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Research article

Integrating science and mathematics at secondary school level: Key elements of practice

Kevin Manunure[1,]*, Nicholas Zezekwa[2], Gladys Sunzuma[2] and Christopher Mutseekwa[3]

[1] Department of Education Studies, Hong Kong Baptist University, Kowloon Tong, Hong Kong
[2] Department of Science and Mathematics Education, Bindura University of Science Education, Bindura, Zimbabwe
[3] Department of Education, Rwanda Basic Education Board, Kigali, Rwanda

* **Correspondence:** Email: manunurek@gmail.com; Tel: +852 84821044.

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Abstract: Integrated STEM education is critical for developing 21st-century skills, interest, engagement, and outcomes. However, in-service teachers often struggle with implementation due to inadequate training and a lack of clear, practical models for blending the disciplines. In this study, we investigated the key factors that can enhance the effective integration of mathematics and science in a real secondary classroom setting, employing a qualitative case study approach that drew on data from classroom observations, student and teacher interviews, student artifacts, and teacher reflections. A science and mathematics teacher collaborated to teach the concept of density in two lesson study cycles, integrating mathematical modeling and science inquiry learning approaches. Our findings revealed four dynamically interdependent factors critical to successful integration: Teacher knowledge, such as Pedagogical Content Knowledge (PCK), integration strategies, and confidence; student autonomy; time allocation; and contingency management. For instance, fostering student autonomy during lessons often requires significant time allocation to accomplish the set objectives and it pauses challenging contingency strategies. How these factors interact and influence the nature of science and mathematics integration was represented in the proposed Dynamic Interdependence Framework (DIF). The framework positioned the teacher as the central agent whose negotiation of this web determines the nature of the integrated lesson, which in turn reinforces their knowledge through a feedback loop of reflective practice. We conclude the study with implications for research, teacher training, and classroom practice in diverse educational contexts, highlighting the need for professional development focused on collaborative planning.

Keywords: integrated STEM, science and mathematics integration, lesson study, teacher knowledge, student autonomy, time allocation, contingency management

1. Introduction

Integrated Science, Technology, Engineering, and Mathematics (STEM) education involves teaching two or more STEM disciplines through shared practices in authentic contexts to deepen student learning [1]. In this study, we focus on integrating science and mathematics, two of the four STEM domains, rather than pursuing full STEM integration. Research on combining these two disciplines provides a strong foundation for broader STEM education [2]. A targeted approach is particularly practical in examination-driven, large-class settings [3], such as those in Zimbabwean schools, where curriculum rigidity and logistical constraints such as exam-centric curricula and large class sizes often hinder more comprehensive integration efforts.

Integrated STEM education is critical for fostering 21st-century skills such as communication, teamwork and collaboration [4,5], addressing workforce demands [6], and improving student engagement and achievement [7]. However, many in-service teachers struggle to implement integrated curricula due to inadequate training and unclear guidelines on effective practices [8]. While research highlights the benefits of integrated STEM education, ambiguity persists regarding the critical aspects of practice and how they mediate in real classroom settings to enable seamless science-mathematics integration [9–11]. We address these gaps by examining the key factors that influence science-mathematics integration during collaborative lesson planning and teaching. Through two Lesson Study cycles, paired science and mathematics teachers co-planned and taught the concept of density in a natural classroom context. Lesson Study is a collaborative, iterative professional development approach that focuses on teachers collaborating with their peers to plan, observe, and reflect on their teaching practices [12,13]. We leverage this approach to create an authentic context for observing the dynamic factors of integration in practice. With our findings, we aim to equip teachers, teacher trainers, and researchers with an evidence-based model, the Dynamic Interdependence Framework (DIF), for cross-disciplinary STEM teaching and training.

2. Theoretical background

Inquiry-based learning is widely recognized as effective in science education, promoting skills such as hypothesis generation, data collection, and evidence-based reasoning [14,15], while problem-solving and mathematical modeling are central to mathematics education, fostering creative thinking and real-world application skills [16,17]. The two learning approaches are closely similar in that they both begin with the problematization of a real-life situation, followed by a process to look for the answers using different inquiry processes [18,19]. The similarity is also evident in the Inquiry-Based Modeling Pedagogical Cycle framework [20]. The framework reveals that elements of mathematical modeling (authentic real problem, domain of inquiry, mathematical model, model results, insight conjecture, and action validation) and inquiry-based learning (engagement, exploration, explanation, elaboration, and evaluation) have a strong synergy and can be used as a bridge between the epistemological and pedagogical approaches in the mathematics and science classrooms. This synergy has also been empirically demonstrated by the researchers in [19,21], affirming the efficacy of connecting the STEM disciplines, mathematics and science. However, a critical analysis revealed that, while demonstrating efficacy, these studies overlooked the practical mediation of this synergy. They tend to present an idealized view of integration without sufficiently addressing the role of teacher expertise in navigating the aforementioned challenges or the significant time constraints that shape implementation in real classrooms. This creates a conceptual gap between pedagogical theory and classroom practice. Consequently, the lack of documentation of the nuanced aspects of practice that facilitate successful integration of the two domains leaves a gap this study seeks to address.

Integrating Inquiry-Based Learning (IBL) and Mathematical Modeling (MM) presents distinct challenges that extend beyond their individual implementation difficulties. IBL in science faces hurdles such as teachers' preparedness for open-ended exploration and aligning inquiry with standardized assessments. Similarly, MM in mathematics is often hindered by students' struggles with abstraction and teachers' insufficient modeling expertise [22]. In an integrated context, these challenges are compounded. A fundamental tension arises from their differing epistemologies and processes: IBL's emergent, iterative nature often clashes with MM's need for structured mathematization and validation [20]. This can create epistemological conflicts for students, such as when empirical data from an IBL activity contradicts the idealized assumptions of a mathematical model [21]. Furthermore, both approaches are inherently time-intensive, and without careful orchestration, lessons can become unbalanced, favoring one discipline at the expense of the other. Consequently, the idealized synergy between IBL and MM may heavily be mediated by practical factors. Successful integration hinges on teacher expertise, particularly their Pedagogical Content Knowledge (PCK) to navigate these epistemological shifts, and the strategic allocation of time to accommodate emergent discoveries without sacrificing learning objectives. This underscores the critical need to move beyond theoretical synergy and empirically investigate how these factors dynamically interact to enable or hinder integration in authentic classroom settings.

3. Science and mathematics integration

Integrated mathematics and science teaching and learning promotes the development of student motivation, engagement, problem-solving skills, criticality, and relevance of concepts studied [9]. The link between mathematics and science has long been posited. The two domains have similar fields of application and a mutual scientific approach towards problem-solving [23,24]. The subjects complement each other. For instance, science can offer students tangible instances of abstract mathematical concepts, while mathematics can empower students to gain a more profound comprehension of scientific principles by offering methods to quantify and elucidate relationships within science [25]. In this regard, commonalities serve to bridge and establish communication while differences serve to maintain the integrity of the individual domain and provide multiple perspectives [20]. Consequently, schools have been encouraged to teach the two subjects in an integrated way [26].

To move beyond a generic skills list, there is need to employ the Technological Pedagogical Content Knowledge (TPACK) framework [27]. TPACK is particularly valuable as it extends [28] foundational idea of Pedagogical Content Knowledge (PCK) by explicitly incorporating technology and, more critically for this context, framing the specialized knowledge required for interdisciplinary teaching. Effective science-mathematics integration requires teachers to synthesize Content

Knowledge (CK) from both disciplines with Pedagogical Knowledge (PK) for inquiry-modeling cycles and Technological Knowledge (TK) to leverage tools like simulations. The focus of integration, however, lies in Integrative TPACK [1]; the unique ability to represent concepts from one discipline using the pedagogical tools of another and to anticipate student struggles with epistemological shifts, such as moving from empirical evidence in IBL to abstract validation in MM. The TPACK lens directly addresses a gap in the literature by pinpointing the competencies needed to manage practical classroom factors. For instance, a teacher with a robust Integrative TPACK can dynamically reallocate time to balance disciplinary coverage without sacrificing inquiry depth [29] and can better anticipate and respond to contingent challenges like student confusion or resource limitations [30]. Thus, the TPACK framework directly supports our investigation by providing a structured way to analyze how teacher competencies interact with and mediate the key practical factors of integration.

Teachers require strong content knowledge in at least one STEM discipline and a working understanding of others to create meaningful interdisciplinary connections [31]. They use this knowledge to align learning objectives from different domains into a single curricular activity, articulating the overarching concepts that span these domains [32]. They must be able to design, implement, and adapt student-centered, inquiry-based, and project-based lesson plans that incorporate real-world problems and design challenges and evaluating student progress using tailored formative and summative assessments [33]. Moreover, collaboration with colleagues across disciplines and strong communication skills are critical for planning and delivering cohesive STEM lessons. Teachers should foster creativity, critical thinking, collaboration, and communication in students, modeling these skills [34]. Furthermore, teachers need to participate in ongoing professional development and reflective practice, and they need to develop positive attitudes, high self-efficacy, and enthusiasm for innovation and student engagement [35,36].

In an integrated science and mathematics lesson, students are exposed to a cohesive learning experience where concepts and skills from both disciplines within a single topic are made explicit [9,37,38]. The lessons typically center on real-world problems, hands-on activities, and encourage students to use mathematics as a tool to explore scientific phenomena [37,38]. Lessons often involve group work, discussion, and reflection, helping students see the value of integrating knowledge from both subjects [9].

Despite this understanding of required competencies, the literature presents a significant gap. Studies often enumerate the knowledge, and skills teachers need but fail to articulate how these factors, such as teacher knowledge, time constraints, student readiness, and unforeseen classroom events, dynamically interact and mediate each other in real classroom settings [9–11]. For instance, the following questions arise: How does a teacher's pedagogical knowledge directly influence their management of time during an inquiry-modeling lesson? How do student queries (contingency) reveal gaps in a teacher's Integrative TPACK? The mediation between these factors remains underexplored, leaving teachers with a list of "what’s" but little guidance on the "how’s" of navigating their interdependence. To develop a model for cross-disciplinary STEM training and teaching of science and mathematics, it is essential to identify the practices that support effective integration. Addressing this need, we investigate the following research question:

What are the key factors that enhance the integration of mathematics (particularly mathematical modeling) and science (through inquiry-based learning) in secondary classrooms?

4. Methodology

4.1. Case study

We employed a qualitative, instrumental, single case study design [39–41] for an in-depth exploration of the process of integrating mathematics and science. The case was defined as the collaborative journey of a teacher pair through two structured cycles of Lesson Study. While a classical case study often examines an existing phenomenon in its natural context, we utilized the case study design to investigate a process-in-action within its authentic setting (a typical Zimbabwean school). The design was instrumental because the specific case (this teacher pair's experience) was examined primarily to provide insight into the broader issue of cross-disciplinary integration [39]. Thus, we incorporated an intervention-based, two-phase structure (two lesson study cycles) to facilitate a focused examination of how teachers navigated integration over time. This approach aligned with the use of a case study to trace how a process unfolds and evolves in response to planned activities and reflection [41]. The single-case design was selected to enable the collection of rich, multi-faceted data (video and audio recordings, field notes, interviews, and artifacts) and a strong rationale for sample size and selection. Two teachers, a science teacher and a mathematics teacher, participated as the core case. They engaged in two cycles of lesson study, collaborating to plan, teach, and reflect on a research lesson focused on the concept of density for junior secondary students in Zimbabwe. We focused on the detailed process of collaborative integration of science and mathematics at a secondary school level rather than measuring comparative outcomes (see [41,42]). The data collection spanned one year, punctuated by intervals, enabling the natural capture of teacher progression and authentic reflection across the intervention phases.

4.2. Participants and school setting

A purposive and convenient sampling technique was used to recruit two in-service teachers: A science teacher and a mathematics teacher. The male science teacher had a Bachelor of Science Education Degree, a Diploma in Counselling, and 32 years of teaching experience. The female mathematics teacher had a Bachelor of Science Honors Degree in Statistics and Operations Research, a Diploma in Education, and 15 years of teaching experience. The teachers taught the same classes and were willing to collaborate to plan, teach, reflect, and amend their research lessons on density. The teachers selected two classes of Grade 8 (average age of 14 years old) learners that were taught by both teachers, ensuring that their joint attendance for lessons would not disrupt the daily schedules of either the teachers or the learners. It was the first time for the learners to formally learn the concept of density. Each class had 40 students, mixed gender and ability. The school was a publicly owned urban, day school, located in Harare, Zimbabwe.

4.3. Data collection procedure

We adapted a lesson study format to collect data. Lesson Study offers opportunities for the participating teachers to collaborate in lesson planning, delivering live lessons, and lesson reflection (see [43]). The study involved two cycles of the lesson study, using the same research lesson on density to facilitate a progressive conceptualization of the research lesson, refine integrating and teaching practices, and to enhance learning outcomes for teachers and students.

Data collection was divided into five phases, which were punctuated by gaps due to participant availability and institutional calendars. The potential impact of these gaps on the research validity are addressed in the Implications and future directions section. In the first phase, participating teachers and some researchers had three planning meetings, which were held in the science laboratory office. Each meeting was approximately 90 minutes long. The meetings were video and audio recorded. The first meeting was facilitated by the researcher, using the proposed skeletal research lesson to introduce teachers to the concept of STEM education, lesson study, and the scope of the study. In the second meeting, teachers discussed and added comments to the research lesson and contextualized the research lesson into their curriculum, school, and classroom context. In the third meeting, the teachers conducted a trial run of the research lesson amongst themselves and the researchers, in the laboratory office, to assess the practical set-up, conceptual development and to predict student behavior. At the end of the third meeting, the teachers allocated duties in preparation for live lessons. In the second and third meetings, the researchers took a participant observer role, providing valuable insights and perspectives to enhance the comprehension of the topic under discussion as needed. This initial data collection stage lasted one week, after which a scheduled two-week period was allocated. This interval served two critical functions: It enabled teachers to fulfil prior commitments and finalize their instructional topics, and provided essential time for them to familiarize themselves with the foundational concepts of cross-disciplinary collaboration, integrated teaching, and the Lesson Study process. Crucially, it enabled teachers to thoroughly understand the research lesson plan and gather all necessary materials, thereby ensuring the subsequent research could focus purely on the cognitive and pedagogical processes of integration rather than being hindered by logistical challenges such as material shortages or a lack of foundational understanding of the concepts by participants.

The second phase started after a two- week break, during which teachers internalized the worksheets, prepared lesson materials, and finished their current topics with their students. As detailed in the description of the lesson sequence below, teachers collaboratively implemented the research lesson by delivering six live instructional sessions within a selected class, maintaining the authentic setting of a regular classroom. Three lessons were delivered in the first lesson study cycle and were dominantly led by the science teacher while the mathematics teacher assumed an assisting role. The first lesson cycle lasted two weeks, then a two-month festive holiday break intervened. The second lesson cycle began with a meeting to reflect on the first three lessons and to prepare for the second lesson cycle, which also lasted two weeks. In the second lesson cycle, the mathematics teacher led the delivery of the additional three lessons with a different class. This time, the science teacher assumed an assisting role. All the lessons were held in the Junior Science Laboratory and were video and audio recorded. Each lesson was approximately 70 minutes long. During live lessons, the primary teacher gave most of the instructions, and the assisting teacher helped with lesson preparation, scaffolding the students during practical activities, and assisting in maintaining student discipline. The researchers observed all the lessons and took field notes.

The third phase involved collaborative discussion and reflection on the lessons, which was done soon after every lesson. Each meeting lasted around 20 minutes and was audio recorded. The teachers reflected on whether the integration of math and science concepts was explicit, aspects that were effectively done, and those that required improvements. The insights gained were used to inform and improve the conduction of succeeding lessons.

In the fourth phase, the researchers conducted four individual, face-to-face, semi-structured teacher interviews. Each teacher interview lasted approximately one hour. Two interviews were done one week after the first three lessons and the other two interviews were done in the second study cycle, one week after the sixth lesson. Additionally, seven student interviews from the first class were held one week after the third lesson, and six students from the second class were interviewed in the second lesson study cycle one week after the sixth lesson. The interviews were done in the science teacher's office housed in the junior science laboratory, and all the interviews were video and audio recorded.

In the fifth phase, two online follow-up telephone teacher interviews were held six months after the second interviews. In particular, the WhatsApp platform was used because it was cost-effective and convenient in the prevailing context, as the interviews were conducted during the school holidays. The interviews were held to shed more light on issues that emanated from the preliminary data analysis, providing essential context and depth to the final findings. The interviews were approximately 90 minutes per interview and were audio recorded. The nineteen interviews (two teachers and thirteen students) were conducted using the interview protocols that were designed by the researchers and evaluated by an external expert.

4.3.1. Description of lesson sequence

The lesson sequence followed the research lesson on density, which consisted of three worksheets (see [44]). The worksheets promoted hands-on activities and were conducted within the framework of scientific inquiry and mathematical modeling using the tragedy of the Titanic ship as the contextual problem.

In the first lesson, teachers followed the procedure of the first worksheet. Students worked in small groups to measure and record the mass of different sets of containers. For each set, the containers were of the same size and made of the same material. One container was filled with salt solution and the other with distilled water, and thus they had different masses. Students made and recorded predictions on whether each container would sink or float when placed in a bucket of water. After making predictions, students placed the containers in buckets of water and recorded and explained their observations. During the activities, teachers moved around to different groups, checking progress and giving objective guidance. Teachers were also seen discussing what they would have seen in different groups, and their discussions informed them of the interventions made. During feedback, teachers led the class to discuss how they can use the insight from the lessons to real-life situations. The overarching first learning objective was for learners to deduce that as the mass of the container increases, it increases the chances of the container to sink in water; formula (1).

The second lesson followed the same procedure as the first, with the only change being the exchange of container sets. Each new set consisted of containers of different sizes but with the same mass. The primary learning objective was for students to deduce that as the volume of a container increases, the likelihood of the container sinking in water decreases; formula (2).

In the third lesson, teachers facilitated class discussion to synthesize insights from Lessons 1 and 2, leading to the development of the density equation; formula (3). Students used this knowledge to explain how the Titanic sank and to explore how the concept of density can be applied to address real-life challenges. The lesson concluded with a written exercise designed to evaluate the effectiveness of the intervention and the student's understanding of the concept.

After each lesson, a reflection session was held, and the insights gained from these reflections informed the subsequent lessons. The research lesson was then implemented again with a second group of students.

$$mass \quad \propto \quad sinking \tag{1}$$

$$volume \quad \propto \quad 1/sinking \tag{2}$$

$$Density = \frac{mass}{volume} \tag{3}$$

5. Data analysis

For the data analysis, we employed an inductive thematic analysis approach (see [45]). The process began with familiarization, where interview transcripts, lesson videos, and student artifacts were thoroughly reviewed to gain an in-depth understanding of the dataset. Line-by-line open coding identified preliminary patterns, prioritizing participants' own words. For instance, the statement "This research was an- eye opener for me. I realized that there quite a lot of concepts that I didn't understand about density and also when it comes to linking it to mathematics" was coded as both 'teacher content gap' and 'integrating skills gap,' while the reflection "I knew I know the stuff (content) but I was not sure if I will be able to handle lessons with practical activities, so I decided to learn from the science teacher first " yielded codes of 'teacher content strong,' 'teacher confidence low,' and 'teacher need to learn.' Through an iterative process of code comparison and refinement, related codes were progressively clustered into meaningful sub-themes - for example, merging 'teacher content gap' and 'teacher content strong' formed the 'content knowledge' sub-theme. Axial coding then established connections between these sub-themes, such as linking 'content knowledge' with 'integrating skills' to develop the overarching theme of 'Teacher Knowledge.' Through this iterative and recursive process, four themes emerged: (1) Teacher Knowledge (encompassing pedagogical knowledge, content knowledge, and attitudes/confidence), (2) Student Autonomy (balancing guided instruction and unguided inquiry), (3) Time Allocation (instruction time dedicated for each phase of science inquiry and mathematical modeling), and (4) Contingency (adaptive responses to planned and unplanned lesson developments). The analysis maintained rigorous documentation of this conceptual development through audit trails, ensuring the emergent framework remained firmly grounded in the empirical data while capturing the complexity of integrated STEM teaching practices.

6. Results

We identified four aspects that can influence the integration of mathematics and science: Teacher knowledge, autonomy, time, and contingency.

6.1. Teacher knowledge

The teachers' attitudes can influence how the STEM lessons develop. During lesson planning, the science teacher was overconfident in his 32 years of experience, assuming that he gained enough content and skills to teach the topic of density. He focused only on ensuring smooth practical activity, while overlooking the need for conceptual reinforcement or proactive addressing of potential student misconceptions. This overconfidence in content and experiential knowledge led to unpreparedness for student queries and technical challenges during live lessons.

> "I thought the topic of density is easy. I have taught it many times and conducted numerous experiments, so I never anticipated challenges. My main concern during planning was ensuring smooth practical execution".

During lesson delivery, the science teacher posited that he realized that he lacked sufficient knowledge about some concepts related to density; for instance, the relationship of surface area and density. He believed that if he had all the relevant content knowledge about density at the start of the study, his teaching could have been much better.

> "This research was an eye-opener for me. I realized that there are quite a lot of concepts that I didn't understand about density and also when it comes to linking it to mathematics".

In the first lesson, the science teacher showed a lack of sufficient realistic knowledge of the problem context. The teacher was unaware of the events that led the ship to sink. During the engagement, the teacher claimed that the ship sank because it had carried an extra number of people than recommended.

> "I thought it was overloaded. If it was meant to carry 2000 people, and then it carries 2001 people, then it means the mass has increased. If mass increases, then it sinks".

During the third lesson, students were asked to link what they had learned to the Titanic ship and to everyday life. One learner responded by saying the ship was small, yet it carried too many people, hence it became overloaded, and it sank. 'The ship sank because it carried too many people than what was recommended'. This student's explanation closely mirrored the initial, scientifically inaccurate explanation offered by the science teacher during the lesson's introduction, which also attributed the sinking to overloading rather than a change in average density. These parallel raises important questions about the transmission of conceptual understanding. While it is possible this similarity indicates the detrimental effect of insufficient teacher content knowledge on student learning, the data from this study does not enable a definitive causal claim. Without interview data from the student to trace the genesis of their idea, alternative interpretations remain equally plausible. For instance, the student may have arrived at this common misconception independently, or the teacher's statement may have simply reinforced a pre-existing belief. Therefore, this instance primarily serves as a powerful illustration of a shared conceptual difficulty around the concept of density, held by the teacher and the student, which hindered a scientifically accurate understanding of the phenomenon. A shift in the science teacher's understanding of specific scientific terms became evident during the second phase of the study. He reflected on this development, stating, "I started doubting myself when the math teacher constantly refused to accept student explanations that included the words surface area and capacity. No wonder these terms are not in the density equation." This realization followed the first learning cycle, during which the teacher had utilized the terms "capacity" and "surface area" in his explanations of density. Subsequently, students also incorporated this terminology into their own conceptual explanations, illustrating how the language used by the teacher in the classroom possibly influence student discourse and concept development.

The mathematics teacher led the class instruction in the second lesson study cycle because she lacked confidence in teaching through practical activities and wanted to learn from the science teacher first. She gained confidence in handling practical science activities after interacting with students while assisting them during group work in the first learning cycle. This experience proved vital when it was her turn to lead the class instruction.

> "I knew I know the stuff (content) but I was not sure if I will be able to handle lessons with practical activities, so I decided to learn from the science teacher first".

In the sixth lesson, the mathematics teacher led a class discussion where she demonstrated high expert knowledge by explicitly connecting science and mathematics concepts. For instance, the concept of proportion, control of variables, use of real-life examples and denseness of particles in an object were articulated in depth. Students were very engaged, recognizing the interplay between different subject areas. When the concept of proportion was elucidated with clarity, accompanied by concrete examples that demonstrate its relevance in both abstract and real-world contexts, the teachers themselves felt the robustness of the connection between mathematics and scientific principles. Students had an AHA! moment when they finally discovered that the two models developed in lesson four, formula (1), and lesson five, formula (2), can be connected to form the density model/formulae, formula (3).

> Math teacher: In the sixth lesson, I now understood how to clearly connect the subjects. The repeated exposure helped me. I enjoyed the lesson and I have never done it the way I did it so I could even see the joy in learners too.
>
> Science Teacher: Yaa, the last lesson really showed me that we never cease to learn. The lesson was executed perfectly, and you could clearly see the link of math and science. I was motivated.

6.2. Autonomy

The level of freedom the teachers gave students during the lessons influenced the form of integration for those lessons. 'I realized that even though we were teaching the same topic, using the same research lesson, the development of our lessons was very different depending on what students were allowed to do on their own during the lessons'. In the first three lessons, students had much freedom. Students followed the worksheet instructions and explored practical materials with minimum guidance from the teachers. Consequently, they made several errors during their practical activities; For instance, some groups overlooked or neglected to formulate and document their predictions. 'Class, you are moving too fast, there is no need to rush. Please read the instructions carefully. Most of you have omitted the prediction stage'. Making predictions was the first instruction and was crucial for the development of the lesson and for linking science and mathematics concepts. With much freedom to explore with the practical activities, students showed a lot of enthusiasm, they made many errors and queries. Comparatively, in lessons 4 to 6, the mathematics teacher led the students step by step, following the worksheet instructions. The lessons were more structured, student's autonomy was restricted, and the lessons progressed smoothly as the teacher wished.

> "The first group was slow in understanding the concepts, moved through the worksheet instructions very fast, they were very explorative, made a lot of mistakes and queries but had enthusiasm, and they seemed to enjoy the lessons. The second group was fast to grasp the concepts, less energetic, low morale, students produced quality work, and the lessons were much better than the first series".

The difference caused cognitive conflict in teachers of finding a balance between fostering student autonomy and maintaining student engagement while ensuring positive learning outcomes. 'I think there is a need to come up with a balance because we need students to understand the concepts and give good answers with little errors, but we also want students to enjoy the lessons, right?' Teachers acknowledged that limited lesson time and syllabus demands restrict their flexibility in enhancing student autonomy to explore and allowing effective student-cantered activities. They emphasized that with more time, they could facilitate deeper engagement. However, they also stressed the need to come up with strategies that equally engage the students in the limited time, for instance using modern digital tools vs analogue tools to maximize efficiency during limited class hours: 'You see, as much as we want students to explore, we don't have time. I think the other thing is to throw away these old tools and start using digital ones so that we won't waste unnecessary time'.

6.3. Time

Time allocation and time spent on different phases of the lessons, i.e., the specific phases of science inquiry and mathematical modeling, determined how the integration of the two learning approaches was shaped. The science teacher expressed that in his third lesson, he could not develop other mathematical concepts (e.g., mathematical analysis and insight conjecturing) because of limited time since learners were also supposed to write an exercise.

> "In my third lesson, I had a lot of things to cover yet, I also wanted learners to write an exercise, so it left me with no option but just to dictate through some concepts and ask students to write the test. You will therefore realize that the aspect of mathematical analysis and insight conjecturing, which are mostly mathematical was heavily affected".

On the other hand, the mathematics teacher claimed that she resorted to guiding the learners for the most part of the lessons, claiming that the strategy ensured that she covered all the components of mathematical modeling and science inquiry: 'I had to strictly guide the students otherwise one of the subjects was going to be trivialized, and we were also not going to finish the topic'. During teacher reflection, teachers expressed that for a true or balanced science-math integration, instructional approaches (e.g., mathematical modeling and science inquiry) must be intentionally segmented into discrete components. By allocating specific time to each segment during lesson planning, teachers are compelled to engage with, rather than avoid unfamiliar or challenging content.

> "I think true integration should start with planning. Each segment should have allotted time and activities specified, otherwise the stressful parts will be ignored or shielded by those that I am very familiar with".

6.4. Contingency

Contingency influenced the integration of mathematics and science. Because of the nature of science and mathematics, several events were experienced during the lessons. The teachers' response to these incidents determined the trajectory of the lessons.

> "In many cases, our lesson plans were changed by simple queries that students asked, and we were not ready for such questions. Even though they appeared to be simple questions, they were challenging, and they changed a lot in the way we intended our lessons to be".

In the first lesson activity, students placed two similar containers of different densities into a bucket of water. Each set was assumed to have the same volume, i.e., the containers were of the same size and were all filled to the brim with salt solution and with distilled water. Containers with salt solutions had a higher mass than those filled with distilled water, containers with salt solution/higher mass sank, and those with distilled water/lower mass floated, which students were able to determine. However, during explanations, some students claimed that some containers sank because they had a large volume of contents. Students argued that salt solutions had more particles than distilled water and therefore had more volume. This line of argument was not expected by the teachers.

Student: Containers with salt solutions had higher volume because they have more molecules, so they sink.

Teacher: Class, what can you say about the size of containers A1 and A2?

Students: They are the same size

Teacher: What about the volumes of the solutions in these containers?

Students 1: They are the same

Students 2: They are different

Teacher: Why are you saying they are the same?

Students: All the containers are full, and the containers are of the same size

Teacher: Those saying the volumes are different, why is the volume different?

Students: It has substances in it

Teacher: What substances

Student: It has a combination of salt and water

Teacher: Clarify

Student: It has 2 molecules, water molecules and salt molecules so it takes more volume.

To resolve this dilemma, the teachers then exhibited contingency skills by providing small measuring cylinders and syringes to each group for them to measure the volume of contents of one of the containers. The teachers intended to let students see that each set of containers had the same container size hence the same volume of content, thereby directing their attention to the mass variable, i.e., employing the control of variable strategy. The teachers also wanted students to discover that the experimental activity involved controlling one variable, i.e., volume. However, when students measured the volumes they obtained mixed results i.e., some groups recorded slightly higher volumes of distilled water, while some groups obtained slightly higher volumes of salt solution, and some groups' recordings had equal volumes.

Teacher: So what did you find?

Student 1: A1 has more solution

Teacher: Any group with a different result

Student 2: A2 has more solution

Teacher: Any other group that found different results

Student 3: They all have the same volume

Student 4: Volumes are slightly different

Although it is typical for a new object to be introduced to students during an activity, what is noteworthy in this instance is that the new object did not convey the scientific principle as intended; instead, it became part of the students' non-scientific reasoning. The additional activity therefore cemented the student's argument that even though the containers are of the same size, and they are all filled to the brim, they may have different volumes of solutions depending on the composition of the solution; a conclusion that conflated mass with volume. To redirect student thinking toward the target concept, the teachers explicitly introduced the concept of margin of error. They explained the discrepancy in measurements not as a property of the solution, but as a result of potential errors in preparation, measurement, or spillage, thereby framing variation as an expected part of empirical work rather than as evidence for their alternative theory. This intervention successfully helped students move past their confusion, and afterward, they demonstrated an understanding of the intended principle that mass, not perceived volume, determines sinking for objects of identical size.

Another important contingent event happened in the second lesson when students were doing practical activity to show that when the volume is large, the container floats and when the volume is small, the container sinks. The practical activity controlled for mass. Instead of concentrating on the volume of containers, some students argued that small containers sank because they had higher mass and that they were full. Teachers were taken by surprise because when they prepared the containers, they ensured that each container set had the same mass. To confirm the student's submissions, teachers moved around to different groups, re-measuring the mass of containers to confirm and they noticed that most of the container sets had different masses in milligrams. This conflict between perceptual and concrete observation was resolved through teacher-student discourse and applying the concept of numerical precision, whereby students were asked to round off the readings to the nearest whole number.

> "Okay, we have noticed that some sets of containers have slightly different masses. So, what I want you to do is to round off the mass readings to the nearest whole number. After that, you explain your observations".

Again, after the teacher's intervention, the students seemed to have understood the concept that was being developed.

Furthermore, when students confronted teachers with difficult conceptual questions, teachers tended to pay little attention, authoritatively dismiss the question, or just ignore it. For instance, students mixed the container sets and asked for an explanation of why the discussed principles did not apply. The teachers did not have an immediate scientific explanation for that. Instead of being open with the students, the teacher chose to treat the query as a procedural anomaly rather than a conceptual query, and, hence, the teacher instantly instructed the students to just follow the worksheet instructions. In other words, the teacher insinuated that if the students followed the worksheet procedure, then the student's query was resolved.

In most of the contingent scenarios, it was interesting to note the collaborative effort the teachers were making in helping each other to solve the student queries using their expert knowledge. This in turn cemented the students' view of the relationship between mathematics and science. For instance, queries that had to do with mensuration, the science teacher would guide learners through practical tools and hands-on techniques, while the mathematics teacher supplemented this with conceptual frameworks like rounding off and error margins, creating a dynamic problem-solving interplay where the limitations of one approach (such as concrete experimentation) were often resolved by the strengths of the other (such as abstract mathematical reasoning).

7. Discussion

In this study, we examined the key factors influencing the integration of mathematics and science through collaborative lesson planning and teaching in a real classroom setting. Four factors emerged as critical to successful integration: Teacher knowledge, student autonomy, time allocation, and contingency. These factors do not operate in isolation but interact dynamically, shaping the effectiveness of interdisciplinary instruction. Below, we discuss these findings in relation to the literature and propose a Dynamic Interdependence Framework (DIF), Figure 1, to guide future research and practice in integrated STEM education.

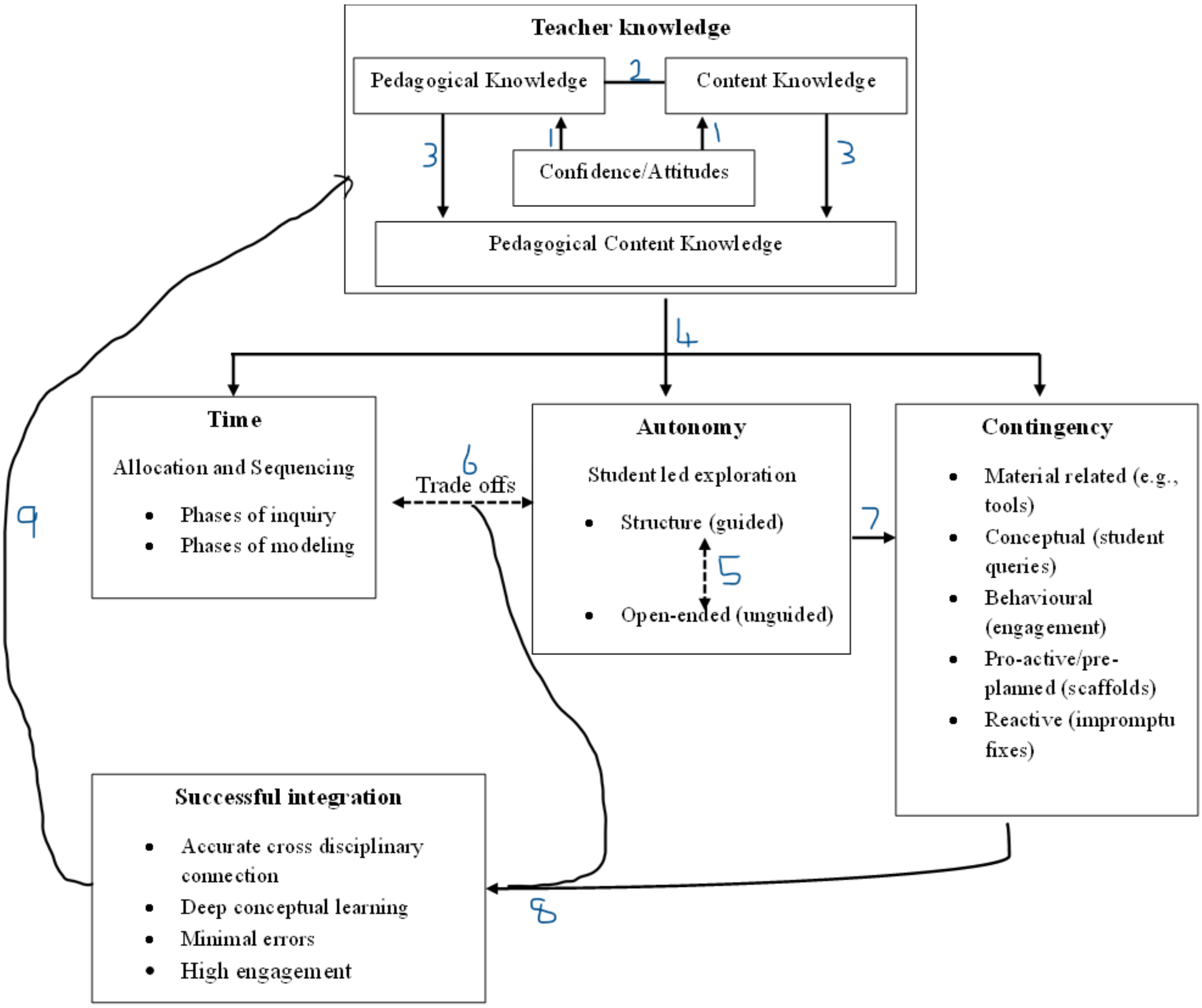

Figure 1. The Dynamic Interdependence Framework (DIF).

The findings highlight the centrality of teacher knowledge, attitude, content mastery, and pedagogical skills, in shaping integrated lessons. Consistent with other research [11,46], the science teacher's initial overconfidence in his experience masked gaps in his conceptual understanding of density, such as confusing surface area with volume (Arrow 1). These gaps inadvertently reinforced student misconceptions, as seen when learners mirrored his erroneous explanation when students consistently used the terms surface area, volume, and capacity interchangeably in their explanations, highlighting the detrimental effect of teacher knowledge gaps on learning [47,48]. In contrast, the mathematics teacher, despite her initial lack of confidence in handling practical activities, compensated through observational learning during the first lesson study cycle where she took the opportunity to learn the connection of the content and the handling of practical activities (Arrow 1-3). By the second cycle, she demonstrated robust integration by explicitly linking mathematical modeling (e.g., proportional reasoning) with scientific inquiry (e.g., controlling variables). This aligns with studies emphasizing that teachers need strong foundational knowledge in both disciplines to create meaningful connections [49,50]. Strong teacher knowledge informed strategic autonomy, time prioritization, and enhanced effective contingency responses (Arrow 4). Knowledge gaps in one area amplify challenges in others, such as contingency management. For instance, the science teacher's limited content knowledge led to improvised, less effective responses to student queries (e.g., dismissing questions about mixed container sets as procedural issues), whereas the mathematics teacher's dual expertise enabled her to address contingencies with conceptual coherence (e.g., using rounding rules to resolve measurement discrepancies).

Student autonomy significantly shaped the nature of integration. Lessons led by the science teacher enabled more freedom, aligning with STEM tenets of embracing errors and hands-on exploration [51–53]. However, this posed challenges in an exam-oriented Zimbabwean curriculum that prioritizes error minimization. In contrast, the mathematics teacher's structured approach in lessons 4–6 ensured conceptual clarity but throttled student curiosity and engagement. This creates a tension on the best approach to take (Arrow 5), reflecting a core challenge of adapting integrated STEM education within exam-driven systems that prioritize correct answers over exploratory learning. Addressing this mismatch requires research into pragmatic models that achieve curricular efficiency without sacrificing authentic inquiry.

Greater autonomy increased engagement but disrupted time management, necessitating structured exploration (Arrows 6). Time allocation further shaped integration quality, particularly in integrating and balancing disciplinary emphases. The science teacher's third lesson rushed to accommodate a summative exercise and restricted opportunities for mathematical analysis. The mathematics teacher's strict time management ensured coverage and integration of all segments of mathematical modeling and science inquiry but reduced exploration depth. This echoes [54] and [55] assertion that effective teachers prioritize equitable time distribution across all learning phases. Equitable time distribution across lesson phases is essential for meaningful integration, requiring teachers to prioritize conceptual links over procedural tasks. Without such planning, teachers default to familiar or less challenging content, undermining integration. The link between autonomy and contingency was evident: Greater student freedom generated more contingent moments (Arrow 7), requiring teachers to devise impromptu strategies to address unanticipated student queries.

Contingency played a pivotal role, particularly in navigating epistemic tensions between disciplines. Tools (e.g., measuring tools) frequently triggered unplanned moments, such as students obtaining conflicting volume measurements. Collaborative teacher efforts, using additional tools and mathematical concepts (e.g., rounding and error margins) resolved these issues, reinforcing the complementary nature of mathematics and science [25] in providing multiple pathways to understand a concept. Yet, contingency management also exposed vulnerabilities; the science teacher's avoidance of student queries revealed how knowledge gaps limit adaptive teaching. Prior research frames contingency as student-centered [56], but our findings argue for teacher-centered contingency readiness, where anticipating disciplinary tensions (e.g., mathematical precision vs. scientific plausibility) becomes a core component for the integration of mathematics and science. The teacher's negotiation of this complex web of factors determines the nature of the integrated lesson (Arrow 8). Furthermore, a successfully implemented lesson creates a feedback loop, reinforcing and expanding teacher knowledge through subsequent reflective practice (Arrow 9).

7.1. Implications and future directions

Our findings of this study have significant implications for theory and practice in integrated

STEM education. Primarily, they show that a shift in professional development from a sole focus on discrete content knowledge toward cultivating integrative pedagogical content knowledge (PCK) is needed, which empowers teachers to anticipate disciplinary connections and tensions. This study underscores the need for collaborative professional development (PD) where science and mathematics teachers jointly rehearse lessons, anticipate contingencies, and refine integrating strategies. Pairing teachers during PD, as in our lesson study cycles, can foster mutual capacity-building, as seen in the mathematics teacher's growth. For curriculum design, the proposed DIF provides a practical model for consciously accounting for the dynamic interplay of core factors: Teacher knowledge, student autonomy, time, and contingency when integrating science and mathematics. Moreover, a key operational skill emerging from this study is "teacher-centered contingency readiness," which should be integrated into teacher training through rehearsals of responses to un/predictable interdisciplinary problems. Furthermore, this study opens several avenues for future research. Investigators should longitudinally track how teacher knowledge evolves through repeated cycles of planning, teaching, and reflective practice, as suggested by the model's feedback loop (Arrow 9). In future work, researchers could also explore how the two knowledge dimensions can be effectively addressed not only at the study's grade level but across other secondary education levels and the applicability of the findings to the wider STEM context, which typically includes technology and engineering. Crucially, there is a pressing need to develop and test pragmatic models of integration that can successfully balance the exploratory nature of STEM with the demands of exam-oriented curricula, ensuring such initiatives are effective and sustainable in diverse educational contexts. Finally, it is important to acknowledge the limitations of this work. As a confined single-case study, the generalizability of the findings is limited to the context of this study. The effective application of the proposed frameworks and strategies in other settings requires further investigation. Furthermore, the extended data collection period, punctuated by breaks due to participant schedules and school holidays, introduced a potential limitation. While these gaps provided valuable reflection time, they could have influenced the continuity of the collaborative process and teacher recall, factors which were mitigated through structured follow-up interviews. It is therefore important for researchers to consider the balance between logistical practicality and the ideal continuity of data collection to further strengthen the validity of longitudinal qualitative research.

8. Conclusions

Integrating mathematics and science is not merely about combining content but navigating a web of interdependent factors. The DIF offers a roadmap for this journey, emphasizing that effective integration hinges on teachers' ability to diagnose, adapt, and equilibrate knowledge, autonomy, time, and contingency. In doing so, educators can transform STEM classrooms into spaces where disciplinary boundaries blur, and students grasp the profound synergies between mathematics and science. While this study provides a credible and valuable proof-of-concept illustration of the DIF in action, it establishes a foundation for further research rather than definitive claims. The conclusions are most credible for understanding the nature of the challenges and adaptive processes involved in integration. Future research involving multiple cases, control groups, and more continuous data collection is necessary to strengthen the inferential validity and confirm the broader applicability of the framework across educational contexts.

Author contributions

Kevin Manunure: Conceptualization, Formal analysis, Writing – review & editing
Nicholas Zezekwa: Resources, Methodology
Gladys Sunzuma: Resources, Methodology
Christopher Mutseekwa: Formal analysis, Writing – original draft

Use of Generative-AI tools declaration

The authors declare(s) that they have used Artificial Intelligence (AI) tools in the creation of this article.

AI tools used: AI Grammar Checker & Paraphraser and DeepSeek.

The AI tools were used for English language editing to enhance language quality of the whole document.

Conflict of interest

We have no conflicts of interest to disclose.

Ethics declaration

The study had the Hong Kong Baptist University's Research Ethics Committee (REC) approval. Informed consent was obtained from all teacher and student participants included in the study.

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Author's biography

Dr. Kevin Manunure is a Science and Mathematics educator. His research interest is in Science education, STEM integration, and Teacher Professional Development.

Dr. Gladys Sunzuma is an academic researcher from Bindura University of Science Education. Her research interest is in Ethnomathematics & Medicine and Science education.

Dr. Nicholas Zezekwa is a senior lecturer and researcher in science education at Bindura University of Science Education (BUSE) in Zimbabwe. His expertise is in researching on and teaching general science and physics education, with significant experience across junior secondary, 'O' Level, 'A' Level, undergraduate, and postgraduate levels.

Dr. Christopher Mutseekwa is a senior lecturer and researcher in the Department of Education, Rwanda Basic Education Board, Kigali, Rwanda. He is a researcher with a focus on Robotics in Education and STEM education.