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Research article

Inequality, mobility, and growth

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Abstract: Many prior studies on inequality and growth have shown that inequality is harmful to growth. In contrast, employing an overlapping-generations model with missing capital markets, educational investment, and intergenerational mobility, this paper shows that inequality characterized by skill-biased technological change is good for growth.

Keywords: economic growth; inequality; intergenerational mobility; skill-biased technological change

JEL Codes: J31, J62, O30, O40

1. Introduction

The relationship between inequality and growth has been a major concern of macroeconomics. Most of the literature of the 1990s, e.g., Alesina and Rodrik (1994), Perotti (1996), and Persson and Tabellini (1994), theoretically and empirically shows that inequality negatively affects growth. On the other hand, Barro (2000) empirically shows that although higher inequality reduces growth in poorer countries, it encourages growth in richer countries. Although the literature on inequality and growth has expanded enormously in recent years, the positive relationship between inequality and growth in advanced economies remains insufficiently studied from a theoretical viewpoint. We therefore try to develop a theoretical model that can explain it.

In this paper, the relationship between inequality and growth is analyzed jointly with intergenerational mobility. Intergenerational mobility refers to the correlation of economic status between parents and their children. The reason why we use mobility is that inequality and mobility are strongly related. Inequality—in particular, wage inequality—has both a negative effect and a positive effect on educational decisions that determine, to a substantial degree, future economic status. The negative effect is unequal opportunity: rich parents invest more in the education of their children than poor ones; hence, inequality persists across generations and mobility is low. On the positive side, greater wage inequality (meaning a greater skill premium) increases incentives for education acquisition and therefore increases upward intergenerational mobility flow (the measure of poor-born children that become rich) and decreases downward intergenerational mobility flow (the measure of rich-born children that become poor). Empirical studies have shown both a positive correlation between inequality and mobility (Rodriguez et al., 2008) and a negative correlation (Andrews and Leigh, 2009). Theoretically, Hassler et al. (2007) give an interesting and tractable explanation for these findings. This paper extends the scope of investigation to the relationship between three factors: inequality, mobility, and growth.

To serve our purpose, our model is based on the earlier work of Maoz and Moav (1999), which studied the dynamic relationship between wage inequality, mobility, and growth. The defining features of their model can be summarized as follows. First, there is heterogeneity in the learning ability of individuals that can be represented as differences in the cost of acquiring the education necessary to become an educated worker. Second, the decision to invest in education is determined by the net benefit of acquiring an education. The threshold level of the cost of education below which individuals will seek to acquire an education is endogenous. Third, capital markets are assumed to be imperfect: educational loans are unavailable, and individuals cannot accumulate wealth. Therefore, some children may not be able to acquire an education.

We assume that wage inequality between educated and uneducated workers widens because of skill-biased technological change, which is not considered in Maoz and Moav (1999), but is a topic of much recent research. There is a consensus that recent technological progress such as the development of information technology raises the productivity of educated workers relative to uneducated workers, increasing the wage gap between them in advanced countries. In addition, we introduce education subsidies, since many governments offer these. Using this model, we are able to show that higher inequality increases upward mobility and decreases downward mobility, and thus increases the number of educated workers and thereby growth.

The remainder of the paper is organized as follows: Section 2 presents the model; Section 3 analyzes the evolution of the economy; Section 4 examines the impact of inequality; and Section 5 summarizes the results.

2. The model

2.1. Production sector

In the proposed model, the economy produces a single homogeneous good under conditions of perfect competition with a CES (constant elasticity of substitution) production function that uses educated and uneducated labor as input. The goods can be used for either consumption or investment in education. We define the number of educated and uneducated workers in period t as E_t and U_t, respectively. We normalize the total number of individuals supplying labor to one and assume that each individual supplies one unit of labor inelastically. Therefore, E_t and $U_t = 1 - E_t$ represent the amounts of educated and uneducated labor, respectively.

The production function takes the form

$$Y_t = F\left(A, E_t, U_t\right), \tag{1}$$

where Y_t is output in period t and $A>1$ is a productivity parameter that is assumed to affect educated labor.

Let us assume that educated and uneducated labor are perfect substitutes.[1] Then, Equation (1) takes the following specific form:

$$Y_t = AE_t + U_t = (A-1)E_t + 1, \tag{2}$$

Wages are given by the marginal productivities. If we define w_t^e (w_t^u) as the wage of an educated (uneducated) worker in period t, then

$$w_t^e = w^e = A, \qquad w_t^u = w^u = 1, \tag{3}$$

where the superscripts e and u indicate educated and uneducated, respectively. From Equation (3), the ratio of educated to uneducated wages, w^e/w^u, which is our measure of wage inequality, is A.

2.2. Individuals

Individuals live for two periods in overlapping generations (see Figure 1). There is no population growth, and in every time period a generation of size one is born. Individuals are heterogeneous with respect to their innate learning ability and parental education level.

In the first period of life (childhood), the individual is a consumer; during this period, he/she decides whether to acquire education and does not work. The consumption and education costs are financed by a bequest from the parent and through education subsidies provided by the government. Education subsidies are available to all children who choose to acquire education. Payment is proportional to the education costs of the child and is financed by a proportional tax on wages. In the second period of life (adulthood), the individual works and makes a bequest of all wealth to his/her child. The individual can work as an educated worker if and only if he/she acquires education in the first period of life.

Individuals gain utility from their consumption in the first period of life and their bequest to their child in the second period of life. The parents' bequest motive takes the form of *paternalistic altruism*: they are concerned for the wealth of their children, apart from their own consumption, in the second period of life. All individuals have identical preferences:

$$u_t^i = \log c_t^i + \log x_{t+1}^i \tag{4}$$

[1].This differs from Maoz and Moav's (1999) model, which uses the Cobb-Douglas production function. If we use the Cobb-Douglas one such that the inputs are complements, the analysis becomes more complicated, since the number of educated workers in the future affects the individual's decision regarding the acquisition of education, as well as intergenerational mobility. For details, see footnote 4.

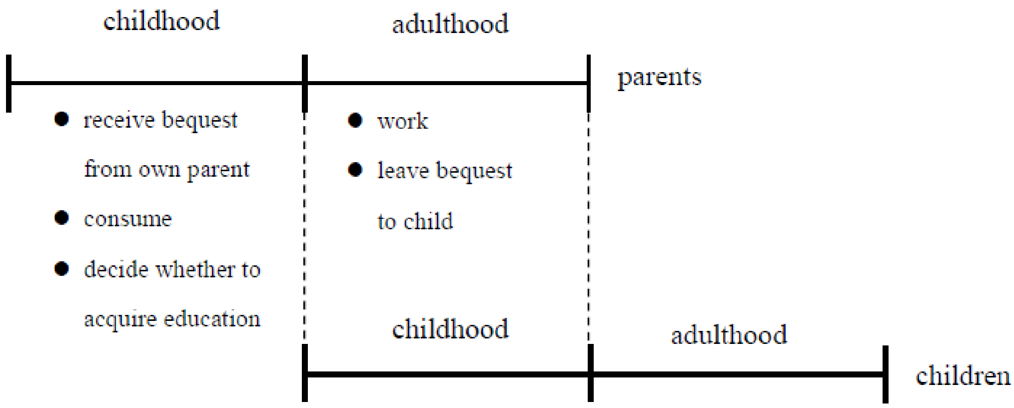

Figure 1. Timing of events.

where c_t^i is consumption in period t and x_{t+1}^i is the bequest to the child born in period $t+1$.

Let θ^i, $s_t \in (0,1)$, and $\tau \in (0,1)$ denote the education cost of individual i, the rate of education subsidy in period t, and a proportional tax rate on wages, respectively. θ^i represents the learning ability of the child in that the higher the ability, the lower the θ^i cost.[2] These costs are uniformly distributed in the interval $(\underline{\theta}, \overline{\theta})$, where $\underline{\theta} \geq 0$, and the ability of a child does not depend on the ability of his/her parent.

We assume that capital markets are imperfect: Educational loans are unavailable and individuals cannot accumulate wealth.[3] If an individual acquires education, then his/her budget constraints are

$$c_t^i + (1-s_t)\theta^i = x_t^i, \qquad x_{t+1}^i = (1-\tau)w^e, \tag{5}$$

where x_t^i is the bequest from the individual's parent when he/she is born in period t. If he/she does not acquire education, then the constraints are

$$c_t^i = x_t^i, \qquad x_{t+1}^i = (1-\tau)w^u. \tag{6}$$

Young individual i will acquire education if and only if the utility derived from investing in education, $\log\left[x_t^i - (1-s_t)\theta^i\right] + \log(1-\tau)w^e$, is greater than or equal to the utility derived from not investing in education, $\log x_t^i + \log(1-\tau)w^u$:

$$\log\left[x_t^i - (1-s_t)\theta^i\right] + \log(1-\tau)w^e \geq \log x_t^i + \log(1-\tau)w^u. \tag{7}$$

[2] In reality, higher-ability children learn faster than lower-ability children. Thus, from a theoretical viewpoint, the education cost required for higher-ability children to become educated is lower than the cost for lower-ability children.

[3] For example, Getachew (2016) remarks that even in advanced economies, there can be a borrowing constraint because of the substantially high cost of administrating credit.

From Equation (7), we obtain the critical value of the education cost for individual i of generation t, $\hat{\theta}_t^i$, such that only those individuals with an education cost below the threshold, $\theta^i \le \hat{\theta}_t^i$, acquire education.

2.3. Government

We assume that the individual's learning ability and educational investment are observable through school education: the government knows θ^i and $\hat{\theta}_t^i$. Then, the government's budget constraint is

$$\int_{\underline{\theta}}^{\hat{\theta}_t^i} s_t \theta^i f\left(\theta^i\right) d\theta^i = \frac{s_t}{\overline{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\theta_t^i} \theta^i d\theta^i = \tau Y_t , \tag{8}$$

where $f\left(\theta^i\right)$ is the density function of θ^i.

Equation (8) can be rewritten as follows:

$$s_t = \frac{2\left(\overline{\theta} - \underline{\theta}\right)\tau Y_t}{\left(\hat{\theta}_t^i\right)^2 - \underline{\theta}^2} . \tag{9}$$

3. The evolution of the economy

From Equation (7), using Equation (2) and Equation (3), and (9), $\hat{\theta}_t^i$ is defined as the solution of

$$\left(\hat{\theta}_t^i\right)^3 - x_t^i\left(1 - \frac{1}{A}\right)\left(\theta_t^i\right)^2 - \left[\underline{\theta}^2 + 2\left(\overline{\theta} - \underline{\theta}\right)\tau\left\{\left(A-1\right)E_t + 1\right\}\right]\theta_t^i + x_t^i\left(1 - \frac{1}{A}\right)\underline{\theta}^2 = 0 . \tag{10}$$

In order to simplify the analysis, the remainder of the paper makes the following two assumptions.

Assumption 1 $\underline{\theta} = 0$.

This assumption implies that the acquisition of education does not involve any cost for the highest ability individual. Under this assumption, $\hat{\theta}_t^i$ is given by

$$\hat{\theta}_t^i = \frac{x_t^i\left(1 - 1/A\right) \pm \sqrt{\left(x_t^i\right)^2\left(1 - 1/A\right)^2 + 8\overline{\theta}\tau\left\{\left(A-1\right)E_t + 1\right\}}}{2} . \tag{11}$$

The bequest from his/her parent, x_t^i, is $x_t^e = \left(1-\tau\right)w^e = \left(1-\tau\right)A$ if the individual is born to an educated parent, and $x_t^u = \left(1-\tau\right)w^u = 1-\tau$ if born to an uneducated parent. Suppose that $\hat{\theta}_t^e$ ($\hat{\theta}_t^u$) is the critical value of the education cost for the individual born to an educated (uneducated) parent.

The temporal evolution of E_t is explained by two types of intergenerational mobility: upward and downward. Upward mobility means that an individual born to an uneducated parent acquires education, whereas downward mobility means that an individual born to an educated parent does not acquire education. The level of upward mobility in period t, UM_t, and the level of downward mobility in period t, DM_t, are respectively given by

$$UM_t = \left(1-E_t\right) F\left(\hat{\theta}_t^u\right) = \left(1-E_t\right)\frac{\hat{\theta}_t^u}{\overline{\theta}}, \qquad DM_t = E_t\left[1-F\left(\hat{\theta}_t^e\right)\right] = E_t\frac{\overline{\theta}-\hat{\theta}_t^e}{\overline{\theta}}, \tag{12}$$

where $F\left(\cdot\right)$ is the cumulative distribution function of θ^i. E_t will increase if and only if UM_t exceeds DM_t. Therefore, the dynamics of E_t can be expressed as $E_{t+1} - E_t = UM_t - DM_t$. The upward and downward mobility levels are equal in the steady state: $UM_t = DM_t$. We define the steady-state level of E_t as E^*, where $E_t = E_{t+1} = E^*$.

We make the following assumption in order to avoid a condition of no economic development.

Assumption 2 $\hat{\theta}_t^i > 0$.

If $\hat{\theta}_t^i = 0$, the most talented child will be indifferent to education investment, and no others will choose to invest. When $\hat{\theta}_t^i < 0$, no one will invest in acquiring education. Thus, if $\hat{\theta}_t^i \le 0$, all children born to educated parents move downward and all children born to uneducated parents remain uneducated. Since only downward mobility occurs, the number of educated workers approaches 0, which is the steady-state level. This case might be expressed as a *poverty trap*, where the number of educated workers and output decrease to their lowest levels of 0 and 1, respectively.

Based on Assumption 2, Equation (11) is rewritten as[4]

$$\hat{\theta}_t^i = \frac{x_t^i\left(1-1/A\right) + \sqrt{\left(x_t^i\right)^2\left(1-1/A\right)^2 + 8\overline{\theta}\tau\left\{\left(A-1\right)E_t+1\right\}}}{2}. \tag{13}$$

Equation (13) implies that since education subsidies, $\tau Y_t = \tau\left\{\left(A-1\right)E_t+1\right\}$, increase with E_t, $\hat{\theta}_t^i$ also increases with E_t.

From Equation (13), considering $x_t^e = \left(1-\tau\right)A$ and $x_t^u = 1-\tau$, the thresholds for children of educated and uneducated parents are

$$\hat{\theta}_t^e = \frac{\left(1-\tau\right)A\left(1-1/A\right) + \sqrt{\left\{\left(1-\tau\right)A\right\}^2\left(1-1/A\right)^2 + 8\overline{\theta}\tau\left\{\left(A-1\right)E_t+1\right\}}}{2}, \tag{14}$$

$$\hat{\theta}_t^u = \frac{\left(1-\tau\right)\left(1-1/A\right) + \sqrt{\left(1-\tau\right)^2\left(1-1/A\right)^2 + 8\overline{\theta}\tau\left\{\left(A-1\right)E_t+1\right\}}}{2}, \tag{15}$$

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[4] If we suppose a Cobb-Douglas production function, $Y_t = AE_t^{1-\alpha}U_t^{\alpha}$, then w^e and w^u in Equation (7) are replaced with $w_{t+1}^e = \left(1-\alpha\right)A\left[\left(1-E_{t+1}\right)/E_{t+1}\right]^{\alpha}$ and $w_{t+1}^u = \alpha A\left[\left(1-E_{t+1}\right)/E_{t+1}\right]^{\alpha-1}$, respectively. Since E_{t+1} affects $\hat{\theta}_t^i$, the dynamics of mobility become more complicated.

respectively. Since $x_t^e > x_t^u$, it follows that $\hat{\theta}_t^e > \theta_t^u$. This implies that children of educated parents are more likely to acquire education than children of uneducated parents.

Because $\hat{\theta}_t^e$ and $\hat{\theta}_t^u$ increase with E_t, $UM_t = (1-E_t)\hat{\theta}_t^u / \bar{\theta}$ and $DM_t = E_t\left(\bar{\theta} - \hat{\theta}_t^e\right) / \bar{\theta}$ are concave. Examples of dynamic paths of UM_t, DM_t, and E_{t+1} are illustrated in Figure 2.

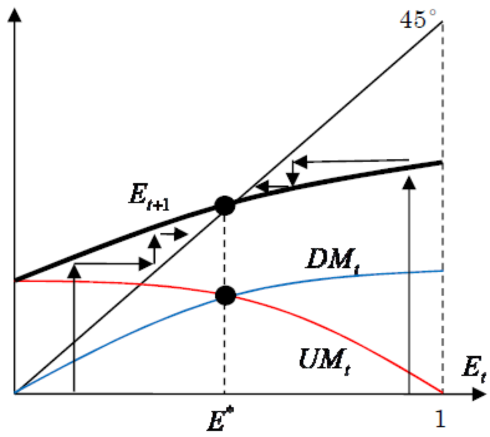

Figure 2. Convergent dynamics: $E^* \in (0,1)$.

Figure 2 shows that E_t approaches its interior steady-state value, $E^* \in (0,1)$: in the range $E_t \in \left[0, E^*\right)$ $\left(E_t \in \left(E^*, 1\right]\right)$, E_t increases (decreases) monotonically toward E^* because $UM_t > DM_t$ $\left(UM_t < DM_t\right)$.

4. Effects of inequality

Next, we consider how higher wage inequality caused by skill-biased technological change affects mobility and growth. In our model, skill-biased technological change is described by an increase in productivity parameter A. First, an increase in A increases x_t^e or $\hat{\theta}_t^e$. Second, from Equation (13), $\frac{\partial \hat{\theta}_t^i}{\partial (1-1/A)} > 0$; higher wage inequality (an increase in $1-1/A$) affects $\hat{\theta}_t^i$ positively. Therefore, higher wage inequality serves as an additional incentive to invest in education. Third, an increase in A increases education subsidies, as well as $\hat{\theta}_t^i$. We summarize this in the following proposition.

Proposition 1 *Higher wage inequality increases* $\hat{\theta}_t^e$ *and* $\hat{\theta}_t^u$*: it increases* UM_t *and decreases* DM_t*, and therefore increases* E_{t+1}*, that is, growth.*

If wage inequality is sufficiently high such that $DM_t = 0$ holds at $E_t = 1$, as illustrated in Figure 3, E_t increases monotonically toward its corner steady-state value, $E^* = 1$, as time passes because $UM_t > DM_t$ for every value of E_t.

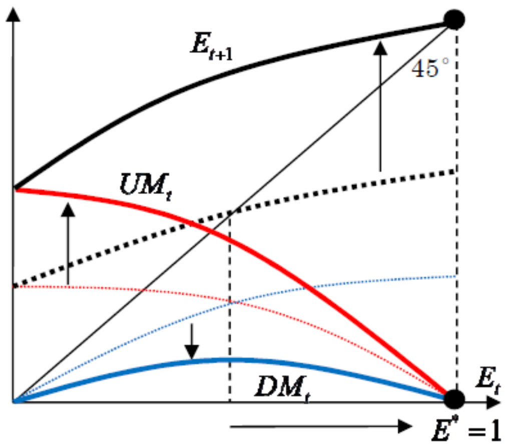

Figure 3. The effect of an increase in A (convergent dynamics: $E^* = 1$).

5. Conclusion

By introducing skill-biased technological change and education subsidies into the model by Maoz and Moav (1999), this paper presents a simple theoretical model to explain the positive relationship between inequality and growth observed in advanced countries.

The analysis has several limitations from being based on a simplified model. The first is that the assumption that the government has full information about the innate ability of a child is unrealistic. This raises the question as to how education subsidies in our model should be modified when the government cannot observe an individual's ability.

A second limitation is that wages in our model are constant regardless of ability. Mobility may affect the average ability in educated and uneducated groups, and hence change wages in each of the groups if wages depend on ability.

These are subjects for further research.

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Conflict of interest

The author declares no conflict of interest in this paper.

References

Alesina A, Rodrik D (1994) Distributive Politics and Economic Growth. *Q J Econ* 109: 465–490.
Andrews D, Leigh A (2009) More Inequality, Less Social Mobility. *Appl Econ Lett* 16: 1489–1492.
Barro RJ (2000) Inequality and Growth in a Panel of Countries. *J Econ Growth* 5: 5–32.
Getachew YY (2016) Credit constraints, growth and inequality dynamics. *Econ Model* 54: 364–376.
Hassler J, Rodríguez Mora JV, Zeira J (2007) Inequality and mobility. *J Econ Growth* 12: 235–259.

Maoz YD, Moav O (1999) Intergenerational mobility and the process of development. *Econ J* 109: 677–697.

Perotti R (1996) Growth, Income Distribution, and Democracy: What the Data Say. *J Econ Growth* 1: 149–187.

Persson T, Tabellini G (1994) Is Inequality Harmful for Growth? *Am Econ Rev* 84: 600–621.

Rodriguez JP, Rodriguez JG, Salas R (2008) A study on the relationship between economic inequality and mobility. *Econ Lett* 99: 111–114.

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