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*Research article*

## Supply-side perspective for carbon pricing

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**Abstract:** This paper theoretically and empirically revisits carbon pricing from the supply-side perspective for carbon assets to solve the recent low price issue which may delay the development of emission reduction technologies in the sense of marginal abatement costs. We propose a carbon pricing model linked to crude oil prices, which has historically been employed in supply-side driven pricing of long-term contracts for early-stage energy trading. Since the model is designed to hold carbon prices between certain lower and upper boundaries using S-shaped carbon price linkage to crude oil prices, it can be useful to overcome a recent low carbon price issue. In addition, it is shown that the model can alleviate the difficulties of carbon derivative pricing in selecting market price of risk. Empirical studies using EUA and Brent crude oil futures prices estimate the parameters of the Brent crude oil-linked EUA price model. The comparison of EUA prices simulated from the model with historical EUA prices suggests that simulated EUA prices be kept relatively higher than historical EUA prices. This is preferable for accelerating carbon emission reductions in that it can make emission reduction technologies with high marginal abatement costs affordable. It may imply that EUA must be priced using a crude oil-linked carbon price model in the early stage of EUA trading until EUA markets mature. This is a sharp contrast to current carbon markets employing premature market-based or supply and demand based pricing models. To show usefulness of crude oil-linked carbon pricing, we also give a numerical example of European carbon option pricing based on the Brent crude oil-linked EUA price model by using the Crank-Nicolson finite difference method. Finally we discuss the relation between crude oil-linked carbon pricing and emission reduction risk. These studies may suggest carbon policy makers should take account of crude oil-linked carbon pricing to tackle low price and low liquidity issues of carbon assets.

**Keywords:** EUA market; Brent crude oil market; crude oil-linked carbon price model

**JEL codes:** C51, G13, Q41, Q54

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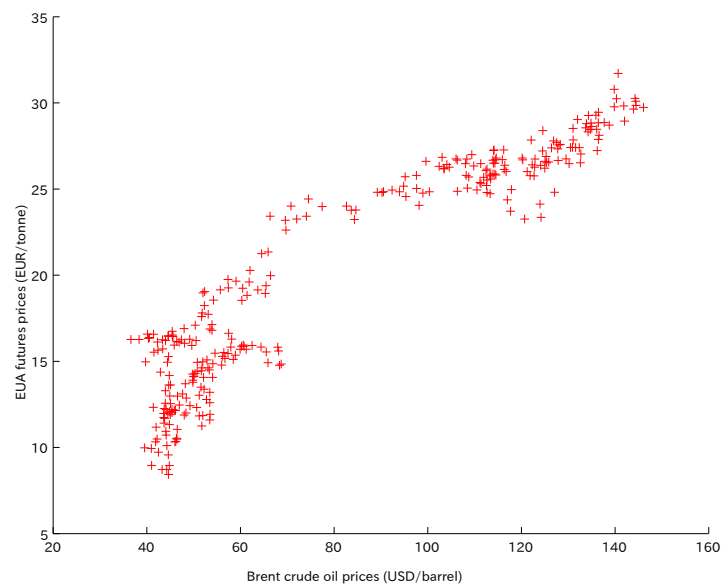
## 1. Introduction

Commodity transaction including energy often starts with long-term contracts from supply-side perspective of commodities. This is reasonable because the supply sides certainly attempt to recover the production costs in the early stage of commodity trading. It also fits well to the demand side preference to desire the commodity purchase with a stable constant price in the absence of the highly liquid spot markets even if the constant price is more or less expensive for consumers. For example, natural gas trading is known to employ long-term contracts between supply and demand sides, resulting in crude oil-linked pricing (see, e.g., Stern (2007)). When the liquidity of energy trading fully increases, as the next step of long-term contracts energy tends to be evaluated in the market using supply and demand-based pricing. For example, the US natural gas prices are determined by the supply and demand in the relatively highly liquid market (see, e.g., Kanamura (2009b)). On the other hand in contrast to energy commodities, emissions trading politically and artificially starts with the markets whose prices are determined by the supply and demand relations for emissions, as observed in the EU ETS. That is to say, carbon assets are transacted in a supply and demand-based market regardless of low liquidity of carbon trading, which is different from market development processes of energy commodities. This can be a reason why carbon markets do not work well, resulting in low prices less than 10 EUR/tonne (see the recent years of Figure 1), i.e., a recent low price issue in carbon markets. Low carbon prices are quite problematic for accelerating the developments of carbon reduction technologies because inexpensive carbon assets are favored by EU ETS covered or compliance entities more than the introduction of new carbon emission reduction technologies with high marginal abatement costs. We revisit carbon pricing to solve a recent low carbon price issue. Crude oil markets are closely related to economic fundamentals as in e.g., Chevallier (2011b). It implies that high crude oil prices can be driven by strong economy, resulting in large carbon emissions and high carbon prices. Thus in a first order approximation it is assumed that carbon prices can positively be affected by crude oil prices. In order to examine the relation between carbon and energy, we show scatter plots between Brent crude oil and EUA futures prices in Figure 2. It seems an upward-sloping nonlinear relation between Brent crude oil and EUA futures prices, resulting in the positive support of crude oil-linked pricing of carbon products.

In the last decade, the development of carbon markets has attracted great academic interest. Fehr and Hinz (2006) propose an equilibrium price model for EUA prices taking into account fuel switching between natural gas and coal fired power plants. Benz and Trück (2009) employ an AR-GARCH Markov switching price return model to capture regime changes between different phases of the EU ETS and heteroskedasticity. Daskalakis, Psychoyios, and Markellos (2009) compare existing popular diffusion and jump diffusion models, with the results in favor of the Geometric Brownian motion with jumps to fit historical EUA spot prices, unlike the mean-reverting processes often used for commodity price modeling. Seifert, Uhrig-Homburg, and Wagner (2008) propose a stochastic model of CO<sub>2</sub> prices which does not have any seasonal pattern, as often observed in commodity markets. Paoletta and Taschini (2008) also propose mixed normal and mixed stable GARCH models to capture the heavy tails and volatility clustering in the U.S. SO<sub>2</sub> permits and EUA price returns, which are not modeled using any mean-reversion and seasonality. Uhrig-Homburg and Wagner (2009) examine the relation between carbon spot and futures prices traded on the Powernext and the European Climate Exchange. Trück, Härdle, and Weron (2015) conduct empirical analyses of EUA convenience yields using the



**Figure 1.** EUA futures and Brent crude oil prices: Note carbon markets do not work well, resulting in low prices less than 10 EUR/tonne, i.e., a recent low price issue in carbon markets.



**Figure 2.** Scatter plots between Brent crude oil and EUA futures prices (April, 2008 to June, 2009): Note the figure seems an upward-sloping nonlinear relation between the two variables, resulting in the positive support of crude oil-linked pricing of carbon.

spot and futures prices traded on the EEX and presenting a convenience yield model based on the spot prices and volatilities. Kanamura (2009a) also investigates the characteristics of carbon asset prices, resulting in the possibility of classifying carbon assets into non commodity assets. Gorenflo (2013) analyzes the pricing and lead–lag relation between EUA spot and futures prices.

Other papers for carbon markets focus on price determination in carbon markets. The relations between EUA futures prices and macroeconomic factors, including stock and bond market variables, are found in Chevallier (2009). Fezzi and Bunn (2009) show that carbon prices accompanied by natural gas prices drive electricity prices in the UK. Hintermann (2010) investigates whether marginal abatement costs explain EUA prices in the first phase of the EU ETS or not. Bredin and Muckley (2011) examine the impacts of economic growth, energy prices, and weather conditions on EUA futures prices. Chevallier (2011a) suggests that yearly compliance events and growing uncertainties in post-Kyoto international agreements may explain the instability in carbon price volatilities. Chevallier (2011b) also develops a carbon pricing model considering two fundamental EUA price drivers of economic activity and energy prices. Gronwald, Ketterer, and Trück (2011) find a strong dependence between EUA futures price returns and those of other financial assets and commodities during the period of the financial crisis. Aatola, Ollikainen, and Toppinen (2013) discover a strong relation between EUA prices and energy prices, including German electricity prices and gas and coal prices. Koch, Fuss, Grosjean, and Edenhofer (2014) conduct an ex-post empirical analysis of the reasons for the EUA price slump. Kanamura (2016) theoretically and empirically examines the role of carbon swap trading and energy prices in volatilities and price correlations between the EU and Kyoto Protocol emissions trading schemes. Gil-Alana, Gupta, and de Gracia (2016) analyze the persistence property of CO<sub>2</sub> allowance price accounting for structural breaks and non-linearities with CO<sub>2</sub> data from the three Phases of the EU ETS. Ji, Zhang, and Geng (2018) investigate information linkages and dynamic spillover effects between the carbon and energy markets by using a systemic time-series approach. Dutta, Bouri, and Noor (2018) study return and volatility linkages between EUA and clean energy stock indices using the bivariate VAR-GARCH approach. Dutta (2018) assesses whether outliers or extreme observations occur in EUA data, examines if time-varying jumps are present in the carbon emission market using the GARCH-jump models, and investigates the effect of oil market uncertainty on the emission prices using the crude oil volatility index.

While these studies keep eyes on carbon price models and the empirical analyses of carbon markets, they do not seem to pay attention to direct modeling of carbon prices using the linkage of carbon prices to crude oil prices. This paper theoretically and empirically revisits carbon pricing from the supply-side perspective for carbon assets to solve the recent low price issue which may delay the development of emission reduction technologies in the sense of marginal abatement costs. We propose a carbon pricing model linked to crude oil prices, which is employed in supply-side driven pricing of long-term contracts for early-stage energy trading. Since the model is designed to hold carbon prices between certain lower and upper boundaries using S-shaped carbon price linkage to crude oil prices, it can be useful to overcome a recent low carbon price issue. In addition, it is shown that the model can alleviate the difficulties of carbon derivative pricing in selecting market price of risk. Empirical studies using EUA and Brent crude oil futures prices estimate the parameters of the Brent crude oil-linked EUA price model. The comparison of EUA prices simulated from the model with historical EUA prices suggests that simulated EUA prices be kept relatively higher than historical EUA prices. This is preferable for accelerating carbon emission reductions in that it can make emission reduction technologies with

high marginal abatement costs affordable. It may imply that EUA must be priced using a crude oil-linked carbon price model in the early stage of EUA trading until EUA markets mature. This is a sharp contrast to current carbon markets employing premature market-based or supply and demand based pricing models. To show usefulness of crude oil-linked carbon pricing, we also give a numerical example of European carbon option pricing based on the Brent crude oil-linked EUA price model by using the Crank-Nicolson finite difference method. Finally we discuss the relation between crude oil-linked carbon pricing and emission reduction risk.

The remainder of this paper is organized as follows. Section 2 proposes a carbon pricing model which is linked to crude oil prices. Section 3 conducts empirical studies of EUA and Brent crude oil prices using the crude oil-linked carbon price model. Section 4 discusses the relation between crude oil-linked carbon pricing and emission reduction risk. Section 5 concludes.

## 2. The Model

### 2.1. A crude oil-linked carbon price model

It is observed that energy transaction including natural gas is conducted using long-term contracts in the early stage where prices are linked to crude oil prices from the supply-side perspective for energy. For example in practice, Japanese liquefied natural gas (LNG) is priced using the linkage to crude oil prices. Then after the liquidity of energy transaction increases, the pricing mechanism is transferred from the oil-linked pricing to market-based pricing where the supply and demand for energy determine the prices, e.g., the US natural gas market. On the other hand, while carbon assets are considered as energy-related assets, the pricing employs market-based pricing from the beginning where the supply and demand for emissions determine the prices, not based on energy-linked pricing. We consider that the development process of carbon markets is different from the other energy markets in the sense of pricing mechanism. When energy prices increase, the world economy is likely to grow rapidly as observed during commodity boom period from 2005 to 2007. It will increase the output of emissions in the world, resulting in high appreciation of carbon assets. Thus carbon prices may also have positive linkage to energy prices. Following this idea, we propose a carbon pricing model which is linked to crude oil prices. Crude oil price  $P_t$  model is given by

$$P_t = \left(1 + a \frac{V_t}{c}\right)^{\frac{1}{a}}, \quad (1)$$

$$dV_t = \mu_V dt + \sigma_V dw_t. \quad (2)$$

The inverse Box-Cox transformation function in Equation (1) and the stochastic process in Equation (2) represent a crude oil supply curve and the demand fluctuation of  $V_t$ , respectively. Assuming demand inelasticity to prices, i.e., a vertical demand curve, the equilibrium prices are obtained from the intersections of the supply and demand curves.

By using Ito's lemma, we have crude oil price model:

$$\frac{dP_t}{P_t} = \mu_P dt + \sigma_P dw_t, \quad (3)$$

$$\mu_P = \frac{\mu_V}{\sigma_V} \sigma_P + \frac{1}{2}(1 - a)\sigma_P^2, \quad (4)$$

$$\sigma_P = \frac{\sigma_V}{c} P_t^{-a}. \quad (5)$$

Carbon price  $C_t$  is defined using S-shaped logit model which has price cap (A) and floor (B).

$$C_t = \frac{Ae^{\alpha+\beta P_t} + B}{1 + e^{\alpha+\beta P_t}} \quad (6)$$

This is referred to as “a crude oil-linked carbon price model” we propose. Since the model holds carbon prices between certain lower and upper boundaries, it can accelerate to develop emission reduction technologies in the sense of marginal abatement costs by resolving a recent low carbon price issue. Note that

$$\frac{\partial C_t}{\partial P_t} = \frac{(A - B)\beta e^{\alpha+\beta P_t}}{(1 + e^{\alpha+\beta P_t})^2}. \quad (7)$$

Taking into account  $A - B > 0$  by definition, if  $\beta > 0$ , then carbon price  $C_t$  is an increasing function of crude oil price  $P_t$ .\*

## 2.2. Derivative pricing on a crude oil-linked carbon price model

It is in general difficult to price carbon derivatives due to the illiquidity of carbon derivative markets, resulting in incomplete market pricing of the carbon assets and the difficulties in selecting carbon market price of risk.<sup>†</sup> As the advantage of the crude oil-linked carbon price model, the market price of risk for carbon derivatives can be selected as crude oil market price of risk, which can be more reliable than carbon market price of risk in the sense of market liquidity. Thus in a first order approximation we can avoid the issue of incomplete market pricing of carbon assets, i.e., the issue of the selection of carbon market price of risk. The European call option price ( $f_t$ ) on carbon prices from crude oil prices is calculated using the following equation.

$$f_t = E_t \left[ \frac{\Lambda_T}{\Lambda_t} (C_T - K)^+ \right] \quad (8)$$

where  $C_T$  is carbon price at time  $T$  using crude oil price ( $P_T$ ),  $\Lambda_t$  is a stochastic discount factor at time  $t$ , and  $K$  is the strike of the call option. Here we assume the stochastic discount factor is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r dt - \phi_P dw_t. \quad (9)$$

Note that  $\phi_P = \frac{\mu_P - r}{\sigma_P}$  is crude oil market price of risk. By using Ito's Lemma, we have the partial differential equation (PDE) for the option prices:

$$\frac{\partial f_t}{\partial t} + r P_t \frac{\partial f_t}{\partial P_t} + \frac{1}{2} \sigma_P^2 P_t^2 \frac{\partial^2 f_t}{\partial P_t^2} - r f_t = 0, \quad (10)$$

\*The EU ETS does neither cover the oil demand from transportation sector nor the demand from the residential and most commercial oil combustion for heating or small-scale electricity generation purposes. One may put to question the assumed validity of a link between oil and carbon prices. However, since positive correlations of EUA prices with oil prices, which are determined by the supply and demand, have been confirmed as in e.g., Kanamura (2016), this model based on oil demand will hold in the first order approximation.

<sup>†</sup>We recognize the praise to its early liquidity in the EU ETS, e.g., Ellerman and Joskow (2008) and the World Bank's report of the much bigger transaction volume in EUA futures contracts than EUA spot contracts. However, the carbon market does not have enough liquidity to employ complete market pricing based on the perfect replication of risk as assumed in stock market option pricing.

$$f_T = (C_T - K)^+. \quad (11)$$

By solving the PDE from time  $T$  to  $t$  numerically, we can obtain the call option price at time  $t$ . It was shown that the crude oil-linked carbon pricing model can alleviate the difficulties of carbon derivative pricing in the sense of the selection of the market price of risk.

### 3. Empirical Studies

#### 3.1. Data

We use daily Brent crude oil and EUA futures price data of spot-month continuous products obtained from the website of Quandl. It covers from April 8, 2008 to June 5, 2009 whose sample size is 300. The motivation for the selected sample period is that the carbon market is considered to have been successful during the period because the prices are maintained as relatively high prices greater than around 10 EUR/tonne. The basic statistics are reported in Table 1. The skewness of Brent crude oil prices is positive while the skewness of EUA prices is negative, resulting in right-skewed and left-skewed distributions, respectively.

**Table 1.** Basic statistics of Brent crude oil and EUA futures prices: Note the skewness of Brent crude oil prices is positive while the skewness of EUA prices is negative, resulting in right-skewed and left-skewed distributions, respectively.

	Brent	EUA
Mean	81.14	20.14
Maximum	146.08	31.71
Minimum	36.61	8.43
Std. Dev.	35.52	6.44
Skewness	0.37	-0.05
Kurtosis	1.48	1.50

#### 3.2. The model parameter estimation

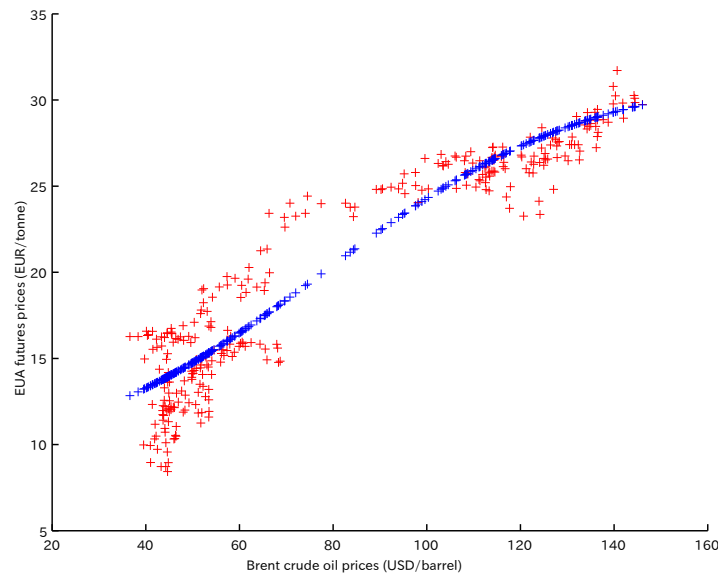
Assuming  $A = 31.71$  and  $B = 8.43$  which are maximum and minimum of the sample carbon price data, respectively in Table 1, we consider the following nonlinear regression model for Equation (6), i.e., a modified logit regression model.

$$C_t = \frac{Ae^{\alpha+\beta P_t} + B}{1 + e^{\alpha+\beta P_t}} + u_t \quad (12)$$

where  $u_t$  represents an error term. The estimation results of the non-linear regression model for Equation (12) are reported in Table 2 based on non-linear least squares. The parameters are statistically significant judging from the standard errors. Since  $\beta > 0$ , carbon prices increase in line with crude oil prices. In addition we draw the model estimation results on the scatter plots between Brent crude oil and EUA prices in Figure 3, which suggests that the model fit well to the historical scatter plots.

**Table 2.** Modified logit regression model parameter estimation for EUA and Brent crude oil prices: Note the parameters are statistically significant judging from the standard errors.

Parameters	$\alpha$	$\beta$
Estimates	-2.733	3.492E-02
Standard errors	0.077	9.787E-04
Log likelihood	290.695	
AIC	-577.389	
SIC	-569.982	



**Figure 3.** Scatter plots between Brent crude oil and historical (red plots) & estimated (blue plots) EUA futures prices (April, 2008 to June, 2009): Note the figure suggests that the model fit well to the historical scatter plots.

Then we estimate crude oil price model in Equation (3) to (5), which are discretized as

$$\log P_{t+1} - \log P_t = (k\sigma_P - \frac{1}{2}a\sigma_P^2)\Delta t + \sigma_P\epsilon_t, \quad (13)$$

$$\sigma_P = \bar{\sigma}_P P_t^{-a}. \quad (14)$$

Note that  $k = \frac{\mu_V}{\sigma_V}$ ,  $\bar{\sigma}_P = \frac{\sigma_V}{c}$ ,  $\Delta t = \frac{1}{252}$ , and  $\epsilon_t \sim N(0, \frac{1}{252})$ . The results using log likelihood estimation are reported in Table 3. Inverse Box-Cox parameter for the supply curve ( $a$ ) and volatility parameter for crude oil price ( $\bar{\sigma}_P$ ) are positive and statistically significant while the drift parameter ( $k$ ) is not statistically significant judging from the standard errors. The positive  $a$  suggests leverage effect for crude oil prices because the volatilities in crude oil prices decrease in line with the prices. This characteristic of crude oil is the same as those of financial products, which may come from the fact that crude oil futures are traded like financial products.

**Table 3.** Brent crude oil price model parameter estimation: Note inverse Box-Cox parameter for the supply curve ( $a$ ) and volatility parameter for crude oil price ( $\bar{\sigma}_P$ ) are positive and statistically significant while the drift parameter ( $k$ ) is not statistically significant judging from the standard errors.

Parameters	$k$	$a$	$\bar{\sigma}_P$
Estimates	−0.682	0.606	7.450
Standard errors	0.865	0.009	0.295
Log likelihood	613		
AIC	−1,220		
SIC	−1,226		

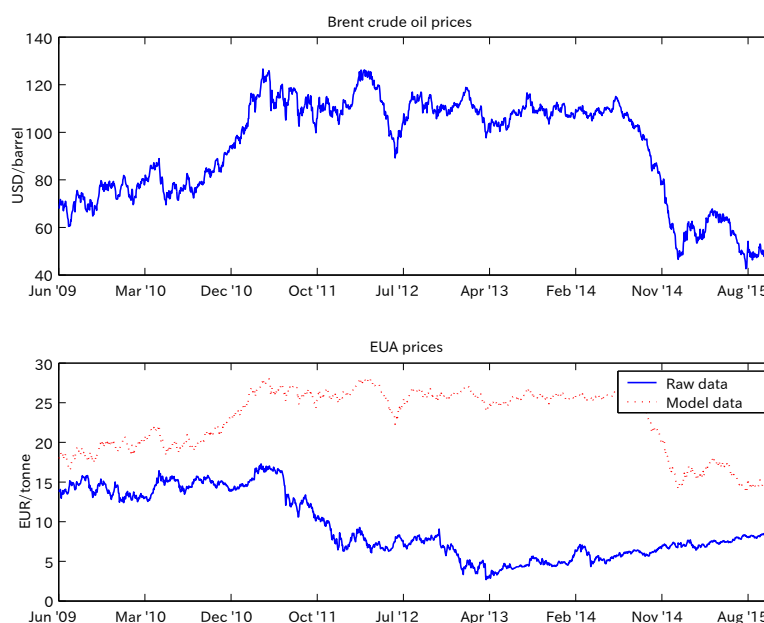
### 3.3. Out of sample data and crude oil-linked carbon price simulation

We simulate EUA prices from crude oil prices by using the Brent crude oil-linked EUA price model with the model parameter estimates obtained in the previous subsection. As out of sample data, we employ the data covering from June 8, 2009 to November 23, 2015. The simulation results accompanied by out of sample carbon price data are shown in Figure 4. It suggests that EUA prices simulated from the Brent crude oil-linked EUA price model are kept relatively higher than historical prices. The characteristics of simulated EUA prices are helpful to accelerate the developments of emission reduction technologies because high carbon prices allow emission reduction technologies with expensive marginal abatement costs. It may imply that EUA must be priced using a crude oil-linked carbon price model, not employing a premature market-based or supply and demand-based carbon price model.

We finally show ten sample paths simulated by using both Brent crude oil price model and the Brent crude oil-linked EUA price model in Equation (3) and (6), respectively. The results are reported in Figure 5. Both figures demonstrate that even if Brent prices are fluctuated dramatically as time goes by, EUA prices are kept between lower and upper boundaries. The simulation results using both the Brent crude oil price model and the Brent crude oil-linked EUA price model also suggest that a carbon price modeling linked to crude oil prices with some boundaries work well to promote the development of emission reduction technologies from the point of the relation between carbon prices and marginal abatement costs of new emission reduction technologies.

Recently, EUA prices have been double that level, recently reaching around 23 EUR/tonne. This rise in EUA prices is largely due to the supply-constraining effect of the market stability reserve (MSR), which is initially only anticipated and now also actual. However, these changing fundamentals and ETS design do not necessarily guarantee the underlying support of carbon prices to avoid carbon price declines in the future. In that sense, the model proposed in this paper will contribute to the robustness of carbon markets.

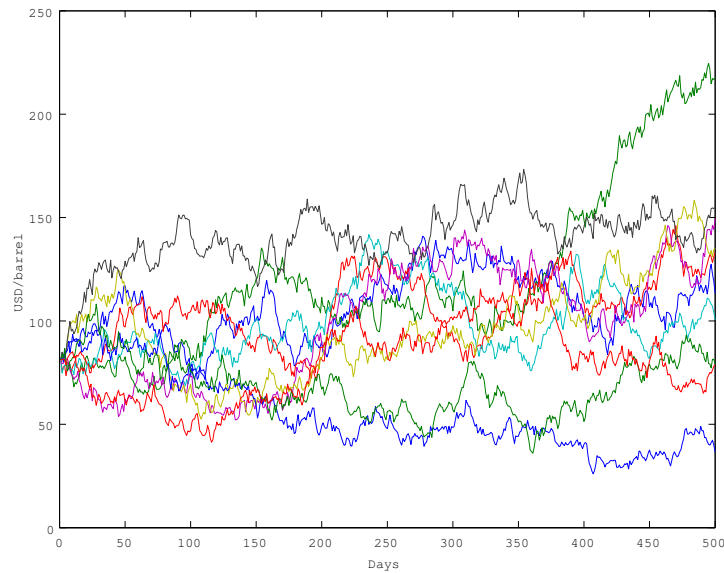
Regarding the link between crude oil and the other commodities prices, including carbon prices, that underlies the “crude oil-linked carbon price model,” the link between natural gas and oil prices has come under pressure because supply and demand fundamentals of both are increasingly divorced from each other: e.g., for supply side hydraulic fracturing in North America and elsewhere has partly disrupted the traditional correlation between oil and gas production while for demand side oil



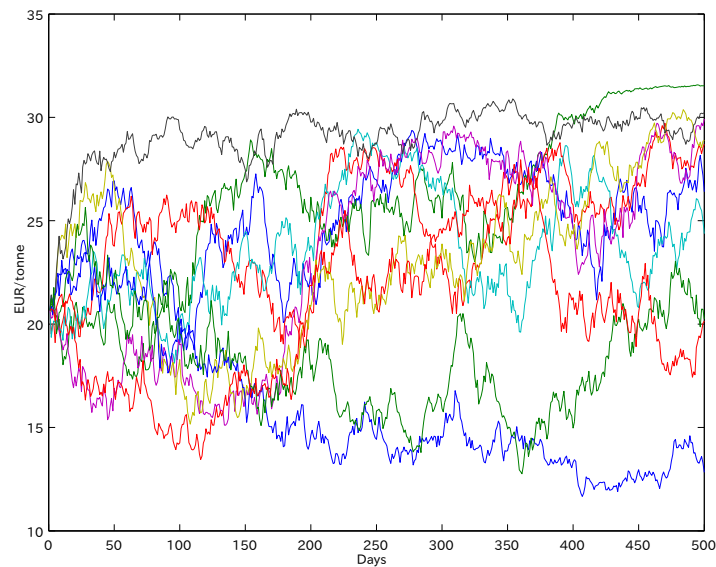
**Figure 4.** Brent crude oil prices and EUA prices (historical raw data & model simulated data): Note they suggest that EUA prices simulated from the Brent crude oil-linked EUA price model be maintained as relatively higher prices than historical prices. It may imply that EUA must be priced using a crude oil-linked carbon price model, not employing a premature market-based or supply and demand-based carbon price model.

consumption may stagnate or drop in some jurisdictions due to fuel efficiency and electric vehicle mandates for transportation, yet natural gas demand may rise if coal to gas switching is encouraged in the power sector. Similarly, the stated link between oil demand growth and carbon emissions growth may seem tenuous. As the power and transportation sectors further decarbonize, the fundamental link between oil prices and carbon prices may weaken even further — in scenarios that see global policy developments aligned with committed decarbonization targets, oil prices should be very low by 2050 due to policy-induced demand constraints, i.e., the supply will be steady and the demand near zero, while carbon prices should be in the triple or even quadruple digits. Additionally, the weak link here is the assumed correlation between economic (and emissions) growth and high oil prices. This may suggest a very pessimistic outlook on the continued innovation in and penetration of low- and zero-carbon technologies that may sever that correlation. While economic growth may continue to go hand in hand with increased energy demand, this increased energy demand may not necessarily translate into emissions growth, as it has in the past. Committed energy efficiency improvements and energy intensity reductions may also change the historical link between economic growth and energy demand. However, the carbon price model proposed in this research is useful on a short-term basis as seen in other commodity markets such as natural gas markets when the carbon market is immature and the imbalance between supply and demand leads to low carbon prices. Therefore, this model is not semi-permanent and if it is possible to maintain an appropriate carbon price level as a result of the supply and demand being reflected in the price, the crude oil-linked carbon pricing will shift to pricing based on supply and demand as with other commodity markets.

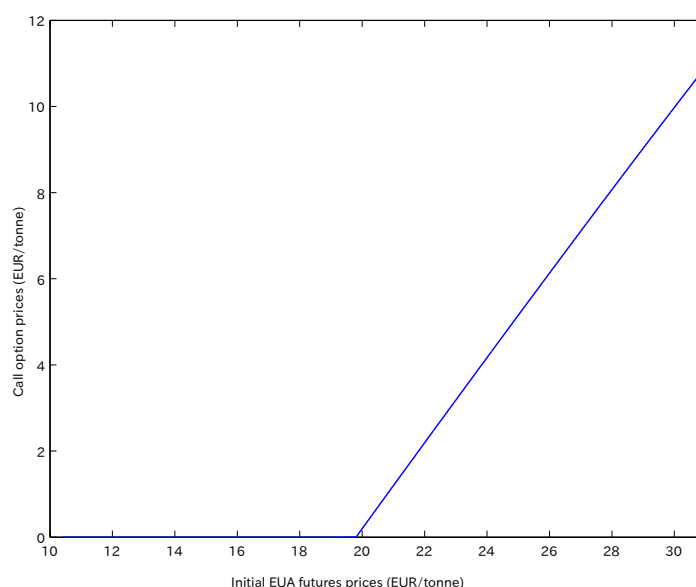
Panel A Brent crude oil price



Panel B EUA price



**Figure 5.** Sample Paths of Brent crude oil prices and EUA prices: Note both figures demonstrate that even if Brent prices are fluctuated dramatically as time goes by, EUA prices are kept between lower and upper boundaries.



**Figure 6.** European carbon option price based on the Brent crude oil-linked EUA price model: Note that the maturity is 3 months and the strike is 20 EUR/tonne. The results empirically suggest that the model can alleviate the difficulties of carbon derivative pricing in selecting the market price of risk by showing that the call option prices are numerically obtained based on the crude oil-linked carbon pricing.

### 3.4. Carbon call option pricing

In order to show usefulness of crude oil-linked carbon pricing, we give a numerical example of European carbon option pricing based on the Brent crude oil-linked EUA price model by using the Crank-Nicolson finite difference method. The PDE in Equation (10) is solved backwards in time from 0.25 year (3 months) to 0 by setting the terminal condition in Equation (11). Note that we assume  $r = 0.05$  per annum. The results are reported in Figure 6 where the strike is 20 EUR/tonne for EUA price, which is close to the sample average of Table 1. The results empirically suggest that the model can alleviate the difficulties of carbon derivative pricing in the sense of selecting the market price of risk by showing that the call option prices are numerically obtained based on the crude oil-linked carbon pricing.

## 4. Discussions

When we look at the track records of the relations between crude oil and carbon prices as in Figure 3, carbon prices move in line with crude oil prices. This is reasonable in the sense of the relations between world economic development and carbon emission outputs. If energy consumption, in particular crude oil consumption, increases, not only will crude oil price rise due to the supply and demand, but carbon prices will also rise due to the increase in energy consumption. It implies that energy consumers will economically have burdens from the rise in carbon prices as well as those from the rise in energy prices. Thus energy consumers have strong incentives to reduce energy usage in order to alleviate both impacts of energy and carbon prices on their businesses as well as to accelerate

new emission reduction technologies with lower marginal abatement costs than carbon prices. In contrast if energy consumption level is low, then emissions themselves become small. In this case we do not need any new emission reduction technologies. In summary, when emissions increase due to energy consumption, the introduction of a crude oil-linked carbon price model to carbon markets forces energy consumers to reduce the emissions. In contrast, when emissions decrease due to energy consumption, the introduction of the model makes energy consumers release their burdens from emission reductions. In this sense, the introduction of a crude oil-linked carbon price model to carbon markets may offer a natural hedge of emission reduction risk for energy consumers.

## 5. Conclusions

This paper theoretically and empirically revisited carbon pricing from the supply-side perspective for carbon assets to solve the recent low price issue which may delay the development of emission reduction technologies in the sense of marginal abatement costs. We proposed a carbon pricing model linked to crude oil prices, which is employed in supply-side driven pricing of long-term contracts for early-stage energy trading. Since the model is designed to hold carbon prices between certain lower and upper boundaries using S-shaped carbon price linkage to crude oil prices, it can be useful to overcome a recent low carbon price issue. In addition, it was shown that the model can alleviate the difficulties of carbon derivative pricing in selecting market price of risk. Empirical studies using EUA and Brent crude oil futures prices estimated the parameters of the Brent crude oil-linked EUA price model. The comparison of EUA prices simulated from the model with historical EUA prices suggests that simulated EUA prices be kept relatively higher than historical EUA prices. This is preferable for accelerating carbon emission reductions in that it can make emission reduction technologies with high marginal abatement costs affordable. It may imply that EUA must be priced using a crude oil-linked carbon price model in the early stage of EUA trading until EUA markets mature. This is a sharp contrast to current carbon markets employing premature market-based or supply and demand based pricing models. To show usefulness of crude oil-linked carbon pricing, we also offered a numerical example of European carbon option pricing based on the Brent crude oil-linked EUA price model by using the Crank-Nicolson finite difference method. Finally we discussed the relation between crude oil-linked carbon pricing and emission reduction risk and showed that the introduction of a crude oil-linked carbon price model to carbon markets may offer a natural hedge of emission reduction risk for energy consumers. These studies may suggest carbon policy makers take account of crude oil-linked carbon pricing to tackle low price and low liquidity issues of carbon assets.

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## Conflict of interest

The author declares no conflict of interest.

## References

- Aatola P, Ollikainen M, Toppinen A (2013) Price determination in the EU ETS market: Theory and econometric analysis with market fundamentals. *Energy Econ* 36: 380–395.
- Benz E, Trück S (2009) Modeling the price dynamics of CO<sub>2</sub> emission allowances. *Energy Econ* 31: 4–15.
- Bredin D, Muckley C (2011) An emerging equilibrium in the EU emissions trading scheme. *Energy Econ* 33: 353–362.
- Chevallier J (2009) Carbon futures and macroeconomic risk factors: A view from the EU ETS. *Energy Econ* 31: 614–625.
- Chevallier J (2011a) Detecting instability in the volatility of carbon prices. *Energy Econ* 33: 99–110.
- Chevallier J (2011b) A model of carbon price interactions with macroeconomic and energy dynamics. *Energy Econ* 33: 1295–1312.
- Daskalakis G, Psychoyios D, Markellos RN (2009) Modeling CO<sub>2</sub> emission allowance prices and derivatives: Evidence from the European trading scheme. *J Bank Financ* 33: 1230–1241.
- Dutta A (2018) Modeling and forecasting the volatility of carbon emission market: The role of outliers, time-varying jumps and oil price risk. *J Clean Prod* 172: 2773–2781.
- Dutta A, Bouri E, Noor MH (2018) Return and volatility linkages between CO<sub>2</sub> emission and clean energy stock prices. *Energy* 164: 803–810.
- Ellerman AD, Joskow PL (2008) *The European Union's Emissions Trading System in Perspective*, Washington, DC: Pew Center on Global Climate Change.
- Fehr M, Hinz J (2006) A quantitative approach to carbon price risk modeling. Working Paper, Institute of Operations Research, ETZ.
- Fezzi C, Bunn DW (2009) Structural interactions of European carbon trading and energy prices. *J Energy Mark* 2: 53–69.
- Gil-Alana LA, Gupta R, de Gracia FP (2016) Modeling persistence of carbon emission allowance prices. *Renew Sustain Energy Rev* 55: 221–226.
- Gorenflo M (2013) Futures price dynamics of CO<sub>2</sub> emission allowances. *Empir Econ* 45: 1025–1047.
- Gronwald M, Ketterer J, Trück S (2011) The relationship between carbon, commodity and financial markets: A copula analysis. *Econ Rec* 87: 105–124.
- Hintermann B (2010) Allowance price drivers in the first phase of the EU ETS. *J Environ Econ Manage* 59: 43–56.

- Ji Q, Zhang D, Geng J (2018) Information linkage, dynamic spillovers in prices and volatility between the carbon and energy markets. *J Clean Prod* 198: 972–978.
- Kanamura T (2009a) A classification study of carbon assets into commodities. Working Paper, SSRN.
- Kanamura T (2009b) A supply and demand based volatility model for energy prices. *Energy Econ* 31: 736–747.
- Kanamura T (2016) Role of carbon swap trading and energy prices in price correlations and volatilities between carbon markets. *Energy Econ* 54: 204–212.
- Koch N, Fuss S, Grosjean G, et al. (2014) Causes of the EU ETS price drop: Recession, CDM, renewable policies or a bit of everything? —New evidence. *Energy Policy* 73: 676–685.
- Paolella MS, Taschini L (2008) An econometric analysis of emission allowance prices. *J Bank Financ* 32: 2022–2032.
- Seifert J, Uhrig-Homburg M, Wagner M (2008) Dynamic behavior of CO<sub>2</sub> spot prices. *J Environ Econ Manage* 56: 180–194.
- Stern JP (2007) Is there a rationale for the continuing link to oil product prices in continental European long-term gas contracts? *Int J Energy Sect Manage* 1: 221–239.
- Trück S, Härdle W, Weron R (2015) The relationship between spot and futures CO<sub>2</sub> emission allowance prices in the EU ETS, *In: Gronwald M, Hintermann B (eds) Emissions Trading as a Policy Instrument: Evaluation and Prospects*, Cambridge MIT Press, 183–212.
- Uhrig-Homburg M, Wagner M (2009) Futures price dynamics of CO<sub>2</sub> emission allowances: An empirical analysis of the trial period. *J Deriv* 17: 73–88.



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