



Research article

A nonlinear adjustment in real exchange rates under transaction costs hypothesis in developed and emerging countries

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Abstract: This paper tries to empirically examine the exchange rate deviations to its level under the purchasing power parity (PPP) and transaction costs hypotheses using a battery of newly developed nonlinear approaches. To explain the persistent differences in exchange rates, we use the half-life function analysis with quarterly data over the period 1988Q1–2018Q2 for a panel of 23 developed and emerging countries. Our results show that some deviations to PPP relation may be governed by non-linear dynamics. The adjustment toward the fundamental equilibrium seems to depend on the size and the sign of the gap from the PPP equilibrium. The foreign exchange rate adjustment can be modeled by symmetric process with the ESTAR model and asymmetric process with LSTAR model.

Keywords: real exchange Rates; STAR models; purchasing power parity; nonlinear adjustments

JEL Codes: C22, F31, E32

1. Introduction

One of the most widely researched and controversial empirical topics in international economics and finance over the past few decades has been the examination of the Purchasing Power Parity (PPP) hypothesis (Rogoff, 1996; Taylor and Taylor, 2004). The PPP are the currency conversion rates that equalize the purchasing power of different currencies by eliminating the price levels differences

between countries. The basic assumption of this theory illustrated by the Law of one price (Dornbusch, 1976).

According to Murad and Hossain (2018), the PPP theory is used to “balance the comparative value of currencies by estimating the adjustment and required for the exchange rate to correspond to countries’ purchasing power”. Theoretically, the PPP hypothesis is at the centre of an extensive literature, which so far has not provided exact answers as to whether it holds in long term or not (Froot and Rogoff, 1995; Taylor and Sarno, 1998; Breitung and Candelon, 2005). This strand of literature has presented the theoretical background and the empirical evidence of the relationship between PPP and real exchange rates.

On the empirical side, although huge literature emerged attempting to test validity of the PPP, no consensus has occurred due to the conflicting. The basic idea of these studies is to prove the existence of a long-term relationship between domestic and foreign price indices and nominal exchange rates (Pippenger and Goering, 1998).

The rehabilitation of the PPP was then affirmed in several works, in this case Teräsvirta (1994) through nonlinear models of the STAR type: MacDonald (1995) by introducing other approaches of the exchange rate and Rogoff (1996) by introducing the notion of “half-life”. Other studies such as Engel (2000), Murray and Papell (2002) have failed to prove this linear cointegration relationship even over long periods. So, the puzzling results of PPP may be due to the potential nonlinearity of real exchange rates missed by traditional approaches.

Accordingly, a multiplicity of recent studies utilizing nonlinear econometric techniques have emerged providing substantial evidence for the empirical validity of the PPP hypothesis (Nobay et al., 1997; Sercu et al., 1995; Baum et al., 2001; Apte et al., 2004; Param et al., 2004; Van Dijk et al., 2002; Smallwood, 2005; Chowdhury, 2007; Liew et al., 2009; Phiri, 2014; Chen, 2017; Machobani et al., 2017; Yıldırım et al., 2009; Saadon and Sussman, 2018; Cavallo et al., 2018; Murad and Hossain, 2018; etc.). Several of them were based on the non-linear adjustment of real exchange rates under the assumption of transaction costs, and especially with the smooth transition autoregressive family processes (STAR, ESTAR, LSTAR, AESTAR, etc.)

These studies revive interest in reexamining the exchange rate deviations to its level under the PPP transaction costs hypothesis using a battery of newly developed nonlinear approaches. The relationship between transaction costs, nominal exchange rates and PPP is well documented in the literature. In fact, according to Sercu et al. (1995), with high transaction costs (such as tariffs, transportation costs, insurance costs and information costs), “the nominal exchange rate can deviate from the nominal price-parity value, but not more than the transaction cost” creating consequently an inaction-band for the real exchange rate. Under the hypothesis of instantaneous goods arbitrage at the borders of the inaction-band, barriers can be assimilated to thresholds.

Given these considerations, several authors have suggested that, although the non-arbitrage Dumas’s model (1992) explained the nonlinear behavior of exchange rate adjustments toward PPP, it remains unable to explain the real exchange rate behavior inside the “inaction-band”. Moreover, the model of Nobay et al. (1997) has given important empirical implications relative to the nonlinearity of deviations from PPP. This hypothesis has been initially exploited by MacDonald (1995).

The objective of this paper consists firstly at applying the STAR methodology to analyses the dynamic behavior of deviations from PPP under the transaction costs hypothesis in international arbitrage. In this framework, the smooth adjustment leads to the smooth function. A distinction must therefore be made between logistic and exponential transition functions. Then, and in order to better

explain the persistent differences in exchange rates, we use the half-life function analysis. Our sample is composed of 23 developed and emerging countries over the period 1988–2018. We show that some PPP deviations can be driven by a non-linear dynamic. Also, the real exchange rate deviations at the PPP appear to depend on the magnitude as illustrated by the exponential smooth transition autoregressive (ESTAR) models and the sign of the equilibrium gap with logistic smooth transition autoregressive (LSTAR) models. Moreover, these types of processes can identify the exchange rate adjustments.

The remainder of this paper is organized as follows. Section 2 reviews the main theoretical and empirical literature related to the PPP. In section 3, we present the empirical methodology as well as the data. In section 4, we discuss the empirical results. The last section concludes.

2. Related literature

Although PPP theory is one of the important assumptions in international finance, rigorous empirical studies did not emerge until 1960, with the productivity bias hypothesis of Balassa (1964) and Samuelson (1964). However, serious and numerous works on the validity of PPP appeared in the late 1970s (Dornbush, 1976, Meese and Rogoff, 1983, etc.). In the late 1980s, unit root tests failed to validate Absolute PPP. To overcome this problem, researchers used long-term data (Kilian, 1999, Engel, 2000) and panel data techniques (Higgins and Zakrajsek, 2000; Guimarães and Karacadag, 2004). At the same time, other empirical works have been explored to test the PPP hypothesis in the last decade, using cointegration techniques, variance ratio tests, structural-break tests, mean reversion tests etc. Later, many authors have developed theoretical models of the nonlinear exchange rate adjustment (Sercu et al., 1995) where the transportation costs create an arbitration band for the exchange, in which the marginal cost of adjustment exceeds the marginal benefit. Assuming a system of instantaneous arbitrage of goods within the band, the thresholds reflect barriers as mentioned in Figure 1 below:

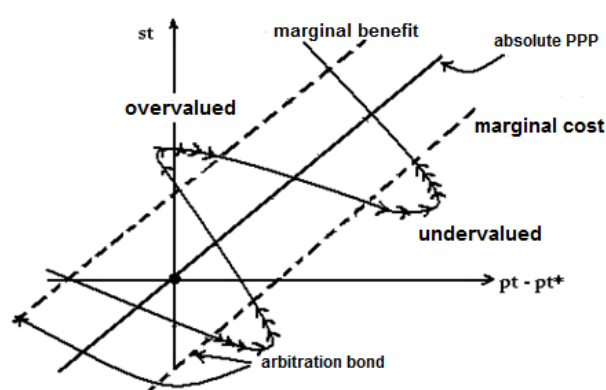


Figure 1. The arbitration bands.

In the same context, Nobay et al. (1997) show that standard cointegration tests ignoring the effect of transactions costs, may be biased against the long run PPP hypothesis. In the presence of transaction costs, the equilibrium models of real exchange rate determination imply a nonlinear adjustment process toward PPP (ESTAR process). The authors put in evidence the presence of mean—reverting behavior for PPP deviations; this may help explain the mixed results of previous studies.

Param et al. (2004) model the dynamics of the adjustment process of Indonesian PPP relative to US, Singapore and Japan in the presence of transaction costs. Using monthly observations over the period 1979:01–2003:06, where exchange rate regimes are managed- and free-floating, the authors apply a non-linear ESTAR to test for the mean-reverting properties of real exchange rates for small and large deviations from the long-run equilibrium. The results show that small deviations are non-stationary, persistent and may be explosive, while large deviations are stationary with a mean reverting adjustment process.

Chowdhory (2007) examines the long run PPP hypothesis in the context of Bangladesh and four trading collaborates (US, Euro area, India and Japan) over the period 1994–2002. Applying nonlinear econometric techniques, the results provide strong support for the long-run PPP validity as well as for theoretical models predicting nonlinear adjustment in real exchange rates.

Phiri (2014) investigates the asymmetric adjustment effects of the PPP hypothesis between South Africa and her main trading partners (the Euro area, the UK, the US, Japan and China). Investigating threshold cointegration, error correction effects, and granger causal effects between nominal exchange rates and aggregate price differentials, the results show a nonlinear significant PPP behavior between South Africa and all trading partners.

The Chen (2017)'s study applies nonlinear KSS and Asymmetric ESTAR (AESTAR) unit root tests, proposed respectively by Kapetanios et al. (2003) and Sollis (2009) in order to test for the validity of long run PPP in a panel of six high-growth countries. The results indicate that PPP holds for five from the six countries (Brazil, Mexico, Indonesia, China and South Korea). Additionally, using the AESTAR unit root test, the author finds nonlinear and asymmetric adjustment toward PPP.

Machobani et al. (2017) test the validity of the Uncovered Interest Rate Parity (UIRP), the International Fisher Effect (IFE), and the PPP hypothesis in the context of South Africa using monthly data over the period 1999–2014. Empirical results reject the UIRP, the IFE and the PPP hypothesis for South Africa. This could be due to the existence of transaction costs and taxes between countries. Moreover, investors also tend to hunt higher yields for their assets. Interest rates rambles in the trading partner countries leading to capital flight out of South Africa, resulting in local currency depreciation (the rand). Accordingly, capital markets may still be inefficient, justifying the government intervention towards more capital controls to avoid excessive outflows during periods of economic collapses.

Using a battery of recent nonlinear unit root tests, Yıldırım (2017) proposes an empirical investigation of the PPP hypothesis between Turkey and four major trading partners (US, Russia, China, and European Union) over the floating exchange rate period 200:03–2015:10. The results issued from unit root tests in a stationary ESTAR process (Kılıç, 2011), provide stronger evidence in favor of the PPP hypothesis when compared to the standard unit root tests. Besides, they reveal that real exchange rates follow a linear path in the absence of transaction costs and trade barriers.

In a very recent study, Saadon and Sussman (2018) test the PPP and the UIRP in advanced small open economies, Israel and the United States. The most important results show that the PPP and the UIRP equilibrium relationships hold in the short run when the necessary conditions are met.

Murad and Hossain (2018), investigate the PPP hypothesis in ten ASEAN countries over the period 1973–2015 using panel data techniques. The authors use the Pesaran (2007)'s cross-sectionally augmented panel unit root test and the Westerlund (2007)'s panel cointegration tests. The results overpoweringly corroborate the relative PPP hypothesis. Consequently, the monetary authority would be able to implement a self-regulating monetary policy and control the exchange rates.

In a nutshell, this relatively recent literature on the nonlinear adjustments in real exchange rates and long run PPP can be considered as a possible solution to such puzzles. Our study subscribes in the continuation of the above-mentioned works, by extending the empirical investigation to a sample of 23 developed and emerging countries. However, our contribution is essentially empirical. We are exploring the possibility that the adjustment of real exchange rates is itself explicitly nonlinear due to the presence of transactions costs in international arbitrage. We are then testing the nonlinearity mean-reverting hypothesis using the STAR framework. We are using a new dataset with quarterly data (1988Q1–2018Q2) and we are dividing the time period in two sub-samples covering the pre- and the post-euro periods. Finally, we are using the half-life function to examine the dynamic response of deviations from PPP to innovations, highlighting thus the importance of nonlinear modeling.

3. Methodology and data

3.1. Empirical methodology

It should be emphasized that the non-linear forecast is much more complicated than that performed with a linear model, thus allowing a more detailed discussion of the forecasts from non-linear models. The STAR models are based on the idea that the economy can be in two different regimes. They are mainly used to characterize the evolution of certain macroeconomic or financial variables and for analyzing economic cycles. They nest a linear autoregressive model, and the additional parameters give it more flexibility, which could be useful in econometric modeling and forecasting. According to Sarantis (1999), the STAR models produced more precise forecasts than a pure random walk. It is also better than another nonlinear model, a hidden Markov model with a switching interception as in Hamilton (1989). It should be noted that these forecast comparisons are based on accurate forecasts. One can argue accordingly that the real value of forecasts from nonlinear models may be prone with forecast densities where the shape can provide important information to decision makers.

Like Michael et al. (1997) and Taylor et al. (2001), we introduce the assumption of transaction costs. The STAR model of order p can be written in the following way:

$$y_t = \alpha' + \lambda' d_{t-1} + \sum_{i=1}^p \phi_i' y_{t-i} + [\alpha'' + \lambda'' d_{t-1} + \sum_{i=1}^p \phi_i'' y_{t-i}] \Phi(y_{t-d}; c; \gamma) + v_t \quad (1)$$

Or y_t represent the series of deviations to the first difference PPA and d_{t-1} the series of deviations in delayed level, $\Phi(y_{t-d}; c; \gamma)$ is the transition function determines the degree of mean reversion which is a function of the parameter “ γ ”, giving the speed of the return to the mean or adjustment; the parameter “ c ” is the equilibrium level of y_t which gives the concentration area of the transition function, also called the parameter “location parameter”; (y_{t-d}) is the transition variable “variable transition”.

$$\Phi_L(y_{t-d}; c; \gamma) = \{1 + \exp(-\gamma^2 [y_{t-d} - c] / \sigma_{yi}^2)\}^{-1} \quad (2)$$

$$\Phi_E(y_{t-d}; c; \gamma) = 1 - \exp(-\gamma^2 [y_{t-d} - c]^2 / \sigma_{yi}^2) \quad (3)$$

These two transition functions suggest different dynamics of the mean reversion process: the logistic function is characterized by an asymmetric adjustment of (y_t) with respect to its past values, as a function of the transition variable (y_{t-d}) , whether it is below or above the threshold “ c ”: from

which it detects the sign of deviations (sign effect). On the other hand, the exponential function suggests a symmetric adjustment in the same direction as $(y_{t-d} - c)$: it gives the magnitude of the deviations (size effect). In other words, using a logistic function is assumed that the positive and negative deviations return average at different speeds, while with the exponential function, the speed of mean reversion is the same as it either positive or negative deviations.

$\Phi_L(y_{t-d}; c; \gamma)$ is symmetric and U-shaped for the exponential function, and it varies slowly from 0 to 1 depending on the transition variable and having the shape of (S) extended for the logistic function. The transition variable can be a delayed endogenous variable (y_{t-d} being the delay parameter). It is also possible to use other transition variables (exogenous variables). These properties of the STAR model are interesting in our modeling since they allow a smooth transition between schemes and a symmetrical or asymmetrical adjustment of the deviations of the exchange rate from its equilibrium level. The transition parameter (γ) determines the transition speed between the two low and high regimes by going through an intermediate regime that depends on the nature of the transition function:

(i) The low regime corresponds to $y_{t-d} = c$ and equation (1) becomes an $AR(p)$ model for LSTAR and ESTAR.

$$y_t = \alpha' + \lambda' d_{t-1} + \sum_{j=1}^p \phi_j' y_{t-j} + h_t \quad (4)$$

(ii) The high regime corresponds, for a given γ , with, $\lim_{[(t-d)-c] \rightarrow \pm\infty} \Phi(y_{t-d}; c; \gamma) = 1$ where equation (1) becomes a different model $AR(p)$ for LSTAR and ESTAR:

$$y_t = (\alpha' + \alpha'') + (\lambda' + \lambda'') d_{t-1} + \sum_{j=1}^p (\phi_j' + \phi_j'') y_{t-j} + \eta_t \quad (5)$$

(iii) The STAR model intermediate regime for which: $y_{t-d} = c$

For the ESTAR model:

$$\Phi_E(y_{t-d}; c; \gamma) = 0; \text{ and } y_t = \alpha' + \lambda' d_{t-1} + \sum_{i=1}^p \phi_i' y_{t-i} + \varepsilon_t \quad (6)$$

For the LSTAR model:

$$y_t = \alpha' + \lambda' d_{t-1} + \sum_{i=1}^p \phi_i' y_{t-i} + \frac{1}{2} [\alpha'' + \lambda'' d_{t-1} + \sum_{i=1}^p \phi_i'' y_{t-i}] + \nu_t \quad (7)$$

To estimate the STAR processes, we must test the presence of non-linearity in the autoregressive processes developed by Luukkonen et al. (1988). The null hypothesis states that the deviation adjustment follows a linear process $AR(p)$ against the alternative hypothesis of non-linear STAR processes. This version of the linearity test has been adopted by Baum et al. (2001), Liew et al. (2009), Lundbergh et al. (2003b), and Van Dijk et al. (2002).

This linearity test has an auxiliary regression of the form:

$$y_t = \phi_0 + \sum_{i=1}^p (\phi_{1i} y_{t-i} + \phi_{2i} y_{t-i} y_{t-p} + \phi_{3i} y_{t-i} y_{t-p}^2 + \phi_{4i} y_{t-i} y_{t-p}^3) + \eta_t \quad (8)$$

where $\phi_0, \phi_{1i}, \phi_{2i}, \phi_{3i}$, and ϕ_{4i} are the parameters to be estimated and η_t the residual term. The parameter “p” is the $AR(p)$ delay order and “d” is the delay parameter. To identify the type of nonlinear model, reference is made to Teräsvirta (1994) which gives specification tests of choice

between the ESTAR and LSTAR models. To do this, Teräsvirta and Anderson (1992) present a series of hypothesis tests of the coefficients of Equation (8) which are:

$$H_{01}: \phi_{4i} = 0 \quad (i = 1, \dots, p)$$

$$H_{02}: \phi_{3i} = 0 \mid \phi_{4i} = 0 \quad (i = 1, \dots, p)$$

$$H_{03}: \phi_{2i} = 0 \mid \phi_{3i} = \phi_{4i} = 0 \quad (i = 1, \dots, p)$$

They propose the following decision rule:

- i. If H_{01} rejected, we choose LSTAR.
- ii. If H_{01} accepted and H_{02} rejected, ESTAR is chosen.
- iii. If H_{01} and H_{02} accepted but H_{03} rejected, in this case we choose LSTAR.

It is important to note that, if none of the null hypothesis can be rejected, then the linearity cannot be rejected, and we will hold in this case the linear model. In addition, Teräsvirta (1994) recommends testing hypothesis by examining the probability values of the Fisher statistics (F_1, F_2 , and F_3) associated with the hypotheses, (H_{01}, H_{02} , and H_{03}) respectively: If the value of the probability of the statistic F_2 of H_{02} is the smallest, one chooses the model ESTAR, if not the model LSTAR will be retained.

3.2. The data

Data sets were built from the International Financial Statistics (IFS, IMF 2019). They cover the nominal exchange rate of 23 currencies compared to the US dollar and the consumer price indices (CPI). The analysis of the PPP is based on the CPI is carried out for 23 countries grouped as follows: developed countries (the G7: Germany, Canada, France, Great Britain, Italy, Japan, New Zealand), other industrialized countries (Australia, Belgium, Denmark, Norway, Luxembourg, Sweden, Switzerland), and emerging countries (India, Indonesia, Korea, Malaysia, Philippines, Singapore, Thailand, Mexico, Paraguay).

The variables used are:

- (i) The nominal exchange rate is the number of units of the national currency that are needed to purchase a unit of a given foreign currency (the US dollar).
- (ii) The consumer price index: Referring to the empirical literature, we are led to normalize the real exchange rate series at the beginning of the sample (1988: 01 and 1999: 01) for the euro.
- (iii) The real exchange rate is expressed as follows:

$$R_t = \frac{P_t/P_t^*}{S_t} \quad (9)$$

where (S_t) the nominal exchange rate; (P_t) the domestic price level, and (P_t^*) the level of foreign (US) prices. In its logarithmic form, the real exchange rate is represented as follows:

$$R_t \equiv \log\left(\frac{P_t/P_t^*}{S_t}\right) = \log\left(\frac{P_t}{P_t^*}\right) - \log S_t \quad (10)$$

Absolute PPP implies that the nominal exchange rate is equal to the price ratio between two countries; while the relative PPP implies that the change in the exchange rate is equal to the change in the price ratio. The empirical verification of Absolute PPP is often based on the following model:

$$R_t = \alpha + \lambda Z_t + \varepsilon_t \quad (11)$$

R_t represents the real effective exchange rate (in logarithm). Z_t the vector of economic fundamentals of the model (in logarithmic); (λ) represents the restoring force, ε_t an error term describing deviations from the PPP.

4. Empirical findings and discussion

The empirical analysis is based on quarterly observations of spot exchange rates and consumer price indices for the 23 developed and emerging countries. The data period runs from 1988Q1 to 2018Q2 depending on the availability of statistics for the different countries. As a preliminary exercise, we should test for the unit root in real exchange rates behavior. Moreover, to test for the validity of PPP hypothesis as a long-run equilibrium, one needs to establish the stationarity of the deviation from PPP. However, the test for cointegration will be affected by the nonlinearity. We tested, to this end, for the unit root behavior of each of the CPI and nominal exchange rate series expressed in level and first difference using ADF and PP tests. In the case of non-stationary process of exchange rates, we conclude that they follow a nonlinear LSTAR process, which is globally stationary. Consequently, the nominal exchange rate series are nonlinear cointegrated with the relative price.

4.1. Linear adjustment of real exchange rate deviations to PPP

The usual Akaike (*AIC*), Schwartz (*SIC*) and Ljung-Box (*Q*) information criteria are used to select the optimal lag order. The results show a lag order of ($p = 1$) for all countries.

The autoregressive linear process *AR*(1) is written:

$$y_t = \alpha + \lambda d_{t-1} + \phi y_{t-1} + \varepsilon_t \quad (12)$$

where d_{t-1} designed the deviations series from the PPP delayed by one period; y_t the first difference of d_t ; ε_t are normally and independently distributed *niid*; (α) , (λ) and (ϕ) the coefficients to be estimated; (λ) is the restoring force.

The results in the Table 1, show the presence of a statistically significant coefficient (λ) at a 99% confidence level for pre-euro Germany, Denmark, pre-euro France, and G. Britain, Italy, Japan, Norway, New Zealand, Switzerland, Korea and Mexico. This confirms the presence of an extremely important restoring force of deviations from APP. Since the coefficient (ϕ) is also significant at 5% level, this implies that the historical information seems to contribute significantly to the specification of exchange rate dynamics. This suggests an inability of exchange rate movements to offset the inflation differential between each country and the United States. As for the constant (α) , it is statistically different from zero, suggesting the absence of deviation drifts from the PPP.

The results are typically similar for all countries. In fact, we note the absence of deviation drift. Similarly, deviations expressed in terms of first differences contribute significantly to the specification of the foreign currencies dynamics relative to the exchange rate. Specifying the dynamics of the different currencies relative to the exchange rate.

Table 1. Estimation results of linear models.

	Model: $y_t = \alpha + \lambda d_{t-1} + \phi y_{t-1} + \varepsilon_t$					
	α		λ		ϕ	
	$\hat{\alpha}$	<i>t-stud</i>	$\hat{\lambda}$	<i>t-stud</i>	$\hat{\phi}$	<i>t-stud</i>
Developed countries						
Germany	0.03	2.04*	-0.06	-2.10*	0.27	2.89*
Germany (P-Euro)	0.001	0.54	-0.01	-1.93	0.27	3.20*
Australia	0.02	2.34	-0.07	-2.31*	0.35	3.79*
Belgium	0.001	0.54	-0.01	-1.93	0.27	2.98*
Belgium (P-Euro)	-0.007	-0.52	-0.01	-0.45	0.31	0.73
Canada	0.005	1.83	-0.03	-1.75	0.37	2.42*
Denmark	0.23	3.14*	-0.04	-1.98*	0.38	1.84*
France	0.31	1.57*	-0.06	-1.45*	0.25	4.86*
France (P-Euro)	-0.005	-0.33	-0.02	-0.35	0.41	0.50
G-Britain	0.02	2.44*	-0.04	-3.26*	0.19	2.38*
Italy	0.53	2.29*	-0.08	-2.29*	0.32	3.35*
Italy (P-Euro)	-0.07	-0.78	-0.02	-0.31	0.17	0.79
Japan	0.21	2.04*	-0.05	-1.97*	0.31	3.01*
Luxembourg	0.23	0.98	-0.03	-1.32	0.29	2.87*
Luxembourg (P-Euro)	-0.16	-0.61	-0.19	-0.39	0.41	2.74
Norway	0.13	1.32*	-0.06	-1.41*	0.23	2.69*
N-Zealand	0.03	2.22*	-0.05	-1.98*	0.36	4.38*
Sweden	0.07	1.81	-0.03	-1.78	0.25	2.96*
Switzerland	0.03	2.29*	-0.07	-2.55*	0.26	3.15*
Emerging countries						
Korea	0.45	2.45*	-0.06	-2.45*	0.30	3.55*
India	0.04	1.54	-0.01	-1.38	0.30	3.73*
Indonesia	0.15	1.21	-0.01	-1.17	0.22	2.55*
Malaysia	0.01	1.22	-0.009	-0.89	0.37	4.59*
Mexico	0.14	2.37*	-0.09	-2.36*	-0.11	-1.25
Paraguay	0.15	1.24	-0.01	-1.02	0.40	4.88*
Philippines	0.08	1.34	-0.02	-1.32	0.26	3.09*
Singapore	0.01	1.74	-0.03	-1.64	0.27	3.43*
Thailand	0.07	1.22	-0.02	-1.18	0.029	3.48*

Note: * significance at 5%. The expression (P-Euro) refers to the post-euro period (1999:01–2018:02)

4.2. The half-life functions analysis

To analyze the dynamic behavior of the PPP exchange rate, we estimate the persistence of exchange rate deviations following a permanent shock using the half-life function. It is defined as the number of times the deviations from the PPP take to halve after an exchange rate shock. According to Rogoff (1996), half-life is always calculated from an $AR(1)$ real exchange rate process:

$$d_t = \alpha + \lambda d_{t-1} + \varepsilon_t \quad (13)$$

The half-life is a simple function of the autoregressive parameter (λ) which is expressed as follows:

$$hl = \frac{\log(0.5)}{\log(\lambda)} \quad (14)$$

The results are presented in the Table 2.

Table 2. Half-life estimation.

	<i>half-life</i> in quarters	<i>half-life</i> in years	Adjustment speed
Developed countries			
Germany	13.04	3.26	5.2
Germany (P-Euro)	12.73	3.18	5.3
Australia	20.74	5.18	3.3
Belgium	17.08	4.27	4
Belgium (P-Euro)	23.36	5.84	2.9
Canada	35.28	8.82	1.95
Denmark	14.14	3.53	4.8
France	12.74	3.18	5.3
France (P-Euro)	17.93	4.48	3.8
G-Britain	15.33	3.83	4.4
Italy	12.24	3.061	5.5
Italy (P-Euro)	40.37	10.09	1.7
Japan	15.24	3.81	4.45
Luxembourg	26.86	6.71	2.6
Luxembourg (P-Euro)	89.42	22.35	0.8
Norway	11.58	2.89	5.8
N-Zealand	17.35	4.33	3.92
Sweden	24.02	6.005	2.9
Switzerland	10.01	2.50	6.6
Emerging countries			
Korea	13.25	3.31	5.1
India	77.33	19.33	0.9
Indonesia	48.22	12.05	1.4
Malaysia	485.93	121.48	0.14
Mexico	6.67	1.66	9.86
Paraguay	100.63	25.15	0.7
Philippines	40.79	10.19	1.7
Singapore	24.60	6.15	2.8
Thailand	57.12	14.27	1.2

Note: The expression (P-Euro) refers to the post-euro period (1999:01–2018:02).

Therefore, the value of 1.6 years for Mexico is typically like that of Garces-Diaz (2003) concluding to the presence of a half-life of 1.8. In the case of Malaysia, it is 121 years old, which is not surprising since the exchange rate follows a non-stationary process and therefore does not respect the mean reversion process. As for the other countries, the exchange rate is governed by return-to-average processes that verify the PPP in the long term, with a half-life of 14 years for the Thailand and an adjustment rate of 1.2% per quarter. The case of Thailand is like other emerging countries where the half-life value is very high compared to Western countries. The results obtained for the Denmark and the Great Britain and other advanced countries seem to be the most reasonable with half-life between 3 and 5 years which seem consistent with the findings of Rogoff (1996), Taylor (2001), Mark (2001), Andrews (1993), Kapetanios and Shin (2002) and , Gharib (2007).

Finally, we conclude that the deviations from PPP are smaller in the long term and that the true adjustment process is non-linear. Consequently, we apply the Luukkonen et al. (1988).

4.3. Luukkonen et al. (1988)'s nonlinearity test

The calculated values of the $LM(d)$ statistic reported in the Table 3 show the null-hypothesis for linear test, which only accepted for India, Japan, Mexico, Canada and European countries for the post-euro period. Thus, PPP exchange rate deviations appear to be well governed by non-linear processes. Moreover, the hypothesis (H0) is rejected for the rest of the countries, and this, for the benefit of STAR nonlinear processes. The non-rejection of linearity for European countries in the post-euro period seems to be due to the short period of study (26 quarters).

Table 3. Results of the linearity test.

Countries	$Max - LM(d)$	p	d	Process
Developed countries				
Germany	24.21	1	11	LSTAR
Australia	21.24	1	11	TAR
Belgium	16.61	1	10	LSTAR
Canada	16.66	1	7	ESTAR
Denmark	15.41	1	8	AR
France	18.71	1	10	LSTAR
G-Britain	9.86	1	10	AR
Italy	10.76	1	12	ESTAR
Japan	4.048	1	8	AR
Luxembourg	19.50	1	12	ESTAR
Norway	15.33	1	10	ESTAR
N-Zealand	21.80	1	11	LSTAR
Sweden	35.42	1	10	ESTAR
Swiss	11.63	1	12	ESTAR
Germany (P-Euro)	17.07	1	13	ESTAR
Belgium- (P-Euro)	17.00	1	11	AR
France- (P-Euro)	15.79	1	9	ESTAR
Italy- (P-Euro)	19.81	1	13	AR
Luxembourg- (P-Euro)	19.04	1	11	AR
Emerging countries				
Mexico	2.75	1	3	LSTAR
India	26.16	1	12	LSTAR
Singapore	41.70	1	12	TAR
Thailand	38.26	1	3	ESTAR
Paraguay	35.50	1	10	ESTAR
Korea	37.06	1	2	ESTAR
Indonesia	19.54	1	3	ESTAR
Malaysia	29.72	1	3	LSTAR
Philippines	27.07	1	4	LSTAR

Note: (d) is the delay parameter that maximizes the LM(d) statistic. (p) is the delay order.

The expression (P-Euro) refers to the post-euro period (1999: 01–2018: 02).

As shown by Schnabl (2001), yen fluctuations are influenced by interest rates, prices and exchange rate policy. The interest rate can only explain the short-term deviations while prices affect the yen in the long run. In fact, in 1998, following the failure of the sterilization policy, the banks are controlled by the finance ministry and exchange rate fluctuations follow that of the interest rate (non-sterilized interventions) resulting in monetary expansion. In 1999, authorities buy dollars and investors convert them into yen to control the foreign exchange market that has caused an economic recession. The Japanese authorities have been forced to lower the interest rate to increase the quantity of money. All these factors are at the origin of the total dependence of the Japanese yen on the US dollar, explaining the non-rejection of a linear adjustment process in the short-term with a half-life of 3.8 years.

As for Canada, the non-rejection of the linearity assumption can be explained by several factors, including a period of increasing trade liberalization between Canada and the United States and the existence of productivity bias effects. The real exchange rate reflects the bilateral differences in productivity for tradable and non-tradable goods between the two countries, commonly known as the Harrod-Balassa-Samuelson effect (HBS effect).

Regarding Mexico and India, the rejection of linearity is not surprising and corroborates the half-life results analysis. As proved by Guimaraes and Karacadag (2004), the Mexican economy is governed by the monetary authorities' interventions that affect the exchange rate volatility and the adoption of an adequate exchange rate regime. These two economies are distinguished by high inflation affecting the validity of the PPP. This result has been proven, among others, by Gharib (2007), Garces-Diaz (2003) and Drine and Rault (2002).

4.4. STAR models and nonlinear adjustments towards PPP real exchange rate deviations

4.4.1. Eklund (2003)'s test

To test stationarity and prove the existence of nonlinear cointegration in the LSTAR model, we use the nonlinear cointegration tests proposed by Eklund (2003). The results of linear and nonlinear stationarity tests of real exchange rates are reported in the Table 4.

Table 4. Stationarity at the LSTAR.

Countries	ADF	PP	KPSS	F _{ND}	F _D
Developed countries					
Denmark	-3.16	-1.36	1.06	10.95	11.80
Sweden	-1.34	-2.14	0.12	11.30	7.59
Sweden	-2.80	-2.85	0.51	12.80	8.84
France	-1.58	-1.73	0.34	12.34	10.13
Emerging countries					
Paraguay	-2.47	-2.12	0.32	27.35	24.65
Singapur	-0.90	-0.94	1.24	13.45	3.45
Korea	-0.60	-0.62	1.08	25.18	21.00
Indonesia	-1.23	-1.78	0.36	21.45	16.98

Note: The critical values of F_{ND} and F_D are 3.66 and 4.96 respectively with $\alpha = 5\%$ and 3.04 and 4.04 at 10%.

The linear stationarity is rejected at levels of 1, 5 and 10% unlike non-linear stationarity tests. Eklund (2003) proposes to test the presence of non-stationarity against a non-linear but generally stationary LSTAR process. The results of this test on the real exchange rate series show that they are stationary for all countries. This test implies that there is a non-linear cointegration relationship between the exchange rate and the price index. This implies the validation of the PPP hypothesis according to a non-linear dynamic, i.e. that PPP deviations are well described by an LSTAR model.

4.4.2. Kapetanios et al. (2003)'s test

To test stationarity and cointegration in the ESTAR model, we perform the usual KPSS unit root test and the KSS and AKSS (Augmented KSS) tests proposed by Kapitanios et al. (2003). The results of the linear and non-linear stationarity tests applied to the real exchange rate series by the ESTAR regression are reported in the Table 5.

Table 5. Stationarity at the ESTAR.

Countries	ADF	PP	KPSS	KSS	AKSS
Developed countries					
Norway	-2.09	-2.31	0.17	-6.78	-5.10
Luxembourg	-1.88	-1.61	0.68	-6.18	-4.78
Belgium	-1.91	-1.95	0.15	-6.16	-4.96
G-Britain	-3.11	-2.92	0.95	-5.47	-3.33
Italy	-1.41	-2.13	0.26	-3.73	-2.48
Germany	-3.08	-2.08	0.15	-7.65	-6.86
Australia	-1.42	-1.83	0.81	-8.08	-5.77
N-Zealand	-2.51	-2.17	0.10	-5.01	-3.21
Emerging countries					
Thailand	-0.98	-0.34	0.28	-2.62	-3.08
Malaysia	-1.32	-1.30	1.77	-7.24	-7.20
Philippines	-1.18	-0.93	0.99	-3.22	-2.40

Note: The critical values of KSS and AKSS are -3.48 to 1%, -2.93 to 5% and -2.66 to 10%.

The usual unit root tests provide similar results for the stationarity of exchange rate series. Linear stationarity is rejected at the levels of 1, 5 and 10% respectively. Thus, the KSS and AKSS tests make it possible to test the null hypothesis of nonstationary against a non-linear but globally stationary ESTAR process. The results show that real exchange rate series are stationary and more precisely stationarity is non-linear.

This result implies that there is a non-linear cointegration relationship between exchange rates and price indexes, which implies the validation of the PPP hypothesis, i.e. the real exchange rate process is an ESTAR process.

5. Conclusion

While a vast empirical literature is available on testing the validity of the PPP, no consensus has registered due to contradictory evidences. Recently, it has been supported that the puzzling results of

PPP may be due to the potential nonlinear nature of real exchange rates which is ignored by the standard approaches. Accordingly, some recent studies utilizing nonlinear econometric methods have provided fairly convincing evidence for the empirical validity of the PPP hypothesis.

With the motivation of these recent studies, this work attempts to model the dynamics of exchange rate adjustments toward PPP as a fundamental long-run equilibrium relationship applying nonlinear specifications. The empirical application to a selection of developed and emerging countries indicates that one should carefully interpret evidence from standard tests. With the presence of transaction costs, we show that the deviations from PPP are, for major countries, nonlinear, following either ESTAR or LSTAR processes. Our results further reveal the existence of a mean-reverting process in real exchange rate adjustments which tendency varying with sign and magnitude of deviations. The crucial estimated parameters of ESTAR and LSTAR processes reveal that exchange rate dynamics have two speeds inside and outside an inaction band within which international price differentials are not arbitrated away. These results are consistent with a large number of recent studies on the nature of exchange rate dynamics in the presence of international arbitrage costs. Finally, in line with previous empirical studies, we have evaluated the forecasting performance of the STAR model by taking the linear process as a benchmark.

Our conclusions have major policy implications, the most important of which consists at giving some credence to emerging countries Central Banks in managing their domestic currencies in certain fluctuations bands. Also, in the short run the nominal exchange rate may deviate from PPP equilibrium inside the inaction arbitration band.

Acknowledgments

This manuscript benefited from the constructive comments of two anonymous reviewers.

Conflict of interest

All authors declare no conflict of interest.

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