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Research article

Bitcoin-based triangular arbitrage with the Euro/U.S. dollar as a foreign futures hedge: modeling with a bivariate GARCH model

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Abstract: This paper proposes a bitcoin-based triangular arbitrage, combining foreign exchanges in the bitcoin market and reverse foreign exchange spot transactions. An FX futures contract is used to reduce exposure to risk as a hedging instrument. The returns of the portfolio are jointly modeled using a bivariate DCC-GARCH model with multivariate standardized student's t disturbances due to the presence of leptokurtosis and fat tails observed. Based on the time-dependent covariance matrix, a dynamic optimal hedge ratio is formed, with a conditional correlation series as a by-product. Empirical results are obtained using Euros and U.S. dollars over the period from 21 April 2014 to 21 September 2018. Multiple rolling one-step-ahead forecasts are generated. The empirical results present bitcoin-based currency strategies dominate bitcoin trading in terms of risk management.

Keywords: bitcoin; bitcoin exchange rate; triangular arbitrage; optimal hedge ratio; DCC-GARCH model

JEL Codes: G11, G12, G17

1. Introduction

Bitcoin, the first widely used cryptocurrency, was devised with the intent of becoming a virtual currency free of any financial authority to facilitate payments from peer to peer in its network (Nakamoto, 2008). Its decentralized architecture that differentiates bitcoin from e-commerce payment is seen by many as an innovation in the established money system (Böhme et al., 2015). On the other hand, it was soon discovered that those dealing in digital currencies were primarily interested in an alternative investment rather than an alternative transaction system (Glaser et al., 2014). Bitcoin investment shows both very high volatility and potentially high returns, and bitcoin can generally sever as an effective diversifier because of its weak positive correlations with other assets (Bouri et al., 2017; Briere et al., 2013; Yermack, 2013). The returns of bitcoin present significant time-series monument which can enhance the expected return, though its loop of continuations and reversals appears to be shorter than other assets (Hong, 2017). Bitcoin, as a unique investment asset, has experienced rapid growth and occupies a first market place among all cryptocurrencies, partially due to its first-mover advantage (Luther, 2016). Data from Blockchain.info show that the major market capitalization of bitcoin in USD increased from $0.26 million in August 2010 to $114.08 billion in September 2018, peaking at 323.07 billion on 17 December 2017.

However, bitcoin's two major roles, as a currency and as an investment asset, have been questioned by a number of researchers. As a currency, bitcoin needs to perform the three functions of money: It must serve as a unit of account, a medium of exchange, and a store of value. Yermack (2013) and Ali et al. (2014) posit that bitcoin perform poorly in all three of these functions: i) It is extremely inconvenient when people quote the price of a good or service in bitcoins due to all the decimal places in the price; ii) Continual changes in the bitcoin price necessitate the frequent updating of bitcoin quotations for a good or service; iii) The wide range of bitcoin prices indicates a large risk in holding bitcoins. Moreover, no broad use of bitcoins being seen in retail transactions also implies that the current usage of bitcoin does not meet its purpose as money (Hong, 2017). As a financial asset, bitcoin is expected to bring a positive risk-adjusted return by investors who employ a buy-and-hold strategy. Without any intrinsic value, bitcoin is said to carry a zero fundamental price and to have a substantial speculative bubble component in its price (Cheah and Fry, 2015). Dong and Dong (2015) point out that bitcoin investing has characteristics such as a liquidity discount, an unsystematic risk premium, and a low return that are associated with higher risk.

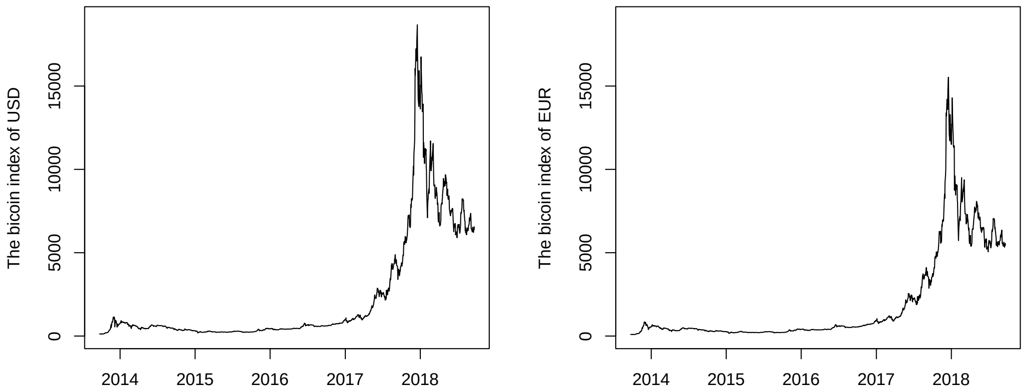

Figure 1. Bitcoin price indices of USD and EUR.

Though several factors contribute to bitcoin's immaturity as a currency and high risk as an investment, bitcoin's wildly fluctuating prices and excessive volatility appear to have a substantial influence. During our period of observation, from 21 April 2014 to 21 September 2018, the bitcoin price index in U.S. dollars ranged from $183.07 to $18,674.48; its price index in Euros varied between €156.97 and €15,528.90 (see Figure 1). Such a wide price variation brings the extreme risk for investors and a consequent increase in the cost of managing risk.

Bitcoin price is said to follow the law of supply and demand. However, because bitcoin supply is set by a rigid exogenous algorithm, bitcoin demand is the only market force determining the equilibrium price (Buchholz et al., 2012; Kristoufek, 2013; van Wijk, 2013). On the demand side, bitcoin's attractiveness as a medium of exchange, a store of value, and a favorable investment tool—or even fondness for its niche—is critical to determining its price. The alternation of positive and negative news has generated highly volatile price cycles (Ciaian et al., 2016).

The risk of bitcoin is difficult to handle. Bitcoin is only found to perform as a strong hedge and safe (hedging) heaven in very few cases (Bouri et al., 2017). Low correlation with other assets implies ineffective hedging to manage the risk of the portfolio (Yermack, 2013). The situation doesn't become better even after the bitcoin futures contracts were introduced by the Chicago Mercantile Exchange (CME) and the Chicago Board Options Exchange (CBOE) in December 2017. Corbet et al. (2018) show that the bitcoin futures cannot mitigate the pricing risk in the spot bitcoin market; reversely, hedging strategies constructed with them lead to an increase in volatility. Li et al. (2018) find that factors such as speculation, investor attention, and market interoperability affect the risk of bitcoin, but their effects present asymmetric features under different risk regimes. Accordingly, bitcoin risk management becomes complicated due to regime switching.

In short, direct bitcoin trading is facing problems such as wide price variation, excess volatility, and poor risk management. To have these problems addressed, this paper proposes a bitcoin-based triangular arbitrage strategy with the bitcoin market and the FX spot market. Since the underlying assets in this strategy are fiat currencies—bitcoins only act as a medium for exchange—the risks of holding bitcoin has been reduced exceedingly. Furthermore, it is possible that the risk of performing this triangular arbitrage can be reduced by using a hedging instrument. In fact, the FX futures contract well serves this purpose. So far, academic work on bitcoin-based foreign exchange is limited, and for the triangular arbitrage with the bitcoin and FX markets and its FX futures hedge, the results appear to be scarce.

Pichl and Kaizoji (2017) studied the arbitrage opportunities in Bitcoin prices in different currencies. In this paper we investigate the Bitcoin arbitrage opportunities in detail. For instance, a speculator, on 22 April 2014, used 100 Euros to buy 0.2755 units of a bitcoin according to the previous day's closing bitcoin price in EUR, and then he immediately sells the bitcoins at the previous day's closing price in USD for 138.6261 U.S. dollars. (Bitcoin prices on 21 April 2014 were retrieved from Bloomberg: USD/BTC = 503.18 and EUR/BTC = 363.00, where BTC denotes the bitcoin.) Suppose now that the user brings his/her 138.6261 U.S. dollars to the FX spot market and trades for Euros at the price of 1.3793 (USD/EUR). This reverse operation in the FX spot market produces 100.5048 Euros. The one-round return based on this triangular arbitrage is approximately (100.5048 – 100)/100 = 0.50%. In employing such a triangular arbitrage—exchange euros for bitcoins, sell the bitcoins for dollars, and covert the dollars back to euros, denoted EUR -> BTC -> USD -> EUR, speculators want to minimize their exposure to Euro exchange rate risk by going short in c unit Euros-worth of futures contracts. This issue is addressed by finding c that optimizes a specific utility function. The whole process can be repeatedly performed if there exists profitability.

The profitability of the proposed triangular arbitrage relies on the differential between the two USD/EUR exchange rates and the transaction fees. After this two-step transaction, 100 Euros has been exchanged for 138.6261 Euros at the rate implied by 138.6261/100 = 1.3863 USD/EUR. This cross rate between U.S. dollars and Euros formed in the bitcoin market is defined as the bitcoin exchange rate of USD/EUR (Nan and Kaizoji, 2018a). The corresponding spot FX rate (the spot rate) was 1.3793. The one-day spread between the two exchange rates was 0.0070, which takes account of 0.51% of the spot rate.

Dong and Dong (2015) found that there exists an opportunity for arbitrage by “spending bitcoin as currency” and that “arbitrage stickiness is persistent over time” potentially due to people’s preference for a buy-and-hold strategy with bitcoins. Nan and Kaizoji (2018a) found that the bitcoin exchange rate of USD/EUR is in long-run equilibrium with the spot rate of USD/EUR, but in the short run, an arbitrage opportunity may exist. The arbitrage paradox states that the market is efficient, yet a short-run arbitrage opportunity is simultaneously created when investors may not have sufficient incentives to observe the market (Grossman and Stiglitz, 1980; Akram et al., 2008).

Bitcoin’s low transaction cost, efficiency, and freedom are attractive features for the strategy of exchanging fiat currency in the bitcoin market. Nakamoto (2008) who claims to have created bitcoin wrote in a white paper that having a central authority working as a trusted third party “increases the transaction cost, limiting the minimum practical transaction size and cutting off possible small casual transactions.” In the bitcoin system, trust is replaced by cryptographic proof to keep transactions decentralized from any authority so that bitcoin trading can be conducted from peer to peer with greater efficiency and lower cost. Moreover, worldwide 24-7 bitcoin transactions provide both abundant liquidity and a broad variety of fiat currencies for currency exchanges.

Nan and Kaizoji (2018b) conducted a study of the bitcoin exchange rate and its optimal FX futures hedge. Three hedge ratios calculated by different approaches were compared in order to identify the optimal one. Results suggested that, in terms of hedging effectiveness, the time-varying conditional model was competitive with the naïve and conventional methods implemented in the static model. However, an arbitrage strategy that relies on the bitcoin exchange rate appears rather primitive due to the one-at-a-time nature of the unidirectional exchanges; for example, after trading Euros for U.S. dollars in the bitcoin market, speculators must wait for another arbitrage opportunity to change their dollars back to Euros. To do otherwise would mean facing the problem of a limited budget and the risk of holding dollars.

This study proposes a triangular arbitrage in which investors who sell Euros and buy U.S. dollars in the bitcoin market execute a reverse transaction in the FX spot market and compares triangular arbitrage with different assets from the bitcoin and FX markets. A time-dependent bivariate GACCH model is employed to measure the joint density of the two returns in the portfolio because of its performance on risk management. Based on the proposed model, three important time-varying series—the covariance matrix, optimal hedge ratio, and correlation—are forecast on a one-step-ahead basis.

The remainder of the paper is divided into two sections: Section 2 introduces the data set and explains the methodology; Section 3 gives empirical results and conclusions.

2. Date and methodology

The data set in this study consists of four daily closing prices series: (1) the USD bitcoin index, (2) the Euro bitcoin index, (3) the USD/EUR spot rate, and (4) the USD/EUR European-style FX monthly futures contract.

The data period extends from 21 April 2014 to 21 September 2018. For performing the triangular arbitrage, market liquidity is needed. The previous bitcoin trading is said to be dominated by few pioneer miners and traders for speculation purpose until that many individuals and institutional investors started to invest in bitcoins as early as late 2013 (Hong, 2017). Moreover, we are intended to avoid the turmoil period brought by Mt. Gox's bankruptcy. The company which once was handling over 70% of all bitcoin transactions worldwide, suspended trading in February 2014 and started liquidation proceedings in April 2014, which resulted in a 36% plunge in bitcoin prices.

After merging the data by date, we were left with 1111 observations for each series. The last 110 observations were reserved for out-sample forecasting. The forecasting horizon covers 18 April 2018 to 21 September 2018.

2.1. Calculations of the bitcoin exchange rate and the triangular arbitrage

Equation 1 is used to calculate the bitcoin USD/EUR exchange rate, $(\mathrm{USD}/\mathrm{EUR})_{\mathrm{BX}}$:

$$(USD/EUR)_{BX} = \frac{USD/BTC}{EUR/BTC} \tag{1}$$

where USD/BTC and EUR/BTC are the prices of bitcoins in U.S. dollars and Euros, respectively (Nan and Kaizoji, 2018a).

The triangular arbitrage is expressed by

$$Triangular\ arbitrage = \frac{USD/BTC}{EUR/BTC} \times EUR/USD \tag{2}$$

where EUR/USD = 1/(USD/EUR) is the reciprocal of the USD/EUR spot rate.

The logarithmic return of the triangular arbitrage is r_{TA} , given by

$$r_{TA} = \log\left(\frac{USD/BTC}{EUR/BTC} \times \frac{1}{USD/EUR}\right) = BX_t - SP_t \tag{3}$$

where BX_t and SP_t denote the logarithm of the USD/EUR bitcoin exchange rate and the logarithm of the USD/EUR spot rate, respectively.

2.2. The futures hedge and the optimal hedge ratio

The reason for incorporating a futures contract into a portfolio is to reduce the risk of performing the triangular arbitrage. The logarithmic return of the hedged portfolio composed of l_{TA} units of a long triangular arbitrage position and l_{Fu} units of a short futures position is given by

$$r_{HP} = l_{TA}\, r_{TA} - l_{Fu}\, r_{Fu} \tag{4}$$

where r_{HP}, r_{TA} and r_{Fu} denote the logarithmic returns of the hedged portfolio, the triangular arbitrage, and the futures contract, respectively. The logarithmic return is normalized by making the units of the triangular arbitrage position equal to unity and has the form $r_{HP}/l_{TA} = r_{TA} - (l_{Fu}/l_{TA})r_{Fu}$. The coefficient $c = l_{Fu}/l_{TA}$ is called the hedge ratio.

The optimal hedge ratio is obtained by optimizing a specific objective function. One of the often-used objective functions is the variance of the portfolio, which, in this case, has the form

$$Var(R_{HP}) = l_{TA}^2\, Var(R_{TA}) + l_{Fu}^2\, Var(R_{Fu}) - 2\, l_{TA}\, l_{Fu}\, Cov(R_{TA}, R_{Fu}) \tag{5}$$

By minimizing the variance of the portfolio (or the portfolio risk) in terms of the hedge ratio $c = P_{Fu}/P_{TA}$, the quadratic function has a minimum at which

$$c^{*} = \frac{l_{Fu}}{l_{TA}} = \frac{Cov(R_{TA}, R_{Fu})}{Var(R_{Fu})} = \rho\frac{\sigma_{TA}}{\sigma_{Fu}} \tag{6}$$

where c^{*} denotes the optimal hedge ratio using the minimum variance method, ρ denotes the correlation coefficient between R_{TA} and R_{Fu}, and σ_{TA} and σ_{Fu} denote the standard deviation of R_{TA} and R_{Fu}, respectively.

As the minimum variance hedge ratio does not take into account the return of the hedged portfolio, the mean-variance hedge ratio is proposed as a remedy to this problem (Hsln et al., 1994). The mean-variance expected utility function is given by

$$EU(\boldsymbol{R}_t) = E(\boldsymbol{R}_t) - \gamma\, Var(\boldsymbol{R}_t) \tag{7}$$

where $\boldsymbol{R}_t$ is the 2×1 dimensional return vector containing two entries, R_{TA} and R_{Fu}; $\gamma > 0$ represents the risk aversion parameter. Speculators maximize their expected utility by solving equation 7 for; the optimal mean-variance hedge ratio c^{**} is shown by

$$c^{**} = -\left[\frac{E(R_{Fu})}{2\,\gamma\, Var(R_{Fu})} - \frac{Cov(R_{TA}, R_{Fu})}{Var(R_{Fu})}\right]. \tag{8}$$

When $(R_{Fu}) = 0$, indicating that the futures series follows a martingale, Equation 8 reduces to Equation 6.

2.3. Measuring the joint density and the time-dependent variance-covariance matrix

To calculate c* in Equation 6 and c** in Equation 8, it is necessary to model the joint density of R_{TA} and R_{Fu}. Time-varying variances and covariances are more attractive to practitioners than static ones, as practitioners alter their portfolio over time rather than holding a fixed position throughout the period. Moreover, financial return series often show clustering of the volatility after a tranquil period. This heteroskedastic characteristic appears to cause the probability density of the return to show excess kurtosis and fat tails (Baillie and Myers, 1991). A wide range of multivariate GARCH models has been devised to estimate density with the aforementioned features. This study proposes using the bivariate Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedastic (DCC-GARCH) model introduced by Engle and Sheppard (2001) and Engle (2002). Using the DCC (1, 1)-GARCH (1, 1) model, our specification is given by

$$\boldsymbol{e}_t|\mathcal{F}_{t-1} \sim Std(0, \boldsymbol{H}_t, \boldsymbol{v}) \tag{9}$$

$$\boldsymbol{H}_t \equiv \boldsymbol{D}_t \mathbf{P}_t \boldsymbol{D}_t \tag{10}$$

$$\boldsymbol{D}_t^2 = diag(\boldsymbol{\omega}) + diag(\boldsymbol{\alpha})\, \boldsymbol{e}_t\, \boldsymbol{e}_t' + diag(\boldsymbol{\beta})\boldsymbol{D}_{t-1}^2 \tag{11}$$

$$\boldsymbol{\varepsilon}_t = \boldsymbol{D}_t^{-1}\boldsymbol{e}_t \tag{12}$$

$$\boldsymbol{Q}_t = (1-\varphi-\psi)\overline{\boldsymbol{Q}} + \varphi\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1} + \psi\boldsymbol{Q}_{t-1} \tag{13}$$

$$\mathbf{P}_t = diag\left(\boldsymbol{Q}_t^{1/2}\right)^{-1}\boldsymbol{Q}_t\, diag\left(\boldsymbol{Q}_t^{1/2}\right)^{-1} \tag{14}$$

In Equation 9, $\boldsymbol{e}_t$ is the 2×1 dimensional residual vector comprising the two residuals produced by a regression in which each of the returns, R_{TA} and R_{Fu}, has been inserted into the ARMA (1, 1) model. Equation 9 indicates that after the time-varying means are removed from the two returns, the conditional distribution of $\boldsymbol{e}_t$ is assumed to be a standardized student's t distribution with mean zero, covariance matrix $\boldsymbol{H}_t$, and shape parameter vector $\boldsymbol{v}$. It is well known that the student's t distribution nests the normal distribution, i.e., when shape parameter ν goes into infinity, the t-distribution approaches a normal distribution. The $\mathscr{F}_{t-1}$ term denotes the previous information set conditioned on $\boldsymbol{e}_t$.

Equation 10 shows that $\boldsymbol{H}_t$ can be decomposed into $\boldsymbol{D}_t\mathbf{P}_t\boldsymbol{D}_t$, where $\boldsymbol{D}_t$ is the diagonal matrix with time-dependent standard deviations as described in (11) in the diagonal, and $\mathbf{P}_t$ is the time-dependent conditional correlation matrix.

Equation 11 captures the univariate GARCH effect for each return. One merit of the DCC model lies in its two-stage estimations (Engle and Sheppard, 2001). Firstly, univariate GARCH models are estimated for each residual series e_i for $i = TA, FU$ where e_i are two entries in residual vector $\boldsymbol{e}_t$ in (9). Secondly, the conditional standard deviations obtained during the first stage are used to create transformed residuals described in (12), and then the parameters of the conditional correlation are estimated using these transformed residuals. Hence, the results of the multivariate GARCH part are easy to interpret as univariate GARCH models. Equation 11 can be rewritten as $\boldsymbol{D}_t = diag(\sqrt{h_{i,t}})$ for $i = TA, FU$ with

$$h_{i,t} = \omega_{i,t} + \alpha_i e_{i,t-1}^2 + \beta_i h_{i,t-1} \tag{15}$$

so that each element of $\boldsymbol{D}_t$ is a univariate GARCH model with the restrictions for non-negativity of variances and stationarity $\alpha_i + \beta_i < 1$ being imposed.

The standard deviation matrix $\boldsymbol{D}_t^{-1}$ is then used to standardize the residual vector $\boldsymbol{e}_t$ to obtain the standardized innovation vector $\boldsymbol{\varepsilon}_t$, as specified in (12).

In equation 13, $\overline{\boldsymbol{Q}}$ denotes the unconditional correlation matrix of $\boldsymbol{\varepsilon}_t$ resulting from the first stage estimation, and $(1-\varphi-\psi)\overline{\boldsymbol{Q}}$ serves as a covariance targeting part where φ and ψ are non-negative scalars with the constraint that $\varphi + \psi < 1$. The specification in (13) indicates that the proxy process $\boldsymbol{Q}_t$ has exponential smoothing structure related not only to its one-period-ahead lag but also the lagged standardized deviation.

The conditional correlation matrix $\mathbf{P}_t$ is then calculated through rescaling $\boldsymbol{Q}_t$, as shown in (14).

2.4. Volatility forecasting

Normally, the r-step-ahead forecast of a multivariate GARCH (1, 1) has the form

$$\boldsymbol{D}_{t+r} = \sum_{i=1}^{r-2} diag(\boldsymbol{\omega})\,[diag(\boldsymbol{\alpha}) + diag(\boldsymbol{\beta})]^i + [diag(\boldsymbol{\alpha}) + diag(\boldsymbol{\beta})]^{r-1}\boldsymbol{D}_{t+1}, \tag{16}$$

However, due to the non-linearity of the DCC evolution process shown in (13) and (14), the multi-step forecast of correlation does not have a direct solution because of the nonlinearity of the DCC evolution. In such a case, Engle and Sheppard (2001) suggest two ways to make approximations: The first technique is to make an approximation that $E_t[\boldsymbol{\varepsilon}_{t+1}\,\boldsymbol{\varepsilon}'_{t+1}] \approx \boldsymbol{Q}_{t+1}$, namely the Q_t forecast; The alternative is said to provide for the least bias, based on their findings, is that $\overline{\boldsymbol{Q}} \approx \overline{\boldsymbol{P}}$ and $E_t[\boldsymbol{Q}_{t+1}] = E_t[\boldsymbol{P}_{t+1}]$, namely the P_t forecast.

Consider the r-step-ahead evolution of the proxy process

$$\boldsymbol{Q}_{t+r} = (1-\varphi-\psi)\overline{\boldsymbol{Q}} + \varphi\, E_t[\boldsymbol{\varepsilon}_{t+r-1}\,\boldsymbol{\varepsilon}'_{t+r-1}] + \psi\, \boldsymbol{Q}_{t+r-1} \tag{17}$$

and its approxiamted r-step-ahead P_t forecast given by

$$E_t[\boldsymbol{P}_{t+r}] = \sum_{i=0}^{r-2}(1-\varphi-\psi)\,\overline{\boldsymbol{P}}(\varphi+\psi)^i + (\varphi+\psi)^{r-1}\boldsymbol{P}_{r+1} \tag{18}$$

where $\overline{\boldsymbol{P}}$ denotes the constant correlation matrix of returns. In practice, $\overline{\boldsymbol{P}}$ is often used as the covariance matrix in place of the correlation matrix since the standardized innovation is usually unobservable (Engle and Sheppard, 2001).

In this study, to minimize bias, a rolling one-step-ahead approach is used, since, for r-step-ahead forecasting, forecast error based on a fixed value for $\overline{\boldsymbol{P}}$ becomes quite large when the value of r is large.

2.5. Forecast evaluation

Brownlees et al. (2011) point out that volatility forecast comparison must be made against an ex-post proxy of volatility, rather than its true, latent value. They suggest Patton's (2011) loss functions, which are said to be robust in generating the same ranking of models as long as the proxy is unbiased. Two classes commonly used are the mean squared error (MSE) loss and the quasi-likelihood (QL) loss specified by

$$MSE\ :\ L(\hat{\sigma}_t^2, h_{t|t-1}) = \left(\hat{\sigma}_t^2 - h_{t|t-1}\right)^2 \tag{19}$$

$$QL\ :\ L\left(\hat{\sigma}_t^2, h_{t|t-1}\right) = \frac{\hat{\sigma}_t^2}{h_{t|t-1}} - log\,\frac{\hat{\sigma}_t^2}{h_{t|t-1}} - 1 \tag{20}$$

where $\hat{\sigma}_t^2$ denotes an unbiased ex-post proxy, and $h_{t|t-1}$ is the one-step-ahead conditional variance forecast (Brownlees et al., 2011).

In this paper, we use the squared returns of the triangular arbitrage and the FX futures as the unbiased ex-post proxies.

3. Empirical results and conclusion

3.1. Statistical characteristics of returns

The USD/EUR bitcoin exchange rate is computed by (1). Figure 2 plots the respective time series: the bitcoin exchange rate series, BX_t; the FX spot rate series, SP_t; and the FX futures rate series, FU_t, all transformed into natural logarithms. As Figure 2 indicates, BX_t shows an intertwining with SP_t but seemingly has more spikes. Surprisingly, BX_t appears not to deviate far from SP_t , even during the period at the end of 2018 when the bitcoin prices of USD and EUR skyrocketed. As for FU_t, it runs above the BX_t and SP_t series; however, the discrepancy tends to narrow, indicating a diminishing risk premium.

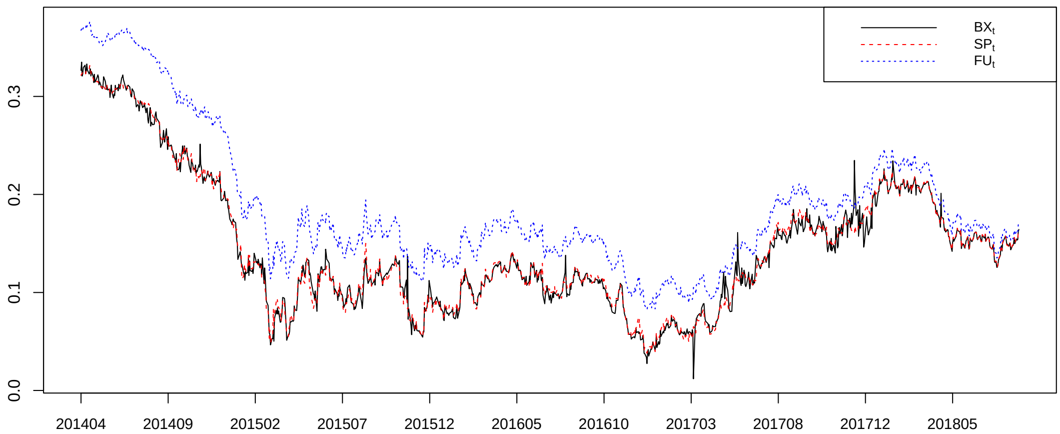

Figure 2. The bitcoin exchange rate of USD/EUR (BX_t), the FX spot of USD/EUR (SP_t) and the FX futures of USD/EUR (FU_t). All series are in natural logarithm form.

Figure 3 plots the six logarithmic return series: (1) the return of the USD/BTC index, $R_{USD/BTC}$; (2) the return of the EUR/BTC index, $R_{EUR/BTC}$; (3) the return of the USD/EUR bitcoin exchange rate, R_{BX}; (4) the return of the USD/EUR spot rate, R_{SP}; (5) the return of USD/EUR futures rate, R_{FU}, and (6) the return of the triangular arbitrage, R_{TA}. Although all of these return series appear to oscillate around their mean and behave rather noisily, R_{TA} presents a cyclic fluctuating pattern that indicates short-run arbitrage opportunities.

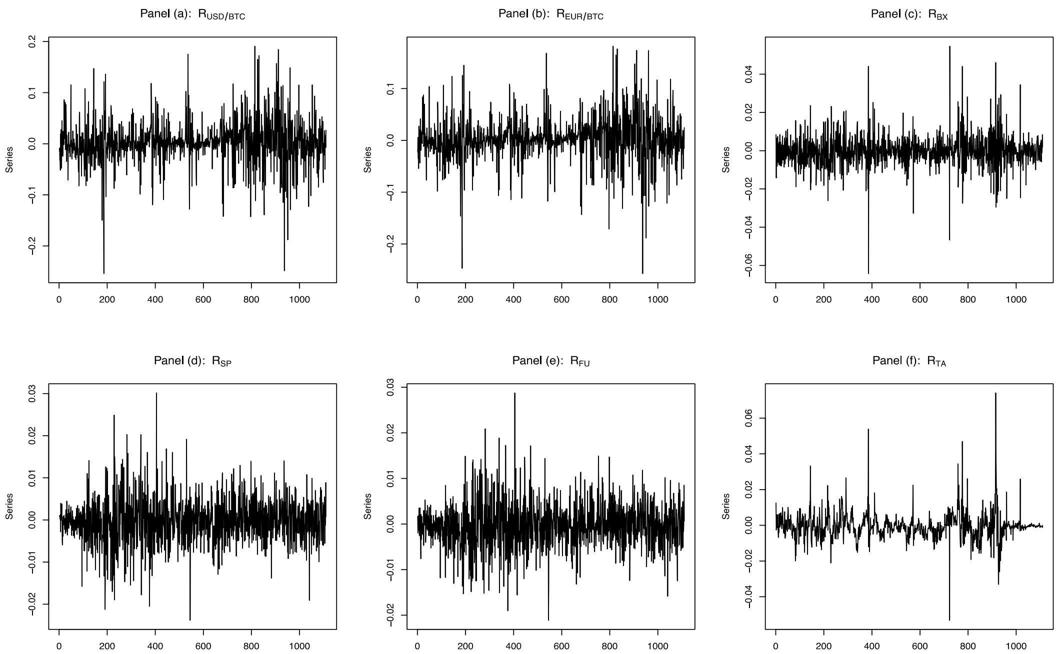

Figure 3. The return series of the six assets (or portfolios): the returns of USD/BTC ($R_{USD/BTC}$), the returns of EUR/BTC ($R_{EUR/BTC}$), the returns of the USD/EUR bitcoin exchange rate (R_{BX}), the returns of the FX spot rates (R_{SP}), the returns of the FX futures rate (R_{FU}), and the returns of the triangular arbitrage (R_{TA}).

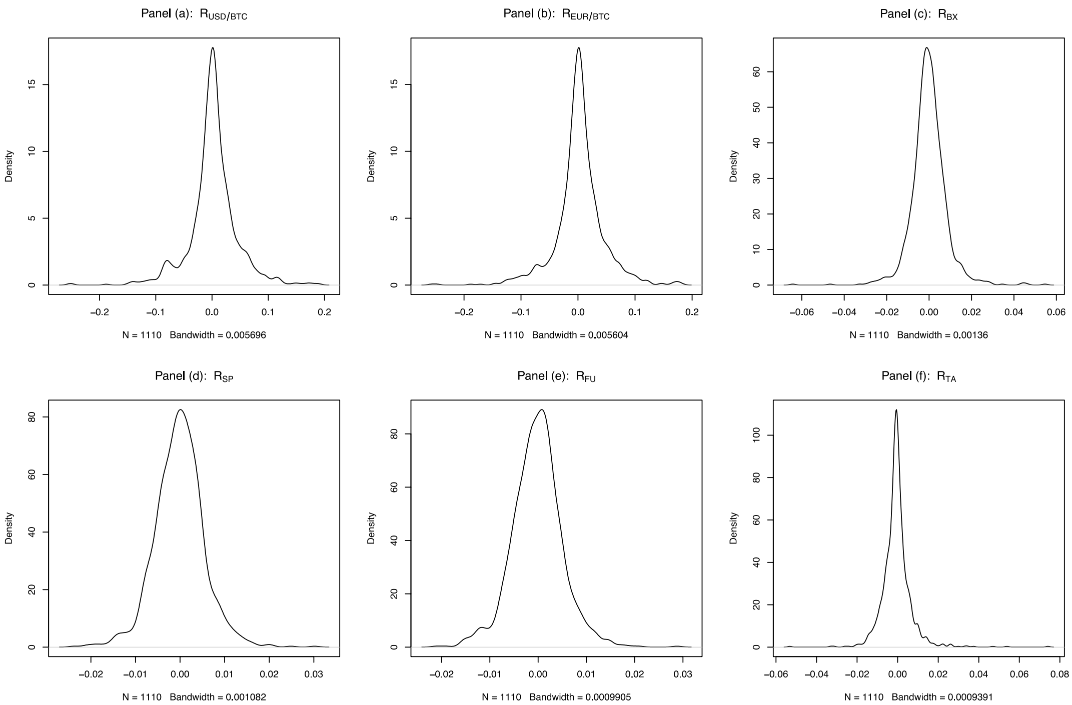

Figure 4. The sample probability densities of the six return-series.

Figure 4 plots the sample probability densities of the six logarithmic return series. As Panel (a) and Panel (b) of Figure 3 show, the graphs of the densities of $R_{USD/BTC}$ and $R_{EUR/BTC}$ have long and fat left tails extending negatively beyond −0.2 on the x-axis. These phenomena suggest the potential for more than 20% daily losses in terms of either the U.S. dollar or the Euro when investors take a positioning trading strategy on bitcoins. That both left tails are seemingly longer than the right tails appears to support Dong and Dong's (2015) finding that bitcoin investment presents higher risk and lower return. In contrast, Panel (c) of Figure 3 suggests a reduction in daily losses to around 6% or so when using the bitcoin to make currency exchanges. The triangular arbitrage strategy appears to improve the currency exchange strategy by means of a more centered and symmetric probability density.

Table 1. Summary statistics.

Returns	Mean	Minimum	Maximum	S.D.	Skewness	Kurtosis	Normality
$R_{USD/BTC}$	0.0023	−0.2542	0.1909	0.0430	−0.23	4.44	928.05**
$R_{EUR/BTC}$	0.0025	−0.2573	0.1818	0.0427	−0.33	4.63	1015.00**
R_{BX}	−0.0001	−0.0643	0.0546	0.0084	0.16	8.21	3136.90**
R_{SP}	−0.0001	−0.0238	0.0301	0.0056	0.12	2.15	219.26**
R_{FU}	−0.0001	−0.0212	0.0288	0.0052	0.15	1.87	167.67**
R_{TA}	−0.0005	−0.0532	0.0741	0.0074	1.59	17.66	14949**

Note: The sample period contains 1110 observations. Kurtosis refers to the excess kurtosis. Normality refers to the Jarque-Beta test.

The summary statistics in Table 1 affirm our conjectures based on the observed plots. First, bitcoin trading had a much wider range of returns than did currency exchange in either the FX market or the bitcoin market. This result indicates that bitcoin investors employing a buy-and-hold strategy face exceedingly significant losses or profits over time. Second, bitcoin-based currency exchange strategies had much less static variance during the sample period. The reduction in the unconditional variances is substantial when speculators perform bitcoin-based currency exchange strategies, while the triangular arbitrage reduces the variance as compared to the bitcoin exchange rate strategy. Third, the density of the return on bitcoin investment shows negative skewness, suggesting that most bitcoin holders can expect only a slight profit, at the price of a very large loss. Furthermore, all six series show excess kurtosis; hence, they all lead to rejecting the assumption of normality (the Jarque-Bera test) in a very significant way. Usually, non-normal densities of returns are the result of weak dependence in the return series. Here, the Student's t distribution functions better than the normal distribution hypothesis for the GARCH family (Baillie and Myers, 1991). An interesting phenomenon is that the density of R_{TA} shows a very high excess kurtosis, which may imply a strong dependence in the return series. Moreover, since the data are more centrally distributed, the areas in the tails of the distribution decrease, indicating a reduction in risk.

3.2. The DCC-GARCH model and the estimated optimal hedge ratio

Time-series features of the six series of returns are summarized in Table 2. First, to test stationarity, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Elliott-Rothenberg-Stock (ERS) tests were conducted. The null hypothesis for the KPSS test of stationarity could not be rejected in all cases using models with a lag of 21. Meanwhile, the null of a unit root process for the ERS test was rejected in all cases at the 1% significance level. Based on these results, all six returns appear to be stationary—i.e., a long-run martingale process. The Ljung-box test (or the Q-statistic) using a lag of 20 indicated that there was no serial correlation in $R_{USD/BTC}$, $R_{EUR/BTC}$, R_{SP}, and R_{FU}; however, it identified serial correlations in R_{BX} and R_{TA}. The order of the ARMA model is suggested by the AIC criterion. For R_{TA}, although AIC recommended the ARMA (2, 1) model, the coefficient of the second-order autoregressive term is not significant. Thus, the ARMA (1, 1) model may be appropriate for parsimony.

Table 2. Time-series features.

Returns	KPSS	ERS	Ljung-Box	ARMA
$R_{USD/BTC}$	0.14	−8.16**	14.27	(1, 1)
$R_{EUR/BTC}$	0.12	−7.15**	12.20	(1, 1)
R_{BX}	0.08	−4.17**	93.91**	(2, 3)
R_{SP}	0.09	−9.20**	18.25	(0, 0)
R_{FU}	0.09	−8.03**	15.04	(0, 0)
R_{TA}	0.04	−4.85**	559.37**	(2, 1)

Note: Ljung-Box refers to the Q-statistic using a lag of 20. The order of the ARMA model is suggested by the AIC criterion.

Based on the obtained information, we propose a framework to model the joint density of the return of triangular arbitrage, R_{TA}, and the return of futures, R_{FU}, i.e., to capture the time-varying conditional variance matrix and the time-varying conditional correlation. The proposed framework combines the ARMA (1, 1) model with the DCC (1, 1)-bivariate GARCH (1, 1) model. A total of 1000 observations were used to fit the proposed model; results are presented in Table 3.

First, the ARMA (1, 1) model was used to model autoregression in the return series, especially in R_{TA}. All three coefficients related to R_{TA} were significant. Comparing the values of these coefficients with those estimated from the univariate ARMA (1, 1) model that was applied to R_{TA}, the values are quite similar. The residuals from the univariate ARMA (1, 1) model applied to R_{TA} passed the Ljung-Box test using a lag of 20, which shows ARMA (1, 1) is sufficient for capturing the autoregressive effect. Second, the GARCH (1, 1) model with Student's t distribution as the underlying assumption was used to measure the volatility clustering effect. Except for the constant terms, all coefficients were significantly different from zero. The sums $\alpha_{TA} + \beta_{TA} = 0.9957$ and $\alpha_{FU} + \beta_{FU} = 0.9970$ suggest a near integrating process for the volatility of the triangular arbitrage returns and the FX futures returns, respectively. The shape parameters for the Student's t distribution were all significant: $v_{TA} = 3.2055$ implies much heavier tails in the density of R_{TA} than in the case of R_{FU} , where $v_{FU} = 6.4826$. Finally, the 0.9413 sum of $\varphi + \psi$ tends to suggest a comparatively persistent conditional correlation process with the shape parameter $\nu = 4.5616$.

Table 3. Estimation of the bivariate ARMA plus DCC-GARCH model.

ARMA (1, 1)		Bivariate GARCH (1, 1)		DCC (1, 1) and others	
μ_{TA}	−0.0009**	ω_{TA}	0.0000	φ	0.0121
	(0.0003)		(0.0000)		(0.0174)
μ_{FU}	−0.0002	ω_{FU}	0.0000	ψ	0.9292**
	(0.0001)		(0.0000)		(0.1092)
$AR1_{TA}$	0.7905**	α_{TA}	0.3553**	ν	4.5616**
	(0.0399)		(0.1150)		(0.2613)
$AR1_{FU}$	0.4146	α_{FU}	0.0501**		
	(0.4112)		(0.0047)		
$MA1_{TA}$	−0.3826**	β_{TA}	0.6404**	LL	7786.798
	(0.0680)		(0.1030)		
$MA1_{FU}$	−0.4520	β_{FU}	0.9469**	AIC	−15.538
	(0.4022)		(0.0072)		
		v_{TA}	3.1055**		
			(0.3780)		
		v_{FU}	6.4826**		
			(1.2713)		

Note: The ARMA (1, 1) model is specified as $R_{i,t} = \mu_i + AR1_i R_{i,t-1} + e_{i,t} + MA1_i e_{i,t-1}$ for $i = TA, FU$ as the subscript. Subscript TA denotes the coefficient concerns the returns of the triangular arbitrage, while subscript FU concerns the returns of the FX futures.
$e_{i,t}|\mathcal{F}_{t-1} \sim Std(0, h_{i,t}, v_i)$ indicates the residuals from the ARMA (1, 1) model have the conditional density following the standardized student's t distribution with mean zero, variance $h_{i,t}$, and degrees of freedom v_i. The bivariate GARCH (1, 1) component is specified as $h_{i,t} = \omega_{i,t} + \alpha_i e_{i,t-1}^2 + \beta_i h_{i,t-1}$. The conditional correlation is modelled by $Q_t = \overline{Q}(1 - \varphi - \psi) + \varphi\varepsilon_{t-1}\varepsilon_{t-1}^{'} + \psi Q_{t-1}$ where vector ε_t has two entries in its column specified as $\varepsilon_{i,t} = e_{i,t}/h_{i,t}^{1/2}$ and $\varepsilon_{i,t}|\mathcal{F}_{t-1} \sim Std(0, 1, v)$. LL denotes Log-Likelihood, and AIC denotes the Akaike Information Criterion for the framework. ** significant at 1%.

Figure 5 plots the conditional variance of the return of the triangular arbitrage, $h_{tt,t}$, the conditional variance of the return of the FX futures, $h_{ff,t}$, and the conditional covariance of the two returns, $h_{tf,t}$. The time-varying conditional optimal hedge ratio is calculated as $c_t^* = h_{tf,t}/h_{ff,t}$ according to Equation 6.

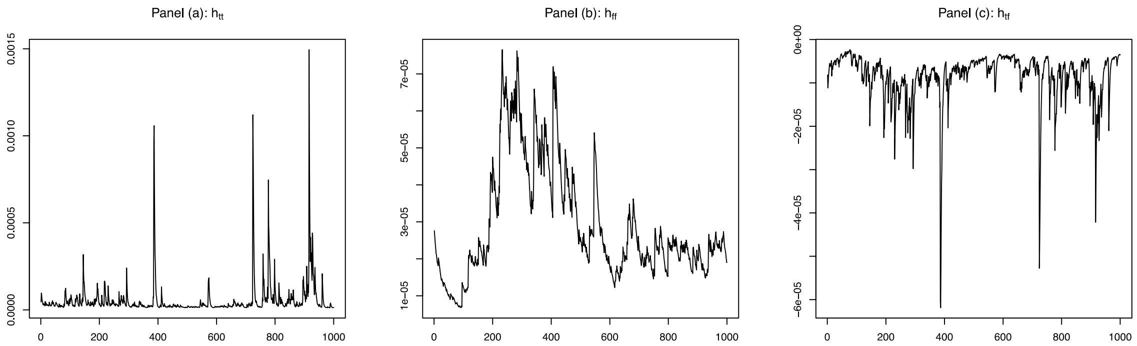

Figure 5. The conditional variance of the returns of the triangular arbitrage ($h_{tt,t}$), the conditional variance of the returns of the futures rate ($h_{ff,t}$), and the conditional covariance ($h_{tf,t}$).

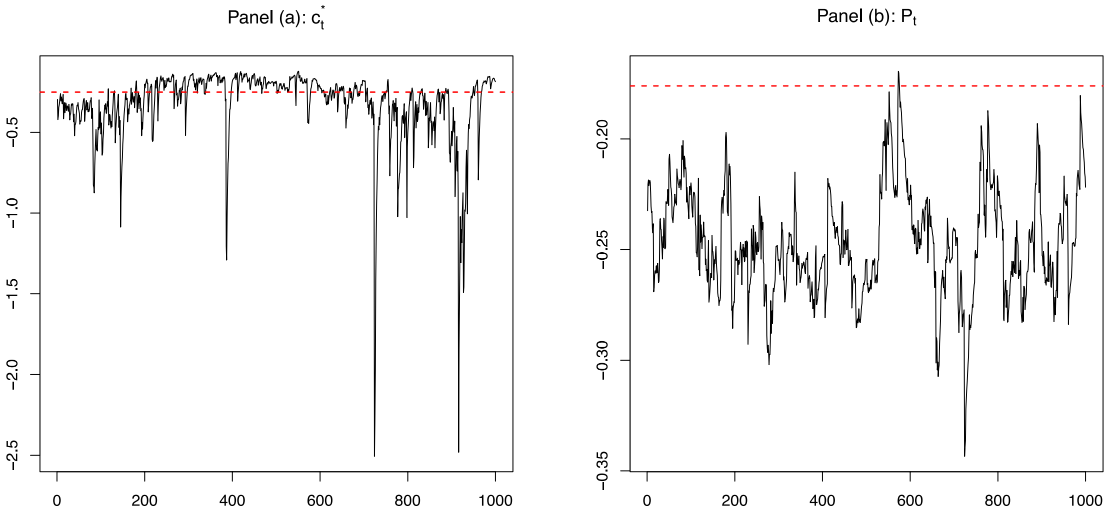

Figure 6. The conditional optimal hedge ratio series (c_t^*) and the conditional correlation series (P_t). The red dashed line in Panel (a) is the optimal hedge ratio obtained from the OLS method (c_{OLS}); the red dashed line in Panel (b) is the unconditional correlation coefficient.

Figure 6 shows a plot of the conditional optimal hedge ratio series, c_t^*, and the conditional correlation series, P_t . In Panel (a) of Figure 6, the dashed line is the unconditional optimal hedge ratio, $c_{OLS}^* = -0.2510$, obtained from the OLS regression. By comparison, the conditional optimal hedge ratio has mean −0.32 and varies from −2.5065 to −0.1221. The dashed line in Panel (b) of Figure 6 is the unconditional correlation between R_{TA} and R_{FU}. The unconditional correlation value is −0.1760, whereas the conditional correlation series varies from −0.3435 to −0.1694.

3.3. Rolling one-step-ahead forecasting

The estimated coefficients of our specification—the ARMA (1, 1) plus the DCC- bivariate GARCH—were used to produce rolling one-step-ahead forecasts. The rolling window was fixed at a length of 1000. This approach makes a one-step-ahead forecast, and then adds a new observation to the end of the window and removes the first observation. In this fashion, the process was repeated 110 times until all 110 bits of data were exhausted.

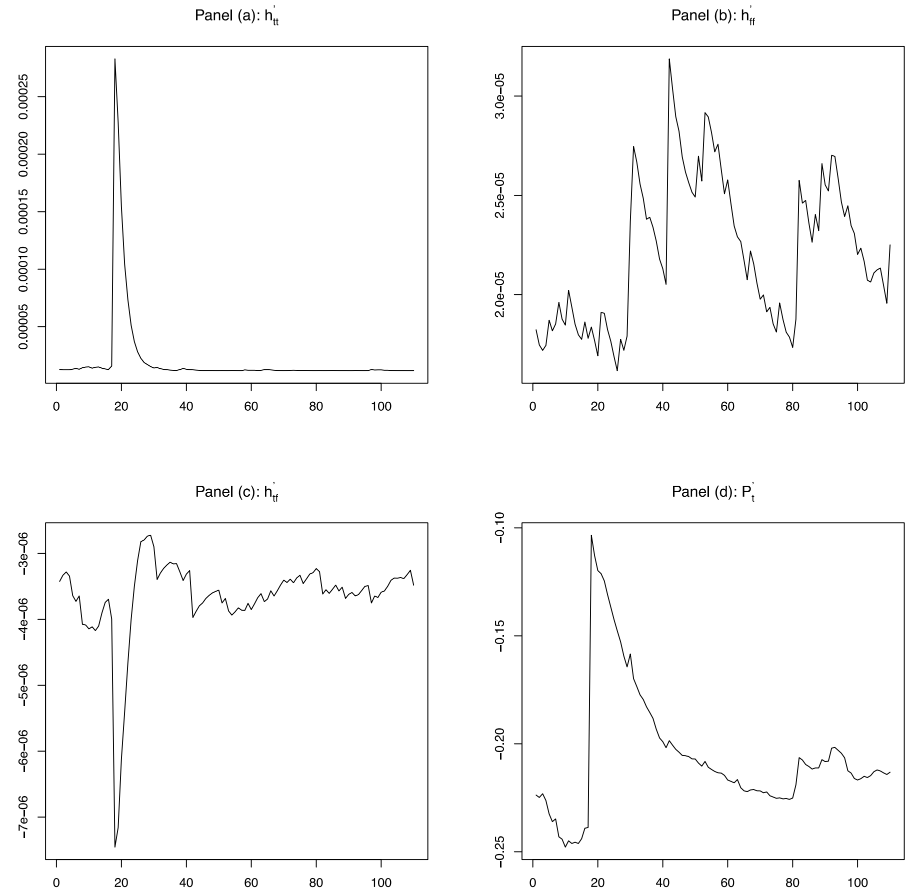

Figure 7. The forecasted conditional variances and covariance and the forecasted conditional correlation based on the ARMA plus DCC- GARCH model.

The forecasted conditional variances and covariance, $h'_{tt,t}$, $h'_{ff,t}$, and $h'_{tf,t}$, and the forecasted conditional correlation P'_t are plotted in Figure 7. Figure 8 combines the conditional optimal hedge ratio c^*_t obtained from our sample (black line) and the forecasted optimal hedge ratio (red dashed line) into a single plot.

To access the forecasted conditional variances $h'_{tt,t}$ for the triangular arbitrage and $h'_{ff,t}$ for the FX futures, we use the corresponding squared returns as the proxy for each true and latent conditional variance. Figure 9 plots the forecast and the proxy together. The mean of the MSE losses and the mean of the QL losses for the forecasted triangular arbitrage variance are 6.0343×10^{-9} and 3.8968 calculated by (17) and (18), respectively, while the mean of the MSE losses and the mean of the QL losses for the forecasted FX futures variance are 1.2293×10^{-9} and 1.4974. Hence, predictive accuracy can be evaluated based on average MSE and QL forecast losses achieved by selections of models and specifications (Brownlees et al., 2011).

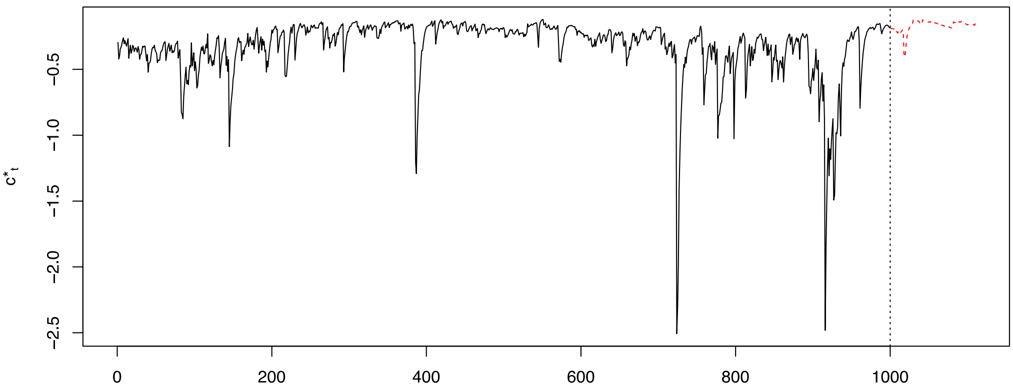

Figure 8. The sample conditional optimal hedge ratio (black line) and the forecasted conditional optimal hedge ratio (red dashed line).

Panel (a): the triangular arbitrage

Panel (b): the FX futures

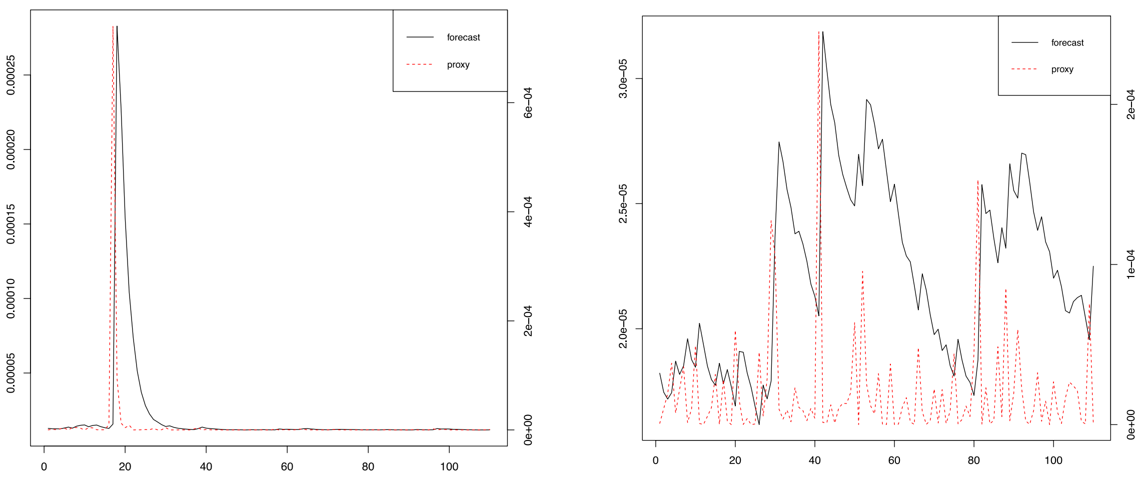

Figure 9. The forecasted triangular arbitrage conditional variance $h'_{tt,t}$ in the black line and the proxy for its true variance in the red dashed line (panel a). The forecasted FX futures conditional variance $h'_{ff,t}$ in the black line and its true variance in the red dashed line (panel b). The proxies are rescaled according to the y-axes on the right-hand side.

3.4. Arbitrage strategy evaluations

To make an evaluation, the unconditional variance (multiplied by the number of the observations) and Value-at-Risk (VaR) were used as the criteria to illustrate the performance of all the arbitrage strategies. Table 4 gives a summary of the results for the sample of 1000 observations. As shown, the returns of the triangular arbitrage and the returns of the hedged portfolio have much smaller unconditional variances than the returns of USD/BTC and EUR/BTC, although they are not the smallest when compared to the FX spot and futures returns. The returns of the USD/EUR bitcoin exchange rate also show a smaller variance than the returns of USD/BTC and EUR/BTC. The VaR criteria indicate that the hedge portfolio appears to have the least negative value of the returns at the high quantile 99.5 using either the Historical approach or the Cornish-Fisher (denoting Modified-) approach, which is modified to adapt to the leptokurtic density of returns. For R_{HP}, the one-day losses at the 0.5% confidence level are −1.94% and −4.10%, respectively, according to the historical and the modified approaches. In contrast, $R_{USD/BTC}$ has daily losses of −14.23% and −19.62, respectively; for $R_{EUR/BTC}$, the losses are −14.35% and 20.07%, respectively.

Table 4. Comparison of assets or portfolios based on the unconditional variances and the VaR values in the sample period.

Assets	Variances	Historical VaR (0.5%)	Modified VaR (0.5%)
$R_{USD/BTC}$	1.8478	−0.1423	−0.1962
$R_{EUR/BTC}$	1.8203	−0.1435	−0.2007
R_{BX}	0.0708	−0.0272	−0.0477
R_{SP}	0.0310	−0.0170	−0.0185
R_{FU}	0.0269	−0.0151	−0.0166
R_{TA}	0.0548	−0.0211	−0.0476
R_{HP}	0.0593	−0.0194	−0.0410

Note: VaR denotes the Value-at-Risk where Modified VaR concerns the Cornish-Fisher estimate of VaR. The sample period starts on 22 April 2014 and ends on 17 April 2017; it includes 1000 observations.

Table 5. Comparison of assets or portfolios based on the unconditional variances and the VaR values in the forecasted period.

Assets	Variances	Historical VaR (1%)	Modified VaR (1%)
$R_{USD/BTC}$	0.1557	−0.1001	−0.1033
$R_{EUR/BTC}$	0.1581	−0.1039	−0.1018
R_{BX}	0.0040	−0.0172	−0.0234
R_{SP}	0.0024	−0.0100	−0.0139
R_{FU}	0.0023	−0.0124	−0.0127
R_{TA}	0.0008	−0.0029	
R_{HP}	0.0009	−0.0034	−0.0280

Note: VaR denotes the Value-at-Risk where Modified VaR concerns the Cornish-Fisher estimate of VaR. The forecasted period starts on 18 April 2017 and ends on 21 September 2018; it includes 110 observations. The Modified VaR is blank in the R_{TA} row because of the unreliable result (inverse risk) produced by the calculation.

Table 5 gives the results using the same criteria as in Table 4 but involves a forecast period of 18 April 2017 to 21 September 2018. The returns of the hedged portfolio using the forecasted optimal hedge ratio R_{HP} shows the second lowest daily risks among the bitcoin assets with respect to the VaR approaches. An interesting outcome is that R_{TA} and R_{HP} during the observation period have smaller variances than the returns from the FX market.

4. Conclusion

Based on empirical results, this paper finds that bitcoin-based currency exchange strategies are competitive with direct bitcoin trading in terms of risk management. In particular, the triangular arbitrage strategy expanding bitcoin-based currency exchange to the FX spot market appears to be most attractive due to the existence of arbitrage opportunities and FX futures hedging. The ARMA plus DDC-GARCH model is suggested to measure the joint density of the hedged portfolio containing the returns of the bitcoin-based triangular arbitrage and the returns of the FX futures, to capture the time-varying conditional covariance matrix and correlation, and to produce one-step-ahead rolling window forecasts. Hence, a dynamic future hedged portfolio was formed, the return of which was then compared to the returns of the bitcoin and FX rates. The empirical and forecasted results tend to show that the portfolio with the bitcoin-based triangular arbitrage and its FX futures is superior to the USD/BTC and EUR/BTC assets in terms of risk management. Despite the importance of our empirical work to the market, it is worth further investigating on the latent volatility and forecast accuracy. This leaves a space for studying high-frequency data using methods such as the realized GARCH model and the neural network.

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Conflict of interest

The authors declare no conflict interest.

References

Akram QF, Rime D, Sarno L (2008) Arbitrage in the foreign exchange market: Turning on the microscope. *J Int Econ* 76: 237–253.

Ali R, Barrdear J, Clews R, et al. (2014) The Economics of Digital Currencies (SSRN Scholarly Paper No. ID 2499418). Available from: https://papers.ssrn.com/abstract=2499418.

Baillie RT, Myers RJ (1991) Bivariate GARCH estimation of the optimal commodity futures hedge. *J Appl Econ* 6: 109–124.

Böhme R, Christin N, Edelman B, et al. (2015) Bitcoin: Economics, technology, and governance. *J Econ Perspect* 29: 213–238.

Bouri E, Molnár P, Azzi G, et al. (2017) On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier?. *Financ Res Lett* 20: 192–198.

Briere M, Oosterlinck K, Szafarz A (2013) Virtual Currency, Tangible Return: Portfolio Diversification with Bitcoins (SSRN Scholarly Paper No. ID 2324780). Available from: https://papers.ssrn.com/abstract=2324780.

Brownlees CT, Engle RF, Kelly BT (2011) A practical guide to volatility forecasting through calm and storm. Available at SSRN 1502915.

Buchholz M, Delaney J, Warren J, et al. (2012) Bits and Bets, Information, Price Volatility, and Demand for Bitcoin. *Economics* 312: 2–48.

Cheah ET, Fry J (2015) Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Econ Lett* 130: 32–36.

Ciaian P, Rajcaniova M, Kancs d'Artis (2016) The economics of BitCoin price formation. *Appl Econ* 48: 1799–1815.

Corbet S, Lucey B, Peat M, et al. (2018) Bitcoin Futures—What use are they? *Econ Lett* 172: 23–27.

Dong H, Dong W (2015) Bitcoin: Exchange rate parity, risk premium, and arbitrage stickiness.

Engle R (2002) Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *J Bus Econ Stat* 20: 339–350.

Engle RF, Sheppard K (2001) Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. *Nat Bur Econ Res.*

Glaser F, Zimmermann K, Haferkorn M, et al. (2014) Bitcoin-asset or currency? revealing users' hidden intentions. Available from: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2425247.

Grossman SJ, Stiglitz JE (1980) On the impossibility of informationally efficient markets. *Am Econ Rev* 70: 393–408.

Hong K (2017) Bitcoin as an alternative investment vehicle. *Inf Technol Manage* 18: 265–275.

Hsln CW, Kuo J, Lee CF (1994) A new measure to compare the hedging effectiveness of foreign currency futures versus options. *J Futures Mark* 14: 685–707.

Kristoufek L (2013) BitCoin meets Google Trends and Wikipedia: Quantifying the relationship between phenomena of the Internet era. *Sci Rep* 3: 3415.

Li Z, Dong H, Huang Z, et al. (2018) Asymmetric effects on risks of Virtual Financial Assets (VFAs) in different regimes: A Case of Bitcoin. *Quant Financ Econ* 2: 860–883.

Luther WJ (2016) Bitcoin and the Future of Digital Payments. *Indep Rev* 20: 397.

Nakamoto S (2008) Bitcoin: A peer-to-peer electronic cash system.

Nan Z, Kaizoji T (2018a) Market Efficiency of the Bitcoin Exchange Rate: Weak and Semi-Strong Form Tests with the Spot, Futures and Forward Foreign Exchange Rates of the U.S. Dollar against the Euro. *Int Rev Financ Anal*: In press.

Nan Z, Kaizoji T (2018b) The optimal foreign exchange futures hedge on the bitcoin exchange rate: an application to the U.S. dollar and the Euro, In press.

Pichl L, Kaizoji T (2017) Volatility Analysis of Bitcoin Price Time Series. *Quant Econ Financ* 1: 474–485.

Patton AJ (2011) Volatility forecast comparison using imperfect volatility proxies. *J Econometrics* 160: 246–256.

Van Wijk D (2013) What can be expected from the BitCoin. Erasmus Universiteit Rotterdam. Available from: https://thesis.eur.nl/pub/14100/Final-version-Thesis-Dennis-van-Wijk.pdf.

Yermack D (2013) Is Bitcoin a real currency? An economic appraisal. Available from: http://www.nber.org/papers/w19747.

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