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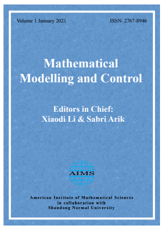

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Research article

Optimal control of dengue fever model with a logistically growing human population

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Abstract: Dengue is a deadly illness in almost all parts of Indonesia, including East Java. This paper analyzes the dengue transmission model by considering the recruitment rate in the form of the logistic growth of the human population. The model parameters were estimated using least-squares methods based on dengue data in East Java, Indonesia. The model analysis results obtained two equilibria, namely the diseases-free and the endemic equilibria. The disease-free equilibrium is asymptotically stable if the basic reproduction number $R_0 < 1$, while the endemic equilibrium is asymptotically stable if $R_0 > 1$. The control variables were incorporated, and an optimal control problem was analyzed using the Pontryagin maximum principle. Finally, the cost-effectiveness analysis suggests that prevention only is the the most cost-effective strategy required to control dengue disease.

Keywords: infectious disease; dengue; logistic growth model; optimal control

1. Introduction

Dengue is an infectious illness that is spread in almost all countries, including in Indonesia [1]. Severe dengue is a prominent leader of critical disease and death in some Asian and Latin American countries. Every year, dengue cases occur in East Java province, Indonesia. Based on the Ministry of Health of the Republic of Indonesia, dengue cases in Indonesia until July 2020 reached 71,633. East Java is one of the provinces in Indonesia with high endemic potential from year to year [2].

Dengue is a viral infection spread to humans through the bite of infected mosquitoes. Aedes aegypti mosquitoes are the major vectors that transmit the disease. The virus responsible for bringing dengue is called dengue virus (DENV). There are four DENV serotypes, and there is the potential to be infected four times. Currently, there is no particular treatment for dengue/severe dengue. Early detection of dengue disease progression and access to appropriate medical care reduce fatality rates of severe dengue to below 1%. Dengue prevention and regulation depends on effective vector control measures [1].

Mathematical modeling can play essential roles in helping understand the complex phenomenon of epidemic spread including the malaria [3, 4], tuberculosis [5, 6], measles [7], meningitis [8], monkeypox [9], zika [10], and COVID-19 [11, 12]. With a mathematical model, the relationship between the disease spread and epidemiological parameters can be identified. Several mathematical models of dengue transmission with various types compartments have been studied by researchers [13, 14]. In [15, 16], the authors designed mathematical model to study the effect of temperature on the spread of dengue fever. The impact of reinfection with the same serotype on the dynamics of dengue transmission has been studied in [17]. Researchers at [18, 19] developed a fractional order model to understand the dynamics of dengue fever transmission. The authors in [20] presented a dynamical model of dengue with hospitalization to analyze the infection in East

Java province, Indonesia. Ndii [21] formulated a dengue model by incorporating seasonally varying mosquitoes, vaccination, and media. The authors of [22] discussed a dengue model that accommodates releasing Wolbachia into wild mosquitoes and vaccination. The authors in [23] presented the dengue model by considering the recruitment rate of the human population in terms of logistic growth. The proposed model is more realistic because the growth rate of the human population is not always constant.

This work aims to extend the previous study of [23] by formulating a dengue model that captures the control variables, such as fumigation and prevention, by applying the optimal control theory to the model. The model parameters will be estimated based on the dengue data in East Java Province, Indonesia. We employ efficiency and economic methods to examine the most cost-effective strategy among the implemented control scenarios. The remaining part of this paper proceeds as follows. Section 2 explains the model formulation. The analysis of the model is discussed in Section 3. The optimal control problem is studied in Section 4. Section 5 summarizes the main findings of this work.

2. Model formulation

In this study, we consider the dengue transmission model with the assumption that the recruitment rate in the human population is logistic growth as proposed by [23]. The total human population (N_h) is divided into three sub-populations: the susceptible (S_h), the infectious (I_h), and the recovered (R_h). The total mosquitoes population (N_m) is divided into two sub-populations: the susceptible mosquitoes (S_m) and the infectious mosquitoes (I_m). The flow diagram of the dengue model is displayed in Figure 1. As the diagram in Figure 1 shows, the dengue model is governed by an autonomous system of ODEs as follows:

$$\begin{aligned}
\frac{d}{dt}S_h &= rN_h\left(1-\frac{N_h}{K}\right)-\left(B\beta_{mh}\frac{I_m}{N_h}+\mu_h\right)S_h,\\
\frac{d}{dt}I_h &= B\beta_{mh}\frac{I_m}{N_h}S_h-(\eta_h+\mu_h)I_h,\\
\frac{d}{dt}R_h &= \eta_h I_h-\mu_h R_h,\\
\frac{d}{dt}S_m &= \mu_m N_m-\left(B\beta_{hm}\frac{I_h}{N_h}+\mu_m\right)S_m,\\
\frac{d}{dt}I_m &= B\beta_{hm}\frac{I_h}{N_h}S_m-\mu_m I_m,
\end{aligned} \tag{2.1}$$

with

$$N_h = S_h + I_h + R_h$$

and

$$N_m = S_m + I_m.$$

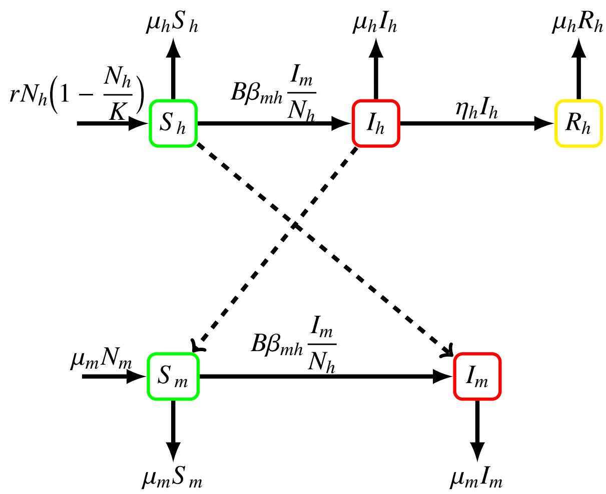


Figure 1. Transmission diagram of dengue model.

The biologically feasible region of the model (2.1) is set out by

$$\Gamma = \Gamma_h \cup \Gamma_m \subset R^3_+ \times R^2_+, \tag{2.2}$$

where

$$\Gamma_h = \left\{(S_h, I_h, R_h) \in R^3_+ : N_h \le K\right\}$$

and

$$\Gamma_m = \left\{(S_m, I_m) \in R^2_+ : S_m + I_m \le N_m(0)\right\}.$$

The description of the variables and parameters of the dengue model can be set out in Tables 1 and 2, respectively.

Table 1. Descriptions of the model state variables.

State variables	Biological interpretation
S_h	The susceptible of the human population
I_h	The infectious of the human population
R_h	The recovery of human population
N_h	The total number of human population
S_m	The susceptible of the mosquito population
I_m	The infectious of the mosquito population
N_m	The total number of the mosquito population

Table 2. Descriptions of model parameters.

Symbol	Description
B	The biting rate of mosquito
β_{mh}	The transmission probability from I_m to S_h
β_{hm}	The transmission probability from I_h to S_m
μ_h	The natural death rate in human
η_h	The recovery rate in human
μ_m	The birth/death rate in the mosquito
K	Carrying capacity of the environment
r	Intrinsic growth rate

The change rate of the total human population can be represented by

$$\frac{dN_h}{dt} = \left[r\left(1 - \frac{N_h}{K}\right) - \mu_h\right]N_h.$$

The total of the mosquito population is constant. Hence, the system (2.1) can be reduced into the following system:

$$\begin{aligned}
\frac{d}{dt}N_h &= \left[r\left(1 - \frac{N_h}{K}\right) - \mu_h\right]N_h, \\
\frac{d}{dt}S_h &= rN_h\left(1 - \frac{N_h}{K}\right) - \left(B\beta_{mh}\frac{I_m}{N_h} + \mu_h\right)S_h, \\
\frac{d}{dt}I_h &= B\beta_{mh}\frac{I_m}{N_h}S_h - (\eta_h + \mu_h)I_h, \\
\frac{d}{dt}I_m &= B\beta_{hm}\frac{I_h}{N_h}(N_m - I_m) - \mu_m I_m.
\end{aligned} \tag{2.3}$$

2.1. *Estimation of the parameters model*

In this present section, we devote the parameters estimation of the model (2.3) using the fitting least-squares method [24, 25]. The monthly cumulative data of dengue in East Java Province, Indonesia, from 2018 until 2020 [26] is utilized in this study. The parameters μ_h, μ_m, and r are estimated taking the demographic information. The natural death rate of human (μ_h) is observed from inverse of the average lifespan in East Java Province. The average of lifespan in East Java Province from 2018 until 2020 is 71.15 years. Therefore,

$$\mu_h = \frac{1}{71.15}$$

per year [27, 28]. Next, the death rate of mosquito (μ_m), is found from the inverse of the average lifespan of *Aedes* mosquitoes. Based on [29], the average of lifespan of *Aedes* mosquitoes is 25 days, that is

$$\mu_m = \frac{1}{25}$$

per day. Furthermore, for the growth rate of human (r), is obtained as follows. The average population in East Java Province from 2018 to 2020 is 39, 955, 059 [30]. Hence,

$$\frac{r}{\mu_h} = 39,955,059,$$

which is the total human population without the disease, so that $r = 561{,}560.91$ per year. The other parameters, such as B, β_{mh}, β_{hm}, η_h, K, and N_m, are estimated with the objective function that minimize

$$\min_{B,\,\beta_{mh},\,\beta_{hm},\,\eta_h,\,K,\,N_m} \sum_{i=0}^{t_f}\left(I_i - I_i^{data}\right)^2,$$

where t_f is the end time of the cumulative dengue cases data I_i^{data} and I_i, $\left(i = 0,\ 1,\ 2, \ldots,\ t_f\right)$ are the cumulative numerical solutions of infected humans from system (2.3). Thus, the initial populations are

$$(N_{h0}, S_{h0}, I_{h0}, I_{m_0}) = \left(39,500,851, N_{h0} - I_{h0}, I_0^{data},\ 100\right),$$

while the initial parameter values for estimation are

$$(B_0, \beta_{mh_0}, \beta_{hm_0}, \eta_{h0}, K_0, N_{m0}) = (0.4, 0.4, 0.4, 0.4, 1.5N_{h0}, 2.5N_{h_0}).$$

The lower bound parameter values are $\left(10^{-3},\ 10^{-3},\ 10^{-3},\ 10^{-3},\ 1.2N_{h0},\ 2N_{h_0}\right)$ and upper bound parameter values are $(1,\ 1,\ 1,\ 1,\ 2N_{h_0},\ 3N_{h_0})$. The result of the estimation and parameter value can be seen in Figure 2 and Table 3.

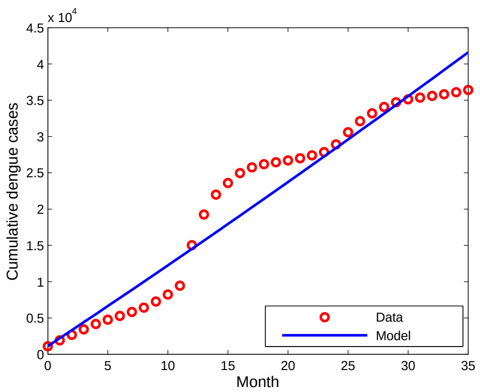


Figure 2. Comparison of dengue data and the model.

Table 3. The parameter value of the model.

Parameter	Value (months)	Source
B	0.3848	Fitted
β_{mh}	0.3632	Fitted
β_{hm}	0.3252	Fitted
μ_h	$\frac{1}{12*71.15}$	Estimated
η_h	0.0198	Fitted
μ_m	$\frac{1}{\frac{12*25}{365}}$	Estimated
K	59,229,000	Fitted
r	$\frac{39,955,059}{12*71.15}$	Estimated
N_m	98,782,000	Fitted

3. Analysis model

In this section, we present the equilibria and the local stability of the DHF model.

3.1. Equilibria of model

Model (2.3) has the dynamic characteristic of model (2.1), so to understand the dynamics behaviour of the model (2.1), we carry out the analysis of the model (2.3). Likewise, the biological domain in both models is the same. Based on [23], two equilibria are obtained, namely the disease-free equilibrium (DFE) and the endemic equilibrium. The DFE of the system (2.3) is provided by

$$E_0(N_h^*, S_h^*, 0, 0) = \left(K\left(1 - \frac{\mu_h}{r}\right), K\left(1 - \frac{\mu_h}{r}\right), 0, 0\right).$$

The DFE E_0 is exist if $r > \mu_h$.

Meanwhile, the endemic equilibrium is

$$E_1(N_h, S_h, I_h, I_m) = (\tilde{N}_h, \tilde{S}_h, \tilde{I}_h, \tilde{I}_m)$$

with

$$\begin{aligned}
\tilde{N}_h &= K\left(1 - \frac{\mu_h}{r}\right), \\
\tilde{S}_h &= (\eta_h + \mu_h)\tilde{N}_h \frac{\left(B\beta_{hm}\tilde{I}_h + \mu_m\tilde{N}_h\right)}{B^2\beta_{mh}\beta_{hm}N_m}, \\
\tilde{I}_h &= \frac{\mu_h\left(B^2\beta_{mh}\beta_{hm}N_m - \mu_m(\eta_h + \mu_h)\tilde{N}_h\right)}{B\beta_{hm}(\eta_h + \mu_h)\left(\frac{B\beta_{mh}N_m}{\tilde{N}_h} + \mu_h\right)}, \\
\tilde{I}_m &= \frac{B\beta_{hm}N_m\tilde{I}_h}{B\beta_{hm}\tilde{I}_h + \mu_m\tilde{N}_h}.
\end{aligned}$$

The endemic equilibrium exists if

$$\frac{B^2\beta_{mh}\beta_{hm}N_m}{\mu_m(\eta_h + \mu_h)\tilde{N}_h} > 1$$

or

$$\frac{B^2\beta_{mh}\beta_{hm}N_m}{\mu_m(\eta_h + \mu_h)K\left(1 - \frac{\mu_h}{r}\right)} > 1, \ \ r > \mu_h.$$

Using the the next-generation matrix method [31], we yield the basic reproduction number as follows:

$$R_0 = \sqrt{\frac{B^2\beta_{hm}\beta_{mh}N_m}{\mu_m K\left(1 - \frac{\mu_h}{r}\right)(\eta_h + \mu_h)}}, \ \ r > \mu_h. \tag{3.1}$$

By using the formulation of R_0, the endemic equilibrium exists if

$$R_0^2 = \frac{B^2\beta_{mh}\beta_{hm}N_m}{\mu_m(\eta_h + \mu_h)K\left(1 - \frac{\mu_h}{r}\right)} > 1$$

or $R_0 > 1$ and $r > \mu_h$.

3.2. Local stability analysis

The local stability analysis of dengue model (2.3) can be expressed by the following theorem.

Theorem 3.1. *The diseases-free equilibrium $E_0\left(N_h^*, S_h^*, 0, 0\right)$ of the model* (2.3) *is locally asymptotically stable if*

$$R_0 < 1.$$

Proof. The evaluation of Jacobian matrix of the system (2.3) at DFE E_0, we have

$$J_{E_0} = \begin{pmatrix} \mu_h - r & 0 & 0 & 0 \\ 2\mu_h - r & -\mu_h & 0 & -B\beta_{mh} \\ 0 & 0 & -(\eta_h + \mu_h) & B\beta_{mh} \\ 0 & 0 & \frac{B\beta_{mh}N_m}{K\left(1 - \frac{\mu_h}{r}\right)} & -\mu_m \end{pmatrix}. \tag{3.2}$$

The two eigenvalues of Jacobian matrix (3.2) are $-\mu_h, -(r - \mu_h)$ that are obviously negative. The other two remaining eigenvalues can be found through the following characteristics equation:

$$\lambda^2 + a_1\lambda + a_2 = 0 \tag{3.3}$$

for

$$a_1 = (\mu_m + \eta_h + \mu_h)$$

and

$$a_2 = (\eta_h + \mu_h)\mu_m - \frac{B^2\beta_{mh}\beta_{hm}N_m}{K\left(1 - \frac{\mu_h}{r}\right)}.$$

Using Routh-Hurwitz criterion, the roots of the Eq (3.3) are negative if $a_1, a_2 > 0$. The coefficient $a_1 > 0$, while $a_2 > 0$ if

$$\frac{B^2\beta_{mh}\beta_{hm}N_m}{\mu_m(\eta_h + \mu_h)K\left(1 - \frac{\mu_h}{r}\right)} < 1$$

or

$$R_0 < 1.$$

So, the theorem is proven. □

The endemic equilibrium E_1 in model (2.3) will be asymptotically stable if

$$R_0 > 1.$$

By Substituting the endemic equilibrium E_1 to Jacobian matrix of the system (2.3), we find that the eigenvalue is $-(r - \mu_h)$ and the others are the roots of the equation

$$\begin{aligned}
&\lambda^3 + \left[\frac{B\beta_{mh}}{\tilde{N}_h}\left(\tilde{I}_m + \tilde{I}_h\right) + 2\mu_h + \mu_m + \eta_h\right]\lambda^2 \\
&+ \left[\left(\frac{B\beta_{mh}\tilde{I}_m}{\tilde{N}_h} + \mu_h\right)\left(\frac{B\beta_{hm}\tilde{I}_h}{\tilde{N}_h} + \mu_m + \eta_h + \mu_h\right)\right. \\
&\left. + (\eta_h + \mu_h)\left(\frac{B\beta_{hm}\tilde{I}_h}{\tilde{N}_h} + \mu_m\right) - \frac{B^2\beta_{mh}\beta_{hm}\tilde{S}_h}{\tilde{N}_h^2}\left(N_m - \tilde{I}_m\right)\right]\lambda \\
&+ \left(\frac{B\beta_{mh}\tilde{I}_m}{\tilde{N}_h} + \mu_h\right)\left[(\eta_h + \mu_h)\left(\frac{B\beta_{hm}\tilde{I}_h}{\tilde{N}_h} + \mu_m\right)\right. \\
&\left. - \frac{B^2\beta_{mh}\beta_{hm}\tilde{S}_h}{\tilde{N}_h^2}\left(N_m - \tilde{I}_m\right)\right] + \frac{B^3\beta_{mh}^2\beta_{hm}\tilde{I}_h\tilde{S}_h}{\tilde{N}_h^3}\left(N_m - \tilde{I}_m\right) = 0.
\end{aligned} \tag{3.4}$$

Equation (3.4) can be expressed by

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$$

with

$$\begin{aligned}
b_1 &= \frac{B\beta_{mh}}{\tilde{N}_h}\left(\tilde{I}_m + \tilde{I}_h\right) + 2\mu_h + \mu_m + \eta_h, \\
b_2 &= \left(\frac{B\beta_{mh}\tilde{I}_m}{\tilde{N}_h} + \mu_h\right)\left(\frac{B\beta_{hm}\tilde{I}_h}{\tilde{N}_h} + \mu_m + \eta_h + \mu_h\right) \\
&\quad + (\eta_h + \mu_h)\left(\frac{B\beta_{hm}\tilde{I}_h}{\tilde{N}_h} + \mu_m\right) - \frac{B^2\beta_{mh}\beta_{hm}\tilde{S}_h}{\tilde{N}_h^2}\left(N_m - \tilde{I}_m\right), \\
b_3 &= \left(\frac{B\beta_{mh}\tilde{I}_m}{\tilde{N}_h} + \mu_h\right)\left[(\eta_h + \mu_h)\left(\frac{B\beta_{hm}\tilde{I}_h}{\tilde{N}_h} + \mu_m\right)\right] \\
&\quad \left[-\frac{B^2\beta_{mh}\beta_{hm}\tilde{S}_h}{\tilde{N}_h^2}\left(N_m - \tilde{I}_m\right)\right] + \frac{B^3\beta_{mh}^2\beta_{hm}\tilde{I}_h\tilde{S}_h}{\tilde{N}_h^3}\left(N_m - \tilde{I}_m\right).
\end{aligned}$$

According to the Ruth-Hurwitz criteria, the roots of the characteristic equation will be negative or have a negative real part if

$$b_1, b_2, b_3 > 0$$

and

$$b_1 b_2 - b_3 > 0.$$

The coefficients b_1–b_3 containing $\tilde{N}_h, \tilde{S}_h, \tilde{I}_h, \tilde{I}_m$ so showing that the condition for the root must be positive will be difficult to do analytically. Therefore, the numerical simulation is implemented to analyze the stability of the endemic equilibrium. The parameter values used are given in Table 3.

The phase plane graph between the infected mosquito population and the infected human population is presented in Figure 3. Based on Figure 3, it can be seen that the three graphs converges to the endemic equilibrium E_1 as time evolves when

$$R_0 > 1.$$

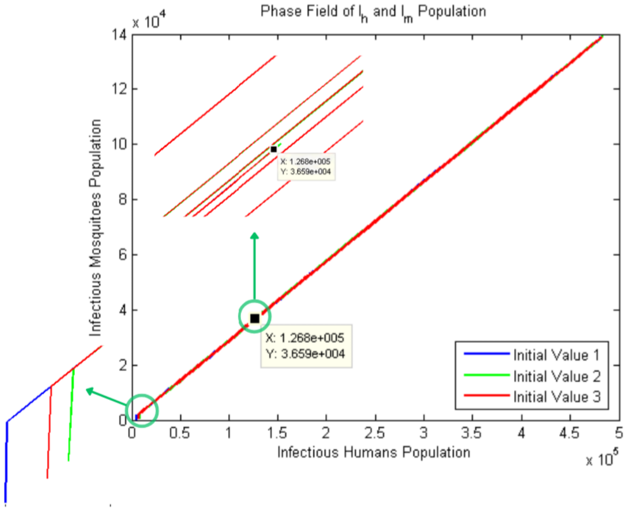


Figure 3. Phase field of (I_m) and (I_h).

3.3. Sensitivity analysis

In this work, we discuss the sensitivity analysis to recognize the parameter that can influence the threshold R_0. To assign the sensitivity index, we refer to [32]. The calculation of the sensitivity index R_0 towards to some parameter, say a, is given by

$$\Upsilon_a^{R_0} = \frac{\partial R_0}{\partial a} \times \frac{a}{R_0}.$$

Using the parameter values in Table 3, the sensitivity indexes of R_0 are summarized in Table 4.

Table 4. Sensitivity index of the parameters.

Parameter	Sensitivity index
B	1
β_{mh}	0.5
β_{hm}	0.5
μ_h	−0.028
η_h	−0.472
μ_m	−0.5
K	−0.5
r	-1.266×10^{-8}
N_m	0.5

The interpretation of the sensitivity index in Table 4 can be explained as follows. The positive sign reveals that when the parameter values are raised, the value of R_0 will also increase. Conversely, the negative sign reveals that when the parameter's value is raised, the value of R_0 will be decreased. For example, for

$$\Upsilon_B^{R_0} = 1,$$

increasing the value of the biting rate of mosquito by 10%, causes R_0 to increase by 10%. Thus, for

$$\Upsilon_K^{R_0} = -0.5,$$

increasing the carrying capacity by 10%, causes R_0 to decrease by 5%. Likewise, for the other indexes. The highly sensitive parameters should be considered carefully because small variations in these parameters will cause large quantitative changes. Table 4 shows that the parameter with the greatest influence on the threshold is parameter B. The implication is that an increase in the value of the biting rate of mosquitoes increases the spread of dengue disease in the population. Hence, we can anticipate it with preventive control by using a bed net and mosquito repellent lotion or fumigation control to reduce the mosquito population. Furthermore, the value of the biting rate of mosquitoes will decrease, which implies dengue transmission in the community also decreases.

Based on the contour plot in Figures 4 and 5, the basic reproduction number (R_0) will increase in proportion to the results of the sensitivity analysis in Table 4. The parameters B and β_{mh} have a positive relation; however the parameters μ_m and η_h have a negative relation.

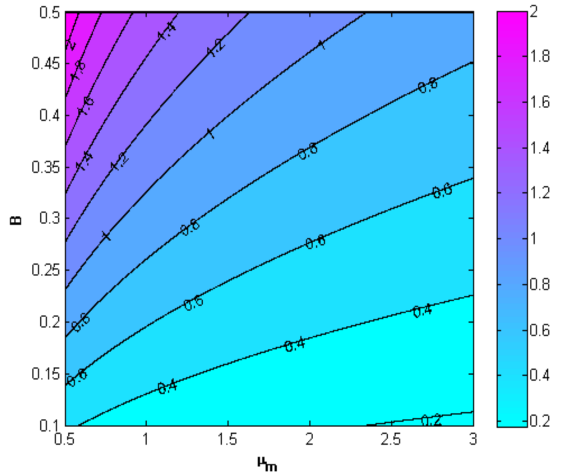


Figure 4. The behaviour of R_0 to the parameters μ_m and B.

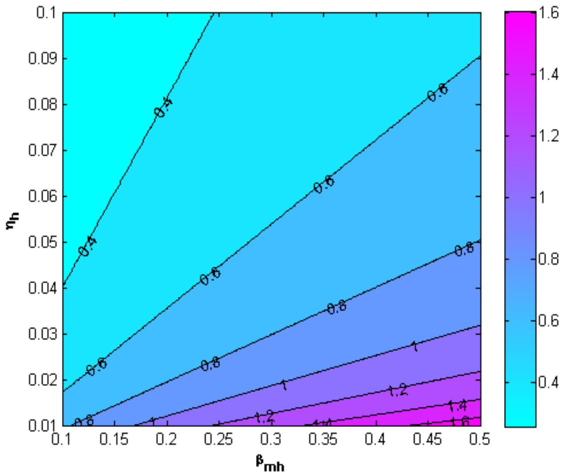


Figure 5. The behaviour of R_0 to the parameters β_{mh} and η_h.

4. Optimal control

In this section, we apply the optimal control (OC) strategy to the dengue model. Based on the sensitivity analysis in the previous section, we can reduce the number of mosquito biting rates with fumigation and prevention efforts. Therefore, we define two control variables, namely u_1 as fumigation and u_2 as prevention. The dengue model with control variables can be written as follows

$$\frac{dS_h}{dt} = rN_h\left(1-\frac{N_h}{K}\right)-(1-u_2)\frac{B\beta_{mh}I_m}{N_h}S_h-\mu_h S_h,$$

$$\frac{dI_h}{dt} = (1-u_2)\frac{B\beta_{mh}I_m}{N_h}S_h - (\eta_h+\mu_h)I_h,$$
$$\frac{dR_h}{dt} = \eta_h I_h - \mu_h R_h,$$
$$\frac{dS_m}{dt} = \mu_m N_m - \frac{B\beta_{hm}I_h}{N_h}S_m - \mu_m S_m - \kappa u_1 S_m,$$
$$\frac{dI_m}{dt} = \frac{B\beta_{hm}I_h}{N_h}S_m - \mu_m I_m - \kappa u_1 I_m.$$

The control variables u_1 and u_2 are established on interval $\left[0, t_f\right]$, where

$$0 \le u_i(t) \le 1, \quad t \in \left[0, t_f\right], \quad i = 1, 2,$$

and t_f represent the final time of the controls. We aim to minimize the number of infected human populations and mosquito populations and keep the cost of employing fumigation and prevention controls as low as possible. For this, we state the objective function

$$J(u_1, u_2) = \int_0^{t_f}\left(A_1 I_h + A_2 I_m + \frac{1}{2}c_1 u_1^2 + \frac{1}{2}c_2 u_2^2\right)dt,$$

where A_1 and A_2 are weights of the objective function for I_h and I_m, respectively, and c_1, and c_2 are weight parameters for fumigation and prevention respectively. We utilize the quadratic cost function for J to depict the cost of control efforts. This quadratic function can explain a nonlinear cost increase associated with the performance of control attempts in the field [33, 34].

Then, to solve this optimal control problem, we assumed that N_h is constant. Using Pontryagin's maximum principle [35], the optimal solutions of u_1 and u_2 are provided by

$$u_1^* = \min\left\{1, max\left(0, \kappa\left(\frac{\lambda_4 S_m + \lambda_5 I_m}{c_1}\right)\right)\right\},$$
$$u_2^* = \min\left\{1, max\left(0, (\lambda_2-\lambda_1)\frac{B\beta_{mh}I_m}{c_2 N_h}S_h\right)\right\}.$$

The variables λ_i, $i = 1, 2, 3, 4, 5$, are represented as adjoint variables or co-state which satisfies the following equations

$$\frac{d\lambda_1}{dt} = (\lambda_1-\lambda_2)(1-u_2)\frac{B\beta_{mh}I_m}{N_h} + \lambda_1\mu_h,$$
$$\frac{d\lambda_2}{dt} = -A_1 + (\lambda_2-\lambda_3)\eta_h + (\lambda_4-\lambda_5)\frac{B\beta_{hm}S_m}{N_h} + \lambda_2\mu_h,$$
$$\frac{d\lambda_3}{dt} = \lambda_3\mu_h,$$
$$\frac{d\lambda_4}{dt} = (\lambda_4-\lambda_5)\frac{B\beta_{hm}I_h}{N_h} + \lambda_4(\mu_m+\kappa u_1),$$
$$\frac{d\lambda_5}{dt} = -A_2 + (\lambda_1-\lambda_2)(1-u_2)\frac{B\beta_{mh}S_h}{N_h} + \lambda_5(\mu_m+\kappa u_1),$$

where the transversality conditions

$$\lambda_i\left(t_f\right) = 0, \quad i = 1, 2, 3, 4, 5.$$

In this optimal control problem, because N_h is assumed to be constant,

$$\frac{dN_h}{dt} = 0 \Leftrightarrow r = \frac{K}{K-N_h}\mu_h,$$

as N_h is the population in East Java Province in 2018. We assume that the values of the parameters are given by

$$A_1 = A_2 = 1, \; c_1 = 15, \; c_2 = 10, \; \kappa = 0.7$$

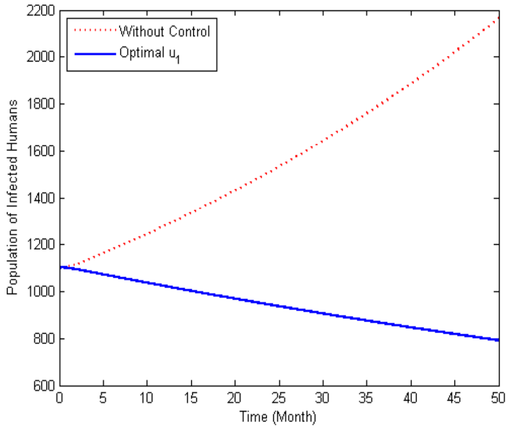
and

$$r = \frac{K}{K-N_h}\mu_h$$

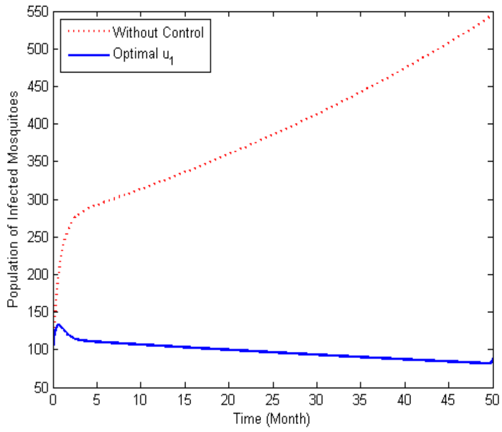
and the other parameters are referred to Table 3. We solve the numerical optimal control simulation using the backward and forward iteration technique as stated in [36]. To determine which strategy or combination provides useful methods of controlling dengue, we addressed the following strategies for our simulation result, as enumerated below.

4.1. Strategy 1. Single intervention: use of fumigation only

The result of the simulation of comparison I_h and I_m without and with control when fumigation usage only is given in Figure 6. We can observe that using this strategy, the infections of humans and mosquitoes tend to decrease compared to no control. The profile of the optimal control u_1 only is presented in Figure 7. It is apparent that fumigation control should be carried out intensively for a maximum of 50 months.



(a)



(b)

Figure 6. Comparison I_h and I_m without and with control.

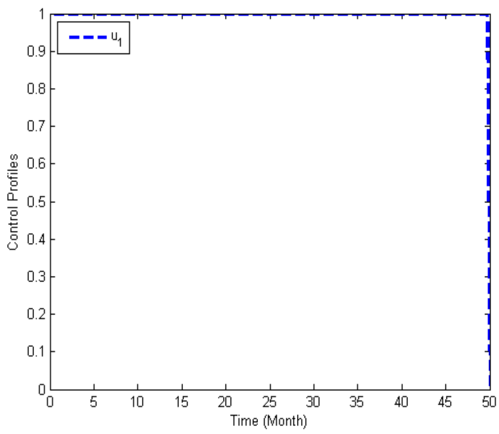
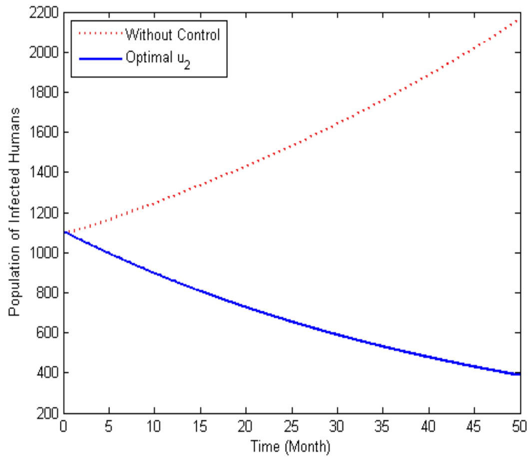


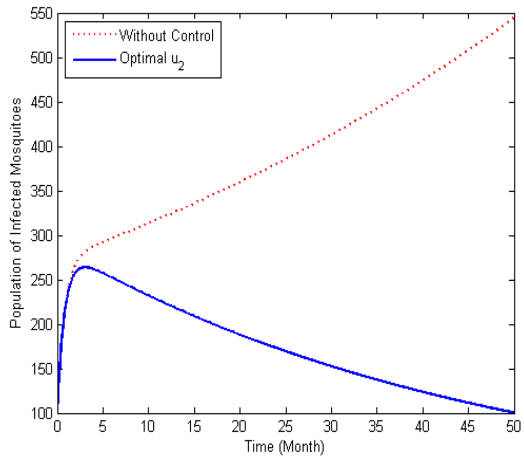
Figure 7. Control profile of u_1.

4.2. Strategy 2. Single intervention: use of prevention only

The result of the simulation of comparison I_h and I_m without and with control when prevention usage only is given in Figure 8. From Figure 8, it can be seen that both infection of humans and mosquitoes significantly reduce using this strategy. The profile of the optimal control u_2 only is displayed in Figure 9. This figure shows that prevention control should be provided with full effort for almost 50 months.



(a)



(b)

Figure 8. Comparison I_h and I_m without and with control.

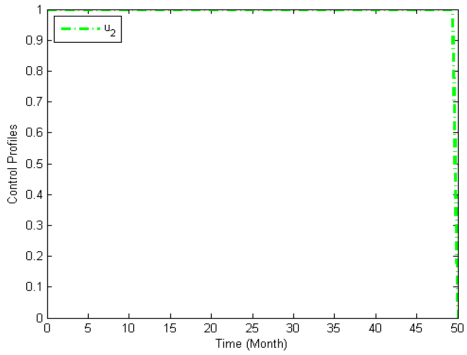
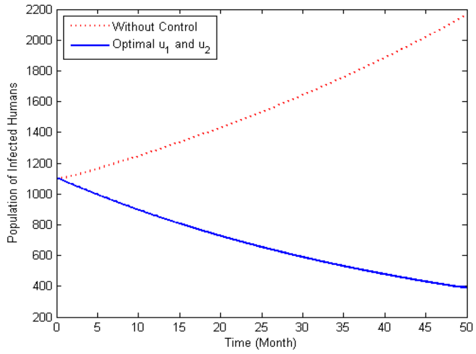


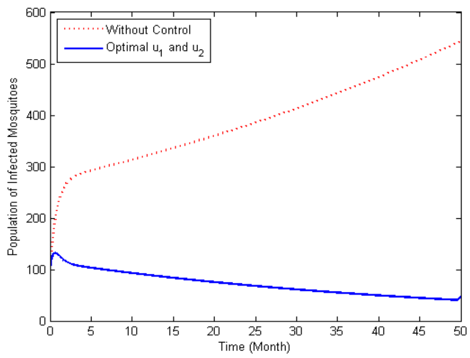
Figure 9. Control profile of u_2.

4.3. Strategy 3. Double intervention: combination of fumigation and prevention

The result of the simulation of comparison I_h and I_m without and with control when implementing both fumigation and prevention are given in Figure 10.



(a)



(b)

Figure 10. Comparison I_h and I_m without and with controls.

Using the last strategy, it is found that both infectious humans and mosquitoes diminish over to compare to without controls. The profile of the optimal control u_1 and u_2 simultaneously is demonstrated in Figure 11. As depicted in Figure 11, we found that fumigation and preventive control were both maintaining at maximum levels for nearly 50 months before decaying gradually to the lower bound.

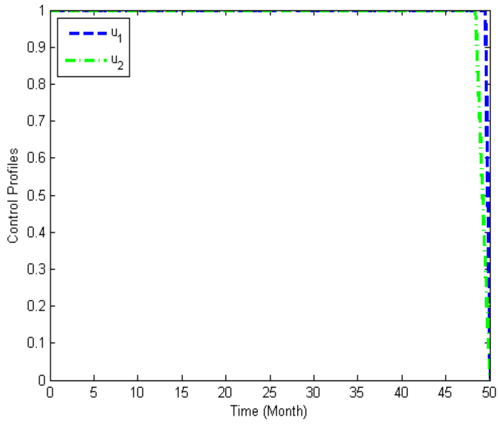


Figure 11. Control profiles of u_1 and u_2 simultaneously.

Furthermore, to evaluate the most proper strategy, we assess and compare the merit and the costs related to the control measures by adopting the infection averted ratio (IAR), average cost-effectiveness ratio (ACER) and incremental cost-effectiveness ratio (ICER). These economic verify are mathematically established in [37, 38] as follows:

$$\text{IAR} = \frac{\text{Cumulative infected averted}}{\text{Cumulative recovered}}.$$

The cumulative infected averted is presented by the sum of the difference between the total infectious individuals without and with control. More explicitly

$$\text{IAR} = \frac{\int_0^{t_f} \left(I_h(t) - I_h^*(t)\right)\, dt}{\int_0^{t_f} R_h^*(t)\, dt},$$

where the symbol with subscript * is employed to represent the optimal solutions associated with the corresponding strategy. The most powerful strategy when taking IAR is the strategy with the highest ratio

$$\text{ACER} = \frac{\text{Total cost invested on the intervention}}{\text{Total number of infectious averted}}.$$

It proposes a single intervention and evaluates it against its baseline desire. The aim of fumigation control (u_1) is the reduction of mosquitoes, while prevention control (u_2) is to protect susceptible humans by diminishing the contact rate. Therefore, we deal with

$$\text{ACER} = \frac{\int_0^{t_f} \left(C_1 u_1^* (t) N_m^* (t) + C_2 u_2^* (t) S_h^* (t) \right) dt}{\int_0^{t_f} \left(I_h (t) - I_h^* (t) \right) dt},$$

where C_i represents the cost of the 2 possible interventions. The most cost-effective strategy when using ACER is the strategy with the smallest ratio.

$$\text{ICER} = \frac{\text{Difference in cost produced by strategies } i \text{ and } j}{\text{Difference in the total of infections averted in strategies } i \text{ and } j}.$$

Hence, ICER is utilized to compare two different strategies, namely i and j. We calculate the cost of a strategy as undertaken for ACER. When comparing two or more competing intervention strategies incrementally, one intervention is compared with the next effective alternative in increasing order of total infected averted [37]. In computing ACER and ICER, we address the same cost for both interventions

$$C_1 = C_2 = 1.$$

The result of the calculation of the total infection and total cost is summarised in Table 5, while IAR and ACER is set out in Table 6.

Table 5. Total cost for each intervention strategy.

Strategies	Total Infection averted	Total cost
1	3.112406×10^4	3.1460×10^9
3	4.394527×10^4	5.0813×10^9
2	4.394682×10^4	1.9622×10^9

Table 6. Comparison of IAR and ACER for each intervention strategy.

Strategies	IAR	ACER
1	1.2901	1.0108×10^5
3	2.2588	1.1563×10^5
2	2.2589	4.4650×10^4

Based on Table 5, Strategy 2 averts the greatest number of infections and yields the largest ratio

$$IAR = 2.2589.$$

Hence, it is the most effective. This strategy is also the most cost-effective because it is associated with the smallest average cost-effectiveness ratio

$$ACER = 4.4650 \times 10^4.$$

Using the total infection and total cost in Table 5, the ICER indexes, as summarised in Table 7, are computed as follows:

$$\begin{aligned}
\text{ICER}(1) &= \frac{3.1460 \times 10^9 - 0}{3.112406 \times 10^4 - 0} \\
&= 1.0108 \times 10^5, \\
\text{ICER}(2) &= \frac{1.9622 \times 10^9 - 5.0813 \times 10^9}{4.394682 \times 10^4 - 4.394527 \times 10^4} \\
&= -2.0123 \times 10^9, \\
\text{ICER}(3) &= \frac{5.0813 \times 10^9 - 3.1460 \times 10^9}{4.394527 \times 10^4 - 3.112406 \times 10^4} \\
&= 1.5095 \times 10^5.
\end{aligned}$$

Table 7. Comparison of ICER for each intervention strategy.

Strategies	ICER	ICER recalculated
1	1.0108×10^5	1.8917×10^4
3	1.5095×10^5	-
2	-2.0123×10^9	-0.9232×10^5

Comparing Strategies 1 and 3, the utilization of Strategy 1 is cost-saving over Strategy 3. This means that Strategy 3 is less effective and more costly than the other strategy. Hence, Strategy 3 is eliminated. Furthermore, we recalculate the index of ICER as follows:

$$\begin{aligned}
\text{ICER}(1) &= \frac{3.1460 \times 10^9 - 0}{3.112406 \times 10^4 - 0} \\
&= 1.0108 \times 10^5, \\
\text{ICER}(2) &= \frac{1.9622 \times 10^9 - 3.1460 \times 10^9}{4.394682 \times 10^4 - 3.112406 \times 10^4} \\
&= -0.9232 \times 10^5.
\end{aligned}$$

Comparing Strategies 1 and 2, the application of Strategy 2 is cost-saving over Strategy 1. This indicates that Strategy 1 is less effective and more costly than the other strategy. Hence, Strategy 1 is removed. Our results recommend that Strategy 2 is the most cost-effective intervention associated with the ICER.

5. Conclusions

This study has investigated the dengue model by incorporating the logistic growth on the recruitment rate in the human population. The model analyses exhibited that the disease-free equilibrium is locally stable when the reproduction number is less than one, while the endemic equilibrium tends to be asymptotically stable when the threshold is greater than one. The parameters of the dengue model are estimated using data on cases of dengue in East Java Province, Indonesia, from 2018 to 2020. The findings indicate that the spread of dengue is persistent in the population.

Next, the sensitivity analysis shows that mosquito biting rate is the most influential parameter in the spread of dengue. Therefore, we carried out the control strategy in the form of fumigation and prevention. Pontryagin's maximum principle was adopted to accomplish the optimal control that minimizes the spread of dengue. The results of the optimal control problem show that the spread of dengue can be controlled by implementing sustainable control over a short period of time, which shows that the optimal control strategy is effective on humans and mosquitoes. Based on the cost-effectiveness evaluation, prevention is the most effective step to reduce dengue transmission using IAR, ACER, and ICER cost analysis.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare that they have no conflicts of interest in this paper.

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