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Research article

Digital currency price formation: A production cost perspective

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Abstract: The paper investigates the long-run relationship between bitcoin and its marginal cost between July 2010 and July 2022. We derive Bitcoin's marginal cost of production from a model of Bitcoin mining grounded in the Bitcoin code, and show that its production cost is a function of only two variables, the electricity price and the mining hardware efficiency. We then estimate a time-varying vector error correction model, and also the cointegration between bitcoin's price and Bitcoin network's hash rate, a commonly used production cost proxy. Our results show that the time-varying cointegration between bitcoin's price and its hash rate is permanently in disequilibrium, bar a short time interval between March 2017 and January 2018. Consequently, although bitcoin's price and the hash rate are cointegrated, it is clear that the latter does not function as a stable long-run explanatory variable for bitcoin price dynamics. On the contrary, we found that bitcoin's price and its marginal cost of production have been cointegrated since its inception, and that their time-varying long-run relationship always reverts towards equilibrium - and often *to* equilibrium- after long periods of divergence. These results contrast with most of the empirical literature that attempted to model the relationship betweeen bitcoin and its fundamentals in a time-invariant framework, but are consistent with recent research showing a significant role for production cost in the determination of bitcoin's price dynamics.

Keywords: bitcoin mining; cost of production; asset pricing; time-varying VECM; Bitcoin hardware efficiency

JEL Codes: G12, G20; G32

1. Introduction

Modelling bitcoin's price dynamics is uniquely challenging, owing to its extreme volatility and its idiosyncratic origin. Most of the initial literature on bitcoin prices highlighted the exponential increases in its value that appeared totally unrelated to the underlying fundamentals. Those fundamentals were in addition hard to discern, given that a bitcoin is produced by solving a cryptographic problem requiring only a computer and energy. As pointed out by the creator of Bitcoin[1], " The steady addition of a constant of [sic] amount of new coins is analogous to gold miners expending resources to add gold to circulation. In our case, it is CPU time and electricity that is expended."Nakamoto (2008, section 6). Nonetheless, the empirical literature has failed to find a significant link between electricity prices - and other production cost proxies- and the price of the bitcoin-USD exchange rate (Marthinsen and Gordon, 2022; Kjærland et al., 2018; Fantazzini and Kolodin, 2020). This has led to the conclusion that bitcoin has no intrinsic value and is merely a bubble (Cheah and Fry, 2015; Xiong et al., 2020; Caferra et al., 2021; Kyriazis et al., 2020). Consequently, most of the literature in economics and finance has focused on sucessfully modelling the volatility of bitcoin prices with various GARCH models (Fantazzini and Kolodin, 2020; Gyamerah, 2019; Katsiampa, 2017; Magtanggol III De Guzman and So, 2018; Caporale and Timur, 2019; Siu and Elliott, 2021; Zhou, 2021).

However, with the exception of O'Dwyer and Malone (2014); Hayes (2017); Li and Wang (2017); Delgado-Mohatar et al. (2019); Derks et al. (2018), most of this empirical literature overlooks the fact that bitcoins have to be mined at a non-negligible cost, which should theoretically imply a long-run relationship between bitcoin price and its production cost. Hayes (2019), Kristoufek (2020), and Fantazzini and Kolodin (2020) investigate the existence of cointegration between bitcoin and cost of production, and whilst the first paper found some evidence of a long-run equilibrium, Kristoufek (2020) and Fantazzini and Kolodin (2020) conclude that there is none over the period January 2014-August 2018 (Kristoufek, 2020), or August 2016-December 2017 (Fantazzini and Kolodin, 2020). However, when analysing cointegration in subsamples between December 2017 and February 2020, both Kristoufek (2020) and Fantazzini and Kolodin (2020) found evidence of a long-run relationship between bitcoin price and its fundamentals. This suggests that time-invariant methodologies are inadequate to analyse the behaviour of bitcoin prices, and that time-invariant cointegration cannot bring to light the long-term equilibrium of bitcoin prices.

The paper investigates the determinants of the bitcoin-USD exchange rate between July 2010 and July 2022. We derive Bitcoin's marginal cost of production from a model of Bitcoin mining grounded in the Bitcoin code, and show that Bitcoin's production cost is a function of only two variables, the electricity price and the mining hardware efficiency. We then estimate a time-varying cointegration vector model, based on Bierens and Martins (2010), who generalise Park and Hahn (1999), where we test whether. bitcoin's price and its marginal cost are cointegrated. We also estimate the cointegration between bitcoin's price and Bitcoin network's hash rate, which has been considered an alternative production cost proxy in the literature (Fantazzini and Kolodin, 2020; Aoyagi and Hattori, 2019). Our results show that the long-run relationship between bitcoin's price and its fundamentals is not constant, as evidenced by the Bierens and Martins (2010)'s test of time-varying cointegration. Moreover, we found that the time-varying cointegration between bitcoin's price and its hash rate is permamently in disequilibrium, bar a short time interval between March 2017 and January 2018. Consequently, although bitcoin's price and the hash rate are cointegrated, it is clear that the former never reverts to the latter after an exogenous shock, suggesting that the hash rate does not function as a stable long-run explanatory variable for bitcoin price dynamics. On the contrary, we found that bitcoin's price and its marginal cost of production have been cointegrated since bitcoin's inception, and that their time-varying

[1]By convention, the name of a cryptoasset protocol and distributed ledger is capitalised, whilst the name of the associated cryptocurrency is spelt with a lower case initial. Moreover, in most of the empirical literature, bitcoin price refers to the btc-USD exchange rate. This paper uses both terms interchangeably.

long-run relationship always reverts towards equilbrium - and often *to* equilibrium- after long periods of divergence.

Our paper extends the existing literature on the impacts of Bitcoin's production fundamentals on its price formation. This literature has been mostly developed to account for Bitcoin's energy consumption, and the profitability of bitcoin mining (O'Dwyer and Malone, 2014; Hayes, 2017; Li and Wang, 2017; Delgado-Mohatar et al., 2019; Derks et al., 2018). Our results show that fundamentals may also account for price dynamics, as anticipated in Kristoufek (2019) and in Hayes (2019, 2017). In addition, the time-varying cointegration analysis conducted in our paper formalises the empirical approach adopted in Li and Wang (2017), and Fantazzini and Kolodin (2020), who obtain differentiated impacts from explanatory variables by splitting their main sample into various sub-periods, and corroborates the intuition that the bitcoin price dynamics cannot be analysed in a time-invariant econometric framework.

Following this Introduction, Section 2 will review the relevant empirical literature on bitcoin's price formation. Section 3 introduces the cost-of-production model that is empirically tested and analysed in Section 4. A discussion of Bitcoin hardware technological changes is presented in Section 3.2, and a survey of changes in electricity costs in Section 3.3. Section 5 concludes. A detailed review of the evolution of the specifications of Bitcoin mining hardware between 2004 and 2022 is available in the Supplement.

2. Background Literature

Models of Bitcoin production cost have first been developed by researchers interested in assessing the sustainability of cryptocurrencies based on proof-of-work (PoW), and specifically, of bitcoin (O'Dwyer and Malone, 2014; Derks et al., 2018; Kristoufek, 2020; Zadé et al., 2019; Stoll et al., 2019; Song and Aste, 2020). For the period 2012-2018, Derks et al. (2018) constructed a detailed model of expenses and rewards of mining, and evaluated it using data on mining hardware, electricity prices, and on rewards such as transactions fees and mined bitcoins. The authors concluded that the structural characteristics of Bitcoin, viz., the proof-of-work (PoW), made it financially unsustainable in the long term. Derks et al. (2018) estimated that by 2016 the marginal profit of mining a bitcoin had become negative. An analogous conclusion is reached by Delgado-Mohatar et al. (2019), who focus mostly on the impact of electricity prices on Bitcoin's marginal cost of production. They estimate the energy consumption of the Bitcoin network, and the production cost of mining between July 2017 and November 2019. The authors conclude that since June 2018 countries (or regions) where the electricity price is above 0.14 $/kWh are not profitable for bitcoin mining, and this accounts for the increasing centralization of mining activity in China, where electricity costs are as low as 0.05 $/kWh (Xiong et al., 2020). The predominance of China in bitcoin mining has also been evidenced by Makarov and Schoar (2021) who analyse the location of individual miners based on their addresses on the Bitcoin blockchain, and the geographical location of the exchanges where they cash out their rewards. The authors found that China -and in particular the province of Xinjiang- has dominated bitcoin mining, with a mining capacity between 60% and 80% over the period 2015 to 2017. The Cambridge Centre for Alternative Finance, which produces a Bitcoin Electricity Consumption index[2], and investigates the environmental sustainability of the Bitcoin network, corroborates those findings.

Recent research on Bitcoin's sustainability has mostly concentrated on evaluating the environmental

[2]https://ccaf.io/cbeci/index.

impacts of proof-of-work and particularly the immense energy requirements of the Bitcoin network (de Vries, 2018; Stoll et al., 2019; Zadé et al., 2019). Song and Aste (2020) estimate the lower bound for the global mining energy cost between 2010 to 2020, taking into account changes in energy costs, improvements in hashing technologies and hashing activity. Given that disaggregated electricity prices are not always available over an extended period of time, particularly for China, the authors estimate the energy cost for bitcoin mining using the Brent Crude oil prices as a global standard and regional industrial electricity prices weighted by the share of hashing activity. They show that hashing activity has increased a 10-billion-fold in a decade and that total energy consumption rose by a 10-million-fold. However, Song and Aste (2020) found that, rather counter-intuitively, the mining cost relative to the volume of transactions has remained stable since 2010, which they account for by the constant difficulty adjustment embedded in the Bitcoin network. In effect, in order to prevent double spending attacks on the Bitcoin network, the proof of work must represent a sizable fraction of the value that miners transfer through the network (Nakamoto, 2008). Zadé et al. (2019) and Stoll et al. (2019) attempt to estimate the future power consumption of the Bitcoin and Ethereum networks. Their analysis emphasize the recent -and estimated future- technological changes in mining hardware, and their impact on the mining difficulty and network power demand. Stoll et al. (2019) estimate that in November 2018 Bitcoin's annual electricity demand was 45.8 TWh, and that it fluctuates between 35.0 and 72.7 TWh. Its carbon footprint ranges between that of Jordan and Sri Lanka (22.0 and 22.9 MtCO2). Zadé et al. (2019) construct several scenarios of technological change of blockchain mining hardware, and quantify the power demand of the Bitcoin and Ethereum blockchains. Interestingly, their results indicate that an increase of the mining hardware efficiency will only have a limited impact on the overall power demand of blockchain networks, a result similar to that of Song and Aste (2020). The authors estimate that, under a scenario of linear growth of the block difficulty and sigmoidal increase of the hardware efficiency until the year of 2025, the mining power demand for the Bitcoin blockchain will be approximately 8 GW in 2025, up from 0.6 to 3 GW in 2018.

Although the literature on Bitcoin sustainability builds production cost models, its ultimate objective is not pricing the bitcoin-USD exchange rate, and most of the emprical literature on the determinants of bitcoin prices has failed to find clear links between the cost of production - and other fundamentals- and prices (Marthinsen and Gordon, 2022). The main drivers of bitcoins prices were found to be macroeconomic variables or behavioural trends (e.g. internet searches, social media), suggesting that the price of bitcoin is in fact a bubble. For instance, Kjærland et al. (2018)'s findings indicate that the price of Bitcoin is affected by returns on the Standard and Poor's 500 (S&P 500) and Google searches, but not by the hashrate, or other characteristics of the Bitcoin protocol. Similar results are found in Lee and Rhee (2022), who examine the relationship between the Bitcoin price and five macroeconomic variables, namely, the S&P 500 volatility index (VIX), US treasury 10-year yield, US consumer price index, gold price and dollar index, over the period August 2010 to February 2022. However, their results contrast with those of Kjærland et al. (2018) who find that the volatility index (VIX), oil, gold, and Bitcoin transaction volume are not significant determinants of bitcoin prices. Lee and Rhee (2022) also investigate the direction of causality between the bitcoin price and the macroeconomics variables, and found that the bitcoin price granger-causes the US consumer price index and dollar index, but that the former is only granger-caused by the the dollar index. The direction of causality between bitcoin and macroeconomic and cost variables has also been tested by Fantazzini and Kolodin (2020) and Hayes (2019). The former found that the bitcoin price is the determinant of production cost variables, whereas

Hayes (2019)'s results indicate that the marginal cost of bitcoins' production granger-cause its price.

The difficulty in explaining bitcoin prices from its fundamentals has led to the development of a vast empirical literature showing that bitcoin prices are a bubble. Kyriazis et al. (2020) survey the extensive literature on the formation of pricing bubbles in digital currency markets, and show that several bubble phases have taken place in Bitcoin prices, mostly during the years 2013 and 2017. Bitcoin appears to have been intermittantly in a bubble-phase since June 2015, e.g., between 1st December 2016 and 16th January 2018(Bianchetti et al., 2018); but there is also evidence that bubbles existed prior to 2015, viz., from early 2013 to mid 2014 (Chaim and Laurini, 2019). Cheung et al. (2015) detected a number of short-lived bubbles over the period 2010–2014, and found three huge bubbles in the latter part of the period 2011–2013 lasting from 66 to 106 days. Xiong et al. (2020) construct a testable model of bubble formation based on bitcoin's production cost and succeed in replicating episodes of excessive volatility in bitcoin prices. Hafner (2018) has successfully run bubble tests on 11 cryptocurrencies, and adapted the tests to the case where volatility is time varying and clustering, which is a characteristic of bitcoin prices. The author confirms the existence of bubbles in cryptocurrencies, but the time-varying approach that was adopted in Hafner (2018) shows that they are much less pronounced than previously verified by models assuming constant volatility. In fact, there are several indications in the empirical literature that time-varying approaches are more effective in capturing the volatility of bitcoin prices. Gyamerah (2019); Katsiampa (2017); Chu et al. (2017) successfully estimate the volatility of various cryptocurrencies' returns using several GARCH models. Gyamerah (2019) found that tGARCH-NIG was the best model as it described the asymmetric occurrence of shocks in the bitcoin market. Chu et al. (2017), on the other hand, found that the IGARCH and GJR-GARCH models provided the best fits for most of the cryptocurrencies. More recently, Lee and Rhee (2022) and Fantazzini and Kolodin (2020) have estimated time-varying VECMs where the cointegrating vector includes the bitcoin price and 5 macroeconomic variables, and the bitcoin price and the hash rate, respectively.

Bubbles in asset prices often result from irrational behaviour of investors (Shiller, 2000), and some empirical research has highlighted the role of herding in the price formation of bitcoin and other cryptocurrencies. Youssef (2022) and Bouri et al. (2019) analyse the static and time-varying cross-sectional absolute standard deviations (CSAD) of 14 (Bouri et al., 2019) and 18 (Youssef, 2022) cryptocurrencies. Both papers confirm the presence of herd behaviour in the cryptocurrency market between April 2013 and November 2019. In addition, Youssef (2022) finds that the level of herding in the cryptocurrency market rises as market volatility, the S&P500, and the dollar index increase, whilst the rise in the trading volume, gold price, and the economic policy uncertainty index (EPU) reduce herding. Finally, Garcia et al. (2014) and Dodd (2018) investigate the role of social interactions in the creation of price bubbles. Dodd (2018) shows that cryptocurrencies, and Bitcoin particularly, have generated a thriving community around its political ideals, and rely on a high degree of social organization in order to be produced. Garcia et al. (2014) show that four socio-economic signals about Bitcoin have a significant influence on its prices: price on online exchanges, volume of word-of-mouth communication in online social media, volume of information search, and user base growth. These results are in line with Kjærland et al. (2018) who also found that Google searches are a significant explanator of bitcoin prices.

Recent research has revisited the relationship between fundamentals and bitcoin prices (Aoyagi and Hattori, 2019; Kristoufek, 2019, 2020; Fantazzini and Kolodin, 2020; Xiong et al., 2020; Hayes, 2019, 2017; Li and Wang, 2017). Kristoufek (2019), Fantazzini and Kolodin (2020), and Hayes (2019) investigated whether there is cointegration between bitcoin prices and its cost of production. Kristoufek (2019) found no evidence of cointegration between January 2014 and August 2018, but showed that the fundamental prices derived from his cost of production model were close to market prices in December 2018. Fantazzini and Kolodin (2020) examine the relationship between the bitcoin production cost proxied by the hashrate (and the Nordpool electricity price) and bitcoin price for the period 01/08/2016–04/12/2017 and for 11/12/2017–24/02/2020. The authors found no evidence of long-run relationship between bitcoin price and cost of production between 01/08/2016 and 04/12/2017. However, a significant cointegration relationship was found in their second subsample, 11/12/2017–24/02/2020.

Kristoufek (2020)'s main result indicate that the bitcoin ecosystem is now entering "[..] a new era of Bitcoin mining where marginal (electricity) costs and mining efficiency play the prime role". In effect, rising electricity costs and China's decision to prohibit bitcoin mining on its territory in July 2021 have had a profound impact on bitcoin mining. Section 3.3 details the changes in energy prices and the relocation of mining to countries with variable, and higher, electricity costs. We simply note here that the empirical literature has observed that increased production costs had started to impact bitcoin prices at least since 2019. Hayes (2019) shows that the marginal cost of production plays an important role in explaining bitcoin prices, even if bubbles occasionally occur in cryptocurrencies markets. The author's premise is that bubbles eventually burst and cryptocurrencies' prices converge to their lower bound represented by their marginal cost. These results were anticipated by Li and Wang (2017). Their analysis suggested that Bitcoin, as an ecosystem, had matured over the 2010 decade and its valuation was now driven more by technology factors and by public recognition than by speculation and social media trends.

Two conclusions can be drawn from the current literature on the determinants of bitcoin prices. Firstly, there is an enormous price volatility stemming from investors' knowledge deficit about a new asset, leading to the creation of bubbles. A similar phenomenon occurred in the so-called "dot.com" bubble, where stocks of internet companies were overvalued and eventually collpased in the late 1990s. This implies that static econometric models will not fully capture the determinants of bitcoin prices, and that the time-varying approach adopted by this paper will be more adequate. We use the same methodology as Lee and Rhee (2022), which is based on Bierens and Martins (2010), with the crucial difference that our model will investigate the existence of a long-term relationship between production costs and bitcoin prices. As was seen above, Lee and Rhee (2022) only consider macroeconomic variables as explanatory factors. Our modelling hypothesis is grounded on the second main finding in the literature, namely, that fundamentals of bitcoin production have become more relevant since 2019, and particularly, since the relocation of bitcoin production away from China, where electricity prices were constant and low, towards the US and other countries where electricity prices fluctuate and are considerably higher (Section 3.3). Finally, our analysis draws on the literature on environmental sustainability of Bitcoin and other proof-of-work protocols to create estimates of mining hardware technological progress.

3. Model

Models of Bitcoin's cost of production first appeared in the information technology literature, e.g., O'Dwyer and Malone (2014), Hayes (2017, 2019), and were recently adopted in economics and finance research (Delgado-Mohatar et al., 2019; Xiong et al., 2020).

3.1. Bitcoin's production cost

The miner's aim is to find a number N such that

$$H(B \circ N) < H^* \tag{1}$$

where H^* is a given target value, B the string representing recent transactions on the blockchain, and $\circ$ is the concatenation operator. N is called a nonce, from "number used once". $H(S) := \text{SHA256}(\text{SHA256}(S))$ is the Bitcoin hash function, where SHA256 is a patented cryptographic hash function that outputs a number that is 256 bits long.The proof of work (PoW) consists of randomly testing values for N until $H(B \circ N) < H^*$. When a suitable N is found, the resulting block is sent to the Bitcoin network and added to the blockchain. Finding a block results in a reward of bitcoins for the miner. The process of finding N such that $H(B \circ N) < H^*$ is referred to as Bitcoin mining.

The rate at which bitcoins are mined is controlled by the Bitcoin network's choice of the value of the target H^*. Given that the Bitcoin hash function is mathematically bounded from above by $H^*_{max} = (2^{16} - 1)2^{208} \approx 2^{224}$, the Bitcoin code[3] defines a difficulty function as

$$D = \frac{H^*_{max}}{H^*} \tag{2}$$

Furthermore, since $H(S)$ is analogous to a uniform probability distribution with support $[0, 2^{256})$, the probability that a nonce satisfies (1) is

$$p = \frac{H^*}{2^{256}} = \frac{H^*_{max}}{D2^{256}} \approx \frac{1}{D2^{32}} \tag{3}$$

Assuming that the mining hardware calculates hashes at a rate R_t, and that trials are independent, the number of trials until a success is geometrically distributed. The expected time to find a block is given by

$$\mathbb{E}[time] = \frac{1}{p} \approx \frac{D * 2^{32}}{R_t} \tag{4}$$

The expected time hard-coded in the Bitcoin blockchain is exactly 10 minutes. From equation (4) it is clear that, given R_t, the difficulty D can be derived, and from equation (2), a target H^* obtained.

However, unlike all the parameters considered so far, the hash rate R_t is a variable beyond the control of the Bitcoin blockchain. The actual time taken to mine a block depends on the current number of miners and mining speed of the hardware. Data from BitcoinVisuals[4] show that the median speed of successfully mining a block has rarely been equal to 10 minutes, and has reached a maximum (daily median) time of 360 minutes on 18/07/2009, and a minimum (daily median) time of 2.08 minutes on 13/07/2010.

Consequently, and in order to maintain the target time to 10 minutes, the Bitcoin code recalculates the difficulty D every 2016 blocks, which should theoretically be mined in two weeks. The relationship between the time-varying difficulty D_t and the target time T_{10} is thus

[3] The Bitcoin code can be found at `https://github.com/bitcoin/bitcoin/blob/d62a1947be5350ed60066ccacc7aba43bbdf48fb/src/main.cpp`.

[4] `https://bitcoinvisuals.com/chain-speed`.

$$D_t = D_{t-1}\frac{T_{10} \times 2016}{T_{2016}} \tag{5}$$

The correction factor is the quotient of the hard-coded time to mine 2016 blocks, $T_{10} \times 2016 = 10$ minutes $\times$ 2016=20160 minutes, and the actual time the network took to mine the last 2016 blocks, T_{2016}. (5) shows that the block difficulty increases (decreases) when the actual time T_{2016} is smaller (higher) than the intended 20160 minutes. The correction factor is also hard-coded to be in the interval $[0.25, 4]$[5].

In the light of the above, it is clear that the major production determinants in Bitcoin mining outside the control of the Bitcoin code are the hash rate of the mining hardware, the amount of energy used to mine a block, and the cost of running the hardware. Energy consumption and hash rate processing can be combined into one variable, the mining hardware's efficiency $\mathcal{E} = R_t/P_t$ (in Mhash/ J), where P_t is the power used (in Watts), and R_t is in Mhash/s[6].

The marginal cost of producing a block depends on the time taken to mine it, the amount of power used to do so, the efficiency of the mining hardware and the price of electricity U_t,

$$MC_t(D_t, U_t, \mathcal{E}_t) = \mathbb{E}[time]P_tU_t \approx \frac{D_t 2^{32} P_t U_t}{R_t} = \frac{D_t 2^{32} U_t}{\mathcal{E}_t} \tag{6}$$

The reward for discovering the block halves every 210000 blocks, and has been equal to 6.25 bitcoins since 12 May 2020. Unlike Hayes (2017) and Delgado-Mohatar et al. (2019), we opted for conducting the analysis in terms of blocks rather than bitcoins, since there is no clarity in the literature regarding the impact of halving on mining decisions (see, for instance, Fantazzini and Kolodin (2020) for discussion). The network power usage P_{BTC} can be estimated from the hash rate of the entire network, the energy efficiency of the mining hardware, $\mathcal{E}$, and the difficulty

$$R_{BTC,t} = \frac{D_t 2^{32}}{600} \quad P_{BTC,t} = \frac{R_{BTC,t}}{\mathcal{E}_t} \tag{7}$$

where 600 is the number of seconds in 10 minutes.

The empirical analysis in Section 4 is grounded on the marginal cost of production (6), which is a fundamental decision variable for producers, in line with Hayes (2017), Delgado-Mohatar et al. (2019), Xiong et al. (2020), and Kristoufek (2020). Research on the sustainability of the Bitcoin ecosystem, e.g. Derks et al. (2018) and Vranken (2017), tends to focus on total profitability of the mining process. Consequently, production cost models in this literature includes the fixed cost of the mining hardware in the total cost evaluations, and calculates total revenue.

3.2. Evolution of hardware efficiency

Table 1 summarizes effficiency data for a selection of mining hardware since 2010. Data sources can be found in Table 2 in the Supplement. The data on these tables were sourced from the literature until 2019, and for data between 2020 and 2022, from the websites of manufacturers of mining hardware and from industry publications. A few papers collated data on hardware efficiency, e.g., O'Dwyer

[5]https://github.com/bitcoin/bitcoin/blob/d62a1947be5350ed60066ccacc7aba43bbdf48fb/src/main.cpp L844 ComputeMinWork() and L865 Getnetworkrequired(). Details regarding mining can be found in the code at `https://github.com/bitcoin/bitcoin/blob/master/src/rpc/mining.cpp`.

[6]1W=1J/sec.

Table 1. Bitcoin mining hardware efficiency

Type	Release date	Source	Manufacturer	Product	Efficiency in Mh/J
GPU	2-Apr-2004	http://karlodwyer.com/publications/ /pdf/bitcoin_KJOD_2014.pdf	Sapphire	ATI 4850	1
FPGA	18-Aug-2011	https://en.bitcoin.it/wiki/Mining_hardware_comparison	Unknown	Bitcoin Dominator X5000	15
FPGA	17-Jul-2012	https://en.bitcoin.it/wiki/Mining_hardware_comparison	Black Arrow	Lancelot	15
ASIC	30-Jan-2013	https://en.bitcoin.it/wiki/Mining_hardware_comparison	Canaan	Avalon Batch 3	117
ASIC	16-Apr-2013	https://bitcointalk.org/index.php?topic=204030.0	ASICMiner	ASICMiner BE Blade	130
ASIC	25-May-2013	https://www.bitcoinmining.com/bitcoin-mining-hardware/	Bitmine	Avalon Clone 85GH	131
ASIC	31-May-2013	https://en.bitcoin.it/wiki/Mining_hardware_comparison	Metabank	Bitfury - Metabank	704
ASIC	13-Sep-2013	https://www.bitcoinmining.com/bitcoin-mining-hardware/	HashFast	HashFast Sierra	909
ASIC	12-Apr-2014	https://www.bitcoinmining.com/bitcoin-mining-hardware/	Canaan	Avalon USB Nano 3	1176
ASIC	1-Sep-2014	https://cdn.shopify.com/s/files/1/0389/3233/files/SP31 _User_Guide.pdf	Spondoolies-Tech	Spondooliestech SP31 Yukon	1633
ASIC	27-Dec-2014	https://www.ccn.com/bitmain-unleashing-antminer-s5/	Bitmain	AntMiner S5	1958
ASIC	1-Jul-2015	https://asicnews.com/asic-miners/top-6-best-bitcoin-asic-miners-2017/	Bitmain	AntMiner S7	4017
ASIC	1-Jun-2016	https://asicnews.com/asic-miners/top-6-best-bitcoin-asic-miners-2017/	Bitmain	AntMiner S9	9310
ASIC	1-Aug-2017	https://asicnews.com/asic-miners/top-6-best-bitcoin-asic-miners-2017/	Bitmain	AntMiner R4	10178
ASIC	20-Dec-2017	http://news.8btc.com/ebang-releasee10-miner-with-samsung-10nm-chips	Hangzhou-Ebang	Ebit E10	11111
ASIC	1-Mar-2018	https://blockchaind.net/dragonmint-halongmining-releases-competitor-bitmain/	Halong Mining	DragonMint 16T	11173
ASIC	Jul-18	https://www.asicminervalue.com/efficiency/sha-260	Bitfily	Snow Panther B1	11594
ASIC	Aug-18	https://www.asicminervalue.com/efficiency/sha-262	Innosil	icon T2 Turbo	12121
ASIC	Sep-18	https://www.asicminervalue.com/efficiency/sha-266	MicroBT	Whatsminer M10S	15714
ASIC	Oct-18	https://www.asicminervalue.com/efficiency/sha-271	Ebang	Ebit E11++	22222
ASIC	Apr-19	https://www.asicminervalue.com/efficiency/sha-285	Bitmain	Antminer S17 Pro (50Th)	25316
ASIC	Apr-20	https://www.asicminervalue.com/efficiency/sha-308	MicroBT	Whatsminer M30S	26316
ASIC	May-20	https://www.asicminervalue.com/efficiency/sha-310	Bitmain	Antminer S19 Pro (110Th)	33846
ASIC	Jul-21	https://www.asicminervalue.com/efficiency/sha-326	Bitmain	Antminer S19j Pro (104Th)	33898
ASIC	May-22	https://www.asicminervalue.com/efficiency/sha-336	Bitmain	Antminer S19 Pro+ Hyd (198Th)	36364
ASIC	Jul-22	https://www.asicminervalue.com/efficiency/sha-337	Bitmain	Antminer S19 XP (140Th)	46512

and Malone (2014), Delgado-Mohatar et al. (2019), but mostly Zadé et al. (2019), and associated database in Zadé and Myklebost (2018). Bedford Taylor (2017) and Delgado-Mohatar et al. (2019) detail the evolution of the Bitcoin hardware from basic computer CPUs to 6th generation purpose-built Application-Specific-Integrated Circuits (ASICs). CUDA-based GPU[7], appeared in September 2010, and Bedford Taylor (2017) considers this as the first Bitcoin -dedicated hardware. By June 2011, GPUs had become obsolete and were replaced by field-programmable gate arrays (FPGAs). In 2013, the first ASIC was introduced, and Tables 1 and Table 1 in the Supplement make it clear that this technology has dominated the Bitcoin mining process ever since.

Aside from the dominance of ASIC, Table 1 in the Supplement shows that the evolution of mining rigs is not linear, and newly released products are not necessarily more efficient than existing ones. For instance, on 13/06/2013 Butterfly lauched the BFL Bitforce S60 with an efficiency of 222Mh/J, two weeks after Metabank had launched the Bitfury - Metabank with an efficiency of 704 Mh/J. Evidently, the former was quickly replaced on the Bitcoin network, as Bedford Taylor (2017) makes it clear that the daily mining revenue has had a decreasing trend since 2010. Considering a flat electricity cost of 0.20$/kWh, Bedford Taylor (2017) shows that CPUs first became unprofitable in late 2010, GPUs in late 2013, FPGAs in mid-2013, and the first ASIC rig, ASICMiner, became unprofitable in early 2014. In the light of the non-linear evolution of Bitcoin technology, the efficiency rate considered in this paper for the construction of the marginal cost of production (6) is only the highest available on Table 1 (Supplement) at any given date. The relevant rigs and efficiency data are condensed in Table 1. This implies that the technological progress of the mining hardware is smoother than it actually was, and in particular, it assumes that only miners using the most efficient rig operate on the network. This assumption is probably not realistic, but it is currently impossible to know the distribution of mining rigs among miners and thus the average energy efficiency of the network. Moreover, the profitability of mining hardware depends to a great extent on the local electricity price of the miner, which is frequently an assumption in most of the literature, even though some authors, such as Fantazzini and Kolodin (2020), have attempted to propose a methodology based on exponential smoothing to model the dynamics of the Bitcoin network energy efficiency.

3.3. Energy consumption and price

The second variable relevant for the marginal cost is the price of electricity. Different authors make different assumptions. O'Dwyer and Malone (2014) use a cost of 0.10 US dollars per kWh. Xiong et al. (2020) assume that most of the production is located in China and thus use a price of 0.05 $/kWh in their empricial analysis. Fantazzini and Kolodin (2020) consider a cost of 0.13 $/kWh, but test the robustness of their result using the NordPool Exchange electricity prices, which vary from 0.01 and 0.06$/ kWk over the period 2016Q3 and 2020Q1. Bedford Taylor (2017) considers a price of 0.2 $/kWh , whilst Delgado-Mohatar et al. (2019) a range of prices between 0.05 $/kWh and 0.14 $/kWh. For this paper, we will assume a constant price of 0.05 $ kWh throughout, given that most of the bitcoin mining was located in Mainland China until July 2021, as can be seen in Figure 2. Although the data cover only the period September 2019 to January 2022, there is ample evidence in the empirical literature that the overwhelming generation of bitcoin was taking place in China during the past decade. Since the prohibition of mining in China, announced in July 2021, the production of bitcoin has shifted to the

[7]Compute Unified Device Architecture (CUDA) is a paralel computing platform that allows simultaneous calculations to be performed. The fundamental technology remains the graphics processing unit.

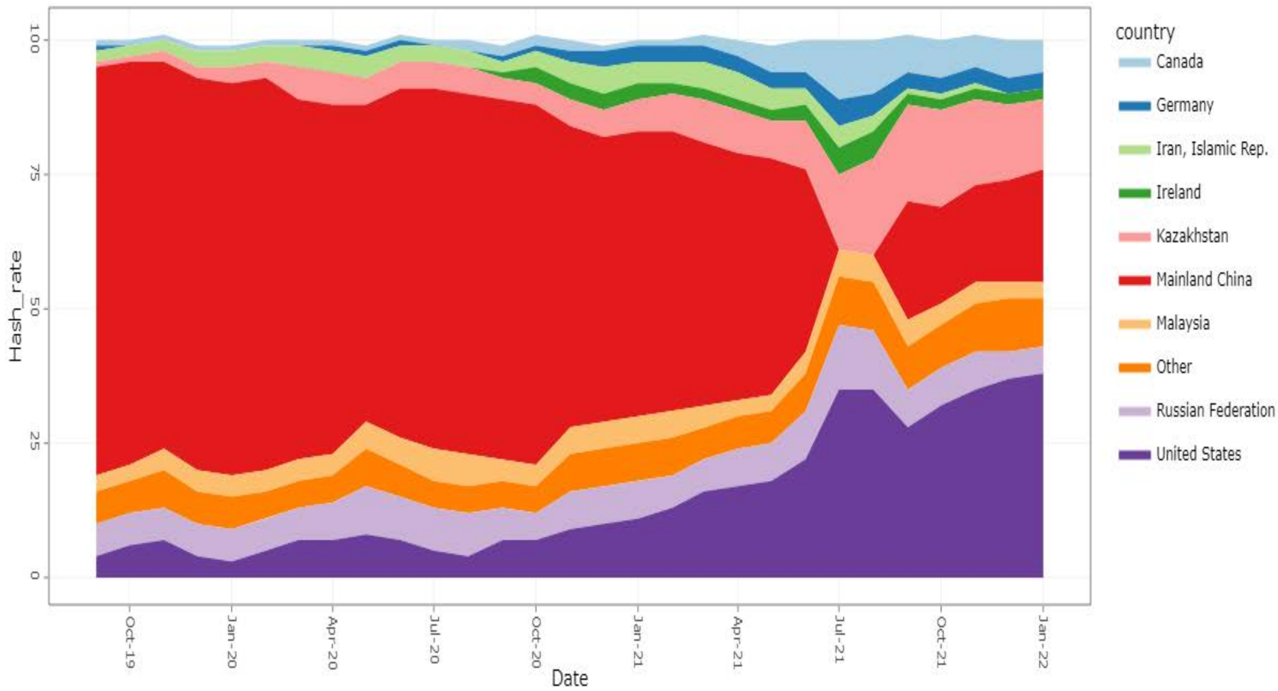

Figure 1. Bitcoin hash rate by country 2019–2021.

Source: Data from Cambridge Centre for Alternative Finance (https://ccaf.io/cbeci/index). All data in percent of Bitcoin's network total hash rate.

US, and in particular Texas, where the price of electricity is the lowest in that country (IEA 2022). Our assumption is in line with Xiong et al. (2020) and Kaiser et al. (2018).

4. Empirical analysis

4.1. Descriptive analysis

Figure 2 shows the main variables used to estimate the cost of producing a bitcoin from 18 July 2010 until 22 July 2022. Although most of the variables of the Bitcoin ecosystem are available from its inception, the total energy usage has only been calculated since 18 July 2010. All data in Figure 2 are obtained from BitcoinVisuals (https://bitcoinvisuals.com/) and energy consumption from Cambridge Centre for Alternative Finance (https://ccaf.io/cbeci/index), and are in logs except the bitcoin price (*btc*) and the median mining speed. The units of the variables are USD for the cost of production, number of hashes per second for the hash rate. The number of bitcoins produced by the network per day is the total supply. Energy consumption (power) is in GWh, and is an estimate of the total electricity consumption of the network. Finally, the median mining speed is in minutes, the reward per block is in bitcoins, and its price in dollars is given by *btcfx*. The difficulty has no unit by definition. Figure 2 shows that the graphs of the hash rate and the difficulty are practically identical. In effect, BitcoinVisuals calculates the hash rate from the observed difficulty[8] - which is a parameter of the Bitcoin code freely available on

[8]https://bitcoinvisuals.com/chain-hash-rate.

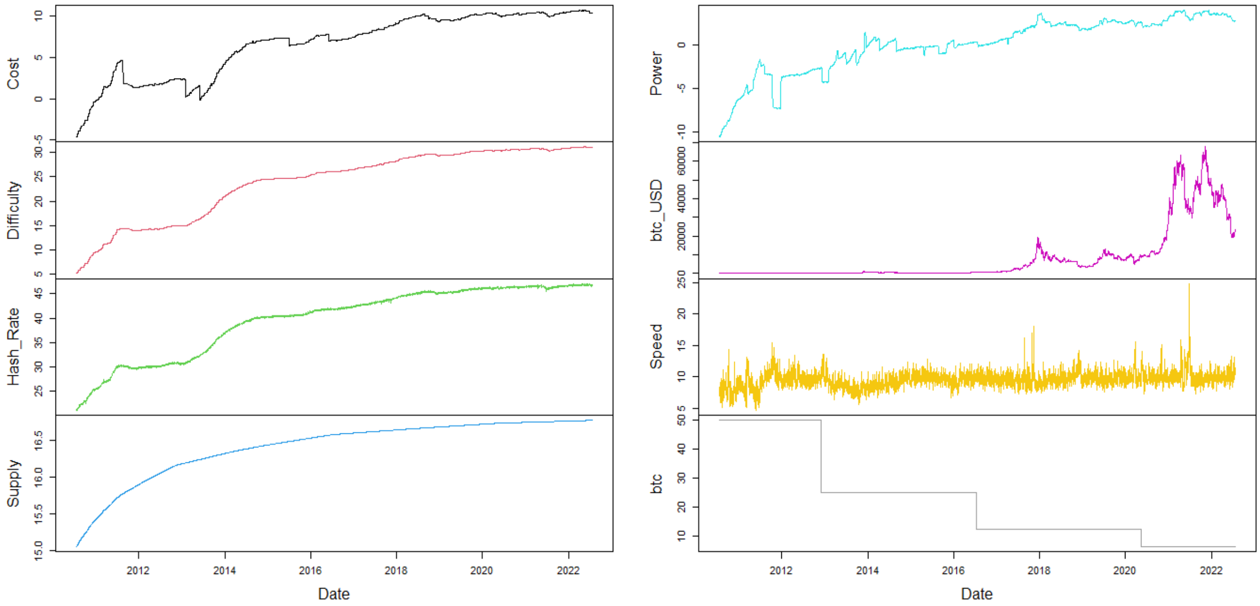

Figure 2. Bitcoin production variables.

Source: Data from BitcoinVisuals (https://bitcoinvisuals.com/) and Cambridge Centre for Alternative Finance (https://ccaf.io/cbeci/index). In logs except btc, btc_usd, speed. Units: cost (USD); difficulty(none); hash rate(hash/sec); speed (mins.); supply (bitcoin); power(GWh); btc_usd (USD).

Github- because the hash rate of the network is not observable. The hash rate is then defined by

$$\text{hash\_rate}_t = \frac{D_t 2^{32}}{T_t} \tag{8}$$

where T_t is the observed median time taken to mine a block at time t. Although the hash rate is used as an independent variable in the econometric analysis of Fantazzini and Kolodin (2020), Kristoufek (2020), and Hayes (2017), it cannot be considered an explanatory variable for the marginal cost function (6), which depends on the mining difficulty.

The total supply and the block reward (*Supply* and *btc*) are determined by the Bitcoin code. The latter halves when 210000 blocks are mined, starting from a value of 50 btcs. The total supply of bitcoins is limited by the code to 21 millions, of which 18.925 millions have been mined up until 2022. The concave supply curve, with decreasing growth rate, reflects these fundamentals. The two interesting plots are those of the median mining speed and the marginal cost of production. It is clear from the graph that the mining speed fluctuates around 10 minutes, as required by the Bicoin code, but is rarely equal to this time. Table 2, which shows the descriptive statistics of the variables graphed in Figure 2, confirms that the mean speed of mining is 9.52 minutes, and its median 9.47 minutes. The absolute minimum and maximum occured before the sample considered in the empirical analysis, viz., 2.08 minutes on 13/07/2010 and 360 minutes on 18/07/2009. The extreme values for the sample period considered here are a maximum of 24.83 minutes on 27 June 2021 and a minimum of 4.69 minutes on 25/05/2011 (Table 2). The plot of the marginal cost (*Cost*) reflects the technological innovation on the

Bitcoin network since the early 2010s. The period prior to 18 August 2011, the date of the first drop in the graph, is dominated by GPU computers (Bedford Taylor, 2017). Around 18 August 2011, the first FPGA computers were used in mining, resulting in a drop of production costs of 95% (Table 1). Between 18 August 2011 and 29 January 2013, the cost of production remains relatively flat, with two drops on 24 January 2013 and 30 January 2013. Table 1 shows that during this period there was a mix of FPGA and ASIC mining rigs being used in production, which accounts for the stationarity of mining efficiency, and hence, of the marginal cost of production. From February 2013 onwards, ASICs start to dominate Bitcoin's production technology, and very few falls in cost are observed, viz., on 01/07/2015, when the efficiency rate rose from 1099 to 4017 Mh/J, and on 01/06/2016 when the efficiency rose from 4017 to 9310 Mh/J. The most efficient mining hardware is currently Micro BT's Whatsminer M50, which has an efficiency of 34483 Mh/J and was realeased on 1 July 2022 (Table 1). Overall, it is clear that technological innovation has been tremendous in the past decade, but its impact on production costs has been dampened by the adjustable difficulty embedded in the Bitcoin code (equation (5)). As a result, the cost of production has consistently increased since February 2013.

Table 2. Descriptive statistics - Units: Cost (USD); Difficulty(none); Hash Rate(hash/sec); Speed (mins.); Supply (bitcoin); Power(GWh); btcfx (USD); Costtx (USD).

	Cost	Difficulty	Hash Rate	Supply
Min.	0.01	1.82×10^{2}	1.55×10^{9}	3439250
1st Qu.	13.29	2.62×10^{7}	2.24×10^{14}	11431109
Median	1513.67	2.13×10^{11}	1.53×10^{18}	15767715
Mean	10166.88	5.64×10^{12}	4.07×10^{19}	14256593
3rd Qu.	21949.95	9.02×10^{12}	6.73×10^{19}	17829174
Max.	43946.75	3.13×10^{13}	2.68×10^{20}	19099696
	btcfx	btc	Speed	Costtx
Min.	0.05	6.25	4.69	0.01
1st Qu.	119.67	12.5	8.73	12
Median	667.18	12.5	9.47	885
Mean	8201.49	22.5	9.524	12864
3rd Qu.	8728.38	25	10.29	11378
Max.	7620.18	50	24.83	5572943

Table 2 presents the descriptive statistics of the variables graphed in Figure 2, and of an additional marginal cost, *Costtx*. (6) was calculated using two different values for the price of electricity. Firstly, we assume a flat rate of \$0.05/kWh (Xiong et al., 2020; Kaiser et al., 2018), resulting in the variable *cost*. Secondly, given that the US was the second major producer of bitcoins after China until July 2021, and has since become the single world largest producer, we consider the electricity price of the state of Texas, where most of the US mining rigs are located. As a result, *Costtx* is bitcoin's marginal cost of production (6), where U_t is variable. Table 2 shows that the two marginal costs of production, *Cost*, and *Costx*, have the same minimum value, \$0.01 kWh, and median and mean values in the same order of magnitude. The median of the marginal cost calculated with a flat electricity price is \$1514, and that of the marginal

Table 3. Correlation matrix.

	Cost	Hash Rate	Supply	Power	btcfx	Speed
Cost	1	0.98003	0.920737	0.939622	0.942729	0.360319
Hash Rate	0.98003	1	0.963909	0.955368	0.967806	0.323625
Supply	0.920737	0.963909	1	0.946559	0.950928	0.34752
Power	0.939622	0.955368	0.946559	1	0.982791	0.268152
btcfx	0.942729	0.967806	0.950928	0.982791	1	0.283924
Speed	0.360319	0.323625	0.34752	0.268152	0.283924	1

Table 4. Variables - Texas electricity price data are SNL US Electricity ERCOT-North-Datastream code: ELENSNL.

Variable name	Description	Source
D_t	Mining difficulty	BitcoinVisuals
$\mathcal{E}_t$	Mining hardware efficiency	Table 1
U_t	Electricity price	Eikon Datastream[1]
hash_t	Hash rate of the Bitcoin network	BitcoinVisuals
btcfx_t	US-bitcoin exchange rate	BitcoinVisuals
costtx_t	Marginal cost of bitcoin production based on Texas electricity prices	Calculated from model
cost_t	Marginal cost of bitcoin production based on China's electricity prices	Calculated from model

cost based on Texas' electricity prices is \$885, whilst the means are \$10167 and \$12864, respectively. The maximum of the cost based on Texas' electricity prices is extremely high, \$5,572,943. This outlier occurred on 16/02/2021, when the electricity price peaked at \$8.8 kWh in Texas due to extreme weather, before falling down to \$8.28 kWh on 17 February, \$4.41 kWh on 18 February, and \$0.0515 kWh on 19 February. Finally, the values of *Difficulty* and *Hash Rate* highlight the tremendous activity on the Bitcoin network between July 2010 and July 2022. *Difficulty* varies from a minimum of 182 to a maximum of 31,300,000,000,000, evidencing the increasing protocol adjustment to mining efforts. *Hash rate*, on the other hand, clearly showcases the scale of bitcoin production, which results from the network's hash rate. The minimum daily hash rate was 1,550,000,000 hashes and the daily maximum 268000000000000000000000 hashes. The discrepant scale of magnitude between minimum and the first quartile -from billions to 100s of trillions- shows that the hash rate has a significant right-hand tail, and its distribution is asymmetric to the right.

Table 4 gathers the main variables used in the analysis in Section 4.2, which investigates the existence of a long-run relationship between bitcoin price and its fundamentals, cost_t, and costtx_t. In addition, to contrast our results with those in the literature, we also estimate the cointegration between bitcoin price and the hash rate. It should be noted that there is a strong endogeneity between the data on the hash rate and those on the network difficulty, as evidenced in equation (8). The correlation matrix shows that all variables are highly correlated, with the exception of *Speed*, which has a correlation of less than 40% with *cost*, *hash rate*, and *supply*, and less than 30% with *power*, and the bitcoin price *btcfx*.

4.2. Econometric analysis

In order to assess the underlying long-run dynamics, we employ two cointegration methodologies, Johansen (1991)'s trace test and Bierens and Martins (2010)'s time-varying vector error correction model (TV-VECM), in which the cointegrating relationship varies smoothly over time and the adjustment can be nonlinear. Johansen (1991)'s cointegration procedure assumes that the cointegrating vector is constant and the adjustment is linear, and can be thus considered as a special case of Bierens and Martins (2010)'s methodology, a time-invariant error correction model (TI-VECM).

The time-invariant Vector Error Correction model (TI-VECM) of order p -with intercept and time trend-, used to construct the Johansen tests can be written as:

$$\Delta Y_t = \alpha(\beta' Y_{t-1} - \beta_0 - \beta_1 t) - \gamma_0 - \gamma_1 t + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \epsilon_t \tag{9}$$

For $t = 1, \ldots, T$, where T is the total number of observations, and $\Pi = \alpha\beta'$[9].

The time-varying vector error correction model (TV-VECM) is derived trivially from equation (9),

$$\Delta Y_t = \alpha(\beta_t' Y_{t-1} - \beta_0 - \beta_1 t) - \gamma_0 - \gamma_1 t + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \epsilon_t \tag{10}$$

where $\epsilon_t \sim N(0, \Omega)$. Y_t is a $k \times 1$ vector of observations, Ω and $\Gamma_j,\ j = 1, \ldots, p-1$ are $k \times k$ matrices. α and β are $k \times r$ matrices of coefficients, r the cointegrating rank of the system. β_0, β_1 are $r \times 1$ vectors of coefficients, γ_0 and γ_1 are $k \times 1$ vectors of the VECM's intercepts, and linear trend, respectively. In equation (10), β_t is a $T \times (r * k)$ matrix of coefficients.

The null and the alternative hypotheses are that cointegration is time-invariant against time-varying cointegration, i.e.,

$$H_0: \quad \Pi_t' = \Pi' = \alpha\beta' \tag{11}$$

$$H_1: \quad \Pi_t = \alpha\beta_t' \tag{12}$$

In order to test the null hypothesis (11), the test statistic must allow for time-varying β_t'. Bierens and Martins (2010) do so by assuming that the time-varying cointegrating vectors β_t can be approximated by a finite sum of m Chebychev polynomials $P_{it}(t)$, such that

$$\beta_t = \beta_m(t/T) = \sum_{i=0}^{m} \xi_{iT} P_{iT}(t) \tag{13}$$

For $t = 1, \ldots, T$. $1 \leq m \leq T-1$ and $\xi_{iT} = \frac{1}{T}\sum_{t=1}^{T} \beta_t P_{iT}(t)$ for $i = 0, \ldots, T-1$ are unknown $k \times r$ matrices. Moreover, $\frac{1}{T}\sum_{t=1}^{r} P_{i,T}(t) P_{jT}(t) = 1$ for $i = j$ and $\frac{1}{T}\sum_{t=1}^{r} P_{i,T}(t) P_{jT}(t) = 0$ for $i \neq j$. Substituting (13) in (10) yields,

[9] The symbol $'$ represents the transpose.

Table 5. TI-VECM - bitcoin price and cost (constant electricity price)

	$\Delta\ln(\text{btcfx}_t)$	std. error	significance	$\Delta\ln(\text{cost}_t)$	std. error	significance
ECT	0.0002	9.9×10^{-5}	*	0.0005	0.0002	***
Intercept	0.0043	0.0012	***	0.0078	0.0018	***
$\Delta\ln(\text{btcfx}_{t-1})$	0.1467	0.0149	***	0.004	0.0227	
$\Delta\ln(\text{cost}_{t-1})$	- 0.0039	0.0099		-0.011	0.0151	

Note: Cointegrating vector:($\ln(\text{btcfx}_t)$ $\ln(\text{cost}_t)$)= (1 -2.94139); Sample size:4388;Number of variables: 2; AIC -49309.19; BIC -49251.71; SSR 34.52721. Estimation of equation $\Delta Y_t = \alpha(\beta' Y_{t-1}) + I + \Gamma_j \Delta Y_{t-j} + \epsilon_t$, where $Y_t = (ln.(btcfx_t)\ ln(cost_t))'$ and I a column vector of intercepts- ECT : Error-Correction term . *,**,*** represent significance at 10%, 5%, and 1%, respectively.

$$\Delta Y_t = \alpha\left(\left(\sum_{i=0}^{m} \xi_{iT} P_{iT}(t)\right)' Y_{t-1} - \beta_0 - \beta_1 t\right) - \gamma_0 - \gamma_1 t + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \epsilon_t \tag{14}$$

Note that the time-invariant vector error-correction model (TI-VECM) shown in equation (9) can be retrieved from the time-varying vector error-correction model shown in equation (14) by setting $m = 0$. In fact, TI-VECM is simply a particular case of TV-VECM. After defining β_t' as a time-varying coefficient in equation (13), testing for the null of time-varying cointegration, and its alternative, corresponds to testing a hypothesis on β_t', namely:

$$H_0 : \qquad \xi_{iT} = 0_{k\times r} \quad i = 1, \dots, m \tag{15}$$

$$H_1 : \quad \lim_{T\to\infty} \xi_{iT} \neq 0_{k\times r} \text{ for some } i = 1, \dots, m \text{ and } \xi_{iT} = 0_{k\times r} \text{ for all } i > m \tag{16}$$

The null hypothesis of standard cointegration then corresponds to the hypothesis that the parameters in the VECM that are related to Chebyshev time polynomials are jointly zero. This can be tested via a likelihood ratio (LR) test. (14) and (9) are estimated by maximum likelihood. The test statistic is the likelihood ratio (LR) test

$$LR_T^{TVC} = -2\left(\hat{l}_T(r,0) - \hat{l}_T(r,m)\right) \ m > 0 \tag{17}$$

where $\hat{l}_T(r,0)$ is the loglikelihood of (9), and $\hat{l}_T(r,0)$ that of (10). Bierens and Martins (2010) prove that given $m, r \geq 1$, LR_T^{TVC} is asymptotically distributed as $\chi^2(rmk)$, under the null hypothesis (15).

4.3. Empirical results

The estimated vector-error correction model is

$$\begin{bmatrix} \ln(\text{btcfx}_t) \\ \ln(\text{cost}_t) \end{bmatrix} = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} \ln(\text{btcfx}_{t-1}) \\ \ln(\text{cost}_{t-1}) \end{bmatrix} + \begin{bmatrix} \text{Intercept}_1 \\ \text{Intercept}_2 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} \Delta\ln(\text{btcfx}_{t-1}) \\ \Delta\ln(\text{cost}_{t-1}) \end{bmatrix} \tag{18}$$

where

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} \ln(\text{btcfx}_{t-1}) \\ \ln(\text{cost}_{t-1}) \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} 1 \\ -\beta \end{bmatrix}' \begin{bmatrix} \ln(\text{btcfx}_{t-1}) \\ \ln(\text{cost}_{t-1}) \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \left[\ln(\text{btcfx}_t) - \beta\ln(\text{cost}_t)\right] = \alpha\text{ECT}_t \tag{19}$$

Table 6. TI-VECM - bitcoin price and cost (variable electricity price)

	Δln(btcfx$_t$)	std. error	significance	Δln(costtx$_t$)	std. error	significance
ECT	-0.0006	0.0005		-0.0197	0.0029	***
Intercept	0.0023	0.0007	***	0.0033	0.0037	
Δln(costtx$_{t-1}$)	- 0.0015	0.0028		-0.0378	0.0151	*
Δln(btcfx$_{t-1}$)	0.1733	0.0149	***	0.0117	0.081	

Note: Cointegrating vector:(ln(btcfx$_t$) ln(costtx$_t$))= (1 -0.9756123); Sample size:4388;Number of variables: 2; AIC -39446.64; BIC -39389.16; SSR 274.972.Estimation of equation $\Delta Y_t = \alpha(\beta' Y_{t-1}) + I + \Gamma_j \Delta Y_{t-j} + \epsilon_t$, where $Y_t = (ln(btcfx_t)\ ln(costtx_t))'$ and I a column vector of intercepts. ECT : Error-Correction term . *,**,*** represent significance at 10%, 5%, and 1%, respectively.

Table 7. TI-VECM - bitcoin price and hash rate.

	Δln(btcfx$_t$)	std. error	significance	Δln(hash$_t$)	std. error	significance
ECT	-0.0022	0.0009	*	0.0037	0.002	
Intercept	0.0007	0.0023		0.0262	0.005	***
Trend	4.1×10^{-7}	9.3×10^{-7}	***	-7.9×10^{-6}	2.0×10^{-6}	
Δln(btcfx$_{t-1}$)	0.1483	0.0149		0.0963	0.0322	**
Δln(hash$_{t-1}$)	0.0029	0.0062		-0.4498	0.0135	***

Note: Cointegrating vector:(ln(btcfx$_t$) ln(hash$_t$))= (1 -0.2373591); Sample size:4388;Number of variables: 2; AIC -46224.7; BIC -46154.45; SSR 59.12488. Estimation of equation $\Delta Y_t = \alpha(\beta' Y_{t-1}) + I + T + \Gamma_j \Delta Y_{t-j} + \epsilon_t$ where $Y_t = (ln(btcfx_t)\ ln(hash_t))'$, T and I are column vectors of trends and intercepts, respectively. ECT : Error-Correction term . *,**,*** represent significance at 10%, 5%, and 1%, respectively.

The time-varying cointegration (TV-VECM) relationships for column vectors $Y_t = (ln(btcfx_t)\ ln(cost_t))'$, $Y_t = (ln(btcfx_t)\ ln(costtx_t))'$, and $Y_t = (ln(btcfx_t)\ ln(hash_t))'$, that were estimated are

$$\beta_t' Y_t = \beta_{1t} ln(btcfx_t) + \beta_{2t} ln(cost_t) \tag{20}$$

$$\beta_t' Y_t = \beta_{1t} ln(btcfx_t) + \beta_{2t} ln(costtx_t) \tag{21}$$

$$\beta_t' Y_t = \beta_{1t} ln(btcfx_t) + \beta_{2t} ln(hash_t) \tag{22}$$

where $ln(btcfx_t)$ is the log of the bitcoin/USD exchange rate, ln($hash_t$) is the log of hash rate of the bitcoin network. $ln(cost_t)$ is the log of marginal cost of production (6), calculated with the flat electricity price of 0.05 \$kWh, and $ln(costtx_t)$ the log of the marginal cost of production (6), assuming time-varying electricity price in Texas (see Table 4).

Tables 5 to 11 present the results of the Johannsen trace and the time invariant cointegration (TI-VECM) analyses. Unit root tests are shown in Table A.1 in the Appendix. The long-run relationship of bitcoin price and the hash rate can be found in Table 7; that of bitcoin prices and marginal costs in Table 5 for the cost based on the constant electricity price, and in Table 6 for the cost based on variable electricity price. The error-correction term (ECT) of the $\Delta\ln(\text{btcfx}_t)$ equation in Table 5 is significant at 10%. However, its sign is positive, indicating that there is no long-run relationship between bitcoin price and its marginal cost of production (constant electricity price). Analogously, the error-correction term (ECT) of the $\Delta\ln(\text{cost})_t$ equation in Table 5 is significant at 10%, also with a positive sign. The short-run dynamics of the cost of production, $\Delta\ln(\text{cost}_{t-1})$, have no impact on the price of bitcoin ($\Delta\ln(\text{btcfx}_t)$), as suggested by the lack of signifcance of its coefficient. The short-run dynamics of the price of bitcoin itself ($\Delta\ln(\text{btcfx}_{t-1})$) are significant at 1%. Looking at the equation of $\Delta\ln(\text{cost})_t$, it is clear that the short-run dynamics of the bitcoin price and of the cost of production have no impact on production cost, as suggested by the lack of signifcance of the coefficients of $\Delta\ln(\text{btcfx}_{t-1})$ and $\Delta\ln(\text{cost}_{t-1})$.

Table 6 presents very similar results. Firstly, looking at the equation for the bitcoin price, $\Delta\ln(\text{btcfx}_t)$, it is clear that the coefficient of the error-correction term is not significant, albeit with the right negative sign. Analogously, the short-run dynamics of the bitcoin price itself is the only significant variable of the equation, at a 1% level. Moreover, the equation of $\Delta\ln(\text{cost}_t)$ shows that the error correction term is signifcant at 1%, with a negative sign. The short-run dynamics of the cost of production have an impact on production cost, as suggested by the 10% signifcance of the coefficient of $\Delta\ln(\text{cost}_{t-1})$. Nonetheless, in accordance with the results of Table 5, the short-run dynamics of the bitcoin price have no impact on production cost.

There is evidence of cointegration between the hash rate and bitcoin prices in Table 7, given that the coefficient of ECT in equation $\Delta\ln(\text{btcfx}_t)$ is significant at 10%. As was seen above, the only significant explanatory variable of bitcoin prices are the short-run dynamics, viz., past values of bitcoin. The ECT coefficient in the equation for the hash rate ($\Delta\ln(\text{hash}_t)$) is not significant and is positive, indicating essentially that there is no long-run relationship between the hash rate and bitcoin prices. However, the short-run dynamics of the bitcoin prices and the hash rate are significant at 1%. The coefficient of $\Delta\ln(\text{hash}_{t-1})$ is negative, as would be expected from the Bitcoin protocol, according to which the mining difficulty is increased when bitcoins are mined in less than 10 minutes. Other things equal, the difficulty adjustment leads to a decrease in the hash rate. The coefficient of $\Delta\ln(\text{btcfx}_{t-1})$ is positive and significant at 1%, suggesting that the dynamics of the price of bitcoin lead to an increase of hash rate. This result is consistent with the empirical evidence that the higher the bitcoin price, the higher the number of miners, and hence the higher the hash rate.

Three conclusions can be drawn from Tables 5 to 7. Firstly, the results for the bitcoin price are consistent irrespective of the assumption on electricity prices. Namely, there is no cointegration between the price of bitcoin and its production cost between 2010 and 2022, and only bitcoin's short-run dynamics affects its current price. It is also noteworthy that in all cases the coefficients of the error-correction term are very close to zero, suggesting that -where there exists a long-run relationship- the speed of convergence to the equilibrium is very slow. These results corroborate the empirical literature that fails to find a consistent link between bitcoin prices and its fundamentals, namely,

Marthinsen and Gordon (2022), Kjærland et al. (2018), and Fantazzini and Kolodin (2020), although Fantazzini and Kolodin (2020) and Aoyagi and Hattori (2019) found evidence of a (constant) cointegration between the hash rate and bitcoin prices, for the subsample 11/12/2017–24/02/2020 (Fantazzini and Kolodin, 2020), and July 2010 to June 2019 (Aoyagi and Hattori, 2019). Secondly, we found no evidence that bitcoin prices are determinants of production costs, as suggested in Fantazzini and Kolodin (2020), whose conclusions were based on the assumption that hash rate can be seen as a proxy for production costs. Tables 5 and 6 clearly indicate that past values of productions costs are the best

Table 8. Granger causality test with 1 lag- Wald statistic.

	$\ln(\text{btcfx}_t)$ caused by			$\ln(\text{btcfx}_t)$ causing	
	F	p-value		F	p-value
$\ln(\text{cost}_t)$	0.8731	0.3502	$\ln(\text{cost}_t)$	6.3615	0.0117^{*}
$\ln(\text{costtx}_t)$	2.2076	0.1374	$\ln(\text{costtx}_t)$	41.515	0.0000^{***}
$\ln(\text{hash}_t)$	0.344	0.5575	$\ln(\text{hash}_t)$	11.189	0.0008^{***}
$\ln(\text{speed}_t)$	6.3793	0.0116^{***}	$\ln(\text{speed}_t)$	88.345	0.000^{***}

Note: *,**, and *** indicate rejection of the null hypothesis at significance levels 10%,5% and 1%, respectively.

Table 9. Time-varying VECM (p=1, r=1) - $\ln(\text{btcfx}_t)$ and $\ln(\text{cost}_t)$.

	LR TVC Statistic	p-value	Loglikelihood	AIC	BIC	HQIC
m=1	3.2638	0.5147	14167.72	−6.4522	−6.4260	−6.4430
m=2	21.0875	0.0069^{***}	14176.63	−6.4545	−6.4224	−6.4432
m=3	32.8700	0.0010^{***}	14182.52	−6.4553	−6.4175	−6.4420
m=4	46.8092	0.0001^{***}	14189.49	−6.4567	−6.4130	−6.4413

Note: *,**, and *** indicate rejection of the null hypothesis at significance levels 10%,5% and 1%, respectively.

predictor of their current values, and that the bitcoin price never is. Finally, the time-invariant VECM and cointegration analysis does not support the hypothesis that bitcoin prices are cointegrated with their fundamentals, viz., the cost of production. As was seen in Section 3, bitcoins have to be mined at a non-negligible cost, and it would be theoretically unusual that the latter would not determine the former to a certain extent. The contention of our empirical analysis is that time-invariant methodologies are inadequate to analyse the behaviour of bitcoin prices, and the success of GARCH models in explaining them suggests that time-varying models are more likely to evidence bitcoin's long-run determinants.

Tables 9 to 11 show the results of the test of the null hypothesis of constant cointegration for the cointegrating relationships (20) to (22). More specifically, Tables 9 to 11 present the values of the LR test statistic (17). Rejection of the null hypothesis (15) indicates time-varying VECM, and its implies that it is rejected for the Chebyshev polynomial expansion up to order m, $m = 1, \ldots, 4$.

In Table 9, the null hypothesis for $m = 1$ is accepted, but rejected for $m = 2, 3, 4$ at 1 percent

Table 10. Time-varying VECM (p=1, r=1) - $\ln(\text{btcfx}_t)$ and $\ln(\text{costtx}_t)$.

	LR TVC Statistic	P-Value	loglikelihood	AIC	BIC	HQIC
m=1	78.7248	0.000^{***}	8849.226	−4.0270	−4.0008	−4.01777
m=2	100.8595	0.000^{***}	8860.293	−4.0302	−3.9982	−4.01893
m=3	112.1364	0.000^{***}	8865.931	−4.03098	−3.9931	−4.01762
m=4	126.9468	0.000^{***}	8873.337	−4.03253	−3.9888	−4.01712

Note: *,**, and *** indicate rejection of the null hypothesis at significance levels 10%,5% and 1%, respectively.

Table 11. Time-varying VECM (p=1, r=1) - ln(btcfx$_t$) and ln(hash$_t$).

	LR TVC Statistic	P-Value	loglikelihood	AIC	BIC	HQIC
m=1	12.2924	0.0153**	12541.19	−5.7105	−5.6843	-5.7013
m=2	22.7983	0.0036***	12546.44	−5.7111	−5.6791	-5.6998
m=3	34.2119	0.0006***	12552.15	−5.7119	−5.6740	-5.6985
m=4	42.4280	0.0003***	12556.25	−5.7119	−5.6682	−5.6965

Note: *,**, and *** indicate rejection of the null hypothesis at significance levels 10%,5% and 1%, respectively.

significance level. The optimal m value is determined by the lowest Hannan-Quinn information criterion (HQIC). In Table 9, the lowest HQIC is -6.4432, which corresponds to $m = 2$. Consequently, the relevant time-varying vectors β_t' for the cointegration relationship between bitcoin price and production cost (20) are given by the sum (13) for $m = 2$. The time-varying cointegration $\beta_t' Y_t$ is then calculated for every t, and plotted in Figure 3.

Analogously, the results of the LR test (17) for the cointegrations between bitcoin and cost (variable electricity price), given in (21), and between bitcoin and the hash rate defined in (22), are shown in Tables 10 and 11, respectively. In both cases, the null hypothesis of constant cointegration is rejected at 1 percent level of significance for $m = 1, 2, 3, 4$. However, the lowest Hannan-Quinn Criterion (HQIC) is -5.6998, for $m = 2$ in Table 11, and -4.01893, also for $m = 2$, in Table 10. These results imply that the relevant time-varying vectors β_t' for the cointegration relationship between bitcoin price and the production cost (21) and for the cointegration relationship between bitcoin price and the hash rate (22) are given by the sum (13) for $m = 2$. The time-varying cointegrations $\beta_t' Y_t$ are then calculated for every t, and plotted in Figures 4 (bitcoin/cost) and 5 (bitcoin/hash rate).

The clear conclusion of the Bierens and Martins (2010)'s LR test of time-varying cointegration is that the long-term relationship between the bitcoin price and its cost cannot be considered constant. Even when a proxy for production cost is used, viz., the hash rate, there is a clear indication of the existence of a time-varying long-run cointegration. This result contrasts with the literature mentioned above, but is congruent with some of Lee and Rhee (2022)' results, who suggest the existence of time-varying cointegration between bitcoin prices and macroeconomic variables. Our main finding, though, is that the marginal production cost and bitcoin prices have a long-run relationship, as indicated by Hayes (2019) and Kristoufek (2020), but refuted by Fantazzini and Kolodin (2020) for most of their sample.

In order to interpret the plots of the cointegrating relationships in Figures 3 to 5, it should be kept in mind that $\beta_t' Y_t$ should be integrated of order zero, and its plot should fluctuate around zero, since by definition, $\beta_t' Y_t = e_t$, where the process e_t represents the short-run deviations from equilibrium. The unit root tests in Table A.1 in the Appendix show that the residuals of the three time-varying cointegration relationships are stationary, since the null hypothesis of a unit root is rejected at all levels of significance. This confirms the existence of a long-run relationship between the price of bitcoin and the cost of production, and between bitcoin price and the hash rate.

Looking at Figure 5, it is clear that the time-varying cointegration relationship between bitcoin price and the hash rate is completely out of equilibrium during most of the sample period. Its graph is mostly monotonic and never fluctuates around zero over the period 22/07/2010 and 22/07/2022, except for a brief time interval between 13/3/2017 and 13/01/2018. According to the literature, this is a period of time where there is no bubble in the bitcoin market (Kyriazis et al., 2020), and this could account for the long-run relationship oscillating around zero. Figures 3 and 4 show that the graphs of the long-run relationship between bitcoin price and its marginal cost fluctuate far more than that of price and the hash rate, even though it cannot be said that the relationships fluctuate around zero, which would indicate that the relationship is in equilibrium. However, in both Figures 3 and 4, it can be seen that the long run relationship fluctuates around zero in the few years after Bitcoin's launch, more specifically, between 12/7/2015 and 30/5/2016. For the long-run relationship between bitcoin price and cost based on variable electricity cost, btcfx$_t$-costtx$_t$, in Figure 4, it can be seen that there are two periods where it is in equilibrium, 15/07/2015 and 15/10/2016, and around the peak on 21/07/2018.

For the remainder of the sample, the cointegrating relationship of the bitcoin price and its marginal cost is in disequilibrium. For the long-run relationship between bitcoin price and cost based on constant electricity cost btcfx$_t$-cost$_t$, Figure 3 shows that it increases for a sustained period of time between 21/11/2011 and 09/04/2013, before reverting towards zero. This behaviour, which is never found in the graph of the the long-run relationship btcfx$_t$-hash$_t$, suggests that bitcoin's price eventually reverts towards its marginal cost of production, after displaying bubble-like behaviour. A similar pattern is found between 30/05/2016 and 01/01/2018, when the cointegrating relationship diverted strongly from the origin, but reverted towards it in January 2018.

The marginal cost considered in Figure 3 is based on a flat electricity price, as was the case in China during the sample period under investigation. In Figure 4, the marginal cost is calculated using the variable electricity price available in the second largest mining country, the USA, and specifically in Texas. The graph shows a similar pattern as in Figure 3 before 2015, with a huge disequilibrium peaking in 03/03/2013, before reverting towards equilibrium around 15/07/2015. Accordingly, after October 2016, the cointegration relationship diverges from equilibrium until 18/12/2017, before reverting to equilibrium on 21/07/2018. The period following the prohibition of bitcoin mining in China, which occured in July 2021, shows a discrepancy between the behaviour of the two cointegration relationships. In Figure 3 the cointegration tends to revert to equilibrium, whilst in Figure 4, the long-run relationship is showing signs of increased disequilibrium from 06/01/2022 onwards.

Finally, the plots of the time- varying coefficients β_{1t}, and β_{2t} in the cointegrating relation can found in Fig. A.1, A.2, and A.3 in the Appendix. The patterns of the graphs suggest that, approximately, $\beta_{1t} + \beta_{2t} = \delta$ for some constant δ.

5. Conclusions and further research

This paper investigated the value formation of the bitcoin-USD exchange rate and, more specifically, whether there exists a long-run relationship between bitcoin's price and its fundamentals over the period 22 July 2010 and 22 July 2022. From a model of bitcoin mining grounded in the Bitcoin code, we derived Bitcoin's marginal cost of production, a function in which the main variables are the electricity price and the mining rig efficiency. We then estimated a time-varying vector error correction model, where we investigated whether bitcoin's price and its marginal cost are cointegrated. In order to compare our results with the existing literature, we also estimated a bitcoin price-hash rate cointegration vector model. Our results first confirm the existence of a time-varying cointegration relationship between bitcoin's price and its cost of production and between bitcoin's price and the hash rate. Bierens and

Figure 3. Estimates of $\beta_t' Y$ for cointegrating relationship btc-USD/Cost.

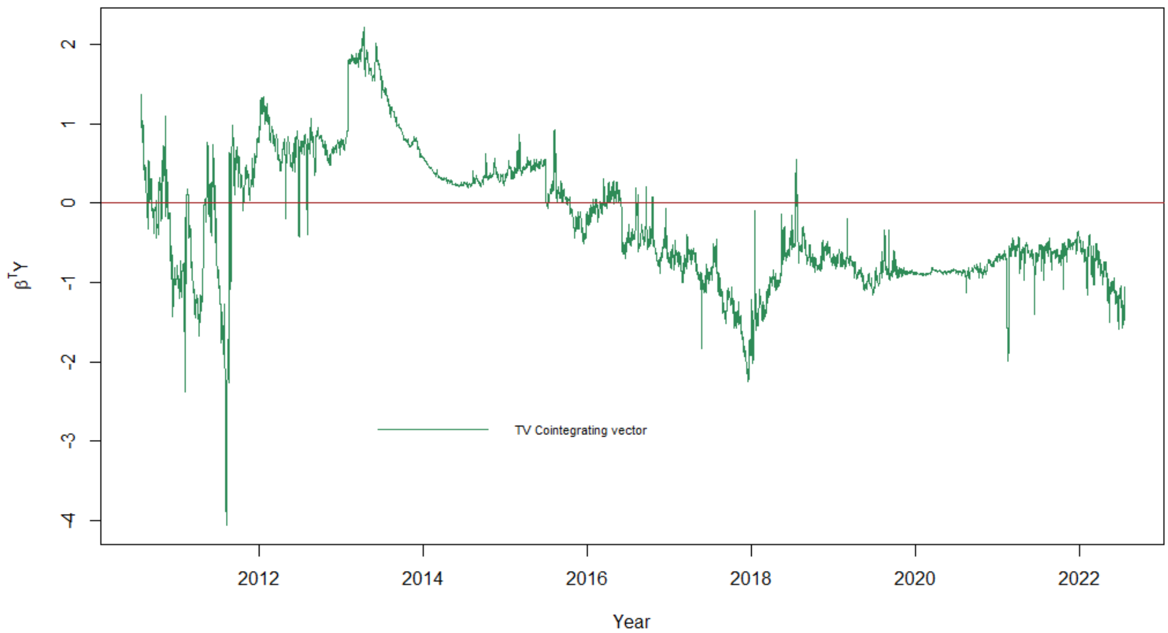

Figure 4. Estimates of $\beta_t' Y$ for cointegrating relationship btc-USD/Cost_TX.

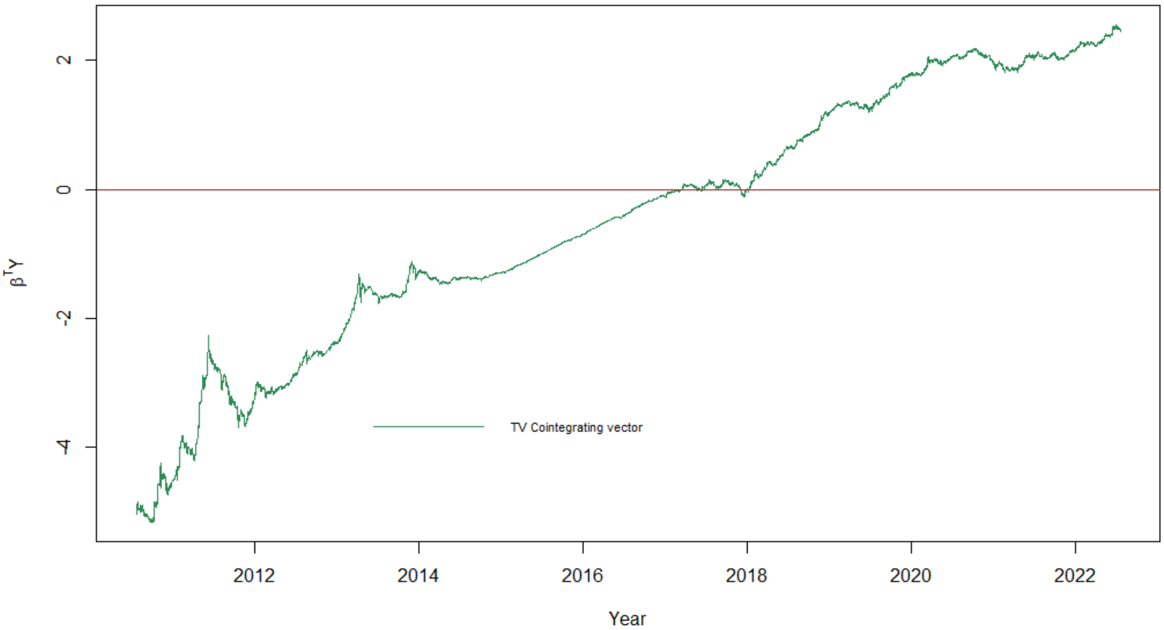

Figure 5. Estimates of $\beta_t' Y$ for cointegrating relationship btc-USD/Hash Rate.

Martins (2010)'s test of time-varying cointegration shows that the long-run relationships cannot be considered constant. Secondly, we found evidence that the time-varying cointegrating relationship between bitcoin's price and its hash rate is permamently in disequilibrium, bar a short time interval between March 2017 and January 2018. Consequently, although bitcoin's price and the hash rate are cointegrated, it is clear that the former never reverts to the latter after an exogenous shock, as would be expected from a long-run relationship. Our main findings are that bitcoin's price and its marginal cost have been cointegrated since bitcoin's inception, and that the long-run relationship always reverts towards equilibrium - and often *to* equilibrium- after long periods of divergence. These results contrast with most of the empirical literature that attempted to model the relationship betweeen bitcoin and its fundamentals in a time-invariant model, but are in line with recent research showing a significant role for production cost in the determination of bitcoin's price dynamics.

The empirical analysis above relied on three measures of production costs, i.e., two marginal costs, and a cost proxy, the hash rate. The two marginal costs were obtained assuming either a constant electricity price or the variable electricity price in the US. Owing to the lack of precise data on the location of most bitcoin mining activities, making different assumptions regarding electricity costs has been recurrent in the empirical literature, as was highlighted in Section 3.3. We adopted the same strategy, and chose a flat rate of USD 0.05kWh as in Xiong et al. (2020), who assume that most mining pools are located in China. This assumption was corroborated in Makarov and Schoar (2021), who analysed the location of individual miners based on their addresses on the Bitcoin blockchain, and the geographical location of the exchanges where they cashed out their rewards. The authors found that China -and in particular the province of Xinjiang- has dominated bitcoin mining, with a mining capacity between 60% and 80% over the period 2015 to 2017 (Makarov and Schoar, 2021, Section 4). As far as the conclusions of this study are concerned, the assumptions regarding electricity prices do not affect the existence of a time-varying cointegration between bitcoin and its fundamentals. Nonetheless, more granular data on geographical location of bitcoin mining, and of electricity costs in mining regions would greatly benefit empirical research on bitcoin price dynamics.

Conflict of interest

The author declares no conflicts of interest in this paper.

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