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*Research article*

# Real options bargaining games

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**Abstract:** The real options approach has been supporting investment decision-making processes for years. However, on competitive markets, the real options games approach is the more suitable way. In this article, a real options game subject to analysis is the situation in which two companies with different market share are exploring an opportunity to implement a new investment project. It is known that competition on the market reduces the scope of benefits a company can gain whilst implementing the project. In this paper, we show that this reduction can be mitigated by taking into account payoff transfer designated as a bargaining solution. We discuss three main types of games between companies that we can observe on the market; then we analyze their bargaining solutions, and finally—come up with recommendations to companies. A firm that dominates its respective market usually benefits by implementing the most advantageous strategy, but in certain situations it should pay special attention to its weaker competitor's opportunities and try to anticipate its movements. In the paper, we show that with high project risk and significant asymmetry in the firms' market share, a weaker company may still hold all the cards.

**Keywords:** investment option; real options games; bargaining game; non-symmetric Nash bargaining solution; cooperative-competitive value; coco value

**JEL Codes:** G13, D81, C78

## 1. Introduction

Operating on a competitive market is not easy: some companies achieve success and are able to keep it; other go out of business. In the paper we would like to look at one piece of such firm's success puzzle.

Without any doubt, effective investment is an important part of a successful strategy. Methods of its evaluation play a vital role when deciding whether to accept or reject an investment project. Among many methods used in economic practice, the concept of real options holds a special position. Not only is it a valuation method, but also a way of looking at investment projects. Many international corporations have used the concept of real options to analyze their investment projects (e.g. Merck, GlaxoSmithKline, Boeing, General Motors, Hewlett Packard, Morgan Stanley, KGHM), but the application of real options in business practice still falls below the expectations of academics. For managers, an ambiguous feature of real options is the fact that they treat project risk as an opportunity rather than a threat. Another factor limiting the use of the real options approach when valuating projects is the proprietorship of many options. Many real (investment) options are not *proprietary* but *shared*, i.e. held by several entities simultaneously (Smit and Trigeorgis, 2004). Firms can compete as they seek the most advantageous position on the market. When analyzing such situations, the game theory is usually applied. The game theory provides tools to describe and examine the situation in which the effect of competitors' behaviors should be incorporated into firm's decision-making process. Investment decision-making can be described as a game between two players. And if, on top of that, the investment project is evaluated as the investment option, then we are dealing with the real options game. In general, depending on the decision-making moment, we can distinguish—in the game theory as well as in the real options games approach—two main types of games: simultaneous games and sequential games. In the first type of games, players make decisions independently and simultaneously; in the second type of games—the concepts of a leader and a follower are used. Both types of games are fundamental to study the strategic behavior of economic operators; however, the second type (a leader-follower game) is most frequently referred to in the literature on real options. One of the first papers in this field of study was written by Smit and Ankum (1993). They examine the extent to which the degree of non-exclusiveness of an investment opportunity influences the investment strategy. They show (using simple numerical examples) that a firm will postpone an investment project when: its net present value is low, market demand is uncertain, and when interest rates are high. They also demonstrate that with many competitors on the market the values of their investment projects may diminish, while few competitors pose a threat of complete preemption. Many key studies on a structured framework for analyzing strategic games in relation to competition, and on the theoretical and applied approach of game theory and real options, can be found in the book edited by Grenadier (2000). Smit and Trigeorgis (2004) emphasize the strategic aspect and valuation technics in the real options games and provide a wide range of valuation examples. Investment decisions made within an uncertain dynamic and competitive framework in monopoly and duopoly settings are also a topic of deliberations by Huisman and Kort (2015). Chevalier-Roignant and Trigeorgis (2011) provide original theoretical discrete and continuous time frameworks to analyze firms' strategic decisions, as well as quantitative guidance for firms. A comprehensive overview of real options games models is offered by Azevedo and Paxson (2014); they also study the leader-follower non-preemption duopoly market model (2018), where they analyze a combined effect of uncertainty and competition on the timing optimization of investments in complementary inputs. Competition and optimal entries into an asymmetric duopoly are studied by Leung and Kwok (2016). An input spillover and a hazard rate of arrival of innovation are important factors in their model. It is worth mentioning that under preemptive equilibrium, real options values are reduced by the fear of preemption. Investment strategies under competitive interactions in an oligopolistic market structure are studied by Arasteh (2017). In his model, as in the papers mentioned above, the threat of a competitor entering a market encourages other firms to speed up the execution of their investment options, destroying the option value of waiting.

In the course of their continuous research on the impact exerted by market competition on the investment option value and the decisions of competitors, Hellmann and Thijssen (2018) pose a crucial question about the motives for decision-making on a competitive market: is it the fear of the market or rather the fear of rival companies? The measure of both "fears" is the expected growth rate of the firm's future cash flows. The authors look for an answer (formally) in balancing the two motives. They also investigate the impact of ambiguity on equilibrium investment behavior.

Trigeorgis and Baldi (2017) propose a different approach to describe a game between market players. They consider real options games and strategies enabling a firm to switch between compete, cooperate, or wait modes in dealing with a patent. They apply normal form games and the Nash equilibrium to offer a solution to games. The bargaining perspective—in combination with real options—is described in the paper by Trigeorgis et al. (2019). They analyze the key factors affecting the allocation of value between the licensor and licensee in the Biopharmaceutical Industry. But there is, however, no general theoretical approach to the bargaining problem in the real options games in their paper. The bargaining problem, so important nowadays, is rarely debated in the articles on the real options games.

Therefore, our paper is part of the research effort devoted to analyzing behaviors of companies on the competitive market in terms of real options games. However, its main part concentrates on the bargaining problem. On the one hand, bargaining is a task for managers; on the other—providing managers with guidance resulting from research may be challenging for academics. So the main challenges in the paper are the following: (1) to describe, in the form of a real options game, a situation in which two firms (with unequal market share) face a new investment opportunity and to specify the types of potential games between them, (2) to consider possible investment strategies (invest, wait or abandon) and indicate ramifications of the choice made for the firms (players), (3) to present proposals of tools to support bargaining (the non-symmetric Nash bargaining solution and the cooperative-competitive value) and compare the companies' benefits resulting from the use of these tools. We also analyze how the features of the investment project (its risk) and size of companies (their market share) affect the type of game between the companies and, on this basis, offer recommendations to players. The paper contributes to the existing literature on real options and bargaining games by combining these two areas to arrive at a more suitable analysis of relationship between the firms that share an investment option.

Given the model formulation, the study closest to this paper is the one by Trigeorgis and Baldi (2017), but there are some essential differences between the Trigeorgis and Baldi's model concept and ours. They consider games between firms in the end-of-period nodes of the binominal tree; the payoffs are multiplied by the respective probabilities and discounted back at the riskless rate. By contrast, we are going to consider a game at its initial moment and payoffs constituted at this moment. Furthermore, for a common investment opportunity (a shared option) we will assume that only the project benefits can be shared between firms but not expenditures (the authors quoted above assume the division of the entire option value). Additionally, we analyze a general range of present values of an investment project. This enables us to find answers to our research questions.

It is worth emphasizing that in this article, unlike in other works, we consider the relationship between model parameters (see formula (1)). This is of particular importance when it comes to the impact of project risk on firms' payoffs, and thus, on the form of the game, its solution and recommendations offered to players. The use of formula (1) means that for realistic parameter values ($\beta > 0$), an increase in project risk causes both an increase in the value of an investment option, and on the other hand, a decrease in the NPV of the project (by increasing the interest rate to discount project cash flows).

The remaining parts of the paper cover the following content: Section 2 provides a theoretical introduction to the real options bargaining games, such as: assumptions about an investment project and relationships between model parameters, firms' payments, a general form of a game, and concepts of game solutions; Section 3 covers the types of real options bargaining games and it proposes solutions to them. In Section 4, based on a numerical example, we discuss what conditions are required for each game to gain importance. Section 5 offers a glimpse into the actual business practice. Section 6 is a conclusion.

## 2. Theoretical introduction to the real options bargaining games

### *2.1. The model*

In order to keep the game simple, but focused on the crucial issues, a duopoly case is considered. Two risk-neutral firms (A and B) operate on a competitive market and divide it between them into two unequal parts. At time $t = 0$ both competitors identify the same investment opportunity considered as a shared investment real option (Smit and Trigeorgis 2004); this investment opportunity is available to them for one period up to $T$. The project requires from each of the parties an irreversible investment outlay $I, I > 0$ and the lifetime of the investment project is infinite.

The investment project generates cash flows ($Y_t$), which evolve in accordance with the geometric Brownian motion, with drift $\alpha$, $\alpha > 0$ and volatility $\sigma$, $\sigma > 0$. A risk-free asset yields a constant rate of return $r$, is a convenience yield (>0) and it reflects an opportunity cost of delaying construction of the project and keeping the option to invest alive instead (Dixit and Pindyck, 1994). The present value of the project is determined by the discounting and accumulating of its future cash flows. It is equal to $V(Y_0) = \frac{Y_0}{\delta}, Y_0 > 0$ (Dixit and Pindyck, 1994).

To analyze the model, we should establish links between model parameters. According to (Dixit and Pindyck, 1994), we assume that the total expected rate of return on owning a completed project is the sum of the expected percentage rate of growth of $Y_t$ ($\alpha$) and the convenience yield ($\delta$), and it is equal to the expected rate of return on a financial asset (non-dividend paying) perfectly correlated with $Y_t$ (according to CAPM), so:

$$\alpha + \delta = r + (r_m - r) \cdot \beta, \quad (1)$$

where, additionally: $r_m$ is the expected return on the market and the coefficient $\beta$ indicates whether the asset is more ($\beta > 1$) or less ($0 < \beta < 1$) volatile than the market, $\beta = \frac{\sigma \cdot \rho_m}{\sigma_m}$ (where $\rho_m$ is the correlation of the asset with the market portfolio and $\sigma_m$ is the standard deviation of $r_m$).

In the face of an investment opportunity ($t = 0$), a firm has three possible decisions to choose from: it can invest immediately, wait to collect more information from the market and thereby reduce the risk of failure, or it can abandon the investment project altogether. The conventional concept of real options required that before an investment decision is taken a comparison of the value of the investment option with the project net present value (the NPV) should be made. However, if the investment option is a shared one, the firm should take into account how its decision influences its competitor's decision, and how such firm itself may be impacted by the rival's reactions. Therefore, both firms' strategic choices may be described as a non-zero sum game and their payoffs in this game are as follows (Rychłowska-Musiał, 2018):

1. Firms A and B invest immediately and simultaneously. They share the project benefits correspondingly to their market shares; the payoff for either of the firms is the net present value of the project:

$$NPV_0^A := NPV(u \cdot Y_t)|_{t=0} = V(u \cdot Y_0) - I \quad (2)$$

$$NPV_0^B := NPV((1-u) \cdot Y_t)|_{t=0} = V((1-u) \cdot Y_0) - I. \quad (3)$$

where: $u$—firm A's market share, $0.5 \le u < 1$ (firm A dominates a market), firm B is left with $(1-u)$ share.

2. Firms A and B defer and keep the investment option. The payoff for either of them is the call option value calculated based on the Black-Scholes-Merton model (the underlying asset is the project present value determined with an appropriate part of project benefits ($u \cdot Y_t$ for firm A or $(1-u) \cdot Y_t$ for firm B), and the exercise price is the investment expenditure $I$):

$$F_0^A := F(u \cdot Y_t)|_{t=0} \quad (4)$$

$$F_0^B := F((1-u) \cdot Y_t)|_{t=0} \quad (5)$$

3. While only one firm decides to invest, it corners the whole market and the firm that is deferring the investment loses its market share towards the investing firm. The investing firm's payoff is the project net present value:

$$NPV_0 := NPV(Y_t)|_{t=0} = V(Y_0) - I \quad (6)$$

and the payoff of the deferring firm is zero. It is forced to abandon the investment project.

### 2.2. A payoff matrix

For various initial values of the cash flows generated by the project $Y_0 > 0$ (and hence for various present values of the project $V_0 > 0$) we can identify four types of relationships between possible payments: the project net present value and the investment option value. These are:

$$I_x. \quad NPV_0^x < NPV_0 < F_0^x, F_0^x > 0 \tag{7}$$

$$II_x. \; NPV_0^x < 0 < F_0^x < NPV_0 \tag{8}$$

$$III_x. \; 0 < NPV_0^x < F_0^x < NPV_0 \tag{9}$$

$$IV_x. \; 0 < F_0^x < NPV_0^x < NPV_0 \tag{10}$$

where $x = A$ (firm $A$) or $x = B$ (firm $B$).

Depending on the area that the value of the project for the company belongs to, we observe different types of games between companies. Even so, these games have one general standard form (presented in Table 1).

**Table 1.** Payoff matrix of the game between competing firms.

| | | firm $B$ | |
|---|---|---|---|
| | | *WAIT (W)* | *INVEST (I)* |
| firm $A$ | *WAIT (W)* | $(F_0^A; F_0^B)$ | $(0;\ NPV_0)$ |
| | *INVEST (I)* | $(NPV_0; 0)$ | $(NPV_0^A;\ NPV_0^B)$ |

We are going to identify the bargaining games that occur between competitors. We would like to work out a solution concept for each game (i.e. a rule used to predict how a game would be played by rational players) and formulate recommendations to firms. The most commonly used solution concept is the Nash equilibrium: no player has anything to gain by changing only its own strategy. If the other player is rational, it is reasonable for each of them to expect its opponent to follow the NE recommendation as well (Watson, 2013, 82). So, we are going to determine a dominant strategy (if one exists) for each player in each game and to indicate the Nash equilibria. However, it is a well-known fact that the NE may—but does not need to—give players the highest possible payoffs. In these cases, firms could consider negotiations as a way to achieve better results and the game becomes a bargaining one.

Various concepts have been proposed to find a reasonable and fair solution to a bargaining game. We are going to analyze and compare two interesting approaches on how to determine a solution to a bargaining real options game: the non-symmetric Nash bargaining solution and the cooperative-competitive value.

*2.3. Concepts of solutions*

In terms of arbitrating the game, the common approach is the Nash arbitration scheme and the Nash bargaining solution (the NBS). It is a unique solution to a two-person bargaining game that satisfies four axioms: rationality, linear invariance, symmetry, and independence of irrelevant alternatives (Nash 1950). It should maximize the product:

$$\max(x - x_{SQ})\,(y - y_{SQ}) \tag{11}$$

where $x \geq x_{SQ}$ and $y \geq y_{SQ}$ and $SQ(x_{SQ}, y_{SQ})$. $SQ$ is the *status quo* point, which means that payoffs are obtained if one decides not to bargain with the other player. The $SQ$ point can be found in several ways. One possibility is to assume that when there is no agreement among players, they can assure security at their minimum levels. Another proposition is to assume the lowest possible payments for both parties as the $SQ$ point. (Straffin, 1993).

In the original NBS, both negotiators have equal rights in negotiations. But if there is a market power asymmetry between companies, their negotiation positions are not the same. Kalai (1977) introduced generalization of the Nash bargaining solution by a measure of the relative bargaining power of the two players. In this proposition, players' negotiation positions may be non-symmetric, so it is the non-symmetric Nash bargaining solution (the NSNBS). The maximized product has taken the following form:

$$\max(x - x_{SQ})^{\eta}\,(y - y_{SQ})^{1-\eta} \tag{12}$$

where $\eta \in < 0{,}1 >$ and it is the relative bargaining power of the firm A. If $\eta = 0.5$ the NSNBS is equal to the NBS. Firms' payoffs will be denoted as $(Payoff_{NS}^{A}, Payoff_{NS}^{B})$.

The cooperative-competitive value (the coco value) may be an interesting alternative to the bargaining game's solution (Kalai and Kalai, 2013). The calculation of the coco value relies on the natural decomposition of a strategic game into two component games. If $(X, Y)$ are payoff matrices for a (complete information) game, the decomposition has the following form:

$$(X, Y) = \left(\frac{X+Y}{2}, \frac{X+Y}{2}\right) + \left(X - \frac{X+Y}{2}, Y - \frac{X+Y}{2}\right) \tag{13}$$

The first term is the cooperative component, the highest possible payoffs that the players can mutually arrange under the agreement to share their payoffs equally. The second term is the competitive component that recognizes the firms' strategic positions.

The coco value is the sum of the *maxmax* payoff for the cooperative team game (equal for either players) and the *minmax* payoff for the competitive game (the value of the zero-sum game), which is an adjustment compensating transfer from the strategically weaker player to the stronger one (Kalai and Kalai, 2009).

$$coco\text{–}\,value\ (X, Y) \equiv maxmax\left(\frac{X+Y}{2}, \frac{X+Y}{2}\right) + minmax\left(\frac{X-Y}{2}, \frac{Y-X}{2}\right) = (Payoff_{coco}^{A}, Payoff_{coco}^{B}) \tag{14}$$

The *coco* decomposition of the game from Table 1 is presented in Table 2.

**Table 2.** The *coco* decomposition of a game from Table 1.

| | |
|---|---|
| Cooperative component | |
| $\left(\frac{F_0^A + F_0^B}{2}, \frac{F_0^A + F_0^B}{2}\right)$ | $\left(\frac{NPV_0}{2}, \frac{NPV_0}{2}\right)$ |
| $\left(\frac{NPV_0}{2}, \frac{NPV_0}{2}\right)$ | $\left(\frac{NPV_0^A + NPV_0^B}{2}, \frac{NPV_0^A + NPV_0^B}{2}\right)$ |
| Competitive component | |
| $\left(\frac{F_0^A - F_0^B}{2}, -\frac{F_0^A - F_0^B}{2}\right)$ | $\left(-\frac{NPV_0}{2}, \frac{NPV_0}{2}\right)$ |
| $\left(\frac{NPV_0}{2}, -\frac{NPV_0}{2}\right)$ | $\left(\frac{NPV_0^A - NPV_0^B}{2}, \frac{NPV_0^B - NPV_0^A}{2}\right)$ |

## 3. Types of the real options bargaining games and their solutions

Let us notice that when the project value is very low and when inequalities for ranges $I_A$ (firm A) and $I_B$(firm B) are fulfilled for both firms, the solution to the game is very easy to find and there is no need to bargain. The investment option values exceed the net present values whenever there is only one investor, or two investors, on the market. In this case waiting is the dominant strategy for both competitors; it is the optimal decision for both of them.

Also, when the project value is very high and inequalities for ranges $IV_A$ (firm A) and $IV_B$ (firm B) are fulfilled for both firms, the game solution is indisputable, and the firms can find it without bargaining. The project net present values (in the case of simultaneous investment) exceed the investment option values for both competitors. In this scenario, instantaneous investment is the dominant strategy and optimal decision for both firms.

However, for a wide range of project values between these extreme intervals we can observe interesting games.

### *3.1. Game 1: "Bargaining chip of a weaker firm"*

When assumptions in the ranges $I_A$ for firm A and $II_B$ for firm B are met, then in the case of the stronger firm A, the value of the investment option is higher than the net present value of the whole project (for the only investor) ($NPV_0 < F_0^A$). At the same time, however, in the case of the weaker firm B, the value of the investment option is lower than the net present value of the whole project (for the only investor) ($F_0^B < NPV_0$). Under these assumptions, the game described in Table 1 is a motivating situation for the weaker firm B and a thought-provoking one for the stronger firm A.

Firm A has a dominant strategy—*Wait*, so if it is a rational player, it delays its investment decision. Firm B has no dominant strategy, but under the assumption of common knowledge, it seeks its best response to the A's dominant strategy *Wait*. It turns to be the *Invest* strategy and the strategy profile $(W; I)$ is the Nash equilibrium. Therefore, firm B invests and corners the market.

The assumption of common knowledge also means that firm A anticipates firm B's decision. Hence, there arises a problem of finding a way to arbitrate the game. The argument in these negotiations is the fact that if both firms invest, they will both suffer losses. However, A's loss is smaller than that of B's. Therefore, the firms ought to undertake negotiations regarding the delay of investment while the initiating party should be firm A.

It is worth mentioning that the A's dominant strategy occurs only if the project net present value for firm A when both firms invest is negative ($NPV_0^A < 0$). Otherwise there is no dominant strategy in game 1. This does not, however, affect the form of solutions.

As it has been found above, we are going to propose and compare two approaches to determine a reasonable and fair solution to a bargaining real options game: the non-symmetric Nash bargaining solution (the NSNBS) and the cooperative-competitive value (the coco value).

We start with the Nash bargaining scheme. The security levels of both players are (0,0) so the *status quo* point is $SQ(0{,}0)$. Then we maximize the formula (12) under the following condition: $y = \frac{F_0^B - NPV_0}{F_0^A} \cdot x + NPV_0$, where $0 \leq x \leq F_0^A$ and $0 \leq y \leq F_0^B$.

The non-symmetric Nash arbitrated solution turns out to be $p \cdot (W, W) + (1 - p) \cdot (W, I)$ and it provides the following payoffs:

$$Payoff_{NS}^A = p \cdot F_0^A + (1 - p) \cdot 0 \tag{15}$$

$$Payoff_{NS}^B = p \cdot F_0^B + (1 - p) \cdot NPV_0. \tag{16}$$

This means that we can recommend to the competitors that they play the strategy $(W, W)$ with probability $p$ and the strategy $(W, I)$ with probability $1 - p$. The outcomes are determined as expected values.

Obviously for different $SQ$ points there will be other solutions and recommendations. If $SQ$ is $(NPV_0^A, NPV_0^B)$, i.e. the lowest possible payments if both parties invest, the NSNBS is $(W, W)$ and it provides the following payoffs:

$$Payoff_{NS}^A = F_0^A \tag{17}$$

$$Payoff_{NS}^B = F_0^B. \tag{18}$$

However, these recommendations may turn out to be difficult to put into practice, especially in real options games, when firms have to choose only one strategy once. The Nash bargaining solution formulates recommendations as mixed strategies and payoffs as expected values. Moreover, in this game, there is no incentive to force the weaker firm to refrain from investing.

The coco-value calculation requires decomposition of a game into two parts: a cooperative component and a competitive one (see Table 2). The coco value is the sum of the *maxmax* solution to the cooperative game and the value of the competitive zero-sum game.

The *maxmax* solution to cooperative matrix is $\left(\frac{F_0^A+F_0^B}{2}, \frac{F_0^A+F_0^B}{2}\right)$ and the value of the zero-sum competitive game, which we have to add (or subtract) to these values, is $\left(q \cdot \frac{F_0^A-F_0^B}{2} + (1-q) \cdot \left(-\frac{NPV_0}{2}\right)\right)$, where $q$ is the probability of playing *the Wait* strategy in this game).

Based on (14) we receive for Game 1:

$$Payoff_{coco}^A = \frac{F_0^A+F_0^B}{2} + q \cdot \frac{F_0^A-F_0^B}{2} + (1-q) \cdot \left(-\frac{NPV_0}{2}\right) \tag{19}$$

$$Payoff_{coco}^B = \frac{F_0^A+F_0^B}{2} - q \cdot \frac{F_0^A-F_0^B}{2} - (1-q) \cdot \left(-\frac{NPV_0}{2}\right) \tag{20}$$

The recommendation to the competitors is to play strategy $(W, W)$ with payoff transfer of $(F_0^A - Payoff_{coco}^A)$ from firm A to firm B.

However, when assumptions in the ranges $I_A$ for firm A and $II_B$ for firm B are met, the coco payoff for firm B is smaller than the payoff that firm B could obtain as its best response to the A's dominant strategy ($Payoff_{coco}^B \leq NPV_0$). On the surface, it may seem strange that firm B would be willing to accept the lower $Payoff_{coco}^B$ instead of the higher payoff $NPV_0$. The reason is simple: the difference $(NPV_0 - Payoff_{coco}^B)$ is the price (an opportunity cost) to be paid by firm B to secure against loss ($NPV_0^B < 0$), which could hit firm B hard if both firms play the *Invest* strategies.

The payoff transfer $(F_0^A - Payoff_{coco}^A)$ is the price (a real cost) firm A is to pay firm B to ensure that firm B will delay the investment decision.

### *3.2. Game 2: "Asymmetric chicken game"*

When the project net present values for the two investing firms are negative ($NPV_0^A < 0$ and $NPV_0^B < 0$) and for both competitors the investment option values are lower than the net present value of the whole project (for the only investor) ($F_0^A < NPV_0$ and $F_0^B < NPV_0$), it means that the assumptions in the ranges $II_A$ for firm A and $II_B$ for firm B are met and the game described in Table 1 creates a very difficult situation for the two entities.

The game in this case has no dominant strategy for any player. However, there are two pure non-equivalent and non-interchangeable equilibria of the game: $(W, I)$ and $(I, W)$, as well as a mixed strategy equilibrium where each player waits with probability $p$ $(p \cdot W, (1-p) \cdot I;\ p \cdot W, (1-p) \cdot I)$. The decisions taken by competitors without any coordination may lead to a strategy profile $(I, I)$, which gives them both the worst possible payments.

This result indicates that negotiations can be a good idea in this case.

Support for negotiations can be: the Nash bargaining solution and the cooperative-competitive value.

In this case, the security levels of both players are (0,0) and the *status quo* point is $SQ(0,0)$, just as in the previous case. Then we have to maximize the formula (12) under condition: $y = -x + NPV_0$, where $0 \leq x \leq NPV_0$ and $0 \leq y \leq NPV_0$.

The non-symmetric Nash arbitrated solution turns out to be $p \cdot (I, W) + (1-p) \cdot (W, I)$ and it provides the following payoffs:

$$Payoff_{NS}^{A} = p \cdot NPV_0 \tag{21}$$

$$Payoff_{NS}^{B} = (1-p) \cdot NPV_0 \tag{22}$$

This means that we could recommend that the two players should play strategy $(I, W)$ with probability $p$ and strategy $(W, I)$ with probability $1-p$. The outcomes are determined as expected values.

If $SQ$ is $(NPV_0^A, NPV_0^B)$, the lowest possible payments if both parties invest, namely the NSNBS, is $(I, W)$ and it provides the following payoffs:

$$Payoff_{NS}^{A} = NPV_0 \tag{23}$$

$$Payoff_{NS}^{B} = 0 \tag{24}$$

As we know, both these recommendations may turn out to be difficult to put into practice. It is hard to indicate an incentive that could induce the weaker firm not to invest.

To calculate the coco-value of Game 2 we need to decompose the game into two parts: cooperative and competitive components (see Table 2).

The cooperative payoffs are the *maxmax* solution $\left(\frac{NPV_0}{2}, \frac{NPV_0}{2}\right)$ to the cooperative matrix and the min-max value of the competitive matrix, the classic solution to the zero-sum game is $min\left\{\frac{NPV_0^A - NPV_0^B}{2}, \frac{NPV_0}{2}\right\}$.

Thus, the coco-value (see: formula (14)) of Game 2 provides the following payoffs:

$$Payoff_{coco}^{A} = \frac{NPV_0}{2} + min\left\{\frac{NPV_0^A - NPV_0^B}{2}, \frac{NPV_0}{2}\right\} \tag{25}$$

$$Payoff_{coco}^{B} = \frac{NPV_0}{2} - min\left\{\frac{NPV_0^A - NPV_0^B}{2}, \frac{NPV_0}{2}\right\} \tag{26}$$

It means that the best recommendation to the competitors is to play strategy $(I, W)$ with (when $NPV_0^B \geq -\frac{I}{2}$) or without (when $NPV_0^B < -\frac{I}{2}$) payoff transfer from firm A to firm B. The strategic position of firm A is stronger than the strategic position of firm B. Firm B has not much to offer when it comes to the very low project net present value for firm B ($NPV_0^B < -\frac{I}{2}$) when negotiations on delaying the investment decision are open. In this case, firm A should invest, and firm B is forced to abandon the project. When the project net present value for firm B is not low ($NPV_0^B \geq -\frac{I}{2}$), it may be a party in negotiations and can expect compensation for delaying the investment decision.

### 3.3. *Game 3: "Asymmetric prisoner's dilemma"*

One more case in which negotiations are advisable when the interest of both competitors is at stake is as follows: the project net present values when both firms invest are positive, but lower than the investment options values (the range $III_A$ for firm A and $III_B$ for firm B).

When assumptions in the ranges $III_A$ for firm A and $III_B$ for firm B are met, the game described in Table 1 is the prisoner's dilemma type of game.

The game has one dominant strategy – *Invest*, and only one Nash equilibrium—the strategy profile $(I, I)$. In this case, simultaneous investing does not result in losses, but nonetheless, both companies would benefit from delaying the investment decision and observing the market. The payoff amounts in the strategy profile $(W; W)$ are higher than in the profile $(I, I)$.

As for the prisoner's dilemma, the security levels are $(NPV_0^A, NPV_0^B)$ (simultaneously, they are the worst possible payments) and the *status quo* point is $SQ(NPV_0^A, NPV_0^B)$. The formula (12) is then maximized under condition: $y = -x + NPV_0$, where $NPV_0^A \leq x \leq NPV_0$ and $NPV_0^B \leq y \leq NPV_0$.

The NSNBS is the strategy profile $p \cdot (I, W) + (1 - p) \cdot (W, I)$ and it provides the following payoffs:

$$Payoff_{NS}^A = p \cdot NPV_0 \tag{27}$$

$$Payoff_{NS}^B = (1 - p) \cdot NPV_0 \tag{28}$$

This means that we could recommend that the two players should play strategy $(I, W)$ with probability $p$ and strategy $(W, I)$ with probability $1 - p$. The outcomes are determined as expected values.

The coco-decomposition of Game 3 into two parts, the cooperative and competitive component, are presented in Table 2.

The cooperative payoffs are the *maxmax* solution $\left(\frac{NPV_0}{2}, \frac{NPV_0}{2}\right)$ to the cooperative matrix. Obviously, these equal payments have to be adjusted in order to take into account the strategic positions of firms. So, we add (or subtract) the competitive payoffs $\left(\frac{NPV_0^A - NPV_0^B}{2}\right)$ which are the min-max value of the competitive matrix, the classic solution to the zero-sum game.

The coco-value (see: formula (14)) of the Game 3 provides payoffs:

$$Payoff_{coco}^A = \frac{NPV_0}{2} + \frac{NPV_0^A - NPV_0^B}{2} \tag{30}$$

$$Payoff_{coco}^B = \frac{NPV_0}{2} - \frac{NPV_0^A - NPV_0^B}{2} \tag{31}$$

It means that the best recommendation to the competitors is to play strategy $(I, W)$

with payoff transfer of $NPV_0 - Payoff_{coco}^A$ from firm A to firm B, or to play strategy $(W, I)$ with payoff transfer of $NPV_0 - Payoff_{coco}^B$ from firm B to firm A (although it seems less likely when firm A has a dominant position on the market).

What is the conclusion? The simultaneous implementation of the project by both firms independently is not fortunate. Firms should cooperate in the implementation of the project. In the real economy, payoff transfer might not necessarily mean a cash transfer, but, for example, share of the profits in return for cooperation.

What is the significance of values of the investment (real) options in this analysis?

Let us look at the relationship between the value of the immediate project implementation $(NPV_0^x)$, and the value of the investment option $(F_0^x)$ for each of the companies $(x = A, B)$.

In both cases, we observe $NPV_0^x < F_0^x$ (regardless of whether $NPV_0^x$ is positive or negative). It would mean that if the impact of the competitor's decision is neglected, the companies should delay the project implementation and keep the real options open. However, on a competitive market the optimal strategy is a bit more complicated. We can find, as in previous papers (Smit and Ankum 1993, Leung and Kwok 2016, Arasteh 2017), that the fear of being preempted by competitors reduces the value of payments for companies through non-optimal decisions. Our findings show that both players can receive the highest possible payouts as a solution to the real options bargaining game. Taking into account payoff transfers determined as the bargaining solution can mitigate the negative impact that the irrational decisions exert on the firms' payments.

## 4. Numerical example and the impact of model parameters

So far, we have discussed general forms of three games and their solutions. Now we would like to answer a few questions: under which conditions does each game matter? Which game is more likely in certain situations and what will be the suggested solution to it? What conclusions and recommendations are important to large corporations dominating the market, and what are important to companies that are fighting for their piece of cake?

To analyze games and find their solutions numerically, let us assume a basic set of parameters given in Table 3.

Note that when $\sigma \geq 30\%$ and other parameters are from the base set, then the coefficient $\beta$ ($\beta = \frac{\sigma \cdot \rho_m}{\sigma_m}$) is greater than 1, which means the project is more volatile than the market.

Let us now analyze which games considered before are relevant for various project values, project risk, and different market share of the competing firms.

**Table 3.** The base set of parameters.

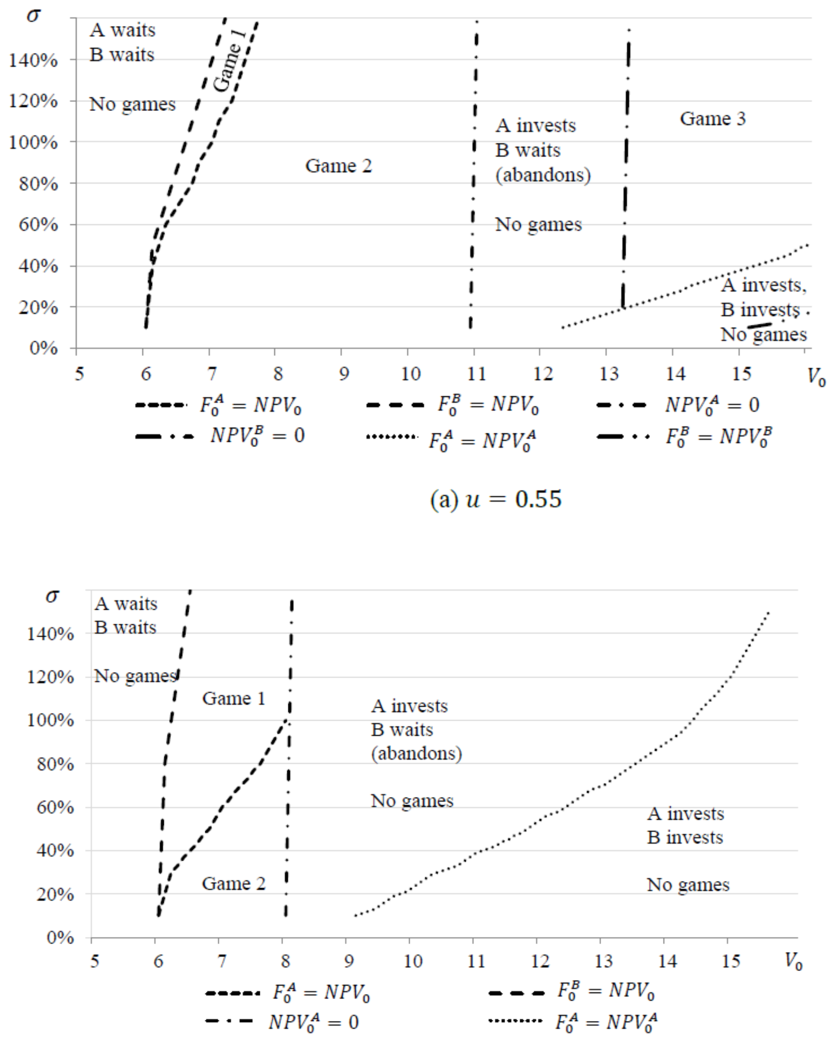
| Parameter | Description | Value |
|---|---|---|
| $\boldsymbol{I}$ | investment expenditure (in monetary unit) | 6 |
| $\boldsymbol{T}$ | expiration date (years) | 2 |
| $\boldsymbol{r}$ | risk free rate (i.e. *YTM* of treasury bonds with maturity date equal to the expiration date of investment option) | 1.96%[a] |
| $\boldsymbol{\alpha}$ | expected percentage rate of change of project cash flows (expert prediction) | 1% |
| $\boldsymbol{\sigma}$ | volatility of the project benefits (calculated based on historical data of spanning asset or expert prediction) | $\langle 10\%, 160\% \rangle$ |
| $\boldsymbol{r_m}$ | expected return on the market (i.e. rate of return on market index) | 6.03%[b] |
| $\boldsymbol{\sigma_m}$ | standard deviation of $r_m$ | 14.65%[b] |
| $\boldsymbol{\rho_m}$ | correlation of the asset with the market portfolio (expert calculations) | 0.5 |
| $\boldsymbol{u}$ | market share of a dominant firm (firm A) | $\langle 0.5, 1)$ |
| $\boldsymbol{\eta}$ | relative bargaining power of a dominant firm (firm A) | $= u$ |

Note: Source: author's own; [a] YTM of 2-year treasury bonds; [b] WIG 2012–2016, data from stooq. Assumptions about the parameters reflect a situation of a real company (a similar approach is used by authors of the cited papers).

*4.1. Game 1: "Bargaining chip of a weaker firm"*

It is obvious that for symmetric firms games of this type do not occur. As the degree of asymmetry between companies increases, the importance of this game is growing. This means that the range of project values for which we observe the game of this type becomes wider (Figure 1a,b).

The impact of project risk is similar. The game of this type does not occur for low-risk projects. When the risk of a project increases, the game appears for a wider range of project values (Figure 1a, b).




(b) $u = 0.75$

**Figure 1.** The impact of the project risk on firms' investment strategies (and the game type if it occurs) for a wide range of project values. Note: Panel (a) small asymmetry in the market share of firms ($u = 0.55$), panel (b) large asymmetry in the market share of firms ($u = 0.75$). In the panel (b), lines $F_0^B = NPV_0^B$ and $NPV_0^B = 0$ are outside the graph area.

Let us now analyze the recommended solutions (see Table 4). Of the two NSNBS proposals, the bargaining solution in which the $SQ$ point is the worst possible payoff is the most advantageous for the dominant company. However, for the weaker company, the most advantageous solution is that one in which the $SQ$ point matches the security levels. In such case, the compromise seems to be the strategy proposed as the coco solution.

Besides, in the coco solution, the larger the asymmetry between firms, the greater the transfer of payment from firm A to firm B. It means that in the case of large asymmetry, the strategic importance of the weaker firm's decision is greater. A company with larger share is willing to pay more in order to ensure the execution of its optimal strategy.

**Table 4.** Game 1. Recommended strategies and firms' payments in the NSNBS (for two possible types of the *status quo* points) and in the coco solution for various market share ($u$) and project risk ($\sigma$).

| $\boldsymbol{u}$, $\boldsymbol{\eta}$ | $\boldsymbol{\sigma}$ | Non-Symmetric Nash Bargaining Solution (NSNBS) | | Cooperative-Competitive (coco) solution |
|---|---|---|---|---|
| | | $SQ$—security levels | $SQ$—the worst possible payments | |
| | | Impact of market share ($\boldsymbol{u}$) | | |
| 0.55 | 60% | $(W,W) = (0.37{,}0.22)$ | $(W,W) = (0.37{,}0.22)$ | $(W,W)$+payoff transfer= $(0.37{,}0.22) + (0.05,-0.05) = (0.42{,}0.17)$ |
| 0.65 | 60% | $(W,W) = (0.57{,}0.10)$ | $(W,W) = (0.57{,}0.10)$ | $(W,W)$+payoff transfer= $(0.57{,}0.10) + (-0.03, 0.03) = (0.54{,}0.13)$ |
| 0.75 | 60% | $0.85\,(W,W) + 0.15\,(W,I) = (0.68{,}0.063)$ | $(W,W) = (0.80{,}0.03)$ | $(W,W)$+payoff transfer= $(0.80{,}0.03) + (-0.068, 0.068) = (0.732{,}0.098)$ |
| 0.85 | 60% | $0.87\,(W,W) + 0.13\,(W,I)$ =$(0.928{,}0.038)$ | $(W,W) = (1.07{,}0.005)$ | $(W,W)$+payoff transfer= $(1.07{,}0.005) + (-0.099, 0.099) = (0.971{,}0.104)$ |
| 0.95 | 60% | $0.95\,(W,W) + 0.05\,(W,I)$ =$(1.292{,}0.013)$ | $(W,W) = (1.36, 0^{+})$ | $(W,W)$+payoff transfer= $(1.36, 0^{+}) + (-0.128{,}0.128) = (1.232{,}0.128)$ |
| | | Impact of project risk ($\boldsymbol{\sigma}$) | | |
| 0.75 | 80% | $(W,W) = (1.12{,}0.11)$ | $(W,W) = (1.12{,}0.11)$ | $(W,W)$+payoff transfer= $(1.12{,}0.11) + (-0.116, 0.116) = (1.004{,}0.226)$ |
| 0.75 | 100% | $(W,W)$ =$(1.39{,}0.2)$ | $(W,W) = (1.39{,}0.2)$ | $(W,W)$+payoff transfer= $(1.39{,}0.2) + (-0.157, 0.157) = (1.233{,}0.357)$ |

Note: The firm's market share $u$ is equal to its relative bargaining power $\eta$. Values were calculated for $V_0 = 6.25$.

It is a very strong signal to companies that dominate the market when, in the face of new investment opportunities, their market positions could be threatened by seemingly irrelevant players. Such players should be subject to a particularly insightful observation. (The drastic examples of a game of this type and the dramatic consequences of opting for the wrong strategy by the dominant player are Nokia and Kodak, as described below in this section).

At the same time, however, a very high project risk can reduce the foolhardiness of the weaker firm (in the recommended solution $p \cdot (W, W) + (1 - p) \cdot (W, I)$; with the higher project risk comes the higher $p$, which means the lower $1 - p$, probability that the weaker firm chooses strategy $I$). For a very high risk project, the NSNBS recommends strategy $(W, W)$. However, in the coco concept, high risk is also associated with high payoff transfer so that the strategy $(W, W)$ could be implemented. It is related to the fact that the increase in the risk of the project causes the increase in the value of the investment option and boosts the motivation to retain it.

To sum up, Game 1 ("Bargaining chip of a weaker firm") is of greatest importance when the asymmetry between the companies' market share is large, and the risk of the project is high. In particular, it should be given proper consideration by companies that dominate the market.

### *4.2. Game 2: "Asymmetric chicken game"*

Game 2 occurs between competitors with similar market share or not very strong asymmetry. When the asymmetry in market share increases, the range of the project values for which Game 2 occurs shrinks (Figure 1a,b). In the base case, if $u \geq 0.8$, Game 2 does not appear in its pure form.

The impact of project risk is similar. For low risk projects, Game 2 appears for a broad range of project values, but this area shrinks as the project risk grows. In the base case, if the project risk $\sigma \geq 100\%$, there is no Game 2 between the competitors. (Figure 1a,b).

What solutions are recommended to companies? Let us first notice that in this type of games, regardless of the size of the company's market share, the non-symmetric Nash arbitrated solution (the $SQ$ point—security levels) turns out to be the mixed strategy profile $p \cdot (I, W) + (1 - p) \cdot (W, I)$. The probability ($p$) of playing the strategy profile $(I, W)$ is the stronger the bigger the firm's market share and its bargaining power are (Table 5).

When the $SQ$ point is the worst possible payments, the Nash solution is strategy $(I, W)$. This solution (with the $SQ$ point as the worst possible payments) can be observed for any market share. Similarly, with greater asymmetry in market share, the coco-value recommends that the competitors play strategy $(I, W)$ without any payoff transfer. This is the madman's strategy described in the literature: firm A should make a commitment convincing firm B that it is going to choose the *Invest* strategy. If the firm market share is large, it can afford this strategy.

**Table 5.** Game 2. Recommended strategies and firms' payments in the NSNBS (for two possible types of the *status quo* points) and in the coco solution for various market share ($u$) and project risk ($\sigma$).

| $\boldsymbol{u}$, $\boldsymbol{\eta}$ | $\boldsymbol{\sigma}$ | Non-Symmetric Nash Bargaining Solution (NSNBS) | | Cooperative-Competitive (coco) solution |
|---|---|---|---|---|
| | | $SQ$—security levels | $SQ$—the worst possible payments | |
| | | Impact of market shares ($\boldsymbol{u}$) | | |
| 0.55 | 60% | $0.55\ (I, W) + 0.45\ (W, I) = (0.935{,}0.765)$ | $(I, W) = (1.7{,}0)$ | $(I, W)$+payoff transfer= $(1.7{,}0) + (-0.465{,}0.465) = (1.235{,}0.465)$ |
| 0.75 | 60% | $0.75\ (I, W) + 0.25\ (W, I) = (1.275{,}0.425)$ | $(I, W) = (1.7{,}0)$ | $(I, W) = (1.7{,}0)$ |

Note: The firm's market share $u$ is equal to its relative bargaining power $\eta$.Values were calculated for $V_0 = 7.7$.

With lower asymmetry, the coco concept recommends cooperation and implementation $(I, W)$ strategy with proportional payoff transfer (Table 5) [strategy $(W, I)$ with proportional payoff transfer is also acceptable, although unlikely]. Of course, the greater the asymmetry in market share is, the smaller the payoff transfer.

In Game 2, the change of the project risk level does not affect the recommended solutions.

To sum up, Game 2 ("Asymmetric chicken game") is of greatest importance when the asymmetry in the companies' market share is not very large, and the risk of the project is not very high.

### 4.3. *Game 3: "Asymmetric prisoner's dilemma"*

Game 3 occurs between competitors with a similar size of market share or with very small asymmetry between them. Moreover, when the asymmetry in market share increases, the range of the project values for which Game 3 occurs shrinks. In the base case, if $u > 0.6$, Game 3 does not appear (Figure 1a,b).

The impact of project risk is quite opposite. For low risk projects, there is no Game 3 between the competitors. In the base case, when the risk of the project $\sigma < 20\%$ , the game is played for a very narrow range of project values, or it is not played at all. But if the risk grows, the range of project values in which the game is played expands vastly (Figure 1a,b).

Does changing the degree of asymmetry between competitors or changing the project risk affect the recommended solutions? It appears that only to a small extent.

When firms are symmetric, the NBS turns out to be the strategy profile $0.5 \cdot\ (I, W) + 0.5 \cdot (W, I)$. In this type of game, the security levels are identical to the worst possible payments. The coco-value provides the same equal payoffs for both players as in the NS, but these payoffs are the consequence of playing strategy $(I, W)$ or $(W, I)$ with transfer of 0.5 payment from the investing player to the abstaining one.

When the asymmetry between the companies increases, the general profile of the recommended strategy is $p \cdot (I, W) + (1 - p) \cdot (W, I)$. At the same time, with the greater asymmetry in the firms' market share (their bargaining power), the probability of choosing strategy $(I, W)$ increases (see Table 6). Payments in the NSNBS and coco payoffs are equal.

**Table 6.** Game 3. Recommended strategies and firms' payments in the NSNBS and in the coco solution for various market share ($u$) and project risk ($\sigma$).

| $\boldsymbol{u}$, $\boldsymbol{\eta}$ | $\boldsymbol{\sigma}$ | Non-Symmetric Nash Bargaining Solution (NSNBS)<br>$SQ$—security levels = the worst possible payments | Cooperative-Competitive (coco) solution |
|---|---|---|---|
| Impact of market shares ($\boldsymbol{u}$) | | | |
| 0.50 | 60% | $0.5\ (I, W) + 0.5\ (W, I)$ =(4.5,4.5) | $(I, W)$+payoff transfer= $(9{,}0) + (-4.5, 4.5) = (4.5{,}4.5)$<br>Or<br>$(W, I)$+payoff transfer= $(0{,}9) + (4.5, -4.5) = (4.5{,}4.5)$ |
| 0.55 | 60% | $0.6\ (I, W) + 0.4\ (W, I)$ =(5.5,3.5) | $(I, W)$+payoff transfer= $(9{,}0) + (-3.75, 3.75) = (5.25{,}3.75)$ |
| 0.58 | 60% | $0.7\ (I, W) + 0.3\ (W, I)$ =(6.2,2.8) | $(I, W)$+payoff transfer= $(9{,}0) + (-3.3{,}3.3) = (5.7{,}3{,}3)$ |

Note: The firm's market share $u$ is equal to its relative bargaining power $\eta$. Values were calculated for $V_0 = 15$.

In Game 3, the project risk does not affect the strategies and amounts of payment. In this game, the NSNBS and the coco concept recommend cooperation in the project execution, while the distribution of profits (possible payoff transfer) depends on the firms' market share.

To sum up, Game 3 ("Asymmetric prisoner's dilemma") is of greatest importance when the companies' market share is almost equal, and the risk of the project is rather high.

## 5. A look at the business practice

We would like to finish this paper with business examples of companies with large market share that seemed to have been too big to fail – Nokia and Kodak. However, as a result of wrong strategic decisions, they were driven out of their markets. In both cases Game 1 can illustrate and offer some explanation to the stories of the fallen giants.

Nokia was the leader of the mobile telephony market at the turn of the centuries. But early in the first decade of the 21st century everything changed. The global smartphone market share held by Nokia was dwindling on a quarterly basis starting from 2007, while its competitors, Apple and Samsung, grew in importance. In Q2 2007, Nokia's market share was 50.8%; by Q2 2013 it dropped to a meager 3.1%; in Q4 2017 Nokia smartphones captured only 1% of the global market share. At the same time, Samsung's share increased from 3.3% at the end of 2009, to over 30% in 2012–2013 and reached a stable level of approx. 20% in 2015–2017. (Figure 2)

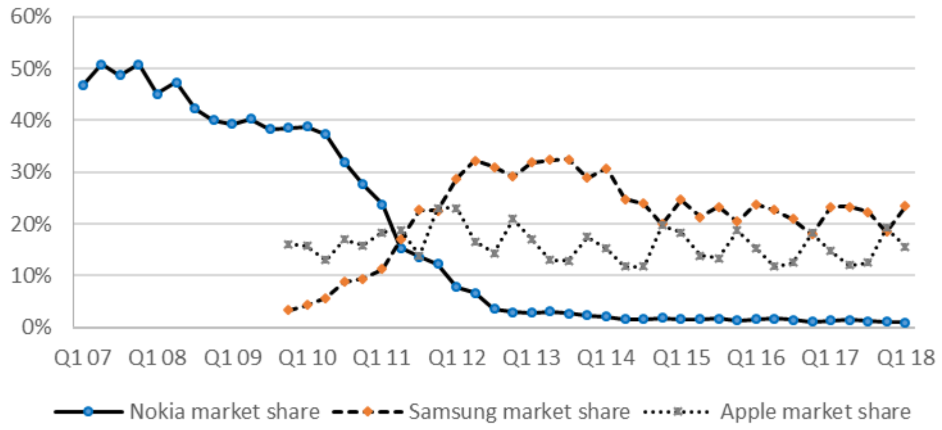



**Figure 2.** Global market share held by smartphones: Nokia 2007–2017, Apple 2009–2017, Samsung 2009–2017. Source: Author's own based on Statista.com/statistics.

Kodak was one of the greatest companies of the 20th century. The company's new technologies in the field of photography were unmatched. In the 20th century, almost 20,000 patents were awarded to Kodak engineers. In 1975, it was Kodak who created the first digital camera and developed this technology in super-professional photographic equipment and technical devices for medical and industrial applications. In 1976, Kodak had 90% of the movie market and 85% of the camera market. But the beginning of the 21st century brought a series of problems for the company: a drastic decline in profits, a drop in brand value, and a reduction in employment. Finally, in 2012, Kodak declared bankruptcy.

You can tell similar sad stories about Xerox, Atari and other former market giants.

The reasons for all these descends are clearly very complex, but a common feature can be indicated. Both Nokia and Kodak had an investment option to implement an innovative solution (a smartphone or a digital camera for mass-market customers). However, when strategic decisions were taken by the companies, they either failed to take into account the moves of its competitors or they took suboptimal decisions out of fear of failure. Nokia and Kodak were aware of the direction the market was going in, but the high risk of innovation made them wait. In 2007, the first iPhone was released by Steve Jobs. Similar solutions (based on other operating systems) were readily available to Nokia and Samsung (they had got a shared option). Samsung immediately joined the game in this market, but Nokia was convinced that the risk of failure was too high and the mass market for smartphones was a matter of distant future. Kodak developed its technology mostly for professional devices. The firm had the opportunity to release a digital camera to the mass market, but they considered the investment too risky and kept their investment option open. In the mid-1990s, Kodak's director predicted that analogue photography would not be surpassed by digital photography for the following 20 years. In both cases, excessive delays of the investment decision resulted in a loss of market share.

## 6. Final remarks

The real options approach (ROA) plays an important role among decision-support tools. The conditions in which this concept can be used have been thoroughly described—usually on a monopolistic market. However, as it has been shown in this paper, on the competitive market, where most of the identified real options are the shared ones, decisions made solely on the ROA basis can lead to catastrophic consequences.

If the firm's investment opportunity is an investment shared option, it has to take into account the impact exerted by its investment decision on its competitor, as well as the impact exerted by the rival's reactions. This situation can be seen as a real options game. In the paper, it is a case of two firms with unequal market share that face a new investment opportunity. Depending on the value of the project and its risk, we can identify different types of games between these companies and make different recommendations to them. In some cases (especially for very low or very high project present values), there is no problem with finding the solution to the game which would be fair and satisfying to both players. Optimal decisions are clear: wait (when project present values are very low) or invest (when project present values are very high). However, when project net present values for both competitors are slightly below or above zero, the game between competitors is the bargaining one. As in the previous papers, we note that the presence of competition reduces payoffs for firms (see, e.g., Smit and Ankum, 1993, Leung and Kwok, 2016, Arasteh, 2017). However, our findings show that payoff transfers determined as a result of the bargaining solution can mitigate the negative impact of firms' irrational decisions on their payments.

The decision to launch negotiations should be taken by the dominant party, case-by-case, based on the Nash bargaining solution or cooperative-competitive value of the real options game. However, the dominant party should pay particular attention to the behavior of its competitor when the value of the investment option suggests delaying an investment decision or when the risk of a project is high. If there is a large disproportion between firms' market share and an investment (shared) option involves a high risk project, the dominant firm is forced to negotiate with its weaker competitor. The dominant firm has to offer compensation to the weaker firm to compel it to implement a strategy that is beneficial to both of them. It is the game referred to in this paper as the "Bargaining chip of a weaker firm".

If there is a small disproportion between the firms' market share and an investment (shared) option involves a high risk project, the game known as the "Asymmetric prisoner's dilemma" may take place. The bargaining solution in this case suggests cooperation in the project execution while distribution of profits depends on the market share held by the firms.

If the project risk is not too high while the difference in firms' market share is substantial, the madman's strategy is available as the solution to the bargaining game. This game is referred to herein as the "Asymmetric chicken game".

The analysis presented in the paper offers some important findings, especially for market-dominating firms. The correct valuation of the project is important, the real options approach can be helpful, but only with the real options games, or even the real options bargaining games, we get a complete picture of the situation and can devise comprehensive strategies.

**Conflict of interest**

The author declares no conflicts of interest in this paper.

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