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Research article

Scarcity of resources as a determining factor of value in input-output models (objectivist concept of capital)

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Abstract: Conducting global trade in national currencies greatly increases complexity and fragility of the modern financial system and, therefore, requires creation of new, much more accurate methods of macroeconomic and monetary regulation than those available today. The current practice of macroeconomic regulation relies on the system of national accounts (SNA) based on the Leontief input-output method. Its analytical tools are explained by the possibility of calculating the volume of output of a product and its cost in natural units. Moreover, it offers no explanation to the relationship between the composition of the output and relative prices. This disadvantage significantly complicates practical application of the Leontief method, since primary accounting reports operate with cost rather than physical indicators, forcing the introduction of various kinds of simplifications into the input-output model, which significantly reduces its analytical capabilities. The article presents a physical concept of value, on the basis of which the input-output model is supplemented by the definition of the material law of relative price formation. This addition turns the input-output method from an applied analysis tool into a complete theory of production, and in the future opens up fundamentally new, previously non-existent opportunities for the empirical studies of economic development and creation of highly effective methods of macroeconomic regulation. The price formation model, methodologically explained in the article, is a synthesis of the W. Leontief's concept of economy as a circular flow and P. Sraffa's model of the price mechanism of income distribution. It is basically our own concept of economic reproduction viewed as sharing by the producers of the common material resource of the production system. We claim that our findings regarding single-product industries in W. Leontief's and P. Sraffa's models can be generalized and applied to J. von Neumann's model of the balanced economic growth in multi-product industries.

Keywords: input-output model; capital theory; value theory; reproduction; economic circulation; neo-cardianism; sruffianism

JEL Codes: B16, B51, D57

1. Introduction

The most important idea of V. V. Leontief's dissertation work (Leontief, 1928) was that the relative prices of goods could be determined solely in terms of their quantities created and expended in production during the year, without any reference to supply and demand (Kurz et al., 2007). He assumed the structural interrelationships of the production system as the objective cause that shapes the relative prices of products of production (Leontief, 1987). When the input-output model was constructed, it turned out that "structural interrelationships" themselves had no effect on prices (Leontief, 1928). It became clear that objective mechanisms of value formation must rely on more fundamental physical principles of the realisation of the production process than simple circulation (Klukin, 2008, 2013). Leontief was well aware of this problem, but probably considered it too complex, and therefore abandoned it, devoting himself to adapting the tools he created to solve practical problems (Leontief, 1936, 1941).

Modern macroeconomic models of countries and regions are based on Leontief's model, and take into account production of several dozens of products. Products in these models are aggregates, indices constructed from real goods using prices, exchange rates, payment flows and accounting estimates (Pospelov, 2009). The use of value indices negates the analytical advantages of the intersectoral approach, originally focused on natural indicators, making it necessary to introduce various kinds of simplifications into balance models, which significantly reduce their quality. Therefore, explaining the relationship between natural and value characteristics of production is one of the fundamental problems of modern macroeconomic theory and practice of macroeconomic regulation.

The main difficulty in explaining the objective nature of value is the need to find a price-independent way of measuring the material costs of producing different products. The need for such an explanation follows from the obvious idea that our judgements about the value of material things do not arise from nothing but are shaped by the conditions of our lives, and thus the physical laws of production cannot be merely a passive instrument of our arbitrariness but can be the objective basis of our judgements and, therefore, must be considered on a par with the laws of competition and demand in analyzing the mechanism of price formation (Sraffa, 1926).

The problem of finding a non-price measure of costs breaks down into two interrelated tasks:

1. the problem of determining a material measure of costing for a given technological production system, and,
2. the problem of determining an absolute material measure of costing to compare different production systems. The method of solving the first of these problems was stated by P. Sraffa in his concept of the "standard commodity" (Sraffa, 1960). The method for solving the second problem was formulated by J. von Neumann who constructed a mathematical model of balanced economic growth in 1932 (Neumann, 1937, 1945; Champernowne, 1945).

V. V. Leontief, P. Sraffa and J. von Neumann were pioneers of reproductive analysis. They created the language, developed the method and laid the foundations of the subject matter of modern production theory (Samuelson, 1991; Garegnani, 2012; Fratini, 2019), but it has not yet been possible to build a consistent concept of value on this basis. The purpose of this article is to try to formulate the physical content of the concepts of capital and costs of the production system, and on the basis of them, to reveal the objective mechanism of formation of the prices of products of production in context the analytical approaches of V. V. Leontief, P. Sraffa and J. von Neumann.

2. Rarity of resources as an objective cause of value

According to Gossen's second law (Blaug, 1994) balance of supply and demand forces is reached at the point where marginal utility equals marginal cost of producing products. At this point, the ratio of equilibrium prices of products is equal to the ratio of their marginal utility, or in this case, marginal cost. The weakness of this theory is the variable marginal cost of production, as its existence is not necessary, in terms of the logic of organizing the production process, nor plausible, in terms of common sense and business experience (Sraffa, 1926; Garegnani, 2010; Lazarini, 2010). The hypothesis of disproportionate costs to output is necessary to give realism to the marginalist theory of equilibrium itself. If this assumption is abandoned, the failure of the subjectivist concept of value becomes clear, because, assuming that the value of production costs varies in proportion to output, the value of marginal costs will be constant, and thus the relative prices of products will also be constant, i.e., demand will not affect prices. This conclusion clarifies the need to develop an objectivist conception of economic value because it shows that subjective preferences, with necessity, determine only the composition of consumption, i.e., the relative output of products, but not their relative prices.

Thus, the problem of value theory is not to explain how our subjective preferences are formed, but to answer the question of why the costs of production change as the structure of output changes. Classical economists intuitively linked this phenomenon to the rarity of resources, but could not define the concept of rarity in relation to production. The only result here is Ricardo's theory of differential rent, which later became the basis for the marginalist approach. However, the theory of differential rent analyzes the case of using of resources which supply is constant or limited (land), and it, in general, makes it inapplicable to the analysis of the rarity of the products of production, the supply of which may vary arbitrarily. The concept of rarity that we propose is based on the principles of conservation of mass and constancy of matter composition of the production system (Kurz et al., 2007; Pantaleoni, 1894).

The principle of conservation of mass says that the quantity of matter does not change in the process of production. Its essence is reflected in James Mill's famous statement that man cannot create matter, but can only split it and transform it, change its form and move it around (Mill, 1826). Substantively, our conception of sparseness consists in developing the principle of the constancy of the composition of matter in a productive system. This principle asserts that matter is heterogeneous, but composed of different parts in nature. The mass of these parts remains constant regardless of the transformations to which they are subjected in the process of production. In other words, production cannot transform some kinds of matter into others, but can only change their form and combine them. The energy necessary for this is extracted through the combination of different kinds of matter. In this sense, the principle of constancy of composition is equivalent to the law of conservation of energy: in order to resume the process of production, the potential energy of the production system must remain unchanged after the completion of each cycle of production. It means that all the transformations of matter that occurred during the production process must, by the beginning of a new cycle of production, undergo reverse transformations, bringing the potential energy of the production system back to its original state. From this point of view, the content of the production process would look like this.

Physically, production is the transformation of matter from an initial state (initial substrate) into a product. The original substratum potentially contains all the possible products. The process of production consists in taking the desired constituent part from the substrate and transforming it into a product. The energy for this transformation is extracted through the interaction in the production process of the various products, called the means of production. The means of production are destroyed (expended) in the production process, returning back to the original substratum. The product, on the other hand, accumulates the energy derived from the destruction of the means of production. This energy is extracted from the product in the next production cycle, when it is used as the means of production. This is how the circulation of matter and energy takes place in the production system. Thus, the means of production constitute the stock of matter that provides the energy potential of the production system. We shall hereafter refer to this stock as capital. The cost of reproduction of capital determines the cost (price) of production. The magnitude of these costs is determined by two reasons: the number of resources and time spent in the process of producing capital. The mechanism of these reasons is as follows.

Each production process uses only part of the material substrate of the production system. However, the material composition of the means of production and the product are not generally the same. Taken together, these features of the realization of production give rise to the problem of the renewal of the material substratum of the production system. The essence of this problem is the division (differentiation) of the aggregate material substrate of production system in the process of production into parts that do not have the same composition. It happens because the composition of the material substratum being transformed into the product, i.e., the input of each production process as raw material, and the composition of the material substratum produced as a result of the destruction of the means of production, i.e., the output of each production process, turn out to be unequal. It makes it impossible to repeat the production cycle without first ensuring that the composition of the output substrate of each production process is identical to that at the start of the production cycle.

This problem can be solved in two ways: 1) by mixing all the different constituents of the total material substrate until it reaches a homogeneous state; 2) by making each unit of the total material substrate equally involved in all the material transformations of the production system, so that all the units of the substrate undergo the same changes, which results in it appearing at the end of the production cycle in the same state as it was at the beginning. The first way, although seemingly simpler, is unrealistic because its implementation requires centralized coordination of all producers. The second way requires a division (differentiation) in time, i.e., the sequential execution of all the production processes, which differ, in terms of changes in the composition of the substrate. This differentiation occurs spontaneously in the course of production, because those production processes, which were started before the necessary transformation of the substrate, can only end after these transformations have been completed. Thus, the substrate renewal process organizes itself. Therefore, the partitioning of the production process over time is a necessary natural mechanism for the renewal of the total material substrate of the production system. The content of the problem of the renewal of the material substratum is outlined by us in (Kurishev, 2022).

In the case of sequential productions, the output of the different products in the production system takes place at different times. In all production processes, the composition of the product and the means of production is generally not the same. It is clear that the production process can only begin once all the necessary resources and means of production are available to the producer. Since the means of production are the products of production for the case of successive production, this means that the process of production of each product produced in the production system can only begin after the production of all the inputs required for it has been completed. Taken together, these conditions give rise to the necessity of saving the means of production. Saving means producing and preserving the means of production in order to use them to produce a product in the next production cycle.

$$P_j = C_j t_j \tag{1}$$

where P_j—the cost of saving to produce the j product; C_j—the amount of saving; t_j—the saving time.

In a production system producing n products, the savings time for the j product will be calculated according to the formula,

$$t_j = \sum_{\substack{i=1 \\ i \neq j}}^{n} T_i \tag{2}$$

where t_j—the saving time of the means of production for the product j; T_i—the production time of the i—product.

Saving ensures the process of reproduction of the capital of the production system. The value of capital is determined by the cost of saving it, and since the volume of production of each product depends directly on the amount of capital used in its production, the value of the products of production is determined by the cost of saving capital. It follows from expressions (1) and (2) that the cost of capital saving in the production of all products is interrelated, as the time of saving t_j for each j product ($j \in \{1, 2, \ldots, n\}$) depends on the time of production of all other products in the production system. This relationship constitutes the physical content of the concept of sparsity of resources in production.

In sequential production, the increase in production of each product is realized by increasing the amount of capital used and the time of production. In contrast, a decrease in production occurs by decreasing the amount of capital used and the time of production. In this case, obviously, an increase in the production time of some products will have the consequence of an increase in the saving time for others. Conversely, a decrease in the production time of some products will result in a decrease in the saving time for others. According to Equations (1) and (2), it means that an increase in products production results in a decrease of the saving cost of each unit of capital used in their production and, vice versa, the decrease in product production results in the increase of the saving cost of each unit of capital used in their production. In other words, the increase in the output of a product reduces its value because the resources used in its production become abundant. Conversely, a reduction in the output of a product increases its value because the resources used in its production become rarer. This effect in the sequential production of products arises because of the need for time-separated access to the common material resource of the production system—its initial material substrate. Thus, the physical meaning of the concept of rarity lies in the sharing of the limited common productive resource of the production system in the renewal of the material means of production.

3. The measure of value (Objectivist concept of capital)

In the cost analysis, we have operated with quantities of different kinds of matter as if they could be directly compared with one another. In reality, of course, this is not the case. In order to be able to compare quantities of different kinds of matter with one another, it is necessary to find their common content. We consider matter as a means of production, so it is natural to take their contribution to the result of production as their common content in order to compare the different kinds of matter. In fact, it means that we will assume equal quantities of different kinds of matter contributing equally to the production of all the products produced in the production system.

Defining quantities of different kinds of matter contributing equally to the production of all products means defining such units of measurement, each of which would contribute equally to the production of all the products produced in the production system. We will call the units of quantity of the products of production in which the coefficients of the direct cost matrix $\{a_{ij}\}$ are expressed as arbitrary units of quantity of products in the production system. The units of the quantity of the products of production making the same contribution to the production of all the products produced in the production system are the natural units of the quantity of products in the production system.

For a given matrix A the ratio of arbitrary and natural units is established by the system of equations:

$$\begin{cases}(a_{11}c_1 + a_{21}c_2)(1 + R) = c_1 \\ (a_{12}c_1 + a_{22}c_2)(1 + R) = c_2\end{cases} \tag{3}$$

where c_i—the number of natural units i of the product contained in each arbitrary unit; $(1 + R)$—a multiplier equal to the amount of product contained in each unit of inputs. In matrix form,

$$\boldsymbol{Ac} = \boldsymbol{\lambda c} \tag{4}$$

where $\boldsymbol{A} = \boldsymbol{D}^T$—the transposed direct cost matrix; $\lambda = {}^{1}/_{(1+R)}$—the largest positive eigenvalue of matrix **A**, $\boldsymbol{c} = \binom{c_1}{c_2}$—a strictly positive eigenvector of matrix **A** corresponding to the eigenvalue of λ.

Let us denote Q_1, Q_2—the output volumes of products 1 and 2, expressed in arbitrary units, and G_1, G_2—the output volumes of products 1 and 2, expressed in natural units. Then the ratio of these quantities will be given by the following equations:

$$G_1 = c_1 Q_1,\ G_2 = c_2 Q_2 \tag{5}$$

Denote also,

$$\alpha = \frac{Q_1}{Q_2},\ N = \frac{G_1}{G_2} \tag{6}$$

Then it follows directly from (5),

$$N = \nu\alpha \tag{7}$$

where $\nu = {}^{c_1}/_{c_2}$—the ratio of the elements of the eigenvector $\boldsymbol{c}$.

It follows from (4) that in sequential production, the output of a product is equal to the product of the amount of capital used in production by the time of production. For the example of production of 2 products we are considering, these products will look like this,

$$\lambda G_1 = C_1 T_1,\ \lambda G_2 = C_2 T_2 \tag{8}$$

The total income generated in the production of products 1 and 2 according to (3) is,

$$\begin{cases} V_1 = G_1(w_1 - a_{11}c_1w_1 - a_{21}c_2w_2) \\ V_2 = G_2(w_2 - a_{12}c_1w_1 - a_{22}c_2w_2) \end{cases} \tag{9}$$

where V_1 and V_2—the total income received by producers of products 1 and 2 respectively; w_1 and w_2—the natural prices of products 1 and 2, i.e., the value of each natural unit of the quantity of products 1 and 2 respectively. Considering that,

$$p_1 = c_1w_1,\ p_2 = c_2w_2 \tag{10}$$

For arbitrary output units, let us write,

$$\begin{cases} V_1 = Q_1(p_1 - a_{11}p_1 - a_{21}p_2) \\ V_2 = Q_2(p_2 - a_{12}p_1 - a_{22}p_2) \end{cases} \tag{11}$$

where p_1 and p_2—the arbitrary prices of products 1 and 2, i.e., the cost of each arbitrary unit of products 1 and 2 respectively.

The equal contribution of each unit of capital to the production of all the products produced in an economic system means that the amount of capital used in production is proportional to the amount of income generated from production, i.e., the fulfilment of the equality,

$$\frac{V_1}{V_2} = \frac{C_1}{C_2} \tag{12}$$

Denote,

$$R_1(\gamma) = 1 - a_{11} - a_{21}\frac{1}{\gamma},\ R_2(\gamma) = 1 - a_{22} - a_{12}\gamma \tag{13}$$

where $\gamma = {}^{p_1}\!/_{p_2}$—the ratio of the prices of products 1 and 2. Then expression (12) can be rewritten as:

$$\alpha\gamma\xi(\gamma) = \upsilon \tag{14}$$

where $\xi(\gamma) = {}^{R_1(\gamma)}\!/_{R_2(\gamma)}$, $\upsilon = {}^{C_1}\!/_{C_2}$.

It is clear that the specific composition of the input material substrate of a production system makes it capable of producing different products to different degrees. The composition of the substrate changes as the output structure changes. It happens because, on the one hand, of differences in the composition of the means of production used in the production of different products. On the other hand, because of the need for less or, conversely, more processing of the substrate in order to extract from it the raw materials required for production. Thus, an increase in the use of some inputs as compared to others will reduce the content of the inputs in the substrate. Increasing outputs of some products relative to others will increase the cost of extracting some raw materials from each unit of substrate, and decrease the cost of extracting others. As a result, a change in the composition of output changes the cost structure of production, i.e., a change in the coefficients of the direct cost matrix $\{a_{ij}\}$.

In this sense, the initial values of the coefficients of the direct cost matrix found for some arbitrary output structure can be seen as the coordinate system relative to which the amount of material inputs in the production system is measured. The deviation of the output structure from this point means the change in the amount of capital required to produce each unit of each product. This deviation can be expressed by means of a function which establishes the relationship between the ratio of the cost of capital to produce each unit of various products and the ratio of the volume of their outputs.

Let $B_1(\alpha)$—a function determining the quantity of capital expended in the production of a unit of product 1 depending on the ratio of volumes of outputs of products 1 and 2 (α); $B_2(\alpha)$—a function determining the quantity of capital expended in the production of a unit of product 2 depending on the ratio of outputs of products 1 and 2 (α). The values of the functions and are computed in the units of the quantity of capital as given by the coefficients of matrix **A**. Let us denote by the ratio of the volumes of outputs of products 1 and 2 for which the values of the coefficients of matrix were fixed $\boldsymbol{A} - \alpha_0$. It follows from equation (5) that,

$$\frac{B_1(\alpha_0)}{B_2(\alpha_0)} = 1 \tag{15}$$

Let us denote by $\pi(\alpha) = {B_1(\alpha)}/{B_2(\alpha)}$, where $\pi(\alpha) = {B_1(\alpha)}/{B_2(\alpha)}$ the ratio of specific capital costs in the production of products 1 and 2 depending on the ratio of the volumes of their output α. Then the relationship between the amount of capital costs calculated on the basis of the coefficients of direct costs (υ), and the amount of real capital costs taking into account changes in the composition of material substance of production system when changing the output structure (Υ) will be described by the equation,

$$\Upsilon = \pi(\alpha)\upsilon \tag{16}$$

where $\Upsilon = {I_1}/{I_2}$; I_1, I_2—the cost of capital in the production of products 1 and 2 taking into account the structure of output α.

The function $\pi(\alpha)$ reflects the dependence of the value of production costs on the structure of the initial material substrate, i.e., it characterises the specifics of the resource base of the production system. Given (17), Equation (14) would take the form of:

$$\alpha\gamma\xi(\gamma) = \pi(\alpha)\upsilon \tag{17}$$

Equation (17) describes the physical conditions of economic equilibrium in terms of the cost of capital. Equilibrium is reached when the return on capital in all production is equal.

Equation (17) establishes the relationship between output (α) and prices (γ) of products for a given cost structure (υ). In this case, costs are computed in natural units of the quantity of capital, i.e., capital is treated here as a measure of the value of production. Thus, equation (17) solves the problem of finding a price-independent objective measure of value. However, it does not reveal the nature of the causes determining the value of production, namely the nature of the relationship between the output of products (α) and the amount of capital expended in their production (υ). To answer this question, let us look in more detail at the process of product production.

4. Allocation

In sequential production, each unit of material substrate participates in the creation of each product produced in the production system, i.e., the capital used in the production of each product passes through (recycles) the entire material substrate of the production system during the production cycle. Since each unit of capital must have equal access to the total material substrate of the production system, i.e., it must undergo equal, in terms of energy inputs, transformations, this means that the same amount of substrate – J, must be transformed per unit of time in the production system. The degree, i.e., the energy equivalent of this transformation, is proportional to the time of interaction of each unit of capital with the material substrate. Thus, time is proportional to the amount of capital simultaneously coming into contact with the material substrate of the production system. For the case of the production of 2 products that we are considering, this proportion, in terms of the notations we have introduced, would look like this:

$$\frac{T_1}{T_2} = \frac{C_1}{C_2} \tag{18}$$

Equation (19) defines the ratio of the access time of capital employed in the production of products 1 and 2 to the total material substrate of the production system. The deviation from this proportion means that each unit of capital employed in the production of one product processes more substrate per unit of time than each unit of capital employed in the production of the other, i.e., that part of the capital in the production system is not fully used. To measure this deviation, let us rewrite equality (19) as follows:

$$\frac{T_1}{T_2} = \rho \frac{C_1}{C_2} \tag{19}$$

where ρ—the coefficient equal to the deviation of the ratio of capital to time spent in the production of products 1 and 2 from the optimum proportion (18).

If the coefficient in equation (19) is $\rho > 1$, it means that each unit of capital employed in the production of product 1 processes a larger volume of material substrate per unit of time than each unit of capital employed in the production of product 2. In contrast, if the coefficient in equation (19) is $\rho < 1$, it means that each unit of capital employed in the production of product 2 processes a larger volume of material substrate per unit of time than each unit of capital employed in the production of product 1. In short, the deviation of the coefficient ρ from one in equation (19) indicates that there is spare capacity in the production system, which obviously leads to an increase in its total costs and a reduction in output. To determine the relationship of the coefficient to aggregate cost and output, we introduce the following notations,

$$C = C_1 + C_2,\ T = T_1 + T_2 \tag{20}$$

where C—the total capital of the production system; T—the total production time of products, i.e., the duration of the production cycle in the production system.

It follows from (4) that,

$$C = \lambda J \tag{21}$$

where J—the total amount of material substrate in the production system.

Equalities (19), (20) and (21) can be represented as a system:

$$\begin{cases} \upsilon = \dfrac{C_1}{C_2} \\ C = C_1 + C_2 \\ C = \lambda J \\ \dfrac{T_1}{T_2} = \rho\upsilon \\ T = T_1 + T_2 \end{cases} \tag{22}$$

Expressing from the 1st, 2nd and 3rd equations of system (22) C_1 and C_2 by λ, J and υ we have,

$$C_1 = \frac{\lambda J\upsilon}{1+\upsilon},\ C_2 = \frac{\lambda J}{1+\upsilon} \tag{23}$$

Then, expressing from the 4th and 5th equations of system (22) T_1 and T_2 by ρ, υ and T we obtain,

$$T_1 = \frac{T\rho\upsilon}{1+\rho\upsilon},\ T_2 = \frac{T}{1+\rho\upsilon} \tag{24}$$

According to (1) and (2), for the production system we are considering producing 2 products, the production costs of the first and second products will be,

$$P_1 = C_1T_2,\ P_2 = C_2T_1 \tag{25}$$

where P_1, P_2—the cost of producing the 1st and 2nd product respectively.

The total cost of a production system producing 2 products is equal to the sum of,

$$P = P_1 + P_2 \tag{26}$$

Substituting in expressions (25) instead of C_1, C_2 and T_1, T_2 their values from (23) and (24) we obtain,

$$P_1(\rho) = \frac{\lambda J\upsilon}{1+\upsilon} \times \frac{T}{1+\rho\upsilon},\ P_2(\rho) = \frac{\lambda J}{1+\upsilon} \times \frac{T\rho\upsilon}{1+\rho\upsilon} \tag{27}$$

Substituting in (26) the values $P_1(\rho)$ and $P_2(\rho)$ from (27) after elementary transformations we obtain the value of the total costs of the production system as a function of the coefficient ρ,

$$P(\rho) = \frac{\lambda JT\upsilon(1+\rho)}{(1+\upsilon)(1+\upsilon\rho)} \tag{28}$$

The parameter of the function $P(\rho)$ is the ratio of the amounts of capital employed in the production of products 1 and 2 – υ. To make our analysis clearer, we express the parameter in terms of the ratio of the quantities of products 1 and 2 – N. To do this, we denote by,

$$\tau = \frac{T_1}{T_2} \tag{29}$$

Taking into account notations (6) and (29), equations (8) and (19) can be rewritten as:

$$N = \tau\upsilon,\ \tau = \rho\upsilon \tag{30}$$

Substituting the value τ from the 2nd equality of the system (30) into the 1st one, we obtain the relationship between the costs of capital υ and the volumes of output N,

$$N = \rho \upsilon^2 \tag{31}$$

whence,

$$\upsilon = \sqrt{\frac{N}{\rho}} \tag{32}$$

Let us denote,

$$X = \sqrt{N}, \ \chi = \sqrt{\rho} \tag{33}$$

Given (32) and notation (33), equality (28) can be rewritten as

$$P(\chi) = \frac{\lambda JTX(1+\chi^2)}{(\chi + X)(1 + X\chi)} \tag{34}$$

Function (34) determines the dependence of the value of total costs of the production system P on the coefficient χ in terms of the ratio of product outputs X. The graph of function (34) for the case of $J = 9$, $T = 3$, $a_{11} = 0{,}4$, $a_{21} = 0{,}2$, $a_{12} = 0{,}4$, $a_{22} = 0{,}5$, $X = 1$ is shown in figure 1 (left axis of ordinates). In the following, we will use the same parameters when plotting the graphs.

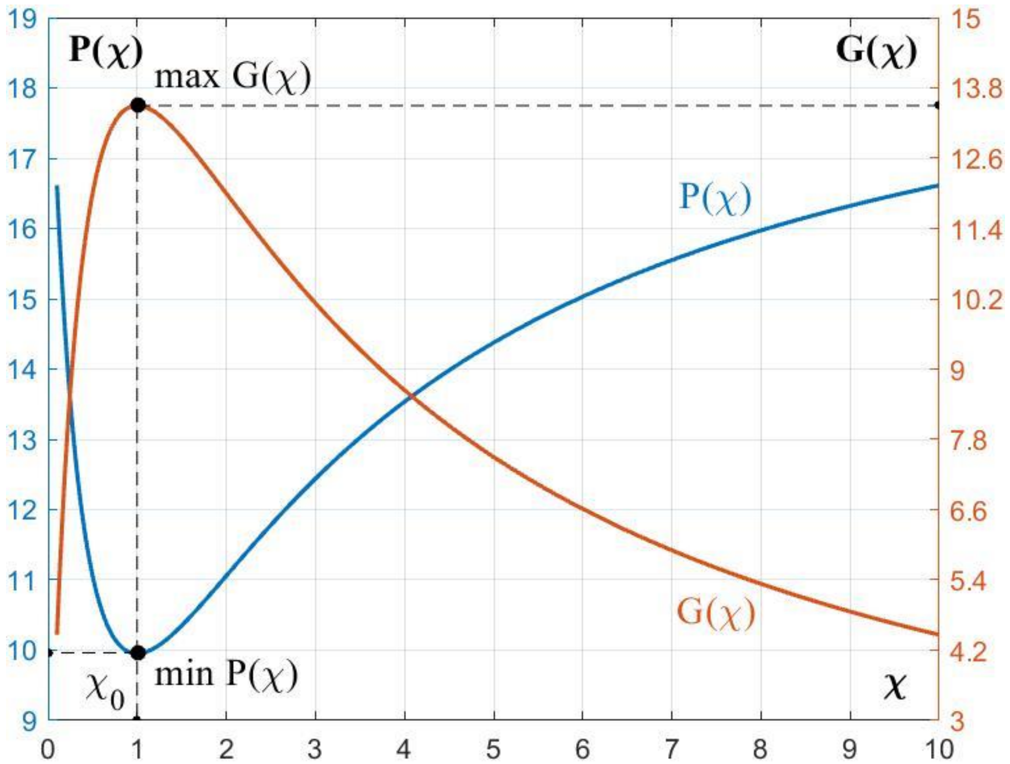

Figure 1. Dependences of aggregate cost P and aggregate output G on the ratio of capital (C_1, C_2) to time (T_1, T_2) in production χ.

Figure 1 shows that function (34) has a minimum at the point $\chi = 1$ denoted by χ_0. According to the notation introduced in (34), the point of minimum corresponds to the value of $\rho = 1$. A deviation of the coefficient ρ from unity leads to an increase in the total cost of production. This increase is due to the emergence of capital surplus in one of the productions in the system in question. If the value of the coefficient ρ shifts along the abscissa axis to the right of unity, i.e., if $\rho > 1$, then a capital surplus occurs in the production of product 2. On the contrary, when moving along the abscissa axis to the left of unity, i.e., in the case when $\rho < 1$, the capital surplus arises in the production of product 1. The reason for the emergence of idle production capacity is obviously a surplus of supply over demand, either because of reduced demand or excess investment. The parameter ρ characterises the imbalance in the distribution of productive resources (allocation) formed by the movement of market conditions, due to the impossibility of instant transformation of capital from one material form to another, which leads to its deficit in productions whose demand for products increases, and its surplus in productions whose demand for products decreases. Physically, these imbalances in the production system manifest themselves in the appearance of idle capacity and, as a consequence, in the reduction of aggregate output.

The total output in natural units, for the system we are considering, producing 2 products according to (8) is,

$$G = \frac{1}{\lambda}(C_1T_1 + C_2T_2) \tag{35}$$

where *G*—the total output of the production system in natural units. By substituting in equality (35) the values of C_1, C_2 and T_1, T_2 from (23) and (24) after elementary transformations we obtain the dependence of *G* on ρ.

$$G(\rho) = \frac{JT(1 + \upsilon^2\rho)}{(1 + \upsilon)(1 + \upsilon\rho)} \tag{36}$$

Expressing υ by N , i.e., replacing the variables in function (36) according to notation (33), we finally have,

$$G(\chi) = \frac{JT(1 + X^2)\chi}{(\chi + X)(1 + X\chi)} \tag{37}$$

The graph of function (38) is shown in figure 1 (right-hand axis of ordinates).

Figure 1 shows that function (37) has a maximum at the point $\chi = 1$ denoted by χ_0. This point corresponds to the value $\rho = 1$, at which full employment of the capital of the production system is achieved. If the coefficient ρ deviates from unity, this means that there is idle capacity and the aggregate output of the production system *G* declines.

Let us determine the impact of the structure of the initial material substrate of the production system given by the function $\pi(\alpha)$, on the values of its total inputs and outputs. Let us express the cost of capital in the production of products 1 and 2 in terms of Y , by adding equality (16) to system (22),

$$\begin{cases} \upsilon = \dfrac{C_1}{C_2} \\ \lambda J = C_1 + C_2 \\ \Upsilon = \pi(\alpha)\upsilon \\ \dfrac{T_1}{T_2} = \rho\Upsilon \\ T = T_1 + T_2 \end{cases} \tag{38}$$

Expressing from (38) $C_1,\ C_2$ and $T_1,\ T_2$ we obtain,

$$C_1 = \frac{\lambda J\Upsilon}{\pi(\alpha)+\Upsilon},\ C_2 = \frac{\lambda J\pi(\alpha)}{\pi(\alpha)+\Upsilon} \tag{39}$$

$$T_1 = \frac{T\Upsilon\rho}{1+\Upsilon\rho},\ T_2 = \frac{T}{1+\Upsilon\upsilon} \tag{40}$$

Given the structure of the initial material substrate, the cost of capital in the production of products 1 and 2, would be,

$$I_1 = B_1(\alpha)C_1,\ I_2 = B_2(\alpha)C_2 \tag{41}$$

Production costs of the 1st and 2nd products respectively,

$$P_1 = I_1T_2,\ P_2 = I_2T_1 \tag{42}$$

Volumes of 1st and 2nd product releases,

$$G_1 = \frac{1}{\lambda}I_1T_1,\ G_2 = \frac{1}{\lambda}I_2T_2 \tag{43}$$

From there we obtain the values of total production costs and total outputs, taking into account the structure of the initial material substrate of the production system,

$$P(\rho) = \frac{\lambda JT\Upsilon B_1(\alpha)(1+\rho)}{(\pi(\alpha)+\Upsilon)(1+\Upsilon\rho)} \tag{44}$$

$$G(\rho) = \frac{JTB_1(\alpha)(1+\Upsilon^2\rho)}{(\pi(\alpha)+\Upsilon)(1+\Upsilon\rho)} \tag{45}$$

The appearance of the coefficients $B_1(\alpha)$ and $\pi(\alpha)$ in functions (44) and (45) has the consequence of shifting the point of the optimal ratio of capital and time in production to the left or right of unity. This shift reflects shifts in the distribution of income between the different types of capital after a restructuring of the production cost structure, as a result of changes in the composition of the initial material substrate of the production system. The shift in the optimum point occurs because cost is measured in the coordinate system defined by the coefficients of matrix **A**, calculated for a particular composition of the material substrate, without regard to possible changes in the material substrate. In other words, if we recalculated matrix **A** after each change in the composition of the material substrate, the point of the optimal ratio of capital to time in production would always be unity. Thus, the coefficients of the Leontief direct cost matrix in natural terms set the coordinate system for measuring the physical (natural) values of material inputs and output in production.

Equations (44) and (45) establish the allocation of resources of the production system, i.e., the efficiency of their allocation in terms of demand. The degree to which the current structure of production α corresponds to consumer demand is characterized by the coefficient ρ which is an indicator of the impact of market forces on the allocation of resources. The allocation (coefficient ρ) determines the value of production costs for a given production structure α. The natural costs of production of products 1 and 2 for all possible values α are determined by equations (42) for optimal allocation of resources ($\rho = 1$). The natural costs of production are the material basis that forms the subjective estimates of the consumers.

5. Production costs

The action of physical forces that form subjective estimates of consumers can be visualized by constructing graphs of dependences: 1) the aggregate material inputs of the production system P from the output composition α; and 2) the square of the difference between the unit production costs of the products 1 and 2 Δ and the output composition α. Graphs of these dependencies $\alpha \in [1,\ 2{,}5]$ are shown in figure 2 (left and right axis of ordinates, respectively).

Figure 2 shows the dependence of aggregate material costs of the production system on the composition of output $P(\alpha)$. This dependence is obtained from function (34), if we assume a coefficient $\chi = 1$ and express the variable X through α taking into account (7) and (33), i.e., assume,

$$X = \sqrt{\nu\alpha} \tag{46}$$

Then,

$$P(\alpha) = \frac{2\lambda JT\sqrt{\nu\alpha}}{\left(1 + \sqrt{\nu\alpha}\right)^2} \tag{47}$$

The graph of function (47) in figure 2 shows that the total cost of the production system changes as the composition of output changes. This relationship means that the aggregate amount of product consumed changes as the composition of consumption changes. The difference in the aggregate amount of matter consumed for different output compositions constitutes the material price of our subjective preferences.

Another characteristic reflecting changes in the conditions under which products are produced when the composition of output changes is the difference in their unit cost. The unit cost is equal to the cost of producing each unit. In the example of production of 2 products we are considering, these costs, given (8) and (25), will be:

$$\sigma_1 = \frac{P_1}{G_1} = \lambda\frac{T_2}{T_1},\ \sigma_2 = \frac{P_2}{C_2} = \lambda\frac{T_1}{T_2} \tag{48}$$

where σ_1, σ_2—the unit costs of production of products 1 and 2, respectively. Substituting values T_1 and T_2 from (24) into equations (48), we finally obtain the dependence of unit production costs of products 1 and 2 on the output composition α,

$$\sigma_1(\alpha) = \lambda\frac{(1+\sqrt{\rho\nu\alpha})}{\sqrt{\rho\nu\alpha}(1+\sqrt{\rho\nu\alpha})},\ \sigma_2(\alpha) = \lambda\frac{\sqrt{\rho\nu\alpha}(1+\sqrt{\rho\nu\alpha})}{(1+\sqrt{\rho\nu\alpha})} \tag{49}$$

To demonstrate the change in the ratio of unit production costs for products 1 and 2 when the output composition changes α, examine the difference,

$$\Delta(\alpha) = \sigma_1(\alpha) - \sigma_2(\alpha) \tag{50}$$

In terms of analysis, it is more convenient to consider the square of this difference,

$$\Delta^2(\alpha) = \left(\sigma_1(\alpha) - \sigma_2(\alpha)\right)^2 \tag{51}$$

The graph of function (51) is shown in figure 2 (right axis of ordinates).

The function (52) reaches its minimum at the point $\alpha'_0 = 1{,}619$. At this point $\sigma_1(\alpha_0) = \sigma_2(\alpha_0)$. On the left of the point α'_0, the unit cost of producing product 1 is greater than the unit cost of producing product 2, i.e., $\sigma_1(\alpha) > \sigma_2(\alpha)$. On the right of the point α'_0, on the contrary, the unit cost of production of product 2 is greater than the unit cost of production of product 1, i.e., $\sigma_1(\alpha) < \sigma_2(\alpha)$. As seen from the graph of the function $\Delta^2(\alpha)$, the difference in the unit cost of production of the product increases as the difference in output increases. At the same time, the unit cost of producing the product whose output decreases, increases, and the unit cost of producing the product whose output increases, decreases.

In other words, the increase in the rarity of a product leads to the increase in the cost of producing it, i.e., the increase in its value. Conversely, the decrease in the rarity of a product leads to the decrease in the cost of producing it, i.e., a decrease in its value. This law is consistent with the law of demand which states that the consumption of products decreases as their value increases. It can therefore be argued that demand, i.e., the subjective valuations of consumers, is shaped by the material conditions of production along the curve (51). Of course, this does not mean that consumers' spontaneous subjective evaluations cannot deviate from this curve, but under the influence of objective laws of production realization they eventually agree with it. In other words, curve (51) acts as a gravitational centre for spontaneous consumer evaluations, setting the natural mechanism of production movement. We have approached to the formulation of the law of formation of natural prices (value) of the products of production.

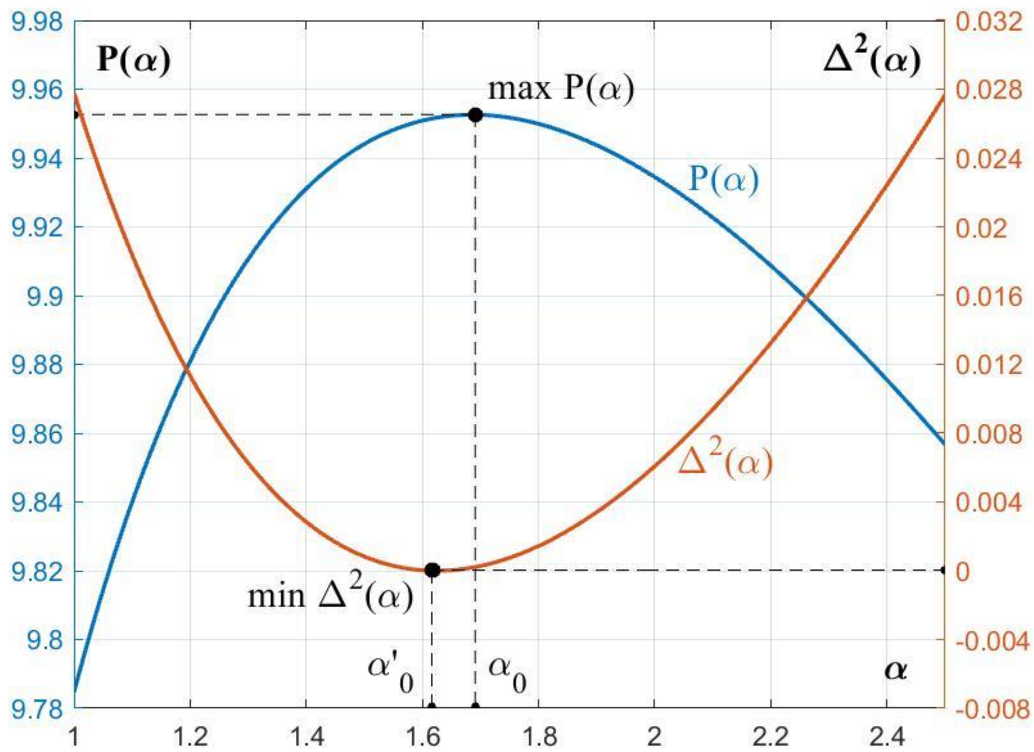

Figure 2. Dependencies of the values of total costs of the production system P and the square of the difference Δ^2 between the unit cost of production of products 1 and 2 on the output composition α.

6. Prices and distribution

We determine the prices of products on the basis of the distribution of income among their producers in proportion to the amount of capital they spend in production. In the coordinate system defined by the coefficients of matrix **A**, in terms of the notations we have introduced, this proportion would look as follows,

$$\frac{N}{\upsilon}\omega = \zeta \tag{52}$$

where $\omega = {}^{w_1}/_{w_2}$—the ratio of natural prices of products 1 and 2; ζ—the coefficient determining the distribution of income between producers of products 1 and 2. If each unit of capital employed in the production of products 1 and 2 receives the same income, then $\zeta = 1$. In general case,

$$\zeta = \frac{1 + r_1}{1 + r_2} \tag{53}$$

where r_1, r_2—the values of the profits obtained in the production of products 1 and 2, expressed in fractions of one. Let us replace in the proportion (52) the ratio of natural prices ω by the ratio of arbitrary prices γ. This substitution is necessary because in reality the quantities of products are measured in arbitrary rather than natural units. According to (7) and (10) we obtain,

$$\frac{N}{\upsilon} \times \frac{\gamma}{\nu} = \zeta \tag{54}$$

Then, substituting in (54) instead of N its value from (31), we finally have,

$$\rho\upsilon\gamma = \nu\zeta \tag{55}$$

From which it obviously follows,

$$\upsilon = \frac{\nu\zeta}{\rho\gamma} \tag{56}$$

Equation (56) establishes the relationship between the inputs of capital (matter) in production υ, the distribution of income ζ and the ratio of product prices γ. Substituting in equation (17) instead of υ its value from (56), we obtain the sought law of the relationship between outputs (α) and prices (γ) of products of production,

$$\rho\alpha\gamma^2\xi(\gamma) = \nu\pi(\alpha)\zeta \tag{57}$$

The graph of the relationship between the prices (γ) of the products of production and their output for the case $\zeta = 1$, $\rho = 1$, $\pi(\alpha) = 1$ and $\alpha \in [0{,}67,\ 2{,}5]$ is shown in figure 3.

We have obtained the law of commodity price formation γ depending on the given technological $\xi(\gamma)$, ν and $\pi(\alpha)$ resource base of production, taking into account market ρ and social ζ conditions of its implementation.

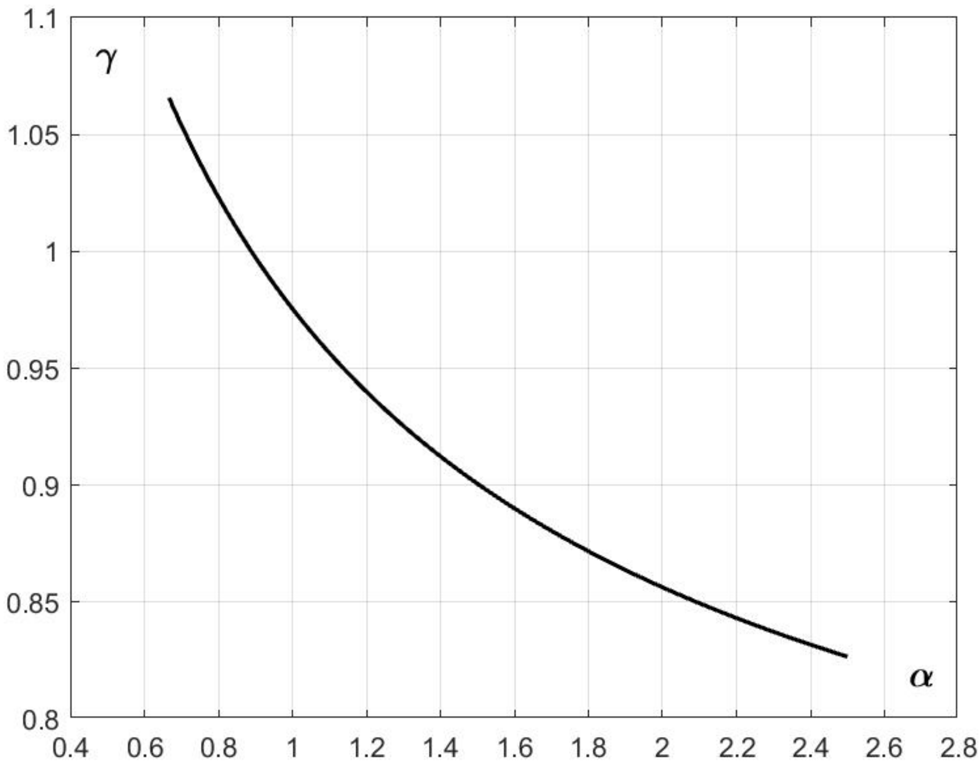

Figure 3. Law of natural price formation of products 1 and 2.

7. Co-production

We believe it is fundamentally important to show how the fundamental proposition of the above theory can be generalised to the case of multi-product industries of J. von Neumann's model.

Substantively, von Neumann's approach boils down to the following system of equations,

$$\begin{cases} q^T A + y = q^T B \\ Ap + v = Bp \end{cases} \tag{58}$$

where A and B—cost and output matrices of size respectively $m \times n$; $q-m$—dimensional vector of levels of "intensities" of production; $p-n$—dimensional vector of product prices; $y-n$—dimensional vector of final consumption; $v-m$—dimensional vector of income.

It is necessary to find the natural units of the quantity of the products of production, i.e., to construct for (58) a system of equations similar to (4). To do this, let us rewrite the second equation of system (58) for the case of the same output of all inputs used,

$$Ac = \lambda Bc \tag{59}$$

whence,

$$AB^{-1}c = \lambda c \tag{60}$$

Thus, all the conclusions we have outlined above can easily be generalised to the case of multi-product industries of the J. von Neumann model.

8. Conclusions

We have shown the nature of the material causes that determine the value of products, regardless of the subjective motives of supply and demand. The approach we propose, in line with the Russian tradition of economic thought (Abalkin, 2000; Kurz et al., 2000; Klukin, 2014), solves the fundamental problem of input-output analysis—the problem of explaining the relationship between natural and value units of product and production costs and, thus, completes the idea of W. W. Leontief's idea of constructing a naturalistic theory of value, free from the need to use notions of supply and demand (Kurz et al., 2007; Leontief, 1928). This decision is of fundamental importance for the development of cross-sectoral analysis, bringing the analytical possibilities of the empirical economics research to a new, previously inaccessible level.

In this article, we focused on the presentation of the general provisions of the naturalistic concept of value and did not set ourselves the goal of demonstrating the possibilities of constructing specific empirical calculation algorithms on its basis, since the issues related to this are not directly related to the development of the theoretical foundations of the objectivist analysis of production. It was of primary importance for us to show that an objectivist explanation of the nature of economic value is possible in principle. Development and verification of the empirical research tools is a separate serious scientific task, the solution of which goes far beyond the scope of the present work.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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