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Case report

A classroom experience about the application of research-based learning for the teaching of probability in engineering

José Luis Díaz Palencia* and Yanko Ordóñez Ontiveros

Department of Applied Mathematics and its Didactics, Universidad a Distancia de Madrid, Collado Villalba, Madrid, Spain

* **Correspondence:** Email: joseluis.diaz.p@udima.es.

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Abstract: We explored the implementation of a Research-Based Learning (RBL) for the teaching and learning of probability in engineering education. The approach provided a student-centered learning environment, integrating practical activities within an active learning conception. A detailed RBL task in connection with weather forecasting was described, highlighting a methodical 10-day learning process that considers research, practical application, and reflection. The effectiveness of this RBL task was evaluated through thematic analysis of student reflective essays, focusing on their learning experiences and challenges. The analysis revealed enhanced understanding of probability, varied student engagement, and the value of practical learning. It also identified challenges in conceptualizing abstract probability concepts. The findings highlighted the RBL's potential in improving students' conceptual grasp and application of probability in real-world scenarios, providing key insights for future educational strategies in engineering. This study contributes to the broader discourse on innovative teaching methodologies in engineering education, demonstrating the impact of active, inquiry-based learning approaches.

Keywords: engineering education, probability learning, Research-Based Learning (RBL), thematic analysis

1. Introduction

Firstly, our initial goal is to create a theoretical framework that enables us to define and understand the various relationships that exist within the university setting. Due to its significance and its capacity to be applied from multiple perspectives, we consider to approach our didactic proposal through the Anthropological Theory of the Didactic (ATD). As will be further elaborated, this theory has been specifically adapted in didactics focused on research through the process of Study and Research Paths (SRP). However, as we will explore, our approach, despite having similarities with SRP, does not purely align methodologically with SRP itself. Instead, it is more accurately described as Research-Based Learning (RBL).

The ATD, formulated by Yves Chevallard in the 1990s, represents a paradigm shift in the understanding and approach to mathematical education. This theory examines the teaching and learning of mathematics as a cultural and social phenomenon. ATD emphasizes a comprehensive understanding of mathematical practices, focusing on the content of mathematics and also on the context in which it is taught and learned. Central to ATD is the concept of 'praxeologies.' This term refers to the different types of tasks and activities involved in the practice of mathematics. A praxeology consists of four components: Type of tasks (T), techniques (τ), technology (θ), and theory (Θ). These elements combine to form a complete picture of mathematical activity, from practical applications to the theoretical frameworks that justify and explain these practices [1]. The theory's emphasis on the 'didactic transposition' process stands as one of its key contributions. This concept describes how knowledge transitions from being at the expert level to being teachable to students. Chevallard highlights a significant gap between the knowledge produced by mathematicians and the knowledge taught in classrooms. This transformation process is essential for enhancing mathematical education, revealing what gets altered or omitted in the journey from expert to learner [2]. ATD also provides a framework for examining the 'ecology' of mathematical practices. This framework looks into how factors such as educational policies, teacher training, curricular materials, and societal beliefs about mathematics influence teaching and learning. This ecological approach helps educators and researchers identify challenges and opportunities in mathematical education [3]. Another critical aspect of ATD is the importance it places on the 'noosphere.' This concept refers to the sphere of human thought. The noosphere is essential in understanding how mathematical ideas evolve and spread within societies and cultures [4]. Moreover, ATD delves into the notion of 'didactic moments.' These moments are important points in the educational process where significant learning or teaching transformations occur. Understanding these moments is relevant for educators to facilitate more effective and meaningful learning experiences in mathematics [5].

The ATD posits that the research process leading to mathematical knowledge is essential for achieving meaningful understanding. From this perspective, a variant path of the ATD has led to the development of SRP`s. These are worth mentioning in this work due to their relevance and influence on the presented research, although strictly speaking, our work does not completely adhere to the protocols of the SRP but rather follows a RBL process. SRP within the framework of the ATD aims to immerse students in mathematical inquiry, emulating the practices of professional mathematicians (pure and applied). This objective is shared with the RBL developed in this work. It is worth noting that SRP and RBL methods diverge significantly from traditional educational methodologies as it places students at the center of the learning process, engaging them in real research activities. They shall explore open-ended questions and construct their own mathematical knowledge through deep investigation [6]. The essence of a research learning experience lies in its process, starting with the identification of a research question and advancing through stages of exploration, conjecture, and the construction of new mathematical knowledge. We will use this general idea in the design of the activities given the practicality of the definition, even deviating a bit from other implementations of SRP within the framework of ATD. This approach deepens students' comprehension of mathematical concepts and sharpens their skills in research, critical thinking, and problem-solving (very important skills for us). The implementation of a research based learning process in the classroom marks a departure from conventional teaching methods. Here, teachers adopt the role of facilitators, guiding students through their research and encouraging a spirit of creativity and exploration in mathematical research [7]. A research centered approach in educational settings aligns with the current educational philosophy, which underscores the importance of developing higher-order thinking skills. It prepares students for the multifaceted challenges of the modern world. In addition, research centered methods demystifies mathematics and reveals its inherent creativity and elegance [8].

As practical experiences, we highlight the work in [9], where the authors detail the implementation of SRP in a statistics course for ICT Systems Engineering students. Our results exemplify how integrating RBL can make abstract concepts more accessible and relevant to students, thereby improving their overall learning experience. The work in [10] broadens the perspective, discussing the effectiveness of research-centered methodologies (like SRP) in promoting deep learning and developing research skills across various educational contexts. It provides ideas about how a learning focused on research encourages students to undertake mathematical investigations, leading to a richer comprehension of mathematical theories and their practical applications. The authors in [11] provide an analysis of the interplay between the ATD and the Theory of Didactic Situations (TDS) in the context of implementing research-centered teaching conceptions. It explores the challenges and experiences of integrating research learning approaches in educational settings. It emphasizes the importance of considering both ATD and TDS in the effective implementation of SRPs. The aforementioned studies [9–11], have served as a significant source of inspiration in the formulation of a research-centered learning approach. Although we do not regard ourselves as direct successors of the methodologies delineated in these studies, we recognize their influence on our approach, which has been shaped by practical application and the authors' experiential learning.

The work presented in this paper provides a comprehensive exploration of implementing a RBL for teaching probability in engineering education. It begins by highlighting the importance of probability in engineering, especially in risk assessment, quality control, and decision-making under uncertainty. The methodology for designing RBL in mathematics education is then detailed, focusing on creating an interactive, student-centered learning environment. This includes developing a structured framework for inquiry, integrating practical activities, and fostering collaborative learning. This work then gives a detailed description of an RBL task aimed at understanding and applying probability concepts through weather forecasting. This task is structured to span 10 days, encompassing research, practical activities, group presentations, and individual reflections. The research methodology section discusses the use of thematic analysis of student reflective essays to evaluate the effectiveness of the RBL task. This involves a meticulous process of coding the essays, identifying themes, and conducting in-depth analysis to understand students' learning experiences and challenges. Finally, the guide reflects on the classroom experiences of implementing the RBL task. It notes the varied levels of student engagement and the challenges in conceptualizing abstract probability concepts. Despite these challenges, the task was effective in enhancing students' understanding of probability and developing key skills. The insights from this analysis are deemed crucial for refining future tasks, ensuring they are more aligned with student needs and learning objectives.

2. RBL in probability at engineering

Firstly, we highlight that our initial intention was to develop a work that methodically followed the guidelines of the SRPs as outlined in the mentioned studies (see [9–11]). However, according to the past experience of the authors and during classroom implementation, we realized that our work followed a less methodical process than that indicated by the various authors who have employed SRP. Following the works of Artigue ([5,8,12]) and after a detailed analysis, we observed some differences between SRP and RBL that are worth mentioning and that make RBL more suitable for our needs due to its didactic flexibility. The SRP approach is specifically crafted for integrating research processes within mathematics education, emphasizing a structured journey from problem identification to knowledge creation. In contrast, RBL is a more versatile educational strategy applicable across various disciplines, prioritizing student autonomy in research activities and focusing broadly on enhancing research skills and critical thinking. While SRP is designed to reflect genuine academic research sequences and deeply engage students in mathematical contexts, RBL offers a broader, less structured framework aimed at cultivating investigative abilities across diverse subjects.

Our intention of this work is to develop the notion of probability by following the design of a methodology and tasks implemented by RBL. This notion is very relevant as it appears in risk assessment and management, especially in fields like civil, mechanical, and aerospace engineering. And it is precisely this open nature of probability that guided our sessions through less explored areas, which while encouraging the use of probability, led to the students' research ultimately focusing more on specific problems. This has value in itself as it introduced them to areas that were unfamiliar to them and based on probability. Probability helps in evaluating the likelihood and potential impact of various risks, enabling engineers to develop strategies to mitigate these risks effectively. In addition, probability is key to quality control processes. It helps in understanding and controlling the variability in manufacturing processes, to ensure the quality and consistency of products. I remark that engineering often involves making decisions under conditions of uncertainty. Probability provides a framework for understanding and quantifying uncertainty, and allows for more informed and rational decision-making.

Designing RBL tasks for probability in mathematics education involves creating a learning environment where students can actively engage with and explore probabilistic concepts through investigation and inquiry. This approach shifts the educational focus from traditional, lecture-based teaching methods to a more interactive, student-centered learning experience, driven by curiosity and practical application. In order to design the RBL tasks, we shall follow a methodical approach supported by the mentioned literature [7–11], in particular the following key aspects are provided along with some ideas in connection with topics to be considered:

- The first step typically consists in identifying a Central Research Question. The foundation of an effective RBL is a compelling, open-ended research question that encourages exploration and discussion. For probability, this question might focus on real-world scenarios, such as predicting weather patterns, analyzing risk in financial investments, or exploring game theory.
- After establishing the central question, the next aspect involves developing a structured framework to guide students through the research process. This includes identifying relevant resources, methodologies, and tools necessary for conducting meaningful research in probability. Essential tools might include statistical software, access to diverse data sets, and foundational texts in probability theory.
- The integration of practical activities is also important. Indeed, practical, hands-on activities are a cornerstone of RBL as the research activity is normally a collaborative task. In probability,

these activities might include conducting experiments with random sampling, using simulation software to model probabilistic scenarios, or analyzing historical data sets to understand probability distributions and their real-world implications.

- Another relevant issue consists in fostering collaborative learning. Collaboration is a vital component of RBL. Activities like group discussions, peer-review sessions, and team-based projects can significantly enhance the learning experience. Collaborative efforts allow students to gain different perspectives and approaches to problem-solving in probability.
- It is relevant to implement continuous assessment and reflection: RBL should incorporate continuous assessment and reflection to ensure effective learning. Mechanisms like regular feedback sessions, reflective journals, or presentations where students articulate their findings and learning experiences can be particularly beneficial.
- It is important to align with curricular goals. This ensures that the RBL complements the broader curricular objectives. The RBL should reinforce and expand upon the probabilistic concepts taught in regular mathematics classes, providing a cohesive and comprehensive learning experience.

In the modern educational landscape, the incorporation of digital tools and technology can significantly enhance RBL. Utilizing software for statistical analysis, accessing online databases for data retrieval, and leveraging interactive learning platforms can enrich the research experience. These tools provide students with valuable, contemporary skills that are increasingly relevant in my technology-driven world.

3. Description of the tasks

Creating a detailed task within the RBL framework to explain probability involves designing an activity that teaches probabilistic concepts and also engages students in research-based learning. We provide a global activity developed in a total of 10 days (each day consists of a session of one hour and a half, although in the experience of the authors the last sessions may require around one hour thanks to the student practice) and the main objective is to understand and apply basic concepts of probability through the context of weather forecasting, enabling students to grasp how probability is used in real-world scenarios. This RBL task is designed to provide a comprehensive understanding of probability through an engaging, hands-on approach that encourages exploration, collaboration, and practical application. It is important to emphasize that each of the phases that make up the activity design is introduced from a general and descriptive perspective. In this way, the activities are described in a manner not contextualized to a specific educational situation. We believe that this approach, which gains in generality but perhaps loses in concreteness, is more suitable for articles of this structure directed at an international audience. In any case, we emphasize that a teacher with a minimum of experience in the field of statistics and classroom sessions will be able to ground the ideas presented into something concrete and traceable at the classroom level. Let us introduce the phases and activities to be done in a systematic way:

1. Introduction and Context Setting (Day 1):

   - Begin with an introductory session explaining the importance of probability in various fields, focusing on meteorology.
   - Present real-world examples where probability plays a crucial role, such as weather

prediction, insurance, and finance.

- Introduce the central research question: "How can probability help in predicting weather patterns, and what is its significance in meteorological forecasting?"

2. Research Phase (Days 2-4):

- Students form small groups and start their research on basic probability concepts. In particular random events, probability scales, and statistical likelihood.
- Each group explores how these concepts apply to weather forecasting, using resources like academic articles, weather databases, and meteorology textbooks (all these resources should be prepared beforehand and shared with students although the students may follow other paths).
- Encourage students to contact local meteorologists or weather stations for interviews or insights. This step may be optional, but it is highly recommendable and should be done outside of class.

3. Practical Activity (Days 5-7):

- Groups analyze historical weather data (e.g., rainfall patterns, temperature variations) to calculate probabilities of certain weather events (like the chance of rain on a given day). For this, a good resource is given in https://www.visualcrossing.com platform.
- Introduce software tools or online platforms for statistical analysis to aid in their calculations. The teacher can introduce very simple ideas of R, Rstudio or SPSS (Statistical Package for the Social Sciences).
- Students compare their calculated probabilities with actual historical weather data to evaluate the accuracy of their predictions.

4. Presentation and Discussion (Day 8):

- Groups present their findings, focusing on the process of applying probability to weather forecasting.
- Discuss the challenges faced in predicting weather and the role of probability in mitigating uncertainty.
- A question-and-answer session follows, where groups can discuss and critique each other's methodologies and findings.

5. Reflection and Extension (Day 9):

- Students write individual reflective essays on what they learned about probability and its applications.
- As an extension, propose a hypothetical scenario (e.g., planning an outdoor event) and ask students to use their newfound knowledge to make decisions based on probabilistic predictions.

6. Feedback and Assessment (Day 10):

- Provide feedback on group presentations and individual essays.
- Assess students based on their understanding of probability concepts, quality of research, presentation skills, and ability to apply learning to practical scenarios.

Resources Needed:

- Access to historical weather data (online databases like https://www.visualcrossing.com or

local meteorological services).
- Statistical analysis software or online platforms.
- Basic texts on probability and meteorology.

Learning Outcomes:
- Understanding fundamental concepts of probability.
- Ability to apply probability concepts to real-world situations.
- Development of research, collaboration, and presentation skills.
- Enhanced critical thinking and decision-making abilities based on probabilistic reasoning.

4. Research methodology

To thoroughly evaluate the effectiveness of the described task within the RBL framework, a qualitative research methodology, specifically thematic analysis of student reflective essays, was proposed. This methodology is well-suited for educational research, providing deep insights into students' experiences and perceptions. Thematic analysis as a method involves a detailed examination of qualitative data, in this case, the reflective essays written by students after completing the RBL task. This process starts with a reading of these essays to familiarize with the content, followed by generating initial codes that capture key ideas and concepts mentioned by the students. These codes are then grouped into themes that are relevant to the research questions. Such themes include students' perceptions of the difficulty and applicability of probability concepts, the challenges they faced during the learning process, and the overall impact of the task on their understanding of and interest in probability [13]. As the analysis progresses, these initial themes are reviewed and refined to ensure they accurately represent the data. This involves revisiting the coded data extracts and ensuring they form a coherent pattern. The final stage of thematic analysis includes defining and naming the themes and conducting a detailed analysis for each theme. This final analysis is important as it connects the themes to the broader research questions and to the existing literature on research-based approaches and probability education. The goal here is to provide a nuanced understanding of how the RBL task impacted students' learning experiences, focusing on their conceptual understanding, application skills, and attitudes towards learning probability. This qualitative approach, particularly through thematic analysis, allows for the exploration of the effectiveness of the RBL framework in a mathematical context and also highlights students' learning experiences. In conducting the research, we have adhered to the ethical concerns pertinent to educational research. This includes ensuring the confidentiality and anonymity of student participants, obtaining informed consent for the use of their essays in research, and handling the data with the utmost respect for privacy and sensitivity.

5. Descriptions of the experiences

The described task concerning probability was applied in class sessions at an Engineering degree. The implementation of the RBL took place during the period from September to December 2023 in a Statistics and Probability course in a Computer Engineering degree. The total number of participating students was 12. All students came from overly traditional mathematics teaching and learning systems, focused on the teacher and the delivery of the curriculum content. This information was gathered based on informal discussions with each of the 12 students. In this way, we find a student profile without previous contact with active experiences, focused on research and the generation of their own knowledge, as is pursued with the design of the activities in the RBL.

Given the locality of the experience, it is important to discuss the generalizability of the research conducted. Initially, it should be noted that our research is situated within a specific context and with the background of the students described. This type of approach is common in qualitative analysis, where the goal is to understand specific facts rather than general facts as in quantitative research. Expanding on this, qualitative research, such as the study conducted, focuses on understanding individual experiences and contextual realities. It provides deep insights into specific situations, hence in some cases the findings from qualitative studies may not be directly generalizable to a larger population due to the small sample size and the subjective nature of the analysis [14]. Moreover, the context-specific nature of qualitative research means that the findings are deeply embedded in the cultural, educational, and social contexts of the study. This is particularly true in educational research, where local educational policies, cultural norms, and institutional practices can significantly influence the outcomes (refer to [15] for additional details). Therefore, the extrapolation of this research results should take into account these variables and should be made within those engineering degrees that follow the educational principles of the considered degree. In this case, it is a 4-year degree in computer science that has been designed according to the guidelines of the Bologna Process, applicable to the entire higher education space in Europe. Although our study may not be universally applicable, we must highlight, as Stake [16] points out, that it emphasizes the nuances and complexities of educational experiences in a case based approach.

Each class session was conducted with three groups, each consisting of four students. The sessions were developed to foster a collaborative and engaging learning environment, focusing on the application of probability in real-world contexts. The following is a general description from the perspective of the authors (note that a specific narrative with student feedback will be presented later): In the first session, all groups were introduced to the basic concepts of probability and the RBL framework. This session served as a foundation, where students were briefed about the objectives of the RBL task and the importance of probability in everyday life, especially in weather forecasting. The atmosphere was vibrant with curiosity, as students expressed their initial thoughts and questions about probability. Following the introductory session, each group embarked on their research phase. They delved into various resources to understand probability theories and their applications. During this phase, students actively engaged in group discussions, brainstorming sessions, and collaborative research. The groups showed remarkable enthusiasm in exploring different aspects of probability, from theoretical underpinnings to practical applications in weather prediction. One of the most exciting parts of the RBL task was the practical application of probability concepts. Each group analyzed real-world data, such as historical weather patterns, to calculate probabilities of certain events. This hands-on experience allowed students to apply the theoretical knowledge they had gained in a practical setting. They used statistical tools and software for data analysis based on Rstudio and SPSS as previously introduced in the class. Both software were used to import and clean weather-related datasets, ensuring that the data is ready for analysis. In the presentation sessions, each group showcased their findings and experiences. These presentations were followed by a peer review process, where students constructively critiqued each other's work. This stage was particularly insightful as it not only demonstrated the students' learning outcomes but also fostered a sense of community and collaborative learning. The final stage involved reflective writing and feedback. Students individually wrote reflective essays about their experiences, learning outcomes, challenges faced, and how they overcame them. These reflections were profound, indicating a deep engagement with the subject matter. The session concluded with feedback from the instructor, which was important in guiding the students' learning process.

6. Results and discussions

The thematic analysis is based on the reflective essays from the three groups involved in the tasks. To conduct the essays, the students were previously introduced in reflective writing. This involves students articulating their learning experiences, challenges, and growth in a structured format. This type of writing allows students to deeply analyze and reflect on their understanding and application of educational content, in this case, probability. To correctly reflect their ideas, we design some questions on various dimensions of their learning experiences that were presented in the following structured format:

1. Description of Experience:

- What was the main activity or task you engaged in during the research project?
- Describe your initial thoughts and feelings when you started working on the tasks.

2. Application of Concepts:

- How did you apply the concepts of probability in the RBL task?
- Describe any specific instances where you used probability theory to solve a problem or analyze a situation.

3. Challenges and Solutions:

- What challenges did you encounter during the tasks?
- How did you address these challenges? Describe the strategies or solutions you implemented.

4. Learning and Insights:

- What are the key learnings you gained from participating in the research tasks?
- How has your understanding of probability changed or deepened as a result of this experience?

5. Collaboration and Teamwork:

- How did you collaborate with your peers during this project?
- Describe any moments of significant teamwork or shared learning.

6. Personal Growth:

- How has this experience contributed to your personal growth, particularly in terms of skills like - critical thinking, problem-solving, or communication?
- Reflect on any changes in your attitude or approach to learning mathematics or probability.

7. Future Application:

- How do you envision applying the knowledge and skills gained from this research task in future scenarios, both academic and real-world?
- Are there any specific areas or topics you are now more interested in exploring further?

8. Feedback and Improvement:

- What feedback or suggestions do you have for improving the RBL task in the future?
- Reflect on what aspects of the project were most beneficial and which areas could be enhanced.

To conduct the thematic analysis, we have followed the descriptions given in [13] and we have further described them in Section 4. In particular, we began by reading all the reflective essays thoroughly to become familiar with the depth and breadth of the content. We now present a narrative style in relation to the most representative responses extracted from student reports. The responses are provided for each block of questions from 1 to 8 as just presented.

In connection with the first block of questions “Description of the Experience”, one student said, "The main activity we engaged in during the research project was conducting research on probability and statistical concepts because I liked analyzing real-world data sets and applying theoretical knowledge." Another student pointed out, "Initially, I felt quite overwhelmed. The task seemed daunting because it was not just about understanding probability, but also about applying it in a real-world context." A third student shared, "When we started working on the task, my first reaction was excitement. I've always enjoyed practical applications of theoretical concepts, and this project promised just that." Furthermore, another student expressed, "I was a bit anxious at the beginning. The idea of working in groups on complex statistical problems was new to me, and I wasn’t sure how well I would cope." One of the students remarked, "I remember feeling curious about how we were going to use statistical software and data analysis in our project. It was something I had little experience with, so it was both exciting and a bit intimidating." In contrast, another student noted, "My initial thought was about the challenge of working collaboratively. I was concerned about how our group would function and if we could effectively combine our skills." Another student reflected, "At first, I was skeptical about how this project would help us in understanding probability. But as we delved deeper, I began to see the relevance and importance of the task."

Now, we provide some interesting responses for the second block of questions “Application of Concepts”. One student explained, "We applied probability concepts in the research task. This helped us understand how probability theory can predict the weather." Another student added, "We used probability to analyze data trends in the meteorology data and make predictions. It was good to see how theoretical probability could be applied to practical data." This feedback was very revealing, as it is a prelude to the construction of the concept of probability based on the frequentist definition of probability. Another student detailed the experience: "In one part of the project, we had to estimate the likelihood of certain events, like raining and snowing, happening based on historical data. Using probability theory, we were able to calculate these likelihoods and apply them to predict future events." One group of students were far beyond in their research and one student proudly shared, "There was a moment where we used the concepts of conditional probability to analyze the relationship between different variables. For example, the probability that it will rain tomorrow knowing that it rained today and yesterday. Or the probability that it will be sunny knowing that it has been raining for two days. It was challenging but rewarding to see how these concepts helped in making sense."

The third block of questions considered “Challenges and Solutions”. In this regard, one student mentioned, "A major challenge in the research task was interpreting weather data and applying probability theory to forecast weather was not straightforward." To address this, another student said, "We tackled this by breaking down the data into smaller parts and used probability. For example, we focus in seven to ten days”. Regarding strategies, a student shared, "We faced difficulty in predicting rare weather events. Our approach was to use historical weather data to estimate probabilities, which improved the accuracy of our predictions." Another added, "To overcome the challenge of integrating different probability concepts, we held group discussions to brainstorm and solve the problems." In terms of solutions, a student stated, "When we struggled with complex calculations, we utilized statistical software as explained by the teacher which made it easier to apply probability theories".

In connection with the fourth block “Learning and Insights”, one student said, "A key learning from the research task was how probability is not just about some numbers; it's about understanding the reality”- Another one chimed in, "I really got how important data is in probability. Before this, it was just equations, but now I see it's about real-world stuff." Talking about their understanding of probability, a student mentioned, "This project totally changed how I see probability. Before, it felt difficult, but now I get how it applies to everyday things like weather forecasting." Another added, "I’ve learned to appreciate the uncertainties in predictions. Probability is not just about being right or wrong; it is about gauging how likely you are to be right." This last feedback is interesting as it represents a shift in the student conception towards probability.

Regarding the “Collaboration and Teamwork” block, one student shared, "We collaborated a lot! We divided the work based on our strengths – like some of us were better at the math part, while others were good at interpreting the data." Another added, "Group chats were our go-to. We’d bounce ideas off each other, which really helped when we got stuck." Regarding teamwork, a student described an interesting moment "We had diverse viewpoints in our group, so when we combined them, it just clicked. It was cool to see how different perspectives led to a better solution."

The block “Personal growth” provided interesting responses. One student reflected, "This experience really sharpened my critical thinking. I’ve learned to look at problems from different angles and not just take data at face value." Another noted, "In terms of problem-solving, I feel more confident now. Tackling real-world scenarios with probability theories was challenging but it taught me to be more persistent and creative in finding solutions." Regarding communication, a student stated, "I’ve definitely improved in explaining complex ideas. Working in a group required me to communicate my thoughts clearly, which was a valuable skill to develop." In terms of changes in attitude towards learning, one student mentioned, "My approach to learning mathematics has become more practical. I now appreciate how mathematical concepts, especially probability, are relevant in everyday life." Another student added, "I used to find probability quite daunting, but this project made it more relatable and interesting. It’s changed my attitude towards learning it – I’m more curious and engaged now."

For the questions in blocks 7 and 8, one student expressed, "The skills I gained from this research task, especially in data analysis and probability application, will be important in my future academic research. I can also see how these skills are transferable to real-world”. Another student showed newfound interest, stating, "I’m now more intrigued by the role of probability in machine learning and data science. This project has sparked a desire to delve deeper into these areas." Regarding feedback for improvement, a student suggested, "It would be great to have more access to advanced statistical tools and perhaps some expert sessions on their usage. This could enhance our learning experience." Reflecting on the project's benefits and potential enhancements, one student noted, "The practical application of theory was the most beneficial aspect. However, more structured guidance in the initial stages of the project could make it even better, especially for those who find probability concepts challenging at first." Another student added, "Including more real-life case studies for analysis would be great. It would give us more varied perspectives on how probability is used in different industries."

As we went through the essays, we started generating initial codes. After coding the data, the next step was to group these initial codes into potential themes. For example, if multiple students mentioned feeling challenged by abstract probability concepts but also expressed a sense of achievement in understanding these concepts, this could indicate a theme of 'overcoming conceptual challenges.' Afterward, we made a revision of themes merging, splitting, or discarding themes. Once the themes were refined, the next step was to define and name them to explain what it captured and how it related to the research questions.

After following the principles mentioned, we came across several insightful themes:

1. Varied Engagement Levels: A common theme across the groups was the variability in engagement levels. While some students showed high enthusiasm and active participation, others were less involved. This disparity sometimes led to uneven contributions within groups, affecting the group dynamics and possibly the overall learning experience.
2. Understanding and Application of Probability: Many students reflected on their enhanced understanding of probability concepts. They reported that engaging in practical, real-world applications, like weather prediction, made abstract concepts more tangible and easier to comprehend.
3. Challenges in Conceptualization: A recurring challenge mentioned in the essays was the difficulty in grasping and applying abstract probability concepts to practical scenarios. Despite these challenges, students often viewed these struggles as valuable learning experiences, enhancing their problem-solving and critical-thinking skills.
4. Value of Practical Learning: Students frequently expressed appreciation for the practical aspects of the tasks. They found that applying probability to real-world scenarios made the learning process more engaging and meaningful compared to traditional classroom methods.
5. Development of Transferable Skills: Reflective essays often highlighted the development of skills beyond understanding probability, such as teamwork, communication, and research skills. Students recognized these skills as valuable for their academic and future professional lives.
6. Feedback for Improvement: Students provided feedback on aspects of the tasks that could be improved, such as needing more resources for data analysis, more time for research phases, and additional support for understanding complex concepts.

The mentioned thematic analysis reveals significant insights into the learning experiences and challenges faced by the students. The themes uncovered through this process highlight the multifaceted nature of learning probability in an applied context, emphasizing both the benefits and the difficulties encountered by students. The varied engagement levels among students underscore the importance of creating learning environments that cater to diverse learning styles and levels of enthusiasm. This variability points to the need for adaptive teaching strategies that can engage all students effectively. The RBL method, as evidenced by the feedback from students who participated in it, offers several advantages. Initially, it provides students with a hands-on and practical approach to learning complex mathematical concepts, such as probability and data analysis. This experiential learning allows students to see the real-world applications of theoretical knowledge, making it more engaging and relatable. Furthermore, the RBL method promotes collaboration and teamwork among students. Working in groups, students learn to leverage each other's strengths, share ideas, and solve complex problems collectively. This enhances their problem-solving skills and improves their communication and teamwork abilities, which are essential in both academic and real-world settings. Additionally, the RBL method fosters critical thinking and a deeper understanding of the subject matter. Students are encouraged to think critically about data, patterns, and uncertainties, which leads to a more profound appreciation of the nuances involved in probability theory. Moreover, the RBL method has the potential to spark students' interest in related fields, such as machine learning and data science. It can serve as a gateway to exploring broader topics and careers, which is beneficial for their academic and professional growth. However, there are also challenges associated with the RBL method. Some students may initially feel overwhelmed, especially if they are new to collaborative and research-centered learning. Addressing this challenge requires providing structured guidance and support in the early stages of the project. Additionally, the RBL method may require access to specialized resources, such as statistical software and real-world data sets. Ensuring equitable access to these resources for all students can be a logistical challenge. Furthermore, the method's success heavily relies on effective group dynamics. Varying levels of engagement among students can lead to uneven contributions within groups, potentially impacting the overall learning outcomes.

It is important to emphasize that our research results complement those reported in [17], where a research approach was applied in a Mechanical Engineering course on General Linear Elasticity. The findings in [17] suggest that research-centered learning, along with tools like Question-Answer maps (certainly an aspect not developed in our current experience), effectively support the design of project topics and facilitate knowledge management for both teachers and students, similar to what we have found in our research, where we have outlined a proposal for applying probability knowledge in real-world situations with essential ideas related to research-based learning. In addition, in [18] research paths are described as a study format centered on open questions, applicable across educational levels, including teacher education and professional development. They are also noted for their role in analyzing teaching and learning processes by identifying the key questions that guide the study process and the generation of new knowledge. In our study, we specifically explored the application of RBL in engineering education, particularly in the context of probability knowledge. While we focus on the practical implementation and benefits of RBL in a specific academic discipline, the study [18] provides a broader perspective on the evolution and potential of research paths in education at various levels and their role in understanding the didactic ecology of teaching and learning processes. Therefore, our research represents a refinement of the ideas presented in [18], from which our research has also drawn and used as a foundation in defining learning strategies. The text in [19] discusses the challenges of implementing inquiry-oriented instructional proposals at the university level. It highlights constraints related to the traditional pedagogical paradigm, which is often centered on transmitting established knowledge rather than fostering inquiry-based learning. It is worth noting that our students align with this highlighted point, meaning they come from a predominantly traditional system, and it is necessary to develop more active teaching and learning methodologies within this preconceived system. The main challenges encountered in our case follow the trend expressed in [19]. Furthermore, it should be noted that while the mentioned text [19] addresses the challenges and strategies for integrating inquiry-oriented approaches in university settings, we focus on the practical implementation and outcomes of RBL in a specific academic discipline, thereby concretizing some of the ideas presented in [19] to a discipline like teaching and learning probability in the university context.

7. Conclusions

In summary, the application of the Research Based Learning (RBL) framework for teaching probability in engineering education has proven to be a rich and varied experience. The thematic analysis of student reflective essays has provided valuable insights into the effectiveness of this approach. While it has enhanced student engagement and deepened their understanding of complex probability concepts, it has also highlighted areas for improvement, such as addressing diverse learning styles and providing additional support for abstract concepts. The experience highlights the potential of RBL in practical learning and developing essential skills, paving the way for future enhancements in engineering education, and especially with the development of important notions like probability.

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Conflict of interest

The author declares that there are no conflicts of interest in this paper.

Ethics declaration

As described in the text, the ethics principles for conducting research in education have been followed.

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Author’s biography

Dr. José Luis Díaz is a professor of applied mathematics and didactic of mathematics at the Distance University of Madrid (UDIMA). His research interests are connected with the Anthropological Theory of Didactics and how it can be applied to classes in engineering and applied sciences.

Dr. Yanko Ordóñez Ontiveros is a professor of innovation and research in education with an emphasis on physical education and the didactics of physical education at the Distance University of Madrid (UDIMA). His background as an engineer and educator gives him an interdisciplinary profile. His research interests are related to educational innovation and, more particularly, how it can be applied to physical education classes.