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Research article

On rotavirus infectious disease model using piecewise modified *ABC* fractional order derivative

Eiman[1], **Kamal Shah**[1,2,*], **Muhammad Sarwar**[1,2] **and Thabet Abdeljawad**[2,3,4,*]

[1] Department of Mathematics, University of Malakand, Chakdara Dir(L), 18000, Khyber Pakhtunkhwa, Pakistan
[2] Department of Mathematics and Sciences, Prince Sultan University, Riyadh, 11586, Saudi Arabia
[3] Department of Medical Research, China Medical University, Taichung 40402, Taiwan
[4] Department of Mathematics and Applied Mathematics School of Science and Technology, Sefako Makgatho Health Sciences University, Ga-Rankuwa, South Africa

* **Correspondence:** Email: kshah@psu.edu.sa, tabdeljawad@psu.edu.sa; Tel: +923448054428.

Abstract: The goal of this manuscript is to use a mathematical model with four compartments to examine the positive effects of rotavirus vaccinations. Susceptible, vaccinated, infected, and recovered (SVIR) classes are included in the suggested model. Some qualitative conclusions are established for the complicated pediatric disease epidemic model of rotavirus, which travels through a population at an inconsistent rate. The model has been fitted with piecewise equations of non-singular kernel-type derivatives in the modified Atangana-Balaneu-Caputo (*mABC*) sense. Using the Laplace transform and the notion of non-singular-type derivatives, we prove several basic conclusions regarding the solution's feasibility and positivity. We have used the matrix approach to compute the reproductive number further. Also, the sensitivity of the model has been computed. Additionally, we have used an efficient numerical approach to simulate the model by using some numerical values for the nomenclature of the model. Additionally, using the numerical approach, various graphical illustrations are given.

Keywords: SVIR model; piecewise derivative; *mABC*; sensitivity analysis; mumerical technique

1. Introduction

Rotavirus is an acute gastrointestinal pathogen that mainly affect infants and young children under the age of five throughout the world. The virus of the said disease was first identified by Bishop [1], when he found wheel-shaped particles in bowel specimens of a child. It can cause acute gastroenteritis and diarrhea [2, 3]. According to the data on the peak incidence, which occurs between four and 36 months of age, about 95 percent of children worldwide have contracted the disease [4]. Its name comes

from the fact that the virus resembles a wheel when viewed under a microscope. Its symptoms usually appear about two days after a person is exposed to rotavirus and remain for eight days [5–7]. Symptoms include fever, vomiting, pains in the abdomen, nausea, and recurrent watery diarrhea. Recurrent watery diarrhea causes severe dehydration that can be dangerous, especially for infants and young children. Therefore, limiting dehydration is essential for disease control. The oral-fecal pathway, hand-to-mouth contact, infected objects and surfaces, and potentially the respiratory route are the main ways that the aforementioned virus is spread [8–11]. The virus takes around two days to incubate [12–14].

It has been noted that the rate of rotavirus infection is comparable in industrialized and underdeveloped nations, indicating that improvement to sanitation, hygiene, or water supply may not be sufficient to realize adequate control measures. Therefore, it is advised to develop, test, and utilize rotavirus vaccines widely in order to prevent severe and also fatal rotavirus sickness [15]. The World Health Organization recommended in June 2009 that rotavirus vaccination shall be necessitated in every country's immunization program on the basis of recent, persuasive data on the illness load of rotavirus and the ability of vaccinations to stop it in situations with limited resources [16].

Recent studies clarify that mathematical modeling can be used to obtain a better understanding of the dynamics of rotavirus transmission, predict its effects in particular countries, and evaluate the potential benefits of interventions. Epidemiology is being studied for treatment, cure, control, and other outcomes for a number of infectious diseases because of the technological advancements in the field. Consequently, throughout the past few decades, there has been a significant advancement in the mathematical modeling of infectious diseases (see [17, 18]). Mathematical models can be used in secure public health systems to effectively control a variety of diseases, including those listed in a previous paper [19]. Both the dynamic behavior of infections and spatial temporal patterns can be studied by using these mathematical models. Over the past three years, researchers have examined rotavirus sickness from a number of perspectives by utilizing and exploiting the importance of mathematical models. Many approaches are being used by researchers in this field to provide workable protocols to manage this sickness (see [20, 21]). Some researchers [22] have analyzed at a mathematical model that was designed to look into how vaccinations and breastfeeding affect the rotavirus pandemic.

The mentioned infectious disease has been studied through the use of mathematical modeling recently. In addition, recent studies that also examined state-of-the-art techniques for diagnosis and treatment have identified risk factors for severe rotavirus infection. Sustained investigation in these domains is crucial in the fight against rotavirus and the mitigation of fatalities resulting from this pathogen. Focusing on the aforementioned disease, a group of authors studied the following compartmental model [23]:

$$\begin{cases} \dot{\mathcal{S}}(t) = (1-\hat{\vartheta})\mu + \varsigma\mathcal{V}(t) - [\kappa\mathcal{I}(t) + (\rho+\lambda)]\mathcal{S}(t), \\ \dot{\mathcal{V}}(t) = \hat{\vartheta}\mu + \rho\mathcal{S}(t) - [\zeta\kappa\mathcal{I}(t) + (\varsigma+\lambda)]\mathcal{V}(t), \\ \dot{\mathcal{I}}(t) = [\kappa\mathcal{S}(t) + \zeta\kappa\mathcal{V}(t) - (\tau+\omega+\lambda)]\mathcal{I}(t), \\ \dot{\mathcal{R}}(t) = -\lambda\mathcal{R}(t) + \omega\mathcal{I}(t), \\ \mathcal{S}(0) = \mathcal{S}_0,\ \mathcal{V}(0) = \mathcal{V}_0, \mathcal{I}(0) = \mathcal{I}_0,\ \mathcal{R}(0) = \mathcal{R}_0. \end{cases} \tag{1.1}$$

Fractional calculus has garnered much attention by researchers in the last few decades. The mentioned area has been utilized very well to study various problems in the real world.

Here, we remark that researchers [24] studied various biological problems by using fractional calculus to investigate the bifurcation and control analysis. In the same way, other researchers [25] have studied some problems related to neural networks by using fractional derivatives. Another group of authors [26] used tools of fractional calculus to investigate a mixed-controller neural network problem. In addition, regarding numerous problems in neural network models, researchers have used the fractional-order derivatives to investigate bifurcation analysis in detail. In this regard, some recent contributions can be found in [27]. There are several definitions available in the current literature about the fractional calculus. The first notable definition was given by Reimann and Liouville, and it has been used in large numbers of articles. Also, Caputo, Grünwald-Letnikov, Hadamard, Hilfer, and Riesz have defined fractional-order differentiations with their own concepts. For some historical improvement in the mentioned area, readers should read [28]. Recently, in 2015 and 2016, two new differential operators were introduced on the basis of exponential and Mittag-Leffler kernels, and they were called non-singular fractional differential operators. The derivative with an exponential kernel was given by Caputo and Fabrizio [29]. In the same way, Atangana and Baleanu introduced their derivatives on the basis of the Mittag-Leffler kernel [30]. The said nonlocal operators have various applications to the real world problems. The first definition given by Atangana and Baleanu in the Caputo sense is called *ABC* [31]. The said derivative suffers from the lack of initialization condition. Therefore, some modification was needed to upgrade the said operator. Other authors [32] modified the former *ABC* derivative to an updated operator which circumvents the mentioned drawback. The modified ABC (*mABC*) derivative has been used in many research articles for applied analysis. Here, we refer the readers to [33, 34].

Various evolutionary processes suffer from sudden changes in their state of rest or uniform motion. The multi-behavioral effect in dynamics as a result of these abrupt changes cannot be modeled by ordinary derivatives. As a result, a number of researchers have observed recently that piecewise differential equations can be used to design the aforementioned process more efficiently than integer-order equations. Piecewise operators of differentiations have received attention recently. For instance, a group of authors introduced the concepts in terms of fractional order (see [35]), where they used the traditional form of *ABC* derivatives in the piecewise sense. Here, we remark that the piecewise form of the *mABC* derivative has very rarely been used in epidemiological problems.

Therefore, motivated by the aforesaid work, for the analysis of crossover behavior, we chose to investigate the considered problem for qualitative, numerical, and stability analysis in the sense of the *mABC* piecewise derivative. This article is focused on investigating the rotavirus disease model under the said operator, because the problem still has not been studied by using the piecewise version of *mABC* derivatives. Here, we consider the model (1.1) with a vaccination class to analyze the considered rotavirus infection for $0 < \mathrm{p} < 1,\ t \in [0, T]$ by using the *mABC* piecewise derivative as follows:

$$\begin{cases} {}_0^{PABC}D_t^{\mathrm{p}}(\mathcal{S}(t)) = (1-\hat{\vartheta})\mu - [\kappa\mathcal{I}(t) + \varsigma\mathcal{V}(t) + (\rho+\lambda)]\mathcal{S}(t) = \Psi_1(t, \mho(t)), \\ {}_0^{PABC}D_t^{\mathrm{p}}(\mathcal{V}(t)) = \hat{\vartheta}\mu + \rho\mathcal{S}(t) - [\zeta\kappa\mathcal{I}(t) + (\varsigma+\lambda)]\mathcal{V}(t) = \Psi_2(t, \mho(t)), \\ {}_0^{PABC}D_t^{\mathrm{p}}(\mathcal{I}(t)) = [\kappa\mathcal{S}(t) + \zeta\kappa\mathcal{V}(t) - (\tau+\omega+\lambda)]\mathcal{I}(t) = \Psi_3(t, \mho(t)), \\ {}_0^{PABC}D_t^{\mathrm{p}}((\mathcal{R})(t)) = -\lambda\mathcal{R}(t) + \omega\mathrm{I}(t) = \Psi_4(t, \mho(t)), \\ \mathcal{S}(0) = \mathcal{S}_0 > 0, \mathcal{V}(0) = \mathcal{V}_0 \geq 0, \mathcal{I}(0) = \mathcal{I}_0 \geq 0,\ \mathcal{R}(0) = \mathcal{R}_0 \geq 0, \end{cases} \tag{1.2}$$

where $\mho = (\mathcal{S}, \mathcal{V}, \mathcal{I}, \mathcal{R})$, and ${}_0^{PABC}D_t^{\mathrm{p}}$ represents the piecewise *ABC* derivative, which can be defined

in Eq (1.3) for the function g as follows:

$$
{}_{0}^{PABC}D_{t}^{\mathrm{p}}(g(t)) = \begin{cases} {}_{0}^{C}D_{t}^{\mathrm{p}}(g(t)) = \dfrac{1}{\Gamma(1-\mathrm{p})}\displaystyle\int_{0}^{t_1}(t-\zeta)^{-\mathrm{p}}g'(\zeta)d\zeta, \;\; 0 < t \leq t_1, \\ {}_{0}^{mABC}D_{t}^{\mathrm{p}}(g(t)) = \dfrac{ABC(\mathrm{p})}{1-\mathrm{p}}\Big[g(t) - \mathscr{E}_{\mathrm{p}}(\mu_{\mathrm{p}}t^{\mathrm{p}})g(0) \\ -\mu_{\mathrm{p}}\displaystyle\int_{t_1}^{t}(t-\zeta)^{\mathrm{p}-1}\mathscr{E}_{\mathrm{p,p}}(-\mu_{\mathrm{p}}(t-\zeta)^{\mathrm{p}})g(\zeta)d(\zeta)\Big], \;\; t_1 < t \leq T, \end{cases} \tag{1.3}
$$

where $\mu_{\mathrm{p}} = \frac{\mathrm{p}}{1-\mathrm{p}}$, and ${}_{0}^{C}D_{t}^{\mathrm{p}}$ and ${}_{0}^{mABC}D_{t}^{\mathrm{p}}$ are the Caputo and *mABC* derivatives, respectively. Further, the compartments, and parameters are described in Table 1 in detail.

Table 1. The nomenclatures, and the corresponding specifications for the model (1.2).

Symbol	Specification
$\mathcal{S}$	Quantity of susceptible class
$\mathcal{V}$	Quantity of vaccinated class
$\mathcal{I}$	Quantity of infected class
$\mathcal{R}$	Quantity of recovered class
ζ	Expected decrease in risk of infection $\zeta \in (0, 1]$ due to vaccination
μ	Recruitment rate for humans
$\hat{\vartheta}$	Recruitment rate for individuals who have been vaccinated
κ	$\mathcal{S}$ class infection rate
ρ	Individual vaccination rate for susceptible class
ς	Rate at which vaccinated transfer to susceptible
τ	Induced rate of disease
λ	Natural death rate
ω	Rate of recovery

In this case, we prove the positivity of the solution and the feasible region under the theory of the modified piecewise *ABC* abbreviated as *mPABC* fractional-order derivative. To realize the aforementioned outcomes, we chose to adhere the process described in [36, 37]. Existence of a solution and uniqueness can be derived by using some tools of mathematical analysis [38]. Furthermore, we characterize the piecewise dynamics in order to observe crossover behaviors in the solution. The Lagrange's polynomial interpolation method [39] is extended to simulate our results graphically. We use some real values for the parameters in the proposed model and present several graphical illustrations resulting from the numerical simulations.

The outline of this work is follows. A detailed introduction is given in Section 1. The basic results are given in Section 2. The qualitative analysis is given in Section 3. The numerical procedure is described in Section 4. Results and discussion are given in Section 5. Finally, the conclusion of the article is given in Section 6.

2. Preliminaries

Here, we give some definitions that are needed in our work.

Definition 2.1. *[32] The traditional mABC derivative of a function $\phi \in L^1(0,T)$ with order* $\mathrm{p} \in (0,1)$ *is defined in Eq* (2.1) *as follows:*

$$ {}^{mABC}{}_0D_t^{\mathrm{p}}(\phi(t)) = \frac{ABC(\mathrm{p})}{1-\mathrm{p}}\Big[\phi(t) - \mathscr{E}_{\mathrm{p}}(\mu_{\mathrm{p}}t^{\mathrm{p}})\phi(0) - \mu_{\mathrm{p}}\int_0^t (t-\chi)^{\mathrm{p}-1}\mathscr{E}_{\mathrm{p},\mathrm{p}}(-\mu_{\mathrm{p}}(t-\chi)^{\mathrm{p}})\phi(\chi)d\chi\Big], \tag{2.1} $$

where $ABC(\mathrm{p})$ satisfies, $ABC(0) = ABC(1) = 1$. Also, $\mathscr{E}_{\mathrm{p}}$ represents the Mittag-Leffler function. Then, $\Gamma(.)$ is defined by $\Gamma(\mathrm{p}) = \int_0^\infty t^{\mathrm{p}-1}\exp(-t)dt$.

Definition 2.2. *[32] The Riemann-Liouville integral of fractional order* $\mathrm{p} \in (0,1)$ *for the function $\phi \in L[0,t_1]$ as follows:*

$$ {}_0I_t^{\mathrm{p}}[\phi(t)] = \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\phi(\chi)d\chi. $$

For $\phi \in L^1(t_1,T)$ with order $\mathrm{p} \in (0,1)$, *the modified Atangana-Baleanue* (*mAB*) *integral is described in Eq* (2.2) *by*

$$ {}^{mAB}{}_{t_1}I_t^{\mathrm{p}}[\phi(t)] = \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[(\phi(t)-\phi(t_1)) + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}[\phi(\chi)-\phi(0)]d\chi\Big]. \tag{2.2} $$

Lemma 2.3. *[32] For $\phi \in C[0,T]$ and $\psi \in L^1[0,T]$, the solution of the problem with the mABC derivative, i.e.,*

$$ \begin{aligned} {}^{mABC}{}_0D_t^{\mathrm{p}}[\phi(t)] &= \psi(t),\ \mathrm{p} \in (0,1), \\ \phi(0) &= \phi_0 \end{aligned} $$

is given by

$$ \begin{aligned} \phi(t) &= \phi_0 + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[(\psi(t)-\psi(0)) + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}[\psi(\chi)-\psi(0)]d\chi\Big] \\ &= \phi_0 + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\psi(t) - \psi(0)\Big(1 + \frac{\mu_{\mathrm{p}}t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big) + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\psi(\chi)d\chi\Big]. \end{aligned} $$

Definition 2.4. *[33]. The piecewise integral of a differentiable function ϕ is defined as follows:*

$$ {}_0I_t[\phi(t)] = \begin{cases} \displaystyle\int_0^{t_1} \phi(\chi)d\chi, & 0 < t \le t_1, \\ \displaystyle\int_{t_1}^{t_2} \phi(\chi)d\chi & t_1 < t \le T. \end{cases} $$

Definition 2.5. *[32, 35]. From Definition 2.2 and the usual Riemann-Liouville integral of fractional-order for a function $\phi \in L[0,T]$, we define the piecewise integral in the mAB sense as follows:*

$$ {}^{PAB}_0I_t^{\mathrm{p}}[\phi(t)] = \begin{cases} \dfrac{1}{\Gamma(\mathrm{p})}\displaystyle\int_0^t (t-\chi)^{\mathrm{p}-1}\phi(\chi)d(\chi), & 0 < t \le t_1, \\ \dfrac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[(\phi(t)-\phi(t_1)) + \dfrac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\displaystyle\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}[\phi(\chi)-\phi(t_1)]d\chi, & t_1 < t \le T. \end{cases} $$

Definition 2.6. *[32, 35]. From Definition 2.1 and the usual Caputo fractional-order derivative, we define the piecewise mABC derivative with order* $\mathrm{p} \in (0,1)$ *as follows:*

$$
{}^{PABC}{}_0D_t^{\mathrm{p}}[\phi(t)] = \begin{cases} {}^{C}{}_0D_t^{\mathrm{p}}(\phi(t)) = \dfrac{1}{\Gamma(1-\mathrm{p})}\displaystyle\int_0^t \psi(\chi)(t-\chi)^{-\mathrm{p}}d\chi,\ 0 < t \le t_1, \\ {}^{mABC}{}_{t_1}D_t^{\mathrm{p}}(\phi(t)) = \dfrac{ABC(\mathrm{p})}{1-\mathrm{p}}\Big[\phi(t) - \mathscr{E}_{\mathrm{p}}(\mu_{\mathrm{p}}t^{\mathrm{p}})\phi(t_1) \\ -\mu_{\mathrm{p}}\displaystyle\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}\mathscr{E}_{\mathrm{p},\mathrm{p}}(-\mu_{\mathrm{p}}(t-\chi)^{\mathrm{p}})\phi(\chi)d\chi\Big],\ t_1 < t \le T. \end{cases}
$$

Lemma 2.7. *[32, 35]; Let* $\phi \in L^1(0,T)$ *and* $\psi \in L^1[0,T]$; *the solution of the problem with the PABC derivative*

$$
\begin{aligned} {}^{PABC}{}_0D_t^{\mathrm{p}}[\phi(t)] &= \psi(t),\ \mathrm{p} \in (0,1), \\ \phi(0) &= \phi_0 \end{aligned}
$$

is given by

$$
\phi(t) = \begin{cases} \phi_0 + \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\psi(\chi)d\chi,\ 0 < t \le t_1, \\ \phi(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\psi(t) - \psi(t_1)\Big(1 + \frac{\mu_{\mathrm{p}}t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big) + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}\psi(\chi)d\chi\Big],\ t_1 < t \le T. \end{cases}
$$

Let us define a Banach space by $\mathbb{B} = C[0,T]$, with the norm described by $\|\mho\| = \max_{t\in[0,T]}|\mho(t)|$.

Theorem 2.8. *[36] If the following hold true:*

- $\mathbb{A} : \mathbb{B} \to \mathbb{B}$ *is a compact operator;*
- *there exists a constant* $\varepsilon > 0$ *and* $\mho = \delta\mathbb{A}(\mho), \delta \in (0,1)$ *such that* $\|\mathbb{A}(\mho)\| \le \varepsilon$*;*

then the operator equation $\mathbb{A}$ *has at least one solution.*

3. Qualitative results

In this section, we establish some results about positivity, equilibrium points, the basic reproduction number, and its sensitivity. The mentioned results are investigated by using the *mABC* derivative and integral transform.

Theorem 3.1. *The set* $\Upsilon = \{(\mathcal{S},\mathcal{V},\mathcal{I},\mathcal{R}) \in \mathbb{R}_+^4 : N \le \frac{\mu}{\lambda}\}$ *is positive- invariant. Additionally, every solution is drawn to* $\mathbb{R}_+^4$.

Proof. From the model (1.2), and given that N is the total population, we have

$$
\begin{aligned} {}_0^{PABC}D_t^{\mathrm{p}}N(t) &= \mu - \lambda\mathcal{S}(t) - \lambda\mathcal{V}(t) - \lambda\mathcal{I}(t) - \lambda\mathcal{R}(t) - \tau\mathcal{I}(t) \\ &\le \mu - \lambda(\mathcal{S}(t) + \mathcal{V}(t) + \mathcal{I}(t) + \mathcal{R}(t)) \\ &= \mu - \lambda N(t). \end{aligned} \tag{3.1}
$$

We can write Eq (3.1) as follows:

$$
{}_0^{PABC}D_t^{\mathrm{p}}N(t) = \begin{cases} {}^{C}D_t^{\mathrm{p}}N(t) \le \mu - \lambda N, \\ {}^{mABC}D_t^{\mathrm{p}}N(t) \le \mu - \lambda N(t). \end{cases} \tag{3.2}
$$

Taking the Laplace transform, and following [32] by using $\gamma_{\mathrm{p}} = ABC(\mathrm{p}) + \lambda(1-\mathrm{p})$ and $N(0) = N_0$, from Eq (3.2), the result follows:

$$N(t) \leq \begin{cases} \frac{\mu_{\mathrm{p}}}{\lambda}\Big(1 - \mathscr{E}_{\mathrm{p}}(-\lambda t^{\mathrm{p}})\Big) + N_0\mathscr{E}_{\mathrm{p}}(-\lambda t^{\mathrm{p}}), \\ \frac{N_0 ABC(\mathrm{p})}{\gamma_{\mathrm{p}}}\mathscr{E}_{\mathrm{p}}\Big(-\frac{\lambda \mathrm{p} t^{\mathrm{p}}}{\gamma_{\mathrm{p}}}\Big) + \frac{(1-\mathrm{p})\mu}{\gamma_{\mathrm{p}}} + \frac{(1-\mathrm{p})\mu}{\gamma_{\mathrm{p}}}\Big[\mu_{\mathrm{p}} - \frac{\lambda \mathrm{p}}{\gamma_{\mathrm{p}}}\Big(t^{\mathrm{p}-1}\mathscr{E}_{\mathrm{p},\mathrm{p}}\Big(-\frac{\lambda \mathrm{p} t^{\mathrm{p}}}{\gamma_{\mathrm{p}}}\Big)\Big)\Big]. \end{cases} \tag{3.3}$$

If we apply $t \to \infty$ in Eq (3.3), we obtain that

$$N(t) \leq \frac{\mu}{\lambda}.$$

Therefore, the region corresponds to that in which the solutions are bounded and Υ is positively invariant. □

Theorem 3.2. *Let* $\{(\mathcal{S}_0, \mathcal{V}_0, \mathcal{I}_0, \mathcal{R}_0) \geq 0\} \in \mathbb{R}^4_+$*; then, the set* $\{(\mathcal{S}, \mathcal{V}, \mathcal{I}, \mathcal{R})\}$ *of the solution of model* (1.2) *remains positive at all* $t > 0$.

Proof. From the first equation of model (1.2), one has

$$\begin{aligned} {}_0^{PABC}D_t^{\mathrm{p}}[\mathcal{S}(t)] &= (1-\hat{\vartheta})\mu + \varsigma\mathcal{V}(t) - [\kappa\mathcal{S}(t)\mathcal{I}(t) - (\rho+\lambda)\mathcal{S}(t)] \\ &\geq -\Big(\kappa\mathcal{I}(t) - (\rho+\lambda)\Big)\mathcal{S}(t). \end{aligned} \tag{3.4}$$

Let us use $\omega = \kappa\mathcal{I}(t) - (\rho+\lambda)$ in Eq (3.4); we have

$${}_0^{PABC}D_t^{\mathrm{p}}[\mathcal{S}(t)] \geq -\omega\mathcal{S}(t). \tag{3.5}$$

Now, we can write Eq (3.5) as follows:

$${}_0^{PABC}D_t^{\mathrm{p}}[\mathcal{S}(t)] = \begin{cases} {}^{C}D_t^{\mathrm{p}}[\mathcal{S}(t)] \geq -\omega\mathcal{S}(t) \\ {}^{mABC}D_t^{\mathrm{p}}[\mathcal{S}(t)] \geq -\omega\mathcal{S}(t). \end{cases}$$

We establish that

$${}^{C}D_t^{\mathrm{p}}[\mathcal{S}(t)] \geq -\omega\mathcal{S}(t), t \in (0, t_1].$$

Taking the Laplace transform, one has

$$\mathcal{S}(t) \geq \mathcal{S}_0\mathscr{E}_{\mathrm{p}}(-\omega t^{\mathrm{p}}) > 0, \text{ for all } t \in (0, t_1].$$

Now, consider the other case:

$${}^{mABC}D_t^{\mathrm{p}}[\mathcal{S}(t)] \geq -\omega\mathcal{S}(t)$$

which leads to the Laplace transform by using the fact that $\gamma_\omega = ABC(\mathrm{p}) + (1-\mathrm{p})\omega$ and $\sigma = \frac{\mu_{\mathrm{p}}\omega(1-\mathrm{p})}{\gamma_\omega}$

$$\mathscr{L}[\mathcal{S}(t)] \geq \frac{\mathcal{S}_0 ABC(\mathrm{p})}{\gamma_\omega}\Big(\frac{s^{\mathrm{p}-1}}{s^{\mathrm{p}}+\sigma}\Big), \tag{3.6}$$

Equation (3.6) implies that

$$\mathcal{S}(t) \geq \mathcal{S}_0\mathscr{E}_{\mathrm{p}}(-\sigma t^{\mathrm{p}}) > 0, \text{ for all } t \in (t_1, T]. \tag{3.7}$$

Therefore, from Eq (3.7), we have that $\mathcal{S} > 0$ at every $t > 0$. Using the same process, for other compartments, we can prove that $\mathcal{V} > 0$, $\mathcal{I} > 0$, and $\mathcal{R} > 0$. □

Remark 3.3. *Furthermore, it can be easily seen that the disease-free equilibrium point is given by*

$$(\mathcal{S}_0, \mathcal{V}_0, 0, 0) = \left(\frac{(\lambda\mu)(1-\hat{\vartheta}) + \varsigma\mu}{\lambda(\rho+\lambda+\varsigma)}, \frac{\mu(\rho+\lambda\hat{\vartheta})}{\lambda(\rho+\lambda+\varsigma)}, 0, 0\right).$$

Using the third equation of model (1.2)*, the basic reproduction number can be calculated by taking the following matrices as follows:*

$$\mathbf{F} = \begin{bmatrix} \kappa\mathcal{S}_0 + \chi\kappa\mathcal{V}_0 \end{bmatrix}, \ \mathbf{V}^{-1} = \begin{bmatrix} \frac{1}{\tau+\omega+\lambda} \end{bmatrix}.$$

The Jacobian matrices at the trivial equilibrium point of $\mathbf{F}$ *and* $\mathbf{V}$ *are respectively given by*

$$\mathbf{F} = \begin{bmatrix} \frac{\kappa}{\lambda}\left(\frac{(\lambda\mu)(1-\hat{\vartheta})+\varsigma\mu}{(\rho+\lambda+\varsigma)} + \chi\frac{\mu(\rho+\lambda\hat{\vartheta})}{(\rho+\lambda+\varsigma)}\right) \end{bmatrix}, \ \mathbf{V}^{-1} = \begin{bmatrix} \frac{1}{\tau+\omega+\lambda} \end{bmatrix}.$$

We have

$$\mathbf{F}\mathbf{V}^{-1} = \begin{bmatrix} \frac{\kappa}{\lambda(\tau+\omega+\lambda)}\left(\frac{(\lambda\mu)(1-\hat{\vartheta})+\varsigma\mu}{(\rho+\lambda+\varsigma)} + \chi\frac{\mu(\rho+\lambda\hat{\vartheta})}{(\rho+\lambda+\varsigma)}\right) \end{bmatrix}.$$

The total number of rotavirus infections that a single infected person can cause in the presence of vaccination is the basic reproduction number of the proposed model. Furthermore, R_0 *indicates the basic reproduction number in the absence of such intervention. Using the next generation's matrix approach [32, 35], we define* R_v *as follows:*

$$R_v = \frac{\kappa}{\lambda(\tau+\omega+\lambda)}\left[\frac{(\lambda\mu)(1-\hat{\vartheta}) + \varsigma\mu + \zeta\mu(\rho+\lambda\hat{\vartheta})}{\rho+\lambda+\varsigma}\right]. \tag{3.8}$$

In the absence of vaccination, $\rho = \varsigma = \theta = 0$*, we get the basic reproduction number as follows:*

$$R_0 = \frac{\mu\kappa}{\lambda(\tau+\omega+\lambda)}. \tag{3.9}$$

Using Eq (3.9)*, we can write Eq* (3.8) *as follows:*

$$R_v = R_0\left[\frac{\lambda(1-\hat{\vartheta}+\zeta\hat{\vartheta}) + \varsigma + \zeta\rho}{\rho+\lambda+\varsigma}\right]. \tag{3.10}$$

Since we have $0 < \zeta < 1$*, it follows from Eq* (3.10) *that* $\frac{\lambda(1-\hat{\vartheta}+\zeta\hat{\vartheta})+\varsigma+\zeta\rho}{\rho+\lambda+\varsigma} < 1$*, which means that* $R_v < R_0$*. Further, when* $\zeta = 1$ *or* $\rho = \varsigma = \theta = 0$*,* $R_v = R_0$*. The expression of* R_v *demonstrates that vaccinations against new illnesses are beneficial for both susceptible individuals and those infected from birth. Keep in mind that* $\zeta = 1$ *suggests that vaccinations are not necessary, which is not the case.*

Additionally, if $R_0 < 1$*, the disease-free equilibrium will be asymptotically stable; if* $R_0 > 1$*, it will be unstable. We present some 3D profiles for* R_0 *in Figure 1 by using different values of the parameters. This behavior indicates stabile behavior of the disease-free equilibrium point. We chose to take the parameter values from [23], as shown in Table 2.*

Table 2. The parameters and their numerical values for the model (1.2).

Parameters	Values	Parameters	Values
ς	0.001884	λ	0.00002537
ζ	0.001	τ	0.00004466
ρ	0.002778	κ	0.0001
$\hat{\vartheta}$	0.001884	ω	0.099
μ	0.4109		

We see that, for the given parameter values, $R_0 < 1$, which indicates that the disease-free equilibrium point of the suggested model is asymptotically stable.

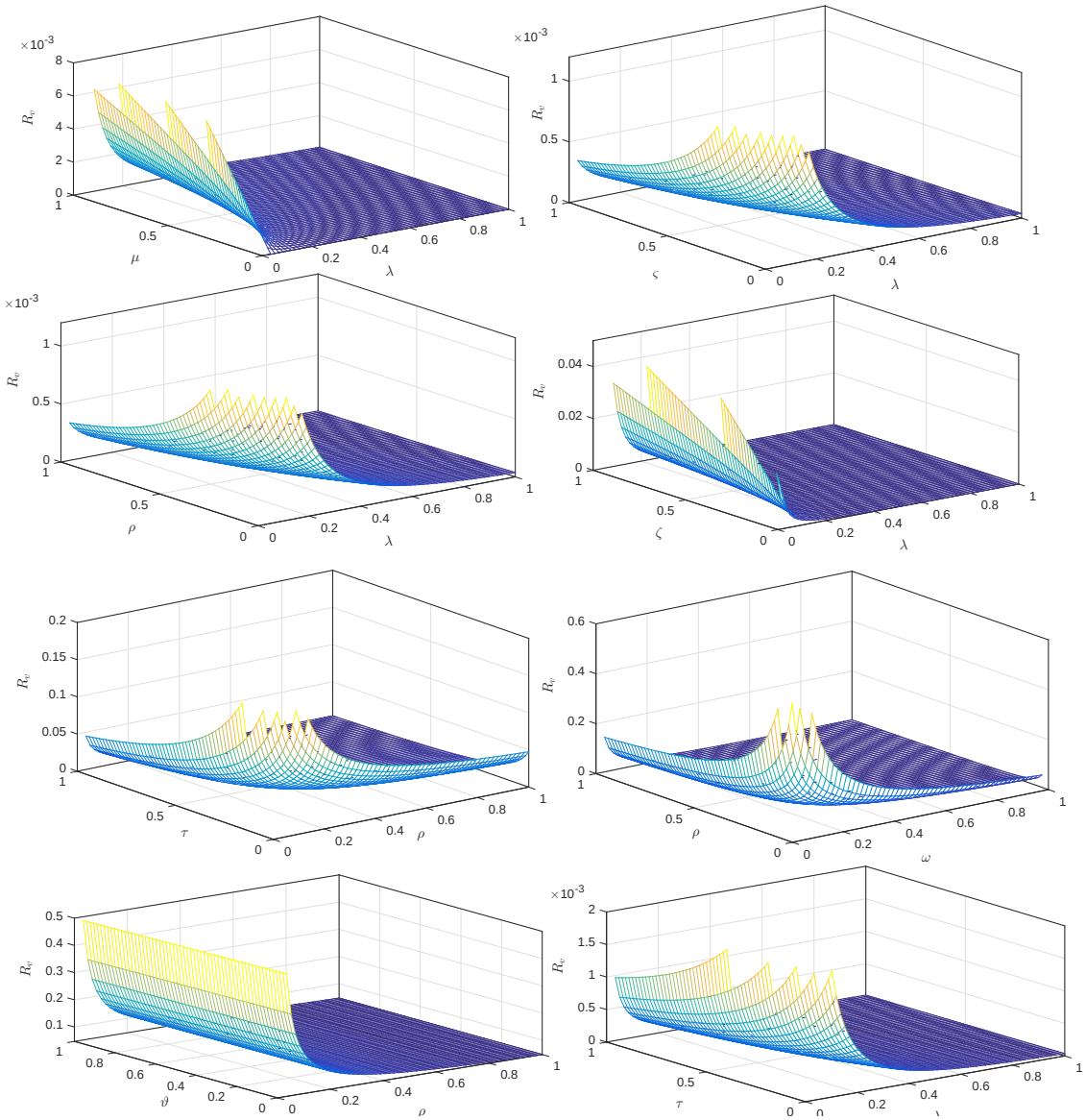

Figure 1. 3D graphical illustration of R_v of the proposed model.

3.1. Sensitivity analysis

Here, we present the sensitivity analysis for R_v on the basis of the significance of parameters by using the procedure given in [40]. These indices emphasize the importance of each component in the development and spread of the disease. Sensitivity analysis has been used to evaluate how well the model's predictions hold up to various parameter values. In this way, we chose to use the following formula depends on the parameters to compute the sensitivity indices p:

$$\mathcal{S}_p^{R_v} = \frac{p}{R_v}\left[\frac{\partial R_v}{\partial p}\right]. \tag{3.11}$$

Using Eq (3.11), we have

$$\begin{aligned}
\mathcal{S}_\mu^{R_v} &= \frac{\mu}{R_v}\left[\frac{\partial R_v}{\partial \mu}\right] = 1 > 0,\\
\mathcal{S}_\kappa^{R_v} &= \frac{\kappa}{R_v}\left[\frac{\partial R_v}{\partial \kappa}\right] = 1 > 0,\\
\mathcal{S}_\lambda^{R_v} &= \frac{\lambda}{R_v}\left[\frac{\partial R_v}{\partial \lambda}\right] = -1.4586 < 0,\\
\mathcal{S}_\tau^{R_v} &= \frac{\tau}{R_v}\left[\frac{\partial R_v}{\partial \tau}\right] = -0.00045079 < 0,\\
\mathcal{S}_\omega^{R_v} &= \frac{\omega}{R_v}\left[\frac{\partial R_v}{\partial \omega}\right] = -0.9992 < 0,\\
\mathcal{S}_{\hat{\vartheta}}^{R_v} &= \frac{\hat{\vartheta}}{R_v}\left[\frac{\partial R_v}{\partial \hat{\vartheta}}\right] = -0.000024919 < 0,\\
\mathcal{S}_\rho^{R_v} &= \frac{\rho}{R_v}\left[\frac{\partial R_v}{\partial \rho}\right] = -0.5912 < 0,\\
\mathcal{S}_\zeta^{R_v} &= \frac{\chi}{R_v}\left[\frac{\partial R_v}{\partial \zeta}\right] = -0.000022872 < 0,\\
\mathcal{S}_\varsigma^{R_v} &= \frac{\varsigma}{R_v}\left[\frac{\partial R_v}{\partial \varsigma}\right] = 0.0015 > 0.
\end{aligned} \tag{3.12}$$

Here in Figure 2, we present the sensitivity indices based on the values in Eq (3.12).

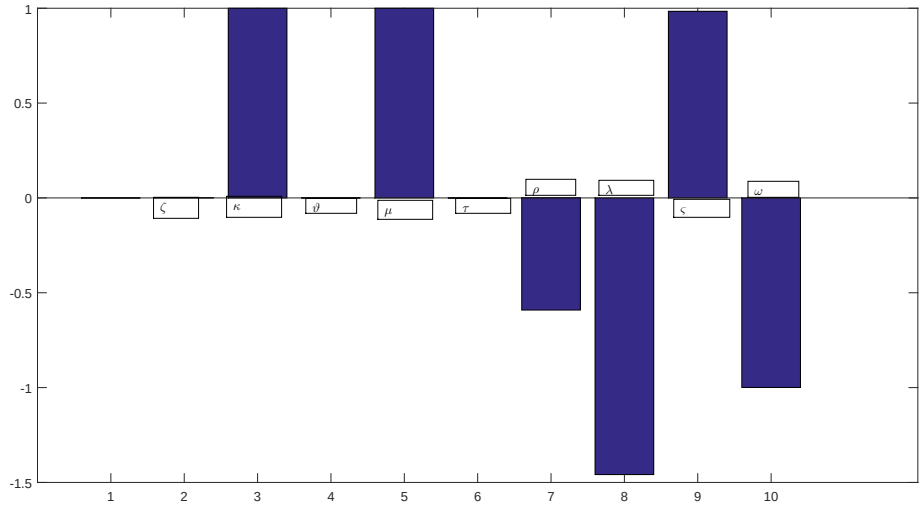

Figure 2. Illustration of sensitivity analysis results.

3.2. Existence of solution

Consider the following possible form for model (1.2)

$$\begin{aligned} {}^{PABC}{}_0D_t^{\mathrm{p}}[\mho(t)] &= \Psi(t,\mho(t)),\ \mathrm{p}\in(0,1), \\ \mho(0) &= \mho_0. \end{aligned} \tag{3.13}$$

Theorem 3.4. *The solution of Eq* (3.13) *is given by*

$$\mho(t)=\begin{cases}\mho_0+\frac{1}{\Gamma(\mathrm{p})}\int_0^t(t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mho(\chi))d\chi,\ 0<t\le t_1,\\ \mho(t_1)+\frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi(t,\mho(t))-\Psi(t_1,\mho(t_1))\Big(1+\frac{\mu_{\mathrm{p}}t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)+\frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t(t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mho(\chi))d\chi\Big],\ t_1<t\le T.\end{cases}$$

Proof. With the help of Lemma 2.7, we can easily obtain the solution. □

We describe some assumptions:

(G_1) For the real values $\mathcal{L}_\Psi>0$, and given that $\mho,\mathrm{z}\in\mathbb{B}$, one has

$$|\Psi(t,\mho(t))-\Psi(t,\mathrm{z}(t))|\le\mathcal{L}_\Psi|\mho-\mathrm{z}|.$$

(G_2) For real values $\mathcal{M}_\Psi,\mathcal{N}_\Psi>0$, one has

$$|\Psi(t,\mho(t))|\le\mathcal{M}_\Psi+\mathcal{N}_\Psi|\mho(t)|.$$

Theorem 3.5. *In view of* (G_1)*, and if*

$$\max\left\{\frac{\mathcal{L}_\Psi t_1^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)},\left(\frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}+\frac{(T^{\mathrm{p}}-{t_1}^{\mathrm{p}})\mu_{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\right)\mathcal{L}_\Psi\right\}=\Lambda<1$$

holds, then the problem described by Eq (3.13) *has a unique solution.*

Proof. Let us define $\mathbb{A} : \mathbb{B} \to \mathbb{B}$ as follows:

$$\mathbb{A}(\mathfrak{V}(t)) = \begin{cases} \mathfrak{V}_0 + \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathfrak{V}(\chi))d\chi, \ 0 < t \le t_1, \\ \mathfrak{V}(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi(t,\mathfrak{V}(t)) - \Psi(t_1,\mathfrak{V}(t_1))\Big(1 + \frac{\mu_{\mathrm{p}} t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big) \\ + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathfrak{V}(\chi))d\chi\Big], \ t_1 < t \le T. \end{cases}$$

For $\mathfrak{V}, \mathrm{z} \in \mathbb{B}$, consider the following:

$$|\mathbb{A}(\mathfrak{V}(t)) - \mathbb{A}(\mathrm{z}(t))| = \begin{cases} \Big|\frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathfrak{V}(\chi))d\chi - \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathrm{z}(\chi))d\chi\Big|, \ 0 < t \le t_1, \\ \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi(t,\mathfrak{V}(t)) - \Psi(t,\mathrm{z}(t)) \\ + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathfrak{V}(\chi))d\chi - \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathrm{z}(\chi))d\chi\Big\|, \ t_1 < t \le T \end{cases}$$

which further implies that

$$|\mathbb{A}(\mathfrak{V}(t)) - \mathbb{A}(\mathrm{z}(t))| \le \begin{cases} \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\,|\Psi(\chi,\mathfrak{V}(\chi)) - \Psi(\chi,\mathrm{z}(\chi))|\,d\chi, \ 0 < t \le t_1, \\ \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[|\Psi(t,\mathfrak{V}(t)) - \Psi(t,\mathrm{z}(t))| \\ + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}\,|\Psi(\chi,\mathfrak{V}(\chi)) - \Psi(\chi,\mathrm{z}(\chi))|\,d\chi\Big], \ t_1 < t \le T. \end{cases}$$

On further simplification of Eq (3.14), we have

$$\|\mathbb{A}(\mathfrak{V}) - \mathbb{A}(\mathrm{z})\| \le \begin{cases} \frac{\mathcal{L}_\Psi t_1^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\|\mathfrak{V} - \mathrm{z}\|, \ 0 < t \le t_1, \\ \Big(\frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})} + \frac{(T^{\mathrm{p}} - t_1{}^{\mathrm{p}})\mu_{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\mathcal{L}_\Psi\|\mathfrak{V} - \mathrm{z}\|, \ t_1 < t \le T. \end{cases} \tag{3.14}$$

Then, from Eq (3.14), we obtain the following:

$$\|\mathbb{A}(\mathfrak{V}) - \mathbb{A}(\mathrm{z})\| \le \begin{cases} \Lambda\|\mathfrak{V} - \mathrm{z}\|, \ 0 < t \le t_1, \\ \Lambda\|\mathfrak{V} - \mathrm{z}\|, \ t_1 < t \le T. \end{cases} \tag{3.15}$$

Therefore, Eq (3.15) yields that $\mathbb{A}$ is a contraction operator. Hence, the problem described by Eq (3.13) has a unique solution. Consequently, the model (1.2) has a unique solution. □

Theorem 3.6. *Using* (G_2)*, the problem described by Eq* (3.13) *has at least one solution.*

Proof. We consider a bounded closed subset $\Omega = \{\mathfrak{V} \in \mathbb{B} : \|\mathfrak{V}\| \le \varepsilon, \ \varepsilon > 0\} \subseteq \mathbb{B}$. Then, we define $\mathbb{C} : \Omega \to \Omega$ such that $\mathfrak{V} \in \Omega$ and we obtain the following:

$$\mathbb{C}(\mathfrak{V}(t)) = \begin{cases} \mathfrak{V}_0 + \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathfrak{V}(\chi))d\chi, \ 0 < t \le t_1, \\ \mathfrak{V}(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi(t,\mathfrak{V}(t)) - \Psi(t_1,\mathfrak{V}(t_1))\Big(1 + \frac{\mu_{\mathrm{p}} t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big) \\ + \frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}\Psi(\chi,\mathfrak{V}(\chi))d\chi\Big], \ t_1 < t \le T, \end{cases}$$

from which we deduce that

$$|\mathbb{C}(\mho(t))| \leq \begin{cases} |\mho_0| + \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}|\Psi(\chi,\mho(\chi))|d\chi,\ 0 < t \leq t_1, \\ |\mho(t_1)| + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[|\Psi(t,\mho(t))| + \Big|\Psi(t_1,\mho(t_1))\Big(1+\frac{\mu_{\mathrm{p}}t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big| \\ +\frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}|\Psi(\chi,\mho(\chi))|d\chi\Big],\ \ t_1 < t \leq T. \end{cases} \tag{3.16}$$

Hence, using (G_2), we get from Eq (3.16) that

$$\|\mathbb{C}(\mho\| \leq \max_{t\in[0,T]} \begin{cases} |\mho_0| + \frac{1}{\Gamma(\mathrm{p})}\int_0^t (t-\chi)^{\mathrm{p}-1}|\Psi(\chi,\mho(\chi))|d\chi,\ 0 < t \leq t_1, \\ |\mho(t_1)| + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[|\Psi(t,\mho(t))| + \Big|\Psi(t_1,\mho(t_1))\Big(1+\frac{\mu_{\mathrm{p}}t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big| \\ +\frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^t (t-\chi)^{\mathrm{p}-1}|\Psi(\chi,\mho(\chi))|d\chi\Big],\ \ t_1 < t \leq T. \end{cases} \tag{3.17}$$

Here, some parameters are presented for simplification:

$$\Delta_1 = |\mho_0| + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\mathcal{M}_\Psi + \Big|\Psi(0,\mho((t_1)))\Big(1+\frac{\mu_{\mathrm{p}}T^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big| + \frac{\mathcal{M}_\Psi\mu_{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big],$$

$$\Delta_2 = \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\mathcal{N}_\Psi\Big[1+\frac{\mu_{\mathrm{p}}T^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big].$$

Therefore, we can write Eq (3.17) as follows:

$$\|\mathbb{C}(\mho\| \leq \begin{cases} |\mho_0| + \frac{t_1^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big[\mathcal{M}_\Psi + \mathcal{N}_\Psi\varepsilon\Big],\ 0 < t \leq t_1, \\ \Delta_1 + \Delta_2\varepsilon,\ \ t_1 < t \leq T. \end{cases} \tag{3.18}$$

Assume that $\varepsilon \geq \max\left\{\frac{|\mho_0|+t_1^{\mathrm{p}}\mathcal{M}_\Psi}{\Gamma(\mathrm{p}+1)-t_1^{\mathrm{p}}\mathcal{N}_\Psi}, \frac{\Delta_1}{1-\Delta_2}\right\}$; then, Eq (3.18) implies that

$$\|\mathbb{C}(\mho\| \leq \begin{cases} \varepsilon,\ 0 < t \leq t_1, \\ \varepsilon,\ \ t_1 < t \leq T. \end{cases} \tag{3.19}$$

Hence, we have from Eq (3.19) $\mathbb{C}(\mho \in \Omega$ which yields that $\mathbb{C}(\Omega) \subseteq \Omega$. Therefore, the operator $\mathbb{C}$ is bounded. We next aim to prove continuity. As we know that, Ψ is continuous over the sub-intervals $[0, t_1]$ and $(t_1, T]$, $\mathbb{C}$ is also continuous over the same sub-intervals $[0, t_1]$ and $(t_1, T]$. To prove uniform continuity, let us take $t_k > t_j$ in the both sub-intervals; then, we have the following:

$$|\mathbb{C}(\mho(t_k)) - \mathbb{C}(\mho(t_j))| \leq \begin{cases} \frac{1}{\Gamma(\mathrm{p})}\int_0^{t_k}[(t_k-\chi)^{\mathrm{p}-1} - (t_j-\chi)^{\mathrm{p}-1}]|\Psi(\chi,\mho(\chi))|d\chi \\ +\frac{1}{\Gamma(\mathrm{p})}\int_{t_j}^{t_k}(t_k-\chi)^{\mathrm{p}-1}]|\Psi(\chi,\mho(\chi))|d\chi,\ 0 < t \leq t_1, \\ \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[|\Psi(t_k,\mho(t_k)) - \Psi(t_j,\mho(t_j))| + \Big|\Psi((t_1),\mho((t_1)))\Big(\frac{\mu_{\mathrm{p}}(t_k^{\mathrm{p}}-t_j^{\mathrm{p}})}{\Gamma(\mathrm{p}+1)}\Big)\Big| \\ +\frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_1}^{t_j}[(t_k-\chi)^{\mathrm{p}-1} - (t_j-\chi)^{\mathrm{p}-1}]|\Psi(\chi,\mho(\chi))|d\chi \\ +\frac{\mu_{\mathrm{p}}}{\Gamma(\mathrm{p})}\int_{t_j}^{t_k}(t_k-\chi)^{\mathrm{p}-1}|\Psi(\chi,\mho(\chi))|d\chi\Big],\ \ t_1 < t \leq T. \end{cases} \tag{3.20}$$

Upon simplification and using (G_2), Eq (3.20) gives the following:

$$|\mathbb{C}(\mho(t_k)) - \mathbb{C}(\mho(t_j))| \leq \begin{cases} \frac{(\mathcal{M}_\Psi + \mathcal{N}_\Psi \varepsilon)}{\Gamma(\mathrm{p}+1)} \left[t_k^{\mathrm{p}} - t_j^{\mathrm{p}}\right], \ 0 < t \leq t_1, \\ \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \Big[|\Psi(t_k, \mho(t_k)) - \Psi(t_j, \mho(t_j))| + \frac{|\Psi((t_1), \mho((t_1)))| \mu_{\mathrm{p}} (t_k^{\mathrm{p}} - t_j^{\mathrm{p}})}{\Gamma(\mathrm{p}+1)} \\ + \frac{\mu_{\mathrm{p}} (\mathcal{M}_\Psi + \mathcal{N}_\Psi \varepsilon)}{\Gamma(\mathrm{p})} \left(t_k^{\mathrm{p}} - t_j^{\mathrm{p}}\right) \Big], \ \ t_1 < t \leq T. \end{cases} \tag{3.21}$$

The right-handside of Eq (3.21) goes to zero when we apply $t_k \to t_j$ over both sub intervals of $[0, T]$. Hence, we conclude that $|\mathbb{C}(\mho(t_k)) - \mathbb{C}(\mho(t_j))| \to 0$ with $t_k \to t_j$ over both sub-intervals of $[0, T]$. Also, $\mathbb{C}$ is a bounded and continuous. Hence, $\mathbb{C}$ is a uniformly continuous operator. Thus, $\mathbb{C}$ is compact. The conditions of Theorem 2.8 are satisfied. So, Eq (3.13) has at least one solution. In view of this theorem, our proposed model has at least one solution. □

4. Numerical procedure

To derive the method for the numerical solution of the proposed model (1.2), let us consider the solution of Eq (3.13) as follows:

$$\mho(t) = \begin{cases} \mho(0) + \frac{1}{\Gamma(\mathrm{p})} \int_0^t (t-\chi)^{\mathrm{p}-1} \Psi(\chi, \mho(\chi)) d\chi, \ 0 < t \leq t_1, \\ \mho(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \Big[\Psi(t, \mho(t)) - \Psi(t_1, \mho(t_1)) \Big(1 + \frac{\mu_{\mathrm{p}} t^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)} \Big) \chi \Big] \\ + \frac{ABC(1-\mathrm{p}) \mu_{\mathrm{p}}}{ABC(\mathrm{p}) \Gamma(\mathrm{p})} \int_{t_1}^t (t-\chi)^{\mathrm{p}-1} \Psi(\chi, \mho(\chi)) d, \ \ t_1 < t \leq T. \end{cases} \tag{4.1}$$

Here, we approximate the integral by taking the step size $h = t_{j+1} - t_j$, where $t_j = t_0 + jh$ at $j = 0$, $t_0 = 0$. Replacing $t = t_{j+1}$ in Eq (4.1) gives the following:

$$\mho(t_{j+1}) = \begin{cases} \mho(t_0) + \frac{1}{\Gamma(\mathrm{p})} \sum_{l=0}^{j} \int_{t_l}^{t_{l+1}} (t_{l+1} - \chi)^{\mathrm{p}-1} \Psi(\chi, \mho(\chi)) d\chi, \ 0 < t_{l+1} \leq t_1, \\ \mho(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \Big[\Psi(t_{j+1}, \mho(t_{j+1})) - \Psi(t_1, \mho(t_1)) \Big(1 + \frac{\mu_{\mathrm{p}} t_{j+1}^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)} \Big) \chi \Big] \\ + \frac{ABC(1-\mathrm{p}) \mu_{\mathrm{p}}}{ABC(\mathrm{p}) \Gamma(\mathrm{p})} \sum_{l=0}^{j} \int_{t_1}^{t_{l+1}} (t_{l+1} - \chi)^{\mathrm{p}-1} \Psi(\chi, \mho(\chi)) d, \ \ t_1 < t_{l+1} \leq T. \end{cases} \tag{4.2}$$

Now, approximating the nonlinear function $\Psi(\chi, \mho(\chi))$ by applying the Lagrange interpolation [39] with equally spaced arguments is achieved as follows:

$$\Psi(\chi, \mho(\chi)) \approx \frac{t_{l-1} - \chi}{h} \Psi(t_l, \mho(t_l)) - \frac{t_l - \chi}{h} \Psi(t_{l-1}, \mho(t_{l-1})). \tag{4.3}$$

Substituting Eq (4.3) in Eq (4.2), one obtains the following:

$$\mho(t_{j+1}) \approx \begin{cases} \mho(t_0) + \frac{1}{\Gamma(\mathrm{p})} \sum_{l=0}^{j} \int_0^{t_{i+1}} (t_{l+1} - \chi)^{\mathrm{p}-1} \Big[\frac{t_{l-1} - \chi}{h} \Psi(t_l, \mho(t_l)) - \frac{t_l - \chi}{h} \Psi(t_{l-1}, \\ \mho(t_{l-1})) \Big] d\chi, \ 0 < t_{l+1} \leq t_1, \\ \mho(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \Big[\Psi(t_{j+1}, \mho(t_{j+1})) - \Psi(t_1, \mho(t_1)) \Big(1 + \frac{\mu_{\mathrm{p}} t_{j+1}^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)} \Big) \chi \Big] \\ + \frac{ABC(1-\mathrm{p}) \mu_{\mathrm{p}}}{ABC(\mathrm{p}) \Gamma(\mathrm{p})} \sum_{l=0}^{j} \int_{t_1}^{t_{l+1}} (t_{l+1} - \chi)^{\mathrm{p}-1} \\ \times \Big[\frac{t_{l-1} - \chi}{h} \Psi(t_l, \mho(t_l)) - \frac{t_l - \chi}{h} \Psi(t_{l-1}, \mho(t_{l-1})) \Big] d\chi, \ \ t_1 < t_{l+1} \leq T. \end{cases} \tag{4.4}$$

Evaluating the integrals in Eq (4.4) and simplifying, we get

$$\mho(t_{j+1}) \approx \begin{cases} \mho(0) + \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=0}^{j} \Big[\Psi(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ 0 < t_{j+1} \le t_1, \\ \mho(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi(t_j, \mho(t_j)) - \Psi(t_1, \mho(t_1))\Big(1 + \frac{\mu_{\mathrm{p}} t_j^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big] \\ + \frac{\mu_{\mathrm{p}} ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=2}^{j} \Big[\Psi(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ t_1 < t_{j+1} \le T, \end{cases} \tag{4.5}$$

where

$$\begin{aligned} \mathbf{a}_{l,j} &= (l+1-j)^{\mathrm{p}}(l-j+2+\mathrm{p}) - (l-j)^{\mathrm{p}}(l-j+2+2\mathrm{p}), \\ \mathbf{b}_{l,j} &= (l+1-j)^{\mathrm{p}+1} - (l-j)^{\mathrm{p}}(l-j+1+\mathrm{p}). \end{aligned}$$

In view of Eq (4.5), the numerical solution of the proposed model (1.2) is described as follows:

$$\mathcal{S}(t_{j+1}) \approx \begin{cases} \mathcal{S}(0) + \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=0}^{j} \Big[\Psi_1(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_1(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ 0 < t_{j+1} \le t_1, \\ \mathcal{S}(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi_1(t_j, \mho(t_j)) - \Psi_1(t_1, \mho(t_1))\Big(1 + \frac{\mu_{\mathrm{p}} t_j^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big] \\ + \frac{\mu_{\mathrm{p}} ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=2}^{j} \Big[\Psi_1(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_1(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ t_1 < t_{j+1} \le T, \end{cases}$$

$$\mathcal{V}(t_{j+1}) \approx \begin{cases} \mathcal{V}(0) + \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=0}^{j} \Big[\Psi_2(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_2(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ 0 < t_{j+1} \le t_1, \\ \mathcal{V}(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi_2(t_j, \mho(t_j)) - \Psi_2(t_1, \mho(t_1))\Big(1 + \frac{\mu_{\mathrm{p}} t_j^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big] \\ + \frac{\mu_{\mathrm{p}} ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=2}^{j} \Big[\Psi_2(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_2(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ t_1 < t_{j+1} \le T, \end{cases}$$

$$\mathcal{I}(t_{j+1}) \approx \begin{cases} \mathcal{I}(0) + \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=0}^{j} \Big[\Psi_3(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_3(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ 0 < t_{j+1} \le t_1, \\ \mathcal{I}(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi_3(t_j, \mho(t_j)) - \Psi_3(t_1, \mho(t_1))\Big(1 + \frac{\mu_{\mathrm{p}} t_j^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big] \\ + \frac{\mu_{\mathrm{p}} ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=2}^{j} \Big[\Psi_3(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_3(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ t_1 < t_{j+1} \le T, \end{cases}$$

and, the last compartment is approximated as follows:

$$\mathcal{R}(t_{j+1}) \approx \begin{cases} \mathcal{R}(0) + \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=0}^{j} \Big[\Psi_4(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_4(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ 0 < t_{j+1} \le t_1, \\ \mathcal{R}(t_1) + \frac{ABC(1-\mathrm{p})}{ABC(\mathrm{p})}\Big[\Psi_4(t_j, \mho(t_j)) - \Psi_4(t_1, \mho(t_1))\Big(1 + \frac{\mu_{\mathrm{p}} t_j^{\mathrm{p}}}{\Gamma(\mathrm{p}+1)}\Big)\Big] \\ + \frac{\mu_{\mathrm{p}} ABC(1-\mathrm{p})}{ABC(\mathrm{p})} \frac{\mathrm{h}^{\mathrm{p}}}{\Gamma(\mathrm{p}+2)} \sum_{l=2}^{j} \Big[\Psi_4(t_l, \mho(t_l))\mathbf{a}_{l,j} - \Psi_4(t_{l-1}, \mho(t_{l-1}))\mathbf{b}_{l,j}\Big], \ t_1 < t_{j+1} \le T. \end{cases}$$

5. Results and discussion

Here, we describe the use of the initial data in Table 3 and the parameters values given in Table 2 for the numerical analysis of model Eq (1.2).

Table 3. The initial values for compartments of model (1.2), as taken from [23].

Symbol	Specification	Symbol	Specification
$\mathcal{S}$	500	$\mathcal{I}$	100
$\mathcal{V}$	300	$\mathcal{R}$	50

In addition, we have divided the domain of t into two sub-intervals, $[0, 10]$ and $[10, 50]$, and plotted the corresponding graphs for various fractional orders by using the relevant numerical scheme. Using the *mABC* derivative of fractional order has significant application, as these type of operators do not force artificial singularities on any model; instead, they exhibit power law, stretched exponential, Brownian motion, Markovian, and non-Markovian properties simultaneously. Further, the derivative's probability distribution was found to be both Gaussian and non-Gaussian, and it can transition from Gaussian to non-Gaussian even without going through the steady state. The mean square displacement is a crossover from normal diffusion to sub-diffusion. This indicates that the non-singular kernel fractional derivatives are both stochastic and deterministic at the same time. Hence, the proposed model has been simulated in the following figures to investigate the mentioned properties.

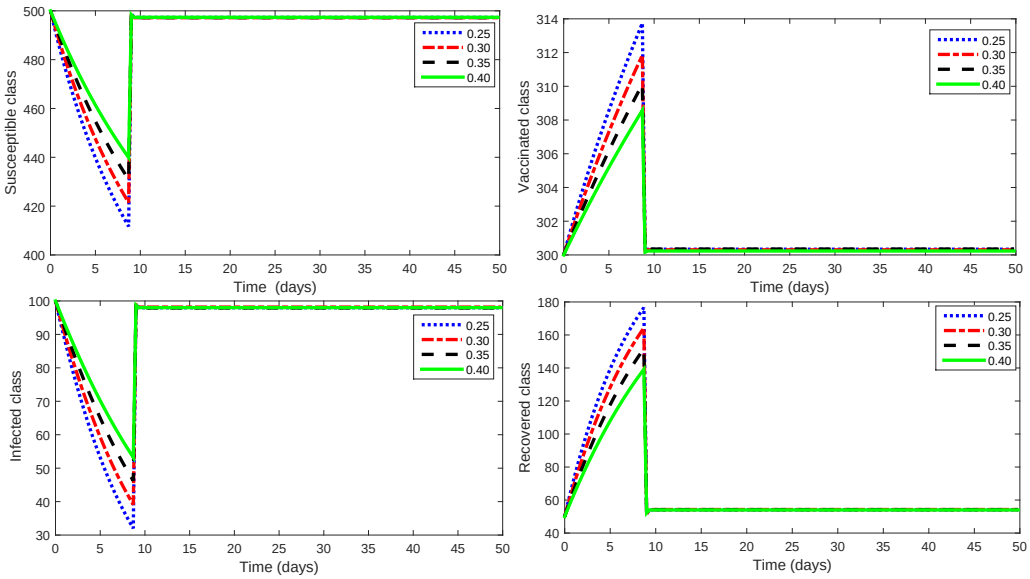

Figure 3. The approximate solutions for different classes of the proposed model (1.2) are presented graphically for various fractional orders $(0, 0.45]$.

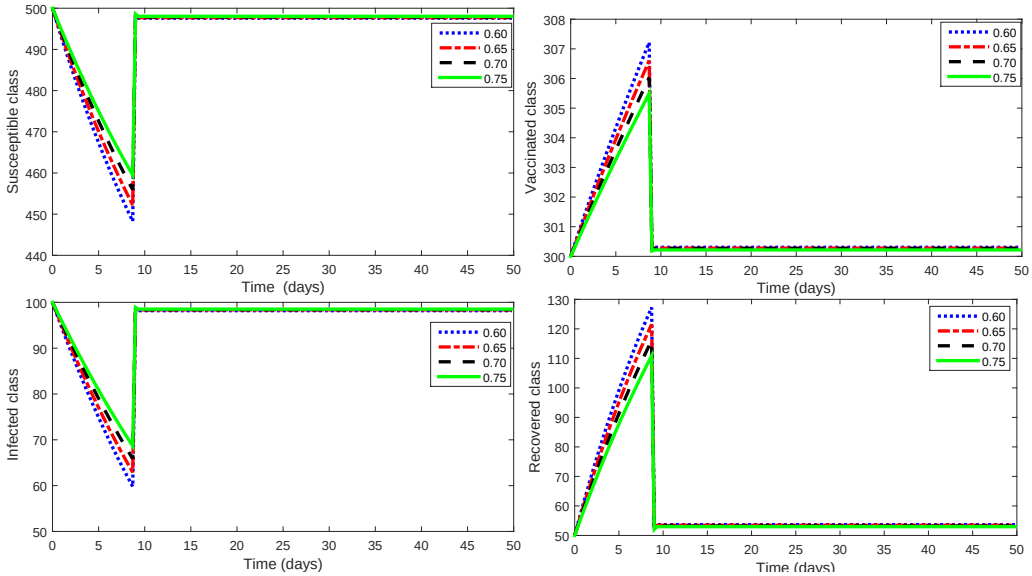

Figure 4. The approximate solutions for different classes of the proposed model (1.2) are presented graphically for various fractional orders $(0.50, 0.75]$.

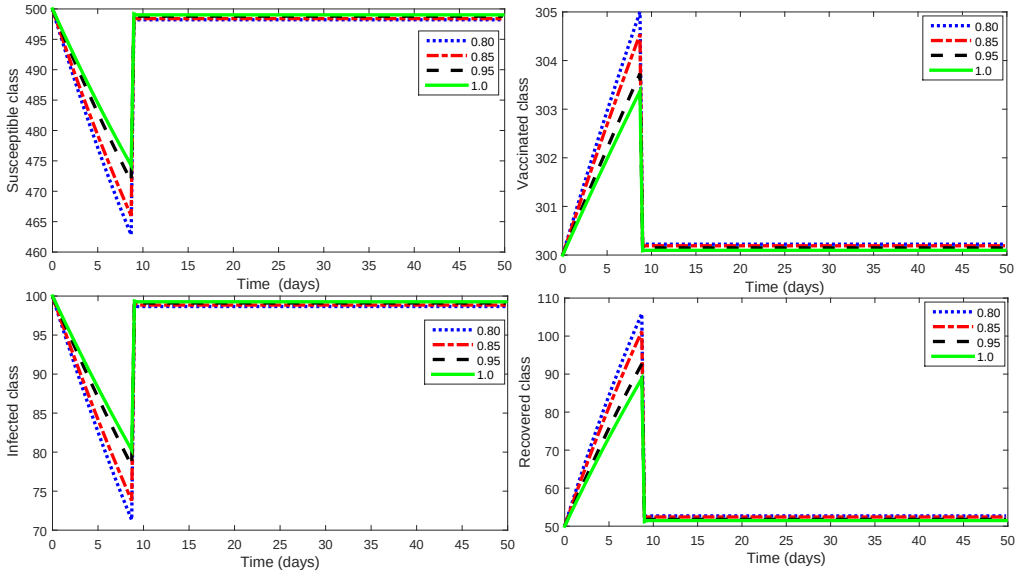

Figure 5. The approximate solutions for different classes of the proposed model (1.2) are presented graphically for various fractional orders $(0.75, 1.0]$.

We have presented the approximate solutions of the proposed model (1.2) by using three different sets of fractional- orders. We observed the crossover behaviors near the point $t_1 = 10$ have been presented for various compartments of the mentioned model. This interesting feature can be detected by using the piecewise derivative of fractional orders. The numerical solutions have been presented for various fractional-order values in Figures 3–5. We point out that the time-fractional derivative has a significant association with memory, and that it has been used in many scientific and technological domains, highlighting its enormous value in epidemiology and in other subjects. It is actually indicated

that the memory function is the kernel of the fractional-order derivative, and the order of the time derivative emphasizes the memory rate. In this research work, we have effectively applied the idea of piecewise differentiation to capture the crossover behaviors in the transmission dynamics of the rotavirus disease. The population dynamics of the susceptible class and the infected class was found to decrease in the presence of the vaccination. Due to the vaccination and decline in the infection, the population density of the recovered class has been shown to increase. Here, near $t = 10$, we see the crossover behaviors in all classes of the proposed model, where a sudden change occurred and the dynamics show multiplicity in the behaviors.

6. Conclusions

In this study, we examined a biological model of rotavirus by using vaccination guidelines. A few qualitative findings were obtained by using the Laplace transform under the *mABC* derivative. Regarding the aforementioned model, the basic reproduction has been computed. We have also performed sensitivity analysis on R_v. Using the *mABC* idea, we have implemented some updated piecewise derivative techniques. Piecewise derivatives can shed light on some of the sudden changes in the evolution of infectious diseases. In order to accommodate the necessary fractional differential operator, we have divided the time domain into two sub-intervals. We have simulated the findings for various fractional orders, p, by using Lagrange's polynomial interpolation criteria. In the same way, we have included a few graphs that show how the rotavirus vaccination model evolved in response to a piecewise derivative. In the future, we will analyze the aforementioned model by combining the first and second derivatives of the Lyapunov function method. Also, to predict the wave in the infection for the future, we will compute the strengthen number in next work. In addition, the aforementioned model will be studied by using the stochastic analysis for the aforementioned model.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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