*Networks and Heterogeneous Media*

http://www.aimspress.com/journal/nhm

NHM, 18(2): 813–841.
DOI: 10.3934/nhm.2023035
Received: 24 November 2022
Revised: 08 February 2023
Accepted: 20 February 2023
Published: 09 March 2023

***Research article***

# Managing consensus based on community classification in opinion dynamics

**Yuntian Zhang**[1], **Xiaoliang Chen**[1,2,*], **Zexia Huang**[1], **Xianyong Li**[1] **and Yajun Du**[1]

[1] School of Computer and Software Engineering, Xihua University, Chengdu, Sichuan, 610039, China

[2] Department of Computer Science and Operations Research, University of Montreal, Montreal, H3C3J7, Canada

* **Correspondence:** Email: chexiaol@iro.umontreal.ca.

**Abstract:** Opinion dynamics in social networks are fast becoming an essential instrument for concentrating on the effect of individual choices on external public information. One of the main challenges in seeing the dynamics is reaching an opinion consensus acceptable to managers in a social network. This issue is referred to as a consensus-reaching process (CRP). Most studies of CRP focus only on network structure and ignore the effect of agent opinions. In addition, existing methods ignore the diversities between divided communities. How to synthesize individual opinions with community diversities to solve CRP issues has remained unclear. Using the DeGroot model for opinion control, this paper considers the effects of network structures and agent opinions when dividing communities, incorporating community classification and targeted opinion control strategies. First, a community classification enhancement approach is utilized, introducing the concept of ambiguous nodes and their division methods. Second, we separate all communities into three levels, *Center*, *Base*, and *Fringe*, according to the logical regions for opinion control. Third, an edge expansion algorithm and three opinion control strategies are proposed based on the community levels, which can significantly reduce the time it takes for the network to reach a consensus. Finally, numerical analysis and comparison are given to verify the feasibility of the proposed opinion control strategy.

**Keywords:** opinion dynamics; social network; consensus strategy; leadership

## 1. Introduction

Social networks play a critical role in our daily life. A wide variety of social network platforms allows people to express their opinions, emotions, and sentiments about events or products, those opinions influence the actions of others. Many studies on social networks including dynamic network embedding [1], user relationships [2], influence maximization [3] and opinion dynamics. The opinion dynamics problem in social networks aims to study the evolution of opinions when a group of people discusses a topic, which has attracted research attention due to its significant challenges and practical value. The research carriers of the problem are usually represented as direct graphs in which nodes and edges symbolize agents and their interactions. Agents constantly update their opinions based on the established rules leading to a consensus. Based on different update rules, a large number of opinion dynamics models, including DeGroot model [4], Friedkin-Johnsen (FJ) model [5, 6], have been presented and revealed the necessity of opinion dynamics. In previous opinion dynamics studies, the process of forming opinions has been analyzed by investigating the conditions leading to consensus or discord among agents. Furthermore, managers are interested in how opinions are formed and guided in reaching a given consensus value.

The effects of group pressure in a dynamic society were investigated by Asch [7]. The task of opinion dynamics was initially introduced by French [8], who accurately explained the influence process in interpersonal relationships. Then, the famous and fundamental DeGroot [4] model was proposed to solve the opinion dynamics problem. Many DeGroot variants and extensions have been developed. For example, the Friedkin-Johnsen (FJ) model [5, 6] introduces the concept of stubbornness degree, the Deffuang-Weisbuch (DW) model [9] defines finite confidence bound, and Hegselmann-Krause (HK) model [10] provides a paradigm for the dynamic evolution of opinions and its further research including [11]. The presence of absolutely stubborn agents in the DeGroot model was detected by Abrahamsson et al. [12]. Zhou et al. [13] studied the influence of partially stubborn agents in tuning the DeGroot model and considering the influence of two jump agent neighbors. Li et al. [14] considered the multi-attribute group decision problem in dynamic opinions. These studies have enriched the theoretical results and contributed to the development of opinion dynamics. SNA and CRP are also hot research topics recently, and they also can be used for conflict elimination, as in [15–18].

The consensus reaching progress (CRP) derived from group decision making [19] is one of the main challenges in opinion dynamics problems. CRP is an iterative and dynamic process guided by a manager consisting of several rounds in which individuals discuss and update their opinions until a consensus is reached. Several researchers have studied the consensus-reaching process (CRP) or its application to group decision-making problems (GDM). Li et al. [20] developed some models to manage incomplete information and consensus for GDM with IHFLPRs. Gai et al. [21] propose a consensus-trust driven framework of bidirectional interaction for social network large-group decision making. Zhang et al. [22] developed a two-consensus-based TOPSIS-Sort-B algorithms to deal with MCS-GDM problems. A social network analysis method based on conflict surveys and group decision-making problems was presented by Ding et al. [23]. Li et al. [24] proposed a two-stage dynamic influence model for achieving consensus among large groups working under incomplete information. Several studies have also examined how network structure and agents affect CRP. Tian and Wang [25] summarized the impact of stubborn agents on opinion formation and suggested criteria for achieving consensus. Specifically, Ding et al. [26] investigated the influence of agent self-adherence on consensus convergence speed. Cho et al. [27] treated informed agents as stubborn agents. Social opinion and self-persistence have also received a lot of attention from researchers. The DeGroot-Friedkin (DF) model is used in Jia et al. [28] to demonstrate the evolutionary process of self-persistence, social power, and interpersonal influence in opinion dynamics. Accordingly, Chen et al. [29] and Ye et al. [30] discuss the DF model with self-persistence.

Existing research suggests that network structures are one of the main factors affecting CRP. Typically, social network agents can be classified into leaders and followers, where the leaders play a decisive role in the evolution of opinions [31]. Dividing the network into subnets(communities) by these two categories allows for studying of opinion dynamics efficiently at a local level, avoiding the loss of important information in the early iterations. In order to delineate reasonable communities, existing studies [24, 30, 31] focus on the structure of social networks. These methods divide a network into different communities according to the aggregation of nodes in the network. However, they all focus on the structure but ignore the opinion of agents. Despite existing studies on CRP, there still exist several challenges that have not yet been fully addressed:

1. **How to divide followers influenced by multiple communities?** Existing research identifies different approaches to simultaneously divide followers who belong to multiple communities. Several studies divide followers into multiple communities simultaneously. Others separate them on the basis of the network structure. However, the influence of agent opinions is ignored by all of the follower divisions.
2. **Hierarchical classification of communities is lacking.** Communities with different characteristics play distinct roles in the CRP problem, and the possess distinct attitudes toward a particular event or topic. However, existing studies of opinion dynamics ignore the differences between communities, managing them uniformly.
3. **How to effectively control the evolution of opinions in a network to reach a consensus?** CRP research aims to assist a network in reaching a consensus. It is necessary to propose a model that optimizes agent opinions and network structure so that the opinion diffusion in the network will stay in control. This optimization process is often insufficiently targeted due to the neglect of community types. The model might be more accurate if community classification is incorporated into it.

Based on these research needs, this paper addresses the CRP problem from a multiview perspective: First, we model the opinions of all agents in the interval [0,1]. Second, to introduce fuzzy agents into CRP to facilitate the guidance and construction of consensus in opinion dynamics, a community identification algorithm is proposed based on the concept of fuzzy agents in social networks to provide a framework for consensus-building strategies in opinion dynamics. Finally, a generalized strategy for guiding the process of opinion dynamics until consensus is reached with the stated goal is proposed

The primary contributions of this study are itemized as follows:

1. We propose a community recognition algorithm with ambiguous node division, which takes into account both the network structure and the agents' opinion values to keep the community classification more rational.
2. Network communities can be differentiated in the CRP problem by introducing community levels *Center*, *Base*, and *Fringe*. The hierarchical division contributes a concise framework for constructing opinion control algorithms.
3. An opinion optimization model is presented, and three community levels, *Center*, *Base* and *Fringe*, are introduced to facilitate network structure expansion to quickly control public opinions to reach the desired consensus.

The remainder of this paper is structured as follows. Section 2 reviews basic concepts in graph theory and opinion dynamics. Section 3 discusses the consensus process on the evolution of opinions in social networks. Section 4 details the adding edges algorithm and opinion control strategy based on community classification. Section 5 provides several numerical analyses to demonstrate the effectiveness of the strategy. Section 6 compares and analyzes the differences and links between the opinion control strategy proposed in this paper and other strategies. Section 7 presents the conclusion.

## 2. Preliminaries

This section formalizes the basic concepts and opinion dynamics addressed in this paper. More details can be found in [4, 31, 32]. Basically, the two-tuple $G(V, E)$ has been applied to represent a directed graph with the finite agent (node) set $V = \{v_1, v_2, \dots, v_n\}$ and the edge set $E \subseteq \{(v_i, v_j)|v_i, v_j \in V\}$. A graph $G(V, E)$ can be denoted by the adjacency matrix pattern $A = (a_{ij})_{n\times n}(i, j = 1, 2, \dots, n)$. For any pair $(v_i, v_j)$, we have $a_{ij} = 1$ if the expression $(v_i, v_j) \in E$ is true; otherwise, $a_{ij} = 0$. The weight matrix $W = (w_1, w_2, \dots, w_n)$ is introduced to represent the tightness of the relationship between the agents in $V$. For any pair $(v_i, v_j)$, $w_{ij} \in \{0, 1\}$ if the relation $(v_i, v_j) \in E$ is satisfied; otherwise, $w_{ij} = 0$. Let square $B = (b_{ij})_{n\times n}(i, j = 1, 2, \dots, n)$ be the accessibility matrix of a directed graph $G(V, E)$. For any pair $(v_i, v_j)$, we define $b_{ij} = 1$ if there is a reachable path from $v_i$ to $v_j$; otherwise, $b_{ij} = 0$. This paper also considers the following boundary condition that any node to itself is assumed reachable. As a result, the main diagonal elements of $B$ equals 1. The accessibility matrix facilitates the calculation of node degrees. The in-degree and out-degree of an agent $v_i$ can be obtained by the simple summation calculations $deg_i^- = \sum_{i=1}^n b_{ij}$ and $v_i$ is $deg_i^+ = \sum_{j=1}^n b_{ij}$, respectively. This study performs a binary division $V = V^l \cup V^f$ for social network $G(V, E)$ with leader set $V^l$ and follower set $V^f$. An agent $v \in V^l \subseteq V$ is called a leader if any other agent $\bar{v} \in V/v$ can reach it. An agent $v \in V^f \subset V$ is called a follower if $v \notin V^l$ is satisfied.

### 2.1. Opinion dynamics

This section formalizes the fundamental elements relating to opinion dynamics. More subtleties can be viewed in [10, 13, 33, 34]. The opinion dynamics describe the opinion evolution of an agent group that discuss the same topic. The investigation of opinion dynamics will contribute to a deeper understanding of the evolution of each agent's opinion when the group's goal is to find a solution to opinion evolution problems. Existing opinion dynamics methods recognize the critical role played by DeGroot models. They assumed that the agents in a network would influence or be influenced by their neighbors. Hence, a stable weight as the standard parameter is applied to represent their influences.

Let $G(V, E)$ be a network with $n$ agents, $x_i(t)$ be the opinion value of an agent $v_i \in V$, $X(t)$ be the opinion value vector of the whole agent set $V$ at time $t$, and $W = (w_{ij})_{n\times n}$ be the weight matrix with non-negative elements for the $n$ agents, where the matrix component $w_{ij}$ denotes the influence weight of agent $v_i$ on agent $v_j$. Suppose that the sum of each row in $W$ equals 1, i.e., $\sum_{j=1}^n w_{ij}$=1. The evolution process of agent opinions can be described as:

$$x_i(t + 1) = w_{i1}x_1(t) + w_{i2}x_2(t) + \dots w_{in}x_n(t), \quad t = 0, 1, \dots \tag{2.1}$$

Their matrix pattern can be represented as:

$$X(t+1) = W \times X(t), \quad t = 0, 1, \ldots \tag{2.2}$$

**Definition 1.** *[10] Let $X(0) = (x_1(0), x_2(0), \ldots, x_n(0))^T$ be an initial opinion vector with respect to n agents in a network. Opinion vector $C = \lim_{t\to\infty} X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is called a consensus reached by the entire network if $x_1(t) = x_2(t) = \cdots = x_n(t) = c$ at a time t from the initial $t = 0$, where c is a constant opinion value and generality called the consensus value.*

The process of all agents reaching consensus in the DeGroot model can be expressed according to Equ. 2.2 and Definition 1:

$$C = \lim_{t\to\infty} X(t) = \prod_{t=1}^{\infty} W \times (W \times X(0)) = (c, c, \ldots, c)^T \tag{2.3}$$

The conditions for reaching consensus in the DeGroot model can be illustrated in the following two Lemmas.

**Lemma 2.1.** *[34] All agents in a network can reach a consensus if and only if the weight matrix $W^*$ contains at least one column of elements strictly positive.*

**Lemma 2.2.** *[35] Let $\mu = (\mu_1, \mu_2, \ldots, \mu_n)$ be the stationary probability vector for the weight matrix W of a network, where $\forall \mu_i \in \mu$, $\mu_i \geq 0$, and $\sum_{i=1}^{n} \mu_i = 1$. If the agents reach a consensus in the DeGroot model, then consensus value c can be expressed by:*

$$c = \sum_{i=1}^{n} \mu_i x_i(0) \tag{2.4}$$

According to Lemma 2.1, agents in the network must have at least one trusted object, and the entire network can reach a consensus only if the initial opinions of all agents are the same if the condition is not satisfied. According to Lemma 2.2, we can derive the following conclusion that an agent is affected by all its neighbors, and the final consensus value $c$ is a linear combination of the initial opinions for all agents.

## 3. Opinion consensus problems in social networks

Consensus issues in social networks are seeking agreement interactively through the evolutionary features of network structures and attributes. Opinion consensus models are the core of decision-making research, which aims to explain the overall tendency of Internet users' opinions on online hot events. These opinions may change depending on the influence of their surroundings due to the subjective nature of agents. They may drive new decisions favorable to the event through opinion iterations based on quantifiable parameters, thus forming a macroscopic evolution of goal-oriented opinions. Several consensus models [14, 36–39] for opinion evolution issues have been proposed. Most of them follow the flow chart of CRP shown in Figure 1. Their primary concepts are depicted as follows:

(i) **Agent's opinion.** The opinion of an agent for a specific topic is normally denoted by a real number $x \in [0, 1]$. A higher value of $x$ indicates that the agent is more supportive of the topic.

(ii) **Social networks.** The graphical representation $G(V, E)$ of social platform data is a class of bigraph computing systems abstracted from the information propagation and the related interaction between agents, known as social networks. The dynamic analysis of a social network has brought benefits to capturing micro and macro agent opinions for its great graphical style, timely dynamic updating, and a wide variety of topics.

(iii) **Opinion evolution.** Opinion evolution is a dynamic procedure of developing opinions among a group of interactive agents in a social network. In general, decision agents recognize the opinions of other adjacent agents in a network to form or evolve their views. All the agents will update their opinions by the pre-designed iteration rules to generate a consensus, polarization, or splitting. This study describes the evolutionary process formally in terms of the DeGroot paradigm. Previous studies [4, 5, 24, 40, 41] primarily defined the ability of an agent $v_i \in V$ in a network $G(V, E)$ to maintain his/her current attitude as "self-persistence" that can be depicted by a real number $\alpha_i \in [0, 1]$. Then, we have the self-persistence vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ for all agents. The higher the self-persistence is, the less the agent's opinions are influenced by others. The values 0 and 1 of $\alpha_i$ indicate the complete inability and permanent preservation of an opinion, respectively. Let $I(v_i) = \{v_j | a_{ij} = 1\}(i, j = 1, 2, \ldots, n)$ be the original trust set of agent $v_i$ in a social network with $n$ agents. This study employs the weight matrix $W$ to represent the degree of mutual influence among all agents and their self-persistence at time $t$, specifically formalized as:

$$W_{ij} = \begin{cases} 1/(|I(v_i)| + 1), v_j \in I(v_i) \\ 0, otherwise. \end{cases} \tag{3.1}$$

$$W_{ii} = \begin{cases} \alpha_i, deg_i^- > 0 \\ 1, deg_i^- = 0. \end{cases} \tag{3.2}$$

where $deg_i^-$ represents the out-degree of agent $v_i$, and $| \bullet |$ denotes the number of set $\bullet$.

(iv) **Final stable opinion.** Agents' opinions eventually evolve over time into three stable states: consensus, polarization, and splitting. A social network reaches a consensus if the final opinions of all agents are in stable agreement. The state of polarization (res. splitting) indicates that the final opinion satisfies binary classification (res. multi-classification) characteristics and remains stable.

(v) **Opinion management.** Opinion management is a pre-designed strategy with specific rules for facilitating a consensus among agents in the opinion evolution process.

(vi) **Opinion control.** Opinion control aims to drive the whole network to the desired consensus by constructing mechanisms to fine-tune agents' opinions.

An essential approach, usually called celebrity endorsements, is currently being adopted in opinion control. The original celebrity endorsement refers to an agreement between influential individuals and products in online marketing [33]. Celebrity influences are widely utilized to promote products because of their abilities to generate positive emotions that can guide product sales [24, 42, 43]. In this

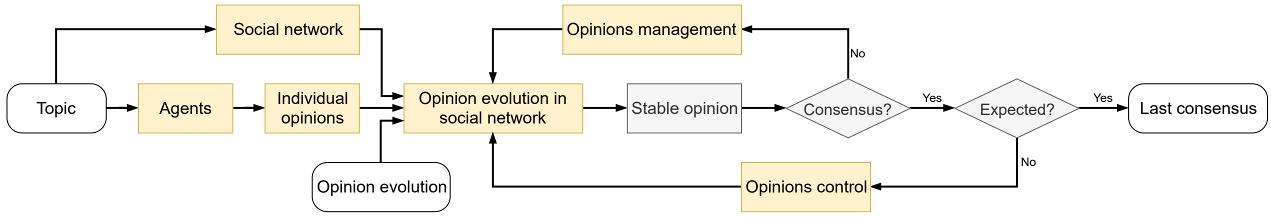



**Figure 1.** Processes of reaching a consensus in a social network

paper, the entities 'leaders' of agents in social network analysis are equated with celebrities. Thus, it is possible to control other agents' opinions in a social network if the leaders are considered the medium to influence opinions.

Before proceeding to examine the opinion control, it is important to highlight the consensus. This study also assumes the sufficient condition [13] of the consensus. A social network $G(V, E)$ can reach a consensus if $V^l \neq \emptyset$. A consensus value $c = \sum_{i=1}^{n} \mu_i x_i(0)$ refers to all agents and their initial opinions in a social network $G(V, E)$ with total $n$ nodes. Existing research [13] recognizes the critical role played by leaders and recommends using the approximate expression of $c = \sum_{v_i \in v^l} \mu_i x_i(0)$ on leaders. In other words, the final consensus value $c$ is a linear combination of the initial opinions of all leaders.

## 4. Opinion control strategies based on community classification

Studies on opinion analysis have emphasized the importance of opinion evolution simulations. However, there are relatively few historical studies on opinion control. The purpose of the control is to allow the agent's opinions to evolve according to the desired (pre-defined) consistency goal, i.e., consensus, polarization, and splitting. Several attempts have been made relating to the assumption that the existence of a leader is a sufficient condition for a social network to reach a consensus. The mechanisms that underpin opinion control without a leader are not fully understood. This study proposes an opinion control strategy based on community classification to establish the expected consensus on the network $G(V, E)$ that does not satisfy the condition $V^l \neq \emptyset$, which will generate fresh insight into opinion control without a leader through the idea of potential leader discovery. The main strategies are described as follows.

Let $G(V, E)$ be an original network with $V^l = \emptyset$ and $\overline{G}(\overline{V}, \overline{E})$ be the expanded network from $G(V, E)$ by supplementing a small number of network relationships and preserving the net structure and initial opinions of $G(V, E)$ as completely as possible, where $V = \overline{V}$, $E \subset \overline{E}$, and $\overline{V}^l \neq \emptyset$. We need to seek the case where the number of extended edges in $\overline{E}$ is minimized to maintain the behaviors of $G(V, E)$. Therefore, an optimal computational model of the extended network $\overline{G}(\overline{V}, \overline{E})$ for the original network $G(V, E)$ is introduced as follows:

$$\begin{aligned} min \ \ & |\overline{E}| - |E| \\ s.t. \ \ & E \subset \overline{E} \\ & V^l \neq \emptyset \end{aligned} \tag{4.1}$$

where $| \bullet |$ represents the cardinality of a set $\bullet$.

One of the core problems with consensus control is to obtain the expanded network $\overline{G}(\overline{V}, \overline{E})$ according to the optimization Equation (4.1). Community division is beneficial for detecting suitable expansion edges. The primary process of applying the consensus control optimization model is described as follows.

(i) We divide the original network $G(V, E)$ with $n$ agents into $s$ communities, denoted by $Com(G) = \{G_1(V_1, E_1), G_2(V_2, E_2), \ldots, G_s(V_s, E_s)\}$, abbreviated as $Com(G) = \{G_1, G_2, \ldots, G_s\}$, according to some regional consensus scenarios, where $Com(G)$ represents the set of the communities, and $1 \leq s \leq n$.

(ii) The concept of communities can be viewed as the collection of agents influenced by the same local leaders (nearby) in a social network. A community is usually seen as a high-level agent containing many original network agents. Our approach will add edges between these communities to create a new network $\overline{G}(\overline{V}, \overline{E})$ with global leaders, i.e., leaders who influence the communities. As a result of our approach, we intend to create a new network $\overline{G}(\overline{V}, \overline{E})$ of global leaders, i.e., leaders who influence communities, by adding edges.

Let $X(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ be an opinion vector with respect to $n$ agents in a network $G(V, E)$ at a moment $t$. Generally, the opinion value of an agent $v_i \in V$ at moment $t$ is denoted by $x_i(t)$. Then, the opinion vector with $m(m \leq n)$ agents that come from a community $G_i(V_i, E_i)$ of $G(V, E)$ is a mapping value, denoted by $X^{G_i}(t)$, of vector $X(t)$ to the nodes in set $V_i$. Similarly, the opinion value of an agent $v_j \in V_i$ in a community $G_i(V_i, E_i)$ at a moment $t$ is represented as $X_j^{G_i}(t)$.

### *4.1. Community Division and Community Classification*

Social networks are a high abstraction of complex online systems. Apart from essential characteristics, such as scale-free and small worlds, social networks also have another key feature: community structure. An entire network is comprised of several communities. Community members are relatively closely connected, but the connections between communities are relatively sparse. The community division in this paper will be accomplished in two steps: subnet recognition and ambiguous nodes division. This section will elaborate on the detailed process of the community division and community classification applied in opinion consensus problems. Figure 2 illustrates the relationship between community division and community classification in CRP.

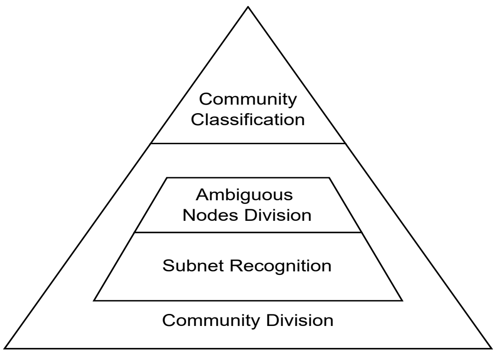



**Figure 2.** The relationship between ambiguous node division and community classification in CRP

4.1.1. Community division

There has been substantial research on opinion evolution, and one of the most significant conclusions is that opinion leaders are crucial to opinion evolution [31]. Consequently, we will divide a social network $G(V, E)$ into multiple communities $Com(G) = \{G_1, G_2, \ldots, G_s\}$ with $1 \leq s \leq n$. Alternatively, this procedure is also known as the subnet recognition and ambiguous nodes division. Several rules must be followed to ensure the smooth running of this process.

(i) There should be a direct interaction between the leaders within the community $G_i$ of a subnet if there are several leaders in it, i.e., $|V_i^l| \geq 2$;

(ii) A leader can only belong to one subnet, meaning that the intersection of leaders in different subnets is empty, i.e., $V_i^l \cap V_j^l = \emptyset$ $(i \neq j)$;

(iii) The relationships between different subnets should satisfy $\cup_{i=1}^{s} V_i = V$, $\cup_{i=1}^{s} E_i = E$, $V_i \cap V_j = \emptyset$ $(i \neq j)$, and $E_i \cap E_j = \emptyset$ $(i \neq j)$.

As demonstrated in Algorithm 1, this study follows a similar approach to the subnet recognition proposed by Zhou et al [13].

**Definition 2.** *Let $G(V, E)$ be a social network with n agents, and $Com(G) = \{G_1, G_2, \ldots, G_s\}$ is the set of subnets of $G(V, E)$ at a specific time t, where $1 \leq s \leq n$. An agent $v \in V$ is called an ambiguous node of $G(V, E)$ at a time t if there exist at least two subsets $G_i$ and $G_j$ satisfying $V_i^l \neq \emptyset$, $V_j^l \neq \emptyset$, $V_i^l \cap V_j^l = \emptyset$, $V_i^f \neq \emptyset$, $V_j^f \neq \emptyset$, $v \in V_i^f$ and $v \in V_j^f$.*

In Definition 2, certain followers are not properly assigned to a particular subnet based on the subnet recognition rules before the division of communities. These followers are called ambiguous nodes and should be investigated further. Therefore, the operations of ambiguous node division are added to the subnet recognition to obtain the desired communities. This type of diagram can be seen in Figure 3, where the red, yellow, and gray nodes represent the leaders, followers, and ambiguous nodes, respectively. The function of our approach is to be able to reasonably categorize any ambiguous node as one of the subnets based on the network structure and the agent opinions. Algorithm 2 provides a detailed representation of this function.

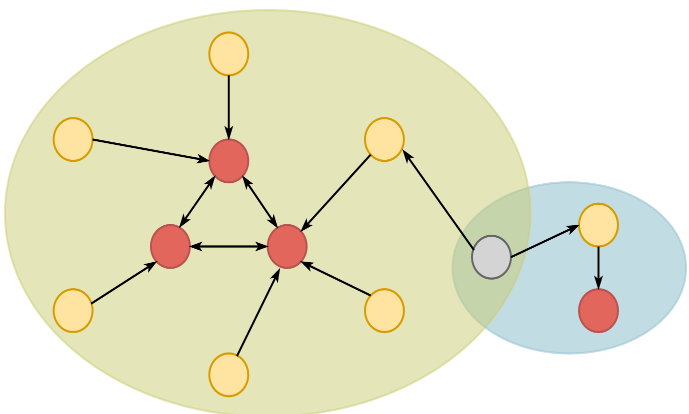


**Figure 3.** Schematic diagram of an ambiguous node between two subnets.

**Algorithm 1** Subnet recognition.

**Input:** The adjacency matrix $A = (a_{ij})_{n\times n}$ of a social network $G(V, E)$ with $n$ agents;
**Output:** The subnets $Com(\tilde{G}) = \{\tilde{G}_1(\tilde{V}_1, \tilde{E}_1), \tilde{G}_2(\tilde{V}_2, \tilde{E}_2), \dots, \tilde{G}_s(\tilde{V}_s, \tilde{E}_s)\}$ with potential ambiguous nodes and the leader sets $V^l = \{V_1^l, V_2^l, \dots, V_s^l\}$.

Initialize $I = I_n = diag(1, 1, \dots, 1)$
Initialize $B = sign((A + I)^{n-1})$
Initialize $H = \{h_1, h_2, \dots, h_j, \dots, h_n\}$, $h$ is the sum of a column in $B$
Initialize $s = 1$
**while** $H \neq \emptyset$ **do**
    $\ddot{V} = \emptyset$
    $q = argmax_j\{H\}$
    $J = \{j \mid q = argmax_j\{H\}\}$
    $s = 1$
    **if** $\mid J \mid \geq 1$ **then**
        $j = Minimum(J)$
    **end if**
    $\ddot{V} = \{v_p | b_{pq} = 1, v_p \in V\}$
    **if** $\ddot{V} = \emptyset$ **then**
        $\tilde{V}_s = \tilde{V}_s^l = \{v_q\}$, $E_s = \emptyset$
    **else**
        $\tilde{V}_s = \ddot{V} \cup \{v_q\}$
        $\tilde{E}_s = \{(v_i, v_j) | v_i, v_j \in \tilde{V}_s, (v_i, v_j) \in E\}$
    **end if**
    $\ddot{V}^l = \{v_p | b_{qp} = 1, v_p \in V\}$
    $\tilde{V}_s^l = \ddot{V}^l \cup v_q$
    $H = H \backslash \{h_m \mid v_m \in V_s\}$
    $s = s + 1$
**end while**
**return** $Com(\tilde{G}) = \{\tilde{G}_1(\tilde{V}_1, \tilde{E}_1), \tilde{G}_2(\tilde{V}_2, \tilde{E}_2), \dots, \tilde{G}_s(\tilde{V}_s, \tilde{E}_s)\}$ and the leader sets $V^l = \{V_1^l, V_2^l, \dots, V_s^l\}$.

#### 4.1.2. Community classification

The operation of ambiguous node division ensures that nodes in a social network belong to no more than one community. Besides, community classifications are intended identify and measure the differences between multiple communities, which provide the structural basis of opinion control. The notation $\mathbb{E} = [\gamma, \eta]$ is introduced to denote the expected opinion interval (final expected consensus interval) for a particular community. Consequently, communities can be divided into three categories:

(i) **supportive communities**. A community is called a supportive community, denoted by $G^{sp}$, if the final opinions of all of its agents fall within the expected opinion interval $\mathbb{E}$.

(ii) **Indecisive communities**. A community is called an indecisive community, denoted by $G^{id}$, if the final opinions of some of its agents fall within the expected opinion interval $\mathbb{E}$.

**Algorithm 2** Ambiguous node partition algorithm.

**Require:** A social network $G(V, E)$ with $n$ agents;
Initial opinion of agents $X(0) = \{x_1(0), x_2(0), \ldots, x_n(0)\}$;
Follower set $V^f$;
Subnet partition $Com(\tilde{G}) = \{\tilde{G}_1(\tilde{V}_1, \tilde{E}_1), \tilde{G}_2(\tilde{V}_2, \tilde{E}_2), \ldots, \tilde{G}_s(\tilde{V}_s, \tilde{E}_s)\}$ with ambiguous nodes (the output of Algorithm 1).

**Ensure:** Subnet partition $Com(G) = \{G_1(V_1, E_1), G_2(V_2, E_2), \ldots, G_s(V_s, E_s)\}$ without ambiguous nodes

$V^{ab} = \emptyset$
**for** $i = 1; i \leq |V^f|; i++$ **do**
  **if** $v_i^f \in \tilde{V}_m, v_i^f \in \tilde{V}_n, m \neq n$ **then**
    $V^{ab} = V^{ab} \cup v_i^f$
  **end if**
**end for**
$P = \{P_1, P_2, \ldots, P_s\} = \{\emptyset, \emptyset, \ldots, \emptyset\}$
$\dot{V} = \{\dot{v} \mid (\forall v \in V^{ab}, \dot{v}) \in E, \dot{v} \in V\}$
$\ddot{V} = \{\ddot{V}_1, \ddot{V}_2, \ldots, \ddot{V}_s \mid \ddot{V}_i = \{\ddot{v} \mid \ddot{v} \in \tilde{V}_j, \ddot{v} \in \dot{V}\}\}$
**for** $i = 1; i \leq | V^{ab} |; i++$ **do**
  **for** $j = 1; i \leq s; j++$ **do**
    $\ddot{X}(0) = \{\ddot{x}(0) \mid \ddot{x}(0)$ is the initial opinion of $\ddot{v}, \ddot{v} \in \ddot{V}_j\}$
    $K = | \ddot{V}_j |$
    $p_{ij} = \frac{\sum_{k=1}^{K} |\ddot{x}_k(0) - x_i(0)|}{K}$
    $P_i = P_i \cup p_{ij}$
  **end for**
  $j = argmax_j\{P_i\}$
  $V_j = \tilde{V}_j \cup v_i^{ab}$
**end for**
**return** $Com(G) = \{G_1(V_1, E_1), G_2(V_2, E_2), \ldots, G_s(V_s, E_s)\}$

(iii) **Opposition communities**. A community is called an opposition community, denoted by $G^{op}$, if the final opinions of all of its agents are not in the expected opinion interval $\mathbb{E}$.

The set $Com(G)$ of communities without ambiguous nodes can be used as a basis for adding edges in opinion control.

### 4.2. *Opinion control strategies*

Returning briefly to the network partition issue, a social network can be well divided into multiple communities without any ambiguous nodes. This study also divides the communities into three categories based on the tendencies of the agent's overall opinions. This section addresses ways of opinion control. It is possible that a spontaneous consensus reached by the public on an online event, e.g., a rumor, without any intervention will not be accepted. Opinion control is intended as a mechanism for guiding multiple confused opinions that fall within an expected range $\mathbb{E} = [\gamma, \eta]$. The control strategies of this paper can be divided into two sequentially related phases:

(i) Adjust the original social network $G(V, E)$ by expanding a few edges to construct a supplementary social network $\overline{G}(V, \overline{E})$ so that its agents can reach a consensus;

(ii) Adjust the leaders' opinions in the updated network $\overline{G}(V, \overline{E})$ to ensure that the final consensus value $c$ falls in an expected opinion interval $\mathbb{E} = [\gamma, \eta]$.

4.2.1. Structure expansions of social networks

Initially, we divide an entire social network $G$ into $s$ subnets $Com(G) = \{G_1(V_1, E_1), G_2(V_2, E_2), \ldots, G_n(V_s, E_s)\}$ by using Algorithm 1 and Algorithm 2. Then, three kinds of communities, i.e., supportive $G^{sp}$, indecisive $G^{id}$, and opposition $G^{op}$, are defined to refer to an opinion coarse-grained classification of all agents within an expected opinion interval. This section separates all communities into three levels, *Center*, *Base* and *Fringe*, according to the logical regions for opinion control. Criteria for the separation are as follows:

(i) One of the supportive communities is chosen as the level *Central*. Formally, $Central = G^{ce}(V^{ce}, E^{ce}) \in Com(G)$ with $V^{ce} = \{v_1^{ce}, v_2^{ce}, \ldots, v_{cn}^{ce}\}$ is a supportive community with $cn$ agents.

(ii) *Base* is a subset of $Com(G)$ with $bs$ communities, which includes all indecisive communities as well as the remaining supportive communities, except for *Central*. Formally, $Base = \{G_1^{ba}(V_1^{ba}, E_1^{ba}), G_2^{ba}(V_2^{ba}, E_2^{ba}), \ldots, G_{bs}^{ba}(V_{bs}^{ba}, E_{bs}^{ba})\} \subset (Com(G) \setminus Central)$.

(iii) All opposition communities constitute the level *Fringe*. Formally, $Fringe = \{G_1^{fr}(V_1^{fr}, E_1^{fr}), G_2^{fr}(V_2^{fr}, E_2^{fr}), \ldots, G_{fs}^{fr}(V_{fs}^{fr}, E_{fs}^{fr})\} \subset Com(G)$ is a subset of $Com(G)$ with $fs$ opposition communities.

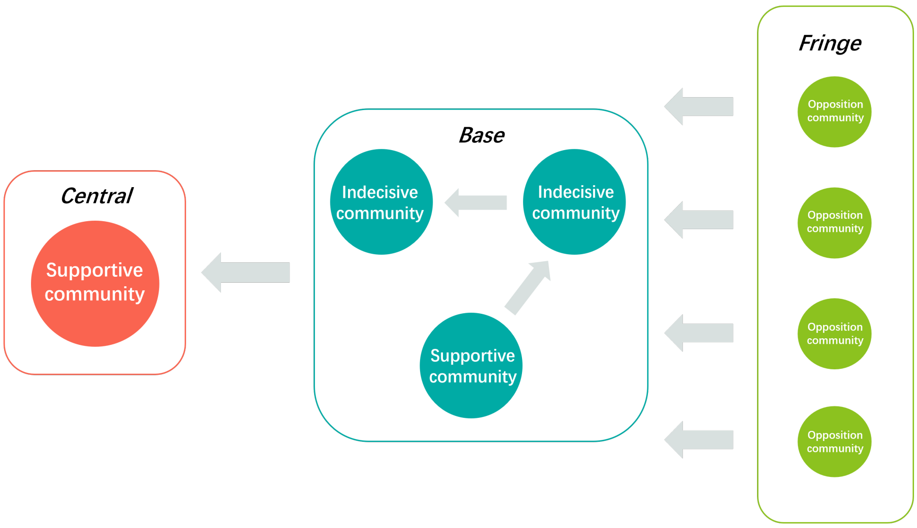



**Figure 4.** Relationship schematic of *Central*, *Base* and *Fringe*.

Figure 4 illustrates the among *Central*, *Base* and *Fringe*, where *Central* is a randomly selected supportive community, *Base* consists of a mixture of indecisive and supportive communities, and

*Fringe* consists of all the opposition communities. Property such as that conducted by Dong et al. [31] have shown that we are merely required to attach an edge between two subnets (at least one leader), and their agents can achieve consensus on their combined network. The algorithm 3 is prepared to generate an updated social network $\overline{G}(V, \overline{E})$ according to the following procedures of edge expansions. First, if there are no indecisive communities in the subnets $Com(G)$, an edge will be added from each leader in opposition communities to the followers in the supportive community of *Central*. This scenario is shown in Figure 5. Second, if a social network does not delineate any supportive community, we first add acyclic one-way edges between the leaders of indecisive communities. Besides, the edge expansion approach randomly selects an indecisive community for each opposition community and creates edges from the opposition community's leaders to the followers of the indecisive one. This case can be illustrated briefly by Figure 6. Third, if we consider the scenario shown in Figure 7 in which there are no opposition communities, the acyclic one-way edges between the leaders of indecisive communities are established. Additionally, we extend the edges between the leaders of the last indecisive community and the supportive community leaders. Fourth, the case presented in Figure 8 illustrates an edge expansion approach where none of the three levels of the community division are empty. The edge expansions of each adjacent two levels are performed by the same strategies as $Central/Fringe$ (Figure 5), $Base/Fringe$ (Figure 6), and $Central/base$ (Figure 7).

**Algorithm 3** Edge expansions among three levels of communities for a social network.

**Input:** A social network $G(V, E)$ and its communities $Com(G)$;
Leader set $V^l$ and follower set $V^f$;
$Central = G^{ce}(V^{ce}, E^{ce})$;
$Base = \{G_1^{ba}(V_1^{ba}, E_1^{ba}), G_2^{ba}(V_2^{ba}, E_2^{ba}), \ldots, G_2^{ba}(V_{bs}^{ba}, E_{bs}^{ba})\}$;
$Fringe = \{G_1^{fr}(V_1^{fr}, E_1^{fr}), G_2^{fr}(V_2^{fr}, E_2^{fr}), \ldots, G_{fs}^{fr}(V_{fs}^{fr}, E_{fs}^{fr})\}$;
Expected consensus interval $\mathbb{E}$;
Initial opinion $X(0) = \{x_1(0), x_2(0), \ldots, x_n(0)\}$ of all agents.
**Output:** An updated social network $\overline{G}(V, \overline{E})$ with edge expansions from $G(V, E)$.

$\overline{E} = E$
$V^{ce\cdot l} = \{v_i \mid v_i \in V^{ce}, v_i \in V^l\}$
$V^{ce\cdot f} = \{v_i \mid v_i \in V^{ce}, v_i \in V^f\}$
$V^{ba\cdot l} = \{v_i \mid v_i \in (V_1^{ba} \cup \cdots \cup V_{bs}^{ba}), v_i \in V^l, x_i(0) \in \mathbb{E}\}$
$V_k^{ba\cdot l} = \{v_i \mid v_i \in V_k^{ba}, v_i \in V^l, x_i^{G_k^{ba}}(0) \in \mathbb{E}\}$
$V_k^{ba\cdot f} = \{v_i \mid v_i \in V_k^{ba}, v_i \in V^f\}$
$V_k^{fr\cdot l} = \{v_i \mid v_i \in V_k^{fr}, v_i \in V^l\}$
**if** $Base = \emptyset$ and $Central \neq \emptyset$ and $Fringe \neq \emptyset$ **then**
  **for** $k = 1; k \leq fs; k++$ **do**
    $\forall v_i \in V_k^{fr\cdot l}, \forall v_j \in V^{ce\cdot f}$, add edge $e = (v_i, v_j)$
    $\overline{E} = \overline{E} \cup e$
  **end for**
**end if**
**if** $Base \neq \emptyset$ and $Central = \emptyset$ and $Fringe \neq \emptyset$ **then**
  **for** $k = 1; k \leq bs - 1; k++$ **do**

$\forall v_i \in V_k^{ba \cdot l}, \forall v_j \in V_{k+1}^{ba \cdot l}$, add edge $e = (v_i, v_j)$
$\overline{E} = \overline{E} \cup e$
**end for**
**for** $k = 1; k \le fs; k++$ **do**
$\exists V_p^{ba \cdot f}, \forall v_i \in V_k^{fr \cdot l}, \forall v_j \in V_p^{ba \cdot f}$, add edge $e = (v_i, v_j)$
$\overline{E} = \overline{E} \cup e$
**end for**
**end if**
**if** $Base \ne \emptyset$ and $Central \ne \emptyset$ and $Fringe = \emptyset$ **then**
**for** $k = 1; k \le bs - 1; k++$ **do**
$\forall v_i \in V_k^{ba \cdot l}, \forall v_j \in V_{k+1}^{ba \cdot l}$, add edge $e = (v_i, v_j)$
$\overline{E} = \overline{E} \cup e$
**end for**
$\forall v_i \in V_{bs}^{ba \cdot l}, \forall v_j \in V^{ce \cdot l}$, add edge $e = (v_i, v_j)$
$\overline{E} = \overline{E} \cup e$
**end if**
**if** $Base \ne \emptyset$ and $Central \ne \emptyset$ and $Fringe \ne \emptyset$ **then**
**for** $k = 1; k \le bs - 1; k++$ **do**
$\forall v_i \in V_k^{ba \cdot l}, \forall v_j \in V_{k+1}^{ba \cdot l}$, add edge $e = (v_i, v_j)$
$\overline{E} = \overline{E} \cup e$
**end for**
$\forall v_i \in V_{bs}^{ba \cdot l}, \forall v_j \in V^{ce \cdot l}$, add edge $e = (v_i, v_j)$
$\overline{E} = \overline{E} \cup e$
**for** $k = 1; k \le fs; k++$ **do**
$\exists V_p^{ba \cdot f}, \forall v_i \in V_k^{fr \cdot l}, \forall v_j \in V_p^{ba \cdot f}$, add edge $e = (v_i, v_j)$
$\overline{E} = \overline{E} \cup e$
**end for**
**end if**
**return** $\overline{G}(V, \overline{E})$

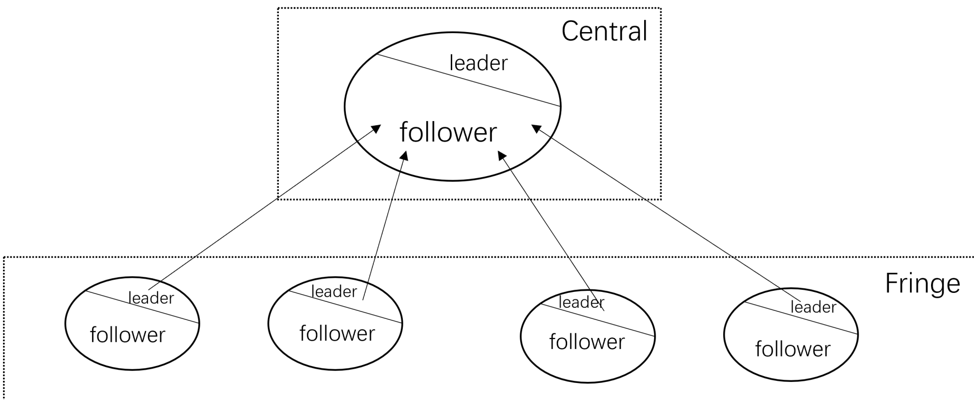



**Figure 5.** *Central* and *Fringe* in the network.

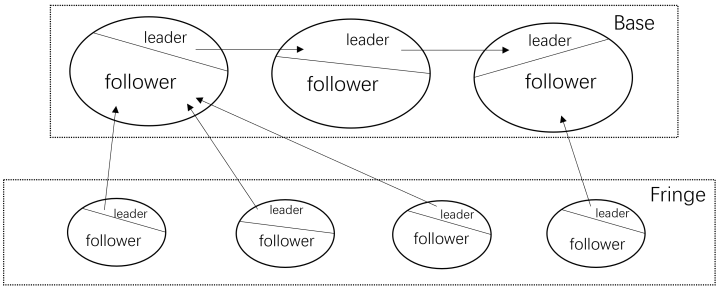



**Figure 6.** *Base* and *Fringe* in the network.

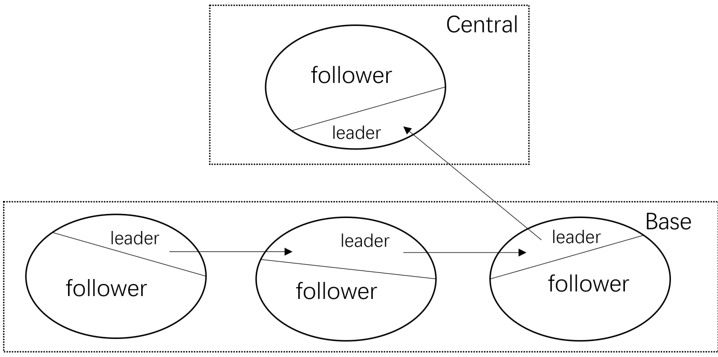



**Figure 7.** *Central*, *Base* in the network.

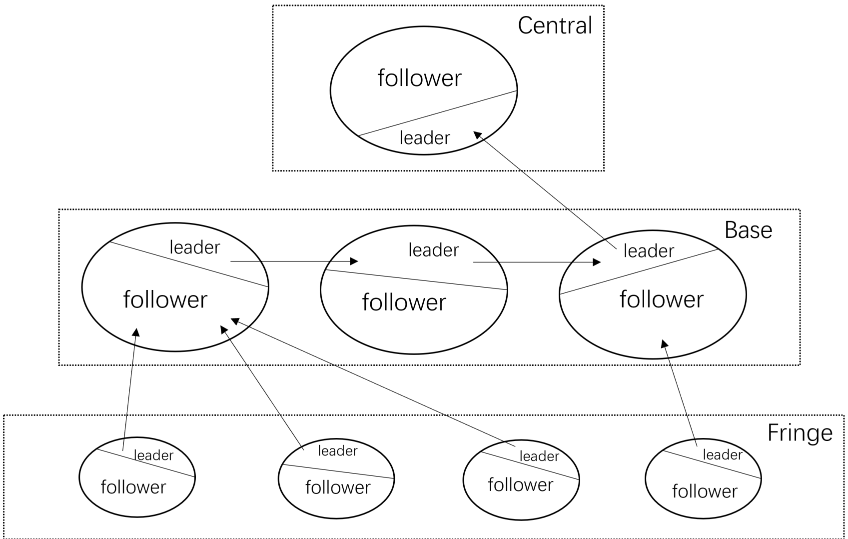



**Figure 8.** *Central*, *Base*, *Fringe* in the network.

#### 4.2.2. Adjustment of opinions and Self-persistence

Section 3 has explained that the final consensus is a linear combination of the leader's initial opinions. We recognize the critical role played by leaders. The research [13, 31, 44] on initial opinion adjustment has tended to focus on all leaders. This paper asserts that the fewer the leaders involved in the initial opinion adjustment, the more similar the initial opinion expressed to that of the original network. Even a mall adjustment to the leader's initial opinions can also result in excellent opinion control. This section establishes three rules for minimized adjusting of the leaders' initial opinions in the expanding network $\overline{G}(V, \overline{E})$ in order to reach an expected consensus value $\overline{c} \in \mathbb{E} = [\gamma, \eta]$. Three rules are provided as follows:

**Rule 1**. Improve the self-persistence of the supportive community's leaders.

**Rule 2**. Rule adjustments for indecisive communities can be divided into three cases: 1) the final opinion $c$ does not have to be adjusted if it falls within the expected opinion range $[\gamma, \eta]$; 2) if the final consensus $c$ is less than the minimum expected consensus $\gamma$, we must enhance and reduce the self-persistence of the leaders in the supportive communities and opposition communities, respectively. Also, the initial opinion of the leaders with the opinion value $c<\gamma$ in the indecisive communities should be adjusted to reach the value $\eta$; and 3) if the final consensus $c$ exceeds the maximum expected consensus $\eta$, we need to improve and reduce the self-persistence of leaders in the supportive communities and opposition communities, respectively. Then, the initial opinion of leaders in the indecisive communities should be adjusted to reach the border value $\eta$.

**Rule 3**. Reduce the self-persistence of the leaders within the opposition communities.

According to Rules 1–3, we can reach the final consensus value of $c$ within the expected consensus interval $\mathbb{E}$. The proposed opinion control strategy is that there should be at least a supportive or indecisive community for a focused social network. Early research on consensus in social networks illustrates the necessity of at least an opinion leader, which recognizes the critical role played by the leader's initial opinions. The final consensus is a linear combination of the opinions. It is now well established from our strategy that adjusting the self-persistence of the opposition communities is enough to achieve opinion control. If the entire network has only opposition communities, then the linear combination of the leader's initial opinions must be opposing. Hence, it is essential to have at least a supportive or indecisive community to reach a consensus.

## 5. Numerical analysis

The numerical analysis will be utilized to test the feasibility and effectiveness of the proposed method for opinion control. We operate the case social network from [31] as the benchmark for performance comparisons. Figure 9 presents the network $G(V, E)$ that contains a total of 26 agents. The initial opinion vector $X(0)$ for all of the agents is depicted below.

$$\begin{aligned} X(0) &= (x_1(0), x_2(0), \ldots, x_{26}(0))^T \\ &=(0.96, 0.64, 0.87, 0.10, 0.74, 0.76, 0.66, 0.79, 0.39, 0.32, 0.13, 0.29, 0.40, \\ &\quad 0.32, 0.73, 0.58, 0.21, 0.64, 0.31, 0.50, 0.36, 0.79, 0.53, 0.24, 0.46, 0.60)^T \end{aligned}$$

The self-persistence vector $\alpha$ for all the agents is shown below.

$$\begin{aligned} \alpha &= W_{ii} = (\alpha_1, \alpha_2, \ldots, \alpha_{26}) \\ &=(0.81, 0.91, 0.91, 0.28, 0.63, 0.10, 0.13, 0.55, 0.96, 0.96, 0.97, 0.42, 0.14, \\ &\quad 0.96, 0.16, 0.80, 0.49, 0.92, 0.79, 0.96, 0.66, 0.68, 0.93, 0.04, 0.85, 0.76) \end{aligned}$$

The consensus reached by the DeGroot model is always at or below 0.5 for random initial opinion values. Two small intervals $\mathbb{E} = [\gamma, \eta] = [0.6, 0.7]$ and $\mathbb{E} = [\gamma, \eta] = [0.3, 0.4]$ are taken separately before and after the opinion midpoint 0.5 to observe the performance of the control strategy. Having defined the network parameters for the simulation, we will now move on to discuss the process of achieving consensus using the proposed method.

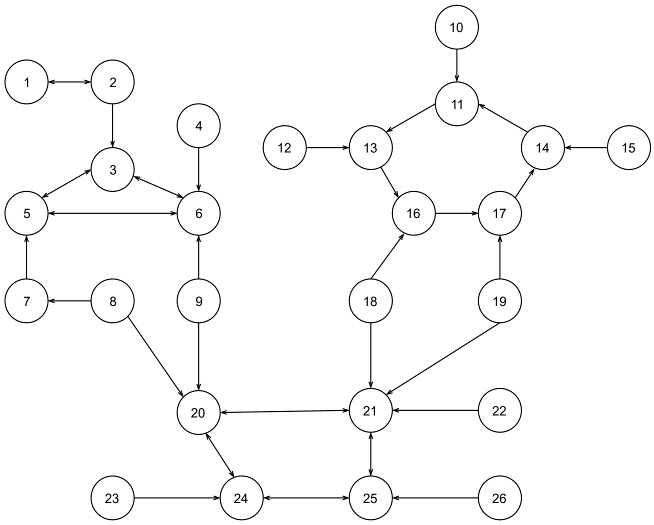



**Figure 9.** Case social network $G(V, E)$.

### 5.1. *Progress of reaching consensus*

This section analyses the progress of reaching consensus in the network $G(V, E)$ shown in Figure 9 by using the initial opinions $X(0)$ and the self-persistence vector $\alpha$. Figure 10 shows the agents' opinion evolution in their natural state. It is apparent from these evolutionary trends that the social network cannot reach a consensus without any adjustment.

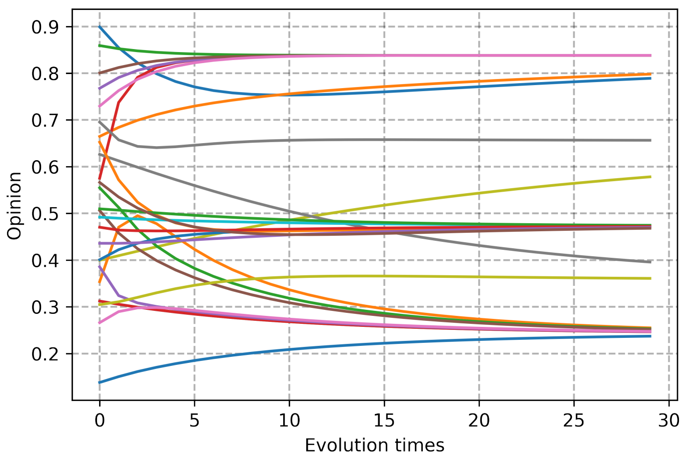



**Figure 10.** Evolution of opinions in the natural state

#### 5.1.1. Community division

Section 4.1 describes the processes of community division and community classification. Algorithm 1 of subnet recognition, together with the Algorithm 2 of ambiguous node partition, have been described in detail in the previous sections. The output results, i.e., $Com(G)$, obtained from the two algorithms are summarized in Table 1. The execution details of the algorithms can be observed through network visualization, where leaders are depicted in red while followers are highlighted in yellow. First, Algorithm 1 ensures that the network is divided into three communities, where there are four ambiguous nodes $v_8$, $v_9$, $v_{18}$, and $v_{19}$. They belong to multiple communities at the same time. The details of the division can be shown in Figure 11. Second, Algorithm 2 categorizes ambiguous nodes into their more biased communities. Figure 12 illustrates the result. There were two indecisive communities and one opposition community among the three.

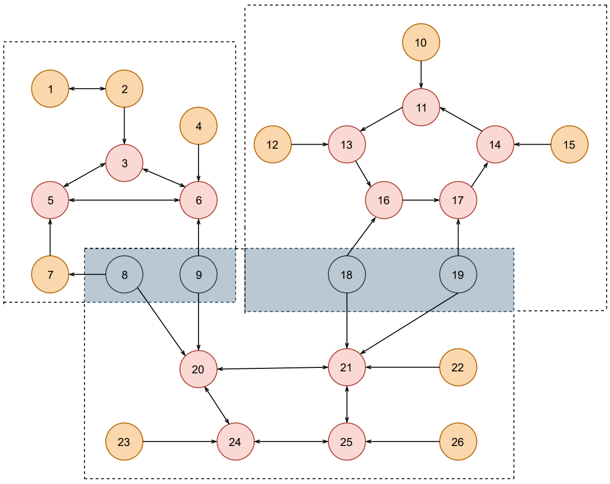



**Figure 11.** Result of subnet recognition.

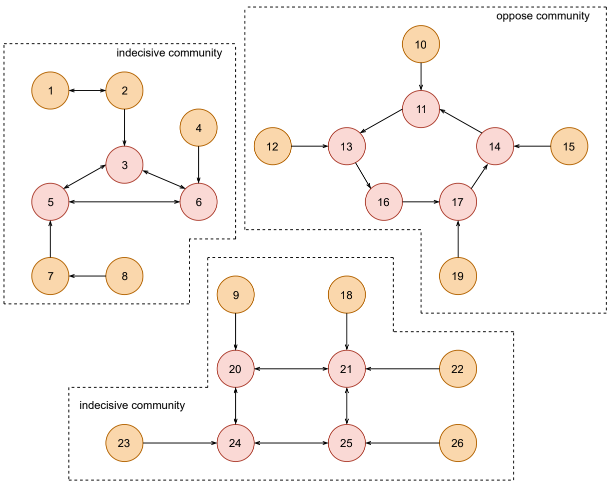



**Figure 12.** Result of ambiguous node partition.

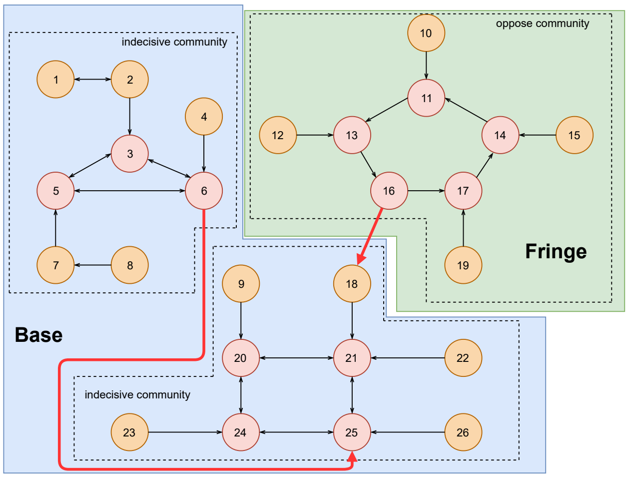



**Figure 13.** Result of adding edges.

**Table 1.** Community division and community classification.

| Communities | $V$ | $V^l$ | types |
|---|---|---|---|
| $G_1(V_1, E_1)$ | $\{v_1, v_2, \ldots, v_8\}$ | $\{v_3, v_5, v_6\}$ | indecisive |
| $G_2(V_2, E_2)$ | $\{v_{10}, v_{11}, \ldots, v_{17}, v_{19}\}$ | $\{v_{11}, v_{13}, v_{14}, v_{16}, v_{17}\}$ | opposition |
| $G_3(V_3, E_3)$ | $\{v_9, v_{18}, v_{20}, \ldots, v_{26}\}$ | $\{v_{20}, v_{21}, v_{24}, v_{25}\}$ | indecisive |

### 5.1.2. Edge expansions

Edge expansions based on the distinctions of *Central*, *Base* and *Fringe* are a continuing concern in Section 4.2. Algorithm 3 describes the steps required for edge expansions. Edge expansions allow for the creation of a new network $\overline{G}(V, \overline{E})$ connected to multiple communities from the original network $G(V, E)$. The simulation of opinion evolution in the new network can achieve the final consensus relative to the initial one, as seen in Figure 14a. The edge expansion strategy satisfies stochasticity in the presence of algorithmic constraints, and the final consensus depends on the edge expansion scheme selected. Table 2 shows the results of testing some edge expansion schemes in this paper, where $T$ represents the time of opinion evolution to reach consensus, and $c$ depicts the consensus value.

**Table 2.** Edge expansion schemes for the expected interval $\mathbb{E} = [0.6, 0.7]$.

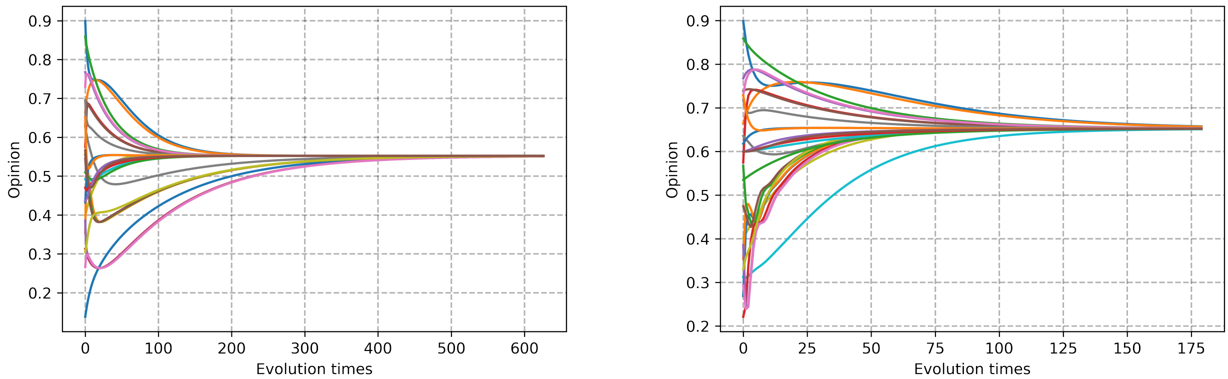
| Scheme | Added edges | $T_{after\_add}$ | $c_{after\_add}$ | $T_{after\_control}$ | $c_{after\_control}$ |
|---|---|---|---|---|---|
| Scheme 1 | $\{v_5, v_{20}\}\{v_{13}, v_8\}$ | 499 | 0.571 | 221 | 0.601 |
| Scheme 2 | $\{v_{20}, v_3\}\{v_{17}, v_{26}\}$ | 806 | 0.837 | 461 | 0.694 |
| Scheme 3 | $\{v_6, v_{21}\}\{v_{13}, v_1\}$ | 607 | 0.573 | 340 | 0.600 |
| Scheme 4 | $\{v_5, v_{23}\}\{v_{20}, v_5\}$ | 621 | 0.562 | 189 | 0.661 |

*5.2. Reach an expected consensus*

The opinion control strategy introduced in Section 4.2 enhances self-persistence in supportive communities, reduces self-persistence in opposition communities, and changes the initial opinions of leaders in indecisive communities. As Table 2 shows, Scheme 4 of edge expansions with $\overline{E} = E \cup \{v_6, v_{25}\} \cup \{v_{16}, v_{18}\}$ and the result of implementing the three opinion control rules in the expansion network is a good illustration of reaching an expected consensus.

As can be seen from Figure 14a, after the expansions of Algorithm 3, the times of evolution required for the network to reach a consensus is $T_{after\_add} = 621$ and the final consensus value of the network is $c_{after\_add} = 0.562$. Compared to the natural evolutionary state of the original network, the expansion one is already capable of achieving a consensus, but it does not satisfy the expectation. The proposed control strategy not only achieves the expected consensus but also further accelerates the achievement of the expected consensus significantly, as shown in Figure14b. Following the opinion control, the time required for the network to reach a consensus is reduced to $T_{after\_add} = 189$ and the final consensus value rises to $c_{after\_control} = 0.66$ that falls in the expected opinion interval $\mathbb{E} = [0.6, 0.7]$. This is evidence of the positive influence these rules have on accelerating public opinion into consensus. The experiment shows the benefits of these rules in addressing public opinion control.

Several experiments have been conducted for different adding edge schemes and expected consensus. The times and opinion values results are shown in Table 3. The data available in Table 3 can be inspected in Figure 15.



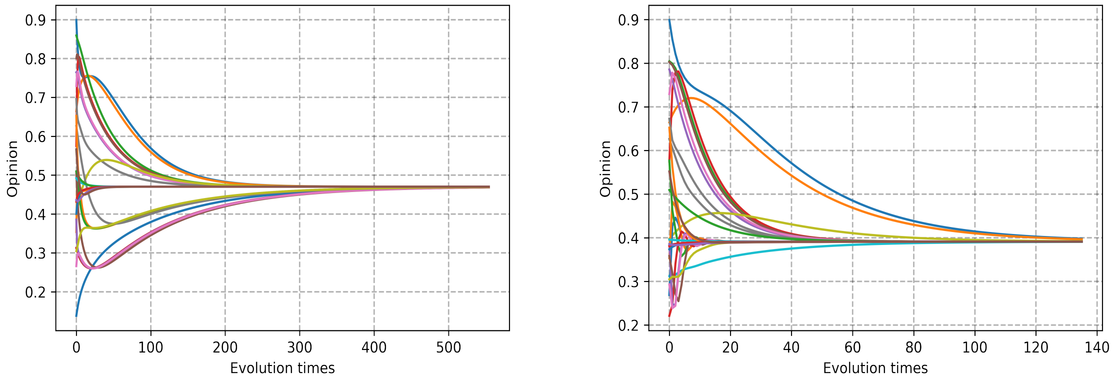

**(a)** Opinion evolution after edge expansions.

**(b)** Opinion evolution after opinion control.

**Figure 14.** Opinion control results for expected interval $\mathbb{E} = [0.6, 0.7]$.

**Table 3.** Edge expansion schemes for the expected interval $\mathbb{E} = [0.3, 0.4]$.

| Scheme | Added edges | $T_{after\_add}$ | $c_{after\_add}$ | $T_{after\_control}$ | $c_{after\_control}$ |
|---|---|---|---|---|---|
| Scheme 1 | $\{v_6, v_{26}\}\{v_{11}, v_{22}\}$ | 552 | 0.473 | 127 | 0.393 |
| Scheme 2 | $\{v_6, v_{23}\}\{v_{16}, v_{22}\}$ | 583 | 0.472 | 138 | 0.391 |
| Scheme 3 | $\{v_5, v_{26}\}\{v_{14}, v_9\}$ | 406 | 0.467 | 194 | 0.392 |
| Scheme 4 | $\{v_6, v_{22}\}\{v_{16}, v_9\}$ | 531 | 0.472 | 185 | 0.392 |




**(a)** Opinion evolution after adding edges.

**(b)** Opinion evolution after opinion control.

**Figure 15.** Opinion control results for the expected interval $\mathbb{E} = [0.3, 0.4]$.

### 5.3. *Experiments conducted using different community levels*

This study separates all communities into three levels, *Center*, *Base* and *Fringe*, according to the logical regions for opinion control. *Central* is one of the supportive communities. All indecisive communities, except for *Central*, are included in the *Base* level. *Fringe* level refers to all opposition communities. The previous sections describe how to expand the edges based on the interaction of three levels. This section examines the impact of adjusting the selection levels on opinion control.

Combination patterns for the three levels include *Center*, *Base*, *Fringe*, *Center* and *Base*, *Center* and *Fringe*, *Base* and *Fringe*, and *Center*, *Base* and *Fringe*. First, edge extension operations cannot be implemented when only one level is contained. Second, it is essential for opinion control to have at least a supportive community in level *Center* and a control objective community in level *Fringe*. Hence, our experiments will examine only two feasible combinations.

#### 5.3.1. *Central* and *Fringe*

This case contains only two levels of *Central* and *Fringe*. Specifically, there are no indecisive communities and more than one support community following community division. Basic experiment parameters are as follows. The initial opinion vector $X(0)$ for all of the agents is depicted below.

$$\begin{gathered} X(0) = (x_1(0), x_2(0), \ldots, x_{26}(0))^T \\ =(0.66, 0.64, 0.67, 0.60, 0.64, 0.66, 0.66, 0.89, 0.89, 0.32, 0.13, 0.29, 0.40, 0.32, 0.73, \\ 0.58, 0.21, 0.64, 0.31, 0.62, 0.36, 0.79, 0.53, 0.24, 0.46, 0.60)^T \end{gathered}$$

The higher value of opinions in $[0, 1]$ indicates that the agents are more supportive of the topic, and vice versa. The self-persistence vector $\alpha$ is set as follows.

$$\begin{gathered} \alpha = W_{ii} = (\alpha_1, \alpha_2, \ldots, \alpha_{26}) \\ =(0.75, 0.81, 0.81, 0.58, 0.73, 0.80, 0.73, 0.95, 0.96, 0.16, 0.47, 0.42, \\ 0.14, 0.96, 0.16, 0.80, 0.39, 0.92, 0.79, 0.96, 0.66, 0.68, 0.93, 0.04, 0.85, 0.76) \end{gathered}$$

This investigation was initially performed with the expected opinion interval $\mathbb{E} = [0.6, 0.7]$ and the edge expansion scheme $\{v_6, v_{12}\}\{v_{21}, v_{15}\}$. Then, the experiment was carried out with the second set of parameters, i.e., the expected opinion interval $\mathbb{E} = [0.3, 0.4]$ and the expansion scheme $\{v_6, v_{12}\}\{v_{21}, v_{15}\}$.

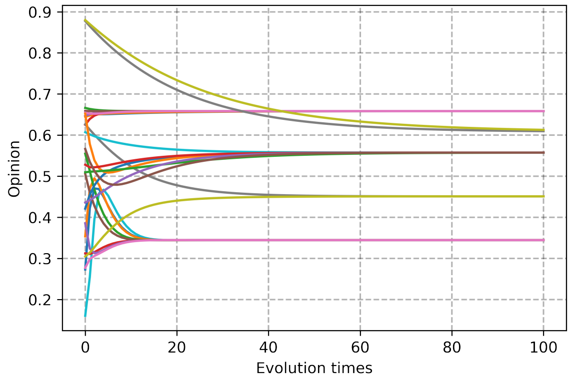
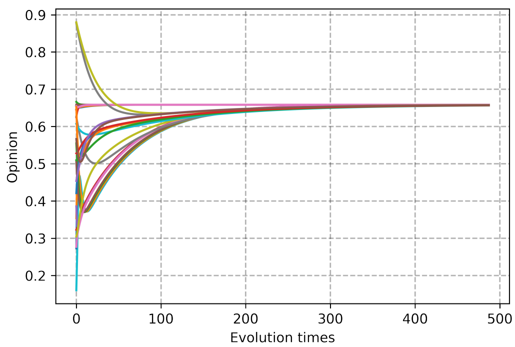
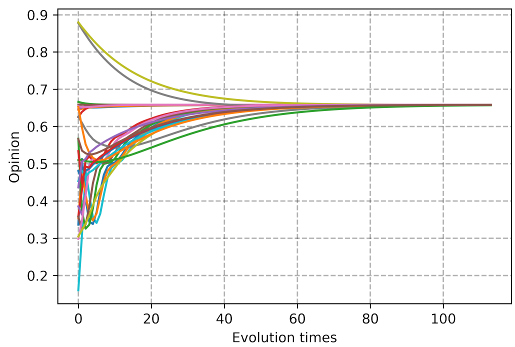


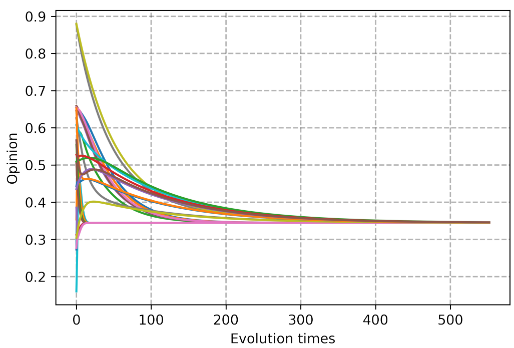

**Figure 16.** Evolution of opinions in the natural state(Only *Central* and *Fringe* in network).



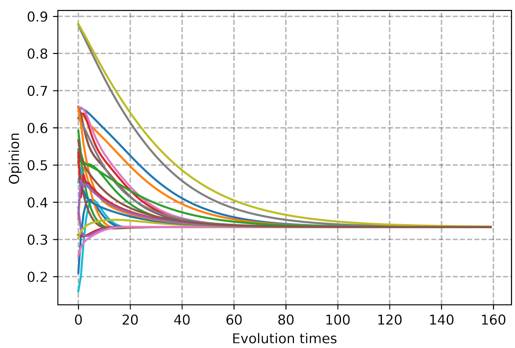

**(a)** Opinion evolution after adding edges ($\mathbb{E} = [0.6, 0.7]$)




**(b)** Opinion evolution after opinion control($\mathbb{E} = [0.6, 0.7]$)




**(c)** Opinion evolution after adding edges($\mathbb{E} = [0.3, 0.4]$)




**(d)** Opinion evolution after opinion control($\mathbb{E} = [0.3, 0.4]$)

**Figure 17.** Only *Central* and *Fringe* in network.

Figure 16 shows the evolution of opinions in the natural state of the original network. It can be seen from the five convergence lines of opinion values in Figure 16 that natural evolution cannot reach any consensus. Then, the edge expansion algorithm is used to obtain an updated network. The opinion evolution analysis results with respect to expected opinion internals $\mathbb{E} = [0.6, 0.7]$ and $\mathbb{E} = [0.3, 0.4]$ are shown in Figure 17a and Figure 17c in comparison with the natural situation, respectively. Consensus can be determined by the existence of a unique convergence line. The final results after the opinion control steps are shown in Figure 17b and Figure 17d, respectively. The time delay from evolution to consensus can be used to measure opinion control. The speed of reaching a consensus has been significantly accelerated.

### 5.3.2. *Base*, *Central* and *Fringe*

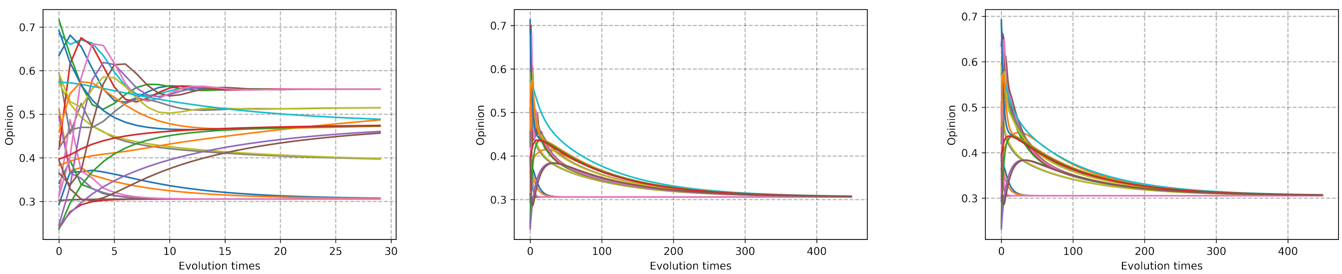
This assessment takes into account the same expected opinion internals, i.e., $\mathbb{E} = [0.3, 0.4]$, when all three levels *Base*, *Central*, and *Fringe* are present. The parameters $X(0)$ and $\alpha$ of opinion evolution are set out below.

$$X(0) = (x_1(0), x_2(0), \ldots, x_{26}(0))^T$$
$$=(0.13, 0.61, 0.82, 0.15, 0.30, 0.30, 0.68, 0.19, 0.65, 0.93, 0.43, 0.37, 0.79, 0.32, 0.17, 0.47, 0.26, 0.28, 0.74, 0.57, 0.80, 0.25, 0.10, 0.47, 0.20, 0.54)^T$$

$$\alpha = W_{ii} = (\alpha_1, \alpha_2, \ldots, \alpha_{26})$$
$$=(0.66, 0.27, 0.08, 0.41, 0.25, 0.99, 0.04, 0.07, 0.64, 0.51, 0.43, 0.97, 0.77, 0.09, 0.49, 0.50, 0.15, 0.6, 0.3, 0.94, 0.74, 0.62, 0.63, 0.14, 0.90, 0.57)$$

The edge expansion scheme can be presented as $\{v_{21}, v_6\}\{v_{11}, v_{26}\}$. Figure 18 depicts the evolution of opinions within the natural state, the edge expansion state, and the opinion control state.

Those results demonstrate that our proposed opinion control strategy is capable of controlling consensus within the expected consensus interval and reducing the time of opinion evolution to achieve consensus. Our opinion control strategy can achieve consensus only at the edge expansion stage. Furthermore, it can reduce the amount of opinion evolution by three rules during the opinion adjustment stage.




**(a)** Opinion evolution in natural state.

**(b)** Opinion evolution after edge expansions.

**(c)** Opinion evolution after control.

**Figure 18.** *Base*, *Central* and *Fringe* in network ($\mathbb{E} = [0.3, 0.4]$).

## 6. Analysis and comparison

This section analyses the key techniques proposed in this paper and compares them to others in order to demonstrate the feasibility of this approach. Community classification is the foundation for introducing the edge expansion algorithm and opinion control strategies. In addition, a community division algorithm including ambiguous node partition is also presented. This algorithm can be applied to different types of networks.

(i) This paper introduces the concept of ambiguous nodes compared with the community division algorithms of other models. Our approach considers agents belonging to multiple communities as ambiguous nodes, and we divide them based on the network structure and opinion values in the communities in which they reside.

(ii) The edge expansion algorithm with community classification is more purposeful than other edge expansion algorithms, and the consensus of the update network following edge expansion will approach the expected consensus. Edge expansion algorithms generally add edges randomly between subnets. The consensus of the new network after edge expansion is random and insufficiently stable. Table 4 displays the consensus achieved by the network generated by the edge expansion algorithm with and without community classification.

(iii) Compared with other strategies reported in [31], opinion control strategies that include community classification steps can reduce changes to the leader's initial opinion, thereby reducing the time necessary for opinion evolution to reach consensus. Table 4 compares opinion evolution times in the same social network using different strategies.

A detailed comparison of the model proposed in this paper and other studies can be found in Table 5.

As seen in Table 4, opinion evolution is estimated to be approximately 500 after edge expansion. Still, our proposed method reaches fewer multipolar opinions, which are all around the expected opinion. After opinion control, our strategy and the approach in [31] can reach a consensus, but our method requires fewer evolution steps.

**Table 4.** Different consensus control optimization model effects.

| | $T_{after\_add}$ | $c_{after\_add}$ | $T_{after\_control}$ | $c_{after\_control}$ |
|---|---|---|---|---|
| [31] | 500 | $0.84, 0.26, 0.48$ | 480 | 0.66 |
| Our proposal | 499 | $0.47, 0.84$ | 221 | 0.60 |

**Table 5.** A detailed comparison of the model proposed in this paper and other studies.

| | Our proposal | Other studies |
|---|---|---|
| Subnet recognition | The concept of ambiguous node is introduced, and the similarity of the opinions of the agent itself and the neighboring agents and the network structure of the agent are considered at the same time. | Consider the network structure in which the agent is located [13, 31]. |
| Adding edges(Network update) | Communities are divided into three types to add edges. Supporting the community as the *center*, indecisive community as the *base*, and opposing community as the *Fringe*. | Update the network structure by centering on the community whose final opinion is within the expected range [13, 31]. Choose leaders with similar perspectives or leaders who are more connected in the network structure to add edges [45]. |
| Opinion control strategies | Increase self-confidence in supporting community leaders, change the opinions of opposing leaders in indecisive communities, and reduce the self-confidence of opposing community leaders. | Change the opinions of leaders in all communities whose opinion values are not in the expected range [13, 31]. |

## 7. Conclusion

The present study was designed to synthesize individual opinions with community differences to solve CRP issues. The results of this investigation show that the subnet recognizes algorithm with ambiguous node division can simultaneously consider the network structure and the ambiguous agent opinions, making the subnet recognition more reasonable. The most apparent finding from this study is that separating communities into three levels, *Center*, *Base* and *Fringe*, facilitates the rapid implementation of edge expansions based on opinion control objectives. Opinion control models can significantly reduce the time from the beginning of opinion evolution to reach a consensus. The optimization model can effectively control the consensus based on preserving the original network structure and leadership opinions as much as possible.

In terms of future research, we will expand the opinion dynamics and fuzzy agents in the community classification to explore their application in more diverse scenarios, as well as CRP problems in social networks without leaders. At the same time, we will try to use membrane computing [46] to study the aggregation of opinions in the network asynchronously or synchronously.

## Acknowledgments

This work is supported by the Science and Technology Program of Sichuan Province (Grant nos. 2023YFS0424) and the National Natural Science Foundation (Grant nos. 61902324, 11426179, and 61872298).

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Yuntian Zhang. All authors read and approved the final manuscript.

## Conflict of interest

The authors have no relevant financial or non-financial interests to disclose.

## References

1. M. Yang, X. Chen, B. Chen, P. Lu, Y. Du, DNETC: dynamic network embedding preserving both triadic closure evolution and community structures, *Knowl Inf Syst*, **65** (2022), 1–29. https://doi.org/10.1007/s10115-022-01792-4

2. B. Chen, X. Chen, MAUIL: Multilevel attribute embedding for semisupervised user identity linkage, *Inf. Sci.*, **593** (2020), 527–545. https://doi.org/10.1016/j.ins.2022.02.023

3. Z. B Wang, X. L Chen, X. Y Li, Y. J Du, X. Lan, Influence maximization based on network representation learning in social network, *Intell. Data Anal.*, **26** (2022), 1321–1340. https://doi.org/10.3233/IDA-216149

4. M. H. Degroot, Reaching a consensu, *J Am Stat Assoc*, **69** (1974), 118–121. https://doi.org/10.1080/01621459.1974.10480137

5. N. E. Friedkin, E. C. Johnsen, Social influence and opinions, *J Math Sociol*, **15** (1990), 193–206. https://doi.org/10.1080/0022250X.1990.9990069

6. N. Friedkin, E. Johnsen, *Social Influence Networks and Opinion Change*, New Yourk: Cambridge University Press, 2011.

7. S. E. Asch, Effects of group pressure upon the modification and distortion of judgments, In: *Groups, leadership and men; research in human relations*, Oxford: Carnegie Press, 1951, 177–190.

8. J. R. P. French Jr, A formal theory of social power, *Psychol Rev*, **63** (1956), 181–194. https://psycnet.apa.org/doi/10.1037/h0046123

9. G. Deffuant, D. Neau, F. Amblard, G. Weisbuch, Mixing beliefs among interacting agents, *Adv Complex Syst*, **03** (2000), 87–98. https://doi.org/10.1142/s0219525900000078 .

10. R. Hegselmann, U. Krause, Opinion dynamics and bounded confidence: models, analysis and simulation, *Jasss*, **5** (2002), 1–33.

11. L. Li, L. Qiu, X. Liu, Y. Xu, E. Herrera-Viedma, An improved HK model-driven consensus reaching for group decision making under interval-valued fuzzy preference relations with self-confidence, *Comput Ind Eng*,**171** (2022), 108438. https://doi.org/10.1016/j.cie.2022.108438

12. O. Abrahamsson, D. Danev, E. G. Larsson, Opinion dynamics with random actions and a stubborn agent, *2019 53rd Asilomar Conference on Signals, Systems, and Computers*, (2019), 1486–1490. https://doi.org/10.1109/IEEECONF44664.2019.9048901

13. Q. Zhou, Z. Wu, A. H. Altalhi, F. Herrera, A two-step communication opinion dynamics model with self-persistence and influence index for social networks based on the DeGroot model, *Inf. Sci.*, **519** (2020), 363–381. https://doi.org/10.1016/j.ins.2020.01.052

14. Y. Li, M. Liu, J. Cao, X. Wang, N. Zhang, Multi-attribute group decision-making considering opinion dynamics, *Expert Syst. Appl.*, **184** (2021), 115479. https://doi.org/10.1016/j.eswa.2021.115479

15. M. Li, Y. Xu, X. Liu, F. Chiclana, F. Herrera, A trust risk dynamic management mechanism based on third-party monitoring for the conflict-eliminating process of social network group decision making, *IEEE Trans Cybern*, (2022), 1–15. 10.1109/TCYB.2022.3159866

16. Y. Lu, Y. Xu, E. Herrera-Viedma, Y. Han, Consensus of large-scale group decision making in social network: the minimum cost model based on robust optimization, *Inf. Sci.*, **547** (2021), 910–930. https://doi.org/10.1016/j.ins.2020.08.022

17. X. Liu, Y. Xu, R. Montes, F. Herrera, Social network group decision making: Managing self-confidence-based consensus model with the dynamic importance degree of experts and trust-based feedback mechanism, *Inf. Sci.*, **505** (2019), 215–232. https://doi.org/10.1016/j.ins.2019.07.050

18. Y. Lu, Y. Xu, J. Huang, J. Wei, E. Herrera-Viedma, Social network clustering and consensus-based distrust behaviors management for large-scale group decision-making with incomplete hesitant fuzzy preference relations, *Appl. Soft Comput.*, **117** (2022), 108373. https://doi.org/10.1016/j.asoc.2021.108373

19. X. Chen, H. Zhang, Y. Dong, The fusion process with heterogeneous preference structures in group decision making: A survey, *Inf Fusion*, **24** (2015), 72–83. https://doi.org/10.1016/j.inffus.2014.11.003

20. Z. Li, Z. Zhang, W. Yu, Consensus reaching with consistency control in group decision making with incomplete hesitant fuzzy linguistic preference relations, *Comput Ind Eng*, **170** (2022), 108311. https://doi.org/10.1016/j.cie.2022.108311

21. T. Gai, M. Cao, F. Chiclana, Z. Zhang, Y. Dong, E. Herrera-Viedma, et al., Consensus-trust driven bidirectional feedback mechanism for improving consensus in social network large-group decision making, *Group Decis Negot*, **32** (2022). https://doi.org/10.1007/s10726-022-09798-7

22. Z. Zhang, Z. Li, Consensus-based TOPSIS-Sort-B for multi-criteria sorting in the context of group decision-making, *Ann. Oper. Res.*, (2022), 1–28. https://doi.org/10.1007/s10479-022-04985-w

23. R. X. Ding, X. Wang, K. Shang, F. Herrera, Social network analysis-based conflict relationship investigation and conflict degree-based consensus reaching process for large scale decision making using sparse representation, *Inf Fusion*, **50** (2019), 251–272. https://doi.org/10.1016/j.inffus.2019.02.004

24. T. Wu, K. Zhang, X. Liu, C. Cao, A two-stage social trust network partition model for large-scale group decision-making problems, *Knowl Based Syst*, **163** (2019), 632–643. https://doi.org/10.1016/j.knosys.2018.09.024

25. Y. Tian, L. Wang, Opinion dynamics in social networks with stubborn agents: An issue-based perspective, *Automatica*, **96** (2018), 213–223. https://doi.org/10.1016/j.automatica.2018.06.041

26. Z. Ding, X. Chen, Y. Dong, F. Herrera, Consensus reaching in social network DeGroot Model: The roles of the Self-confidence and node degree, *Inf. Sci.*, **486** (2019), 62–72. https://doi.org/10.1016/j.ins.2019.02.028

27. J. Cho, Dynamics of uncertain and conflicting opinions in social networks, *IEEE Trans. Comput. Soc. Syst.*, **5** (2018), 518–531. https://doi.org/10.1109/TCSS.2018.2826532

28. P. Jia, A. MirTabatabaei, N. E. Friedkin, F. Bullo, Opinion dynamics and the evolution of social power in influence networks, *SIAM Review*, **57** (2015), 367–397. https://doi.org/10.1137/130913250

29. G. Chen, X. Duan, N. E Friedkin, F. Bullo, Social power dynamics over switching and stochastic influence networks, *IEEE Trans. Automat. Contr.*, **64** (2019), 582–597. https://doi.org/10.1109/TAC.2018.2822182

30. M. Ye, J. Liu, B. D. O. Anderson, C. Yu, T. Başar, Evolution of social power in social networks with dynamic topology, *IEEE Trans. Automat. Contr.*, **63** (2018), 3793–3808. https://doi.org/10.1109/TAC.2018.2805261

31. Y. Dong, Z. Ding, L. Martínez, F. Herrera, Managing consensus based on leadership in opinion dynamics, *Inf. Sci.*, **397** (2017), 187–205. https://doi.org/10.1016/j.ins.2017.02.052

32. J. A. Bondy, U. S. R. Murty, *Graph theory with applications*, London: Macmillan, 1976.

33. C. Godsil, G. F. Royle, *Algebraic graph theory*, Berlin: Springer Science & Business Media, 2001.

34. D. Urbig, J. Lorenz, H. Herzberg, Opinion dynamics: The effect of the number of peers met at once, *Jasss*, **11** (2008), 1–27.

35. R. L. Berger, A necessary and sufficient condition for reaching a consensus using DeGroot's method, *J Am Stat Assoc*, **76** (1981), 415–418. https://doi.org/10.1080/01621459.1981.10477662

36. Y. Dong, Z. Ding, F. Chiclana, E. Herrera-Viedma, Dynamics of public opinions in an online and offline social network, *IEEE Trans. Big Data*, **7** (2021), 610–618. https://doi.org/10.1109/TBDATA.2017.2676810

37. M. Gupta, Consensus building process in group decision making—An adaptive procedure based on group dynamics, *IEEE Trans Fuzzy Syst*, **26** (2018), 1923–1933. https://doi.org/10.1109/TFUZZ.2017.2755581

38. Y. Dong, M. Zhan, G. Kou, Z. Ding, H. Liang, A survey on the fusion process in opinion dynamics, *Inf Fusion*, **43** (2018), 57–65. https://doi.org/10.1016/j.inffus.2017.11.009

39. N. Capuano, F. Chiclana, H. Fujita, E. Herrera-Viedma, V. Loia, Fuzzy group decision making with incomplete information guided by social influence, *IEEE Trans Fuzzy Syst*, **26** (2018), 1704–1718. https://doi.org/10.1109/TFUZZ.2017.2744605

40. Y. Dong, Q. Zha, H. Zhang, G. Kou, H. Fujita, F. Chiclana, et al., Consensus reaching in social network group decision making: Research paradigms and challenges, *Knowl Based Syst*, **162** (2018), 3–13. https://doi.org/10.1016/j.knosys.2018.06.036

41. H. Y. Xu, Y. P. Luo, J. W. Wu, M. C. Huang, Hierarchical centralities of information transmissions in reaching a consensus, *Physics Letters A*, **383** (2019), 432–439. https://doi.org/10.1016/j.physleta.2018.11.013

42. L. Bergkvist, K. Q. Zhou, Celebrity endorsements: a literature review and research agenda, *Int J Advert*, **35** (2016), 642–663. https://doi.org/10.1080/02650487.2015.1137537

43. Z. Cao, F. Jiao, X. Qu, W. X. Wang, M. Yang, X. Yang, et al., Rebels lead to the doctrine of the mean: A heterogeneous DeGroot model, *J Syst Sci Complex*, **31** (2018), 1498–1509. https://doi.org/10.1007/s11424-018-7136-6

44. Y. Liu, H. Liang, L. Gao, Z. Guo, Optimizing consensus reaching in the hybrid opinion dynamics in a social network●, *Inf Fusion*, **72** (2021), 89–99. https://doi.org/10.1016/j.inffus.2021.02.018

45. Z. Wu, Q. Zhou, Y. Dong, J. Xu, A. H. Altalhi, F. Herrera, Mixed opinion dynamics based on DeGroot model and Hegselmann–Krause model in social networks, *IEEE Trans. Syst. Man Cybern. Syst.*, **53** (2023), 296–308. https://doi.org/10.1109/TSMC.2022.3178230

46. X. Chen, H. Peng, J. Wang, F. Hao, Supervisory control of discrete event systems under asynchronous spiking neuron P systems, *Inf. Sci.*, **597** (2022), 253–273.

AIMS Press

© 2023 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0)