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Classroom note

Streamlining applications of integration by parts in teaching applied calculus

William Guo*

School of Engineering and Technology, Central Queensland University, Bruce Highway, North Rockhampton, QLD 4702, Australia

*** Correspondence:** Email: w.guo@cqu.edu.au; Tel: +61-7-49309687

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Abstract: Integration by parts can be applied in various ways for obtaining solutions for different types of integrations and hence it is taught in all calculus courses in the world. However, the coverage and discourse of various applications of integration by parts in most textbooks, often packed into one section, lack a cohesion of progression for solving different types of integrals. Students may be confused by such incohesive presentation of the method and applications in the textbooks. Based on the author's experiences and practices in teaching applied calculus for undergraduate engineering and education students since 2013, a streamlined approach in teaching integration by parts has been gradually developed to the current state and ready to be shared with the mathematics teaching and learning communities. This streamlined approach allows integration by parts to be applied to solve complicated and integrated problems in a progressive way so that students can improve efficacy in their use of integration by parts gradually. This approach also makes communications easier with students on particular problems involving integration by parts.

Keywords: integration by parts, nested approach, cyclic approach, reduction formula, streamline

1. Introduction

Integration by parts is commonly used to obtain solutions for many types of problems with differential equations, integral transforms, and other advanced mathematics [1–3]; hence it is included in all calculus textbooks for most undergraduate STEM programs in the world [4–6].

In teaching applied calculus courses using a few popular textbooks [4–6] to undergraduate engineering and education students at a regional university in Australia since 2013, through responding to numerous questions asked by different cohorts of students, the author realized that the coverage and discourse of various applications of integration by parts in the textbooks lack a cohesion of progression in terms of solving different types of integrals by integration by parts, mostly packed in one section in the textbooks. Students were often confused by this loose discourse in the textbooks. Hence, amending such gaps or missed links for better delivering integration by parts in applied calculus courses became the major motivation of this pedagogical project begun in 2018. With the accumulation of more examples and experiences in applying this teaching approach in recent years, this paper has been gradually enriched to the current status for sharing with the mathematics teaching and learning communities in the world.

In Section 2, the streamlining of various applications of integration by parts is presented using typical examples. Sections 3 presents the student performances in solving more challenging problems under the guidance of this streamlined approach in recent years. Concluding remarks are summarized in Section 4.

2. Streamlining applications of integration by parts

2.1. Direct application of integration by parts

Integration by parts is basically the inverse of the product rule of differentiation. If $u = f(x)$ and $v = g(x)$ are differentiable functions, by the product rule of differentiation

$$\frac{d(uv)}{dx} = \frac{du}{dx}v + u\frac{dv}{dx},$$

multiply dx to both sides of above equation to transfer it to its differential form:

$$\cancel{dx} \times \frac{d(uv)}{\cancel{dx}} = \left(\frac{du}{\cancel{dx}}v + u\frac{dv}{\cancel{dx}}\right) \times \cancel{dx} \longrightarrow d(uv) = vdu + udv.$$

Apply integral to both sides of above differential equation,

$$\int d(uv) = \int vdu + \int udv \longrightarrow uv = \int vdu + \int udv$$

or

$$\int udv = uv - \int vdu\ . \qquad (1)$$

Formula (1) is commonly known as *integration by parts*. It converts one combinative part of an integral to the other combinative part of the integral if the integrand can be treated as a product of two differentiable functions. Note that not all combined integrands can be solved by integration by parts.

In practice, once u and dv on the left of formula (1) are identified, v and du can be obtained through

$$\int udv = uv - \int vdu \longleftarrow v = \int dv, \quad du = u'dx\ . \qquad (2)$$

For simple combinative integrands, one round of integration by parts is sufficient in obtaining the solution.

Example 1: Evaluate $\int x^2 \ln x dx$.

Let $u = \ln x$ and $dv = x^2 dx$. By formula (2),

$$\int \underbrace{\ln x}_{u} \underbrace{x^2 dx}_{dv} = (\underbrace{\ln x}_{u})\left(\underbrace{\frac{1}{3}x^3}_{v}\right) - \int \left(\underbrace{\frac{1}{3}x^3}_{v}\right)\left(\underbrace{\frac{dx}{x}}_{du}\right) \longleftarrow v = \int x^2 dx = \frac{1}{3}x^3, \quad du = (\ln x)' dx = \frac{dx}{x}$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c = \frac{1}{9}x^3(3\ln x - 1) + c.$$

Example 2: Evaluate $\int 4x \sin x dx$.

Let $u = 4x$ and $dv = \sin x dx$. By formula (2),

$$\int \underbrace{4x}_{u} \underbrace{\sin x dx}_{dv} = (\underbrace{4x}_{u})(\underbrace{-\cos x}_{v}) - \int (\underbrace{-\cos x}_{v})(\underbrace{4dx}_{du}) \longleftarrow v = \int \sin x dx = -\cos x, \quad du = (4x)' dx = 4dx$$

$$= -4x\cos x + \int 4\cos x dx = -4x\cos x + 4\sin x + c = 4(\sin x - x\cos x) + c.$$

Example 3: Evaluate $\int x \arctan x dx$.

Let $u = \arctan x$ and $dv = x dx$. By formula (2),

$$\int \underbrace{\arctan x}_{u} \underbrace{(x dx)}_{dv} = (\underbrace{\arctan x}_{u})(\underbrace{\frac{1}{2}x^2}_{v}) - \int \left(\underbrace{\frac{1}{2}x^2}_{v}\right)\left(\underbrace{\frac{dx}{1+x^2}}_{du}\right) \longleftarrow v = \frac{1}{2}x^2, \quad du = (\arctan x)' dx = \frac{dx}{1+x^2}$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx \longleftarrow \frac{x^2}{1+x^2} = \frac{1+x^2-1}{1+x^2} = 1 - \frac{1}{1+x^2}$$

$$= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right) dx = \frac{1}{2}x^2 \arctan x - \frac{1}{2}(x - \arctan x) + c$$

$$= \frac{1}{2}[(x^2+1)\arctan x - x] + c.$$

2.2. Nested applications of integration by parts

For more complicated combinative integrands, the process of integration by parts can be repeated in the subsequent integral(s) nested within the scrolling process of integration. Add constant c to the solution when no further integration is needed.

Example 4: Evaluate $\int x^2 e^x dx$.

Let $u = x^2$ and $dv = e^x dx$. By formula (2),

$$\int \underset{u}{\underline{x^2}}\, \underset{dv}{\underline{e^x dx}} = (\underset{u}{\underline{x^2}})(\underset{v}{\underline{e^x}}) - \int (\underset{v}{\underline{e^x}})(\underset{du}{\underline{2xdx}}) \longleftarrow v = \int e^x dx = e^x,\quad du = (x^2)'dx = 2xdx$$

$$= x^2 e^x - 2\int xe^x dx = x^2 e^x - 2I \longleftarrow I = \int xe^x dx$$

Repeat integration by parts to *I*.

$$I = \int xe^x dx = (x)(e^x) - \int e^x dx = xe^x - e^x .$$

Substitute *I* back to obtain

$$\int x^2 e^x dx = x^2 e^x - 2(xe^x - e^x) + c = x^2 e^x - 2xe^x + 2e^x + c = e^x (x^2 - 2x + 2) + c .$$

This process can be combined together by multiple rounds of integration by parts demonstrated in the following examples.

Example 5: Evaluate $\int (\ln x)^2 xdx$.

Let $u = (\ln x)^2$ and $dv = xdx$. By formula (2),

$$\int \underset{u}{\underline{(\ln x)^2}}\, \underset{dv}{\underline{xdx}} \longleftarrow v = \int xdx = \frac{1}{2}x^2,\quad du = [(\ln x)^2]'dx = 2\ln x(\ln x)'dx = \frac{2\ln xdx}{x}$$

$$= (\underset{u}{\underline{\ln x}})^2 (\underset{v}{\underline{\frac{1}{2}x^2}}) - \int (\underset{v}{\underline{\frac{1}{2}x^2}})(\underset{du}{\underline{\frac{2\ln xdx}{x}}}) \longleftarrow \mathit{1st\ integration\ by\ parts}$$

$$= \frac{1}{2}x^2(\ln x)^2 - \int (\ln x)(xdx)$$

$$= \frac{1}{2}x^2(\ln x)^2 - \left[(\ln x)(\frac{1}{2}x^2) - \int (\frac{1}{2}x^2)(\frac{dx}{x}) \right] \longleftarrow \mathit{2nd\ integration\ by\ parts}$$

$$= \frac{1}{2}x^2(\ln x)^2 - \left(\frac{1}{2}x^2 \ln x - \frac{1}{2}\int xdx \right) = \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + c$$

$$= \frac{1}{4}x^2[2(\ln x)^2 - 2\ln x + 1] + c.$$

Example 6: Evaluate $\int x^2 \sin\frac{x}{2}dx$.

Let $u = x^2$ and $dv = \sin\frac{x}{2}dx$. By formula (2),

$$\int \underset{u}{\underline{x^2}}\, \underset{dv}{\underline{\sin\frac{x}{2}dx}} \longleftarrow v = \int \sin\frac{x}{2}dx = -2\cos\frac{x}{2},\quad du = (x^2)'dx = 2xdx$$

$$= \underset{u}{\underline{x^2}}\underset{v}{\underline{(-2\cos\frac{x}{2})}} - \int \underset{v}{\underline{(-2\cos\frac{x}{2})}}(\frac{2xdx}{du}) \longleftarrow \textit{1st integration by parts}$$

$$= -2x^2\cos\frac{x}{2} + 4\int(x)(\cos\frac{x}{2}dx)$$

$$= -2x^2\cos\frac{x}{2} + 4\left[(x)(2\sin\frac{x}{2}) - \int(2\sin\frac{x}{2})(dx)\right] \longleftarrow \textit{2nd integration by parts}$$

$$= -2x^2\cos\frac{x}{2} + 4\left[2x\sin\frac{x}{2} - 2\int\sin\frac{x}{2}dx\right] = -2x^2\cos\frac{x}{2} + 4\left[2x\sin\frac{x}{2} + 4\cos\frac{x}{2}\right] + c$$

$$= -2x^2\cos\frac{x}{2} + 8x\sin\frac{x}{2} + 16\cos\frac{x}{2} + c = 2(8-x^2)\cos\frac{x}{2} + 8x\sin\frac{x}{2} + c.$$

2.3. Cyclic applications of integration by parts

For some special integrands involving combinations of logarithmic, exponential, and trigonometric functions, or higher-order functions, the process of integration by parts may be repeated in the subsequent integral(s) until the recurrence of the original integral or its lower-order form. If the original integral reoccurs, the integral can be obtained by means of solving algebraic equations from that point. If the lower-order integral occurs, the integral can be obtained as a reduction formula.

Example 7: Evaluate $\int e^x \cos x dx$.

Let $u = \cos x$ and $dv = e^x dx$. By formula (2),

$$I = \int \underset{u}{\underline{\cos x}}\,\underset{dv}{\underline{e^x dx}} = (\underset{u}{\underline{\cos x}})(\underset{v}{\underline{e^x}}) - \int(\underset{v}{\underline{e^x}})(\underset{du}{\underline{-\sin x dx}}) = e^x\cos x + \int \sin x e^x dx$$

$$= e^x\cos x + \left(\sin x e^x - \int \cos x e^x dx\right) = e^x\cos x + e^x\sin x - \underset{I}{\underline{\int \cos x e^x dx}}$$

$$I = e^x\sin x + e^x\cos x - I \longrightarrow 2I = e^x\sin x + e^x\cos x$$

$$I = \int \cos x e^x dx = \frac{1}{2}e^x(\sin x + \cos x) + c.$$

Example 8: Evaluate $I(n) = \int x^n e^x dx$ ($n > 0$). Use the obtained formula to find $I(1)$, $I(2)$ and $I(3)$ respectively.

$$I(n) = \int \underset{u}{\underline{x^n}}\,\underset{dv}{\underline{e^x dx}} = \underset{u}{\underline{(x^n)}}\,\underset{v}{\underline{(e^x)}} - \int \underset{v}{\underline{(e^x)}}\,\underset{du}{\underline{(nx^{n-1}dx)}} = x^n e^x - n\int x^{n-1}e^x dx$$

$$= x^n e^x - nI(n-1) \longleftarrow I(n-1) = \int x^{n-1}e^x dx.$$

This expression is a reduction formula because the integral of a higher-order integrand can be determined by that of a lower-order integrand. The following example shows how to use the reduction formula obtained to find the integrals for higher-order integrands from the lower-order integrals.

$$n=1: \quad I(1)=x^1e^x-\int x^{1-1}e^x dx = xe^x-\int e^x dx = xe^x-e^x+c=e^x(x-1)+c;$$

$$n=2: \quad I(2)=x^2e^x-2I(1)=x^2e^x-2e^x(x-1)+c=e^x(x^2-2x+2)+c;$$

$$n=3: \quad I(3)=x^3e^x-3I(2)=x^3e^x-3e^x(x^2-2x+2)+c=e^x(x^3-3x^2+6x-6)+c.$$

Example 9: Evaluate $I(n)=\int \sin^n ax dx$ (n: positive integers; a: constant).

$$I(n)=\int \sin^n ax dx=\int \underbrace{(\sin^{n-1} ax)}_{u}\underbrace{(\sin ax dx)}_{dv}$$

$$=\underbrace{(\sin^{n-1} ax)}_{u}\underbrace{(-\frac{1}{a}\cos ax)}_{v}-\int \underbrace{(-\frac{1}{a}\cos ax)}_{v}\underbrace{(n-1)\sin^{n-2} ax(a\cos ax)dx}_{du}$$

$$=-\frac{1}{a}\cos ax\sin^{n-1} ax+(n-1)\int \cos^2 ax\sin^{n-2} ax dx$$

$$=-\frac{1}{a}\cos ax\sin^{n-1} ax+(n-1)\int (1-\sin^2 ax)\sin^{n-2} ax dx$$

$$=-\frac{1}{a}\cos ax\sin^{n-1} ax+(n-1)\int (\sin^{n-2} ax-\sin^n ax)dx$$

$$=-\frac{1}{a}\cos ax\sin^{n-1} ax+(n-1)\int \sin^{n-2} ax dx-(n-1)\underbrace{\int \sin^n ax dx}_{I(n)}$$

$$I(n)=-\frac{1}{a}\cos ax\sin^{n-1} ax+(n-1)\int \sin^{n-2} ax dx-(n-1)I(n)$$

$$I(n)+(n-1)I(n)=-\frac{1}{a}\cos ax\sin^{n-1} ax+(n-1)\int \sin^{n-2} ax dx$$

$$nI(n)=-\frac{1}{a}\cos ax\sin^{n-1} ax+(n-1)\int \sin^{n-2} ax dx$$

$$I(n)=-\frac{1}{na}\cos ax\sin^{n-1} ax+\frac{n-1}{n}\int \sin^{n-2} ax dx \quad \text{or}$$

$$I(n)=-\frac{1}{na}\cos ax\sin^{n-1} ax+\frac{n-1}{n}I(n-2)\longleftarrow I(n-2)=\int \sin^{n-2} ax dx.$$

Similar to the output of Example 8, this is a reduction formula, by which the integral of any higher-order integrand can be determined by the integrals of the lower-order integrands progressively.

3. Student learning outcomes in different courses

This streamlined delivery of integration by pars was first introduced to teaching applied calculus to the first-year undergraduate engineering students in 2018. The successful experiment then led to the adoption of this teaching practice for the second-year education students from 2019. The following subsections present the outcomes of solving integrals by combining the nested and cyclic approaches with integration by parts by engineering students in 2018 and education students in 2019 respectively.

3.1. Learning outcomes of engineering students

For the twenty-eight first-year engineering students enrolled in the applied calculus course in 2018, the following question was included in an individual assignment to all students. Note that in the prescribed textbook [4], the equivalent 'nested approach' for integration by parts was covered by a few basic examples and the 'cyclic approach' was not fully covered, with only one simple example to demonstrate obtaining a reduction formula. In teaching, both the nested and cyclic approaches were extended by using the supplementary textbook published in 2018 [7] because students would need to master the cyclic approach to solve the second part of this assigned question.

Question 1: Evaluate $\int (4x^2 \cos 2x - e^{-x} \sin x)dx$.

The step-by-step solution for this question is presented below as a reference. The first part is a simple application of the nested approach with two rounds of integration by parts whereas the second part requires applying the cyclic approach.

$$I = \int (4x^2 \cos 2x - e^{-x} \sin x)dx = \int 4x^2 \cos 2x dx - \int e^{-x} \sin x dx = I_1 - I_2$$

$$I_1 = \int \underbrace{4x^2}_{u} \underbrace{\cos 2x dx}_{dv} = \underbrace{4x^2}_{u} \underbrace{(\frac{1}{2}\sin 2x)}_{v} - \int \underbrace{(\frac{1}{2}\sin 2x)}_{v} (\frac{8x dx}{du})$$

$$= 2x^2 \sin 2x - 4\left[(x)(-\frac{1}{2}\cos 2x) - \int (-\frac{1}{2}\cos 2x)(dx) \right]$$

$$= 2x^2 \sin 2x - 4\left[-\frac{1}{2} x \cos 2x + \frac{1}{2}\int \cos 2x dx \right] = (2x^2 - 1)\sin 2x + 2x\cos 2x + c_1.$$

$$I_2 = \int \underbrace{\sin x}_{u} \underbrace{e^{-x} dx}_{dv} = (\underbrace{\sin x}_{u})(\underbrace{-e^{-x}}_{v}) - \int (\underbrace{-e^{-x}}_{v})(\frac{\cos x dx}{du})$$

$$= -e^{-x} \sin x + \int (\cos x)(e^{-x} dx) = -e^{-x} \sin x + \left(-e^{-x} \cos x - \int e^{-x} \sin x dx \right)$$

$$= -e^{-x} \sin x - e^{-x} \cos x - \underbrace{\int \sin x e^{-x} dx}_{I_2}$$

$$I_2 = -e^{-x}(\sin x + \cos x) - I_2 \longrightarrow 2I_2 = -e^{-x}(\sin x + \cos x)$$

$$I_2 = -\frac{1}{2} e^{-x}(\sin x + \cos x) + c_2.$$

$$I = \int (4x^2 \cos 2x - e^{-x} \sin x)dx = I_1 - I_2$$

$$= (2x^2 - 1)\sin 2x + 2x\cos 2x + c_1 - \left[-\frac{1}{2} e^{-x}(\sin x + \cos x) + c_2 \right]$$

$$= (2x^2 - 1)\sin 2x + 2x\cos 2x + \frac{1}{2} e^{-x}(\sin x + \cos x) + c. \longleftarrow c = c_1 - c_2$$

Twenty-three out of the twenty-eight students solved this question correctly using both the nested and cyclic approaches with integration by pars. The other five students did the nested application correctly but could not resolve the second part by the cyclic approach. These students neither attended the live classes nor watched the recorded classes available on Moodle. Overall, this experiment was a great success because the full cyclic approach was the first time assigned to the students in the history of this course.

3.2. Learning outcomes of education students

For the twenty second-year education students enrolled in the applied integral calculus course in 2019, the following question was included in an individual assignment to all students. Note that in the previously prescribed textbook [5], both the equivalent 'nested and the cyclic approaches' for integration by parts were shown by one example each, despite not being streamlined. However, how to use the obtained reduction formula was not demonstrated. Using a reduction formula to derive integrals of higher-order integrands had been a point of difficulty reported by the previous lecturers for education students. Hence, by recommending students to use the new textbook [7] that demonstrates the use of the obtained reduction formula, the following assigned question aimed to test student's efficacy in not only obtaining the reduction formula by the cyclic approach but also using the obtained reduction formula to derive integrals for the higher-order integrands from the lower-order integrands.

Question 2: Derive a reduction formula for $I(n)=\int \cos^n xdx,$ in which n is 0 or positive integers and a is a constant. Given $I(0)=\int \cos^0 xdx=\int dx=x,$ and $I(1)=\int \cos xdx=\sin x,$ use $I(n)$ to derive integrals for $\int \cos^2 xdx, \int \cos^3 xdx,$ and $\int \cos^4 xdx,$ respectively.

The step-by-step solution for this question is presented below as a reference. The first part is to obtain the reduction formula using the cyclic approach and the second part is to use the obtained formula to derive more integrals for the higher-order integrands.

$$
\begin{aligned}
I(n)=\int \cos^n axdx &= \int \underbrace{(\cos^{n-1} ax)}_{u}\underbrace{(\cos axdx)}_{dv} \\
&= \underbrace{(\cos^{n-1} ax)}_{u}\underbrace{(\frac{1}{a}\sin ax)}_{v} - \int \underbrace{(\frac{1}{a}\sin ax)}_{v}\underbrace{(n-1)\cos^{n-2} ax(-a\sin ax)dx}_{du} \\
&= \frac{1}{a}\sin ax\cos^{n-1} ax+(n-1)\int \sin^2 ax\cos^{n-2} axdx \\
&= \frac{1}{a}\sin ax\cos^{n-1} ax+(n-1)\int (1-\cos^2 ax)\cos^{n-2} axdx \\
&= \frac{1}{a}\sin ax\cos^{n-1} ax+(n-1)\int (\cos^{n-2} ax-\cos^n ax)dx \\
&= \frac{1}{a}\sin ax\cos^{n-1} ax+(n-1)\int \cos^{n-2} axdx-(n-1)\underbrace{\int \cos^n axdx}_{I(n)}
\end{aligned}
$$

$$I(n)=\frac{1}{a}\sin ax\cos^{n-1}ax+(n-1)\int\cos^{n-2}axdx-(n-1)I(n)$$

$$I(n)+(n-1)I(n)=\frac{1}{a}\sin ax\cos^{n-1}ax+(n-1)\int\cos^{n-2}axdx$$

$$nI(n)=\frac{1}{a}\sin ax\cos^{n-1}ax+(n-1)I(n-2)\longleftarrow I(n-2)=\int\cos^{n-2}axdx$$

$$I(n)=\frac{1}{na}\sin ax\cos^{n-1}ax+\frac{n-1}{n}I(n-2).$$

$n=2, a=1:$

$$I(2)=\int\cos^2 xdx=\frac{1}{2}\sin x\cos^{2-1}x+\frac{2-1}{2}I(2-2)=\frac{1}{2}\sin x\cos x+\frac{1}{2}I(0)=\frac{1}{2}\sin x\cos x+\frac{1}{2}x+c$$

$$=\frac{1}{2}(\sin x\cos x+x)+c$$

$n=3, a=1:$

$$I(3)=\int\cos^3 xdx=\frac{1}{3}\sin x\cos^{3-1}x+\frac{3-1}{3}I(3-2)=\frac{1}{3}\sin x\cos^2 x+\frac{2}{3}I(1)$$

$$=\frac{1}{3}\sin x\cos^2 x+\frac{2}{3}\sin x+c$$

$n=4, a=1:$

$$I(4)=\int\cos^4 xdx=\frac{1}{4}\sin x\cos^{4-1}x+\frac{4-1}{4}I(4-2)=\frac{1}{4}\sin x\cos^3 x+\frac{3}{4}I(2)$$

$$=\frac{1}{4}\sin x\cos^3 x+\frac{3}{4}\left[\frac{1}{2}(\sin x\cos x+x)\right]+c=\frac{1}{8}\left[2\sin x\cos^3 x+3(\sin x\cos x+x)\right]+c.$$

Thirteen students out of the twenty solved this question correctly in both deriving the reduction formula and using it to obtain the integral for $I(2)$, $I(3)$, and $I(4)$ from the known $I(0)$ and $I(1)$. Two students were correct in deriving the reduction formula but unclear how to use it to obtain the integral for $I(2)$, $I(3)$, and $I(4)$ from the known $I(0)$ and $I(1)$. Two more students experienced difficulty in deriving the reduction formula by the cyclic approach with integration by pars. These four students used only the previously prescribed textbook without attending the live classes or watching the recorded classes available on Moodle. The remaining three students did not attempt this question at all. Overall, this was a highly satisfactory outcome, given the fact that similar activities had never been assigned to the education students in the previous years as it was thought too difficult for them.

4. Concluding remarks

This streamlined approach allows integration by parts to be applied to solve complicated problems in a progressive way so that students can improve efficacy in their use of integration by parts gradually. This was hardly implementable into student's assignments when using other textbooks with a loose coverage of integration by parts in the past.

This approach also made communications with students on particular problems of integration much more efficient and purposeful as student's question could be easily associated to where it came from by referring to the ‘nested approach’ or the ‘cyclic approach’, instead of a general reference to integration by parts.

The learning outcomes of using this streamlined approach to solve sophisticated problems by integration by parts from both the engineering and education students in 2018 and 2019 were encouraging, but it also confirmed the trend that more students become unwilling to engage with any form of teaching and learning activities, regardless of the pedagogical enhancement made by the teachers. This trend has been observed by many educators around the world for the recent two decades [8,9], particularly in regional universities [10]. Hence, dealing with pedagogical challenges in teaching and learning is not easy, but overcoming social challenges to motivate students to actively engage with teaching and learning is even more challenging.

Acknowledgments

This article was resulted from the reviewers’ suggestions to an earlier article dealing with technical difficulties commonly experienced by many undergraduate students in applying integration by parts to solve problems of integral, particularly those sophisticated problems. That article assumed that all readers should have been capable of applying integration by parts for solving integrals with easy and medium levels of difficulty. The reviewers were concerned that such assumption is likely untrue for many undergraduate students because integration by parts is always a challenging topic for many students. Similar feedback was also received from some students who were learning integral calculus at the time with the author. As a result, this article goes ahead of the earlier article that has been on hold for the next issue. The author much appreciates the referees’ thoughtful suggestions and the students’ feedback and encouragement towards preparing this note.

References

1. Guo, W., Unification of the common methods for solving the first-order linear ordinary differential equations. *STEM Education*, 2021, 1(2): 127–140. https://doi.org/10.3934/steme.2021010
2. Zill, D.G., *A First Course in Differential Equations with Modeling Applications*, 10th ed. 2013, Boston, USA: Cengage Learning.
3. Kreyszig, E., *Advanced Engineering Mathematics*, 10th ed. 2011, USA: Wiley.
4. Croft, A., Davison, R., Hargreaves, M. and Flint J., *Engineering Mathematics*, 5th ed. 2017, Harlow, UK: Pearson.
5. Stewart, J., *Calculus: Concepts and Contexts*, 4th ed. 2019. USA: Cengage.
6. Trim, D., *Calculus for Engineers*. 4th ed. 2008, Toronto, Canada: Pearson.
7. Guo, W.W., *Essentials and Examples of Applied Mathematics*. 2018, Melbourne, Australia: Pearson.
8. Massingham, P. and Herrington, T., Does attendance matter? An examination of student attitudes, participation, performance and attendance. *Journal of University Teaching and Learning Practice*, 2006, 3: 82–103. https://doi.org/10.53761/1.3.2.3
9. Henry, M.A., Shorter, S., Charkoudian, L., Heemstra, L.M. and Corwin, L.A., FAIL is not a four-letter word: A theoretical framework for exploring undergraduate students’ approaches to academic challenge and responses to failure in STEM learning environments. *CBE Life Sciences Education*, 2019, 18: 1–17. https://doi.org/10.1187/cbe.18-06-0108

10. Guo, W., Li, W. and Tisdell, C.C., Effective pedagogy of guiding undergraduate engineering students solving first-order ordinary differential equations. *Mathematics*, 2021, 9(14):1623. https://doi.org/10.3390/math9141623

Author's biography

Dr. William Guo is a professor in mathematics education with Central Queensland University, Australia. He is specialized in teaching applied mathematics for both engineering and education students. His research interests include mathematics education, applied computing, data analysis and numerical modeling. He is a member of IEEE and Australian Mathematical Society.