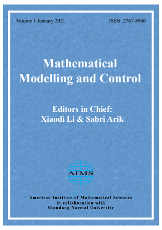
Mathematical Modelling and Control

https://www.aimspress.com/journal/mmc



MMC, 5(1): 121–130.
DOI: 10.3934/mmc.2025009
Received: 01 May 2024
Revised: 31 October 2024
Accepted: 10 November 2024
Published: 31 March 2025

Research article

Position tracking control of nonholonomic mobile robots via H_∞-based adaptive fractional-order sliding mode controller

Naveen Kumar[1,2] **and Km Shelly Chaudhary**[1,3,*]

[1] Department of Mathematics, National Institute of Technology Kurukshetra, Kurukshetra 136119, Haryana, India

[2] Department of Mathematics, Mahatma Jyotiba Phule Rohilkhand University Bareilly, Bareilly 243006, Uttar Pradesh, India

[3] Department of Mathematics, Meerut College Meerut, Meerut 250002, Uttar Pradesh, India

* **Correspondence:** Email: shelly_62000004@nitkkr.ac.in.

Abstract: In this article, the position tracking control problem of a nonholonomic wheeled mobile robot with system uncertainties and external disruptions is examined. In the design control technique, a fractional-order sliding surface is presented for faster response of the dynamical system's states. Based on this sliding surface, a robust H_∞-based adaptive fractional-order sliding mode controller is developed to effectively handle the system's uncertainty and external disruptions. In the structure of the designed control scheme, the radial basis function neural network is utilized to reproduce the nonlinear function of the dynamical structure. The controller's H_∞ part compensates for the negative effects of the external disturbances and uncertainties robustly. The Lyapunov approach is used to determine the stability of the dynamical system. Furthermore, a numerical simulation analysis is carried out to show the effectiveness of the proposed control technique.

Keywords: nonholonomic mobile robots; H_∞-based tracking control; fractional order controllers; neural networks

1. Introduction

Recently, there has been a lot of interest in the position tracking control problem of nonholonomic wheeled mobile robots because of their wide range of applications in the medical areas, organizations, military operations, and many other sectors [1–4]. The dynamical structure of nonholonomic mobile robots are extremely nonlinear, coupled, and time-varying. To manage these nonlinear properties, uncertainties, and outside disturbances, a variety of classical controllers have been developed in the literary texts to govern these dynamical systems, including PID controllers, back-stepping based controllers, adaptive controllers, sliding mode-based controllers, and many others (see [5–10]).

Among these, sliding mode controllers (SMC) (see the works [11–13]) are the most often used control technique due to their inherent robustness against outside disturbances and uncertainty. SMC employs a sliding surface to guarantee that tracking errors converge toward zero. The sliding mode control scheme's inherent adaptability feature is that it operates irrespective of the system's dynamics whenever the system is working on the sliding manifold. In SMCs, linear and nonlinear sliding surfaces are increasingly used for improved controller performance. In the literature, linear sliding surfaces have been used to introduce linear sliding mode controllers (LSMC) [14]. In LSMC, only asymptotic convergence of dynamical states is confirmed but there is no evidance that analyse the fixed time convergence property of dynamical system so to resolve this issue, a terminal sliding mode controllers (TSMC) [15] have been presented using non-linear sliding manifolds. In [16], a fixed-time non-singular fast TSM control for wheeled mobile robots was presented using a disturbance observer.

For efficient performance of the dynamical system, various combinations of sliding mode controllers with fractional calculus have been reported in the literature for improved and accurate controller performance. Fractional-order controllers perform better than integer-order controllers due to their faster order convergence. Applying the fractional-order derivative to LSMC, referred to as the fractional-order sliding mode controller (FoSMC) [17, 18], is the first step in the work on integrating the fractional-order derivative with SMC. The tracking capability of these controllers is better than that of conventional sliding mode controllers due to their higher-order convergence. Eray and Tokat in [19] discusses the design of FoSMC for the robot manipulator trajectory tracking problem with a time-varying sliding surface. The authors of this study demonstrate the tracking errors' asymptotic convergence toward their system states. In [20], a fractional adaptability rule for a sliding mode control approach for a MIMO nonlinear dynamical system was presented while in [17], a linked FoSMC strategy using obstacle avoidance for the control of a four-wheeled steerable mobile robot was designed. A new fractional-order global sliding mode control method for these systems under outside disruptions is presented in [21].

Although fractional-order sliding mode controllers give efficient responses to the dynamical system's states, the negative influence of uncertainties and external disturbances causes a degrading performance of the overall system. So, to overcome these aspects, H_∞-based robust controllers [22] have been designed in the literature. An H_∞-based position tracking control method for robot manipulators employing a wavelet neural network-based TSMC was presented by Panwar [23]. The integration of the H_∞-based controller with sliding mode controller gave an outstanding performance of the dynamical system with controlled gain values and significantly faster response of system states towards their trajectories.

Inspired by the prior work, this study offers a novel combination of H_∞-based controllers, neural networks, and fractional-order SMC controllers for enhanced performance of nonholonomic wheeled mobile robots. A fractional-order sliding manifold is presented first in the control methodology, and then an H_∞-based adaptive fractional-order sliding mode novel controller is designed for the control of nonholonomic wheeled mobile robots with system uncertainties and external disturbances. The main contribution of the presented work is as follows:

(1) The position tracking issue of a nonholonomic wheeled mobile robotic system in the presence of outside disruptions and system uncertainty is examined.

(2) An efficient combination of a fractional-order sliding mode control technique with neural network-based H_∞ trajectory tracking control approach is proposed based on a fractional-order sliding manifold.

(3) The stability of the dynamical system and the resulting convergence of the position's tracking errors toward their equilibrium are examined by using the Lyapunov stability criterion.

(4) A comparative simulation study is conducted on a 3-dof nonholonomic wheeled mobile robot to analyse the performance of the the designed technique with the existing control techniques.

This paper's remaining sections are organized as follows: A dynamical model for a nonholonomic wheeled mobile robot is provided in Section 2. The control technique is given in Section 3, and the stability analysis is presented in Section 4. A simulation study is presented in Section 5, and the article is concluded in Section 6.

2. Dynamic model of nonholonomic wheeled mobile robot

The Euler-Lagrange equation for nonholonomic wheeled mobile robot dynamics is stated as

$$M(\theta)\ddot{\theta} + V_Q(\theta, \dot{\theta})\dot{\theta} + F(\dot{\theta}) + \tau_d(t) = B(\theta)U + Q^T(\theta)\lambda_Q, \quad (2.1)$$

where

$$\theta = [x, y, \Theta]^T \in R^{3\times1}$$

denotes the cordinates for the mobile base,

$$M(\theta) \in R^{3\times3}$$

denotes the inertia matrix,

$$V_Q(\theta, \dot{\theta}) \in R^{3\times3}$$

denotes the centripetal or coriolis matrix,

$$F(\dot{\theta}) \in R^{3\times 1}$$

denotes the friction to the dynamical system,

$$\tau_d(t) \in R^{3\times 1}$$

denotes the bounded disturbance,

$$B(\theta) \in R^{3\times 2}$$

transforms the states of the wheeled mobile robot in the earth's reference frame,

$$U \in R^{3\times 1}$$

denotes the control torque input,

$$Q^T(\theta) \in R^{3\times 1}$$

denotes the constraint matrix associated with

$$\lambda_Q \in R,$$

Langrange multiplier.

The wheeled mobile robots are subjected to the following nonholonomic kinematic constraint:

$$Q(\theta)\dot{\theta} = 0. \tag{2.2}$$

This constraint is the limitations on the dynamical equation of the wheeled mobile robot system to the manifold

$$\Im_Q = \{(\theta, \dot{\theta}) | Q(\theta)\dot{\theta} = 0\}.$$

From Eq (2.2), we can obtain a full-rank matrix

$$T(\theta) \in R^{3\times 2}$$

as

$$T^T(\theta)Q^T(\theta) = 0. \tag{2.3}$$

From the constraints given in Eqs (2.2) and (2.3), we have a new vector

$$\chi = [y, \Theta]^T \in R^2,$$

that satisfies the following condition:

$$\dot{\theta} = T(\theta)\dot{\chi}. \tag{2.4}$$

So, by controlling this new vector

$$\chi = [y, \Theta]^T \in R^2$$

in the task space, the whole position and orientation of the wheeled mobile robot can be controlled.

Differentating Eq (2.4) w.r.t $'t'$, we get

$$\ddot{\theta} = T(\theta)\ddot{\chi} + \dot{T}(\theta)\dot{\chi}. \tag{2.5}$$

Utilizing Eqs (2.4) and (2.5) in Eq (2.1) and multiplying the resultant equation by T^T, we obtain

$$\bar{M}_g\ddot{\chi} + \bar{V}_g\dot{\chi} + \bar{F}_g + \bar{\tau}_{gd} = T^T U \tag{2.6}$$

with

$$\bar{M}_g = T^T M(\theta)T, \quad \bar{F}_g = T^T F(\dot{\theta}), \quad \bar{\tau}_{gd} = T^T \tau_d,$$
$$\bar{V}_g = T^T M(\theta)\dot{T} + T^T V_Q(\theta, \dot{\theta})T.$$

Let the above-constrained dynamics Eq (2.6) of a nonholonomic wheeled mobile robot satisfy the following properties.

Property 1. $\bar{M}_g$ is a bounded, symmetric, and positive-definite matrix.

Property 2. The dynamical terms

$$\|\bar{F}_g\| \leq g_2$$

and

$$\|\bar{\tau}_{gd}\| \leq g_3$$

are bounded for some constants $g_i > 0$ for $i = 2, 3$ with

$$g_3 \in L^2[0, t]$$

for $t > 0$.

Property 3. The term

$$\bar{B} = (\frac{1}{2}\dot{\bar{M}}_g - \bar{V}_g)$$

assures the skew-symmetric property, i.e.,

$$A^T \bar{B} A = 0 \quad \forall A \in R^n.$$

Assumption 1. All the Jacobian matrices are uniformly bounded and continuous if

$$\chi = [y, \Theta]^T \in R^2$$

is consistent and uniformly bounded.

3. Controller structure

3.1. Sliding surface

Consider the following sliding surface:

$$r(t) = D^{\alpha+1}\bar{\chi}(t) + \dot{\bar{\chi}}(t) + A\bar{\chi}(t), \tag{3.1}$$

where

$$0 < \alpha < 1, \quad \bar{\chi}(t) = \chi_d(t) - \chi(t)$$

gives the position tracking errors,

$$\chi_d(t) \in R^2$$

gives the desired trajectory,

$$A = diag[A_1, A_2] \in R^{2\times 2},$$

where A_1, A_2 are positive values, and

$$r(t) = [r_1(t), r_2(t)]^T \in R^2$$

is a sliding variable. Differentiating the above equation, we get

$$\dot{r}(t) = D^{\alpha+2}\bar{\chi}(t) + \ddot{\bar{\chi}}(t) + A\dot{\bar{\chi}}(t).$$

Putting

$$\ddot{\bar{\chi}}(t) = \ddot{\chi}_d(t) - \ddot{\chi}(t)$$

into above equation, we will get

$$\dot{r}(t) = D^{\alpha+2}\bar{\chi}(t) + \ddot{\chi}_d(t) - \ddot{\chi}(t) + A\dot{\bar{\chi}}(t). \tag{3.2}$$

Putting the value of $\ddot{\chi}(t)$ from Eq (2.6), Eq (3.2) is reduced to

$$\begin{aligned}\dot{r}(t) =& D^{\alpha+2}\bar{\chi}(t) + \ddot{\chi}_d(t) + A\dot{\bar{\chi}}(t) \\ &+ \bar{M}_g^{-1}(\bar{V}_g\dot{\chi} + \bar{F}_g + \bar{\tau}_{gd} - T^T U).\end{aligned} \tag{3.3}$$

The reduced dynamical equation in terms of $r(t)$ is given as

$$\begin{aligned}\bar{M}_g\dot{r}(t) =& \bar{M}_g(D^{\alpha+2}\bar{\chi}(t) + \ddot{\chi}_d(t) + A\dot{\bar{\chi}}(t)) \\ &+ \bar{V}_g\dot{\chi} + \bar{F}_g + \bar{\tau}_{gd} - T^T U.\end{aligned} \tag{3.4}$$

Taking

$$\dot{\chi} = \dot{\chi}_d - \dot{\bar{\chi}}$$

and using Eq (3.1), Eq (3.3) is reduced to the following equation:

$$\begin{aligned}\bar{M}_g\dot{r}(t) =& \bar{M}_g(D^{\alpha+2}\bar{\chi}(t) + \ddot{\chi}_d(t) + A\dot{\bar{\chi}}(t)) + \bar{V}_g(\dot{\chi}_d - r(t) \\ &+ D^{\alpha+1}\bar{\chi}(t) + A\bar{\chi}(t)) + \bar{F}_g + \bar{\tau}_{gd} - T^T U.\end{aligned}$$

Rewriting the above equation, we get

$$\bar{M}_g\dot{r}(t) = h(t) - \bar{V}_g r(t) + \bar{F}_g + \bar{\tau}_{gd} - T^T U, \tag{3.5}$$

where

$$\begin{aligned}h(t) =& \bar{M}_g(D^{\alpha+2}\bar{\chi}(t) + \ddot{\chi}_d(t) + A\dot{\bar{\chi}}(t)) \\ &+ \bar{V}_g(\dot{\chi}_d + D^{\alpha+1}\bar{\chi}(t) + A\bar{\chi}(t))\end{aligned}$$

is the nonlinear function of the dynamical structure that will be approximated by a radial basis function neural network (RBFNN). The architecture of the RBFNN is given in the next subsection.

3.2. RBFNN

Due to the adaptive nature of the RBFNN, it is utilized to reproduce the non-linear part of the manipulator's dynamics. Let the function approximation on a simply connected compact set of the continuous function $h(t)$ be

$$h(t) = H^T\xi(t) + \psi(t), \tag{3.6}$$

where,

$$H \in R^{N\times b}$$

demonstrates the weight matrix, and it will be updated online in an adaptive manner

$$\xi(\cdot) : R \to R^N$$

denotes predefine basis array,

$$\psi(t) : R \to R^b$$

denotes reconstruction error, and N denotes the number of nodes used in the structure of neural networks. So, we have

$$||\psi(t)|| < \psi_N$$

for some $\psi_N > 0$.

For larger values of N, $\psi(t)$ may be reduced to be very small. The RBFNN uses the Gaussian function $\xi(t)$, which is given by

$$\xi_i(t) = exp(\frac{-||t - c_i||^2}{\sigma_i^2}), \tag{3.7}$$

where $i = 1, 2, ..., N$ and c_i, σ_i represent the center vectors and width vectors, respectively.

Putting the value of $h(t)$ from (3.6) into (3.5), the reduced-error dynamical equation is then given by

$$\bar{M}_g \dot{r}(t) = H^T \xi(t) + \psi(t) - \bar{V}_g r(t) + \bar{F}_g + \bar{\tau}_{gd} - T^T U. \tag{3.8}$$

In addition to this, it is not sufficient to use only the neural network architecture to handle the dynamical system. The effect of environmental constraints that are unknown can be very serious for the nonholonomic wheeled mobile robot system. Therefore, a switching control law during the sliding phase should be added to control the dynamical system that ensures robustness against external disturbances. To construct the robust dynamical system's control technique, an H_∞-based tracking controller is presented in this work. The structure of the H_∞-based tracking control technique is given in the next subsection.

3.3. *H_∞-based tracking control technique*

To attenuate the effects caused by unmodeled dynamics, disturbances, and approximate errors, the advantages of the H_∞-based tracking control technique are utilized in this work. The H_∞-based tracking control technique is a robust technique and the efficient combination of this technique with neural networks and fractional-order SMC controllers give enhanced performance of the wheeled mobile robot with less chattering in the control phase.

To impose the H_∞-based tracking control technique on the dynamical system, let the overall system satisfy the given condition:

$$\begin{aligned}\int_0^{\mathcal{T}} \bar{\chi}^T P \bar{\chi} dt \leq & r^T(0) \bar{M}_{0g} r(0) + \bar{\chi}^T(0) Q \bar{\chi}(0) \\ & + \int_0^{\mathcal{T}} \pi^2 ||g_3||^2 dt, \end{aligned} \tag{3.9}$$

where P, Q are positive definite matrices with predescribed attenuation level $\pi > 0$ and $\mathcal{T} \in (0, \infty)$.

3.4. *H_∞-based adaptive fractional order sliding mode controller*

To obtain the reference trajectories, the proposed H_∞-based adaptive fractional-order sliding mode controller is given by

$$T^T U = \hat{H}^T \xi(t) + P_1 r(t) + P_2 sign(r(t)) + \Delta, \tag{3.10}$$

where

$$\Delta = Q\bar{\chi} + \frac{r(t)}{2\pi^2},$$

and P_1 and P_2 are positive-definite gain matrices.

4. Stability analysis

Theorem 4.1. The system states of the reduced-error dynamics of the nonholonomic wheeled mobile robot system as given in Eq (3.8) converges toward zero along with the boundedness of signals if the controller is chosen as described in Eq (3.10) with the update law as follows:

$$\dot{\hat{H}} = \Lambda_H \xi(t) r(t)^T, \tag{4.1}$$

where

$$\Lambda_H = \Lambda_H^T \in R^{N \times N}.$$

Proof. Consider the candidate Lyapunov function as

$$V = \frac{1}{2} r(t)^T \bar{M}_g r(t) + \frac{1}{2} tr(\tilde{H}^T \Lambda_H^{-1} \tilde{H}) + \frac{1}{2} \bar{\chi}^T Q \bar{\chi}, \tag{4.2}$$

where

$$\tilde{H} = H - \hat{H}.$$

Differentating Eq (4.2) w.r.t. $'t'$, we get

$$\begin{aligned}\dot{V} =& \frac{1}{2} r(t)^T \dot{\bar{M}}_g r(t) + r(t)^T \bar{M}_g \dot{r}(t) \\ & + tr(\tilde{H}^T \Lambda_H^{-1} \dot{\tilde{H}}) + \bar{\chi}^T Q \dot{\bar{\chi}}. \end{aligned} \tag{4.3}$$

Taking the value of $\bar{M}_g \dot{r}(t)$ from Eq (3.8) with

$$\dot{\tilde{H}} = -\dot{\hat{H}},$$

we have

$$\begin{aligned}\dot{V} =& \frac{1}{2} r(t)^T \dot{\bar{M}}_g r(t) + r(t)^T (H^T \xi(t) + \psi(t) - \bar{V}_g r(t) + \bar{F}_g \\ & + \bar{\tau}_{gd} - T^T U) - tr(\hat{H}^T \Lambda_H^{-1} \dot{\hat{H}}) + \bar{\chi}^T Q \dot{\bar{\chi}}. \end{aligned} \tag{4.4}$$

Using property 3 and taking the value of $T^T U$, Eq (4.4) becomes

$$\dot{V} = r(t)^T(H^T\xi(t) + \psi(t) + \bar{F}_g + \bar{\tau}_{gd} - \hat{H}^T\xi(t) - P_1 r(t) - P_2 sign(r(t)) - Q\bar{\chi} - \frac{r(t)}{2\pi^2}) - tr(\hat{H}^T\Lambda_H^{-1}\dot{\hat{H}}) + \bar{\chi}^T Q\dot{\bar{\chi}}.$$

Using the update law as given in Eq (4.1) and simplifying Eq (4.5), we have

$$\dot{V} = r(t)^T(\psi(t) + \bar{F}_g + \bar{\tau}_{gd}) - r(t)^T P_1 r(t) - r(t)^T P_2 sign(r(t) - r(t)^T Q\bar{\chi} - \frac{r(t)^T r(t)}{2\pi^2} + \bar{\chi}^T Q\dot{\bar{\chi}}. \quad (4.5)$$

Simplifying the above equation, we have

$$\dot{V} = r(t)^T(\psi(t) + \bar{F}_g + \bar{\tau}_{gd} - Q\bar{\chi} - \frac{r(t)}{2\pi^2}) - P_1\|r(t)\|^2 - P_2|r(t)| + \bar{\chi}^T Q\dot{\bar{\chi}} \quad (4.6)$$

and

$$\dot{V} \leq \|r(t)\|\|\psi(t) + \bar{F}_g + \bar{\tau}_{gd}\| - r(t)^T Q\bar{\chi} - \frac{\|r(t)\|^2}{2\pi^2} - P_1\|r(t)\|^2 - P_2|r(t)| + \bar{\chi}^T Q\dot{\bar{\chi}}. \quad (4.7)$$

Rewriting the above equation by using some basic concepts, we have the following equation:

$$\dot{V} \leq \|r(t)\|\|B\| - r(t)^T Q\bar{\chi} - \frac{\|r(t)\|^2}{2\pi^2} - r(t)^T Q\bar{\chi} + \bar{\chi}^T Q\dot{\bar{\chi}}, \quad (4.8)$$

where

$$\psi(t) + \bar{F}_g + \bar{\tau}_{gd} = B \in L^2[0, t]$$

for $t > 0$.

This completes the proof. □

Now, adding and subtracting $\frac{\pi^2\|B\|^2}{2}$ in the above equation, we get

$$\dot{V} \leq \|r(t)\|\|B\| - r(t)^T Q\bar{\chi} - \frac{\|r(t)\|^2}{2\pi^2} + \bar{\chi}^T Q\dot{\bar{\chi}} + \frac{\pi^2\|B\|^2}{2} - \frac{\pi^2\|B\|^2}{2}.$$

After simplification of the above equation,

$$\dot{V} \leq -r(t)^T Q\bar{\chi} - \frac{1}{2}\Big(\frac{\|r(t)\|}{\pi} - \pi\|B\|\Big)^2 + \frac{\pi^2\|B\|^2}{2} + \bar{\chi}^T Q\dot{\bar{\chi}}$$

and

$$\dot{V} \leq -r(t)^T Q\bar{\chi} + \frac{\pi^2\|B\|^2}{2} + \bar{\chi}^T Q\dot{\bar{\chi}}. \quad (4.9)$$

Taking the value of $\dot{\bar{\chi}}$ from Eq (3.1), we get

$$\dot{V} \leq -r(t)^T Q\bar{\chi} + \frac{\pi^2\|B\|^2}{2} + \bar{\chi}^T Q\Big(r(t) - D^{\alpha+1}\bar{\chi}(t) - A\bar{\chi}(t)\Big)$$

and

$$\dot{V} \leq -r(t)^T Q\bar{\chi} + \frac{\pi^2\|B\|^2}{2} + \bar{\chi}^T Q r(t) - \bar{\chi}^T Q D^{\alpha+1}\bar{\chi}(t) - \bar{\chi}^T QA\bar{\chi}(t) \leq \frac{\pi^2\|B\|^2}{2} - \bar{\chi}^T QA\bar{\chi}(t). \quad (4.10)$$

Putting

$$QA = 2P$$

in the above equation, we get

$$\dot{V} \leq \frac{\pi^2\|B\|^2}{2} - 2\bar{\chi}^T P\bar{\chi}(t). \quad (4.11)$$

Now, integrating above equation between the limit 0 to $\mathcal{T}$, we have

$$V(\mathcal{T}) - V(0) \leq \frac{1}{2}\int_0^{\mathcal{T}} \pi^2\|B\|^2 dt - 2\int_0^{\mathcal{T}} \bar{\chi}^T P\bar{\chi}(t). \quad (4.12)$$

Taking the value of $\int_0^{\mathcal{T}} \bar{\chi}^T P\bar{\chi}(t)$ from Eq (3.5), we have

$$V(\mathcal{T}) - V(0) \leq \frac{1}{2}\int_0^{\mathcal{T}} \pi^2\|B\|^2 dt - 2\Big(r^T(0)\bar{M}_{0g}r(0) + \bar{\chi}^T(0)Q\bar{\chi}(0) + \int_0^{\mathcal{T}} \pi^2\|g_3\|^2 dt\Big). \quad (4.13)$$

Since,

$$\|\psi(t)+\bar{F}_g+\bar{\tau}_{gd}\| = \|B\| \ \text{ and } \ \|\psi(t)+\bar{F}_g+\bar{\tau}_{gd}\| \leq g_2+g_3+\psi_N \leq 3\|g_3\|,$$

by choosing

$$\|g_2\| \leq \|g_3\|$$

and

$$\psi_N \leq \|g_3\|,$$

we can conclude that

$$\|B\| \leq 3\|g_3\|,$$

which implies the following equation:

$$V(\mathcal{T}) - V(0) \leq \frac{1}{2}\int_0^{\mathcal{T}} 9\pi^2\|g_3\|^2 dt - 2\Big(r^T(0)\bar{M}_{0g}r(0) + \bar{\chi}^T(0)Q\bar{\chi}(0) + \int_0^{\mathcal{T}} \pi^2\|g_3\|^2 dt\Big) \leq 0. \quad (4.14)$$

As g_3 and B belong to $L^2[0, t]$ with $t > 0$, $|V(t)| < \infty$ implies that sliding variables and trajectory tracking errors are both bounded. Utilizing the Lyapunov criteria, it can be stated that the system states are stable in the Lyapunov sense with convergence of trajectory tracking errors toward their equilibrium.

5. Simulation

A simulation investigation is conducted on a 3-dof nonholonomic wheeled mobile robot to demonstrate the effective performance of the developed controller. The dynamic framework and parameters employed in this investigation for the nonholonomic wheeled mobile robot position tracking issue are taken from [24]. The non-holonomic constraint applied on mobile robot system is considered as:

$$-\dot{x}\sin(\Theta) + \dot{y}\cos(\Theta) = 0.$$

The simulation study on the nonholonomic wheeled mobile robot is carried out using MATLAB. The ODE45 MATLAB solver is utilized to solve ordinary differential equations. For calculating the fractional-order derivative, the definition of the Grunwald-Letnikov (GL) derivative [25] has been used. For comparison with the existing control scheme, a non-singular terminal sliding mode type controller is chosen. The architecture of this controller is taken from [22], and during the simulation study, the name for this controller is written as existing controller. The case "proposed controller with $\Delta = 0$" represents the case in which the estimation of disturbances and parametric uncertainties is negligible, i.e., in this case simulation results are carried out with the adaptive fractional-order sliding mode controller without H_∞ tracking control to show the robustness of the designed control scheme.

To show the efficiency of the proposed control scheme, the performance of the designed controller alongside the existing controller and proposed controller with $\Delta = 0$ for a nonholonomic wheeled mobile robot system is given in Figures 1–5. The position and velocity tracking errors are shown in Figures 1 and 2. From this figure, it can be shown that the trajectory tracking errors converge quickly with fewer fluctuations as compared to the existing controller, while for the case "proposed controller with $\Delta = 0$" a lot of fluctuations are seen that show the robustness of the designed controller as in the designed controller, for which the H_∞ tracking control technique is utilized to handle these negative after effects. In Figure 3, the position tracking performance is given. This figure shows that for the proposed control approach, the dynamical system tracks the desired trajectory very efficiently by taking less time to reach the trajectory.

In Figure 4, velocity tracking performance is given, which shows that the reference velocity is achieved very smoothly. Control input torque for the proposed scheme is shown in Figure 5.

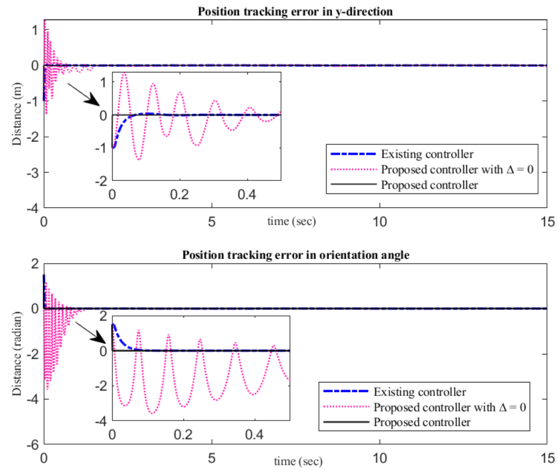


Figure 1. Position tracking errors.

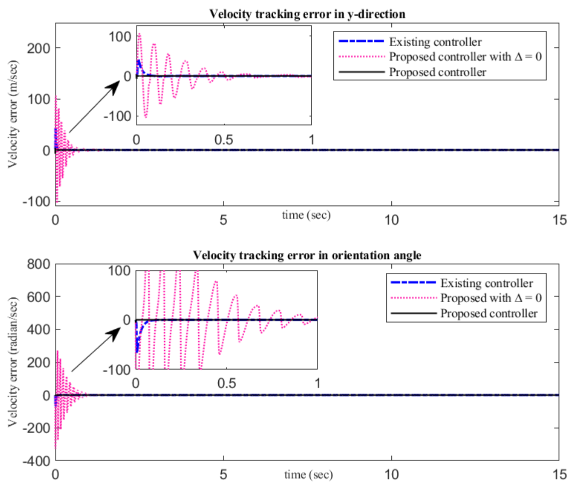


Figure 2. Velocity tracking errors.

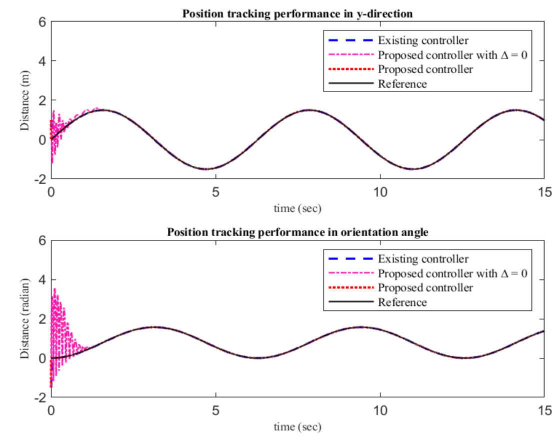


Figure 3. Position tracking performance in the y-direction and orientation angle.

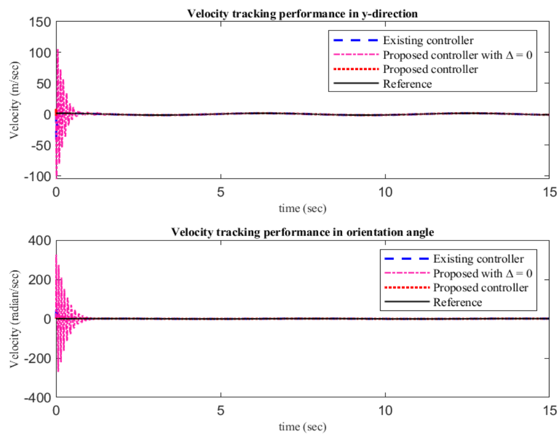


Figure 4. Velocity tracking performance in the y-direction and orientation angle.

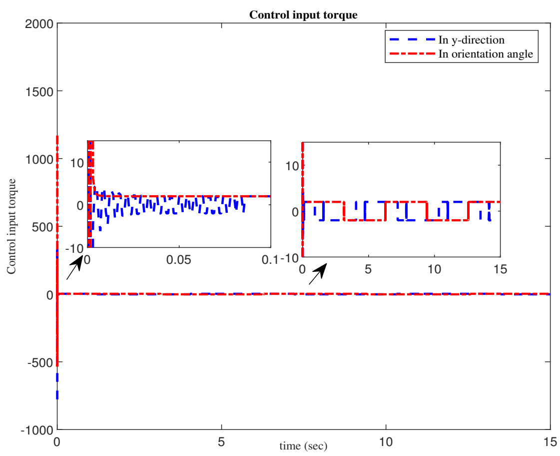


Figure 5. Control input torque.

From these figures, we come to the conclusion that the proposed controller efficiently tracks the reference trajectory in a robust manner as compared to the existing controller.

Further, to compare the performance of the controller statistically, the L^2-norm is presented in tabular form by comparing these parameters with the existing controller. The formula used for the L^2-norm is

$$L^2[\bar{\chi}] = \sqrt{\frac{1}{t_f - t_0} \int_{t_0}^{t_f} \|\bar{\chi}\|^2 dt}. \tag{5.1}$$

Table 1 presents the L^2-norm for position tracking errors for existing controller, proposed controller with $\Delta = 0$, and proposed controller comparatively that shows L^2-norm for proposed controller is very less. The lesser value of L^2-norm shows more efficient performance of the controller. Hence it can be concluded from L^2-norm that the performance of the proposed controller is enhanced as compared to the existing controller given in [22].

Table 1. L^2-norm for position tracking errors.

Controllers	$L^2[\bar{\chi}_1]$	$L^2[\bar{\chi}_2]$
Existing controller	0.0576 m	0.0836 radian
Proposed controller with $\Delta = 0$	0.1904 m	0.6833 radian
Proposed controller	0.0100 m	0.0150 radian

6. Conclusions

In this article, an H_∞-based adaptive fractional-order sliding mode controller was designed for the position tracking problem of nonholonomic wheeled mobile robots. In the designed controller, H_∞-based trajectory tracking control with fractional-order sliding mode control and neural networks are utilized to enhance the performance of the system. The H_∞ trajectory tracking performance enhances the performance of the dynamical system by compensating the negative aspects of external disturbances and parametric uncertainties in a robust way. To reproduce the non-linear dynamical part of the dynamical structure, the radial basis neural network was utilized. In order to analyze the convergence of tracking errors, the Lyapunov stability criterion was used to show the stability of the overall dynamical system. To show the efficiency and robustness of the presented controller, a simulation analysis was carried out by drawing comparisons to the obtained results with the existing techniques. The overall analysis including simulation results and statistical analysis, shows that the designed control method efficiently tracks the intended trajectory with a shorter reaching time.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article

Acknowledgments

The authors would express a special thank to the NIT Kurukshetra, Meerut College Meerut and MJPRU Bareilly for providing the necessary support for this research. The research work is financially supported by the Higher Education Department, U. P. Government under the Research and Development Scheme received vide letter No. 39/2024/242/sattar-04-2024-001-04(33)/2023.

Conflict of interest

There are no conflicts of interest.

References

1. X. Wu, Y. Wang, X. Dang, Robust adaptive sliding-mode control of condenser-cleaning mobile manipulator using fuzzy wavelet neural network, *Fuzzy Sets Syst.*, **235** (2014), 62–82. https://doi.org/10.1016/j.fss.2013.07.009
2. A. Kashkynbayev, R. Rakkiyappan, Sampled-data output tracking control based on T-S fuzzy model for cancer-tumor-immune systems, *Commun. Nonlinear Sci. Numer. Simul.*, **128** (2024), 107642. https://doi.org/10.1016/j.cnsns.2023.107642
3. W. Dong, W. Huo, Tracking control of wheeled mobile robots with unknown dynamics, *Proceedings 1999 IEEE International Conference on Robotics and Automation*, 1999. https://doi.org/10.1109/ROBOT.1999.773997
4. T. Khan, F. A. Rihan, H. Ahmad, Modelling the dynamics of acute and chronic hepatitis B with optimal control, *Sci. Rep.*, **13** (2023), 14980. https://doi.org/10.1038/s41598-023-39582
5. N. Chen, F. Song, G. Li, X. Sun, C. Ai, An adaptive sliding mode backstepping control for the mobile manipulator with nonholonomic constraints, *Commun. Nonlinear Sci. Numer. Simul.*, **18** (2013), 2885–2899. https://doi.org/10.1016/j.cnsns.2013.02.002
6. M. Begnini, D. W. Bertol, N. A. Martins, A robust adaptive fuzzy variable structure tracking control for the wheeled mobile robot: simulation and experimental results, *Control Eng. Pract.*, **64** (2017), 27–43. https://doi.org/10.1016/j.conengprac.2017.04.006
7. K. S. Chaudhary, N. Kumar, Fractional order fast terminal sliding mode control scheme for tracking control of robot manipulators, *ISA Trans.*, **142** (2023), 57–69. https://doi.org/10.1016/j.isatra.2023.08.008
8. N. Kumar, K. S. Chaudhary, Neural network based fractional order sliding mode tracking control of nonholonomic mobile robots, *J. Comput. Anal. Appl.*, **33** (2024), 73–89.
9. K. Liu, P. Yang, R. Wang, L. Jiao, T. Li, J. Zhang, Observer-based adaptive fuzzy finite-time attitude control for quadrotor UAVs, *IEEE Trans. Aerosp. Electron. Syst.*, **59** (2023), 8637–8654. https://doi.org/10.1109/TAES.2023.3308552
10. K. Liu, P. Yang, L. Jiao, R. Wang, Z. Yuan, S. Dong, Antisaturation fixed-time attitude tracking control based low-computation learning for uncertain quadrotor UAVs with external disturbances *Aerosp. Sci. Technol.*, **142** (2023), 108668. https://doi.org/10.1016/j.ast.2023.108668
11. M. Defoort, T. Floquet, A. Kokosy, W. Perruquetti, Sliding-mode formation control for cooperative autonomous mobile robots, *IEEE Trans. Ind. Electron.*, **55** (2008), 2002717. https://doi.org/10.1109/TIE.2008.2002717
12. B. Moudoud, H. Aissaoui, M. Diany, Robust trajectory tracking with adaptive non-singular fast TSM control of a robot manipulator, *2022 19th International Multi-Conference on Systems, Signals & Devices (SSD)*, 2022, 1880–1885. https://doi.org/10.1109/SSD54932.2022.9955926

13. B. Moudoud, H. Aissaoui, M. Diany, Adaptive integral-type terminal sliding mode control: application to trajectory tracking for mobile robot, *Int. J. Adapt. Control Signal Process.*, **37** (2023), 603–616. https://doi.org/10.1002/acs.3540
14. S. J. Gambhire, D. R. Kishore, P. S. Londhe, S. N. Pawar, Review of sliding mode based control techniques for control system applications, *Int. J. Dyn. Control*, **9** (2021), 363–378. https://doi.org/10.1007/s40435-020-00638-7
15. L. Zuo, R. Cui, W. Yan, Terminal sliding mode-based cooperative tracking control for non-linear dynamic systems, *Trans. Inst. Meas. Control*, **39** (2017), 1081–1087. https://doi.org/10.1177/01423312166292
16. B. Moudoud, H. Aissaoui, M. Diany, Fixed-time non-singular fast TSM control for WMR with disturbance observer, *IFAC-Papers Online*, **55** (2022), 647–652. https://doi.org/10.1016/j.ifacol.2022.07.385
17. Y. Xie, X. Zhang, W. Meng, S. Zheng, L. Jiang, J. Meng, et al., Coupled fractional-order sliding mode control and obstacle avoidance of a four-wheeled steerable mobile robot, *ISA Trans.*, **108** (2021), 282–294. https://doi.org/10.1016/j.isatra.2020.08.025
18. N. Kumar, K. S. Chaudhary, Motion control of underactuated cart-double-pendulum system via fractional-order sliding mode controller, In: R. Kumar, A. K. Verma, O. P. Verma, T. Wadehra, *Soft computing: theories and applications*, Springer, 2024, 155–165. https://doi.org/10.1007/978-981-97-2031-6
19. O. Eray, S. Tokat, The design of a fractional-order sliding mode controller with a time-varying sliding surface, *Trans. Inst. Meas. Control*, **42** (2020), 3196–3215. https://doi.org/10.1177/01423312209446
20. M. O. Efe, C. A. Kasnakoglu, A fractional adaptation law for sliding mode control, *Int. J. Adapt. Control Signal Process.*, **22** (2008), 968–986. https://doi.org/0.1002/acs.1062
21. K. S. Chaudhary, N. Kumar, Hybrid neural network-based fractional-order sliding mode controller for tracking control problem of reconfigurable robot manipulators using fast terminal type switching law, *Eng. Appl. Artif. Intell.*, **139** (2025), 109515. https://doi.org/10.1016/j.engappai.2024.109515
22. Ruchika, N. Kumar, Dinanath, Non-singular terminal sliding mode control of robot manipulators with H_∞ trajectory tracking performance, *Arab. J. Sci. Eng.*, **44** (2019), 9057–9065. https://doi.org/10.1007/s13369-019-04049-5
23. V. Panwar, Wavelet neural network-based H_∞ trajectory tracking for robot manipulators using fast terminal sliding mode control, *Robotica*, **35** (2017), 1488–1503. https://doi.org/10.1017/S0263574716000278
24. H. Xie, J. Zheng, Z. Sun, H. Wang, R. Chai, Finite-time tracking control for nonholonomic wheeled mobile robot using adaptive fast nonsingular terminal sliding mode, *Nonlinear Dyn.*, **110** (2022), 1437–1453. https://doi.org/10.1007/s11071-022-07682-2
25. I. Podlubny, *Fractional differential equations*, Mathematics in Science and Engineering/Academic Press, 1999.