A, Hashim MHA (2016) Wavelet multifractal models for transmembrane proteins' series. *Int J Wavelets Multires Inf Process* 14: 36.

In F, Kim S (2006) The hedge ratio and the empirical relationship between the stock and futures markets: A new approach using wavelet analysis. *J Bus* 79: 799–820.

In F, Kim S (2007) A note on the relationship between Fama-French risk factors and innovations of ICAPM state variables. *Financ Res Lett* 4: 165–171.

In F, Kim S, Marisetty V, et al. (2008) Analysing the performance of managed funds using the wavelet multiscaling method. *Rev Quant Financ Accounting* 31: 55–70.

Karan MB, Karadagli E (2001) Risk return and market equilibrium in Istanbul stock exchange: The test of the capital asset pricing model. *J Econ Administrative Sci* 19: 165–177.

Kishor NK, Marfatia HA (2013) The time-varying response of foreign stock markets to US monetary policy surprises: Evidence from the Federal funds futures market. *J Int Financ Mark Inst Money* 24: 1–24.

Kothari S, Shanken J (1998) On defense of beta, J. Stern, and D. Chew, Jr. (Eds.), *The Revolution in Corporate Finance,* 3e, Blackwell Publishers Inc., 52–57.

Levhari D, Levy H (1977) The Capital Asset Pricing Model and the Investment Horizon. *Rev Econ Stat* 59: 92–104.

Lévy H (1978) Equilibrium in an imperfect market: a constraint on the number of securities in the portfolio. *Am Econ Rev* 68: 643–658.

Lintner J (1965a) Security Prices and Maximal Gaines from Diversification. *J Financ* 20: 587–615.

Lintner J (1965b) The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Rev Econ Stat* 47: 13–37.

Litzenberger RH, Ramaswamy K (1979) The effect of personal taxes and dividends on capital asset prices: Theory and empirical evidence. *J Financ Econ* 7: 163–195.

Magni CA (2007a) Project selection and equivalent CAPM-based investment criteria. *Appl Financ Econ Lett* 3: 165–168.

Magni CA (2007b) Project valuation and investment decisions: CAPM versus arbitrage. *Appl Financ Econ Lett* 3: 137–140.

Marfatia HA (2014) Impact of uncertainty on high frequency response of the US stock markets to the Fed's policy surprises. *Q Rev Econ Financ* 54: 382–392.

Marfatia HA (2015) Monetary policy's time-varying impact on the US bond markets: Role of financial stress and risks. *North Am J Econ Financ* 34: 103–123.

Marfatia HA (2017a) A fresh look at integration of risks in the international stock markets: A wavelet approach. *Rev Financ Econ* 34: 33–49.

Marfatia HA (2017b) Wavelet Linkages of Global Housing Markets and macroeconomy. Available at SSRN 3169424.

Marfatia HA (2020) Investors' Risk Perceptions in the US and Global Stock Market Integration. *Res Int Bus Financ* 52: 101169.

Markowitz H (1952) Portfolio Selection. *J Financ* 7: 77–91.

Merton RC (1973) An Intertermporal Capital Asset Pricing Model. *Econometrica: J Econometric Society* 41: 867–887.

Mossin I (1966) Equilibrium in a Capital Asset Market. *Econometrica: J Econometric Society* 34: 768–783.

Perold A (2004) The Capital Asset Pricing Model. *J Econ Perspect* 18: 3–24.

Percival DB, Walden AT (2000) *Wavelet methods for time series analysis,* Camridge University Press, NY.

Rhaiem R, Ammou SB, Mabrouk AB (2007a) Estimation of the systematic risk at different time scales: Application to French stock market. *Int J Appl Econ Financ* 1: 79–87.

Rhaiem R, Ammou SB, Mabrouk AB (2007b) Wavelet estimation of systematic risk at different time scales, Application to French stock markets. *Int J Appl Econ Financ* 1: 113–119.

Roll R (1977) A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory. *J Financ Econ* 4: 129–176.

Selcuk F (2005) Wavelets: A new analysis method (in Turkish). *Bilkent J* 3: 12–14.

Sharkasi A, Crane M, Ruskin HJ, et al.(2006) The reaction of stock markets to crashes and events: A comparison study between emerging and mature markets using wavelet transforms. *Phys A* 368: 511–521.

Sharpe WF (1964) Capital asset prices: A theory of market equilibrium under conditions of risk. *J Financ* 19: 425–442.

Sharpe WF (1970a) Computer-Assisted Economics. *J Financ Quant Anal,* 353–366.

Sharpe WF (1970b) Stock market price behavior. A discussion. *J Financ* 25: 418–420.

Sharpe WF (1970c) *Portfolio theory and capital markets,* McGraw-Hill College.

Soltani S (2002) On the use of the wavelet decomposition for time series prediction. *Neurocomput* 48: 267–277.

Soltani S, Modarres R, Eslamian SS (2007) The use of time series modeling for the determination of rainfall climates of Iran. *Int J Climatol* 27: 819–829.

Vasichek AA, McQuown JA (1972) Le modèle de marché efficace. Analyse financière, 15, 1973, traduit de "The effecient market model". *Financ Anal J.*

Xiong X, Zhang X, Zhang W, et al. (2005) Wavelet-based beta estimation of China stock market, In: Proceedings of 4th International Conference on Machine Learning and Cybernetic, Guangzhou. IEEE: 0-7803-9091-1.

Yamada H (2005) Wavelet-based beta estimation and Japanese industrial stock prices. *Appl Econ Lett* 12: 85–88.

Zemni M, Jallouli M, Mabrouk AB, et al. (2019a) Explicit Haar–Schauder multiwavelet filters and algorithms. Part II: Relative entropy-based estimation for optimal modeling of biomedical signals. *Int J Wavelets Multiresolution Inf Process* 17: 1950038.

Zemni M, Jallouli M, Mabrouk AB, et al. (2019b) ECG Signal Processing with Haar-Schauder Multiwavelet, In: Proceedings of the 9th International Conference on Information Systems and Technologies—Icist 2019.

AIMS Press