STEM Education

STEM Education, 5(4): 515–545
DOI: 10.3934/steme.2025025
Received: 26 February 2025
Revised: 23 April 2025
Accepted: 08 May 2025
Published: 28 May 2025

https://www.aimspress.com/journal/steme

Research article

Rebuilding manipulatives through digital making in teacher education

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Academic Editor: Hepu Deng

Abstract: The integration of digital making and computer-aided design (CAD) technologies, using 3D printing, in mathematics education has opened new opportunities for creating manipulatives that enhance conceptual understanding and problem-solving skills. Here, we explored how pre-service elementary teachers engage with the engineering design (ED) process while using Tinkercad and 3D printing to create customized manipulatives for mathematics education. A qualitative exploratory study was conducted with thirteen pre-service teachers enrolled in a master's program in Mathematics and Science Education at a Portuguese higher education institution. We employed multiple data sources, including classroom observations, participants' artifacts, written reports, and visual records, to capture the learning processes and challenges faced by the participants. Our findings indicated that participants developed a deeper understanding of content knowledge by analyzing mathematical properties and structures in the design of manipulatives, although some struggled with mathematical precision and curricular alignment. The iterative nature of ED reinforced technical knowledge, as participants engaged in CAD modeling and 3D printing. Despite initial difficulties, they refined their models through iterative redesign, recognizing the importance of spatial reasoning and precision. Pedagogical knowledge emerged as participants reflected on the educational use of manipulatives, proposing classroom applications and recognizing digital making's potential for active learning. However, some tasks lacked clear curricular connections, highlighting the need for further pedagogical scaffolding. Overall, the study underscores the potential of integrating ED, CAD environments, and 3D printing into teacher education to foster interdisciplinary STEAM learning and problem-solving skills.

Keywords: STEAM education, engineering design, digital making, manipulatives, CAD, 3D printing, teacher education

1. Introduction

The use of manipulatives in mathematics education has long been recognized as a powerful strategy to enhance student learning by making abstract concepts more tangible and accessible [1]. These physical objects enable learners to develop a deeper conceptual understanding through hands-on interaction, fostering engagement and exploration. While premade manipulatives provide structured learning opportunities, their design is often fixed, limiting adaptability to diverse educational needs. Advancements in digital making and computer-aided design (CAD) technologies, along with 3D printing, have significantly expanded the possibilities for creating manipulatives that are customized, flexible, and responsive to specific learning objectives [2]. Access to digital making technologies has catalyzed the transformation of traditional teaching materials, enabling educators to design and construct instructional resources tailored to their students' needs [3]. This shift aligns with broader educational movements, particularly STEAM (Science, Technology, Engineering, Arts, and Mathematics) education, which emphasizes interdisciplinary learning and hands-on problem-solving. Within STEAM education, Engineering Design (ED) is a key pedagogical approach to promote creativity, critical thinking, and iterative learning [4,5].

Integrating digital making into teacher education programs has the potential to equip pre-service teachers with the technological and pedagogical competencies required to create meaningful learning experiences. However, one critical gap in educational research is the limited focus on how digital making, specifically CAD environments and 3D printing, can be effectively incorporated into teacher training programs. While researchers have explored the role of 3D printing in mathematics education [6–8], further research is needed to understand how pre-service teachers engage with these technologies throughout the ED cycle and how their experiences influence their pedagogical perspectives on manipulatives. We seek to address this gap by examining how the use of Tinkercad and 3D printing, within the context of ED, influence the development of pre-service teachers' knowledge and skills in designing and creating manipulatives for educational purposes. Specifically, we address the following questions: 1) What knowledge and skills are displayed throughout the ED cycle mediated by the use of Tinkercad and 3D printing?; and 2) How do pre-service teachers perceive the use of Tinkercad and 3D printing as tools for designing and creating manipulatives in the context of STEAM education?

By providing pre-service teachers with hands-on experiences in digital making, we aim to contribute to the growing body of research on technology-enhanced learning and teacher education. The findings may have implications for how teacher preparation programs integrate digital making tools to support innovative pedagogical practices and improve mathematics instruction through the development of personalized manipulatives.

2. Theoretical framework

2.1. STEAM Education and its relevance for teacher training

2.1.1. STEAM Education overview

The rapid technological, economic, and societal transformations of the 21st century demand an education system that prepares students with the competencies required to address complex real-world problems. STEAM (Science, Technology, Engineering, Arts, and Mathematics) education, as an evolution of STEM, has emerged as a promising framework to achieve this goal, emphasizing the interconnectedness of disciplines and fostering critical thinking, creativity, collaboration, and communication, often referred to as the 4Cs [9]. These skills align with the global job market's growing emphasis on cognitive, interpersonal, and socio-emotional skills [10].

Unlike traditional approaches that compartmentalize disciplines, STEAM education emphasizes interconnectedness, providing a richer, holistic learning experience. This interdisciplinary nature promotes deeper learning by relating traditionally siloed subjects. This integration not only fosters a richer understanding of concepts but also nurtures skills such as design thinking, which combines artistic expression with scientific and technical rigor [11]. Despite its potential, achieving equitable attention to all disciplines within STEAM remains a challenge. Researchers such as Martin-Páez et al. [12] and Ortiz-Revilla et al. [13] highlight the tendency to prioritize science and mathematics over the arts and engineering, often due to curricular constraints. Effective STEAM integration requires reflection on how each discipline contributes to problem solving and how their contributions can be interwoven to enhance learning. Vasquez et al. [14] propose a continuum of integration, from disciplinary to transdisciplinary approaches, where knowledge and skills from multiple fields are applied collaboratively to address real-world issues. English [15] further argues for the integration of systems thinking, critical mathematical modeling, and philosophical inquiry to enhance adaptive and innovative problem-solving skills.

2.1.2. Pre-service teachers in STEAM contexts

STEAM education offers a transformative opportunity for pre-service teacher training. By equipping future educators with interdisciplinary methodologies and fostering innovation, STEAM empowers them to inspire students to engage actively with these disciplines. Teacher agency is pivotal in this process, as educators must design environments that encourage exploration, support authentic problem-solving, and bridge theoretical knowledge with practical application [15,16]. Integrating STEAM into teacher education programs equips pre-service teachers with the tools to design lessons that reflect interdisciplinary learning. Literature highlights the value of programs that prioritize problem-centered, inquiry-based, and design-based learning, emphasizing STEAM lesson planning, where pre-service teachers engage in designing interdisciplinary lessons that combine science and mathematics with engineering and artistic creativity [13,16]. Such methodologies help (pre-service) teachers bridge theory with practice, fostering both their confidence and pedagogical competence.

Research shows that pre-service teachers often face challenges in integrating STEAM due to limited content knowledge and pedagogical experience. However, hands-on activities and iterative feedback during training programs help address these gaps. For instance, participants in STEAM modules frequently report increased motivation to adopt interdisciplinary approaches and greater confidence in their ability to integrate technology and the arts into their teaching practices [17,18]. While STEAM education offers numerous benefits, its implementation in pre-service teacher training is not without challenges. Teachers often express concerns about the time and resources required to design and deliver STEAM lessons, and the difficulty of achieving balanced integration across all disciplines [17]. Addressing these issues requires robust professional development programs that provide ongoing support, foster collaboration among educators, and emphasize the development of both content knowledge and pedagogical strategies [18,19].

2.2. The ED cycle and its implementation

The Engineering Design (ED) cycle is a systematic, iterative process central to problem-solving in engineering, with significant pedagogical value when integrated into educational contexts. Its emphasis on problem solving, creativity, critical thinking, and collaboration makes it an effective framework for fostering interdisciplinary and transdisciplinary learning in STEM and STEAM education [15,20,21]. As students engage with this process, they develop not only technical competencies but also 21st-century skills crucial for addressing complex real-world challenges.

The ED process focuses on decomposing ill-structured, often complex problems, solving them by generating multiple ideas and accomplishing a goal through an open-ended path, resulting in multiple solutions that must be evaluated [22,23]. Several researchers have presented different ED models, which vary according to their goals, context, and emphasis, divided into prescriptive and descriptive models, represented by charts, recursive diagrams, and decision-making trees [23]. The prescriptive models draw on the conceptualization of the major tasks for each design step, rather than following a sequential design process; descriptive models have a solution-focused nature, emphasizing successful engineering design processes [22,23].

Regardless of the model type, the ED cycle comprises several interconnected stages that have evolved over time. According to the literature, design processes generally follow an iterative structure, consisting of a series of steps, such as: (a) Defining a problem by identifying criteria and constraints to find acceptable solutions; (b) generating possible solutions and assessing them to determine the one(s) that best meet the problem requirements; and (c) optimizing the solution by systematically testing and refining it [4,11,23]. This iterative nature helps students understand that failure is an integral part of the design process, promoting resilience and a growth mindset [11].

In recent studies [6,24,25], we implemented an ED cycle adapted from the one proposed by Hester and Cunningham [5], which is composed of seven steps: problem (define the problem/identify the constraints); imagine (brainstorm ideas/look for possible solutions/choose the best one); design (plan the solution/draw a sketch); (re)build (follow the plan, create and construct the idea); (re)test and evaluate (test and evaluate the idea/the prototype); redesign (discuss what works/what does not work/improve/modify the design to make it better/test it once more); and solution (share and communicate the solution/results/obtained product) [25]. As seen in Figure 1, after the testing and evaluation step, we included the possibility of redesigning, rebuilding, and retesting in case students need to improve the artifact/prototype to meet the required needs.

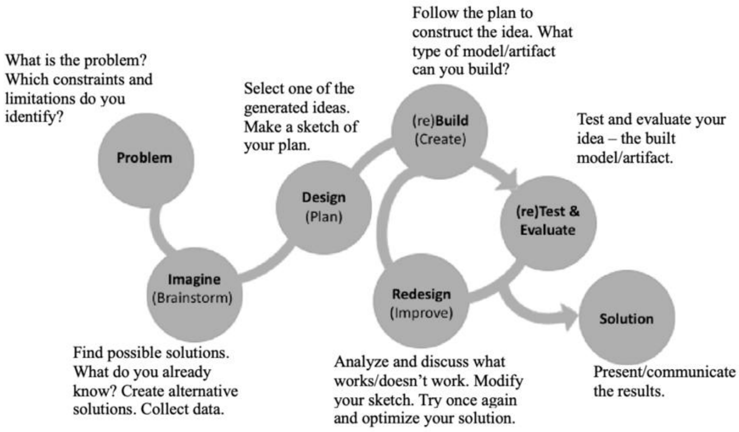

Figure 1. Engineering design process cycle proposed by Vale et al. [25].

This structured approach enhances students' ability to connect engineering concepts with other STEAM disciplines. The educational value of the ED cycle lies in its ability to integrate concepts across disciplines. By incorporating elements from science, technology, engineering, the arts, and mathematics, the process provides a holistic understanding of how these fields interconnect. Cunningham [26] emphasizes that this integration is critical for developing authentic problem-solving skills. Engaging with real-world challenges enhances students' motivation and demonstrates the societal relevance of their learning. Moreover, the ED cycle promotes collaboration and communication, as students often work in teams to articulate and refine their ideas. These experiences mirror professional engineering practices and prepare students for future interdisciplinary endeavors [21]. Therefore, the Engineering Design cycle represents a powerful pedagogical approach that aligns with the goals of contemporary education.

While the ED cycle inherently promotes problem-solving and iterative thinking, its educational relevance is amplified when mapped explicitly to mathematical practices. In this study, each step of the ED process aligns with core mathematical concepts, including geometric visualization, proportional reasoning, and spatial structuring. Participants' manipulation of shapes and dimensions when designing manipulatives helps reinforce their understanding of measurement, symmetry, and scale. Furthermore, the iterative design reflections mirror problem-solving steps and support mathematical reasoning and justification.

2.3. Making in technological-enhanced contexts and STEAM Education

2.3.1. Digital making

The concept of Making has long been connected to experiential and hands-on learning, enabling individuals to engage in creative problem-solving, collaboration, and the development of practical skills. Traditionally, Making involved crafting and constructing physical objects using various materials, but with the appearance of modern technologies, the landscape of Making has expanded significantly [3,27]. The value of the hands-on nature of Making has been widely recognized in education, particularly within STEAM disciplines, as it facilitates conceptual understanding through tangible experiences.

With the advancement of digital tools, Making has evolved to encompass new forms of design and construction that integrate computational processes. This shift, often described as "digital making" or "technology-enhanced making," includes advanced techniques such as computer-aided design (CAD), 3D modeling, laser cutting, and automated manufacturing systems [27,28]. These technologies enable learners to develop, test, and refine their ideas using virtual simulations before creating tangible prototypes, fostering iterative thinking and problem-solving abilities [3,28], which is aligned with the steps of the ED cycle.

A particularly influential aspect of digital making is the incorporation of 3D design and manufacturing into educational settings. By leveraging CAD software and additive manufacturing technologies, students can translate conceptual ideas into physical models, reinforcing their understanding of design principles and spatial reasoning [29]. This approach has proven especially valuable in STE(A)M education, supporting the understanding of core STE(A)M concepts, the exploration of geometric relationships, engineering applications, and real-world problem-solving scenarios [2,6,29]. For instance, the use of 3D printers in mathematics education allows students to visualize abstract concepts such as geometric properties, enabling them to explore the relationship between shapes, dimensions, and volumes in a hands-on manner [2,30].

While digital making presents significant educational benefits, its integration into the classroom is not without challenges. Issues such as access to these technologies, teacher training, and alignment with curricular goals remain key barriers [30]. However, initiatives aimed at professional development and establishing school-based makerspaces have demonstrated promising results in equipping teachers with the necessary skills to incorporate digital Making into their practices. Furthermore, emerging research suggests that digital Making not only enhances STEAM education but also contributes to the broader goal of developing students' 21st-century skills [1]. As digital making continues to evolve, it holds the potential to transform educational practices by bridging the gap between abstract concepts and hands-on, experiential learning.

2.3.2. The use of CAD and 3D printing: Knowledge and skills developed

The integration of digital Making tools into education has gained increasing attention in recent years, particularly in STEAM disciplines. Among these tools, Tinkercad, a widely accessible CAD environment, and 3D printing have demonstrated significant potential in fostering hands-on learning, spatial reasoning, and problem-solving skills. These technologies enable learners to engage in the iterative ED process, facilitating conceptual understanding in science, technology, engineering, arts, and mathematics. The iterative nature of these technologies aligns closely with the engineering design process, which emphasizes cycles of design, testing, and improvement [6,31].

CAD can be seen as a technological extension of traditional hands-on planning, processing, and producing, allowing for the creation of virtual representations of physical objects. Engineers and designers frequently use this type of software to solve ED problems. Additionally, CAD software serves as a predictive tool, enabling users to evaluate the outcome of a design before physically printing it, supporting creative engagement with problem-solving and the communication of ideas through graphical representations [7,31]. This type of software can be used to simulate and experiment with different scenarios, learn by trial and error from the process of making mistakes and correcting them, perform problem solving, and establish decision making with little risk and without wasting resources [6,7,31]. Also, the ability to design, modify, and print 3D models provides a direct and tangible way for students to engage with abstract mathematical and engineering concepts, particularly in geometry, volume calculation, and algebraic transformations [2]. The CAD environment facilitates the teaching and learning of elementary and advanced mathematical and scientific concepts [7,32], supporting the development of a strong foundation in computer-aided design, which is an asset for future careers [29]. Spatial thinking and the use of spatial representations are applied in design practices, especially when the aim is to obtain a tangible object from a virtual prototype, attending to features, like shapes, sizes, spatial relations, spatial arrangements, perception of space, and proportion [32]. Several studies highlight the educational potential of CAD modeling and 3D printing, particularly in facilitating spatial reasoning and mathematical understanding. Research by Ng and Chan [29] and Barbosa et al. [6] emphasizes how 3D CAD environments, such as Tinkercad, enhance students' ability to visualize and manipulate geometric objects, improving their understanding of three-dimensional spatial relationships.

The possibility of seeing the printing product and touching the designed object can be a unique experience for students and spark a sense of satisfaction, giving students the opportunity to plan and create 3D models through design thinking skills, bringing virtual objects to life [33]. This process promotes a more effective learning experience, contrasting theoretical knowledge with practice, recognizing possible misconceptions. The printed physical objects allow students to make concrete the solution to a given problem, examine the success level of their designs, and improve their existing designs with various arrangements [6], as expected in ED. In this sense, 3D printing can also lay the foundation for reflective validation, encouraging students to evaluate their own work [7].

In addition to enhancing conceptual understanding, Tinkercad and 3D printing also promote skills like the 4Cs. Critical thinking is fostered as students engage in iterative problem-solving, refining their designs based on real-world constraints and feedback [2]. Creativity is stimulated through open-ended design tasks, where learners experiment with different modeling techniques to develop unique solutions [7]. Collaboration emerges in maker-based learning environments, as students and teachers work together, sharing CAD models, evaluating technical issues, and collectively refining projects [1]. Finally, communication is strengthened as students articulate their design choices, justify modifications, and present their final prototypes, often using graphical representations and digital storytelling to convey their ideas [30]. These interconnected skills prepare learners for interdisciplinary problem-solving, aligning with the broader goals of STEAM education [33].

Läufer and Ludwig [7] explored the impact of 3D modeling on pre-service mathematics teachers, demonstrating that engaging with digital Making promotes active learning and critical thinking. By designing and customizing their own products, students/teachers transition from passive recipients of knowledge to active creators, fostering a deeper engagement with the learning process [6,7]. Furthermore, engaging pre-service teachers in activities that integrate such tools not only enhances their technical proficiency but also deepens their understanding of the interconnections among STEAM disciplines [33]. However, effective implementation requires a structured approach, not only technological, but also pedagogical and content knowledge, particularly within teacher education programs [1,34]. This perspective provides a valuable lens to analyze the challenges and opportunities associated with digital making in education. While CAD technologies can be an asset to teaching and learning, successful integration requires more than technical proficiency. Educators must develop pedagogical knowledge (PK) to effectively incorporate CAD-based activities into curricula, content knowledge (CK) to align these activities with disciplinary goals, and technological knowledge (TK) to operate with CAD and 3D printing software [34].

However, despite their numerous advantages, challenges remain in the implementation of these tools, particularly concerning accessibility, teacher training, and curriculum integration. One of the most significant barriers is technical and logistical complexity. As noted by Ng and Ye [2], 3D printing requires specialized hardware, software, and consumables, posing financial and operational challenges for schools. Additionally, the printing process itself is time-consuming, with even small objects requiring hours to complete, making it difficult to integrate into standard classroom schedules [7]. Teacher preparedness is another key challenge. Research by Stigberg [8] suggests that many educators lack formal training in CAD software and digital making, hindering the implementation of these tools effectively in their teaching practice. Without adequate professional development, teachers may struggle to align digital making activities with existing curricula, leading to fragmented or superficial integration of these technologies. In particular, Barbosa et al. [6] highlight that pre-service teachers often require structured guidance and hands-on experiences to fully understand the technical use and interdisciplinary applications of CAD modeling and 3D printing.

Furthermore, the learning curve for students must also be considered. While Tinkercad is designed to be an accessible entry-level CAD tool, learners require time to develop proficiency in 3D manipulation, object alignment, and design iteration [6,29]. Studies indicate that students with limited prior experience in spatial reasoning may initially struggle to translate conceptual ideas into digital models, requiring scaffolding to build confidence and competence [29]. Additionally, Ng and Ye [2] identified hardware limitations and software usability as potential obstacles, noting that issues related to file formatting, slicing, and printer calibration can introduce unexpected technical difficulties.

This discussion supports the general need in teacher education programs for information and training about 3D printing, which can support their professional development and enable their ability to teach others about 3D printing and design processes [6,34].

3. Research design and methods

3.1. Methodological options

We adopt a qualitative approach to understand how the use of Tinkercad and 3D printing, within the context of ED, influences the development of pre-service teachers' knowledge and skills in designing and creating manipulatives for educational purposes. This study is inherently qualitative because we focus on understanding experiences, processes, and sense-making, enabling us to capture the complexity of learning processes in context [35]. Given the nature of this research, we performed an exploratory study to provide in-depth insights, uncover patterns, and gain a deep understanding of the problem [36].

The study involved thirteen participants, pre-service teachers from primary education (future teachers of children aged 5-12 years old), from a higher-education institution in Portugal. The group included eleven women and two men, with an average age of 25, who willingly and informedly agreed to take part in the study. They were attending the first semester of the first year of a Masters course in Mathematics and Science Education, with a duration of four semesters. Previously, they completed a BSc in Basic Education with a duration of six semesters. This BSc program was composed of subjects related to the areas of didactics, general education, content knowledge, and practice in formal and non-formal educational contexts. The master's program they are now attending grants them teaching qualifications, with a specialization in Mathematics and Science, providing a more focused training in these subjects from both a scientific and didactic perspective.

The participants were enrolled in a Didactics of Mathematics unit course, managed and lectured by the researchers, which served as the context for the didactical experience and global data collection. The work developed during the semester was based on the current curricular guidelines for the teaching and learning of mathematics, focusing on fundamental skills, such as the 4Cs, and the analysis and discussion of rich and challenging tasks, underpinned by principles of active learning and technology integration.

Data collection involved a combination of methods, like classroom observations, which led to free-flowing notes, documents (records and sketches from the participants' notebooks; written reports – global report and ED steps guide report), the student teachers' artifacts (3D-printed models), and visual records (photos, computer screenshots). These methods allow for triangulation [37], enhancing the validity and reliability of the findings [35], enabling a nuanced understanding of the pre-service teachers' experiences and challenges in adopting digital making.

To analyze and systematically interpret data, we used a qualitative, inductive approach, relying on content analysis [38], drawing from multiple data sources to reinforce credibility. After repeatedly consulting and reading the information, as well as cross-referencing the evidence, we generated categories of analysis, influenced mainly by the research questions, complemented by the theoretical framework and the data collected. These included: Performance across the ED cycle (Problem, Imagine, Design, (Re)Build, (Re)Test and Evaluate, Redesign and Solution); and perceptions regarding the use of Tinkercad and 3D printing.

3.2. The didactical experience

The didactical experience underlying the study was structured into four key stages developed during the lessons of the unit course. Table 1 summarizes these stages, identifying the corresponding modules, the contents associated with each module, and the resources used along the process.

Table 1. Overview of the didactical experience.

Modules	Contents	Resources
Framework	▪ STEAM Education	
	▪ Maker movement	
	▪ ED process	
Technical training	▪ Tinkercad	Tinkercad
	▪ 3D printing	3D printer
Making manipulatives	▪ Geometry and Measurement concepts	Manipulatives
	▪ Number concepts and properties	Measuring tools
	▪ Proportional relations	Tinkercad
	▪ Mathematical connections and representations	Geogebra
		3D printer
Evaluation	▪ Connections among mathematical ideas and between different disciplines	
	▪ Integrative approach to problem solving	

Throughout their training, these student teachers had diverse experiences framed by current trends in mathematics education, particularly focusing on working with tasks, emphasizing problem-solving, and supported by the use of manipulatives in the classroom, given the importance of these resources. Given the importance of authentic problems and interdisciplinarity, we considered STEAM education a meaningful framework to connect these dimensions, specifically highlighting the importance of problem-solving and the potential for establishing connections. With the abovementioned modules, we aimed to enable pre-service teachers to create manipulatives of their choice using digital making and to reflect on and discuss their choices through the ED process, a widely recognized approach to STEAM education.

The first module (one 3-hour lesson) introduced STEAM education as a contemporary trend with the potential to foster the solving of authentic, real-world problems through an interdisciplinary perspective. A particular emphasis was placed on the Maker Movement and its alignment with STEAM principles, highlighting how Making, which involves hands-on, project-based learning, can serve as a powerful tool to engage learners in design thinking. In a more specific outlook, we introduced and analyzed Engineering Design as a structured process that promotes deeper levels of integration between STEAM disciplines, identifying it as a problem-solving model suited for engineering problems, using the Vale et al. [25] model presented in Figure 1.

Before the implementation stage, participants underwent technical training with the tools they would be using, namely Tinkercad and 3D printing. Tinkercad was selected due to its intuitive interface, making it suitable for educational purposes, as well as its seamless integration with 3D printers. Since none of the pre-service teachers had prior experiences with CAD software, particularly Tinkercad, or 3D printing, it was essential to provide them with some exploration time to familiarize themselves with both the software and the printer. This training module (two 3-hour lessons, 6 hours) was a crucial step, as pre-service teachers should not only be aware of available technological resources but also gain firsthand experience in using them effectively. Developing this confidence is key to leveraging digital tools to their full potential in educational contexts. During this stage, participants were challenged to design a personalized keychain with their name in Tinkercad, serving as an engaging introductory task to digital design. Through this activity, they explored several core functionalities of Tinkercad, such as resizing, rotating, changing views, and adding holes to geometrical objects. Additionally, they were introduced to the 3D printing workflow, using Ultimaker Cura to slice the models before observing the printing process. The 3D models were printed using a Creality Ender-3 S1 Plus printer, equipped with a 0.4 mm nozzle and supporting a layer resolution of up to 0.2 mm. The maximum build volume was 300 x 300 x 300 mm, and PLA filament was selected for its safety and ease of use in educational environments. These technical specifications informed participants' design strategies and helped them consider physical constraints such as size, resolution, and structural support when designing manipulatives. The hands-on exposure enabled them to understand how digital designs are transformed into physical objects, providing a concrete experience of the making process.

The third and main stage of the didactical experience (two 3-hour lessons, 6 hours) was the implementation module - “Making manipulatives”. During this module, participants were engaged in problem-solving and finding a solution. The proposed problem was "Build a 3D model of a manipulative of your choice using Tinkercad and 3D printing, applying the ED cycle". The student teachers had complete freedom in selecting their manipulatives; no specific mathematical content or educational level was prescribed. During these lessons, they worked in groups—five pairs and one trio—and received ongoing support from the researchers throughout the process. They were encouraged to use any tools they deemed necessary at different stages of the task. As a result, participants incorporated various measuring instruments and digital resources, such as GeoGebra, to support their design decisions. Additionally, they were asked to document their process methodically to foster deeper reflection about their work. Their records included detailed notes on all steps of the ED cycle, key questions, discoveries, and design decisions, among other relevant aspects. We expected that, throughout the problem-solving process, these pre-service teachers would demonstrate diverse knowledge (CK, PK, TK) and skills, given the nature of the iterative and multidisciplinary procedures involved in Engineering Design.

In the final stage, the evaluation module, participants were asked to produce a group-written report reflecting on their experience. In this report, they were expected to explain the reasons behind their choice of manipulative, sharing the challenges and learning experiences throughout the activity, and propose a classroom task in which the manipulatives could be effectively used. Additionally, they were required to document all stages of the ED cycle, detailing the decisions made along the way. This reflective component aimed to encourage a deeper analysis of the design, implementation, and pedagogical potential of the manipulatives they created.

4. Results and discussion

To ensure clarity regarding our aim and findings, the results are structured according to the Engineering Design cycle, enabling a detailed analysis of how pre-service teachers progressed through each phase, from identifying a mathematical challenge to implementing and evaluating a tangible solution. Particular attention is given to how sketches influenced the accuracy of CAD models and how iterative testing and feedback informed refinements. The findings emphasize learning outcomes such as enhanced mathematical precision, spatial reasoning, and awareness of classroom applicability.

The results are presented and discussed in two sections, starting with the performance along the ED process (the implementation stage of the didactical experience – Making manipulatives) and then advancing to the pre-service teachers' perceptions of Tinkercad and 3D printing. To report the major findings, we used the information from the observational notes, the written reports,the global report, the ED steps guide report, the produced artifacts, and the illustrative photos.

4.1. Emerging knowledge and skills throughout the ED cycle

4.1.1. Problem

The proposed problem was: Build a 3D model of a manipulative of your choice using Tinkercad and 3D printing, applying the ED cycle. Students had the freedom to choose any manipulative material, as long as they justified their selection. The classes of this unit course took place at the Mathematics Education Laboratory (LEM), which comprises a wide variety of manipulatives commonly used by these students in mathematics classes (Figure 2).

Figure 2. Variety of manipulatives available in the LEM.

The pre-service teachers started by choosing a model, which was not limited to the options available in the LEM. To inform their decisions, they explored the resources within this space, conducted online research, and in some cases, drew inspiration from previous didactical experiences. The manipulatives selected by the groups were the following: fractions game (Group 1), tangram (Group 2), geoboard (Group 3), pentominoes (Group 4), tic-tac-toe (Group 5), and the Tower of Hanoi (Group 6).

Participants justified their choices based on different criteria. Some selected a manipulative due to the lack of similar resources available (e.g., fractions game), while others emphasized its importance in supporting the learning of specific mathematical contents (e.g., fractions game, tangram, geoboard, pentominoes). Certain choices were motivated by the potential to enhance problem-solving skills and collaborative work (e.g., pentominoes), whereas familiarity played a key role in selecting tangram and tic-tac-toe. Some groups opted for manipulatives that are less commonly used for learning mathematics concepts (e.g., Tower of Hanoi, tic-tac-toe), aiming to explore their potential in new learning contexts. The following excerpts illustrate these justifications: *"Among the various manipulative materials available, we chose the fractions game because we believe it can be a useful tool to teach fractions, making use of visualization... Moreover, we consider that there is a lack of such materials... For us, it is also a challenge since we do not have a reference to construct our model"* (Group 1); *"The choice of this manipulative (geoboard) is due to its relevance in teaching mathematical concepts, especially in exploring geometric shapes, properties, and spatial relationships"* (Group 3).

Once each group selected a manipulative, they examined a physical reference model (Figure 3), either one from the LEM collection or one created through a hands-on construction by the group (as in the case of the fractions game).

After delimiting the problem and establishing the context in which they would be working, participants identified several key constraints and design considerations that needed to be addressed throughout the development process. One of the primary concerns was ensuring that the dimensions of the model were compatible with the 3D printer's plate, requiring careful scaling and optimization to fit within the available printing area. Additionally, minimizing filament waste was an important factor, as excessive material usage could lead to inefficiencies, increased costs, and environmental impact. Another crucial aspect was the creation of a storage box to accommodate the manipulatives, ensuring both organization and durability. This introduced an additional layer of complexity during the planning process, as participants had to consider not only the dimensions of their model but also how it would fit within the box.

Figure 3. Physical models used by each group.

4.1.2. Imagine

Once the manipulative and reference model were selected, participants began the brainstorming process, engaging in a detailed exploration of the chosen material. This step involved a thorough analysis of each manipulative's characteristics, including its function, composition, dimensions, relationships between its elements, among other aspects. By carefully examining these features, they gathered important information to make informed decisions in the following ED steps. To deepen their understanding, participants interacted with the manipulatives directly, taking measurements and making written records. This hands-on exploration allowed them to become familiar with the materials and their structural properties (Figure 4), fostering a more intentional approach to the design process.

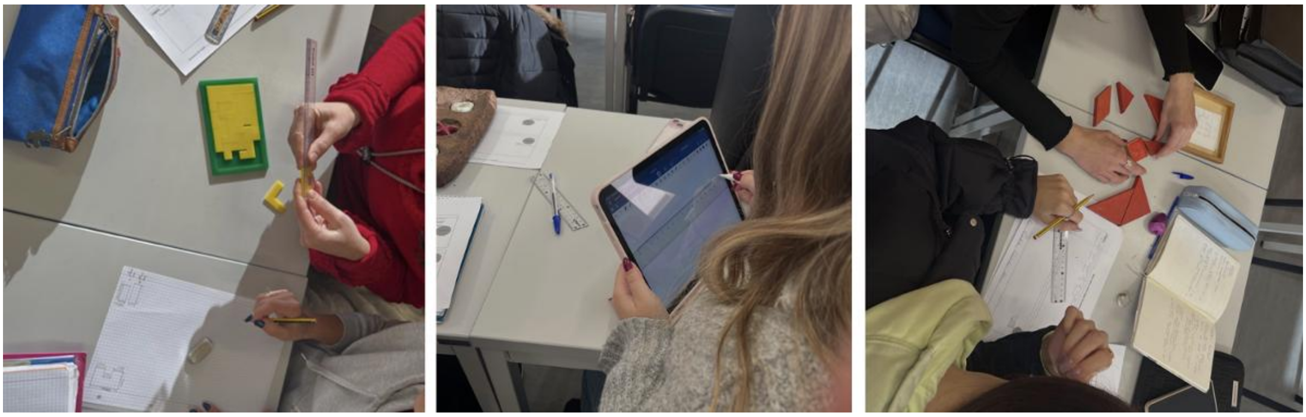

Figure 4. Pre-service teachers getting acquainted with the manipulatives.

Groups 2 (Tangram) and 3 (Geoboard) explored more than one model during this phase, each for different reasons. The student teachers working with the Tangram (Group 2) initially selected a model but soon discovered some flaws in its design, as the pieces did not fit together perfectly. This led them to analyze alternative versions of this manipulative to ensure greater accuracy (Figure 5). Group 3 encountered challenges with their first geoboard, which was opaque and had a raised edge, making precise measurements difficult to perform. To overcome this, they used a translucent geoboard, which enabled better visibility, and applied a measuring tape instead of a ruler, as it offered greater flexibility for their measurements. Even so, they had to explore an additional geoboard model, this time with different dimensions and without the edge, allowing a more precise analysis (Figure 6).

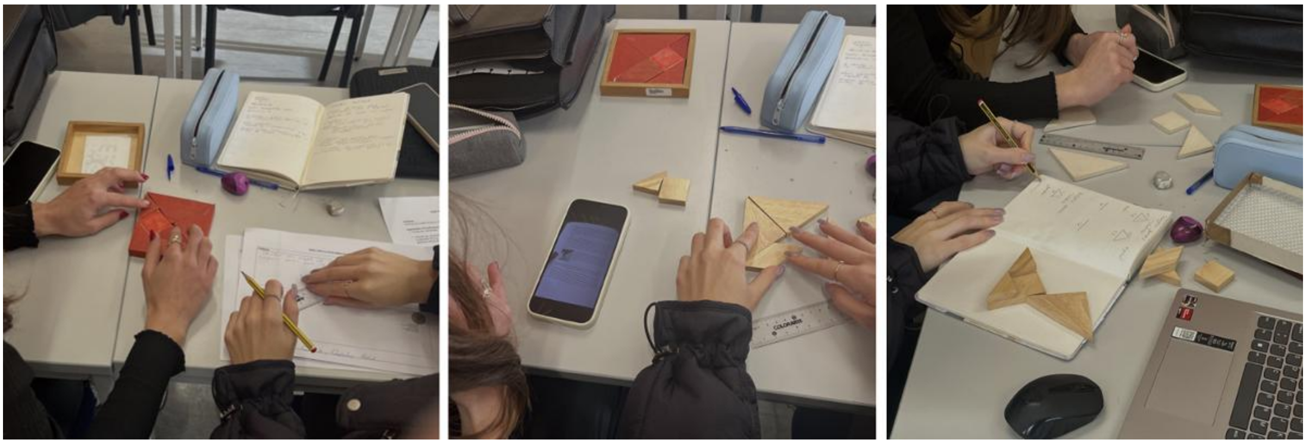

Figure 5. Different Tangram models.

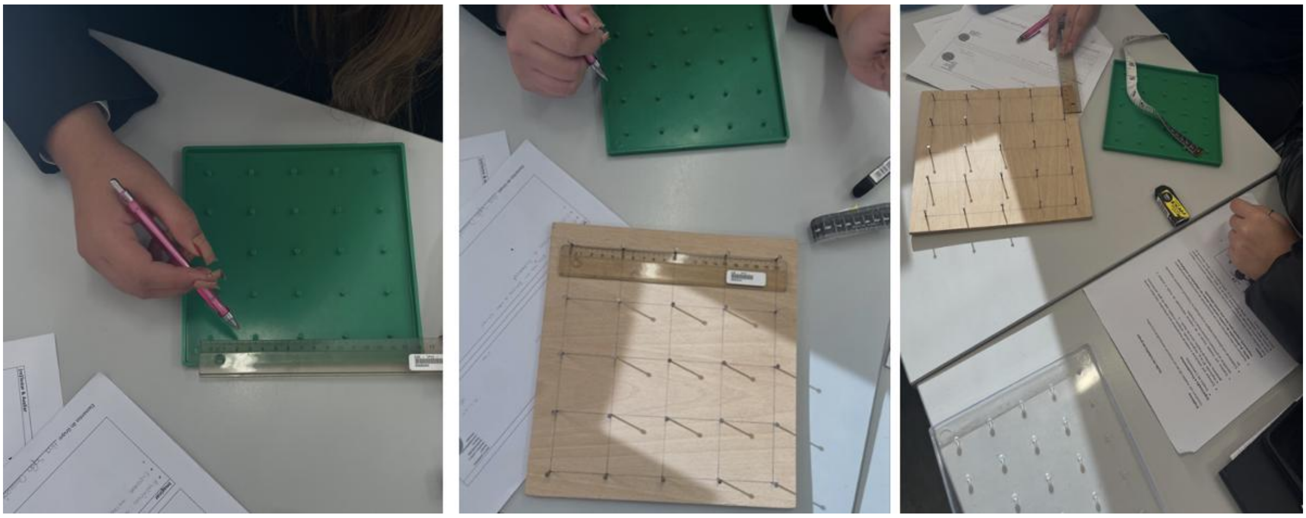

Figure 6. Different Geoboard models.

To develop a more realistic perception of scale and make informed decisions regarding the dimensions of their models, some groups used noteworthy strategies. Group 5 started by analyzing an existing tic-tac-toe game available in the LEM, which they considered excessively large (first image in Figure 7). To address this, they adjusted its dimensions, aiming to reduce the overall area. To better visualize a suitable size, they compared the game board to everyday objects, such as a mobile phone, using it as a reference for scaling (second image in Figure 7). Similarly, some participants, like those in Group 3 (geoboard), took a practical approach by placing the reference manipulative directly on the 3D printer's plate. This helped them visualize the printing constraints and ensure that their design would fit the available printing area (third image in Figure 7).

Figure 7. Strategies used by participants to decide the manipulatives' dimensions.

Most participants were able to gather key information about their models before progressing to the planning phase. Their observations provided valuable insights into the structure and properties of the manipulatives. Although most groups showed a solid understanding of their manipulatives' characteristics, some descriptions lacked mathematical precision and omitted important details, such as geometric relationships and proportionality.

Group 1, working with the fractions game, described their process as follows: *"We performed folds starting from a 12 cm x 12 cm square to obtain different geometric shapes representing parts of the whole, such as 1/6, 1/4, 1/8, and 1/16"*. This hands-on approach enabled them to conceptualize fractional divisions and establish equivalences, visually and structurally. Group 3 analyzed the geoboard, noting that: *"It has a square base with 15 cm sides, an outer edge of 1 cm height, and an inner edge of 0.7 cm height with a thickness of 0.5 cm. The 25 pegs were 0.7 cm high and 0.2 cm wide, arranged in a 5x5 square grid"*. However, their description lacked specific details regarding the position and spacing of pegs, which later impacted their design process. Group 4 focused on the pentominoes, defining them as *"a polyomino composed of five congruent squares connected orthogonally. Reflective and rotational symmetry do not create distinct pentominoes"*. Their explanation demonstrated conceptual understanding of the mathematical principles underlying the manipulative. Group 5, working on the tic-tac-toe game, described their model as *"a square board subdivided into nine uniform squares in a 3x3 grid, requiring nine pieces of two different shapes, one set for each player, designed to fit into the board's cavities"*. The description emphasized the structure and interactive components of the game. Group 6, which selected the Tower of Hanoi, noted that its composition *"consists of three equidistant rods, each 8 cm high, fixed on a base measuring 8 cm x 23 cm, with seven independent discs of different diameters that can be manipulated"*. However, their description lacked details about the relationships between the discs, which are essential to understanding the manipulative's intended functionality. Group 2, working with the tangram, concluded that it *"consists of seven pieces: Two large isosceles triangles, one medium isosceles triangle, two small isosceles triangles, one square, and one parallelogram. Together, these shapes form a 10 cm x 10 cm square and that the angle measures are multiples of 45º, specifically 45º, 90º, and 135º. There is a proportional relationship between the areas of the shapes"*. This description, while informative, lacked mathematical rigor in identifying key geometric relationships, which influenced the subsequent stages of their work.

4.1.3. Design

After completing their exploration and brainstorming within each group, participants reflected on their ideas and moved into the planning step, creating sketches that represented the design of their models. These sketches served as a visual blueprint, allowing them to refine their concepts before proceeding to the construction. With the exception of group 3 (geoboard, Figure 8), all groups produced clear and complete sketches (Figure 9). Contrary to previous studies [6,25], this stage was more highly valued by these future teachers before progressing to the building step.

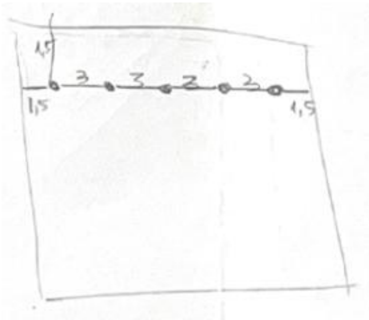

Figure 8. Incomplete sketch produced in the *Design* step.

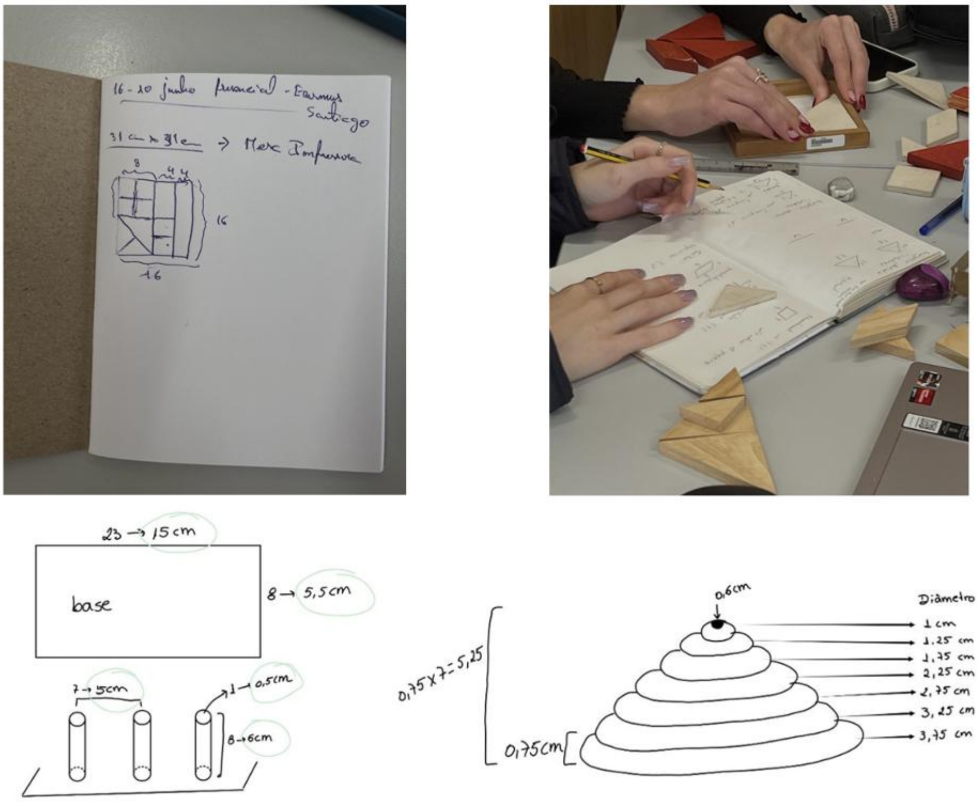

Figure 9. Clear and complete sketches produced in the *Design* step.

Group 4, having decided to maintain the original dimensions of the pentominoes, simplified the sketching process by tracing the shapes onto their record sheet and indicating the respective measurements. They also found it helpful to use graph paper to support the pictorial representation of the material (Figure 10). Moreover, the groups that decided to build a storage box for their manipulatives (groups 1, 2, and 4) also planned the shape and dimensions of the box during this stage. Group 5, needing to reduce the size of the tic-tac-toe game they used as a model, chose to use a tangram box as a reference, adapting its dimensions to fit the dividers and game pieces (Figure 11).

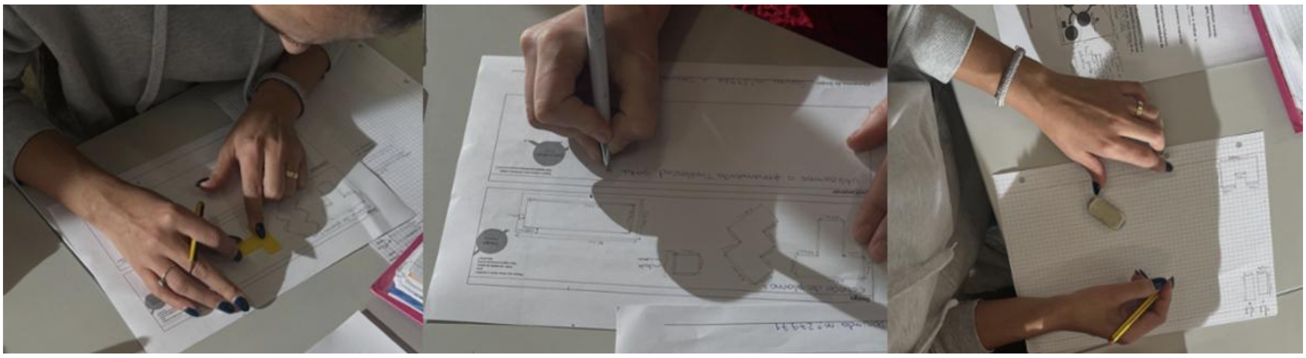

Figure 10. Strategies used by group 4 in the *Design* step.

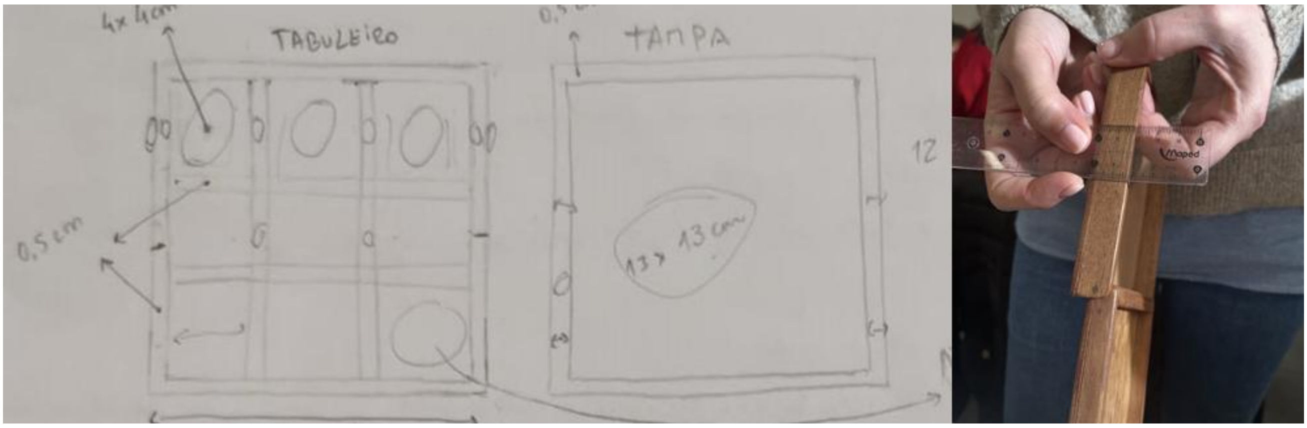

Figure 11. Strategies used by group 5 to design the box.

The hand-drawn sketches played a pivotal role in supporting the 3D printing process. They functioned as visual planning tools, enabling participants to externalize and refine their ideas before transitioning to digital modeling. By explicitly defining dimensions, spatial relationships, and intended functionalities, the sketches served as a blueprint that guided the CAD construction in Tinkercad. Groups that created more accurate and detailed sketches experienced fewer problems during the modeling and slicing stages, indicating that the sketches were essential for bridging conceptual thinking with digital execution.

4.1.4. Build

After imagining and designing the manipulatives, participants moved on to building the 3D models using Tinkercad (Figure 12). As part of the didactical experience, the pre-service teachers previously explored this CAD environment, familiarizing themselves with key features (e.g., drag and drop, resizing, rotating, changing views, grouping), so they would feel comfortable using this tool to solve the problem.

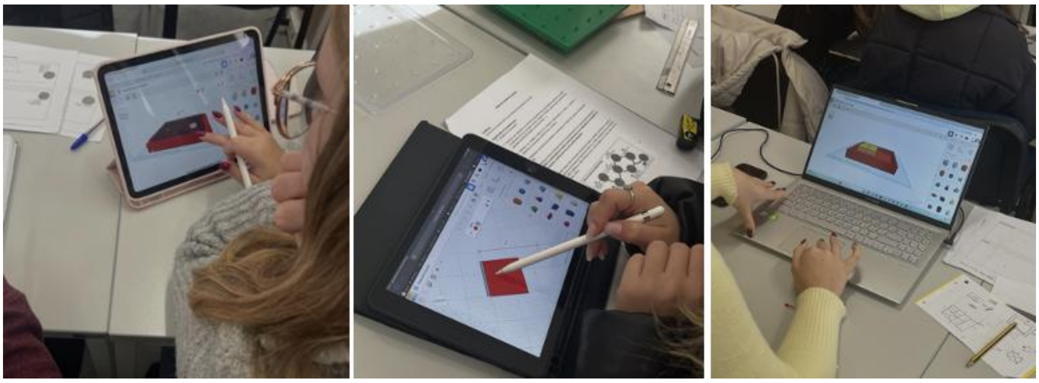

Figure 12. Building the model with Tinkercad.

Overall, they completed the constructions without major difficulties. In their written reports, each group detailed the procedures used in Tinkercad to construct their model, including screenshots of the various stages as evidence (Figure 13).

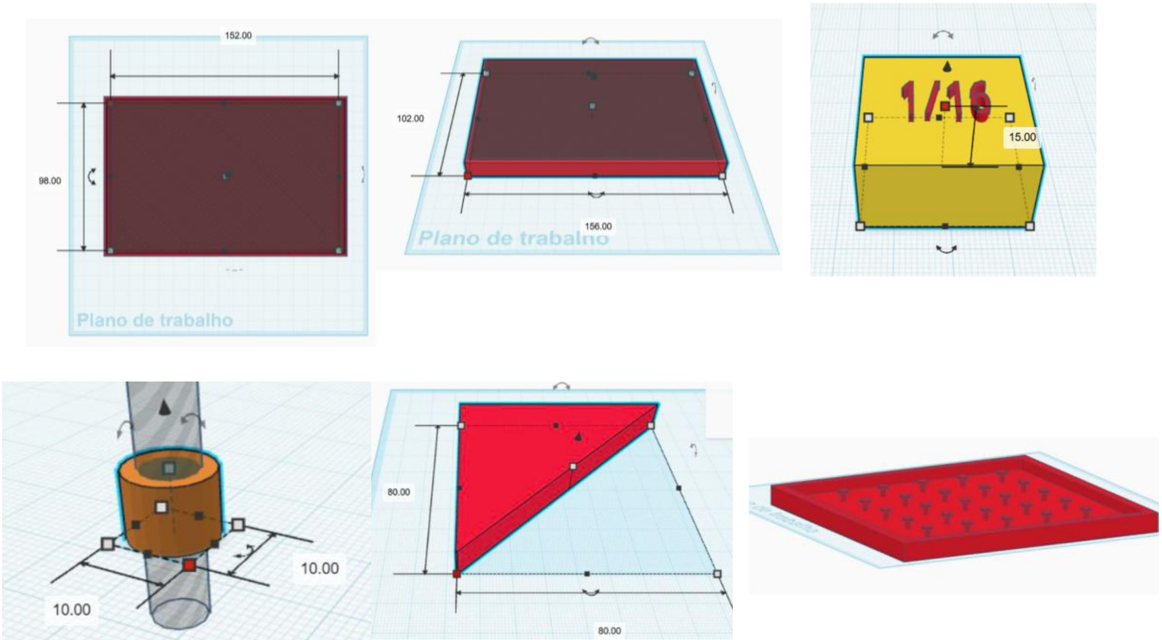

Figure 13. Screenshots of the work developed with Tinkercad.

Some groups encountered occasional challenges when they manipulated their models in Tinkercad, particularly when positioning elements correctly. Some models were positioned outside the workplane, floating above or below it, while others were not parallel to the plane (tilted/misaligned), requiring careful adjustments. The group that faced the greatest difficulties in constructing their model was Group 3, particularly in precisely arranging and aligning the pegs on the geoboard’s square base and grouping them altogether. Group 2 (tangram) used GeoGebra to accurately confirm the measurements of the shapes, constructing them in this software before transferring them to Tinkercad (Figure 14). Group 6 (Tower of Hanoi) built the base and the discs with relative ease; however, during the slicing process, they realized that their model only had six discs instead of the seven required for the Tower of Hanoi, which had direct implications in the next step, Test and Evaluate.

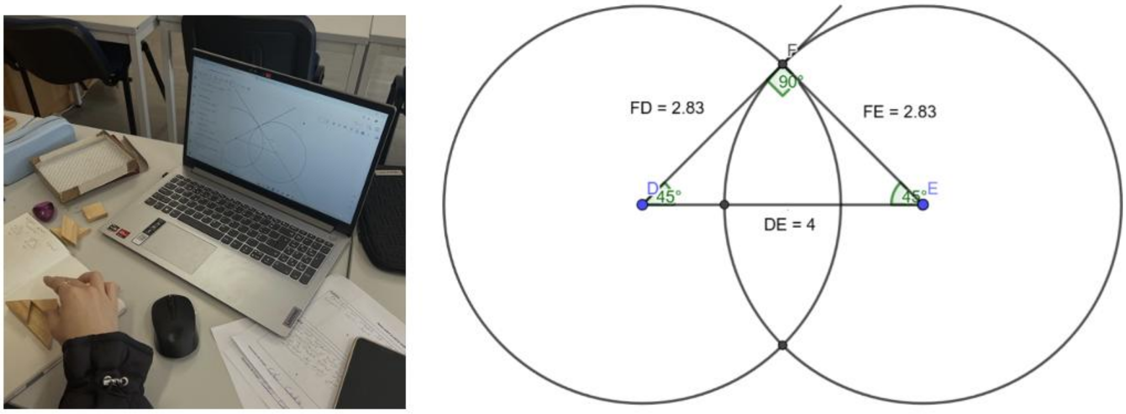

Figure 14. Group 2 using Geogebra to confirm the measurements.

As the groups completed the different parts of their manipulatives, they proceeded with 3D printing (Figure 15), enabling them to assess the adequacy of the product and make improvements as needed during the ED process, and not only at the end.

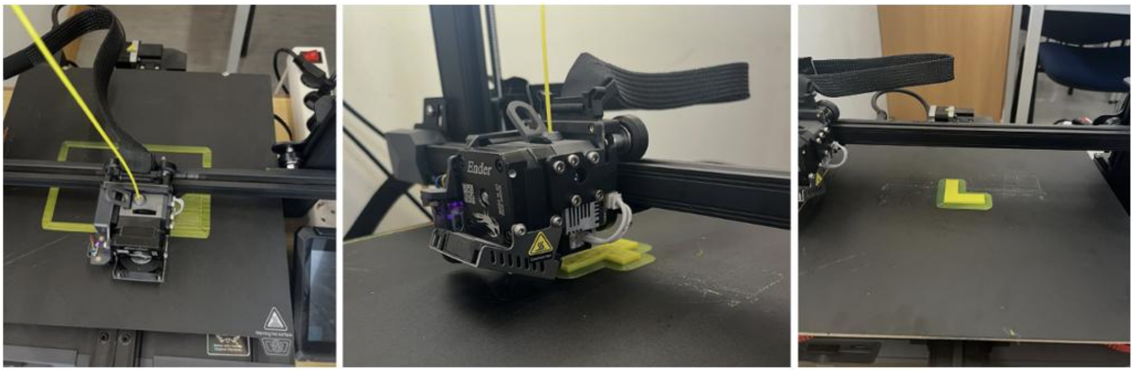

Figure 15. Printing parts of the manipulatives during the *Building* step.

4.1.5. Test & Evaluate and Redesign & Build

As expected in an iterative process, most groups made several changes, either before printing the model or after the prototype was printed, which emerged after testing and evaluating what was done. Only groups 3 (geoboard) and 5 (tic-tac-toe) achieved a solution without making changes. Group 1 (fractions game) initially constructed the shapes individually, proceeding with the 3D printing in parallel. The first printed shape revealed they had made an error in the construction, as it was not a quadrangular prism. This mistake led them to reassess the accuracy of the construction of the remaining pieces. Additionally, the printer did not recognize the fraction line on the shape, requiring them to reinforce the thickness of the line when reconstructing the shape (Figure 16).

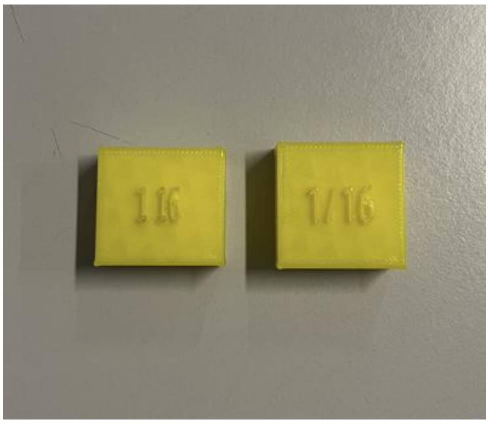

Figure 16. Evaluating and Redesigning after printing parts of the manipulatives (group 1).

Like other groups, group 2 built the tangram pieces while simultaneously proceeding with 3D printing. They started by printing the two smallest triangles of the tangram. After printing the pieces, they found it odd that when joining the triangles, they did not form a square or a larger triangle (Figure 17).

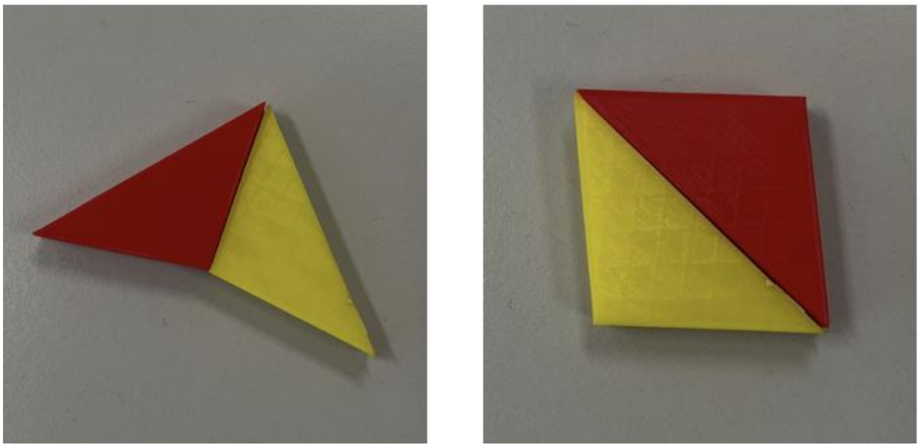

Figure 17. Evaluating and Redesigning after printing parts of the manipulatives (group 2).

Upon closer analysis, they realized that although the triangles were isosceles, they were not right-angled, a mistake that originated in the Imagine step when they failed to identify this characteristic while analyzing the initial model of the tangram. As a result, they had to reconstruct all five tangram triangles, as they all shared the same problem. Using Tinkercad, they recognized that the initially chosen shape, "Roof," was a triangular prism that did not allow them to create the correct tangram pieces. They then opted to use the "Box" shape instead, ensuring the presence of right angles (Figure 18).

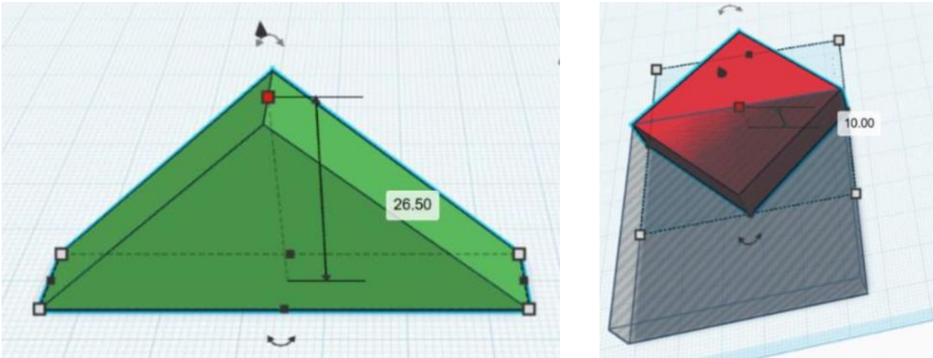

Figure 18. Rebuilding parts of the manipulatives (group 2).

Group 6 (Tower of Hanoi) had built their model in Tinkercad using millimeters as a unit measure. However, they overlooked this feature of Tinkercad, forgetting the unit and assuming they were working in centimeters, like in the planning step, focusing only on the numerical values. When they proceeded to the slicing stage in Ultimaker Cura, they noticed the error. The software estimated two minutes to print the tower's base, which was unrealistic. Although the STL file visually appeared to correspond to the expected model, the slicing program revealed the error made (Figure 19). This step allowed them to verify that the 3D model was not correctly scaled and not suitable for printing. The second image in Figure 19 shows that the model was far too small.

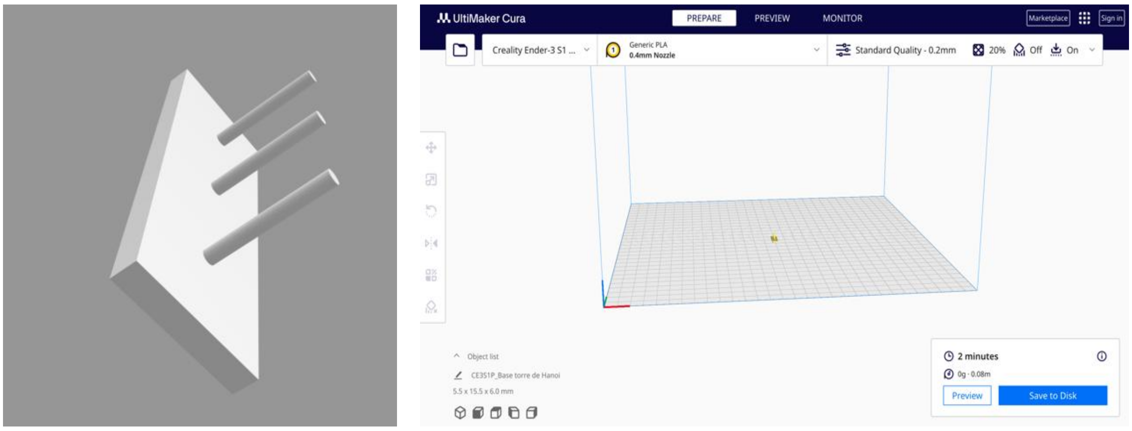

Figure 19. Evaluating and Redesigning after printing parts of the manipulatives (group 2).

Most groups, who decided to build a box to store the materials, had to reassess its dimensions to ensure the pieces fitted properly before proceeding with the 3D printing (groups 1, 2, and 4). In these cases, they could evaluate before printing, benefiting from the observation of unsuccessful experiences of other groups. For example, group 1 (fractions game) printed a box that did not match the dimensions of the pieces, requiring them to redesign the box and reprint it (Figure 20).

Figure 20. Mismatch between printed box and manipulative dimensions (group 1).

In conclusion, it is important to highlight that, in the *Building* step, most groups who selected manipulatives with multiple components found it essential to print the pieces in predefined colors using different filaments. Their choices demonstrated pedagogical awareness by recognizing the importance of visual differentiation to enhance understanding and engagement. This careful consideration underscores the role of visualization in learning, particularly in facilitating conceptual clarity and interaction with the materials.

4.1.6. Solution

The final stage of the didactical experience consisted of a written report where each group described and reflected, among other aspects, on how they solved the problem using the ED process. Through this report, it was possible to access more detailed ideas about the results. Each group organized and presented the data collected throughout the experience, although with different levels of depth in the argumentation. Additionally, as part of the report, they were required to propose a task in which the designed manipulative could be effectively applied in a learning context.

Throughout the process, student teachers faced some challenges, including measurement inaccuracies, misalignment issues in Tinkercad, and difficulties adapting their designs to the constraints of the 3D printer. However, by actively engaging in evaluation, redesign, and rebuild, they successfully improved their models, ensuring that the final outputs met their educational objectives. Although they encountered setbacks, the iterative nature of the ED process supported systematic revisions, testing, and peer collaboration. Each group refined their models through trial and error, demonstrating the importance of hands-on learning in developing both technical and pedagogical skills. Their final presentations (Figure 21) showcased improved visualization and spatial reasoning skills, enhanced by the experience of working with 3D modeling software. Students acknowledged that error analysis and iterative redesign were essential steps, reinforcing a growth mindset in problem-solving.

Figure 21. Final 3D-printed models.

Reflecting on their experiences, the students identified several aspects they would approach differently if given the opportunity to repeat the project. One of the most significant changes would be a more thorough planning phase, particularly in terms of measurement accuracy and unit conversion in Tinkercad.

Regarding the proposed tasks, it was observed that students revealed varying levels of depth in terms of Content Knowledge (CK) and Pedagogical Knowledge (PK). A common issue among almost all groups, except for Group 5, was the lack of curricular alignment, as they did not specify the educational level for which their task was intended. Groups 2 and 4 identified the potential of the tangram and the pentominoes in exploring various geometric concepts, respectively, but they limited their proposals to simple figure composition without further exploration. In the case of the geoboard, group 3 suggested using the material to construct different polygons and compare their properties but made a CK-related error by asking students to calculate the perimeter by counting the number of pegs along the polygonal line. Group 1 proposed using the fraction game in an area problem to highlight part-whole relationships. Group 6's Tower of Hanoi followed its traditional use, focusing on discovering a rule that represents the minimum number of moves required for a given set of disks. Group 5, however, stood out by introducing tic-tac-toe into the classroom as a means to explore mean and mode concepts in 5th grade, using the number of wins or ties in 10 rounds as data. While the task is innovative and engaging, it presents a CK-related error, as the variable in question is qualitative, making the proposed statistical calculations inappropriate.

4.2. Participants' perceptions of Tinkercad and 3D printing

Throughout the experience, pre-service teachers better understood the potentials and limitations of using Tinkercad and 3D printing.

Most considered Tinkercad to be an intuitive tool, with a simple interface, that would seamlessly translate their designs into printable models. However, as they progressed, they realized that precise measurements, correct unit settings, and structural considerations played a crucial role in ensuring a successful print, so they would have to pay more attention to them. One of the most common insights was the need for meticulous attention to measurement accuracy. Some groups encountered issues because they did not immediately recognize that Tinkercad works in millimeters, leading to models that were either too small or incorrectly scaled. This highlighted the importance of double-checking dimensions and conversions before finalizing a design.

Student teachers also noted that Tinkercad's default geometric shapes, while useful, required careful manipulation to fit specific project needs. For example, the tangram group initially selected the "Roof" shape to create triangular pieces but later realized that it did not produce right-angled isosceles triangles, leading to errors in the printed version. This experience reinforced the importance of understanding the software's shape properties and making necessary adjustments early in the design phase. They also recognized this as an opportunity to promote the learning of geometric concepts and the development of spatial thinking.

Regarding 3D printing, the participants reacted positively and enthusiastically to the possibility of virtually building an object and converting it into something tangible. They quickly became aware that what appears correct in the Tinkercad virtual environment may not translate perfectly to a physical print. Some models seemed structurally sound on screen but failed to print as expected due to an incorrect construction, filament thickness, or printer resolution limitations. They also realized that the slicing phase can be an opportunity to evaluate if something is wrong. Another key realization was the importance of designing with 3D printing constraints in mind. Several groups had to redesign their boxes after discovering that filament thickness could alter the final fit of the pieces. These challenges emphasized the need to anticipate material properties and adjust designs accordingly. Students also developed a greater appreciation for iteration and real-world testing. Initially, many assumed their first design attempts would be sufficient, but errors in alignment, sizing, and proportions became evident only after printing. This underscored the trial-and-error nature of 3D modeling, where testing prototypes and refining designs based on real-world feedback is essential.

5. Concluding remarks

We aimed to explore the integration of STEAM education through Engineering Design in teacher education, incorporating a CAD environment and 3D printing as central tools. The focus was on examining knowledge and skills that emerged across the ED process, as well as the perceptions of pre-service teachers about the use of Tinkercad and 3D printing. Understanding (future) teachers' performance and perceptions in such contexts is particularly relevant, as they align with current curricular recommendations and educational trends. Additionally, it is crucial to provide teachers with firsthand experiences that reflect the teaching and learning principles they will be expected to implement in their own classrooms [24]. The study involved thirteen pre-service elementary teachers, who participated in a didactical experience structured into sequential stages. These included theoretical introductions, technical training in digital tools (Tinkercad and 3D printing), followed by implementation (digital making) and evaluation, all within the framework of solving an ED-based problem. The study's key findings are presented and analyzed according to the research questions, using data triangulation and drawing on the theoretical framework to support the interpretations.

The results provide valuable insights into the development of pre-service teachers' knowledge and skills as they progress through the Engineering Design cycle while using Tinkercad and 3D printing. Their learning trajectory highlights the potential of STEAM education contexts [15] and the potential of digital making to foster problem-solving, spatial reasoning, and pedagogical reflection [2,3].

Participants were contacted with an authentic problem, the digital making of a math manipulative, that demanded the application of multidisciplinary concepts in the scope of STEAM Education, resorting to the ED process. The ED model used was the one proposed by Vale et al. [25] (Figure 2), consisting of seven steps: Problem, Imagine, Design, (Re)Build, (Re)Test and Evaluate, Redesign, and Solution. The pre-service teachers strongly engaged with the problem, appreciating the possibility of choosing and making their own manipulatives. The reasons for their selection reflected their CK and PK by highlighting their relevance in mathematical learning. However, some of the participants lacked curricular specificity, reinforcing findings from Boice et al. [17] that pre-service teachers often struggle to connect hands-on projects with explicit curriculum goals. During the Imagine phase, participants conducted a detailed analysis of their chosen manipulative, assessing aspects like structure and mathematical relationships. This step reinforced CK, as they explored geometric properties, proportionality, and spatial relationships. However, some groups struggled with the presentation of accurate mathematical descriptions, such as the misidentification of isosceles right triangles in the tangram, a common issue among pre-service teachers with limited experience in mathematical modeling [29]. The Design step required participants to create sketches and plan their manipulatives, integrating TK as they would be required to engage with digital representations afterwards. CK was also present, as the sketches translated the manipulatives' properties. Most groups successfully organized their ideas into clear and structured designs, reflecting an understanding of spatial constraints and usability. Unlike previous studies performed by the authors in [6,25], where planning was often overlooked, pre-service teachers in this study valued the importance of structured pre-design work, recognizing its role in enhancing efficiency and reducing errors. The Building step marked a shift from conceptual planning to digital making, deepening participants' TK. They effectively applied CAD principles, manipulating Tinkercad's features for resizing, rotating, aligning, and grouping objects. However, some challenges arose, particularly regarding scaling and unit conversion—a common issue in digital making with Tinkercad [6]. One group felt the need to cross-check measurements in GeoGebra during this ED step, demonstrating an adaptive approach to technology integration [31]. Obviously, CK was also required to transfer the manipulatives' properties into the digital environment. The Test & Evaluate phase was critical in reinforcing the iterative nature of learning. Many groups encountered unexpected errors, such as misalignment in 3D-printed pieces or incorrect geometric properties, requiring them to redesign and refine their models, thus engaging both CK and TK. For example, Group 6 mistakenly constructed the Tower of Hanoi in millimeters instead of centimeters, a realization that only surfaced in the slicing stage. This highlights the importance of trial-and-error in digital making [27]. The Solution step required participants to write a report and propose a task using their manipulative. Their reflections demonstrated growing PK, as they considered how the ED process and the digital tools used could be effectively integrated into mathematics education. However, some of the proposed tasks lacked curricular specificity, indicating a need for further emphasis on explicit content-pedagogy connections in teacher education [19].

The didactical experience provided an opportunity for pre-service teachers to develop a range of essential skills, including problem-solving, communication, reasoning, collaboration, and creativity.

Each of these skills emerged at different stages of the ED cycle, reinforced by the integration of Tinkercad and 3D printing throughout the learning process.

Problem-solving was a core skill developed throughout the experience, as participants were required to identify challenges, propose solutions, and refine their models through multiple iterations. This was particularly evident in the Building and Evaluation steps, where participants faced technical difficulties such as misaligned objects in Tinkercad, incorrect scaling, and unexpected 3D printing constraints [27]. Rather than discarding their initial designs, participants analyzed issues, explored alternative solutions, and modified their models accordingly. Effective communication played a crucial role in every step of the ED cycle, particularly in group collaboration, discussions, and report writing. Participants needed to articulate their reasoning, justify their choices, and present their designs clearly, ensuring that their ideas could be understood by their peers and the instructors [6,17]. The quality of their written communication varied, with some groups providing detailed explanations with explicit curricular links, while others struggled to articulate the connections between their models and mathematical learning outcomes [17]. The ability to reason mathematically was particularly evident in the Design and Testing phases, where participants had to translate abstract mathematical concepts into physical models. This process required them to analyze proportions, geometric properties, and numerical relationships, ensuring their manipulatives were accurate, functional, and pedagogically effective [29]. Collaboration was a defining feature of the didactical experience, as participants worked in small groups, engaging in discussion, negotiation, and cooperative problem-solving [6,19,25]. In several cases, peer feedback played a crucial role in detecting and correcting errors. Discussions with other groups helped them identify and correct miscalculations, illustrating how collaborative engagement enhances problem-solving accuracy [1]. Creativity emerged as an essential skill throughout the ED process, with participants having to think beyond existing models and explore innovative possibilities. The open-ended nature of the problem encouraged them to experiment with different representations, alternative materials, and unique modifications to traditional manipulatives [33].

The pre-service teachers' reflections revealed enthusiasm for the creative potential of Tinkercad and 3D printing but also identified technical and pedagogical challenges that need to be addressed to ensure effective classroom implementation. These perceptions align with findings in the literature on digital making in STEAM education [3,6,19,27], highlighting both the opportunities and limitations of using computer-aided design and 3D printing in teacher training programs.

One of the most frequently mentioned advantages of using Tinkercad and 3D printing was their engaging and interactive nature, which pre-service teachers perceived as a highly motivating approach to learning. Many participants highlighted the visual and tangible aspects of digital making, emphasizing that manipulating virtual objects in Tinkercad and subsequently transforming them into physical models enhanced their spatial reasoning and conceptual understanding [2,6]. Participants also recognized the potential of 3D printing to bridge abstract mathematical concepts with real-world applications, reinforcing the idea that making tangible models deepens conceptual understanding [27]. Several groups mentioned that creating their own manipulatives led to a greater appreciation of the mathematical relationships embedded within them, such as proportions in fraction models or symmetry in pentominoes. This aligns with research by Stigberg et al. [1], who emphasize that designing and constructing educational materials enables teachers to better understand the affordances and constraints of hands-on learning resources, preparing them to integrate them more effectively in their teaching. Another commonly reported benefit of Tinkercad was its intuitive interface, which most pre-service teachers found accessible and easy to navigate, even with no CAD experience [6,29].

Despite recognizing the educational potential of Tinkercad and 3D printing, participants also encountered several challenges. One of the most cited difficulties was accuracy in measurement and scaling, which led to design inconsistencies and required multiple iterations. Many groups initially struggled with misalignments, incorrect unit conversions, and difficulty ensuring precise object placement, particularly when transitioning from 2D sketches to 3D models [6,7,21]. Another key concern was the technical complexity of 3D printing itself, particularly in terms of slicing, material constraints, and print failures. Several participants reported frustration with filament waste, long printing times, and errors in the final printed objects, which sometimes required redesigning the model [7]. From a pedagogical standpoint, some pre-service teachers expressed uncertainty about how to meaningfully integrate 3D printing into their future classrooms, citing concerns such as time constraints, access to resources, and the need for structured lesson plans. While participants valued the hands-on nature of digital making, some questioned whether the learning gains justified the logistical effort required. This reflects Boice et al.'s [17] argument that successful integration of emerging technologies in education requires not only technical training but also pedagogical modeling and reflection on best practices.

These findings highlight the potential of integrating Engineering Design, CAD environments, and 3D printing into teacher education. A key aspect of the didactical experience was the focus on designing and constructing mathematical manipulatives, which required participants to deeply engage with mathematical concepts, spatial reasoning, and usability considerations. While the experience strengthened technological, content, and pedagogical knowledge, some participants struggled with measurement accuracy, print failures, and curricular alignment, emphasizing the need for further pedagogical scaffolding and technical training [17]. Despite these challenges, the study underscores how digital making can deepen conceptual understanding and engagement, particularly when linked to the creation of manipulatives that enhance hands-on learning in mathematics.

Based on the results of this study, we present a few practical suggestions for teacher educators aiming to implement a similar approach with pre-service teachers. First, provide access to various physical manipulatives and encourage exploration before the design process begins. Second, introduce hand sketching as a step to support spatial reasoning and design planning. Third, ensure basic technical training in CAD tools such as Tinkercad, emphasizing how to translate conceptual designs into digital models. Fourth, consider grouping participants strategically to promote peer learning, particularly by balancing technical and pedagogical skills. Finally, schedule time for reflection and redesign, allowing students to learn from prototyping errors and reinforce their mathematical understanding through iteration. These recommendations can scaffold the learning experience and improve the effectiveness of integrating Engineering Design in mathematics education.

Therefore, this work brings clarity to how each phase of the ED cycle engages specific forms of teacher knowledge, particularly the transition from conceptual sketches to digital fabrication, and how this process supports both mathematical learning and pedagogical development in the context of STEAM education. The integration of ED and mathematics was maintained throughout the study, from conceptualization and spatial planning through sketching and modeling to final evaluation, ensuring the reflection of a coherent interdisciplinary focus. This connection is not only theoretical but also practical: participants continuously applied mathematical reasoning as part of their ED decisions, reinforcing the role of mathematics as a foundational element within the engineering problem-solving process. Moving forward, teacher education programs should ensure that emerging technologies are embedded in structured, reflective learning experiences, equipping teachers with the confidence and skills to apply 3D printing and CAD tools effectively in their classrooms to create meaningful, interactive learning materials.

Author contributions

Ana Barbosa: Conceptualization, Methodology creation, Investigation, Resources, Validation, Formal analysis, Data curation, Writing - original draft, Writing – review & editing, Supervision; Isabel Vale: Conceptualization, Methodology creation, Investigation, Resources, Validation, Formal analysis, Data curation, Writing – review & editing.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was financially supported by Portuguese national funds through the FCT (Foundation for Science and Technology) within the framework of the inED (Centre for Research and Innovation in Education), under the reference UID/05198, and the CIEC (Research Centre on Child Studies of the University of Minho), under the references UID/317.

Conflict of interest

We declare that there are no conflicts of interests.

Ethics declaration

The ethics principles for conducting research in education have been followed.

References

1. Stigberg, H., Stigberg, S.K. and Maugesten, M., Making manipulatives for mathematics education. In M. Hjorth & S. Ejsing-Duun (Eds.), *Proceedings of 6th FabLearn Europe/MakeEd Conference 2022 - An International Conference on Computing, Design and Making in Education*, 2022, 1–9. Association for Computing Machinery (ACM).
2. Ng, O.L. and Ye, H., Mathematics learning as embodied making: Primary students' investigation of 3D geometry with handheld 3D printing technology. *Asia Pacific Education Review*, 2022, 23(2): 311–323. https://doi.org/10.1007/s12564-022-09755-8
3. Greenstein, S. and Fernández, E., Learning mathematics with mathematical objects: Cases of teacher-made mathematical manipulatives. *Contemporary Issues in Technology and Teacher*

Education, 2023, 23(1).
4. English, L.D., King, D.T. and Smeed, J., Advancing integrated STEM learning through engineering design: sixth-grade students' design and construction of earthquake resistant buildings. *The Journal of Educational Research*, 2017, 110: 255–271. https://doi.org/10.1080/00220671.2016.1264053
5. Hester, K. and Cunningham, C., Engineering is Elementary: An Engineering and Technology Curriculum for Children. *Proceedings of the ASSE Annual Conference and Exposition*, 2007.
6. Barbosa, A., Vale, I. and Alvarenga, D., The use of Tinkercad and 3D printing in interdisciplinary STEAM education: A focus on engineering design. *STEM Education*, 2024, 4(3): 222–246. https://doi.org/10.3934/steme.2024014
7. Läufer, T. and Ludwig, M., "Building" knowledge by creating manipulatives with the 3D printer: A course for mathematics student teachers. *Proceedings of the 9th International Conference on Higher Education Advances (HEAd'23)*, 2023, 19–23. https://doi.org/10.4995/HEAd23.2023.16225
8. Stigberg, H., Digital fabrication for mathematics education: A critical review of the field. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*, 2022, 1–8. ERME.
9. World Economic Forum, *Future of Jobs Report 2023*, WEF, 2023. Available from: https://www.weforum.org/reports/the-future-of-jobs-report-2023/
10. Organisation for Economic Co-operation and Development (OECD), *Future of education and skills 2030: OECD learning compass 2030*, OECD Publishing, 2022.
11. English, L.D. and King, D.T., STEM learning through engineering design: fourth-grade students' investigations in aerospace. *International Journal of STEM Education*, 2015, 2(14): 1–18. http://doi.org/10.1186/s40594-015-0027-7
12. Martín-Páez, T., Aguilera, D., Perales-Palacios, F., and Vílchez-González, J., What are we talking about when we talk about STEM education? A review of literature. *Science Education*, 2019, 103(4): 799–822. https://doi.org/10.1002/sce.21522
13. Ortiz-Revilla, J., Greca, I.M. and Arriassecq, I.A., Theoretical Framework for Integrated STEM Education. *Science & Education*, 2022, 31: 383–404. https://doi.org/10.1007/s11191-021-00242-x
14. Vasquez, J., Sneider, C. and Comer, M., *STEM lesson essentials, grades 3–8: integrating science, technology, engineering, and mathematics*, 2013, Heinemann.
15. English, L.D., STEM education K-12: perspectives on integration. *International Journal of STEM Education*, 2016, 3(3): 1–8. https://doi.org/10.1186/s40594-016-0036-1
16. Thibaut, L., Knipprath, H., Dehaene, W. and Depaepe, F., Integrated STEM education: A systematic review of instructional practices in secondary education. *European Journal of STEM Education*, 2018, 3(1): 2–15. https://doi.org/10.20897/ejsteme/85525
17. Boice, K. L., Alemdar, M., Jackson, J. R., Kessler, T. C., Choi, J., Grossman, S. and Usselman, M., Exploring teachers' understanding and implementation of STEAM: One size does not fit all. *Frontiers in Education*, 2024, 9. https://doi.org/10.3389/feduc.2024.1401191
18. Herro, D. and Quigley, C., Exploring teachers' perceptions of STEAM teaching through professional development. *International Journal of STEM Education*, 2016, 3(1): 1–16. https://doi.org/10.1080/19415257.2016.1205507

19. Quigley, C.F., Herro, D. and Baker, A., Moving Toward Transdisciplinary Instruction: A Longitudinal Examination of STEAM Teaching Practices. In Khine, M.S., & Areepattamannil, S. (Eds), *STEAM Education*, 2019, Springer, Cham. https://doi.org/10.1007/978-3-030-04003-1_8
20. Cunningham, C.M. and Hester, K., Engineering is elementary: An engineering and technology curriculum for children. American Society for Engineering Education Annual Conference & Exposition, 2007.
21. Lin, K. Y., Wu, Y. T., Hsu, Y. S. and Williams, P. J., Effects of infusing the engineering design process into STEM project-based learning to develop preservice technology teachers' engineering design thinking. *International Journal of STEM Education*, 2021, 8(1): 1–15. https://doi.org/10.1186/s40594-020-00258-9
22. Dym, C.L., Agogino, A.M., Eris, O., Frey, D.D. and Leifer, L.J., Engineering design thinking, teaching, and learning. *Journal of Engineering Education*, 2005, 94: 103–120. https://doi.org/10.1002/j.2168-9830.2005.tb00832.x
23. Sung, E. and Kelley, T.R., Elementary Students' Engineering Design Process: How Young Students Solve Engineering Problems. *International Journal of Science and Mathematics Education*, 2023, 21: 1615–1638. https://doi.org/10.1007/s10763-022-10317-y
24. Vale, I. and Barbosa, A., A Educação STEAM e o Design de Engenharia na resolução de problemas autênticos [STEAM Education and Engineering Design in solving authentic problems]. In Neto, S., Silva, A., Coelho, A., & Fortunato, I. (Eds.), *Coletânea do IV Congresso Paulista de Ensino de Ciências: discutindo a Educação em Ciências nos países Ibero-Americanos*, 2024, 9–28. Edições Hipótese.
25. Vale, I., Barbosa, A., Peixoto, A., and Fernandes, F., Solving Problems through Engineering Design: an exploratory study with pre-service teachers. *Education Sciences*, 2022, 12(12): 1–26. https://doi.org/10.3390/educsci12120889
26. Cunningham, C.M., Engineering is elementary. *The Bridge: Linking Engineering and Society*, 2009, 39(3): 11–17.
27. Blikstein, P., Maker movement in education: History and prospects. In V. Svihla & R. Reeve (Eds.), *Constructionism in context: The Art, Theory, and Practice of Learning Designs*, 2018, 419-437. Routledge.
28. Barab, S. and Squire, K., Design-based research: Putting a stake in the ground. *The Journal of the Learning Sciences*, 2004, 13(1): 1–14. https://doi.org/10.1207/s15327809jls1301_1
29. Ng, O. and Chan, T., Learning as Making: Using 3D computer-aided design to enhance the learning of shape and space in STEM-integrated ways. *British Journal of Educational Technology*, 2018, 50(1): 294–308. https://doi.org/10.1111/bjet.12643
30. Harron, J. R., Jin, Y., Hillen, A., Mason, L. and Siegel, L., Maker Math: Exploring Mathematics through Digitally Fabricated Tools with K–12 In-Service Teachers. *Mathematics*, 2022, 10(17): 3069. https://doi.org/10.3390/math10173069
31. Dasgupta, C., Magana, A. and Vieira, C., Investigating the affordances of a CAD enabled learning environment for promoting integrated STEM learning. *Computers & Education*, 2019, 129: 122–142. https://doi.org/10.1016/j.compedu.2018.10.014
32. Zhu, C., Klapwijk, R., Silva-Ordaz, M., Spandaw, J. and de Vries, M., Investigating the role of spatial thinking in children's design ideation through an open-ended design-by-analogy challenge. *International Journal of Technology and Design Education*, 2024.

https://doi.org/10.1007/s10798-024-09877-7

33. Khurma, O., Ali, N. and Khine, M., Exploring the impact of 3D printing integration on STEM attitudes in elementary schools. *Contemporary Educational Technology*, 2023, 15(4): 458. https://doi.org/10.30935/cedtech/13568
34. Ku, C.J., Loh, W.L.L., Lin, K.Y. and Williams, P.J., Development of an instrument for exploring preservice technology teachers' maker-based technological pedagogical content knowledge. *International Journal of Technology and Design Education*, 2021, 31: 1–23. https://doi.org/10.1007/s10798-020-09596-3
35. Denzin, N.K. and Lincoln, Y.S., *The SAGE handbook of qualitative research*, Sage, 2017.
36. Creswell, J.W., *Research design: Qualitative, quantitative, and mixed methods approaches*, Sage, 2014.
37. Yin, R.K., *Case Study Research: Design and Methods*, Sage Publications, 2003.
38. Miles, M.B. and Huberman, A.M., *Qualitative Data Analysis*, Sage Publications, 1994.

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