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Research article

Anti-stabilizing dynamics in fuzzy multidirectional associative memory neural networks with discrete spatiotemporal structure

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Abstract: Understanding and shaping the dynamics of neural architectures with uncertainty, memory, and spatial interactions is a fundamental problem in neural networks and adaptive systems. In particular, controlled destabilization plays an important role in promoting exploration, adaptability, and non-stationary behavior, yet remains far less studied than stabilization and convergence. In this paper, we investigated destabilizing dynamics in a class of space–time discrete fuzzy multidirectional associative memory (MAM) neural networks with time-varying delays and diffusion effects. Such networks integrate fuzzy rule-based representations, delayed feedback, and spatial coupling, and are relevant to adaptive control, associative memory, and multi-agent dynamical systems. We first established the existence of equilibrium states by using topological degree theory, which provides a rigorous foundation for the subsequent analysis. Then, by designing localized Dirichlet boundary feedback mechanisms and constructing novel discrete Lyapunov–Krasovskii functionals with delay-dependent double-sum terms, we derived verifiable sufficient conditions for global asymptotic and exponential anti-stabilization. These results characterize how diffusion intensity, fuzzy parameters, and self-inhibition coefficients influence destabilizing behavior and determine the rate of divergence from equilibrium. The proposed framework provides new theoretical insights into anti-stabilization dynamics in discrete spatiotemporal fuzzy neural networks. Numerical examples further support the theoretical analysis and demonstrate the effectiveness of the proposed approach.

Keywords: multidirectional associative memory; neural networks; fuzzy model; space-time discretizations; anti-stabilization

1. Introduction

Multidirectional associative memory neural networks (MAMNNs) were designed to mimic certain aspects of human memory and cognitive processes. MAMNNs were influenced by neurobiological concepts such as Hebbian learning, which suggests that neurons that fire together, wire together, which were one of the early attempts to model associative memory using neural networks. They were proposed as a way to address some limitations of earlier models like the Hopfield network, such as the inability to handle bidirectional associations effectively. MAMNNs have found applications in various fields such as pattern recognition, information retrieval, content-based image retrieval, and cognitive modeling [1–6]. Their ability to handle bidirectional associations and robustness to noise makes them valuable in these domains. In sum, MAMNNs represent an important milestone in the development of neural network models for mimicking aspects of human memory and cognition, with practical implications in diverse areas of artificial intelligence and computational neuroscience. In 2024, Zhang et al. [1] studied the existence, uniqueness, and global exponential stability of the almost periodic solutions of MAMNNs in the sense of the Filippov solution. According to the theory of differential inclusions, Yan et al. [2] derived some results for fixed-time synchronization for the MAMNNs with discontinuous nonlinear functions. Chaouki and Touati [4] considered the issue of dissipativity for MAMNNs in the Clifford field by adopting the method of the Lyapunov functional.

Fuzzy neural networks (FNNs) combine fuzzy logic, which deals with uncertainty and imprecision, with neural networks, which are powerful tools for learning from data. Fuzzy logic allows for the representation of vague concepts using linguistic variables and fuzzy sets, which can have degrees of membership rather than strict binary values [7, 8]. Besides, fuzzy logic allows for the creation of rule-based systems using linguistic variables and fuzzy rules. FNNs leverage this capability to construct rule-based models that can capture complex relationships between inputs and outputs, making them suitable for applications in control systems, expert systems, and decision support. FNNs are used in a wide range of applications such as control systems, pattern recognition, forecasting, data mining, and medical diagnosis. What is important is that FNNs play a significant role in bridging the gap between fuzzy logic and neural networks, offering solutions to problems that involve uncertainty [9–12], imprecision, and complex relationships in data or knowledge representation. Moreover, they are well-suited to real-world problems with complex and uncertain information due to their ability to handle uncertainty [13–16]. On the other hand, time-delayed neural networks (TDNNs) were developed to address problems [17–21] where data has a sequential or temporal structure, such as time series data, speech signals, sensor data streams, etc. Traditional feedforward neural networks lack the ability to handle temporal dependencies directly. TDNNs capture temporal context by introducing delay elements in the network architecture. These delays allow the network to consider past inputs and outputs, enabling it to learn and model temporal patterns and dependencies in the data. TDNNs are also used in modeling dynamic systems where the behavior evolves over time. By considering time delays, these networks can learn the dynamics of the system and make predictions or control decisions accordingly. TDNNs play a vital role in handling temporal information and sequential data, making them indispensable in various domains ranging from signal processing and time series analysis to dynamic systems modeling and real-time applications [1, 2, 4–6, 13].

Space-time discrete neural networks process data across spatial and temporal dimensions in discrete steps [22, 23]. The model proposed in this paper can be viewed as a discrete-space-time counterpart of reaction–diffusion neural networks that have been widely studied in the literature; see, for example, [24–29]. These are spatiotemporal data networks designed to handle information that varies over space and time. Video sequences, time-series data from sensors, and spatiotemporal patterns in scientific data are just a few examples of data that can be handled by such networks. Space-time discrete networks have been developed to effectively process complex data. While traditional neural networks are capable of handling spatial or temporal data independently, they cannot manage complex data. These are a game-changer in neural network research, allowing us to process and analyze complex spatiotemporal data like never before. Without question, they are essential for handling dynamic data. Their value lies in improving forecasting and providing real-time insights across a wide range of applications. This includes video analytics, environmental monitoring, and autonomous systems. These networks will continue to grow in importance as technology and methodology advance, playing a pivotal role within numerous scientific and applied domains [30, 31]. The work in [30, 31] constructed several discrete time-space networks and conducted qualitative analyses on these systems. For instance, they investigated the concept of almost automorphism [30] and random periodicity [31], among other topics.

Stabilization for neural networks relates to ensuring that these networks exhibit desirable properties such as convergence, boundedness, and robustness during training and inference [32]. During training, neural networks adjust their weights and biases based on input data and optimization algorithms (e.g., backpropagation). Unstable training dynamics can lead to issues like exploding or vanishing gradients, which hinder convergence and learning. Different network architectures (e.g., feedforward, recurrent, convolutional) and activation functions (e.g., ReLU, sigmoid, tanh) can influence the stability of neural networks [33, 34]. For example, certain activation functions may be prone to gradient saturation, impacting stability. By stabilizing training dynamics, neural networks can achieve higher performance and accuracy on tasks such as classification, regression, and pattern recognition. Stable networks are less likely to get stuck in poor local minima or exhibit erratic behavior during inference. Systems operating in dynamic or uncertain environments often benefit from anti-stabilization. These environments may include complex interactions, changing conditions, or unpredictable events where adaptability and flexibility are crucial [35–42]. Further research on anti-stabilization for discrete-time neural networks is still needed; see, for example, [35, 38]. There is a noticeable gap when we look at space-time discrete neural networks. We have identified a pressing need to think about exponential anti-stabilization of space-time discrete MAMNNs with time-varying delays as an important research topic that will make a big impact.

Although anti-stabilization has attracted increasing attention in recent years, the existing results are still mainly restricted to continuous-time neural networks or purely discrete-time neural networks without spatial diffusion structure; see, for example, [35, 38]. For fuzzy MAMNNs with time-varying delays, the anti-stabilization problem in a space-time discrete setting has not yet been adequately studied. This issue is more challenging because the simultaneous presence of spatial diffusion, fuzzy MIN/MAX couplings, time-varying delays, and boundary feedback controllers makes the dynamical analysis substantially different from the existing frameworks. Motivated by this gap, this paper investigates asymptotic and exponential anti-stabilization of fuzzy MAMNNs in a discrete spatiotemporal framework by means of the Lyapunov–Krasovskii technique, and the main contributions are summarized as follows.

1) The completely original structure of discrete fuzzy MAMNNs in time-space schemes represents a significant advancement over previous discrete-time neural networks [35, 38], creating a more comprehensive approach that is worthy of attention.

2) The equilibrium point of the discrete fuzzy MAMNNs in a time-space scheme is investigated in accordance with the theory of topological degree involving the Brouwer degree.

3) We provide a detailed analysis of the innovative subject matter pertaining to asymptotic anti-stabilization for discrete fuzzy MAMNNs, with a focus on the controllers within the Dirichlet boundaries. This approach diverges from the established methodologies outlined in the extant literature, specifically those derived from Neumann boundaries [23].

4) Our method for exponential anti-stabilization of the discrete fuzzy MAMNNs incorporates a Lyapunov-Krasovskii functional involving a double sum of a delay-dependent component. This is a significant step forward and builds upon existing results for the anti-stabilization of discrete-time neural networks [35, 38] and continuous-time neural networks [36, 37, 39–42].

Throughout this article, we denote $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_0 := [0, \infty)$, $\mathbb{Z}_+ := \{1, 2, \ldots\}$, $\mathbb{Z}_0 := \mathbb{Z}_+ \cup \{0\}$, $\mathbb{Z} = \mathbb{Z}_0 \cup (-\mathbb{Z}_+)$, $\mathcal{n}_p := \{1, 2, \ldots, n_p\}$, $\mathcal{m} = \{1, 2, \ldots, m\}$. To describe the spatial interaction and diffusion effect in the space-time discrete neural network, we introduce the forward spatial difference operator and its second-order forms. Here, κ denotes the discrete spatial node, k denotes the discrete time level, and $\iota > 0$ is the spatial mesh size. For a state variable $z_k^{[\kappa]}$, the quantity $\Delta_\iota z_k^{[\kappa]} = \frac{z_k^{[\kappa+1]} - z_k^{[\kappa]}}{\iota}$ represents the local spatial variation between two adjacent nodes, and thus measures how the neural state changes along the spatial direction. Based on this, we define

$$\Delta_\iota^2 z_k^{[\kappa]} = \Delta_\iota(\Delta_\iota z_k^{[\kappa]}), \qquad \Delta_{c,\iota}^2 z_k^{[\kappa]} = \Delta_\iota(\Delta_\iota z_k^{[\kappa-1]}),$$

where $z = z_k^{[\kappa]} : \mathbb{Z}^2 \to \mathbb{R}^n$, $(\kappa, k) \in \mathbb{Z}^2$, $\iota > 0$. In particular, $\Delta_{c,\iota}^2 z_k^{[\kappa]}$ is the central second-order spatial difference, which serves as the discrete analogue of the spatial diffusion operator in the underlying continuous reaction-diffusion model. Hence, this term characterizes the diffusion-driven information exchange among neighboring spatial nodes and plays a key role in modeling the spatiotemporal propagation behavior of the neural network.

2. Space-time discrete fuzzy MAMNNs

Let us discuss the following space-time discrete MAMNNs with time-varying delays:

$$
\begin{aligned}
z^{[\kappa]}_{pi,k+1} = {} & e^{-b_{pi}h} z^{[\kappa]}_{pi,k} + \frac{1-e^{-b_{pi}h}}{b_{pi}} \Big[a_{pi} \Delta^2_{c,\iota} z^{[\kappa]}_{pi,k} \\
& + \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} c^{qj}_{pi} f_{qj}(z^{[\kappa]}_{qj,k}) + \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} d^{qj}_{pi} f_{qj}(z^{[\kappa]}_{qj,\rho_{qj,k}}) \\
& + \sum_{q=1,q\neq p}^{m} \bigvee_{j=1}^{n_q} \beta^{qj}_{pi} f_{qj}(z^{[\kappa]}_{qj,\rho_{qj,k}}) + \sum_{q=1,q\neq p}^{m} \bigwedge_{j=1}^{n_q} \gamma^{qj}_{pi} f_{qj}(z^{[\kappa]}_{qj,\rho_{qj,k}}) + \alpha_{pi} \Big],
\end{aligned} \tag{2.1}
$$

where $(\kappa, k) \in (0, \Bbbk)_{\mathbb{Z}} \times \mathbb{Z}_0$, $\Bbbk > 1$, is a fixed integer; $i \in \mathcal{N}_p$, $p \in \mathcal{M}$; $m \geq 3$; z_{pi} shows the state of the ith neuron of layer p; $a_{pi}, b_{pi} > 0$ represent the diffusive and self-inhibition coefficients; c^{qj}_{pi}, d^{qj}_{pi} display the linking weights; f_{qj} stands for the activation function; $\bigvee, \bigwedge$ show the fuzzy OR and AND calculations, respectively; $\beta^{qj}_{pi}, \gamma^{qj}_{pi}$ denote the parameters corresponding to fuzzy MIN and MAX feedbacks, respectively; $k - \rho_{qj,k} \in [0, \rho_\infty]_{\mathbb{Z}}$ for all $k \in \mathbb{Z}_0$, which denotes the time delay; α_{pi} represents the external disturbance, $i \in \mathcal{N}_p$, $j \in \mathcal{N}_q$, $p \in \mathcal{M}$. In this model, the state variable represents the activity of neurons distributed over discrete spatial nodes and evolving at discrete time instants, the term $\Delta^2_{c,\iota} z^{[\kappa]}_{pi,k}$ describes diffusion-driven information exchange between neighboring spatial sites, the delayed terms characterize memory and transmission effects, and the fuzzy MIN/MAX terms reflect rule-based uncertain interactions. Therefore, model in Eq (2.1) can be interpreted as a discrete spatiotemporal neural dynamical system arising from the discretization of continuous neural fields with diffusion, delay, and fuzzy coupling, which is relevant to distributed associative memory, signal propagation, and networked neural processing systems.

For the MAMNNs in Eq (2.1), several efficacious, discrete methodologies are utilized, encompassing the exponential Euler time difference and the central finite space difference. For more information, please refer to [30, 31]. Additionally, the MAMNNs in Eq (2.1) possess the initial, boundary values as noted below:

$$
\begin{cases}
z^{[\kappa]}_{pi,s} = \varphi^{[\kappa]}_{pi,s}, & \forall s \in [-\rho_\infty, 0]_{\mathbb{Z}}, \kappa \in [0, \Bbbk]_{\mathbb{Z}}; \\
z^{[0]}_{pi,k} = 0 = z^{[\Bbbk]}_{pi,k}, & \forall k \in \mathbb{Z}_0, i \in \mathcal{N}_p, p \in \mathcal{M}.
\end{cases} \tag{2.2}
$$

The MAMNNs in Eq (2.1) are a time-space differencing model of the continuous reaction-diffusion MAMNNs [30, 31] with time-varying delay as shown below:

$$
\begin{aligned}
\frac{\partial z_{pi}(x,t)}{\partial t} = {} & a_{pi} \frac{\partial^2 z_{pi}(x,t)}{\partial x^2} - b_{pi} z_{pi}(x,t) + \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} c^{qj}_{pi} f_{qj}(z_{qj}(x,t)) \\
& + \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} d^{qj}_{pi} f_{qj}(z_{qj}(x, \rho_{qj}(t))) + \sum_{q=1,q\neq p}^{m} \bigvee_{j=1}^{n_q} \beta^{qj}_{pi} f_{qj}(z_{qj}(x, \rho_{qj}(t)))
\end{aligned}
$$

$$+\sum_{q=1,q\neq p}^{m}\bigwedge_{j=1}^{n_q}\gamma_{pi}^{qj}f_{qj}(z_{qj}(x,\rho_{qj}(t)))+a_{pi},\quad \forall (x,t)\in(0,\delta)\times\mathbb{R}_0, \tag{2.3}$$

where $\delta>0$, $i\in\mathcal{N}_p$, $p\in\mathcal{M}$.

Definition 2.1 ([43])**.** *The equilibrium point of the MAMNNs in Eq* (2.1) *is an n_p-dimensional constant vector $z_p^*=(z_{p1}^*,\ldots,z_{pn_p}^*)^T\in\mathbb{R}^{n_p}$ satisfying*

$$z_{pi}^*=\frac{1}{b_{pi}}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}(c_{pi}^{qj}+d_{pi}^{qj})f_{qj}(z_{qj}^*)+\frac{1}{b_{pi}}\sum_{q=1,q\neq p}^{m}\Big(\bigvee_{j=1}^{n_q}\beta_{pi}^{qj}+\bigwedge_{j=1}^{n_q}\gamma_{pi}^{qj}\Big)f_{qj}(z_{qj}^*)+\frac{a_{pi}}{b_{pi}}, \tag{2.4}$$

where $i\in\mathcal{N}_p$, $p\in\mathcal{M}$.

Throughout this article, suppose that the network in Eq (2.1) admits at least one equilibrium point denoted by $z_p^*=(z_{p1}^*,\ldots,z_{pn_p}^*)^T\in\mathbb{R}^{n_p}$, that is, it is valid for

$$\begin{aligned} z_{pi,k+1}^{*[\kappa]} &= e^{-b_{pi}h}z_{pi,k}^{*[\kappa]}+\frac{1-e^{-b_{pi}h}}{b_{pi}}\Big[a_{pi}\Delta_{c,\iota}^2 z_{pi,k}^{*[\kappa]}\\ &\quad+\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}c_{pi}^{qj}f_{qj}(z_{qj,k}^{*[\kappa]})+\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}d_{pi}^{qj}f_{qj}(z_{qj,\rho_{qj,k}}^{*[\kappa]})\\ &\quad+\sum_{q=1,q\neq p}^{m}\bigvee_{j=1}^{n_q}\beta_{pi}^{qj}f_{qj}(z_{qj,\rho_{qj,k}}^{*[\kappa]})+\sum_{q=1,q\neq p}^{m}\bigwedge_{j=1}^{n_q}\gamma_{pi}^{qj}f_{qj}(z_{qj,\rho_{qj,k}}^{*[\kappa]})+a_{pi}\Big], \end{aligned} \tag{2.5}$$

where $(\kappa,k)\in(0,\mathbb{k})_{\mathbb{Z}}\times\mathbb{Z}_0$, and it meets the following initial and boundary values:

$$\begin{cases} z_{pi,s}^{*[\kappa]}=z_{pi}^*, & \forall s\in[-\rho_\infty,0]_{\mathbb{Z}},\kappa\in[0,\mathbb{k}]_{\mathbb{Z}};\\ z_{pi,k}^{*[0]}=z_{pi}^*, z_{pi,k}^{*[\mathbb{k}]}=z_{pi}^*, & \forall k\in\mathbb{Z}_0, i\in\mathcal{N}_p, p\in\mathcal{M}. \end{cases}$$

Consider networks in Eq (2.5) as the master networks and the corresponding slave networks are established as

$$\begin{aligned} \varpi_{pi,k+1}^{[\kappa]} &= e^{-b_{pi}h}\varpi_{pi,k}^{[\kappa]}+\frac{1-e^{-b_{pi}h}}{b_{pi}}\Big[a_{pi}\Delta_{c,\iota}^2 \varpi_{pi,k}^{[\kappa]}\\ &\quad+\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}c_{pi}^{qj}f_{qj}(\varpi_{qj,k}^{[\kappa]})+\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}d_{pi}^{qj}f_{qj}(\varpi_{qj,\rho_{qj,k}}^{[\kappa]})\\ &\quad+\sum_{q=1,q\neq p}^{m}\bigvee_{j=1}^{n_q}\beta_{pi}^{qj}f_{qj}(\varpi_{qj,\rho_{qj,k}}^{[\kappa]})+\sum_{q=1,q\neq p}^{m}\bigwedge_{j=1}^{n_q}\gamma_{pi}^{qj}f_{qj}(\varpi_{qj,\rho_{qj,k}}^{[\kappa]})-a_{pi}\Big], \end{aligned} \tag{2.6}$$

where $(\kappa,k)\in(0,\mathbb{k})_{\mathbb{Z}}\times\mathbb{Z}_0$, $i\in\mathcal{N}_p$, $p\in\mathcal{M}$. Further, the MAMNNs in Eq (2.6) admit the initial and controlled boundary values as noted below:

$$\begin{cases} \varpi_{pi,s}^{[\kappa]}=\phi_{pi,s}^{[\kappa]}, & \forall s\in[-\rho_\infty,0]_{\mathbb{Z}},\kappa\in[0,\mathbb{k}]_{\mathbb{Z}};\\ \varpi_{pi,k}^{[0]}=-z_{pi}^*, \varpi_{pi,k}^{[\mathbb{k}]}=\vartheta_{pi,k}, & \forall k\in\mathbb{Z}_0, \end{cases} \tag{2.7}$$

where $\vartheta_{pi,k}$ denotes the boundary control input of the ith neuron in field p in time $k \in \mathbb{Z}_0$, $i \in \mathcal{N}_p$, $p \in \mathcal{M}$.

We construct an anti-synchronized networks by setting $\omega_{pi} = \varpi_{pi} + z^*_{pi}$, which is govern by the partial difference equations below:

$$\begin{aligned}\omega^{[\kappa]}_{pi,k+1} = {} & e^{-b_{pi}h}\omega^{[\kappa]}_{pi,k} + \frac{1-e^{-b_{pi}h}}{b_{pi}}\Big[a_{pi}\Delta^2_{c,\iota}\omega^{[\kappa]}_{pi,k} \\ & + \sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q} c^{qj}_{pi}\tilde{f}_{qj}(\omega^{[\kappa]}_{qj,k}) + \sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q} d^{qj}_{pi}\tilde{f}_{qj}(\omega^{[\kappa]}_{qj,\rho_{qj,k}}) \\ & + \sum_{q=1,q\neq p}^{m}\bigvee_{j=1}^{n_q}\beta^{qj}_{pi}\tilde{f}_{qj}(\omega^{[\kappa]}_{qj,\rho_{qj,k}}) + \sum_{q=1,q\neq p}^{m}\bigwedge_{j=1}^{n_q}\gamma^{qj}_{pi}\tilde{f}_{qj}(\omega^{[\kappa]}_{qj,\rho_{qj,k}})\Big], \end{aligned} \tag{2.8}$$

where $(\kappa, k) \in (0, \Bbbk)_{\mathbb{Z}} \times \mathbb{Z}_0$, $\tilde{f}_{qj}(\omega) = f_{qj}(z_{qj}) + f_{qj}(z^*_{qj})$, in which $i \in \mathcal{N}_p$, $j = 1, 2, \ldots, n_q$, $p, q = 1, 2, \ldots, m$. The initial and boundary conditions of the anti-synchronized networks in Eq (2.8) are traced as

$$\begin{cases}\omega^{[\kappa]}_{pi,s} = \phi^{[\kappa]}_{pi,s} + z^*_{pi}, & \forall s \in [-\rho_\infty, 0]_{\mathbb{Z}}, \kappa \in [0, \Bbbk]_{\mathbb{Z}};\\ \omega^{[0]}_{pi,k} = 0, \omega^{[\Bbbk]}_{pi,k} = \vartheta_{pi,k} + z^*_{pi}, & \forall k \in \mathbb{Z}_0,\end{cases} \tag{2.9}$$

where $i \in \mathcal{N}_p$, $p \in \mathcal{M}$.

Definition 2.2 ([44]). *If the slave networks in Eq* (2.6) *globally asymptotically anti-synchronize to the master networks in Eq* (2.5) *under the boundary feed-back controls (BFBCs) in Eq* (2.7)*, i.e.,*

$$\lim_{k\to\infty}\sum_{\kappa=0}^{\Bbbk}\left|\omega^{[\kappa]}_{pi,k}\right|^2 = \lim_{k\to\infty}\sum_{\kappa=0}^{\Bbbk}\left|\varpi^{[\kappa]}_{pi} + z^*_{pi}\right|^2 = 0, \quad i \in \mathcal{N}_p, p \in \mathcal{M},$$

then the MAMNNs in Eq (2.1) *are called* ***globally asymptotically anti-stabilized****.*

Definition 2.3 ([38]). *If the slave networks in Eq* (2.6) *globally exponentially anti-synchronize to the master networks in Eq* (2.5) *under the BFBCs in Eq* (2.7)*, i.e., there exist constants* $\alpha > 0$ *and* $M > 1$ *such that*

$$\sum_{\kappa=0}^{\Bbbk}\left|\omega^{[\kappa]}_{pi,k}\right|^2 = \sum_{\kappa=0}^{\Bbbk}\left|\varpi^{[\kappa]}_{pi} + z^*_{pi}\right|^2 \leq M(1-\alpha)^k, \quad \forall k \in \mathbb{Z}_0,$$

then the MAMNNs in Eq (2.1) *are called* ***globally exponentially anti-stabilized****, where* $i \in \mathcal{N}_p, p \in \mathcal{M}$.

Remark 2.1. *The field of research into anti-stabilization for discrete-time neural networks [35, 38] and other topics related to continuous-time MAMNNs [1–6] would benefit from further investigation. I was surprised to discover that there had been no reports on discrete MAMNNs in time or space settings. Consequently, the current work is of significant value.*

Lemma 2.1 ([45]). *If* $\{u_v : v = 0, 1, \ldots, \Bbbk\}$ *is a one-dimensional real-valued sequence, then*

$$\sum_{v=1}^{\Bbbk-1} u_v\Delta^2_\iota u_{v-1} = \frac{1}{\iota}u_v\Delta_\iota u_{v-1}\Big|_1^{\Bbbk} - \sum_{v=1}^{\Bbbk-1}\Delta_\iota u_v\Delta_\iota u_v.$$

Lemma 2.2 ([46])**.** *If* $\{u_v : v = 0, 1, \ldots, \mathbb{k}\}$ *is a one-dimensional real-valued sequence, then*

$$\iota^2 \sum_{v=0}^{\mathbb{k}-2} (\Delta_\iota u_v)^2 \leq 4\cos^2 \frac{\pi}{2\mathbb{k}} \sum_{v=0}^{\mathbb{k}-1} (u_v)^2.$$

If $u_0 = 0$*, then*

$$4\sin^2 \frac{\pi}{2(2\mathbb{k}+1)} \sum_{v=0}^{\mathbb{k}} u_v^2 \leq \iota^2 \sum_{v=0}^{\mathbb{k}-1} (\Delta_\iota u_v)^2 \leq 4\cos^2 \frac{\pi}{2\mathbb{k}+1} \sum_{v=0}^{\mathbb{k}} u_v^2.$$

Lemma 2.3 ([47])**.** *Supposing* $\xi_j, \zeta_j, u, v \in \mathbb{R}$*, then*

$$\Big| \bigvee_{j=1}^{n_q} \xi_j f_{qj}(u) - \bigvee_{j=1}^{n_q} \xi_j f_{qj}(v) \Big| \leq \sum_{j=1}^{n_q} |\xi_j||f_{qj}(u) - f_{qj}(v)|,$$

$$\Big| \bigwedge_{j=1}^{n_q} \zeta_j f_{qj}(u) - \bigwedge_{j=1}^{n_q} \zeta_j f_{qj}(v) \Big| \leq \sum_{j=1}^{n_q} |\zeta_j||f_{qj}(u) - f_{qj}(v)|,$$

where $j \in \mathcal{N}_q$*,* $q \in \mathcal{M}$*.*

3. Existence of an equilibrium state

Prior to discussing the asymptotic or exponential anti-stabilization of MAMNNs in Eq (2.1), it is imperative to ensure the existence of the equilibrium state for MAMNNs in Eq (2.1). The following section will investigate the existence of the equilibrium point in accordance with the theory of topological degree, which involves the Brouwer degree as outlined in Propositions 5.2.2 and 5.2.6 of [48].

Definition 3.1 ([48, Definition 5.2.1])**.** *Let* Ω *be some bounded open set in* $\mathbb{R}^n$*,* $F \in C(\bar{\Omega}, \mathbb{R}^n) \cap C^1(\Omega, \mathbb{R}^n)$*,* $z_0 \in \mathbb{R}^n \setminus F(\partial\Omega)$ *and*

$$F^{-1}(z_0) = \{x \in \Omega : F(x) = z_0\} \text{ is a finite set.}$$

Define the Brouwer degree $\deg(F, \Omega, z_0)$ *by*

$$\deg(F, \Omega, z_0) = \sum_{x \in F^{-1}(z_0)} \operatorname{sgn} J_{F(x)},$$

where $J_{F(x)}$ *represents the Jacobian of* F *at the point* $x \in \Omega$*.*

Lemma 3.1 ([48, Propositions 5.2.2 and 5.2.6])**.** *Let* Ω *be some bounded open set in* $\mathbb{R}^n$ *and the Brouwer degree be shown in* Definition 3.1. *Then the following properties are fulfilled.*

(*a*) (Normality.) $\deg(I, \Omega, z_0) = 1, \forall p \in \Omega$, *where* I *represents the identity operator.*

(*b*) (Kronecker existence theorem.) *If* $\deg(F, \Omega, z_0) \neq 0$*, then* $F(x) = z_0$ *has a solution in* Ω*.*

(*c*) (Homotopy invariance.) *Let* $F, G \in C(\bar{\Omega}, \mathbb{R}^n) \cap C^1(\Omega, \mathbb{R}^n)$ *and*

$$đ_t(x) = (1-t)F(x) + tG(x), \quad \forall (t, x) \in [0,1] \times \bar{\Omega}.$$

If

$$z_0 \in \mathbb{R}^n \backslash \{đ_t(x) : (t, x) \in [0,1] \times \partial\Omega\},$$

then

$$\deg(F, \Omega, z_0) = \deg(G, \Omega, z_0).$$

Define the working space $\mathbb{X} = \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \cdots \times \mathbb{R}^{n_m}$ and its norm is given by

$$\|z\| = \sum_{i=1}^{n'} |z_i|, \quad \forall z = (z_1, z_2, \cdots, z_{n'})^T \in \mathbb{X},$$

where $n' = n_1 \times n_2 \times \cdots \times n_m$. Here it is necessary to define two positive constants, which will be utilized in what follows. Define

$$æ = \sum_{p=1}^{m} \sum_{i=1}^{n_p} \frac{1}{\left|b_{pi}\right|} \max_{(q,j)} \left\{ \left|c_{pi}^{qj} + d_{pi}^{qj}\right| L_{qj}^f + \left|\beta_{pi}^{qj}\right| L_{qj}^f + \left|\gamma_{pi}^{qj}\right| L_{qj}^f \right\},$$

$$œ = \sum_{p=1}^{m} \sum_{i=1}^{n_p} \frac{|a_{pi}|}{|b_{pi}|}.$$

Theorem 3.1. *There exists an equilibrium point* z^* *of the MAMNNs in Eq* (2.1) *satisfying* $\|z^*\| \leq \frac{1+œ}{1-æ}$, *if the following conditions are valid.*

(M_1) *Function* f_{qj} *is odd and there exists* $£_{qj} > 0$ *so that*

$$|f_{qj}(x) - f_{qj}(y)| \leq £_{qj}|x-y|, \quad \forall x, y \in \mathbb{R}, j = 1, 2, \ldots, n_q, q = 1, 2, \ldots, m.$$

(M_2) $æ < 1$.

Proof. For any $z = (z_1, z_2, \ldots, z_m)$ with $z_p = (z_{p1}, \ldots, z_{pn_p})^T \in \mathbb{R}^{n_p}$, $p \in \mathcal{M}$, define operator $\mathbf{F}$ on the product space $\mathbb{X}$ as

$$\mathbf{F}(z) = (\mathbf{F}_1(z), \mathbf{F}_2(z), \ldots, \mathbf{F}_m(z))^T,$$

where

$$\mathbf{F}_p(z) = \left(\mathbf{F}_{p1}(z), \ldots, \mathbf{F}_{pn_p}(z)\right)^T \in \mathbb{R}^{n_p},$$

$$\mathbf{F}_{pi}(z) = \frac{1}{b_{pi}} \sum_{q=1, q\neq p}^{m} \sum_{j=1}^{n_q} (c_{pi}^{qj} + d_{pi}^{qj}) f_{qj}(z_{qj}) + \frac{1}{b_{pi}} \sum_{q=1, q\neq p}^{m} \left(\bigvee_{j=1}^{n_q} \beta_{pi}^{qj} + \bigwedge_{j=1}^{n_q} \gamma_{pi}^{qj} \right) f_{qj}(z_{qj}) + \frac{a_{pi}}{b_{pi}},$$

where $i \in \mathcal{N}_p$, $p \in \mathcal{M}$.

On the basis of Assumption (M_1) and Lemma 2.3, it follows that

$$\|\mathbf{F}(z)\| = \sum_{p=1}^{m} \sum_{i=1}^{n_p} \left|\mathbf{F}_{pi}(z)\right|$$

$$\begin{aligned}
&\le \sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{1}{\left|b_{pi}\right|}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\left|c_{pi}^{qj}+d_{pi}^{qj}\right|L_j^f\left|z_{qj}\right| \\
&\quad+\sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{1}{\left|b_{pi}\right|}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\left|\beta_{pi}^{qj}\right|L_j^f\left|z_{qj}\right| \\
&\quad+\sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{1}{\left|b_{pi}\right|}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\left|\gamma_{pi}^{qj}\right|L_j^f\left|z_{qj}\right|+\sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{|\bar{a}_{pi}|}{|b_{pi}|} \\
&\le \sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{1}{\left|b_{pi}\right|}\max_{(q,j)}\left\{\left|c_{pi}^{qj}+d_{pi}^{qj}\right|L_{qj}^f\right\}\|z\| \\
&\quad+\sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{1}{\left|b_{pi}\right|}\max_{(q,j)}\left\{\left|\beta_{pi}^{qj}\right|L_{qj}^f\right\}\|z\| \\
&\quad+\sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{1}{\left|b_{pi}\right|}\max_{(q,j)}\left\{\left|\gamma_{pi}^{qj}\right|L_{qj}^f\right\}\|z\|+\sum_{p=1}^{m}\sum_{i=1}^{n_p}\frac{|\bar{a}_{pi}|}{|b_{pi}|} \\
&\le æ\|z\|+œ.
\end{aligned} \tag{3.1}$$

We set

$$\Omega=\left\{z\in\mathbb{X}:\|z\|<\frac{1+œ}{1-æ}\right\}.$$

If $z\in\partial\Omega$, then $\|z\|=\frac{1+œ}{1-æ}$ and by Eq (3.1),

$$\|\mathbf{F}(z)\|<æ\frac{1+œ}{1-æ}+œ+1=\frac{1+œ}{1-æ}=\|z\|. \tag{3.2}$$

Define a homotopic mapping by

$$đ_t(z)=z-t\mathbf{F}(z),\quad \forall t\in[0,1], z\in\mathbb{X}.$$

Clearly, $đ_t(\cdot)$ is continuous with respect to variable $t\in[0,1]$ and

$$đ_0(z)=z,\quad đ_1(z)=z-\mathbf{F}(z),\quad \forall z\in\Omega.$$

Next, we claim that $0\notin đ_t(\partial\Omega)$, $\forall t\in[0,1]$. In fact, if there exist some $z\in\partial\Omega$ such that $đ_t(z)=0$, then

$$z-t\mathbf{F}(z)=0\Rightarrow z=t\mathbf{F}(z)\Rightarrow\|z\|=t\|\mathbf{F}(z)\|\le\|\mathbf{F}(z)\|,\quad \forall t\in[0,1].$$

This induces a conflict with Eq (3.2). So $0\notin đ_t(\partial\Omega)$, $\forall t\in[0,1]$. According to the theory of topological degree in Lemma 3.1, we have

$$1=\deg(đ_0(z),\Omega,0)=\deg(đ_1(z),\Omega,0)=\deg(z-\mathbf{F}(z),\Omega,0).$$

By the existence theorem of Kronecker, there exists a fixed point $z^*\in\Omega$ such that $\mathbf{F}(z^*)=z^*$. This is the equilibrium point of the network in Eq (2.1). This completes the proof.

4. Asymptotic and exponential anti-stabilizations

Define $m_* = m \max\{n_1, n_2, \ldots, n_m\}$,

$$\mathfrak{b}_{pi}^{qj} = \tilde{b}_{pi}^2\Big[(d_{pi}^{qj})^2 + (\beta_{pi}^{qj})^2 + (\gamma_{pi}^{qj})^2\Big]\pounds_{qj}^2, \quad \tilde{b}_{pi} = \frac{1 - e^{-b_{pi}h}}{b_{pi}},$$

where $i \in \mathcal{N}_p$, $j = 1, 2, \ldots, n_q$, $p, q = 1, 2, \ldots, m$.

Design the BFBCs in the boundary in Eq (2.9) by

$$\omega_{pi,k}^{[\Bbbk]} = \vartheta_{pi,k} + z_{pi}^*, \quad \vartheta_{pi,k} = \omega_{pi,k}^{[\Bbbk-1]} - z_{pi}^* + \sum_{\kappa=1}^{\Bbbk-1} \varsigma_{pi}\omega_{pi,k}^{[\kappa]}, \tag{4.1}$$

where ς_{pi} stands for the gain constant of the controller, $i \in \mathcal{N}_p$, $p \in \mathcal{M}$.

4.1. Asymptotic anti-stabilization

Theorem 4.1. *Let conditions* (M_1) *and* (M_2) *hold. Assume further that:*

(M_3) *There exist numbers* $\xi_{pi}, \sigma_{pi} > 0$ *so that*

$$\frac{\iota^2}{4\sin^2\frac{\pi}{4\Bbbk-2}}\sigma_{pi} + \frac{20}{\iota^2}\cos^2\frac{\pi}{2\Bbbk}a_{pi}^2\tilde{b}_{pi}^2\xi_{pi} \le 2a_{pi}\tilde{b}_{pi}\xi_{pi}e^{-b_{pi}h},$$

$$0 \le \frac{1}{\iota^2}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi} \le \sigma_{pi}$$

for $i \in \mathcal{N}_p$, $p \in \mathcal{M}$.

(M_4) *It holds that*

$$\boldsymbol{\sigma}_* = \max_{1\le i\le n_p, 1\le p\le m}\Big\{ -\xi_{pi} + (1+4\varepsilon)e^{-2b_{pi}h}\xi_{pi} + (5+\varepsilon^{-1})m_*\sum_{q=1,q\ne p}^{m}\sum_{j=1}^{n_q}(\tilde{b}_{qj}c_{qj}^{pi})^2\pounds_{pi}^2\xi_{qj}$$
$$+ (5+\varepsilon^{-1})m_*(\rho_\infty + 1)\sum_{q=1,q\ne p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{qj}^{pi}\xi_{qj} + \frac{3}{\iota^2}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}\Big\} < 0.$$

Therefore, $\varepsilon > 0$ *is a predetermined constant.*

Then the MAMNNs in Eq (2.1) *are globally asymptotically anti-stabilized under the BFBCs in Eq* (4.1)*, viz.,*

$$\lim_{k\to\infty}\sum_{\kappa=0}^{\Bbbk}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\left|\omega_{pi,k}^{[\kappa]}\right|^2 = 0.$$

Proof. To establish global asymptotic anti-stabilization, we construct the Lyapunov–Krasovskii functional

$$V_k = V_{1,k} + V_{2,k},$$

where

$$V_{1,k}=\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\xi_{pi}(\omega_{pi,k}^{[\kappa]})^2$$

and

$$V_{2,k}=(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\flat_{pi}^{qj}\xi_{pi}(\omega_{qj,s}^{[\kappa]})^2,\qquad k\in\mathbb{Z}_0.$$

The above choice is motivated by the structure of the error system in Eq (2.8). $V_{1,k}$ describes the present-state energy, whereas $V_{2,k}$ is a memory term designed to handle the delay-induced and fuzzy-coupling-induced contributions.

Via the definition of V_1, it follows that

$$\begin{aligned}\Delta V_{1,k}&=V_{1,k+1}-V_{1,k}\\&=\underbrace{\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\xi_{pi}(\omega_{pi,k+1}^{[\kappa]})^2}_{1,k}-\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\xi_{pi}(\omega_{pi,k}^{[\kappa]})^2,\quad\forall k\in\mathbb{Z}_0.\end{aligned}\tag{4.2}$$

According to the error system in Eq (2.8), we have

$$\omega_{pi,k+1}^{[\kappa]}=e^{-b_{pi}h}\omega_{pi,k}^{[\kappa]}+\tilde{b}_{pi}a_{pi}\Delta_{c,\iota}^2\omega_{pi,k}^{[\kappa]}+\tilde{b}_{pi}\mathcal{C}_{pi,k}^{[\kappa]}+\tilde{b}_{pi}\mathcal{D}_{pi,k}^{[\kappa]}+\tilde{b}_{pi}\mathcal{F}_{pi,k}^{[\kappa]},$$

where

$$\mathcal{C}_{pi,k}^{[\kappa]}=\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}c_{pi}^{qj}\tilde{f}_{qj}(\omega_{qj,k}^{[\kappa]}),\quad \mathcal{D}_{pi,k}^{[\kappa]}=\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}d_{pi}^{qj}\tilde{f}_{qj}(\omega_{qj,\rho_{qj,k}}^{[\kappa]}),$$

and

$$\mathcal{F}_{pi,k}^{[\kappa]}=\sum_{q=1,q\neq p}^{m}\bigvee_{j=1}^{n_q}\beta_{pi}^{qj}\tilde{f}_{qj}(\omega_{qj,\rho_{qj,k}}^{[\kappa]})+\sum_{q=1,q\neq p}^{m}\bigwedge_{j=1}^{n_q}\gamma_{pi}^{qj}\tilde{f}_{qj}(\omega_{qj,\rho_{qj,k}}^{[\kappa]}).$$

Then

$$\begin{aligned}{}_{1,k}&=\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\xi_{pi}\Big(\omega_{pi,k+1}^{[\kappa]}\Big)^2\\&=\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\xi_{pi}\Big(e^{-b_{pi}h}\omega_{pi,k}^{[\kappa]}+\tilde{b}_{pi}a_{pi}\Delta_{c,\iota}^2\omega_{pi,k}^{[\kappa]}+\tilde{b}_{pi}\mathcal{C}_{pi,k}^{[\kappa]}+\tilde{b}_{pi}\mathcal{D}_{pi,k}^{[\kappa]}+\tilde{b}_{pi}\mathcal{F}_{pi,k}^{[\kappa]}\Big)^2.\end{aligned}$$

Using the inequality

$$(x_1+x_2+x_3+x_4+x_5)^2\le(1+4\varepsilon)x_1^2+(5+\varepsilon^{-1})\sum_{r=2}^{5}x_r^2+2x_1x_2,$$

together with Assumption (M_1) and Lemma 2.3, we obtain

$${}_{1,k}\le\underbrace{2\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\omega_{pi,k}^{[\kappa]}\xi_{pi}\Delta_{c,\iota}^2\omega_{pi,k}^{[\kappa]}}_{2,k}$$

$$+ (1+4\varepsilon)\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1} e^{-2b_{pi}h}\xi_{pi}(\omega_{pi,k}^{[\kappa]})^2 + \underbrace{5\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1} a_{pi}^2\tilde{b}_{pi}^2\xi_{pi}(\Delta_{c,\iota}^2\omega_{pi,k}^{[\kappa]})^2}_{3,k}$$

$$\underbrace{+ (5+\varepsilon^{-1})m_*\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}(\tilde{b}_{pi}c_{pi}^{qj})^2\pounds_{qj}^2\xi_{pi}(\omega_{qj,k}^{[\kappa]})^2}_{4,k}$$

$$+ (5+\varepsilon^{-1})m_*\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\tilde{b}_{pi}^2\Big[(d_{pi}^{qj})^2 + (\beta_{pi}^{qj})^2 + (\gamma_{pi}^{qj})^2\Big]\pounds_{qj}^2\xi_{pi}(\omega_{qj,\rho_{qj,k}}^{[\kappa]})^2. \qquad (4.3)$$

By right of Lemma 2.1, we attain

$$\begin{aligned}
{}_{2,k} &= 2\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1} a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\omega_{pi,k}^{[\kappa]}\xi_{pi}\Delta_{c,\iota}^2\omega_{pi,k}^{[\kappa]}\\
&= 2\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\xi_{pi}\sum_{\kappa=1}^{\Bbbk-1}\omega_{pi,k}^{[\kappa]}\Delta_{\iota}^2\omega_{pi,k}^{[\kappa-1]}\\
&= \frac{2}{\iota}\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\xi_{pi}\omega_{pi,k}^{[\kappa]}\Delta_{\iota}\omega_{pi,k}^{[\kappa-1]}\Big|_{\kappa=1}^{\kappa=\Bbbk} - 2\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\xi_{pi}\sum_{\kappa=1}^{\Bbbk-1}\Big(\Delta_{\iota}\omega_{pi,k}^{[\kappa]}\Big)^2\\
&= \underbrace{\frac{2}{\iota}\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\xi_{pi}\omega_{pi,k}^{[\Bbbk]}\Delta_{\iota}\omega_{pi,k}^{[\Bbbk-1]}}_{5,k} - \frac{2}{\iota^2}\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\xi_{pi}\Big(\omega_{pi,k}^{[1]}\Big)^2\\
&\quad - 2\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}\tilde{b}_{pi}e^{-b_{pi}h}\xi_{pi}\sum_{\kappa=1}^{\Bbbk-1}\Big(\Delta_{\iota}\omega_{pi,k}^{[\kappa]}\Big)^2, \quad \forall k\in\mathbb{Z}_0. \qquad (4.4)
\end{aligned}$$

Via Lemma 2.2, we have

$$\begin{aligned}
{}_{3,k} &= 5\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1} a_{pi}^2\tilde{b}_{pi}^2\xi_{pi}\Big(\Delta_{c,\iota}^2\omega_{pi,k}^{[\kappa]}\Big)^2\\
&= 5\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}^2\tilde{b}_{pi}^2\xi_{pi}\sum_{\kappa=1}^{\Bbbk-1}\Big(\Delta_{\iota}^2\omega_{pi,k}^{[\kappa-1]}\Big)^2\\
&= 5\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}^2\tilde{b}_{pi}^2\xi_{pi}\sum_{\kappa=0}^{\Bbbk-2}\Big(\Delta_{\iota}(\Delta_{\iota}\omega_{pi,k}^{[\kappa]})\Big)^2\\
&\leq \frac{20}{\iota^2}\cos^2\frac{\pi}{2\Bbbk}\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}^2\tilde{b}_{pi}^2\xi_{pi}\sum_{\kappa=0}^{\Bbbk-1}\Big(\Delta_{\iota}\omega_{pi,k}^{[\kappa]}\Big)^2\\
&= \frac{20}{\iota^4}\cos^2\frac{\pi}{2\Bbbk}\sum_{p=1}^{m}\sum_{i=1}^{n_p} a_{pi}^2\tilde{b}_{pi}^2\xi_{pi}\Big(\omega_{pi,k}^{[1]})\Big)^2
\end{aligned}$$

$$+\frac{20}{\iota^2}\cos^2\frac{\pi}{2\mathbb{k}}\sum_{p=1}^{m}\sum_{i=1}^{n_p}a_{pi}^2\tilde{b}_{pi}^2\xi_{pi}\sum_{\kappa=1}^{\mathbb{k}-1}\left(\Delta_\iota\omega_{pi,k}^{[\kappa]}\right)^2,\quad \forall k\in\mathbb{Z}_0. \tag{4.5}$$

Besides,

$$\begin{aligned}
{}_{4,k} &= 6m_*\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}(\tilde{b}_{pi}c_{pi}^{qj})^2\pounds_{qj}^2\xi_{pi}(\omega_{qj,k}^{[\kappa]})^2\\
&= 6m_*\sum_{q=1}^{m}\sum_{j=1}^{n_q}\sum_{\kappa=1}^{\mathbb{k}-1}\sum_{p=1,p\neq q}^{m}\sum_{i=1}^{n_p}(\tilde{b}_{qj}c_{qj}^{pi})^2\pounds_{pi}^2\xi_{qj}(\omega_{pi,k}^{[\kappa]})^2,\quad \forall k\in\mathbb{Z}_0.
\end{aligned} \tag{4.6}$$

Define $\boldsymbol{\omega}_{pi,k}^{[\kappa]}=\omega_{pi,k}^{[\mathbb{k}]}-\omega_{pi,k}^{[\kappa]}$ for $(\kappa,\mathbb{k})\in[0,\mathbb{k}]_{\mathbb{Z}}\times\mathbb{Z}_0$, $i\in\mathcal{N}_p$, $p\in\mathcal{M}$. Relying on Eq (4.1), we have

$$\begin{aligned}
{}_{5,k} &= \frac{2}{\iota^2}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}\omega_{pi,k}^{[\mathbb{k}]}\omega_{pi,k}^{[\kappa]}\\
&= \frac{2}{\iota^2}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}\boldsymbol{\omega}_{pi,k}^{[\kappa]}\omega_{pi,k}^{[\kappa]}\\
&\quad+\frac{2}{\iota^2}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}(\omega_{pi,k}^{[\kappa]})^2\\
&\le \frac{1}{\iota^2}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}(\boldsymbol{\omega}_{pi,k}^{[\kappa]})^2\\
&\quad+\frac{3}{\iota^2}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}(\omega_{pi,k}^{[\kappa]})^2,\quad \forall k\in\mathbb{Z}_0.
\end{aligned} \tag{4.7}$$

Besides, via Lemma 2.2, we achieve

$$\begin{aligned}
\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}\sigma_{pi}(\boldsymbol{\omega}_{pi,k}^{[\kappa]})^2 &\le \sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}}\sigma_{pi}(\boldsymbol{\omega}_{pi,k}^{[\kappa]})^2\\
&\overset{\kappa'=\mathbb{k}-\kappa}{=\!=} \sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa'=0}^{\mathbb{k}-1}\sigma_{pi}(\boldsymbol{\omega}_{pi,k}^{[\mathbb{k}-\kappa']})^2\\
&\le \frac{\iota^2}{4\sin^2\frac{\pi}{4\mathbb{k}-2}}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa'=0}^{\mathbb{k}-2}\sigma_{pi}(\Delta_\iota\boldsymbol{\omega}_{pi,k}^{[\mathbb{k}-\kappa']})^2\ (\Delta_\iota \text{ w.r.t. } \kappa')\\
&\overset{\kappa=\mathbb{k}-\kappa'}{=\!=} \frac{\iota^2}{4\sin^2\frac{\pi}{4\mathbb{k}-2}}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}\sigma_{pi}(\Delta_\iota\boldsymbol{\omega}_{pi,k}^{[\kappa]})^2\ (\Delta_\iota \text{ w.r.t. } \kappa)\\
&= \frac{\iota^2}{4\sin^2\frac{\pi}{4\mathbb{k}-2}}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\mathbb{k}-1}\sigma_{pi}(\Delta_\iota\omega_{pi,k}^{[\kappa]})^2,\quad \forall k\in\mathbb{Z}_0.
\end{aligned} \tag{4.8}$$

Based on the definition of V_2, we can compute

$$\begin{aligned}
\Delta V_{2,k} &= V_{2,k+1} - V_{2,k} \\
&= (5+\varepsilon^{-1})m_* \sum_{l=0}^{\rho_\infty} \sum_{s=k+1-l}^{k} \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} (\omega_{qj,s}^{[\kappa]})^2 \\
&\quad - (5+\varepsilon^{-1})m_* \sum_{l=0}^{\rho_\infty} \sum_{s=k-l}^{k-1} \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} (\omega_{qj,s}^{[\kappa]})^2 \\
&= (5+\varepsilon^{-1})m_* \sum_{l=0}^{\rho_\infty} \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} (\omega_{qj,k}^{[\kappa]})^2 \\
&\quad - (5+\varepsilon^{-1})m_* \sum_{l=0}^{\rho_\infty} \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} (\omega_{qj,k-l}^{[\kappa]})^2 \\
&\leq (5+\varepsilon^{-1})m_*(\rho_\infty+1) \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} (\omega_{qj,k}^{[\kappa]})^2 \\
&\quad - (5+\varepsilon^{-1})m_* \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} \Big[\omega_{qj,k-(k-\rho_{qj,k})}^{[\kappa]}\Big]^2 \\
&= (5+\varepsilon^{-1})m_*(\rho_\infty+1) \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} (\omega_{qj,k}^{[\kappa]})^2 \\
&\quad - (5+\varepsilon^{-1})m_* \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{pi}^{qj} \xi_{pi} (\omega_{qj,\rho_{qj,k}}^{[\kappa]})^2, \quad \forall k \in \mathbb{Z}_0. \qquad (4.9)
\end{aligned}$$

Based on Eqs (4.2)–(4.9), via Assumption (M_3), we have

$$\begin{aligned}
\Delta V_k &\leq \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \Big\{ -\xi_{pi} + (1+4\varepsilon)e^{-2b_{pi}h}\xi_{pi} + (5+\varepsilon^{-1})m_* \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} (\tilde{b}_{qj} c_{qj}^{pi})^2 £_{pi}^2 \xi_{qj} \\
&\quad + (5+\varepsilon^{-1})m_*(\rho_\infty+1) \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} \mathfrak{b}_{qj}^{pi} \xi_{qj} + \frac{3}{\iota^2} a_{pi} \tilde{b}_{pi} \varsigma_{pi} e^{-b_{pi}h} \xi_{pi} \Big\} (\omega_{pi,k}^{[\kappa]})^2 \\
&\leq \boldsymbol{\sigma}_* \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} (\omega_{pi,k}^{[\kappa]})^2, \quad \forall k \in \mathbb{Z}_0. \qquad (4.10)
\end{aligned}$$

Noting the validness of Assumption (M_4), we have

$$0 \leq -\boldsymbol{\sigma}_* \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} \sum_{k=0}^{T-1} (\omega_{pi,k}^{[\kappa]})^2 \leq V(0) < +\infty,$$

and by letting T tend to $+\infty$, this leads to

$$\sum_{k=0}^{\infty} \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} (\omega_{pi,k}^{[\kappa]})^2 < +\infty \Rightarrow \lim_{k\to\infty} \sum_{p=1}^{m} \sum_{i=1}^{n_p} \sum_{\kappa=1}^{\Bbbk-1} (\omega_{pi,k}^{[\kappa]})^2 = 0.$$

Together with the boundary values in Eq (2.9), the MAMNNs in Eq (2.1) are globally asymptotically anti-stabilized under the BFBCs in Eq (4.1). This completes the proof.

Remark 4.1. *The boundary feedback controller in Eq* (4.1) *acts only on the boundary nodes of the spatially discrete network, and thus it can be interpreted as a boundary actuation mechanism for regulating the spatiotemporal propagation behavior of the network. From a practical viewpoint, the control input depends on the current boundary state together with a finite number of past states, which is consistent with feedback implementations with memory. Moreover, the gain parameters ς_{pi} should be selected such that conditions* (M_3) *and* (M_4) *hold. Therefore, these conditions also serve as explicit and verifiable design criteria for choosing the controller gains.*

In what follows, let us regard the non-fuzzy networks ($\beta_{pi}^{qj} = 0$ and $\gamma_{pi}^{qj} = 0$) corresponding to the MAMNNs in Eq (2.1), which can be written by

$$\begin{aligned} z_{pi,k+1}^{[\kappa]} &= e^{-b_{pi}h} z_{pi,k}^{[\kappa]} + \frac{1-e^{-b_{pi}h}}{b_{pi}} \Big[a_{pi}\Delta_{c,\iota}^2 z_{pi,k}^{[\kappa]} \\ &+ \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} c_{pi}^{qj} f_{qj}(z_{qj,k}^{[\kappa]}) + \sum_{q=1,q\neq p}^{m} \sum_{j=1}^{n_q} d_{pi}^{qj} f_{qj}(z_{qj,\rho_{qj,k}}^{[\kappa]}) \Big], \end{aligned} \tag{4.11}$$

where $(\kappa, \Bbbk) \in (0, \Bbbk)_{\mathbb{Z}} \times \mathbb{Z}_0$, $i \in \mathcal{N}_p$, $p \in \mathcal{M}$. Adopting Theorem 4.1 into the MAMNNs in Eq (4.11), we have the following:

Corollary 4.1. *Let conditions* (M_1)–(M_3) *hold by setting the fuzzy coupling parameters* $\beta_{pi}^{qj} = 0$ *and* $\gamma_{pi}^{qj} = 0$. *Assume further that:*

(M_5) *It holds that*

$$\begin{aligned} \sigma_*' = \max_{1\le i\le n_p, 1\le p\le m} \Big\{ &-\xi_{pi} + (1+4\varepsilon)e^{-2b_{pi}h}\xi_{pi} + (5+\varepsilon^{-1})m_* \sum_{q=1}^{m}\sum_{j=1}^{n_q} (\tilde{b}_{qj} c_{qj}^{pi})^2 £_{pi}^2 \xi_{qj} \\ &+ (5+\varepsilon^{-1})m_*(\rho_\infty + 1)\sum_{q=1}^{m}\sum_{j=1}^{n_q} (\tilde{b}_{qj} d_{qj}^{pi})^2 £_{pi}^2 \xi_{qj} + \frac{3}{\iota^2} a_{pi}\tilde{b}_{pi}\varsigma_{pi} e^{-b_{pi}h}\xi_{pi} \Big\} < 0. \end{aligned}$$

Then the MAMNNs in Eq (4.11) *are globally asymptotically anti-stabilized under the BFBCs in Eq* (4.1).

4.2. Exponential anti-stabilization

Theorem 4.2. *Let conditions* (M_1)–(M_3) *hold. Assume further that:*

(M_6) *Taking* $0 < \alpha \le \frac{1}{1+\rho_\infty}$ *so that*

$$\sigma_\diamond = \max_{1\le i\le n_p, 1\le p\le m} \Big\{ -\xi_{pi} + (1+4\varepsilon)e^{-2b_{pi}h}\xi_{pi} + (5+\varepsilon^{-1})m_* \sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q} (\tilde{b}_{qj} c_{qj}^{pi})^2 £_{pi}^2 \xi_{qj}$$

$$+(5+\varepsilon^{-1})m_*(\rho_\infty+1)^2\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{qj}^{pi}\xi_{qj}+\frac{3}{\iota^2}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}+\alpha\xi_{pi}\Big\}\leq 0,$$

then $\varepsilon>0$ *is a predetermined constant.*

Then the MAMNNs in Eq (2.1) *are globally exponentially anti-stabilized under the BFBCs in Eq* (4.1), *viz., there exists* $M>1$ *such that*

$$\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=0}^{\Bbbk}\left|\omega_{pi,k}^{[\kappa]}\right|^2\leq M(1-\alpha)^k,\quad\forall k\in\mathbb{Z}_0.$$

Proof. Define a Lyapunov–Krasovskii functional as $V_k=V_{1,k}+V_{2,k}+V_{3,k}$, where $V_{1,k},V_{2,k}$ are given as those in the proof of Theorem 4.1, and

$$V_{3,k}=(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{s'=s}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2,\quad\forall k\in\mathbb{Z}_0.$$

By a simple calculation, we have

$$\begin{aligned}\Delta V_{3,k}&=V_{3,k+1}-V_{3,k}\\&=(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k+1-l}^{k}\sum_{s'=s}^{k}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2\\&\quad-(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{s'=s}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2\\&=(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,k}^{[\kappa]})^2\\&\quad+(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k+1-l}^{k-1}\sum_{s'=s}^{k}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2\\&\quad-(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{s'=s}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2\\&=(5+\varepsilon^{-1})m_*(\rho_\infty+1)\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,k}^{[\kappa]})^2\\&\quad+(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k+1-l}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,k}^{[\kappa]})^2\\&\quad+(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k+1-l}^{k-1}\sum_{s'=s}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2\\&\quad-(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{s'=s}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\mathfrak{b}_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2\end{aligned}$$

$$
\begin{aligned}
&\le (5+\varepsilon^{-1})m_*\rho_\infty(\rho_\infty+1)\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\flat_{pi}^{qj}\xi_{pi}(\omega_{qj,k}^{[\kappa]})^2\\
&\quad-(5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\flat_{pi}^{qj}\xi_{pi}(\omega_{qj,s}^{[\kappa]})^2\\
&\le (5+\varepsilon^{-1})m_*\rho_\infty(\rho_\infty+1)\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\flat_{qj}^{pi}\xi_{qj}(\omega_{pi,k}^{[\kappa]})^2-V_{2,k},\ \forall k\in\mathbb{Z}_0. \qquad (4.12)
\end{aligned}
$$

In the meantime,

$$
\begin{aligned}
V_{3,k} &\le (5+\varepsilon^{-1})m_*\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{s'=k-l}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\flat_{pi}^{qj}\xi_{pi}(\omega_{qj,s'}^{[\kappa]})^2\\
&\le (5+\varepsilon^{-1})m_*\rho_\infty\sum_{l=0}^{\rho_\infty}\sum_{s=k-l}^{k-1}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\flat_{pi}^{qj}\xi_{pi}(\omega_{qj,s}^{[\kappa]})^2\\
&=\rho_\infty V_{2,k},\quad \forall k\in\mathbb{Z}_0. \qquad (4.13)
\end{aligned}
$$

Through the facts in Eqs (4.10), (4.12), and (4.13), there exists a constant $0<\alpha\le\frac{1}{1+\rho_\infty}$ such that

$$
\begin{aligned}
\Delta V_k+\alpha V_k &\le \sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\Big\{-\xi_{pi}+(4+\varepsilon)e^{-2b_{pi}h}\xi_{pi}+(5+\varepsilon^{-1})m_*\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}(\tilde{b}_{qj}c_{qj}^{pi})^2\pounds_{pi}^2\xi_{qj}\\
&\quad+(5+\varepsilon^{-1})m_*(\rho_\infty+1)^2\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}\flat_{qj}^{pi}\xi_{qj}+\frac{3}{\iota^2}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}+\alpha\xi_{pi}\Big\}(\omega_{pi,k}^{[\kappa]})^2\\
&\quad-(1-\alpha-\alpha\rho_\infty)V_{2,k}\\
&\le \sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}\boldsymbol{\sigma}_\diamond(\omega_{pi,k}^{[\kappa]})^2
\end{aligned}
$$

implies

$$
\xi_{\min}\sum_{p=1}^{m}\sum_{i=1}^{n_p}\sum_{\kappa=1}^{\Bbbk-1}(\omega_{pi,k}^{[\kappa]})^2\le V_{1,k}\le V_k\le V_0(1-\alpha)^k,\quad \forall k\in\mathbb{Z}_0,
$$

where $\xi_{\min}=\min_{i\in\eta_p,p\in\mathcal{m}}\xi_{pi}$. Noting that V_0 is finite, and by the boundary values in Eq (2.9) and boundary control in Eq (4.1), it easily deduces that the MAMNNs in Eq (2.1) are globally exponentially anti-stabilized under the BFBCs in Eq (4.1). This completes the proof.

Corollary 4.2. *Let conditions* (M_1)–(M_3) *hold by setting the fuzzy coupling parameters* $\beta_{pi}^{qj}=0$ *and* $\gamma_{pi}^{qj}=0$. *Assume further that:*

(M_7) *Taking* $0<\alpha\le\frac{1}{1+\rho_\infty}$ *so that*

$$
\boldsymbol{\sigma}'_\diamond=\max_{1\le i\le n_p,1\le p\le m}\Big\{-\xi_{pi}+(1+4\varepsilon)e^{-2b_{pi}h}\xi_{pi}+(5+\varepsilon^{-1})m_*\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}(\tilde{b}_{qj}c_{qj}^{pi})^2\pounds_{pi}^2\xi_{qj}
$$

$$+ (5+\varepsilon^{-1})m_*(3\rho_\infty+1)\sum_{q=1,q\neq p}^{m}\sum_{j=1}^{n_q}(\tilde{b}_{qj}d_{qj}^{pi})^2\pounds_{pi}^2\xi_{qj}+\frac{3}{\iota^2}a_{pi}\tilde{b}_{pi}\varsigma_{pi}e^{-b_{pi}h}\xi_{pi}+\alpha\xi_{pi}\Big\}\leq 0,$$

then the MAMNNs in Eq (4.11) *are globally exponentially anti-stabilized under the BFBCs in Eq* (4.1).

Remark 4.2. *It is clear that more and more papers on the dynamical behaviors of various types of MAMNNs are being published. For more information, please refer to the recent reports [1–6]. The current work has several outstanding merits in comparison to these other works. (1) The discrete-time and discretized-space networks (MAMNNs) presented in this paper build upon the continuous-time model of the MAMNNs as described in prior publications [1–6]. (2) This paper explores the subjects of anti-stabilizations of the MAMNNs, thereby addressing the lacunae in the extant literature pertaining to MAMNNs [1–6].*

Remark 4.3. *Various studies have been conducted on the subject of anti-stabilization of neural networks [35–42]. In this paper, we present an innovative approach to the time-space discrete model that differs from existing continuous-time neural networks [36, 37, 39–42] and discrete-time neural networks [35, 38] in the academic literature. It is clear that this paper addresses the deficiencies of the previously mentioned reports [35–42] in a meaningful way.*

Remark 4.4. *Compared with the anti-stabilization criteria reported in [35, 38], the present results are theoretically less conservative in two aspects. First, instead of treating the delayed and coupling terms by coarse lumped estimates as in purely discrete-time settings, this paper constructs a discrete Lyapunov–Krasovskii functional with delay-dependent double-sum terms, which preserves more information on the distributed delay trajectory and therefore yields tighter upper bounds in the difference inequalities. Second, the proposed criteria explicitly incorporate the effects of spatial diffusion, Dirichlet boundary feedback gains, and fuzzy MIN/MAX couplings into the sufficient conditions, so that the admissible parameter region is characterized in a more refined way rather than being absorbed into a single aggregated bound. Hence, our approach extends the results of [35, 38] from purely discrete-time neural networks to fuzzy spatiotemporal discrete MAM neural networks, while reducing conservatism at the level of the theoretical estimates.*

5. Illustrative example

We consider the MAMNNs with the time delay and fuzzy operation as noted below:

$$\begin{cases} z_{11,k+1}^{[\kappa]} = e^{-2h}z_{11,k}^{[\kappa]}+\dfrac{1-e^{-2h}}{2}\Big[0.2\Delta_{c,\iota}^2 z_{11,k}^{[\kappa]} \\ \qquad +\displaystyle\sum_{j=1}^{2}c_{11}^{2j}f_{2j}(z_{2j,k}^{[\kappa]})+\bigvee_{j=1}^{2}\beta_{11}^{2j}f_{2j}(z_{2j,\rho_k}^{[\kappa]})+a_{11}\Big], \\ z_{21,k+1}^{[\kappa]} = e^{-3h}z_{21,k}^{[\kappa]}+\dfrac{1-e^{-3h}}{3}\Big[0.1\Delta_{c,\iota}^2 z_{21,k}^{[\kappa]}+0.25f_{11}(z_{11,\rho_k}^{[\kappa]})+a_{21}\Big], \\ z_{22,k+1}^{[\kappa]} = e^{-5h}z_{22,k}^{[\kappa]}+\dfrac{1-e^{-5h}}{5}\Big[0.2\Delta_{c,\iota}^2 z_{22,k}^{[\kappa]}+0.12f_{11}(z_{11,\rho_k}^{[\kappa]})+a_{22}\Big], \end{cases} \tag{5.1}$$

where $(\kappa, k) \in (0,3)_{\mathbb{Z}} \times \mathbb{Z}_0$, $m = 2$, $n_1 = 1$, $n_2 = 2$, $d_{11}^{2j} = 0$, $\gamma_{11}^{2j} = 0$, $j = 1,2$, $c_{21}^{11} = 0$, $\beta_{21}^{11} = 0$, $\gamma_{21}^{11} = 0$, $c_{22}^{11} = 0$, $\beta_{22}^{11} = 0$, $\gamma_{22}^{11} = 0$, $c_{11}^{21} = 0.1$, $c_{11}^{22} = 0.2$, $\beta_{11}^{21} = 0.3$, $\beta_{11}^{22} = 0.1$, $f_{11}(s) = f_{22}(s) = 0.1\sin(0.5s)$, $f_{21}(s) = 0.1\arctan s$, $a_{11} = 2 - 0.01\arctan 1 - 0.02\sin(0.5) - \max(0.03\arctan 1, 0.01\sin(0.5))$, $a_{21} = 3 - 0.025\sin(0.5)$, $a_{22} = 5 - 0.012\sin(0.5)$, $\rho_k = 1$ if $k/2$ is an integer, and $\rho_k = 2$ when $k/2$ is not an integer.

Obviously, the MAMNNs in Eq (5.1) admit an equilibrium point $(1,1,1)^T$. In line with the drive MAMNNs in Eq (5.1), the response MAMNNs are described by

$$\begin{cases} \varpi_{11,k+1}^{[\kappa]} = e^{-2h}\varpi_{11,k}^{[\kappa]} + \dfrac{1-e^{-2h}}{2}\Big[0.2\Delta_{c,\iota}^2\varpi_{11,k}^{[\kappa]} \\ \qquad + \displaystyle\sum_{j=1}^{2} c_{11}^{2j} f_{2j}(\varpi_{2j,k}^{[\kappa]}) + \bigvee_{j=1}^{2} \beta_{11}^{2j} f_{2j}(\varpi_{2j,\rho_k}^{[\kappa]}) - a_{11}\Big], \\ \varpi_{21,k+1}^{[\kappa]} = e^{-3h}\varpi_{21,k}^{[\kappa]} + \dfrac{1-e^{-3h}}{3}\Big[0.1\Delta_{c,\iota}^2\varpi_{21,k}^{[\kappa]} + 0.25 f_{11}(\varpi_{11,\rho_k}^{[\kappa]}) - a_{21}\Big], \\ \varpi_{22,k+1}^{[\kappa]} = e^{-5h}\varpi_{22,k}^{[\kappa]} + \dfrac{1-e^{-5h}}{5}\Big[0.2\Delta_{c,\iota}^2\varpi_{22,k}^{[\kappa]} + 0.12 f_{11}(\varpi_{11,\rho_k}^{[\kappa]}) - a_{22}\Big], \end{cases} \tag{5.2}$$

where $(\kappa, k) \in (0,3)_{\mathbb{Z}} \times \mathbb{Z}_0$. Further,

$$\begin{cases} \varpi_{pi,s}^{[\kappa]} = 1 + pis\kappa\mathrm{r}, \quad \forall s \in [-2,0]_{\mathbb{Z}}, \kappa \in (0,3)_{\mathbb{Z}}; \\ \varpi_{pi,k}^{[0]} = -1, \varpi_{pi,k}^{[3]} = \vartheta_{pi,k}, \quad \forall k \in \mathbb{Z}_0, \end{cases}$$

where $i = 1$ if $p = 1$, $i = 1,2$ if $p = 2$, and $\mathrm{r} \sim \mathcal{N}(0,1)$.

We take $\varepsilon = 0.1$, $\xi_{pi} = 1$, $\sigma_{pi} = 0.001$, $\varsigma_{pi} = 0.01$ for $i = 1$ if $p = 1$, and $i = 1,2$ if $p = 2$. By some easy calculations, all conditions of Theorem 4.2 are fulfilled. Via Theorem 4.2, solution $(1,1,1)^T$ of the MAMNNs in Eq (5.1) are exponentially anti-stabilized through the BFBCs below:

$$\vartheta_{pi,k} = 0.01\omega_{pi,k}^{[1]} + 1.01\omega_{pi,k}^{[2]} - 1, \tag{5.3}$$

where $(\kappa, k) \in (0,3)_{\mathbb{Z}} \times \mathbb{Z}_0$. Hereby, $\omega_{pi} = \varpi_{pi} + 1$, where $i = 1$ if $p = 1$, and $i = 1,2$ if $p = 2$. Figures 1–3 picture the 3D global exponential anti-stabilization of the equilibrium point $(1,1,1)^T$ of the MAMNNs in Eq (5.1) under the BFBCs in Eq (5.3). Especially, by taking the space variable $\kappa = 2$, Figures 4–6 show the 2D curves of global exponential anti-stabilization of the equilibrium point $(1,1,1)^T$ of the MAMNNs in Eq (5.1) under the BFBCs in Eq (5.3).

Remark 5.1. *To further illustrate the advantage of the proposed method, we compare the criterion in* Theorem 4.2 *with the existing works [35, 38]. Since the results in [35, 38] were established for purely discrete-time neural networks without spatial diffusion, fuzzy MIN/MAX operators, or Dirichlet boundary feedback, they cannot be directly applied to the system in Eq* (5.1)*. However, if the diffusion terms and fuzzy feedback terms in Eq* (5.1) *are removed, then the resulting reduced model falls into the framework of [35, 38]. In that case, the admissible parameter region ensured by* Theorem 4.2 *is still valid and is less restrictive because the present criterion retains more detailed delay-dependent information through the double-sum Lyapunov–Krasovskii functional. Therefore, the current method* *not only covers a broader class of spatiotemporal fuzzy MAMNNs, but also provides a more effective anti-stabilization test for the corresponding reduced models.*

Remark 5.2. *Although the illustrative example only presents the state trajectories under one representative parameter set, the influence of the diffusion coefficients and fuzzy parameters is not limited to this specific choice. In fact, for the system in Eq* (5.1)*, these parameters explicitly appear in the quantities involved in condition* (M_6) *of* Theorem 4.2*, and hence their effects on exponential anti-stabilization can be directly understood through the feasibility of the theoretical criterion. More precisely, changing the diffusion intensity or the fuzzy coupling coefficients will alter the estimate of* $\boldsymbol{\sigma}_\diamond$ *and consequently affect whether the sufficient condition in* Theorem 4.2 *remains valid. Therefore, the role of such parameters has already been characterized at the theoretical level, while the present example is mainly included to provide a representative verification of the obtained result.*

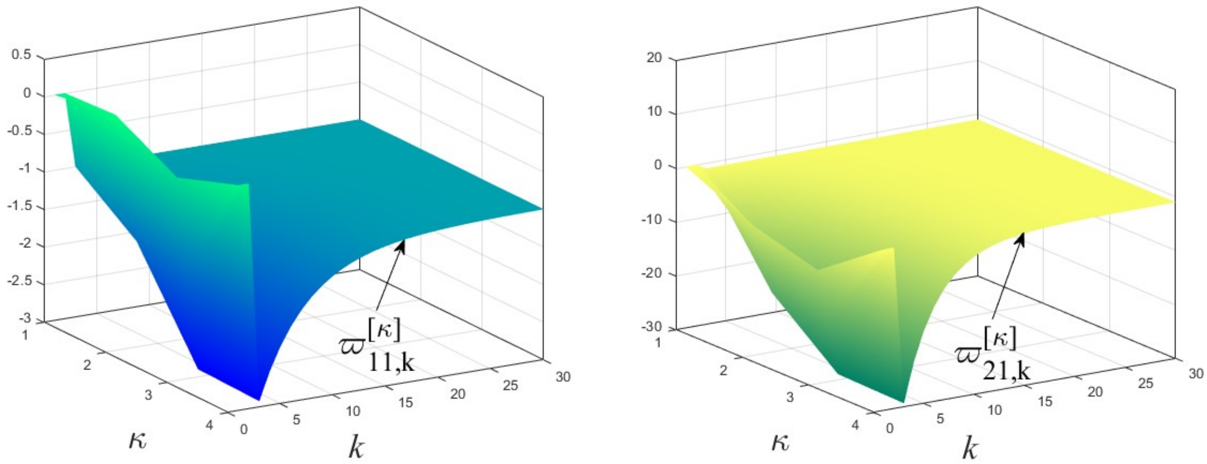

Figure 1. 3D states ϖ_{11}, ϖ_{21} in the response MAMNNs.

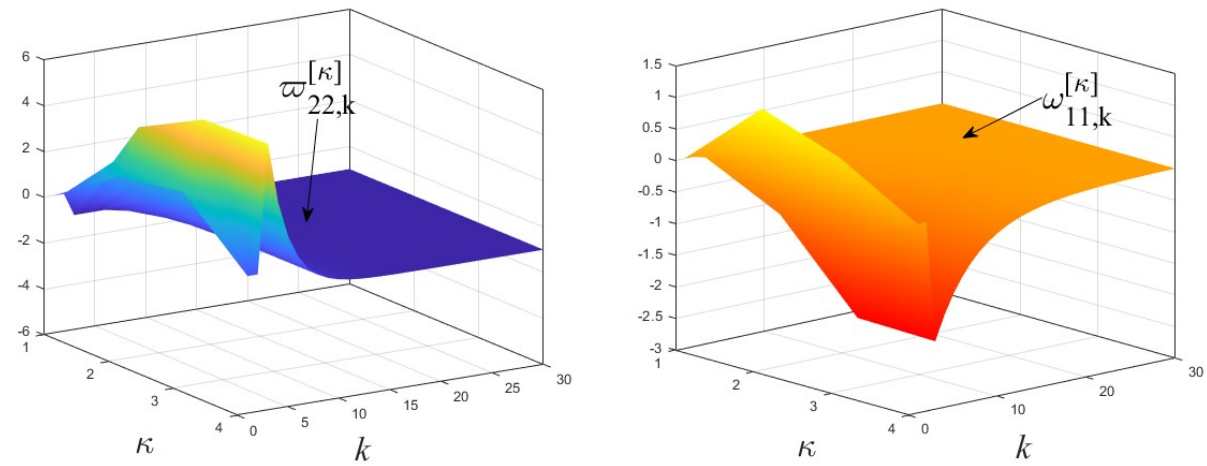

Figure 2. 3D state ϖ_{22} and the error ω_{11} in the response-error MAMNNs.

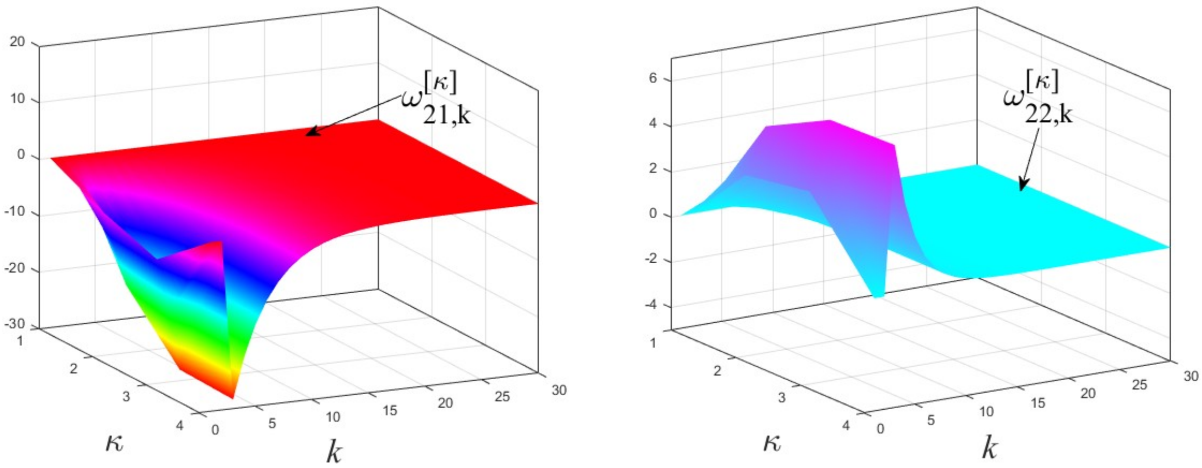

Figure 3. The 3D surfaces of the errors ω_{21} and ω_{22} in the error MAMNNs.

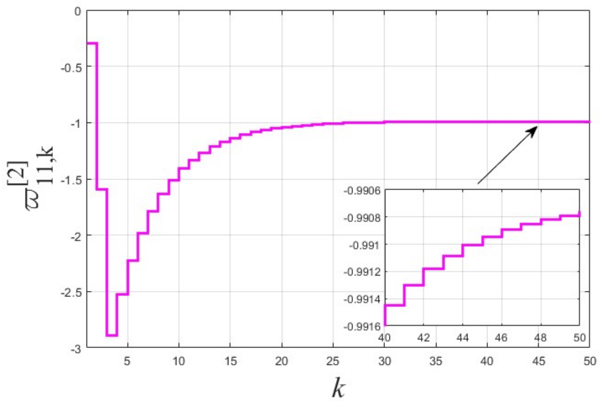

Figure 4. The 2D curve of the state ϖ_{11} with $\kappa = 2$.

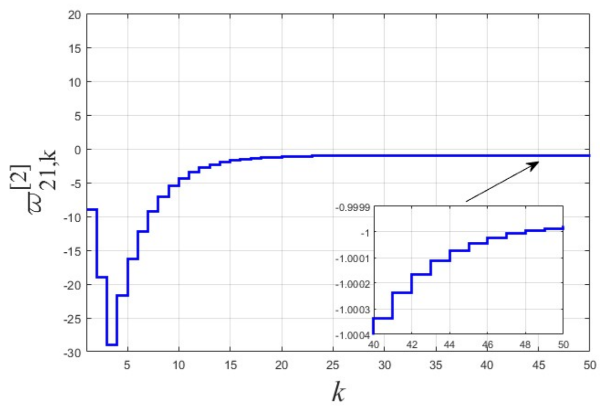

Figure 5. The 2D curve of the state ϖ_{21} with $\kappa = 2$.

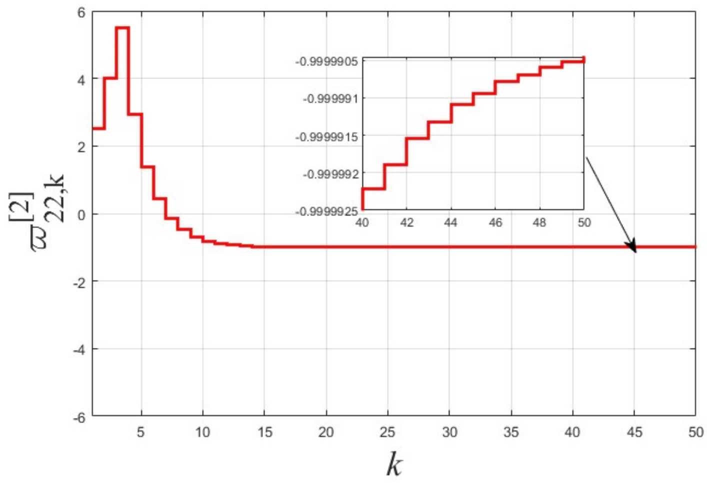

Figure 6. The 2D curve of the state ϖ_{22} with $\kappa = 2$.

6. Conclusions

In this paper, we investigated the problem of global asymptotic and exponential anti-stabilization for a class of space-time discrete fuzzy delayed multidirectional associative memory neural networks under Dirichlet boundary feedback controls. First, by using the Brouwer topological degree theory, we established a sufficient condition to guarantee the existence of the equilibrium point for the considered model. Then, by constructing appropriate discrete Lyapunov–Krasovskii functionals combined with boundary feedback control strategies, we derived verifiable criteria ensuring the global asymptotic and exponential anti-stabilization of the equilibrium state. These results provide a theoretical framework for analyzing anti-stabilizing dynamics in fuzzy MAM neural networks with discrete spatiotemporal structure, time-varying delays, diffusion effects, and fuzzy couplings.

The obtained results enrich the current research on anti-stabilization of neural networks by extending the analysis from conventional continuous-time or purely discrete-time settings to a more general space-time discrete fuzzy MAM neural network model. In particular, the proposed approach reveals how spatial diffusion, time-varying delays, fuzzy MIN/MAX interactions, and boundary feedback mechanisms jointly affect the anti-stabilizing behavior of the network.

There are several meaningful directions for future research. One possible extension is to investigate anti-stabilization for more general classes of discrete spatiotemporal neural networks involving stochastic perturbations, impulsive effects, switching parameters, or Markovian jumping structures. Another interesting topic is to develop less conservative anti-stabilization criteria by constructing more refined Lyapunov–Krasovskii functionals or by introducing event-triggered, adaptive, or distributed control schemes. In addition, it would be valuable to study whether the present theoretical framework can be applied to other neural network models with complex coupling structures, and to explore potential applications in distributed associative memory, signal transmission, and spatiotemporal information processing systems.

Use of AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The author declares there is no conflict of interest.

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