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Research article

Portfolio optimization from a Copulas–GJR–GARCH–EVT–CVAR model: Empirical evidence from ASEAN stock indexes

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Abstract: This study employs several methods to simulate and construct the portfolio from stock indexes of the six Association of Southeast Asian Nations (ASEAN) markets during the period from January 2001 to December 2017, namely, time–varying Copulas; Glosten, Jagannathan and Runkle (GJR); generalised autoregressive conditional heteroskedasticity (GARCH); extreme value theory (EVT); and conditional value at risk (CVaR). Our target is minimising the risk based on CVaR, then achieving the maximal expected return for investors. Our model also sheds further light on the role of the dependence structure among stock indexes by employing elliptical (student *t*) Copulas, which are incorporated for simulating the optimal portfolios. Our findings suggest that the investor should invest in the optimal portfolio, which lies in the efficiency curve. Hence, the optimal portfolio has similar time–varying characteristics across the dependence of Copulas, as well as confidence levels. The research implications can be employed practically by portfolio managers and individual investors who desire to invest in ASEAN equity markets. Therefore, our findings can draw investors' attention to constructing the portfolio with the dependence level via time–varying Copulas and minimise the risk represented by CVaR rather than traditional variance.

Keywords: GARCH models; GJR; EVT; Copulas models; CVaR; portfolio optimization

JEL Codes: C14, C30, G11, G17

1. Introduction

Beginning in 2016, the Vietnamese equity market has experienced miraculous growth, doubling from 520.7 to 1130.1 points. Therefore, the stock market in Vietnam has become much more dynamic, with greater potential, which has attracted more domestic and foreign investors. However, this raises a concern about the necessity of analysing external and internal factors influencing many groups of risks for maintaining the expected return for portfolios. Meanwhile, the Association of Southeast Asian Nations (ASEAN) region has different cultures and perspectives among 10 countries, which conveys normative institutionalism and cultural–finance decisions (Goodell, 2018). Thus, foreign investors face difficult financial situations when it comes to choosing how much of their assets to invest in each market, although they consider the ASEAN region to have immense potential as a market. In addition, as Baele and Inghelbrecht (2009) argued, the financial crisis in 2007–2008 has contributed to the dynamics of the level of integration, leading to higher volatility in stock indices among the ASEAN countries. Foreign investors need to know how strongly these stock markets are interconnected to make the appropriate financial decisions.

Errunza et al. (1992) indicated that there are three types of financial integration, namely, perfect integration, mild segmentation and perfect segmentation, based on macro–economic profile and business characteristics. Thus, the question that is raised is as follows: "*When and how do the capital markets show their integration in ASEAN, a region with a mixture of emerging and developing equity markets?*" Especially, indicating the weight of each stock markets has implications for maximising investors' returns and minimising their risk, and this represents our motivation in the present study. Therefore, this paper aims to investigate the dependence structure and optimal weight for investors choosing to invest in potential, dynamic and emerging markets like those in the ASEAN region.

In the past, many funds applied econometric models, such as Capital Asset Pricing Model (CAPM) (Sharpe, 1964), in the context of determining the relationship between the expected return and systematic risk. However, regarding unsystematic risks, Markowitz's (1952) theorem of portfolio management indicates that diversification can optimise the portfolio efficiency. The prominent theoretical concept is mainly based on the assumption that the expected return and standard deviation for these assets should be linearly dependent. However, in reality, financial assets have asymmetric distribution (and not the usual normal distribution) and do not follow the linearity rules. Based on datasets from six ASEAN economies (Indonesia, Vietnam, Malaysia, Singapore, the Philippines and Thailand), discussed in previous studies like those of Patton (2001, 2006), Wang et al. (2010) and Ling Deng et al. (2011), we work to construct the optimal portfolio for these indexes.

After the period of financial crisis in 2007–2008, many financial models were constructed for estimating and minimising risks, with different risk appetites from investors. Recently, multivariate Copulas have not only been applied in physics and biology as basic science but also in finance, econometrics, economics and risk management. Especially, Morgan (1996) introduced many models, including the expected shortfall and spectral risk measure models. However, there has been a shortage of research considering aspects of these models, for example, the lack of external shocks, contagion risk phenomenon and nonlinearity dependence. Furthermore, the Value–at–Risk (VaR) model, introduced by Morgan (1996), fails to estimate the risks in case of the worst losses happening under the conditional terms. In addition, this approach brings challenges for investors to make a decision due to moral hazard, which is related to behavioural finance. For instance, the financial crisis 2018 experienced that many financial institutions maximized their profits by originate as many loans as possible regardless of financial status of borrowers. In addition, the VaR model does not qualify as sub–additive, and it probably discourages portfolio diversification. In terms of linearity dependence, it requires consistency for the dataset; therefore, there are some techniques to make financial data smooth, which changes data distribution. The current financial dataset also exhibits high dependency over a period of crisis with high volatility or in a downtrend market. Therefore, employing the estimated single–variate distribution normally causes limitations when it comes to constructing the appropriate models. To be more precise, the correlation (normally denoted as ρ) usually suffices for the linear dependency, but it provides biased results, which is the consequence of single–sided observation. Therefore, Xisong Jin et al. (2018), Nasir et al. (2019), Huynh et al. (2018) and Luu Duc Huynh (2019) proposed that using Copulas will generate consistent, unbiased and robust results for financial models, addressing the weaknesses delineated above.

In the context of an uncertain global economy, after the event of Brexit (Britain's declaration that it would exit the European Community), equity markets in the European region and around the world have been adversely influenced (Burdekin et al., 2018). Consequently, many investors have raised the question of whether there is interdependency among these markets. From another perspective, Vietnam and ASEAN countries are aiming to negotiate many treaties and agreements, such as the Comprehensive and Progressive Agreement for Trans–Pacific Partnership (CPTPP) and ASEAN Economic Community (AEC), which contributes to a mechanism of foreseen risks (Desierto, 2017) because these countries will have more connections with each other. ASEAN economies are considered to represent an emerging region that can generate an attractive return for investors, with the existence of some magical economies referred to as the "*Dragons of Asia*", such as Singapore (Mercereau, 2005). Another side that should be considered is that one country—Thailand—faces many crises from the real estate bubble development and 2007 financial crisis (Rigg and Salamanca, 2009). Interestingly, there is also a country with unique institutional politics and ongoing classification in the emerging market list, namely, Vietnam. For the reasons mentioned above, we include the geographical scope of the ASEAN region, with a methodology involving Copulas integrated from other quantitative techniques to construct an optimal portfolio for investors. Our contributions are as follows: (i) integrating the Glosten, Jagannathan and Runkle (GJR), generalised autoregressive conditional heteroskedasticity (GARCH); extreme value theory (EVT); and conditional VaR (CVaR) models for estimation; (ii) employing Monte Carlo simulation with n = 5500 trials for estimation with a main test and back testing for robustness; and (iii) expanding the research data for 16 years for covering sudden shocks, such as the financial crisis of 2007–2008. It is noted that our GJR–GARCH–Copulas–EVT–CVaR models for estimation outperform in terms of quantifying risk appetite of investors by estimating the time–varying dependence structure. In addition, we rely on simulation for generating highly reliable frequency data to construct the optimal portfolio, which indicates the exact weights of indices for investing.

The remainder of this paper is structured as follows: Section 2 reviews the existing literature. The data are systematically summarised in Section 3, while our methodologies are presented in Section 4. The empirical results are analysed and discussed in Section 5, and finally, our conclusions are provided in Section 6.

2. Literature review

Our main concept is based on Markowitz's (1952) portfolio construction, in which there is a utility curve (U) that illustrates the trade–off behaviour between the expected return and risk. Then, based on the conceptual theory from Markowitz (1952), it is finally concluded that the optimal portfolio is the tangential point between the utility curve and trade–off curve from the expected return and standard deviation. The main advantage of Markowitz's (1952) model is that it helps construct an appropriate portfolio with an acceptable level of risk from investors. Thus, the final purpose is maximising the expected return but minimising the risk. However, this model does not generate good results with a large number of assets. It also has external shocks without measurement.

Another concept for our literature review relates to CVaR. Uryasev and Rockafellar (2000, 2002) introduced this expanded version of VaR with the following formula:

$$\mathrm{CVaR} = \mathrm{E}\left[\xi \middle| \xi \geq \zeta_{\alpha}(\xi)\right] = \mathrm{E}[\xi|\xi \geq \mathrm{VaR}]\ ^{1} \tag{1}$$

VaR has limitations, such as being sub–additive and monotonic, and CVaR addresses them. Therefore, CVaR easily supports maximisation regarding asset allocation for one or many portfolios. Similarly, Uryasev and Rockafellar (2000) assumed that the ratio of return follows the normal distribution, and CVaR is calculated by[2]

$$\mathrm{CVaR} = \mu + k_1(\alpha)\sigma, \tag{2}$$

$$k_1(\alpha) = \frac{1}{\sqrt{2\pi}e^{\left(\mathrm{er\,f}^{-1}(2\alpha-1)\right)^2}(1-\alpha)}, \tag{3}$$

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt. \tag{4}$$

Based on our model, CVaR has a value that is approximate to the tail of the distribution. Thus, the α–tail distribution[2] can be used for calculating CVaR without tail dependence.

The distribution function $\Psi_{\alpha}(\xi)$ is obtained by scaling the function with a threshold value ξ for defining CVaR in the interval [0,1], representing its probability. Then, the value of the mean is associated with the decision; this is also known as the mean excess loss in Bassi et al. (1998).

The other theory considered in our paper is EVT. This is also used for analysing the extreme worst case in financial markets because of risks from other markets in many papers (Danielsson and de Vries, 2000; Gilli and Këllezi, 2006; Jondeau and Rockinger, 2003; McNeil and Frey, 2000; Onour, 2010). EVT provides model parameters for capturing the distribution with a heavy tail in the market. This is because EVT can simulate extreme losses to forecast the possibility of the maximum loss happening in the tail distribution for risk. VaR and CVaR are mainly used as inputs for generating EVT in Gilli and Këllezi's (2006) and Embrechts, Frey and McNeil's (2005) methodologies. These studies have mentioned extreme values, including Gumbel, Fréchet or Weibull distribution, which are generalised as the generalised extreme value (GEV) and generalised Pareto distribution (GPD) to evaluate how effective the model is for data simulation under restriction.

[1] Here, α is the quantile distribution for the random variable, while ξ is the threshold excess generalised Pareto distribution (GPD) parameter estimated using the maximum likelihood method.

[2] We consider a confidence level $\alpha \in (0,1)$, which applies for VaR and CVaR at $\alpha = 0.95$ or $\alpha = 0.99$, following Rockafellar and Uryasev (2002). The α–tail distribution is considered as the loss function associated with the loss value over the threshold; "erf" denotes the "error function".

EVT is not only used for quantitative estimation in the VaR and CVaR; rather, it can also incorporate the time series to predict the next 1–day VaR and CVaR. McNeil and Frey (2000) suggested a financial model forecast CVaR based on EVT via the new GARCH to generalise the volatility on the market.

The empirical evidence using the model is wide and practical. For example, Geman and Kharoubi (2008) proposed the function of Copulas to provide a better description regarding the dependence structure for oil future contracts and equity indexes. By analysing the diversification effects of crude oil futures, it is highly recommended to add liquidity assets to the portfolios. Thus, the maturity effect in asset selection in the portfolio is evaluated for benefits under–diversification. The results from Geman and Kharoubi's (2008) study showed that the oil future contracts have a negative correlation among these assets, which are exempt from the movement of prices. Interestingly, this can contribute to giving investors more choices for minimising their risks through future contracts in the context of no–trend stock markets.

In the US stock market, Fernandez (2008a) criticised the applications of Copulas for capturing the dependence structure. This author emphasised the disadvantage of using Copulas, which skips the characteristics of the normal distribution from a group of random variables and represents an isolated dependence structure of single–variate variables. Therefore, Fernandez (2008a) developed his model by generating a generalised approach with Monte Carlo simulations. Hence, the applications of Copulas incorporating Monte Carlo are quite useful and significant statistics. In addition, from the metal inter–trading portfolios in London for 1935–2005, the lack of cross–sectional dependence in variables results in the extreme bias when it comes to calculating the optimal hedging and efficiency in this behaviour. In this model, Fernandez (2008b) selected the optimal ratio of hedging products for portfolios by adding optional quantities with Copulas for calculating co–movement. This study also clarified the features of multivariate GARCH, which bring more benefits to hedging for basic components in portfolios.

Most recent empirical studies have concentrated on equity or stock markets by employing the Gaussian Copulas and t–Copulas. A few studies have focussed on analysing multivariate influence. Especially, Patton (2001) utilises Copulas with time–series data to determine their conditional relationship with the exchange rates' variance under different regimes. Moreover, this study simulated a marginal distribution between each pair of exchange rates independently. Employing the changing formulas by Fisher effect for a pair of EUR/USD, JPY/USD, Patton (2001) concluded that the relationship is statistically significant based on time–varying and historical data. Therefore, its main finding was suggesting the cooperation of time–varying Copulas and dynamic models, which are better than dummy variables. This study contributes new empirical evidence on the dependence structure of a pair of exchange rates. Finally, it is found that the depreciations of the German mark and Japanese yen depend strongly on the movements by the USD.

In China, some researchers incorporate the GARCH model and Copulas for measuring the moments and financial risks for financial assets. Wu et al. (2006) demonstrated how the GARCH–Copulas model describes randomly multivariate variables. Interestingly, this study was constructed to forecast the ability to invest in the Chinese stock markets. In contrast, Wang and Chen (2010) employed the EVT model in combination with Copulas for calculating VaR for portfolios including EUR/CNY and JPY/CNY by Monte Carlo simulation. In comparison with the previous studies, the Wu (2006)'s work only simulates the historical data and measures the variance, whereas Wang and Chen (2010) evaluate the tail distribution by Pareto (GPD) more accurately. These studies initially solve the puzzle regarding the interconnectedness among different markets. However, their perspectives still consider risk as historical and parametric rather than non–parametric and asymmetric distributions.

Deng and Yang (2011) employed a rich set of quantitative techniques, namely, CVAR, EVT, GARCH and Copulas, for calculating the risk threshold. Ultimately, they built a model with Monte Carlo simulation and mean CVaR for optimising their portfolios. However, the scope of this paper is mainly focussed on the Chinese stock markets. When it comes to a research comparison, Deng et al. (2011) used the samples using the simulation procedure without a back–testing process; therefore, we aim to address this limitation by employing the approach from Glyn A. Holton (2015) for further validation of the simulation process. In addition, we use t–Copulas rather than vanilla Copulas to capture the distribution extracted as residuals in the GJR–GARCH for our estimation to ensure the robustness of our model construction. Interestingly, the latest Huang and Hsu (2015) expanded the previous research by carrying out quantitative techniques for the G8 countries. Thus, this study also indicates the advantage of for using this method to enhance the portfolio performance under short–term rebalancing intervals. However, our study includes GJR, which is a new element, in GARCH, offering the same advantages as GARCH plus a leveraging effect. The main reason is that GJR–GARCH obtains empirical findings for reversed effects. Especially, if a negative shock occurs in the $(t - 1)$ period, this effect will have a stronger influence on the variance at time t than a positive shock will. To summarise, this asymmetric concern is called a *leverage effect*, which refers to an increase in risk coming from the increased negative leverage shock.

In their study, Singvejsakul et al. (2019) measured the dependence structure and portfolio optimization among ASEAN countries using the Markov–switching model (MS model), D–vine trees and Markowitz portfolio selection model. However, our approach allows us to see the dependence structure varying over time, as well as using bootstrapping methods for creating a more reliable dataset before constructing a portfolio. Recently, Liu et al. (2019) employed the (Vector Error Correction) VEC Copulas GJR–GARCH–skewed–t model for estimating the dependence structure. Interestingly, this study motivated us to consider the skewed–t as an important element for the Copulas' student t distribution to capture the return distribution more precisely. The current studies still acknowledge the previous fundamental concepts from Rodriguez (2007) for the mixed Copulas analysis method, Baur (2013) for the quantile regression approach and Choe et al. (2012) for the Copulas function to avoid errors that may be generated by static correlation (such as CCC).

Similar research has been conducted on the topics of Copulas–EVT models, portfolio optimization, VaR and CVaR and GJR–GARCH in previous studies, making important contributions to the field (e.g. Cerrato et al., 2015). Such research has also indicated the dynamic and asymmetric dependence structure from beta, co–skewness, co–kurtosis, t–Copulas and generalised autoregressive score (GAS) models from two different markets, such as the United States and United Kingdom. Interestingly, Emamverdi (2018) employed the GARCH–EVT–Copulas for measuring the VaR in terms of the interaction between two markets, including the Tehran Stock Exchange Price Index (TEPIX) and Composite (National Association of Securities Dealers Automated Quotation System) NASDAQ Index. Based on this study, we found that there is still a gap, as the different levels of dependence are underestimated in the relevant period. Therefore, our work attempts to fill the gap by using the time–varying Copulas, which allows the exact level of dependence to be captured in each period. To ensure the theoretical framework is solid, we refer to Embrechts et al.'s (2003) study, which demonstrates the distributional bounds for functions of dependent risks. In addition, the study of

Sabino et al. (2017) encourages our work by putting forward the finding that the mixed Copulas–CVaR approach generates lower values of the minimum average loss than the average performance of the Gaussian Copulas–CVaR. Hence, our work attempts to test this effect by using another Copulas family, specifically, time–varying Copulas. Recently, one appealing study has come from Sampid et al. (2018). These authors introduced the new methodology with Bayesian Markov switching from the traditional method of forecasting financial asset returns, and their outputs are in line with the previous research by Sabino et al. (2017). This means that the VaR model with Bayesian Markov–switching integration outperforms the other models. Finally, the work of Zhang et al. (2015) has contributed to the existing literature by extracting the residuals of logarithmic returns to estimate the distribution function from the GJR–GARCH, EVT and Copulas model. This study also asserted the value of VaR from portfolio construction with equal weights for each individual stock. Therefore, in the scope of this study, we would like to calculate the optimal weights for minimising the risk factor in the portfolio, which means targeting portfolio optimization. Using the described model, there has been limited research for the ASEAN region, which includes some emerging stock markets like Malaysia, Thailand and Vietnam. Therefore, this is also a research opportunity for us to replicate this model by employing GJR and time–varying Copulas in another region (ASEAN countries) with stock indexes to construct the optimal portfolios. Thus, the present research will address the limitations of the previous studies via three main points: (i) the study is based on CVaR instead of VaR estimation, (ii) it employs time–varying Copulas and (iii) it is applied to a new geographical region application.

3. Data

The authors collected the weekly data returns of six ASEAN stock market indexes from January 2001 to December 2017, which were directly gathered from Thomson Reuters[3]. These indices are the VN Index, SET Index, FTSE Straits Times Index, PSEi Index, FTSE Bursa Malaysia KLCI Index and Jakarta SE Composite Index, representing the stock markets of six ASEAN countries, namely, Vietnam, Thailand, Singapore, the Philippines, Malaysia and Indonesia, respectively. The main reason for us choosing their equity markets is to ensure the availability of data (for generating balanced data). Furthermore, other countries, such as Cambodia, Laos, Brunei and Myanmar, have insufficient data because these countries opened their economies quite late in general, especially in the equity market. Finally, our selected ASEAN economies cover up to 70% market capitalisation in this region, which suggests that our findings relate to the total ASEAN community. As an additional point, we employed weekly data instead of daily data because these capital markets have different holiday times and establishment time period. To ensure balanced data and avoid missing data, we employ weekly data, following Click and Plummer (2005), Chung and Liu (1994) and DeFusco et al. (1996). After data collection, we continue data processing for calculating the index return via logarithm return, following the study of Miller and Scholes (1972)[4]. The main reason for choosing this method to calculate index returns for all the ASEAN economies is the continuous compounding return calculation. We then perform the statistical description for our variables to understand the variables' characteristics and distribution.

[3] Our data were collected from Thomson Reuter Eikon software online.

[4] Here, $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, where P_t is index at time *t*.

Table 1. Summary of Statistics description for weekly data return.

Variables	Mean	Std. Dev.	Min	Max	Median	Skewness	Kurtosis
Indonesia	0.0032	0.0302	−0.2330	0.1159	0.0046	−0.9487	9.0844
Philippine	0.0020	0.0282	−0.2015	0.1619	0.0025	−0.4197	8.3207
Singapore	0.0006	0.0254	−0.1647	0.1532	0.0017	−0.3851	9.5812
Thailand	0.0021	0.0282	−0.2666	0.1075	0.0041	−1.4041	14.0140
Vietnam	0.0020	0.0406	−0.2028	0.1570	0.0014	−0.3606	7.3113
Malaysia	0.0011	0.0179	−0.1145	0.0665	0.0018	−0.7839	8.0092

Note: There are total 887 observations for weekly return from 6 ASEAN countries over the period from 2001 to 2017. There are some main criteria in the table such as mean, standard deviation, minimum value, maximum value, media, skewness and kurtosis to understand the characteristics of data as well as data distribution. (Source: The authors with data from Thomson Reuters).

The summary of the statistical description shows that the average index return in the Philippines reaches the highest point (0.0032) compared with the other ASEAN economies. In fact, the Philippines' equity index return increased 15 times from 2001 to 2017, which explains how the Philippines stock market brings more benefit for investors. In contrast, the Singapore market has the lowest average of return (0.0006). Interestingly, Singapore has held the leading position regarding market capitalisation for many years. Hence, the Singaporean financial structure is currently stable, as evident in the low volatility (standard deviation of only 0.0254). In the group of ASEAN economies, Vietnam has the highest volatility (0.0406). Although the Vietnamese and Singaporean markets were established at the same time, the Singaporean market capitalisation is approximately 9.4 times higher than that of Vietnam (2016), and Singapore is classified as an advanced stock market while Vietnam is only a frontier market. Therefore, the Vietnamese stock market experiences unitability in growth with this feature.

We consider the risk factors from the third and fourth moments (skewness and kurtosis) for the dataset. The skewness and kurtosis of the Thai market are higher, at −1.4041 and 14.0140, respectively. These figures demonstrate that the probability of losses (with lower tail) is highest, including the left–skewed feature and fat tail. The practical evidence from historical trading can be taken as an example. Thailand has experienced many shocks, such as the monetary crisis in 1997. Another shock occurred on 19 December 2006, which represents "the darkest time" in the Thai stock exchange. The Thai market dipped after financial crashing event. On this day, the SET index decreased by 108.41 points, which is equivalent to 816 billion baht (23 billion USD; Impavido et al., 2005). The main reason is that this country was suffering from political risk, and Thai government seems to be ineffective in controlling the volatility of the depreciation of the baht. This country attempted to hold the value of Thai Baht (currency); however, the authority had to abandon their control, which strongly and adversely influence stock market.

In conclusion, the statistical description in the table above shows the individual features in each market in the ASEAN region. This quantitative technique can reasonably explain many events in ASEAN economies. Furthermore, this summary indicates that our dataset is not normally distributed because all the figures in kurtosis are higher than the benchmark[5], which is understood as the fat tail.

[5] Platen (2006) also indicated the benchmark for Gaussian distribution in terms of kurtosis (3.0).

From these results, we can choose the most appropriate quantitative methods for correcting the characteristics of the dataset.

4. Methodology

4.1. Copulas approach methodology

The Copulas approach is a useful technique for capturing the dependence structure between assets, and it is widely applied for measuring the non–linear correlation level of two variables[6]. The Copulas concept is mainly based on the study by Sklar (1959). To elucidate this theory, let $F_1(x_1),\ldots, F_n(x_n)$ be given marginal distribution functions while $x_1,\ldots, x_n$[7] are random and continuous variables. Therefore, H is known as the joint distribution of $(x_1,\ldots, x_n)$. Then, there will exist a unique Copulas C such that

$$\exists C: [0,1]^d \to [0,1], \tag{5}$$

where

$$H(x) = C\{F_1(x_1), \ldots, F_d(x_d)\} \quad x \in \mathbb{R}^d \tag{6}$$

According to Sklar's theorem, for an n–dimensional random vector, the inter–dependence structure between the random variables is defined by a Copulas, which is decomposed into a series of marginal distributions. The combination of these parts is called the multivariate density.

The essential component of the Copulas approach is tail dependence, including lower tail dependence and higher tail dependence (λ_l and λ_u). The parameter estimates the Copulas' tail dependence using the following formula:

$$\tau_l \coloneqq \lim_{t\downarrow 0} E\big(Y \le G^-(t) | X \le F^-(t)\big),^8 \tag{7}$$

which can also be represented by Copulas C as

$$\tau_l = \lim_{t\downarrow 0} E\left(\frac{C(t,t)}{t}\right) \tag{8}$$

and

$$\tau_u \coloneqq \lim_{t\uparrow 1} E\big(Y > G^-(t) | X > F^-(t)\big). \tag{9}$$

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[6] Please refer to Nelsen (1998) and Joe (1997) for the formal treatment and definition of copula theory.

[7] This signifies a random vector $X = (X_1, X_2,\ldots, X_n)$ with continuous marginals. Then, let us assume a function F for random vector X with marginal distribution F_i, $X_i \sim F_i$, i from 1 to n. Then, the distribution function C (in Equation 6) with uniform marginals on [0, 1], referring to equation 5, is called a "copula".

[8] It is denoted that F^- is the generalised inverse of F. Then, assume that G is a one–dimensional distribution function with $F_i \le G_i$. In addition, for any $F \in f(F_1 \ldots F_n)$, there exists an element $G \in f(G_1 \ldots G_n)$ with $F_i \le G_i$. Therefore, G^- is the generalised inverse of G.

This can also be represented by the Copulas C via the following formula:

$$\tau_u = \lim_{t \uparrow 1} \mathrm{E}\left(\frac{1-C(t,t)}{1-t}\right), \tag{10}$$

where, if $\tau_l = 0$, then X and Y have lower tail independence. When applied in a Taylor series for $C(t,t)$, this can be written as

$$\tau_l \coloneqq \lim_{t \downarrow 0} \mathrm{E}\left(\frac{\partial}{\partial u} C(u,v) + \frac{\partial}{\partial v} C(u,v)\Big|_{u=v=t}\right), \tag{11}$$

$$\tau_u \coloneqq 2 - \lim_{t \uparrow 1} \mathrm{E}\left(\frac{\partial}{\partial u} C(u,v) + \frac{\partial}{\partial v} C(u,v)\Big|_{u=v=t}\right). \tag{12}$$

These formulas are illustrated for the parameters called τ for the upper and lower bounds under Copulas estimation.

4.2. GJR–GARCH estimation

Following the traditional GARCH approach based on Bollerslev (1986), any developed models are mainly used for estimating the risk under volatility movement. This approach includes the studies of French et al. (1987), Christie (1982) and Black (1976), which indicated that bad news or negative effects, rather than positive effects, have an adverse influence. From these perspectives, some asymmetric models are continuously built from the GARCH basis, including E–GARCH by Nelson (1991), T–GARCH by Zakoian (1994), NA–GARCH from Higgins and Bera (1992) and AV–GARCH from Taylor (1986). Then, Awartani and Corradi (2005) found that asymmetric GARCH generates more efficiency than the traditional GARCH does; especially, Monfared and Enke (2014) proved that GJR–GARCH, validated by Glosten et al. (1993), is one of the most appropriate asymmetric GARCH approaches for forecasting. Because Monfared and Enke (2014) demonstrated that GJR–GARCH could estimate the asymmetric nature of investor response to stock returns, this model could be considered as a better way to solve the portfolio optimization as a maximum likelihood estimator compared with EGARCH or the other models. At a glance, the GJR–GARCH model shows the relationship between the expected return and volatility of residuals by the expected return on the equity market.

The GJR–GARCH(p,o,q) by Brownlees et al. (2011) and Laurent et al. (2012) can be illustrated as follows:

$$r_t = \mu_t + e_t, \tag{13}$$

$$e_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(v_{i,t}, \lambda_{i,t}),[9] \tag{14}$$

[9] Normally, in traditional GARCH, $\varepsilon_t \sim N(0,1)$ represents the mean and variance in the white noise process. There are two parameters, $v_{i,t}$ and $\lambda_{i,t}$, which stand for the degree of freedom and skewness, respectively. The two parameters depend on the lagged dependent variable in the nonlinear shape; therefore, we suggest that employing t–copulas for time series is more effective for evaluating the residual.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{k=1}^{o} \gamma_k \varepsilon_{t-k}^2 I_{[e_{t-k}<0]} + \sum_{j=1}^{q} \beta_j \sigma_{t-j.}^2 \quad [10] \tag{15}$$

Here, ω, α, γ and β are the coefficients for the constant, stochastic factor, indicator function and previous lagged variance, respectively. Furthermore, μ_t is any equation representing the conditional mean and meeting the requirement of the indicator function as follows:

$$I_{[e_{t-k}<0]} = \begin{cases} 1 & e_{t-k} < 0 \\ 0 & e_{t-k} \geq 0 \end{cases}. \quad [10] \tag{16}$$

This means that the extreme value at condition $(t-1)$ has a stronger influence on variance at time t than a positive impact does. Therefore, integrating with an asymmetric shape, this phenomenon is called a "*leverage effect*". To make it shorter, GJR–GARCH(1,1,1) can be written as

$$\sigma_t^2 = \omega + \alpha_1 e_{t-1}^2 + \gamma_1 \epsilon_{t-1}^2 I_{[e_{t-1}<0]} + \beta_1 \sigma_{t-1.}^2 \tag{17}$$

These parameters have values of $\omega > 0$, $\alpha_1 \geq 0$, $\alpha_1 + \gamma_1 \geq 0$ and $\beta_1 \geq 0$. Under normal conditions, the GJR–GARCH model is covariance only if these parameters meet the condition as $\alpha_1 + \frac{1}{2}\gamma_1 + \beta_1 < 1$[11].

Employing the higher co–moments for time series, Hansen (1994) and Jondeau and Rockinger (2003) assumed that the GJR–GARCH model has a skew–t distribution. This validates that skew–t is appropriate with time variance in different models. The assumption is as follows:

$$\varepsilon_t \sim \text{Skewed} - \text{T}\left(\varepsilon_t \middle| v_{i,t}, \lambda_{i,t}\right) \quad [12] \tag{18}$$

Here, constant and independent variance, $v_{i,t}$ as the estimated degree of freedom (DoF), $\lambda_{i,t}$ as the parameter for skewness. These parameters depend on the lagged explanatory values in non–linear and time–series data. For all the reasons mentioned above, we conclude that GJR–GARCH simulation is well fitted to time–varying Copulas for estimating the residual.

4.3. *Time–varying Copulas*

Based on the traditional Copulas, Engle (2002) improved the model with a time–varying correlation matrix under dynamic conditional correlation (DCC) with the following return (R_t):

$$R_t = \text{diag}\{Q_t\}^{-1} Q_t \text{diag}\{Q_t\}^{-1}. \tag{19}$$

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[10] According to previous research (Brownlees et al., 2011; Laurent et al. (2012), the GJR models generally perform better than the GARCH specifications do. Thus, including a *leverage effect* leads to enhanced forecasting performance.

[11] The main reason for choosing this restriction is avoiding the exponential series in our estimation.

[12] As discussed in footnote 7, the residual (for a mean process) of the GARCH model should be considered as the normal distribution $\varepsilon_t \sim N(0,1)$. However, Hansen (1994) and Jondeau and Rockinger (2003) indicated that the GJR–GARCH should generate the residual, which is under skew–t. Therefore, the residual of innovated GARCH is distributed by the skewness of two factors, namely, the parameters of DoF and skewness, given the residual generated by GJR–GARCH. Then, the GJR–GARCH also includes the indicator function.

To incorporate the definite multivariate GARCH models with correlation parameterisation, Ding and Engle (2001) introduced the first–order form for the M–ARCH family (ARCH in mean values), with Q_t for positive semi–definite values as matrix versions of these estimators. Here, α and β are extracted from the GARCH(1,1) model for satisfying $\alpha + \beta < 1$. In addition, Ω is the unconditional correlation matrix of the time series Y_t, which represents residuals of the model (normally known as ε):

$$\mathcal{Q}_t = \Omega(1-\alpha-\beta) + \alpha Y_{t-1}Y_{t-1}' + \beta\mathcal{Q}_{t-1}. \tag{20}$$

Next, we can calculate the result for each dependence level using the parameter τ at time t with the covariates matrix ρ:

$$\tau_t = \frac{2}{\pi}\arcsin(\rho_t), \quad \theta_t = \gamma(\tau_t), \tag{21}$$

where $Y_{it} = \Phi^{-1}(u_{t,i})$ and $Y_t = (Y_{1t}, Y_{2t})'$ for the residual components in the extracted GARCH(1,1) model. From this perspective, Zhang (2014) developed the matrix of dependency by time–varying Copulas:

$$\mathcal{Q}_t = (1-\alpha-\beta)S + \alpha(\varsigma_{t-1}\varsigma_{t-1}') + \beta\mathcal{Q}_{t-1}, \tag{22}$$

in which S is the covariance matrix of ς_t, $\alpha, \beta > 0$, $\alpha + \beta < 1$. Then, we can calculate the covariates matrix with

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}q_{j,j,t}}}, \tag{23}$$

where $q_{i,j,t}$ is a component of matrix Q_t and i and j are the components in the covariance matrix of condition R_t.

Zhang (2014) showed the incorporation between GJR–GARCH and time–varying Copulas with the dependence structure DCC by time. Interestingly, this model demonstrates the efficiency of capturing the flexible parameters for specific individuals among all pairs of variables. Thus, we can employ this model for further investigation to manage risk and allocate assets.

4.4. EVT and CVaR

EVT is mainly based on the distribution function F, which lies in the tail distribution by F only, not for all distribution. Therefore, Wang et al. (2010) indicated that EVT is the most appropriate approach for capturing the tail–structure model, and the selections u and N are the prominent factors for generating EVT. However, the application to EVT sometimes does not match for a random variable r_t. It is hard to determine whether r_t is independent. Hence, the incorporation of GARCH and EVT[13] is more appropriate because GARCH is good for historical data and new standardisation

[13] This is because the conditional distribution of GARCH models is inferred to have a heavier tail than that of a normal distribution, which provides better approximation to actual financial time series. Therefore, we incorporate GARCH–EVT, according to Wang et al.'s (2010) suggestion, to estimate the historical return, volatility and threshold for innovation distribution in equation 24.

by two distributions—GPD in the upper tail and lower tail. Therefore, Coles (2001) proposed the EVT and GARCH model as follows:

$$F(z)=\begin{cases}\frac{N_{u^L}}{N}\left\{1+\xi^L\frac{u^L-z}{\beta^L}\right\}^{-\frac{1}{\xi^L}} & , \quad z<u^L\\ \phi(z) & , \quad u^L<z<u^R,\\ 1-\frac{N_{u^R}}{N}\left\{1+\xi^R\frac{u^R-z}{\beta^R}\right\}^{-\frac{1}{\xi^R}} & , \quad z>u^R\end{cases} \tag{24}$$

where $\beta \;\; and \; \xi$ are parameters for scale and individual distribution, respectively. These parameters are used for choosing the appropriate threshold u^L, u^R, representing the lower and upper tails.

For VaR and CVaR, X is a random variable that includes the loss and parameter $0<\alpha<1$; Fercoq and Richtárik (2015) suggested the measurement for VaR_α by X as

$$\mathrm{VaR}_\alpha(\mathrm{X}) \coloneqq \min\{c: P(X\le c)\ge\alpha\}. \tag{25}$$

Then, $VaR_\alpha(X)$ is the minimum loss, which is not more than the determined value with probability α. The lowest loss in case of $(1-\alpha)\times 100\%$ is $VaR_\alpha(X)$, and this is also the highest loss in terms of $\alpha\times 100\%$. However, the CVaR for the random variable is more visual than VaR is, as proposed in Fercoq and Richtárik's (2015) study:

$$\mathrm{CVaR}_\alpha(\mathrm{X}) \coloneqq \mathrm{E}[\mathrm{X}|\mathrm{X}\ge \mathrm{VaR}_\alpha(\mathrm{X})]. \tag{26}$$

From another perspective, Rockafellar and Uryasev (2002) indicated that CVaR can estimate from the tail dependence by probability α and random variable X. In this case, $F_X(z)$ is the cumulative function of X with $F_X(z)=P(X\le z)$; the general distribution of tail α is calculated by

$$F_X^\alpha(z) \coloneqq \begin{cases}0, & z<\mathrm{VaR}_\alpha(\mathrm{X})\\ \frac{F_X(z)-\alpha}{1-\alpha}, & z\ge \mathrm{VaR}_\alpha(\mathrm{X})\end{cases}. \tag{27}$$

Simultaneously, X^α is a random variable with a cumulative distribution function and structure, F_X^α; then, CVaR is calculated as

$$\mathrm{CVaR}_\alpha(\mathrm{X}) \coloneqq \mathrm{E}[\mathrm{X}^\alpha]=\int_{-\infty}^{\infty} z f_X^\alpha(z)dz = \int_{-\infty}^{\mathrm{VaR}_\alpha(\mathrm{X})} z f_X^\alpha(z)dz+\int_{\mathrm{VaR}_\alpha(\mathrm{X})}^{\infty} z f_X^\alpha(z)dz. \tag{28}$$

Next, we combine the general distribution of tail α from $F_X^\alpha(z)$ in Equation 27 with Equation 28, and we have

$$\mathrm{CVaR}_\alpha(\mathrm{X}) = \int_{\mathrm{VaR}_\alpha(\mathrm{X})}^{\infty} z\frac{f_X(z)}{1-\alpha}dz. \tag{29}$$

According to the formula of Acerbi and Tasche (2002), a new variable means β with $\beta=F_X(z)$, incorporated with z in the following formula:

$$\frac{d}{dz}\beta = f_X(z) \Leftrightarrow f_X(z)dz = d\beta. \tag{30}$$

In addition, three parameters—β, z and $VaR_\alpha(X)$—have a relationship, in which $z = VaR_\beta(X)$. Then, we adjust the limit for integral ($F_X\left(VaR_\beta(X)\right) = \alpha$ and $F_X(\infty) = 1$), obtaining the value

$$\mathrm{CVaR}_\alpha(\mathrm{X}) = \frac{1}{1-\alpha}\int_\alpha^1 \mathrm{VaR}_\beta(\mathrm{X})\, d\beta. \tag{31}$$

To optimise the portfolio by CVaR, we set that S is a set of variables "x" and hypothesis $X(x, r)$ is the convex point of x. Therefore, we have the minimum of $CVaR_\alpha$, which represents the risk factor, values the minimum loss in $\phi_\alpha(x, c)$ by all points $(x, c) \in S \times R$. As a finite convex function for all points of x, we always find an x in S where the value of loss is the minimum, as in Equation 32. In Rockafellar and Uryasev's (2000, 2002) studies, this is represented with the following formula:

$$\min_{x\in S} \mathrm{CVaR}_\alpha(\mathrm{X}) = \min_{(x,c)\in S\times R} \phi_\alpha(x, c). \tag{32}$$

For the decision to invest x for minimising CVaR under a portfolio at confidence level $1-\alpha$ to adjust the significance level of α, we can write how to calculate it on right side ($^+$) as follows:

$$\min_{x\in S} \mathrm{CVaR}_\alpha(\mathrm{X}) = \min_{(x,c)\in S\times R}\left(c + \frac{1}{1-\alpha}E[(X(x,r) - c)^+]\right). \tag{33}$$

Therefore, we determine x as the weight so that the risk is minimal.

5. Results

5.1. Copulas and the multivariate normal approach for calculating VaR and CVaR

When it comes to EVT and time–varying Copulas, we employ an algorithm for extracting residuals in the dependence structure estimation for Monte Carlo simulation. From our return dataset at date *t*, we use Monte Carlo simulation with n = 5500 trials to forecast the return in date (*t* + 1), which is in accordance with the t–Copula dependence structure. Here, there are 5000 simulated observations for measuring VaR estimators and 500 for back testing the VaR and CVaR models. The equal weight for each stock index is determined by $w = [1/6, 1/6, 1/6, 1/6, 1/6, 1/6]$, and we can interpret the portfolio returns. Following this, we aim to determine the value, which stands in the 250^{th} and 50^{th} ranks for VaR 95% and VaR 99%, as well as CVaR 95% and CVaR 99%.

From the multivariate normal approach perspective, portfolios are known as the weight for each stock index P $(w_1, w_2, \ldots, w_n)$. Due to our calculation from the logarithm for the stock price, we assume that the stock return is under normal distribution. We propose the following equations for calculating the return for the portfolio r_p, mean return $\bar{r}_p$ and variance for portfolio σ_p^2, respectively:

$$r_p = \sum_{i=1}^{N} w_i * r_i, \tag{36}$$

$$\bar{r}_p = \sum_{i=1}^{N} w_i * \bar{r}_i\,, \tag{37}$$

$$\sigma_p^2 = W' * V * W, \tag{38}$$

where w_i is the weight of stock index i and r_i is the return for stock index i.

Because $r_i \sim N(\mu, \sigma_i^2)$ with $i = {}^{1}/_{N}$, and therefore, $r_p \sim N(\bar{r}_p, \sigma_p^2)$, the portfolio's VaR is calculated by the following formula:

$$\mathrm{VaR}\,(1\ \mathrm{day}, (1-\alpha) * 100\%) = \bar{\mathrm{r}}_{\mathrm{p}} + \mathrm{N}^{-1}(\alpha)\sigma_{\mathrm{p}}, \tag{39}$$

where $\bar{r}_p$ is the mean return of the portfolio, σ_p is the standard deviation of the portfolio and V is the correlation matrix among these kinds of assets. It is similar to the EVT approach, but we also use Monte Carlo simulation with n = 5500 trials[14] for estimating the expected value input to calculate VaR 95% and 99% and CVaR 95% and 99%. Table 2 illustrates our results from the calculation for 1–day VaR 95% and 99% and 1–day CVaR 95% and 99%.

Table 2. VaR and CVaR estimation from Copulas and Multivariate Normal.

Calculation	95%	99%
Copulas VaR (1 day)	1.46%	2.66%
Copulas CVaR (1 day)	2.23%	3.71%
Multivariate Normal VaR (1 day)	3.01%	4.37%
Multivariate Normal CVaR (1 day)	3.81%	5.10%

The results from Table 2 show that, based on the index stock's return simulation by normal distribution, the values of VaR for the six ASEAN countries are estimated as 3.01% and 4.37% at significance levels of 5% and 1%, respectively. Meanwhile, with EVT–Copulas, the results are 1.46% and 2.66%.

5.2. Back testing for VaR[15]

In the empirical literature, Gneiting (2011) and Ziegel (2016) indicated the lack of elicitability of CVaR. A study by Barone (2016) also showed that one of the difficulties stemming from the elicitability of the CVaR approach is computing the tail expectation of losses, which are mainly based on an unknown shape of distribution, empirically resulting in historical data. After calculating VaR and CVaR based on the EVT–Copulas and multivariate normal approaches, we apply back

[14] Lerche and Mudford (2005) also showed that the trials (over 5500) will generate the cumulative probability more accurately. Hence, we decided to use 5500 trials (with 5000 trials for the training model and 500 for the back–testing process).

[15] We employed the weekly historical data (887 observations) for estimating the parameters for GJR–GARCH and t–copulas for each pair of markets. Then, we simulated the Monte Carlo by generating the return for the next following 5500 days. (These are just simulated data and completely separate from the observation). Next, we divided our simulated data into two parts—the first part (n = 5000) was for training and building up model and the rest (n = 500) was for back testing (10% of the simulated data). Especially, we employed n = 5000 for calculating VaR 95% and VaR 99% (well matched to the 50th and 250th observation in n = 5000). After that, we used these VaR results to check the consistency and fit in the back–test model with n = 500. If they passed, we chose these values of VaR. We have weekly data from historical observation. However, for greater precision, we changed the weekly VaR to daily VaR.

testing for reassessing whether our measurements are appropriate. We choose the 501st to 5500th observations from the Monte Carlo methodology simulation above. To perform this test, we set the fixed estimated intervals and carry out continuously random movements for the observations. Then, we comparison our CVaR between the EVT–Copulas and multivariate normal approaches with the new simulated values. Thus, back testing is a typical approach for simulating a model or strategy on historical data to gauge its inherent accuracy and effectiveness. Chuang et al. (2014) indicated that using back testing for CVaR is one of the methods for considering the level effects and asymmetry in volatility, which will be our input for portfolio construction.

The CVaR model is considered appropriate by Basel II (the second of the Basel Accords, issued by the Basel Committee on Banking Supervision) when the number of over–threshold values after back testing lies in the allowed maximum range. Especially, EVT–copula CVaR (95%) generates a value that is 23 times over the threshold. Meanwhile, the allowed maximum value for these confidence intervals for EVT–Copulas CVaR (95%) ranges from 16 to 35. This can be interpreted as signifying that the EVT–Copulas model witnesses the appropriateness of CVAR. A similar explanation can be given for the other CVaR value.

Table 3. The results of back–testing Value–at–Risk[16].

	Acceptable threshold	Estimated number	Mean Absolute Error (MAE)	Mean Square Error (MSE)
EVT–Copulas CVaR (95%)	[16,35]*	23	0.0176	0.000431
EVT–Copulas CVaR (99%)	[1,9]*	3	0.0294	0.000995
Multivariate Normal (95%)	[16,35]	30	0.0305	0.0013
Multivariate Normal (99%)	[1,9]	7	0.0434	0.0022

Note: *Value–At–Risk, Glyn A.Holton[17].

After careful analysis from Table 3, we recognise that the estimated numbers fall into the acceptable intervals. However, the EVT–Copulas model generates fewer failure values than the multivariate normal approach does. In addition, the mean absolute error (MAE) and mean square error (MSE) extracted by EVT–Copulas are lower than those from the multivariate normal approach. This means that the EVT–Copulas is better than multivariate normal estimation is. Therefore, the marginal distribution does not persist in accordance with the normal distribution. Then, the results represented by EVT–Copulas are appropriate for our further quantitative approaches in portfolio simulation.

[16] We employed the methodology from Glyn A. Holton. In fact, the VaR and CVasR calculated by n = 5000 trials, after that we used this value for back–testing with the threshold acceptance proposed by Glyn A. Holton to count how many values which lie out the number of acceptance. Especially, Glyn A. Holton proposed the value for both cases including acceptance or rejection. In addition, he also introduce how many number of VaR which is acceptable instead of rejectable.

[17] Proportion of Failures coverage test non–rejection intervals $[x_1, x_2]$ for various values of q and $\alpha + 1$. The value–at–risk measure is rejected at the 0.05 significance level if the number of exceedances X is less than x_1 or greater than x_2.

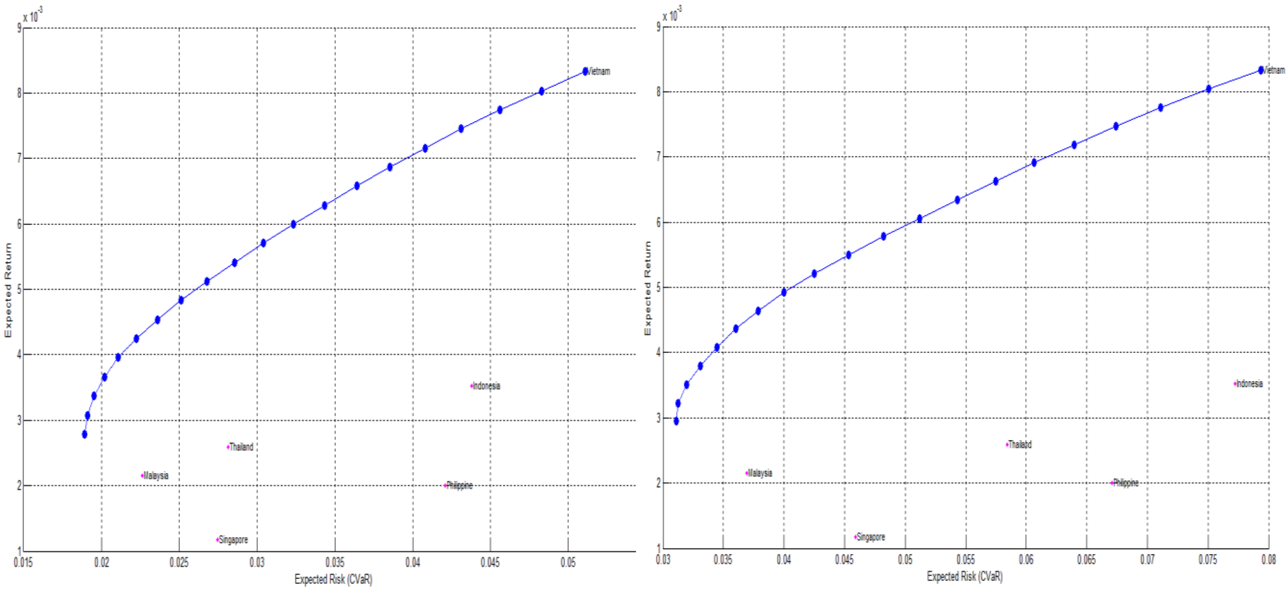
In fact, the back–testing process performed here aims to ensure that our historical data are appropriate for generating the simulation data needed to construct the model. We do not need to perform testing for independence of violations via an independence test and/or conditional coverage test by Christoffersen (1998), because this scholar also pointed out that the risk metric is likely to be overly confident, while GARCH with a t–student distribution forecast is overly cautious. Therefore, we incorporate the risk metric and GJR–GARCH with t–Copulas for correcting this phenomenon in our model.

6. Findings and implications

6.1. Optimal portfolio by t–Copulas and CVaR after GJR–GARCH–EVT processing

Based on the simulated return from six countries in the ASEAN region by the dependence structure with t–Copula, we use the optimal function from CVaR for estimating the efficiency frontier for the portfolio at significance levels of 5% and 1%. As discussed in Section 5.2, the back testing from CVaR, which is built using the EVT–Copulas model, is better. We extract the sample from 20 portfolios at each significance level for representative results for our EVT–Copulas–CVaR approach.

From the results in Table 4 and Figure 1, based on historical returns, if the investors choose to invest with a diversification strategy, they are potentially advised to invest in the six ASEAN stock exchanges, those of Indonesia, the Philippines, Singapore, Thailand, Vietnam and Malaysia, with weights of 10%, 0%, 0%, 17%, 31% and 42%, respectively. The suggested portfolio at CVaR 5% has a risk value of −2.22%, and its expected return is 0.42%. For comparison, with 100% invested in the Malaysian market, Malaysia's return is 0.22% and the risk value under CVaR 5% is −2.26%. Therefore, our simulated portfolio generates more return and less risk compared with a single market investment. According to the EVT–Copulas methodological simulation, the Vietnamese market totally lies in the efficiency frontier, which means that Vietnamese market is very potential to invest. This can be interpreted as showing that international risk–favour investors accept a higher risk at CVaR 5% (equivalent to −5.11%), which brings a higher return, at 0.83%. Our estimation provides many choices for investors with different risk appetites, ranging from risk averse to risk favourable, for investing in ASEAN stock markets. In terms of CVaR 1% for optimal portfolio construction. It means that investors are encouraged to include the Vietnamese and Malaysian indexes in their portfolios. However, most of our simulated portfolios do not have data on the Philippines index. The result is similar to the evaluation for CVaR 5%. Ultimately, when investors intend to invest in ASEAN countries, especially Indonesia, the Philippines, Singapore, Thailand, Vietnam and Malaysia, based on their risk appetite (CVaR accepted by investors), they can use our simulated results to consider their weight for generating the maximum expected return (E_P).



(a) (b)

Figure 1. Efficiency frontier by simulated portfolios by EVT–Copulas–CVaR at significance level 5% and 1%. (a): CVaR (5%); (b): CVaR(1%).

6.2. For investors and policymakers

Our model is mainly used for validating the appropriateness of Copulas–GJR–GARCH–EVT–CVaR in terms of measuring the CVaR for constructing optimal portfolios after passing the qualification of back testing. In addition, our research points to the superiority Copulas–GJR–GARCH–EVT–CVaR compared to the multivariate normal approach in choosing CVaR after evaluating three important criteria, namely, the over–threshold value, MAE and MSE. Therefore, our research paper has addressed the following questions:

(i) Is the Copulas–GJR–GARCH–EVT–CVaR model validated for use?

(ii) What percentage of their investments do investors put in each ASEAN market?

Especially, this paper also suggests that the investors should employ the Copulas–GARCH–EVT–CVaR model rather than multivariate normal model for managing their risks.

Table 4. Optimal weighted for our simulated portfolios by EVT–Copulas–CVaR at significance level 5% and 1%.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20
CVaR (5%)																				
Indonesia	0.00	0.01	0.04	0.05	0.07	0.10	0.10	0.11	0.11	0.14	0.16	0.17	0.19	0.20	0.21	0.19	0.18	0.12	0.06	0.00
Philippine	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Singapore	0.20	0.14	0.09	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Thailand	0.17	0.20	0.21	0.22	0.21	0.17	0.14	0.14	0.14	0.13	0.14	0.13	0.12	0.09	0.07	0.04	0.00	0.00	0.00	0.00
Vietnam	0.12	0.16	0.19	0.22	0.26	0.31	0.35	0.40	0.45	0.49	0.53	0.57	0.62	0.67	0.71	0.77	0.82	0.88	0.94	1.00
Malaysia	0.51	0.49	0.47	0.46	0.46	0.42	0.40	0.35	0.30	0.24	0.17	0.13	0.08	0.04	0.00	0.00	0.00	0.00	0.00	0.00
CVaR (1%)																				
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20
Indonesia	0.00	0.00	0.00	0.00	0.02	0.03	0.06	0.08	0.10	0.11	0.12	0.15	0.16	0.17	0.19	0.20	0.18	0.12	0.06	0.00
Philippine	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Singapore	0.25	0.20	0.12	0.02	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Thailand	0.02	0.05	0.07	0.09	0.11	0.09	0.08	0.07	0.07	0.09	0.11	0.09	0.10	0.12	0.09	0.03	0.00	0.00	0.00	0.00
Vietnam	0.17	0.20	0.23	0.26	0.31	0.35	0.38	0.43	0.47	0.51	0.55	0.59	0.64	0.68	0.72	0.77	0.82	0.88	0.94	1.00
Malaysia	0.56	0.56	0.58	0.62	0.53	0.50	0.48	0.42	0.37	0.30	0.22	0.16	0.10	0.03	0.00	0.00	0.00	0.00	0.00	0.00

Note: P = Portfolio simulated from our data from Portfolio 1 to Portfolio 20.

For policymakers, this paper contributes new empirical evidence regarding financial risk from a CVaR perspective for each country in ASEAN economies. This also provides a stress threshold for them to analyse how much the equity market is adversely incurred under the crisis. In addition, this also shows the dependence structure for policymakers to understand the market direction. Thus, the government can intervene in the markets when they find necessary cases.

Our limitations are the scope of research. We mainly focus on the ASEAN economies without many variables to validate more variables in the models. We suggest employing more markets, such as Europe, the G7 or all emerging markets to be tested. In addition, this paper only uses EVT–t–Copulas–CVaR for building the model. In fact, there are many time–varying Copulas that should be employed, such as t–DCC and DVine, for calculating the dependence structure and CVaR. Then, we suggest replacing CVaR by many modern techniques, such as expectiles VaR (EVaR) or mean–varying CVaR. One of our suggestions for further research is testing for different types of assets, such as cryptocurrencies, corporate bonds or exchange rates in different countries and regions to validate the efficiency of EVT–Copulas–CVaR–GJR–GARCH models.

Finally, our research can make a positive contribution to practice. The current study is not only research paper but also a practical code for commercial banks, investment banks, securities companies or mutual funds. They can replicate the results by adding their microscopic assets, then construct the optimal portfolio for risk management. Moreover, policymakers can use the findings to manage the Government Treasury with T–bills, T–bonds or other exchange currencies. The authors can then replicate the code and use it for further applications. Hence, in the future, it could be an avenue for portfolio optimization for cryptocurrency as the study of Burggraf (2019) to be replicated. This perspective could draw scholars and investors' attention due to the growing return of cryptocurrency market.

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Conflict of interest

All authors declare no conflict of interest.

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