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Case study

Solving worded real-world problems using simultaneous equations by pre-service mathematics teachers in regional Australia: Performances and implications

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Abstract: Studies have shown that solving worded real-world problems is a difficult challenge for pre-service secondary mathematics teachers engaging study in most tertiary institutions. This case study reports the performances of three groups of regional pre-service mathematics teachers in their attempts to solve three different worded real-world problems through simultaneous equations in their formal assessments. These students were the first-year pre-service mathematics teachers enrolled in an undergraduate education program in a regional university in Australia. Their performances are compared by statistical analysis. The result of this study indicates that design of the word questions should consider challenging tasks appropriate for the students to achieve the pedagogical purpose in solving real-life problems that can best facilitate training the students towards becoming knowledgeable and motivated mathematics teachers for secondary schools, rather than ideologically making a levelling field with less challenging problems to achieve a high pass rate mandated by many institutions.

Keywords: pre-service mathematics teacher, worded real-word problem, simultaneous equations, regional university

1. Introduction

Solving simultaneous equations or system of equations is not only an important part in algebra in secondary mathematics curriculum, but also frequently encountered in numerous scientific, engineering, and daily-life problems described in words that require transforming the problems into simultaneous equations. Hence, understanding the properties of simultaneous equations and effectively using techniques of solving simultaneous equations are featured in mathematics curriculum in schools and taught in education mathematics programs for pre-service mathematics students in tertiary institutions [1–5]. The ultimate goal of learning simultaneous equations is to capture appropriate real-world scenarios into correct mathematical formulas so that students can use the learnt techniques to solve the real-world problems, which is particularly important for pre-service mathematics teachers who would teach school students in the future.

However, studies have shown that many pre-service mathematics teachers were struggling in understanding problems described by words and hence were unable to transform the problem to correct equations, despite being able to use a proper mathematical technique to solve explicitly formulated simultaneous equations [2–6]. This finding is similar to pre-service mathematics teachers' difficulties in dealing with worded problems for real-life scenarios associated with other mathematical topics [7–15]. These indicate that efficacy in disciplinary techniques in mathematics does not guarantee an ability to correctly deal with worded problems in the real world for pre-service mathematics teachers.

The only way to improve pre-service mathematics teachers' ability in solving worded problems for the real-world scenarios is to create more opportunities of approaching and solving worded problems for them to gradually build up and enhance skills in critical thinking and problem solving so as to become more competent and confident in dealing with such problems. Apart from creating such opportunities, design of worded problems and associating the problems with appropriate real-life experiences would have a great influence in guiding the students towards the targeted learning goals and expectations. The worded problems either too easy or too difficult for the students would greatly dilute the significance of creating such opportunities in enhancing students' ability in problem solving.

This case study compares the works of three groups of first-year pre-service mathematics teachers (or student mathematics teachers) who enrolled in an undergraduate education program with specialisation in secondary mathematics at Central Queensland University (CQU), a regional university in Australia. The case study is focused on three aspects: the pedagogical requirement in designing word questions which can best facilitate training the students becoming knowledgeable and motivated mathematics teachers for secondary schools, the actual outcomes from students' efforts to solve the word questions with respect to the achievement of the pedagogical design in simultaneous equations to solve real-life problems, and implications in teaching and learning of education mathematics in regional areas.

In the rest of this paper, Section 2 introduces the background information of this case study and the appropriate research method for this study. The three cases of worded problems involving setting up and solving simultaneous equations along with students' performances are presented in Sections 3, 4 and 5 respectively. Section 6 compares and discuses students' results and implications through statistical analysis. Section 7 summarised this study.

2. Background and research method

Australia has a large territory with relatively a small population just over 26 million people, among whom two-thirds live in about 20 major cities mainly along the coastal zones. By the classification of Rural, Remote and Metropolitan Areas (RRMA), Australian regions are classified into five categories of Major Cities, Inner Regional, Outer Regional, Remote, and Very Remote areas [16]. Except Major Cities, the other four regions are commonly referred to as the regional, rural and remote (RRR) areas which cover more than 95% of Australian territory [7,16].

Compared to the richness level of educational resources and the standard of academic achievements in schools located in or near the major cities, schools located in most RRR areas are far behind the marks of the metropolitan schools in almost all aspects, for example, the shortage of experienced STEM teachers, outdated infrastructure, and a low rate of school leavers participating in tertiary education which led to the shortage of teachers in the RRR schools. Research has shown that once the students from the RRR communities enter to a tertiary program with a regional university, most graduates are likely to choose a career in the RRR communities after graduation [17,18].

The low rate of participation of school leavers in tertiary education has a profound impact on the curriculum design, pedagogy, quality assurance, and sustainability of tertiary programs in regional universities, particularly undergraduate degree programs. Running classes with a small number of students is always costly; consequently, a regional university can only run programs as long as they are viable. To make a tertiary program financially viable, regional universities have to both encourage as many eligible school leavers to engage in tertiary education as possible and offer preparatory courses for those people (both new school leavers and adults) who are academically ineligible to enrol in the tertiary program that they really want to take [19,20]. Hence, in a class at a regional university, students' backgrounds are typically diverse in age, academic preparation, study mode and type, time commitment to learning, and so forth, which is particularly common in many STEM programs.

The mathematics specialty in Bachelor of Education at CQU is a typical example that serves the needs of training mathematics teachers for secondary schools, particularly the RRR schools. CQU is a regional university with multiple campuses in RRR areas located in local centres along the north-eastern coast of Australia. Its headquarter is located in Rockhampton, a regional city with a population of around 70,000 people. The mathematics specialty consists of one statistics course and five mathematics courses across multiple academic levels over three years of full-time study. The five mathematics courses have been gradually realigned to the Queensland Senior Mathematics Syllabus [21] since 2019 with one foundation mathematics course, two intermediate mathematics courses, and two advanced mathematics courses.

The foundation course is the prerequisite for the second-level mathematics courses scheduled in the subsequent year, followed by the advanced mathematics courses in the third year for full-time students. The main objective of this foundation mathematics course is to streamline the basic topics in algebra, geometry, and trigonometry that students had learnt in secondary schools through systematic reviews with further conceptual reasoning, logical articulation, and real-world applications. This also provides those students who left school more than five years ago with an opportunity to refresh their previously obtained mathematical knowledge and/or bridge the gaps in their original mathematics learning in secondary schools.

A class of the foundation mathematics has typically 20-40 students aged from 17/18 years old (new school leavers) to the 50s of adults who want a career change after years of highly itinerant services in remote mining and agricultural sectors. Most of these 20-40 students in a class are likely to live in different RRR areas in Queensland, with a few in other states of the country. Effectively, online delivery is the most viable option for such a diverse class.

This setting of a mathematics class in a regional university is vastly different from most existing studies with hundreds of students sitting in multiple rooms or the same venue. Hence, for the purpose of this study focusing on the performances of solving word problems involving simultaneous equations by the pre-service mathematics teachers from RRR areas, the research method of comparative case study is adopted [22], supported by simple statistics.

3. The first case study

3.1. The first real-world problem

The real-world problem shown in Problem 1 was assigned to 21 first-year pre-service mathematics teachers enrolled in the foundation mathematics course. This word problem was related to a simple mixing problem in chemistry that would have been learnt by most students in senior high schools in developed countries and many developing countries. The problem aimed at testing student's basic skills in capturing a simple scientific scenario by two linear equations and then applying either substitution or elimination to solve this problem. Students were reminded of considering both the volume of solutions and the quantity of salt before and after mixing during their attempt at the problem.

Problem 1. The first real-world problem assigned to the pre-service mathematics teachers

Two salt solutions, 21% and 5% of salt respectively, are available. To make 1000 ml of solution with 11% salt using these two solutions, what amounts of solutions from these two solutions must be mixed together?

3.2. The reference solution

Let x and y be the volumes for the 21% and 5% salt solutions respectively required to make the new 11% salt solution for 1000 ml. This means that the total volume before and after mixing should be 1000, i.e., $x + y = 1000$. The quantity of salt in the mixed 11% solution should be $1000 \times 11\% = 110$ g. This amount of salt should equal the total quantity of salt coming from the 21% and 5% solutions together, i.e., $x \times 21\% + y \times 5\% = 0.21x + 0.05y$; hence, $0.21x + 0.05y = 110$. These two linear equations form the following system:

$$\begin{cases} x + y = 1000 \\ 0.21x + 0.05y = 110 \end{cases}$$

From the first equation, $y = 1000 - x$. Substitute y into the second equation.

$$0.21x + 0.05(1000 - x) = 110 \longrightarrow 0.21x + 50 - 0.05x = 110 \longrightarrow 0.16x = 60 \longrightarrow x = 375 \text{ ml.}$$

Substitute $x = 375$ into $y = 1000 - x$ to find y.

$$y = 1000 - 375 = 625 \text{ ml.}$$

Therefore, an appropriate mixing requires 375 ml of the 21% salt solution and 625 ml of the 5% salt solution to make up 1000 ml of 11% salt solution.

3.3. The pre-service teachers' performance

Overall performance

Fourteen out of the twenty-one students solved this problem correctly, which was about 67% of the total. One student formulated the equations correctly but made a mistake in solving the system of equations, which could be avoided if the process was more carefully checked. Four other students were wrong in formulating the equations for this problem, made mistakes in capturing the second equation on the equality in quantity of salt. This indicates their inability to understand the simple mechanism of mixing in chemistry due to their lack of basic scientific knowledge. However, at least, these four students had made effort on solving the problem. The most disappointing fact was that two students did not attempt this problem at all, which counted about 10% of the total. The overall performances of the student teachers are summarized in Table 1.

Table 1. Summary of the overall performances of students in solving the first problem.

Outcome	Number	Percentage (%)
Correct	14	66.67
Partly correct	1	4.76
Incorrect	4	19.05
No attempt	2	9.52
Total	21	100

Informal reflections from some students

After releasing the reference solutions and the marked assignments to the students, the lecturer had informal chats with several students about reflections on their performances in this assignment during the interclass break, in which six students mentioned solving this real-world problem. Four students who correctly solved this problem felt no trouble in formulating the equations and then solving the system of equations. This was because they graduated from high schools less than a year ago and knew the mixing problem in chemistry well. One student who solved the problem incorrectly regretted that she should have refreshed what was learnt in science (chemistry) five years ago in high school. The other student who did not attempt this problem admitted that he had no idea about the mixing problem as he dropped out school at an age of 15. As a certified chef over recent 10 years, he was thinking about a career change to teaching mathematics for young pupils in primary schools. This student later withdrew from this course and admitted to the lecturer that mathematics was not a good choice for him and he would explore other opportunities for a career change.

4. The second case study

4.1. The second real-world problem

The second real-world problem shown in Problem 2 was assigned to 37 student teachers enrolled in the foundation mathematics course in the following semester. Since some students did not have the basic scientific knowledge required to capture a science-related scenario as demonstrated by the past students in the first real-world problem, this second problem was designed to be more related to everyone's daily life so that a levelling field was set for all students, particularly those students without a sound background in science. The problem again aimed at testing student's ability to capture a real-life situation by formulating two equations and then to solve the system of equations by using either substitution or elimination just reviewed. No further clue was offered to the students as the question was described sufficiently by words.

Problem 2. The second real-world problem assigned to new pre-service teachers

The price for a bottle of Coke is \$3.00 and that for a bottle of fruit juice is \$5.50. Kate spent \$37.00 to buy a total of nine bottles of Coke and juice. How many bottles of Coke and how many bottles of juice did Kate buy?

4.2. The reference solution

Let x and y be the numbers of bottle for Coke and juice respectively.

$$\begin{cases} x+y=9 \\ 3x+5.5y=37 \end{cases} \xrightarrow{(1)\times 3} \begin{cases} 3x+3y=27 \\ 3x+5.5y=37 \end{cases} \xrightarrow{(2)-(1)} 2.5y=10 \longrightarrow \begin{cases} x=5\,\text{(Coke)} \\ y=4\,\text{(juice)} \end{cases}$$

Therefore, Kate bought 5 bottles of Coke and 4 bottles of juice.

4.3. The pre-service teachers' performance

Overall performance

Thirty out of the thirty-seven students solved this problem correctly, which was above 80% of the total (Table 2). One student just had a wrong guess on the numbers of bottles for Coke and juice without presenting any mathematical procedure whereas another student missed out this real-world problem. These two students withdrew from the course later. Five others made mistakes in solving the correct system of equations, hence were rewarded half of the mark assigned to this problem. Interesting to note that six students who found the correct numbers were not by formulating a system of linear equations, instead by using a table of different Coke-juice pairs as shown in Table 3. They were given the full mark as a result of the fault in the assessment design that did not mandate students to only use system of linear equations to solve this problem. Should these students not be counted as 'correct', the 'correct rate' would be down to about 65%.

Lessons learnt from this case

It was proven that this real-world problem had no challenge to most students; hence, no valuable reflection on solving this problem was collected from students. Instead, the lecturer learnt a couple of lessons from this case. Firstly, the intent to level the field as equal as possible for all students to work on the real-world problem was correct, but setting the problem without enough challenge lost the purpose of helping students learn the targeted mathematical skills. This led to almost all the students finding the correct solution with ease, less the two students who withdrew later. Secondly, some students lacked motivation to learn real mathematical skills so as to enhance their ability to teach mathematics to future school students by mathematical ways, instead of arithmetic counting for simple cases that is not applicable generically to more complicated cases. Their aim was simple to get enough marks to pass the course, rather than learning mathematics well as a good mathematics teacher for the future. This is a real concern in current mathematics training for pre-service mathematics teachers if they are not self-motivated to become an able and knowledgeable secondary mathematics teacher. Hence, setting assessments with challenge to the pre-service mathematics teachers is better than assigning easy tasks to the students for the sake of equality for all.

Table 2. Summary of the overall performances of students in solving the second problem.

Outcome	Number	Percentage (%)
Correct	30	81.08
Partly correct	5	13.51
Incorrect	1	2.70
No attempt	1	2.70
Total	37	100

Table 3. An example of solving the second problem by trying different Coke-juice pairs.

Coke ($3.0)	Juice ($5.5)	Total
7×$3 = $21	2×$5.5 = $11	$32
6×$3 = $18	3×$5.5 = $16.5	$34.5
5×$3 = $15	**4**×$5.5 = $22	**$37**

5. The third case study

5.1. The third real-world problem

The real-world problem shown in Problem 3 was assigned to 30 first-year pre-service mathematics teachers enrolled in the foundation mathematics course, one year after the group of students who did the second real-world problem. Learnt from the lessons in the previous year, this real-world problem was related to the grade for a student who just completed a mathematics course, similar to the mathematics course all students were currently undertaken. The problem aimed at testing student's reasoning in capturing the simple logic described with two linear equations and then applying either substitution or elimination to solve this problem.

5.2. The reference solution

Let A be the assignment score and E be the exam score.

$$\begin{cases} A+E=75 \\ 3E=2A-5 \end{cases} \longrightarrow \begin{cases} A+E=75 \\ 2A-3E=5 \end{cases} \xrightarrow{(1)\times 2} \begin{cases} 2A+2E=150 \\ 2A-3E=5 \end{cases} \xrightarrow{(1)-(2)} 5E=145$$

$$E=\frac{145}{5}=29 \longrightarrow A=75-E=75-29=46.$$

Hence, Peter obtained 46 marks out of 50 from the assignment and 29 marks out of 50 from the examination.

Problem 3. The third real-world problem assigned to the pre-service mathematics teachers

Peter obtained 75 out of 100 marks from both the assignment (out of 50) and the examination (out of 50) in a mathematics course. Three times of the exam score is still five points less than two times of the assignment score. Find the assignment and exam scores Peter obtained from this course respectively.

5.3. A similar example for the pre-service teachers

To assist the students familiar with this type of problems, a similar example was solved by the lecturer and shared with all the students before their attempt at the assigned real-world problem. This sample is shown in Problem 4 and the process of solving this problem is detailed as follows.

Problem 4. A worked sample problem for the pre-service teachers

A mathematics course is offered to both education students and engineering students. Suppose there are totally 99 students enrolled in the course. The number of engineering students is 3 fewer than the double of the education students. How many education students and engineering students are enrolled in this course respectively?

Let *x* and *y* be the numbers of education and engineering students enrolled respectively. The total number is 99, i.e., $x + y = 99$. The number of engineering students is the double number of education students less 3, i.e., $y = 2x - 3$. These two linear equations form the following system:

$$\begin{cases} x + y = 99 \\ y = 2x - 3 \end{cases} \longrightarrow \begin{cases} x + y = 99 \\ 2x - y = 3 \end{cases}$$

Apply elimination to this system as the *y* terms in both equations are the same but with opposite signs.

$$\begin{cases} x + y = 99 \\ 2x - y = 3 \end{cases} \xrightarrow{(1)+(2)} 3x = 102 \longrightarrow x = 34.$$

Substitute $x = 34$ into the first equation (or the second one) to find *y*:

$$y = 99 - x = 99 - 34 = 65.$$

Alternatively, we can substitute $y = 2x - 3$ directly into $x + y = 99$. Hence,

$$x + 2x - 3 = 99 \longrightarrow 3x = 102 \longrightarrow x = 34$$
$$y = 2 \times 34 - 3 = 68 - 5 = 65.$$

Therefore, there are 34 education students and 65 engineering students in this course.

5.4. The pre-service teachers' performance

Overall performance

Sixteen out of the thirty students solved this problem correctly, which was about 53% of the total. Three students formulated the equations correctly but made a mistake in solving the system of equations, hence obtained half of the mark allocated to this problem. Eight students (or about 27% of the class total) were wrong in formulating the equations for this problem, all being not able to sort out the logical relationship between the numbers, despite availability of the worked example for all students. This indicates not only their weak ability in basic logical reasoning, but also an unwillingness to engage with the learning process and actively utilise available learning materials. Disappointedly, three students did not attempt this problem at all, which makes the total incorrect to about 37% of the class total. The overall performances of the student teachers are summarized in Table 4.

Table 4. Summary of the overall performances of students in solving the third problem.

Outcome	Number	Percentage (%)
Correct	16	53.33
Partly correct	3	10.00
Incorrect	8	26.67
No attempt	3	10.00
Total	30	100

Informal reflections from some students

After releasing the reference solutions and the marked assignments to the students, the lecturer had informal chats with four students during the interclass break about reflections on their performances in this assignment. Three students who correctly solved this problem felt no trouble in formulating the equations and then solving the system of equations with the assistance of the worked example. One student who made a mistake in solving the correctly set system of equations admitted he should be more careful in checking the solution so that the mistake could be identified and fixed. No reflection was collected from any of those eleven students who either solved the problem incorrectly or did not attempt the problem. This was because these students neither attended the live class nor frequently accessed the recordings of the live class and other learning materials on the course website. However, the negative rating and comments during the course evaluation were entered by a few of these students at the end of the teaching semester, including comment like "*no sufficient learning materials were available*".

6. Discussion

6.1. Design strategy to facilitate effective training of secondary school mathematics teachers

Regional universities have an obligation to provide opportunities of tertiary education to potential students living in RRR areas [20,23]. Hence, a large proportion of students admitted to various undergraduate degree programs in regional universities is academically underprepared compared with the academic entry standard required by most metropolitan universities. This measure of relaxing the entry standard has made a positive impact on rising tertiary participation rate in the RRR communities. Meanwhile, admission of many academically underprepared students also saw the highest attrition and lowest completion rates in many of these regional universities [24–26].

The internal grading policy of many institutions somehow mandates a minimum class pass rate for each course to keep as many students progress as possible. Otherwise, the teachers must provide the course committee with acceptable explanations and strategies to improve the pass rate for future offerings. To avoid further scrutiny, some teachers intended to let as many students pass as possible, even though some students made minimum or no effort on their studies, particularly in the first-year courses. The false high pass rate had a severe impact on teaching and learning of other advanced courses. The students who passed a foundation course with bare minimum or no effort would expect to pass other courses with bare minimum or no effort too. Such misperception would be carried on to the subsequent advanced courses, particularly in mathematics, science and engineering. The option to deal with such misperception in advanced courses is either to let the students pass as they wanted, or to fail the students according to the academic assessment criteria for an advanced course. The former would see the students keep going through the system as inferior mathematics or science teachers to teach secondary school students or as incapable engineers to design and implement real engineering projects. The latter would filter those incapable students to prevent them from making further harms to the society. However, such option would likely lead the teacher to be both accused of bad teaching by the students and reprimanded by the management for not having sufficiently supported the students and hence not achieved the required pass rate.

This foundation mathematics course serves as a bridging unit for potential secondary mathematics teachers to help them review basic mathematical concepts and rules they had learnt in Years 7-10 in high schools. As the purpose of this mathematics curriculum is to train the students becoming knowledgeable and able mathematics teachers to teach future generations of secondary school students, the first option of passing as many students as possible should never be adopted for realising such purpose of education. Setting purposeful and challenging assessments at an appropriate level can help identify those willing and motivated students who are determined to become capable mathematics teachers through embracing the challenging nature and recursive progression of mathematics over the entire degree study. For those students who are not prepared to take on this persistent challenge, such purposeful and challenging assessments would also have a positive influence for such students to make an earlier switch to other disciplines more suitable to individuals' capability. Hence, among the three real-world problems for the three groups of students, the first and third problems served this purpose much better than the second problem.

6.2. Pedagogical considerations in designing real-world problems in simultaneous equations

One may argue that the three real-world problems served the students' learning on solving system of linear equations well, and the second problem looked even better with a 'correct rate' over 80% than the other two problems (Table 5). However, these rates should be placed in the individual circumstances respectively. For the first and third problems, no student used a guessing strategy to try the solution as everyone knew such strategy was not going to work easily. The guessing strategy worked for the second problem and six students obtained the correct solution by this strategy. However, they deliberately missed the opportunity to apply the generic approach of system of linear equations, to which the problem was intended. Surely, some people would choose the short-cut if available, but this was against the purpose of training future school mathematics teachers.

Table 5. Rates of students' results from the three real-world problems.

	Correct (%)	Partly (%)	Incorrect (including No attempt) (%)
RWP1	66.67	4.76	28.57
RWP2	81.08	13.51	5.41
RWP3	53.33	10.00	36.67

The difference between the second problem and the first and third problems is statistically significant by Chi-test (Table 6). By counting 'No attempt' as 'Incorrect', at significance level α = 0.05, the chi-square value for the grade distributions between the second problem and the first problem is 6.571 whereas the chi-square value for the grade distributions between the second problem and the third problem is 10.374. Both are greater than the critical chi-square value of 5.992

at the same level for 2 degrees of freedom (*d.f.*). This indicates that the students' grade distributions between the second problem and either the first or the third problem are significantly different.

Table 6. Chi-test results on students' performances in solving the three real-world problems.

RWP1-RWP3	RWP2-RWP1	RWP2-RWP3	Critical chi-square value	*d.f.*
1.048	6.571	10.374	5.992 (a = 0.05)	2

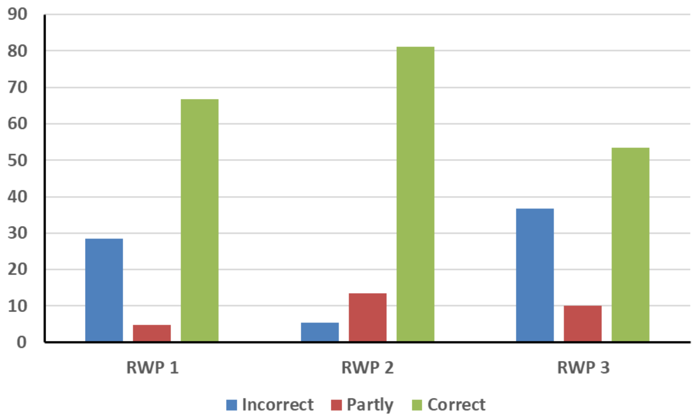

Figure 1. Grade distributions (%) of students' results for the three real-world problems.

In the meantime, the chi-square value for the grade distributions between the first and the third problems is 1.048, well below the critical chi-square vale, which indicates that the pattern of students' grade distributions for the first and third problem is similar to each other. This similarity in grade distribution is visually apparent in Figure 1, in which both share a U-shape for 'Incorrect', 'Partly correct', and 'Correct' categories. In contrast, an ascending grade distribution is shown for the second problem. This difference in grade patterns was caused by the guessing strategy worked in the second problem.

Should those six students who did not follow the process of simultaneous equations in the second real-world problem be counted as 'Incorrect', the grade distribution would share the same pattern as the two other cases with a U-shape (Figure 2). Correspondingly, this similarity among the three cases would be confirmed by chi tests shown in Table 7, in which all chi-square values are smaller than the critical value. Hence, the over simplicity of the second problem would have largely distorted the grade distribution for the problem.

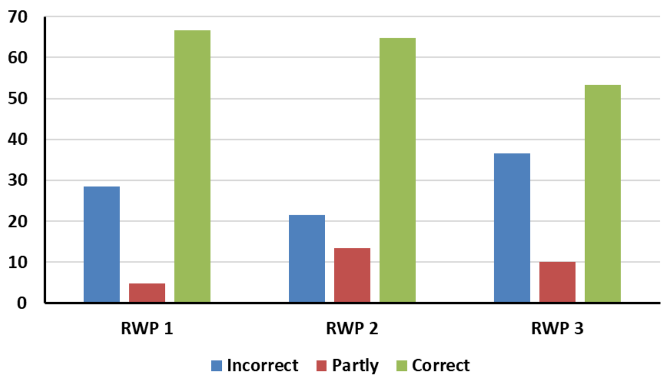

Figure 2. Grade distributions (%) of the reclassified results for the three problems.

Table 7. Chi-test results on students' performances in solving the three real-world problems after reclassification of the second real-world problem.

RWP1-RWP3	RWP2-RWP1	RWP2-RWP3	Critical chi-square value	*d.f.*
1.048	1.267	1.863	5.992 (a = 0.05)	2

This analysis has implications on the design of assessments for the pre-service mathematics students. Firstly, the assessment questions should be designed with appropriate level of difficulty with respect to the targeted purpose so that simple guessing strategy (or any short-cut) cannot be easily undertaken. Secondly, if the level of difficulty is not able to be properly adjusted, clear instructions on the mandatory mathematical approaches must be stated for all students to follow. Otherwise, there should be no mark awarded to the solution obtained by any other means, even correct.

6.3. Implications on teaching and learning of education mathematics in RRR areas

Solving word problems involving real-world scenarios has been one of the most challenging tasks for pre-service mathematics teachers enrolled in both metropolitan and regional universities over the world [7,8,27,28]. Recent study also found that, by excluding no attempts, the performance of RRR pre-service mathematics teachers in solving word problems involving triangles at a regional university seemed above that of students in metropolitan institutions [7]. However, it was also found that the higher proportion of no attempt and the weak background in basic mathematics with the RRR student mathematics teachers were the major factors more likely to affect the progression of most these students in their mathematics study. This new study on solving real-world problems associated with simultaneous equations proves that these two factors are true once again for the RRR pre-service mathematics teachers.

There is no doubt that every effort should be made to provide equal opportunities in higher education to RRR students by all tertiary institutions, particularly the regional universities located in regional cities as local hubs to serve the surrounding rural and remote communities. Unlike learning other social science courses which are based more on social norms and experiences, in which common sense plays an important role, however, mathematics learning is a process of continuous knowledge building from what has been previously learnt to new concepts and techniques, or a recursive process from knowns to unknowns. This also applies to the first-year pre-service mathematics teachers who should have retained most of the foundation mathematics learnt in Years 7-10, if excluding senior mathematics learnt in Years 11-12. Otherwise, the first mathematics course in their university study would be too challenging to cope with for many students even though this first course only serves the purpose of reviewing the foundation mathematics students should have learnt in their Years 7-10 in high school. The tertiary curriculum cannot afford to spend one or two years to simply repeat the junior high school mathematics.

Therefore, institutions should maintain the minimum entry standard for applicants who want to become school mathematics teachers. For those candidates who are determined to become a future mathematics teacher but academically ineligible to be admitted into a formal mathematics program by a long margin, such as the chef in the first real-world problem, the institution may need to consider creating a bridging program, through which the students can rebuild their foundation in basic mathematics and gradually improve their mathematical skills and confidence in embracing new challenges while progressing to the formal mathematics program.

All first-year pre-service mathematics teachers, regardless of directly admitted or progressed from a bridging program, should be prepared to make a persistent effort on each of every new mathematics course throughout their degree study. Unlike school study where both teaching and learning are mainly driven by the teachers during the school time, tertiary education is a process of mutual commitments to teaching and learning with some degree of flexibility and independence. Teaching and assessment design are mainly driven by the teachers whereas learning and fulfilling required learning tasks are mainly driven by students. Both would need to meet somewhere in the middle to forge smooth progression in mathematics study. Solely relying on either side would not prevail in tertiary education, particularly in STEM disciplines where the nature of the disciplines is challenging.

7. Conclusions

Solving worded real-world problems is a difficult challenge for pre-service secondary mathematics teachers engaging study and training in most tertiary institutions, including the pre-service mathematics teachers in RRR areas with regional universities. The results of this study have the following pedagogical implications on training the students in solving real-life problems towards becoming knowledgeable and motivated mathematics teachers for secondary schools.

Setting purposeful and challenging assessments at an appropriate level can help identify those willing and motivated students who are determined to become capable mathematics teachers through embracing the challenging nature and recursive progression of mathematics learning. For those students who are not prepared to take on this persistent challenge, such purposeful and challenging assessments would make them switch to other disciplines more suitable to individuals' capability earlier.

The assessment questions should be designed with appropriate level of difficulty with respect to the targeted purpose of the question so that any short-cut cannot be easily taken. If the level of difficulty is not able to be properly adjusted, clear instructions on the mandatory mathematical approaches must be stated for all students to follow so as to enforce the pedagogical purpose of the assessments.

Maintaining the minimum entry standard for applicants who want to become school mathematics teachers must be enforced too so that only those candidates who are determined to become a future mathematics teacher are accepted into a formal program directly. For the motivated candidates who are academically ineligible to be admitted into a formal mathematics program by a long margin, the institution may provide a bridging program for such students to help them rebuild foundation of basic mathematics and confidence in embracing new challenges when progressing to the formal mathematics program. All first-year pre-service mathematics teachers, regardless of directly admitted or progressed from a bridging program, should be prepared to make a persistent effort on every new mathematics course throughout their degree study.

The major weakness of this case study was that it was not able to make comparisons with other research outcomes from a similar RRR setting in other places in Australia or the world. This also highlights the need for educators and researchers in RRR regions all over the world to pay attention to the training of pre-service mathematics teachers in learning and applying solving worded real-world problems closely associated with regional lives and activities so as to better understand such challenges in the RRR communities. The other limitation of this study is the lack of a formal mechanism to collect students' feedback on their learning journey and experiences in dealing with solving worded real-world problems due to the vastly distributed nature of teaching and learning in RRR regions, which should be addressed in future research projects.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare there is no conflict of interest in any part of this article.

Ethics declaration

The author declared that the ethics committee approval was waived for the study.

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