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Research article

Finite-time stability and applications of positive switched linear delayed impulsive systems

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Abstract: In this paper, we study the finite-time stability and applications of positive switched linear delayed systems under synchronous impulse control, which includes two types of random switching and average dwell time switching. By constructing a type of linear time-varying co-positive Lyapunov functional, we first propose several new finite-time stability criteria. It should be emphasized that the linear term coefficient of the linear vector of the Lyapunov functional is adjusted to the difference between the weighting vector and the given vector. Then, we apply the obtained stability criteria to the linear time-varying delayed systems with impulsive effects. At last, three examples are given to demonstrate the validity of the obtained results, which includes the specific linear programming algorithm process.

Keywords: finite-time stability; impulse control; co-positive Lyapunov functional; average dwell time switching

1. Introduction

Positive systems represent dynamical systems whose states maintain nonnegative states on the condition that the original states are nonnegative [1, 2]. Positive switched systems are complex dynamical systems that consist of several positive subsystems and a switching rule among them. Compared with switched systems, positive switched systems can model some complex systems, so it is of great practical value to study the stability and control problems of positive switched systems. It is commonly known that positive switched systems have a large number of applications in the areas of congestion control [3], chemical processes [4], aerospace engineering [5], multi-agent systems [6], etc. On the other hand, delays are frequently encountered in complex dynamical systems. Time delays bring a period of historical state information to the system dynamics; the solution states of the systems are not only affected by the current states, but also affected by the corresponding time delays. In other words, time delays physically represent the delays between the system's response and the external excitation. Time delays naturally occur in practical engineering systems, which are usually considered an important factor in degrading the performance of dynamic systems. Time delays affect the performance of the systems, which may make the systems unstable or even oscillate. Moreover, it is well known that even small delays may affect or even destroy the stability of systems, which make stability analysis difficult. The study of stability is always an important topic in control systems. Therefore, an increasing number of experts and scholars are taking notice of the stability of positive switched delayed systems [7–10]. For example, by constructing an appropriate co-positive Lyapunov-Krasovskii functional, Xiang and Xiang [7] investigated the exponential stability, L_1-gain performance and controller design problems for a class of positive switched systems with time-varying delay. Liu and Xiang [8] addressed the exponential L_1 output tracking control for positive switched linear systems with time-varying delays. Liu [9] studied the stability problem of delayed nonlinear positive switched systems. Sun [10] considered the stability of positive switched linear systems with time delay.

To our knowledge, the vast majority of existing results on the stability of positive switched systems concern Lyapunov stability, which indicates stability over an infinite time interval. It is well known that asymptotic stability can ensure that the stead-state performance of the systems is good, not necessarily have the corresponding transient performance, and may even make the transient performance of the systems very poor. For example, Zhao and Sun [11] addressed the absolute exponential stability for switching Lurie systems with time-varying delays using the switching time-varying Lyapunov function technique. Compared with Lyapunov stability, some systems do not need to stabilize over finite time interval but only need to stabilize over finite time interval. This stability of the system is named finite time stability (FTS), which means that given a bound on the original conditions, the system's solution states do not transcend a definite threshold during a concrete time range (see [12]). The transient performance of finite-time stability is better. Nevertheless, in some practical circumstances, some control systems usually do not need to be stable at infinite time; these control systems only need to be stable at a finite time interval. Finite-stability problems receive plenty of applications; see [13–18]. For example, Garcia et al. [13] studied the finite-time stabilization of linear time-varying continuous systems by Lyapunov differential matrix equations. Zhao et al. [14] considered the finite-time stability of linear time-varying singular systems with impulsive effects. Wang et al. [15] investigated the input-output finite-time stability for a class of networked control systems with network-induced delay. By a singular value decomposition approach, Thanh et al. [16] studied the finite-time stability of singular nonlinear switched time-delay systems. Wei et al. [17] studied the finite-time stability of linear discrete switched singular systems with finite-time unstable subsystems. Zhang et al. [18] studied the finite-time stability and stabilization of linear discrete time-varying stochastic systems with multiplicative noise.

There are some meaningful FTS results for switched systems [19–25]. Hou et al. [19] provided a sufficient FTS condition for switched linear systems with a new multiple Lyapunov functional. Xiang and Xiao [20] proposed a number of sufficient finite-time boundness and stability conditions as switched linear systems. Liu et al. [21] regarded the FTS for positive switched linear delayed systems (PSLDSs) with piecewise co-positive Lyapunov–Krasovskii functional. With the help of a time-varying co-positive Lyapunov functional, Chen and Yang [22] investigated the FTS as positive switched linear systems. After that, Xu et al. [23] established the FTS conditions of positive switched linear delayed systems by Lyapunov–Krosovskii functional. Zhang and Zhu [24] extended the finite-time input-to-state stability to switched stochastic time-varying nonlinear systems with time delays by the Razumikhin theorem, comparison principle, and average dwell-time approach. Recently, by using a type of linear time-varying co-positive Lyapunov functional, Huang et al. dealt with the FTS of a positive switched linear delayed system (PSLDS) as follows [25]:

$$\begin{cases} \dot{z}(t) = M_{\delta(t)}z(t) + N_{\delta(t)}z(t-\rho), & t \in [0, S], \\ z(t) = \nu(t), & t \in [-\rho, 0], \end{cases} \tag{1.1}$$

where $z(t) \in R^n$ represents the state vector, $\delta(t)$: $[0,\infty) \to \{1, 2, \cdots, q\}$ expresses the switching rule, which implies a piecewise continuous function, $q > 1$ expresses an integer, M_l and N_l symbolize system matrices of the lth subsystem for $l \in \{1, 2, \cdots, q\}$, $\rho > 0$ stands for time delay, $\nu(t)$: $[-\rho, 0] \to R^n$ shows the continuous vector valued original function.

However, in some practical applications, impulsive interference is inevitable. Impulsive behaviors are regarded as a dynamical course that expresses a state that changes abruptly at some time points [26–28]. Nowadays, numerous researchers focus on the stability of positive systems with impulsive interference, and several significant results have emerged in [29–31]. Specifically, by constructing a time-varying co-positive Lyapunov function and utilizing the average impulsive interval approach, Hu et al. [29] investigated the finite-time stability and stabilization problems of positive systems with impulses. Hu et al. [30] addressed the exponential stability and positive stabilization problems of impulsive positive systems with time delay. Based on the linear co-positive Lyapunov function method, Briat [31] obtained the stability and stabilization of linear impulsive positive systems under arbitrary, constant, minimum, maximum, and range dwell-time. Therefore, it's beneficial to study the switched systems with impulsive effects.

Then, inspired by the method of [25], we will further study the FTS for a PSLDS with impulsive effects as follows:

$$\begin{cases} \dot{z}(t) = M_{\delta(t)}z(t) + N_{\delta(t)}z(t-\rho), \quad t \in [0,S],\ t \neq t_r, \\ z(t^+) = g_{\delta(t^+)}(z(t^-)), \quad t = t_r,\ r = 1,2,3,\cdots, \\ z(t) = \nu(t), \quad t \in [-\rho, 0], \end{cases} \tag{1.2}$$

where $g_l(z)$: $R^n \to R^n$ are impulses for $l \in \{1, 2, \cdots, q\}$, t_r, $r = 1, 2, \cdots$, those are not just the switching instants but the impulsive instants, and satisfying $0 < t_r < t_{r+1}$ and

$$\lim_{r\to\infty} t_r = \infty.$$

When $t \in [t_r, t_{r+1})$, the $\delta(t_r)$th subsystem is motivated, $r = 0, 1, 2, \cdots$. At the switching and impulsive instants, let $z(t) = z(t^+)$ at $t = t_r$.

The differences between this paper and the related results lie mainly in the following two aspects: First, we introduce the impulsive effects of PSLDS. Next, we adjust the linear term coefficient of the linear vector function and add the vector constraints at every impulsive instant. The primary contributions are highlighted as follows:

(1) Compared with the existing results [21–23, 25], impulsive effects are introduced to the positive switched systems. By constructing a linear time-varying co-positive Lyapunov functional, we deduce several new finite-time stability criteria for the models in Huang et al. [25] under synchronous impulse control.

(2) Inspired by Chen and Yang [22] and Huang et al. [25], a linear time-varying co-positive Lyapunov functional is established. It should be emphasized that the linear term coefficient of the linear vector of the Lyapunov functional is adjusted to the difference between the weighting vector and the given vector. Then, at every impulsive and switching instant, we use impulsive vector constraints to replace the vector constraints at every switching instant.

(3) We apply the obtained results to the finite-time stability of linear time-varying systems with impulses. We design new linear programming algorithm using Lingo software to better demonstrate the feasibility of our results, and our programming process is relatively easy.

The framework of this paper is organized as follows. Indispensable definitions and preliminaries for this paper are provided in Section 2. Section 3 is dedicated to proving the main FTS criteria. In Section 4, we apply the obtained FTS criteria to the linear time-varying delayed systems with impulsive effects. Three examples are given in Section 5 to support our main theoretical results. A conclusion and some future directions are discussed in Section 6.

2. Preliminaries

Some necessary notations or symbols are given here. R^n represents the group of n-dimensional real vector spaces. $R^{n\times n}$ represents $n \times n$-dimensional real matrices space. Two vectors $p, q \in R^n$, $p \succ 0$ implies its entry $p_l \succ 0$ for $l \in \{1, 2, \cdots, n\}$, $p \succeq q$ (or $q \preceq p$) if $p_l \succeq q_l$ (or $p_l \preceq q_l$) for $l \in \{1, 2, \cdots, n\}$. $p \in R^n$ expresses a positive vector, provided that $p \succ 0$. A vector $p \in R^n$ and a matrix $M \in R^{n\times n}$, $p^\top$ and $M^\top$ characterize the transpose of p and M, respectively. Provided that all off-diagonal elements of matrix N are nonnegative, then N is called a Metzler matrix. If all elements of matrix C are nonnegative, then C is called positive.

Set $t_0 = 0$ and $t_q = S$. The switching and impulsive moments are $0 < t_1 < \cdots < t_r < \cdots < t_{q-1} < S$, where $q > 1$ is an integer.

The next definitions, assumptions, and lemmas are important for the formation of the crucial FTS criteria.

Definition 2.1. ([25]) Provided that $\nu(t) \geq 0, t \in [-\rho, 0]$, for any switching signal $\delta(t)$, the state solution trajectory $z(t)$ fulfills $z(t) \geq 0$, for any $t \geq 0$, then PSLDS (1.2) is positive.

Definition 2.2. ([25]) Given positive scalars S, $b_1 < b_2$ and a positive vector $p \in R^n$, PSLDS (1.2) is called finite-time stable about (S, p, b_1, b_2) provided that

$$\sup_{t\in[-\rho,0]} p^\top z(t) \leq b_1 \Rightarrow p^\top z(t) \leq b_2,$$

for $t \in [0, S]$.

Definition 2.3. ([25]) $N_\delta(0, t)$ defines the number of switching points in the time interval $(0, S)$. Provided that there is a positive number $T_b > 0$ in order that

$$N_\delta(0, S) \leq \frac{S}{T_b}$$

holds, T_b represents the average dwell time (ADT) of $\delta(t)$ in the time interval $[0, S]$.

An assumption is proposed as follows:

(H_1) Impulsive function $g_{fl}(z) \geq 0$ for $z \geq 0$, and there exist a group of positive matrices $D_l \in R^{n\times n}$, satisfying $g_{fl}(z) \leq D_l z$ for $z \in R^n$, where $l, f \in \{1, 2, \cdots, q\}$, $\sigma(t_r^+) = f, \sigma(t_r^-) = l, l \neq f$.

Lemma 2.1. *([25]) PSLDS* (1.2) *is positive if and only if M_l represents a Metzler matrix and $N_l \geq 0$, for any $l \in \{1, 2, \cdots, q\}$.*

3. Main results

Firstly, we propose FTS criteria for PSLDS (1.2) under arbitrary switching.

Theorem 3.1. *Suppose that (H_1) holds. Given positive scalars S, $b_1 < b_2$, a positive vector $p \in R^n$, PSLDS* (1.2) *is finite-time stable about (S, p, b_1, b_2) under arbitrary switching, provided that there are a positive vector $\psi \in R^n$, a free weighting vector $\chi \in R^n$, parameters $\delta_1 > 0$, $\delta_2 > 0$, and $\omega \geq 1$, in order to satisfy the following inequalities:*

$$-\chi^\top + \psi^\top + \psi^\top M_f + [\psi^\top - \rho(\chi^\top - \psi^\top)]N_l \leq 0, \tag{3.1}$$

$$\begin{aligned}&-\chi^\top + \psi^\top + [\psi^\top - S(\chi^\top - \psi^\top)]M_f\\ &+ [\psi^\top - (S+\rho)(\chi^\top - \psi^\top)]N_l\\ &\leq 0,\end{aligned} \tag{3.2}$$

$$p^\top \leq \psi^\top, \quad p^\top \leq [\psi^\top - S(\chi^\top - \psi^\top)], \tag{3.3}$$

$$0 \leq \psi^\top N_l, \quad 0 \leq (\psi^\top - (S+\rho)(\chi^\top - \psi^\top))N_l, \tag{3.4}$$

$$\psi^\top \leq \delta_1 p^\top, \quad \psi^\top N_l \leq \delta_2 p^\top, \quad (\psi^\top - \rho(\chi^\top - \psi^\top))N_l \leq \delta_2 p^\top, \tag{3.5}$$

$$\psi_f^\top D_l^\top - \omega\psi_l^\top \leq 0, \ \omega(\chi_l^\top - \psi_l^\top) - (\chi_f^\top - \psi_f^\top)D_l^\top \leq 0, \ \ l \neq f, \tag{3.6}$$

$$(\delta_1 + \rho\delta_2)b_1 \leq b_2, \tag{3.7}$$

hold, where $l, f \in \{1, 2, \cdots, q\}$.

Proof. For any $v(t) \geq 0$, $t \in [-\rho, 0]$, it is obvious that $z(t) \geq 0$, $t \geq 0$. Construct the linear time-varying co-positive Lyapunov functional as follows:

$$V(t, z(t)) = \eta^\top(t)z(t) + \int_{t-\rho}^{t} \eta^\top(\xi+\rho)N_{\delta(\xi+\rho)}z(\xi)\mathrm{d}\xi, \quad t \in [0, S],$$

where

$$\eta(t) = \psi - t(\chi - \psi), \quad t \in [0, S+\rho], \quad \delta(t) = \delta(S^-)$$

for $t \geq S$, and $\delta(S^-)$ characterizes the left limitation of $\delta(t)$ at $t = S$. For any $t \in [0, S]$, let

$$\delta(t+\rho) = l, \quad \delta(t) = f, l, \quad f \in \{1, 2, \cdots, q\},$$

which are related to t.

Taking the derivative of $V(t, z(t))$ about t along the solution trajectory of PSLDS (1.2), we obtain

$$\begin{aligned}\dot{V}(t, z(t)) =& \dot{\eta}^\top(t)z(t) + \eta^\top(t)M_f z(t) + \eta^\top(t)N_f z(t-\rho)\\ &+ \eta^\top(t+\rho)N_l z(t) - \eta^\top(t)N_f z(t-\rho)\\ =&[\dot{\eta}^T(t) + \eta^T(t)M_f + \eta^\top(t+\rho)N_l]z(t)\\ =&[-\chi^T + \psi^\top + (\psi^T - t(\chi^T - \psi^\top))M_f\\ &+ (\psi^T - (t+\rho)(\chi^T - \psi^\top))N_l]z(t)\\ =&P_{lf}(t)z(t),\end{aligned}$$

where

$$\begin{aligned}P_{lf}(t) =& -\chi^\top + \psi^\top + (\psi^\top - t(\chi^\top - \psi^\top))M_f\\ &+ (\psi^\top - (t+\rho)(\chi^\top - \psi^\top))N_l.\end{aligned}$$

Then, we prove $P_{lf}(t) \leq 0$ for $l, f \in \{1, 2, \cdots, q\}$ and $t \in [0, S]$. Because

$$\dot{P}_{lf}(t) = (-\chi^\top + \psi^\top)(M_f + N_l),$$

in other words, $P_{lf}(t)$ is monotone with t in the time interval $[0, S]$, for $l, f \in \{1, 2, \cdots, q\}$. Furthermore, conditions (3.1) and (3.2) show that $P_{lf}(0) \leq 0$ and $P_{lf}(S) \leq 0$, respectively. After that, it concludes from the monotonicity of $P_{lf}(t)$ that $P_{lf}(t) \leq 0$, for $l, f \in \{1, 2, \cdots, q\}$ and $t \in [0, S]$, so $\dot{V}(t, z) \leq 0$ for $t \in [0, S]$. Condition (3.5) indicates that

$$\eta^\top(t)N_l \leq \delta_2 p^\top$$

for $t \in [0, \rho]$ and $l \in \{1, 2, \cdots, q\}$. Furthermore, condition (3.5) also shows that

$$\begin{aligned}V(t, z(t)) \leq& V(0, z(0))\\ =& \psi^\top z(0) + \int_{-\rho}^{0} \eta^\top(\xi+\rho)N_{\delta(\xi+\rho)}z(\xi)\mathrm{d}\xi\\ \leq& \delta_1 p^\top z(0) + \delta_2 \int_{-\rho}^{0} p^\top z(\xi)\mathrm{d}\xi, t \in [0, S].\end{aligned} \tag{3.8}$$

At every switching and impulsive instant t_r, due to the definition of $V(t, z(t))$, we acquire

$$V(t_r, z(t_r)) = \eta^\top(t_r)z(t_r) + \int_{t_r-\rho}^{t_r} \eta^\top(\xi+\rho)N_{\delta(\xi+\rho)}z(\xi)\mathrm{d}\xi.$$

By condition (3.6), we obtain

$$\begin{aligned}&(\psi_f^\top D_l^\top - \omega\psi_l^\top) + t_r[\omega(\chi_l^\top - \psi_l^\top) - (\chi_f^\top - \psi_f^\top)D_l^\top]\\ &= [\psi_f^\top - t_r(\chi_f^\top - \psi_f^\top)]D_l^\top - \omega[\psi_l^\top - t_r(\chi_l^\top - \psi_l^\top)]\\ &\leq 0.\end{aligned}$$

Therefore,

$$\begin{aligned}&[\psi_f^\top - t_r(\chi_f^\top - \psi_f^\top)]D_l^\top \leq \omega[\psi_l^\top - t_r(\chi_l^\top - \psi_l^\top)]\\ &\Longleftrightarrow \eta(t_r^+) \leq \omega\eta(t_r^-), \quad \delta(t_r^+) = f, \delta(t_r^-) = l.\end{aligned} \tag{3.9}$$

From (3.9), we gain

$$V(t_r^+, z(t_r^+)) \leq \omega V(t_r{}^-, z(t_r{}^-)), \tag{3.10}$$

where $t = t_r,\ r = 0, 1, 2, 3, \cdots, q$. By (3.10), we get

$$V(t_r, z(t_r)) \leq \omega^{N(0,S)}V(0, z(0)), \tag{3.11}$$

where $N(0, S)$ expresses the number of impulses in time interval $(0, S)$.

(i) At every switching and impulsive instant t_r, we obtain

$$V(t_r, z(t_r)) \geq p^\top z(t_r). \tag{3.12}$$

If

$$\sup_{t\in[-\rho,0]} p^\top z(t) \leq b_1,$$

then

$$p^\top z(0) \leq b_1.$$

Since (3.5), (3.8) and (3.11), we have

$$V(t_r, z(t_r)) \leq \omega^{N(0,S)}(\delta_1 + \rho\delta_2)b_1. \tag{3.13}$$

Thus, if

$$p^\top z(0) \leq b_1,$$

we conclude from (3.12) to (3.13) that

$$p^\top z(t_r) \leq V(t_r, z(t_r)) \leq \omega^{N(0,S)}(\delta_1 + \rho\delta_2)b_1 \leq b_2.$$

(ii) When $t \in [0, S]$, $t \neq t_r$, conditions (3.3) and (3.4) indicate that $\eta^\top(t) \geq p^\top$, for $t \in [0, S]$, $t \neq t_r$ and

$$\eta^\top(t)N_l \geq 0,$$

for $t \in [0, S+\rho]$, $t \neq t_r$ and $l \in \{1, 2, \cdots, q\}$. Accordingly, provided that

$$\sup_{t\in[-\rho,0]} p^\top z(t) \leq b_1,$$

from (3.7) and (3.8), we get the conclusion that

$$p^\top z(t) \leq V(t, z(t)) \leq (\delta_1 + \rho\delta_2)b_1 \leq \omega^{N(0,S)}(\delta_1 + \rho\delta_2)b_1 \leq b_2.$$

Consequently, PSLDS (1.2) is finite-time stable about (S, p, b_1, b_2). □

Remark 3.1. In the course of proving Theorem 3.1, compared to [25], we define that the linear term coefficient of the linear vector function $\eta(t)$ is about ψ and the weighting vector χ (not required $\chi > 0$), which makes the results more accurate.

Remark 3.2. Condition (3.6) is a vector constraint at every impulsive instant. If there exists a common positive vector ψ and a free weighting vector χ, the conditions (3.1)–(3.5) hold. If we cannot find the common vectors ψ and χ, we further construct multiple time-varying linear co-positive Lyapunov functionals to deduce the next FTS criteria for PSLDS (1.2).

Second, we will further investigate the FTS of PSLDS (1.2) under ADT switching.

Theorem 3.2. *Suppose that* (H_1) *holds. Given positive scalars* S, $b_1 < b_2$, *and a positive vector* $p \in R^n$, *PSLDS* (1.2) *is finite-time stable about* (S, p, b_1, b_2) *under arbitrary switching, if there exist positive vectors* $\psi_l \in R^n$, *free vectors* $\chi_l \in R^n$, *parameters* $\omega \geq 1$, $\varepsilon \in R$, $\delta_1 > 0$, $\delta_2 > 0$ *in order to satisfy the following inequalities:*

$$\begin{aligned}&-\chi_f^\top + \psi_f^\top + \psi_f^\top M_f + e^{-\varepsilon\rho}\left(\psi_f^\top - \rho(\chi_f^\top - \psi_f^\top)\right)N_f\\ &-\varepsilon\psi_f^\top \leq 0,\end{aligned} \tag{3.14}$$

$$\begin{aligned}&-\chi_f^\top + \psi_f^\top + \left(\psi_f^\top - S(\chi_f^T - \psi_f^\top)\right)M_f - \varepsilon\left(\psi_f^\top - S(\chi_f^\top - \psi_f^\top)\right)\\ &+ e^{-\varepsilon\rho}\left(\psi_f^\top - (S+\rho)(\chi_f^\top - \psi_f^\top)\right)N_f \leq 0,\end{aligned} \tag{3.15}$$

$$\psi_f^\top D_l^\top - \omega\psi_l^\top \leq 0,\ \ \omega(\chi_l^\top - \psi_l^\top) - (\chi_f^\top - \psi_f^\top)D_l^\top \leq 0,\ l \neq f, \tag{3.16}$$

$$p^\top \le \psi_f^\top,\ \ p^\top \le \psi_f^\top - S(\chi_f^\top - \psi_f^\top), \tag{3.17}$$

$$0 \le \psi_f^\top N_f,\ \ 0 \le \left(\psi_f^\top - (S+\rho)(\chi_f^\top - \psi_f^\top)\right) N_f, \tag{3.18}$$

$$\psi_f^\top \le \delta_1 p^\top,\ \ \psi_f^\top N_f \le \delta_2 p^\top,\ \ \left(\psi_f^\top - \rho(\chi_f^\top - \psi_f^\top)\right) N_f \le \delta_2 p^\top, \tag{3.19}$$

$$\omega^{\frac{S}{T_b}} e^{\varepsilon_+ S}\left(\delta_1 + \rho\delta_2 \frac{1-e^{-\varepsilon\rho}}{\varepsilon}\right) b_1 \le b_2, \tag{3.20}$$

hold, where $l, f \in \{1, 2, \cdots, q\}$, $\varepsilon_+ = \max\{\varepsilon, 0\}$ *and*

$$\frac{1-e^{-\varepsilon\rho}}{\varepsilon} = 0$$

when $\varepsilon = 0$.

Proof. For $v(t) \ge 0$, $t \in [-\rho, 0]$, we get that $z(t) \ge 0$ for $t \ge 0$. Construct a piecewise linear time-varying co-positive Lyapunov functional as follows:

$$V_{\delta(t)}(t, z(t)) = \eta_{\delta(t_r)}^\top(t) z(t) + \int_{t-\rho}^{t} e^{\varepsilon(t-\xi-\rho)} \eta_{\delta(t_r)}^\top(\xi+\rho) N_{\delta(t_r)} z(\xi) \mathrm{d}\xi,$$

$t \in [t_r, t_{r+1})$, where

$$\eta_{\delta(t)}(t) = \psi_{\delta(t)} - t(\chi_{\delta(t)} - \psi_{\delta(t)}),$$

$t \in [0, S+\rho]$, and

$$\delta(t) = \delta(t_{p-1})$$

for $t \in [S, S+\rho]$. The proof will be separated into the next three steps.

(I). For any $t \in [t_r, t_{r+1})$, $r = 0, 1, \cdots, q-1$, we prove that

$$V_{\delta(t)}(t, z(t)) \le e^{\varepsilon(t-t_r)} V_{\delta(t_r)}(t_r, z(t_r)). \tag{3.21}$$

Denote $\delta(t) = f$, when $t \in [t_r, t_{r+1})$, $r = 0, 1, \cdots, q-1$. Then, the derivative of $V_f(t, z(t))$ about $t \in [t_r, t_{r+1})$ along the solution trajectory of PSLDS (1.2) shows

$$\begin{aligned}
\dot{V}_f(t, z(t)) =& \dot{\eta}_f^\top(t) z(t) + \eta_f^\top(t) \dot{z}(t) \\
&+ \varepsilon \int_{t-\rho}^{t} e^{\varepsilon(t-\xi-\rho)} g_{\delta(t_r)}^\top(\xi+\rho) N_{\delta(t_r)} z(\xi) \mathrm{d}\xi \\
&+ e^{-\varepsilon\rho} \eta_f^\top(t+\rho) N_f z(t) - \eta_f^\top(t) N_f z(t-\rho) \\
=& (-\chi_f^\top + \psi_f^\top) z(t) + \eta_f^\top(t) M_f z(t) \\
&+ e^{-\varepsilon\rho} \eta_f^\top(t+\rho) N_f z(t) \\
&+ \varepsilon \int_{t-\rho}^{t} e^{\varepsilon(t-\xi-\rho)} \eta_{\delta(t_r)}^\top(\xi+\rho) N_{\delta(t_r)} z(\xi) \mathrm{d}\xi.
\end{aligned}$$

Therefore, we get:

$$\begin{aligned}
&\dot{V}_f(t, z(t)) - \varepsilon V_f(t, z(t)) \\
&= (-\chi_f^\top + \psi_f^\top) z(t) + \left(\psi_f^\top - t(\chi_f^\top - \psi_f^\top)\right) M_f z(t) \\
&\quad - \varepsilon \left(\psi_f^\top - t(\chi_f^\top - \psi_f^\top)\right) z(t) \\
&\quad + e^{-\varepsilon\rho} \left(\psi_f^\top - (t+\rho)(\chi_f^\top - \psi_f^\top)\right) N_f z(t) \\
&= Q_f(t) z(t),
\end{aligned}$$

where

$$\begin{aligned}
Q_f(t) =& -\chi_f^\top + \psi_f^\top + \left(\psi_f^\top - t(\chi_f^\top - \psi_f^\top)\right) M_f \\
&- \varepsilon \left(\psi_f^\top - t(\chi_f^\top - \psi_f^\top)\right) \\
&+ e^{-\varepsilon\rho} \left(\psi_f^\top - (t+\rho)(\chi_f^\top - \psi_f^\top)\right) N_f.
\end{aligned}$$

Since

$$\dot{Q}_f(t) = (\chi_f^\top - \psi_f^\top)(-M_f + \varepsilon - e^{-\varepsilon\rho} N_f),$$

then $Q_f(t)$ is monotone over an interval $[0, S]$, it concludes that $Q_f(t) \le 0$ for $t \in [0, S]$ if and only if $Q_f(0) \le 0$ and $Q_f(S) \le 0$. After that, conditions (3.14) and (3.15) indicate that $Q_f(t) \le 0$, for $t \in [t_r, t_{r+1})$, and hence

$$\dot{V}_f(t, z(t)) \le \varepsilon V_f(t, z(t)), \quad t \in [t_r, t_{r+1}).$$

Thus, inequality (3.21) is held.

(II). We indicate that for any switching signal $\delta(t)$,

$$V_{\delta(t_r)}(t_r, z(t_r)) \le \omega V_{\delta(t_r^-)}(t_r, z(t_r)), \tag{3.22}$$

holds, where $r = 1, 2, \cdots, q-1$. At every switching and impulsive instant t_r, on the basis of the definition of $V(t, z(t))$, we obtain that

$$\begin{aligned}
V_{\delta(t_r)}(t_r, z(t_r)) &= \eta_{\delta(t_r)}^\top(t_r) z(t_r) \\
&+ \int_{t_r-\rho}^{t_r} e^{\varepsilon(t_r-\xi-\rho)} \eta_{\delta(t_r)}^\top(\xi+\rho) N_{\delta(t_r)} z(\xi) \mathrm{d}\xi.
\end{aligned}$$

From condition (3.16), we obtain

$$\begin{aligned}
&(\psi_f^\top D_l^\top - \omega\psi_l^\top) + t_r[\omega(\chi_l^\top - \psi_l^\top) - (\chi_f^\top - \psi_f^\top) D_l^\top] \\
&= [\psi_f^\top - t_r(\chi_f^\top - \psi_f^\top)] D_l^\top - \omega[\psi_l^\top - t_r(\chi_l^\top - \psi_l^\top)] \\
&\le 0,
\end{aligned}$$

where $\delta(t_{l-1}^+) = l$, $\delta(t_{l-1}^-) = l-1$.

Accordingly,

$$
\begin{aligned}
&[\psi_f^\top - t_r(\chi_f^\top - \psi_f^\top)]D_l^\top \le \omega[\psi_l^\top - t_r(\chi_l^\top - \psi_l^\top)]\\
&\Longleftrightarrow g_{\delta(t_r^+)}(t_r^+) \le \omega g_{\delta(t_r^-)}(t_r^-).
\end{aligned} \tag{3.23}
$$

From (3.23), we get

$$V_{\delta(t_r^+)}(t_r^+, z(t_r^+)) \le \omega V_{\delta(t_r^-)}(t_r^-, z(t_r^-)).$$

(III). According to (3.21) and (3.22), the following relations are established:

(1) When $t \in [t_0, t_1)$,

$$V_{\delta(t)}(t, z(t)) \le e^{\varepsilon(t-t_0)} V_{\delta(t_0)}(t_0, z(t_0)) = e^{\varepsilon t} V_{\delta(0)}(0, z(0)).$$

(2) When $t = t_1$,

$$V_{\delta(t_1)}(t_1, z(t_1)) \le \omega V_{\delta(t_1^-)}(t_1, z(t_1)) = \omega e^{\varepsilon t_1} V_{\delta(0)}(0, z(0)).$$

(3) When $t \in [t_1, t_2)$,

$$V_{\delta(t)}(t, z(t)) \le e^{\varepsilon(t-t_1)} V_{\delta(t_1)}(t_1, z(t_1)) \le \omega e^{\varepsilon t} V_{\delta(0)}(0, z(0)).$$

(4) When $t = t_2$,

$$V_{\delta(t_2)}(t_2, z(t_2)) \le \omega V_{\delta(t_2^-)}(t_2, z(t_2)) = \omega^2 e^{\varepsilon t_2} V_{\delta(0)}(0, z(0)), \cdots.$$

By repeating the above process, we obtain

$$V_{\delta(t)}(t, z(t)) \le \omega^{N_\delta(0,t)} e^{\varepsilon t} V_{\delta(0)}(0, z(0)),\ t \in [0, S].$$

Owing to

$$N_\delta(0, t) \le N_\delta(0, S) \le \frac{S}{T_b}$$

and $\omega \ge 1$, it shows that

$$V_{\delta(t)}(t, z(t)) \le \omega^{\frac{S}{T_b}} e^{\varepsilon_+ S} V_{\delta(0)}(0, z(0)),\ \ t \in [0, S]. \tag{3.24}$$

Conditions (3.17) and (3.18) indicate that

$$V_{\delta(t)}(t, z(t)) \ge p^\top z(t),\ \ t \in [0, S], \tag{3.25}$$

condition (3.19) demonstrates that

$$
\begin{aligned}
V_{\delta(0)}(0, z(0)) &\le \delta_1 p^\top z(0) + \delta_2 \int_{-\rho}^{0} e^{\varepsilon(-\xi-\rho)} p^\top z(\xi) \mathrm{d}\xi\\
&\le \left(\delta_1 + \rho\delta_2 \frac{1 - e^{-\varepsilon\rho}}{\varepsilon}\right) b_1.
\end{aligned}
$$

Combining the above inequality, (3.20), (3.24) and (3.25), we get that

$$p^\top z(t) \le b_2,$$

for $t \in [0, S]$. Accordingly, PSLDS (1.2) is finite-time stable about (S, p, b_1, b_2). □

Remark 3.3. Condition (3.16) is a vector constraint at every impulsive instant. Compared to [25], we use impulsive constraints to replace the vector constraints at switching instants in proofing the process (II).

Remark 3.4. In this section, the delays represent constant delays, and when the delays become time-varying delays, we will next consider establishing an appropriate Lyapunov function to obtain the FTS criteria.

4. Applications in FTS of linear time-varying delayed systems

By the same approach, we investigate the linear time-varying delayed system with impulsive effects as follows:

$$
\begin{cases}
\dot{z}(t) = M(t)z(t) + N(t)z(t-\rho),\ \ t \in [0, S],\ t \ne t_r,\\
z(t^+) = g_{\delta(t^+)}(z(t^-)),\ \ t = t_r,\ r = 1, 2, 3 \cdots,\\
z(t) = \nu(t),\ \ t \in [-\rho, 0],
\end{cases} \tag{4.1}
$$

where the definitions of z, ρ and ν are similar to PSLDS (1.2),

$$M(t) = [m_{lf}(t)] \in R^{n\times n}$$

and

$$N(t) = [n_{lf}(t)] \in R^{n\times n}$$

represent piecewise continuous matrix functions. Separate the time interval $[0, S]$ into Q subintervals $[t_{r-1}, t_r]$ for $r = 1, 2, \cdots, Q$, where

$$t_r = \frac{r}{Q} S$$

for $r = 0, 1, \cdots, Q$. We present the next two hypotheses:

(H_2) There exist matrices

$$\bar{M}_r = [\bar{m}_{lf}^{(r)}]$$

in order that

$$m_{ll}(t) \le \bar{m}_{ll}^{(r)} \ \text{ and } \ m_{lf}(t) \le \bar{m}_{lf}^{(r)}$$

for $t \in [t_{r-1}, t_r]$, $r = \{1, 2, \cdots, Q\}$, $l, f \in \{1, 2, \cdots, q\}$, $l \ne f$.

(H_3) There exist matrices

$$\bar{N}_r = [\bar{n}_{lf}^{(r)}]$$

in order that

$$n_{ll}(t) \le \bar{n}_{ll}^{(r)}$$

and

$$n_{lf}(t) \le \bar{n}_{lf}^{(r)}$$

for $t \in [t_{r-1}, t_r]$, $r = \{1, 2, \cdots, Q\}$, $l, f \in \{1, 2, \cdots, q\}$, $l \neq f$.

Note that system (4.1) is not confined to being positive. For $t \in [t_{l-1}, t_l)$, $l \in \{1, 2, \cdots, q\}$, construct the following time-varying Lyapunov functional:

$$V_l(t, z(t)) = \eta_l^\top(t) \mid z(t) \mid + \int_{t-\rho}^{t} e^{\varepsilon(t-\xi-\rho)} \eta_l^\top(\xi+\rho) \bar{N}_l \mid z(\xi) \mid \mathrm{d}\xi,$$

where

$$\mid z(t) \mid = (\mid z_1(t) \mid, \cdots, \mid z_n(t) \mid)^\top,$$

$$\eta_1(t) = \psi_1 - t(\chi_1 - \psi_1), \quad t \in [0, \frac{S}{Q}),$$

$$\eta_2(t) = \psi_2 - (t - \frac{S}{Q})(\chi_2 - \psi_2), \quad t \in [\frac{S}{Q}, \frac{2S}{Q}),$$

$$\cdots$$

$$\eta_Q(t) = \psi_Q - (t - \frac{(Q-1)S}{Q})(\chi_Q - \psi_Q), \quad t \in [\frac{(Q-1)S}{Q}, S).$$

Then, we get the next FTS conditions for system (4.1).

Theorem 4.1. *Suppose that* (H_1)–(H_3) *hold. Given positive scalars* S, $b_1 < b_2$ *and a positive vector* $p \in R^n$, *system* (4.1) *is finite-time stable about* (S, p, b_1, b_2). *If there exist positive vectors* $\psi_l \in R^n$, *free vectors* $\chi_l \in R^n$, *parameters* $\omega \ge 1$, $\varepsilon \in R$, $\delta_1 > 0$, $\delta_2 > 0$, *in order that the following inequalities:*

$$-\chi_l^\top + \psi_l^\top + \psi_l^\top \bar{M}_l + e^{-\varepsilon\rho}\left(\psi_l^\top - \rho(\chi_l^\top - \psi_l^\top)\right)\bar{N}_l - \varepsilon\left(\psi_l^\top - (\chi_l^\top - \psi_l^\top)t\right) \le 0, \tag{4.2}$$

$$-\chi_l^\top + \psi_l^\top + \left(\psi_l^\top - S(\chi_l^\top - \psi_l^\top)\right)\bar{M}_l + e^{-\varepsilon\rho}\left(\psi_l^\top - (S+\rho)(\chi_l^\top - \psi_l^\top)\right)\bar{N}_l - \varepsilon\left(\psi_l^\top - (S+\rho)(\chi_l^\top - \psi_l^\top)\right) \le 0, \tag{4.3}$$

$$\psi_{l-1}^\top D_l^\top - \omega\psi_l^\top \le 0, \quad \omega(\chi_l^\top - \psi_l^\top) - (\chi_{l-1}^\top - \psi_{l-1}^\top)D_l^\top \le 0, \tag{4.4}$$

$$p^\top \le \psi_l^\top, p^\top \le \psi_l^\top - \frac{S}{Q}(\chi_l^\top - \psi_l^\top), \tag{4.5}$$

$$0 \le \psi_l^\top \bar{N}_l, \quad 0 \le \left(\psi_l^\top - (\frac{S}{Q} + \rho)(\chi_l^\top - \psi_l^\top)\right)\bar{N}_l, \tag{4.6}$$

$$\psi_1^\top \le \delta_1 p^\top, \ \psi_1^\top \bar{N}_1 \le \delta_2 p^\top, \tag{4.7}$$

$$\left(\psi_1^\top - (\frac{S}{Q} + \rho)(\chi_1^\top - \psi_1^\top)\right)\bar{N}_1 \le \delta_2 p^\top, \tag{4.8}$$

$$e^{\varepsilon_+ S}\omega^{Q-1}\left(\delta_1 + \frac{1 - e^{-\varepsilon\rho}}{\varepsilon}\delta_2\rho\right)b_1 \le b_2, \tag{4.9}$$

hold, where $l \in \{1, 2, \cdots, q\}$, $\varepsilon_+ = \max\{\varepsilon, 0\}$, *and*

$$\frac{1 - e^{-\varepsilon\rho}}{\varepsilon} = 0$$

when $\varepsilon = 0$.

Proof. The proof is separated into three steps:

(I). For $t \in [t_{l-1}, t_l)$, $l \in \{1, 2, \cdots, q\}$, we first demonstrate that

$$V_l(t, z(t)) \le e^{\varepsilon(t-t_{l-1})} V_l(t_{l-1}, z(t_{l-1})). \tag{4.10}$$

$V_l(t, z(t))$ represents the right derivative of $D^+ V_l(t, z(t))$. By the l-th entry of the state $z(t)$, it concludes that

$$D^+ \mid z_l(t) \mid \le m_{ll}(t) \mid z_l(t) \mid + \sum_{f=1, f\neq l}^{n} \mid m_{lf}(t) \| z_f(t) \| + \sum_{f=1, f\neq l}^{n} \mid n_{lf}(t) \| z_f(t) \|, t \in [t_{l-1}, t_l).$$

Because

$$D^+ \mid z_l(t) \mid = \operatorname{sign} z_l(t) \dot{z}_l(t)$$

if $z_l(t) \neq 0$ and

$$D^+ \mid z_l(t) \mid = \mid \dot{z}_l(t) \mid$$

if $z_l(t) = 0$, it shows

$$\begin{aligned} D^+ V_l(t, z(t)) \le & \dot{\eta}_l^\top(t) \mid z(t) \mid + \eta_l^\top(t)\bar{M}_l \mid z(t) \mid \\ & + \eta_l^\top(t)\bar{N}_l \mid z(t-\rho) \mid + e^{-\varepsilon\rho}\eta_l^\top(t+\rho)\bar{N}_l \mid z(t) \mid \\ & - \eta_l^\top(t)\bar{N}_l \mid z(t-\rho) \mid \\ & + \varepsilon \int_{t-\rho}^{t} e^{\varepsilon(t-\xi-\rho)}\eta_l^\top(\xi+\rho)\bar{N}_l \mid z(\xi) \mid \mathrm{d}\xi. \end{aligned}$$

Therefore, we acquire

$$D^+ V_l(t, z(t)) - \varepsilon V_l(t, z(t)) \le \left(-\chi_l^\top + \psi_l^\top + \eta_l^\top(t)\bar{M}_l - \varepsilon\eta_l^\top(t) + e^{-\varepsilon\rho}\eta_l^\top(t+\rho)\bar{N}_l\right) \mid z(t) \mid.$$

Conditions (4.2) and (4.3) indicate that

$$\dot{V}_f(t, z(t)) - \varepsilon V_f(t, z(t)) \le 0,$$

so inequality (4.10) holds for $\forall\, t \in [t_{l-1}, t_l)$.

(II). At every impulsive instant t_{l-1}, from the definition of $V_l(t, z(t))$, we acquire

$$\begin{aligned}&V_l((t_{l-1}, z(t_{l-1}))\\&= g_l^\top(t_{l-1}) \mid z(t_{l-1}) \mid + \int_{t_{l-1}-\rho}^{t_{l-1}} e^{\varepsilon(t_{l-1}-\xi-\rho)}\eta_l^\top(\xi+\rho)\bar{N}_l \mid z(\xi) \mid \mathrm{d}\xi.\end{aligned}$$

From condition (4.4), we get

$$\begin{aligned}&(\psi_{l-1}^\top D_l^\top - \omega\psi_l^\top) + t_{l-1}[\omega(\chi_l^\top - \psi_l^\top) - (\chi_{l-1}^\top - \psi_{l-1}^\top)D_l^\top]\\&= [\psi_{l-1}^\top - t_{l-1}(\chi_{l-1}^\top - \psi_{l-1}^\top)]D_l^\top - \omega[\psi_l^\top - t_{l-1}(\chi_l^\top - \psi_l^\top)]\\&\le 0,\end{aligned}$$

where $\delta(t_{l-1}^+) = l,\ \delta(t_{l-1}^-) = l-1$. Hence,

$$\begin{aligned}[\psi_{l-1}^\top - t_{l-1}(\chi_{l-1}^\top - \psi_{l-1}^\top)]D_l^\top &\le \omega[\psi_l^\top - t_{l-1}(\chi_l^\top - \psi_l^\top)]\\&\Longleftrightarrow \eta_l(t_{l-1}^+)\\&\le \omega\eta_{l-1}(t_{l-1}^-).\end{aligned} \tag{4.11}$$

According to (4.11), we obtain

$$V_l(t_{l-1}, z(t_{l-1})) \le \omega V_{l-1}(t_{l-1}, z(t_{l-1})). \tag{4.12}$$

(III). Combining (4.10) and (4.12), by mathematical induction, we obtain

$$V_l(t, z(t)) \le e^{\varepsilon S}\omega^{Q-1}V_1(0, z(0)),$$

where $t \in [t_{l-1}, t_l), l \in \{1, 2, \cdots, q\}$. Since $\omega \ge 1$, by conditions (4.5)–(4.9), we gain

$$\begin{aligned}p^\top z(t) &\le V_l(t, z(t))\\&\le e^{\varepsilon_+ S}\omega^{Q-1}\left(\delta_1 + \frac{1-e^{-\varepsilon\rho}}{\varepsilon}\rho\delta_2\right)b_1\\&\le b_2,\end{aligned}$$

for $t \in [0, S]$. Accordingly, system (4.1) is finite-time stable about (S, p, b_1, b_2). □

Remark 4.1. Condition (4.4) is a vector constraint at every impulsive instant. Compared to [25], we add the impulsive vector constraints at the segmented time points in proofing the process (II).

Remark 4.2. We regard the effect of the synchronous impulses on the stability of the systems in this paper, and change the conditions at the switching point of the synchronous impulses. In other words, we limit the impulses jump so that the impulses jump is not so large that the system is unstable. In fact, there may be an asynchronous impulses jump at the switching moment. For the case of asynchronous impulses jumping, the difficulty is how to limit the condition constraints under the influence of asynchronous impulses, which will be the next consideration.

Remark 4.3. Chen et al. [32] studied the mean square exponential stability analysis for itô stochastic systems with aperiodic sampling and multiple time delays by the Razumikhin-type theorems method and looped-functionals method. Chen et al. [33] considered the sampled-data synchronization of stochastic Markovian jump neural networks with time-varying delay by constructing a mode-dependent one-sided loop-based Lyapunov functional method. And Chen et al. [34] investigated the stability analysis and controller design issues for aperiodic sampled-data networked control systems with time-varying delays. The method in this paper cannot directly deduce the sampled-data control of the systems. The main difficulty is how to choose an appropriate Lyapunov functional to stabilize sampled-data control systems. We will next use another effective method to study the sampled-data synchronization of PSLDS with impulsive effects. Furthermore, we will study the controller synthesis of aperiodic sampled data of PSLDS with impulsive effects in the future.

5. Simulation examples

The theoretical results are verified by three simulation examples.

Example 5.1. Consider PSLDS (1.2), where $\rho = 1$,

$$M_1 = \begin{bmatrix} 0 & 0.018 & 0.015\\ 0.016 & 0 & 0.045\\ 0.017 & 0.02 & 0\end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0 & 0.02 & 0.018\\ 0.019 & 0 & 0.035\\ 0.02 & 0.028 & 0\end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.038 & 0.02 & 0.045 \\ 0.02 & 0.018 & 0.02 \\ 0.02 & 0.047 & 0.02 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0.018 & 0.02 & 0.028 \\ 0.02 & 0.016 & 0.02 \\ 0.02 & 0.045 & 0.015 \end{bmatrix}.$$

Impulsive matrices take the following form:

$$D_1 = \begin{bmatrix} 1.12 & 0 & 0 \\ 0 & 1.08 & 0 \\ 0 & 0 & 1.07 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 1.04 & 0 & 0 \\ 0 & 1.125 & 0 \\ 0 & 0 & 1.038 \end{bmatrix}.$$

For given $p = (0.7, 0.3, 0.4)^\top$, $S = 50s$, $\varepsilon = 0$, $\omega = 1.01$, $b_1 = 0.92$, and $b_2 = 651$, by solving inequalities from (3.14) to (3.20), we can gain $\delta_1 = 15$, $\delta_2 = 16$, $T_b = 4.13$ s, and

$$\psi_1 = (0.6479487, 0.2388812, 0.4810801)^\top,$$
$$\chi_1 = (0.6481300, 0.2387812, 0.4839241)^\top,$$
$$\psi_2 = (0.6388812, 0.2389793, 0.4339033)^\top,$$
$$\chi_2 = (0.6331107, 0.2389812, 0.4358038)^\top.$$

Therefore, by applying Theorem 3.2, for any switching signal with ADT $T_b = 4.13$ s, PSLDS (1.2) is finite-time stable about (S, p, b_1, b_2).

Figure 1 depicts the switching signal $\delta(t)$ for PSLDS (1.2) designed as

$$\delta(t) = \begin{cases} 1, t \in [0,2) \cup [9,14) \cup [19,23) \cup [25,29) \cup [34,39) \cup [43,47), \\ 2, t \in [2,9) \cup [14,19) \cup [23,25) \cup [29,34) \cup [39,43) \cup [47,50), \end{cases}$$

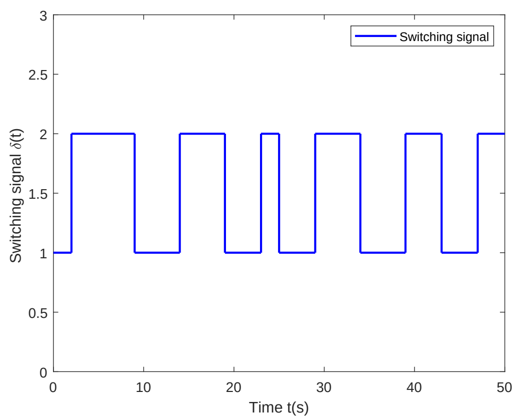

Figure 1. The switching signal of PSLDS (1.2).

Figure 2 implies that the impulsive signal $p(t)$ is designed as $p(t) = 1$, when $t = [2, 9, 14, 19, 23, 25, 29, 34, 39, 43, 47]$, where $p(t) = 1$ indicates that the impulse jumps occur at these moments.

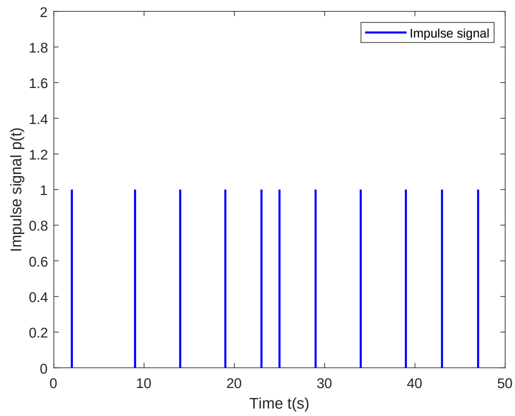

Figure 2. The impulsive signal of PSLDS (1.2).

Choosing the initial value $z(0) = (0.6, 1.1, 0.4)^\top$, Figure 3 signifies that each solution of PSLDS (1.2) is finite-time stable. What's more, Figure 4 fulfills the response of PSLDS (1.2) from 0 to 50 s. Furthermore, PSLDS (1.2) contains the impulses, so the results in [25] can-not be applied to the example. The solution and response states of PSLDS (1.2) are all discontinuous due to the impulse jumping effects.

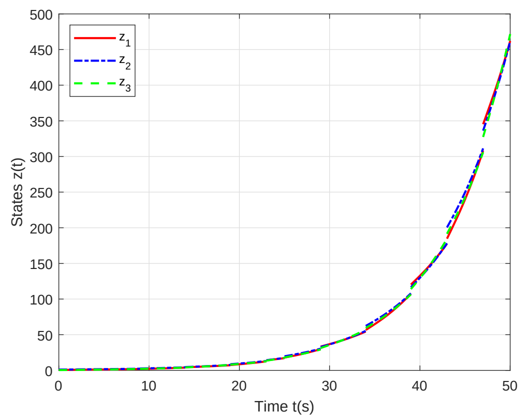

Figure 3. The state solution trajectory of PSLDS (1.2) under signals in Figures 1 and 2.

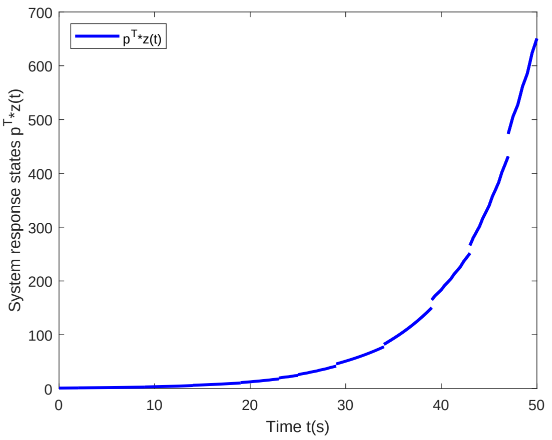

Figure 4. The response of PSLDS (1.2) from 0 to 50 s.

Example 5.2. Let z_1, z_2, z_3 represent the positions of three vehicles, respectively, whose dynamics satisfy PSLDS (1.2), where $\rho = 2$,

$$M_1 = \begin{bmatrix} 0 & 0.03 & 0.01 \\ 0.01 & 0 & 0.05 \\ 0.02 & 0.01 & 0 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0 & 0.01 & 0.02 \\ 0.02 & 0 & 0.04 \\ 0.01 & 0.03 & 0 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.04 & 0.01 & 0.05 \\ 0.01 & 0.02 & 0.01 \\ 0.01 & 0.05 & 0.01 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0.02 & 0.01 & 0.03 \\ 0.01 & 0.02 & 0.01 \\ 0.01 & 0.05 & 0.02 \end{bmatrix}.$$

Impulsive matrices take the following form:

$$D_1 = \begin{bmatrix} 1.02 & 0 & 0 \\ 0 & 1.01 & 0 \\ 0 & 0 & 1.03 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 1.01 & 0 & 0 \\ 0 & 1.02 & 0 \\ 0 & 0 & 1.05 \end{bmatrix}.$$

We now discuss the positional relationship of three vehicles related to the hyperplane

$$p^\top z = 0$$

with

$$p = (0.8, 0.4, 0.5)^\top.$$

For given $S = 90$ s, $\varepsilon = 0$, $\omega = 1.2$, $b_1 = 1.3$, and $b_2 = 10001$, by solving inequalities from (3.14) to (3.20), we can gain $\delta_1 = 12.4$, $\delta_2 = 13.5$, $T_b = 4.73$ s, and

$$\psi_1 = (0.7319823, 0.3319823, 0.5369209)^\top,$$
$$\chi_1 = (0.7319821, 0.3319821, 0.5380869)^\top,$$
$$\psi_2 = (0.7319822, 0.3319822, 0.4319822)^\top,$$
$$\chi_2 = (0.7302752, 0.3319822, 0.4319821)^\top.$$

Therefore, by applying Theorem 3.2, for any switching signal with ADT $T_b = 4.73$ s, PSLDS (1.2) is finite-time stable about (S, p, b_1, b_2).

Figure 5 depicts the switching signal $\delta(t)$ for PSLDS (1.2) designed as

$$\delta(t) = \begin{cases} 1, t \in [0,3) \cup [8,13) \cup [18,22) \cup [26,30) \cup [35,40) \\ \cup [44,48) \cup [52,60) \cup [66,70), \cup[75,80) \cup [86,90), \\ 2, t \in [3,8) \cup [13,18) \cup [22,26) \cup [30,35) \cup [40,44) \\ \cup [48,52) \cup [60,66) \cup [70,75), \cup[80,86), \end{cases}$$

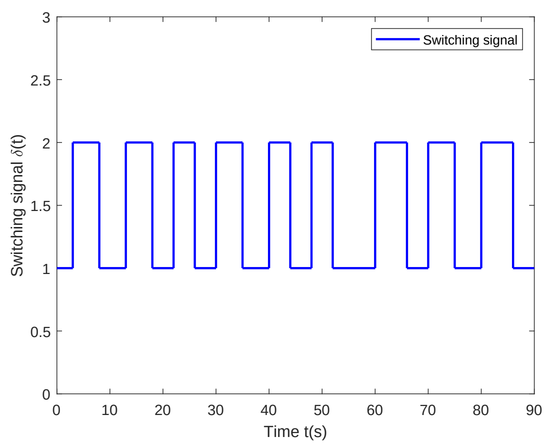

Figure 5. The switching signal of PSLDS (1.2).

Figure 6 implies the impulsive signal $p(t)$ is designed as $p(t) = 1$, when

$$t = [3, 8, 13, 18, 22, 26, 30, 35, 40, 44, 48, 52, 60, 70, 75, 80, 86],$$

where $p(t) = 1$ indicates that the impulses jumps occur at these moments.

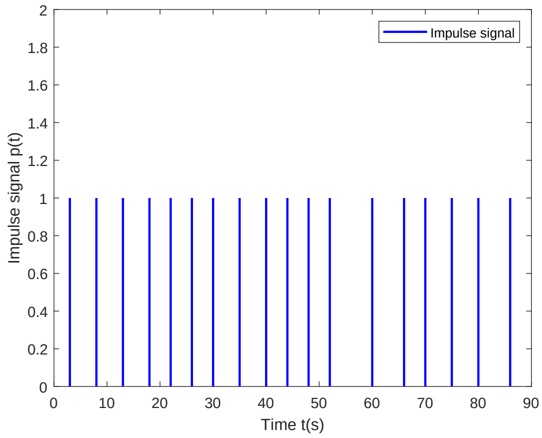

Figure 6. The impulsive signal of PSLDS (1.2).

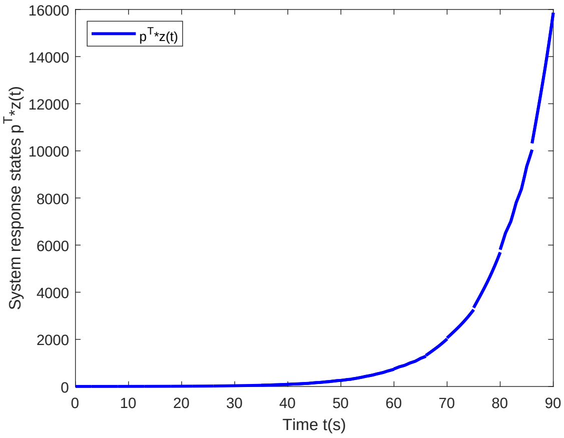

Figure 8. The response of PSLDS (1.2) from 0 to 90 s.

Choosing the initial value

$$z(0) = (0.7, 1.2, 0.5)^{\top},$$

Figure 7 signifies that each solution of PSLDS (1.2) is finite-time stable.

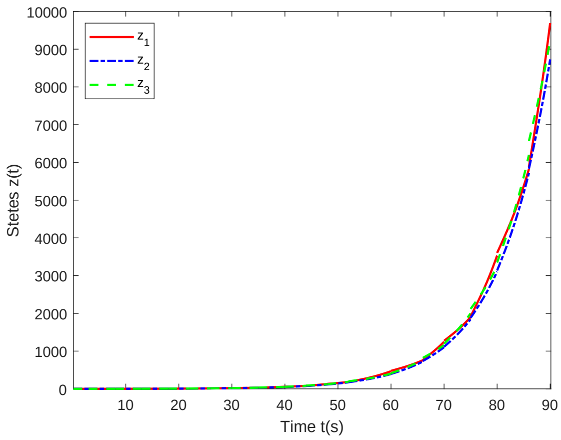

Figure 7. The state solution trajectory of PSLDS (1.2) under signals in Figures 5 and 6.

What is more, Figure 8 fulfills the response of PSLDS (1.2) from 0 to 90 s. Furthermore, because PSLDS (1.2) contains impulses, the results in [25] are invalid. The solution and response states of PSLDS (1.2) are all discontinuous due to the impulse jumping effects.

Example 5.3. Consider system (4.1), where $\rho = 3$. The time interval is divided into three subintervals,

$$M(t) = \begin{cases} \begin{bmatrix} 0 & -\frac{3}{1000}\sin t & \frac{1}{1000} \\ \frac{1}{1000} & 0 & \frac{1}{200} \\ \frac{1}{500}\sin t & \frac{1}{1000} & 0 \end{bmatrix}, 0 \le t \le 30, \\ \begin{bmatrix} 0 & \frac{1}{1000} & \frac{1}{500}\cos t \\ -\frac{1}{500}\sin t & 0 & \frac{1}{250} \\ -\frac{1}{1000}\cos t & \frac{3}{1000} & 0 \end{bmatrix}, 30 \le t \le 60, \\ \begin{bmatrix} 0 & -\frac{1}{1000}\cos t & \frac{1}{500} \\ \frac{1}{500} & 0 & \frac{1}{500} \\ \frac{3}{1000}\sin t & \frac{1}{500} & 0 \end{bmatrix}, 60 \le t \le 90, \end{cases}$$

$$N(t) = \begin{cases} \begin{bmatrix} \frac{1}{25}\sin t & \frac{1}{100} & \frac{1}{20} \\ \frac{1}{100} & \frac{1}{50}\sin t & \frac{1}{100} \\ \frac{1}{100}\cos t & \frac{3}{50} & \frac{1}{100} \end{bmatrix}, 0 \le t \le 30, \\ \begin{bmatrix} \frac{1}{50} & \frac{1}{100} & \frac{3}{100}\sin t \\ \frac{1}{100}\cos t & \frac{3}{100} & \frac{1}{50} \\ \frac{1}{100}\cos t & \frac{1}{20} & \frac{1}{50} \end{bmatrix}, 30 \le t \le 60, \\ \begin{bmatrix} \frac{3}{100}\cos t & \frac{1}{100} & \frac{1}{25} \\ \frac{1}{50} & \frac{3}{100}\sin t & \frac{1}{50} \\ \frac{1}{50}\cos t & \frac{1}{20} & \frac{1}{100} \end{bmatrix}, 60 \le t \le 90. \end{cases}$$

Choose $Q = 3$. According to assumptions (H_2) and (H_3), we have

$$\bar{M}_1 = \begin{bmatrix} 0 & 0.003 & 0.001 \\ 0.001 & 0 & 0.005 \\ 0.002 & 0.001 & 0 \end{bmatrix},$$

$$\bar{M}_2 = \begin{bmatrix} 0 & 0.001 & 0.002 \\ 0.002 & 0 & 0.004 \\ 0.001 & 0.003 & 0 \end{bmatrix},$$

$$\bar{M}_3 = \begin{bmatrix} 0 & 0.001 & 0.002 \\ 0.002 & 0 & 0.002 \\ 0.003 & 0.002 & 0 \end{bmatrix},$$

and

$$\bar{N}_1 = \begin{bmatrix} 0.04 & 0.01 & 0.05 \\ 0.01 & 0.02 & 0.01 \\ 0.01 & 0.06 & 0.01 \end{bmatrix},$$

$$\bar{N}_2 = \begin{bmatrix} 0.02 & 0.01 & 0.03 \\ 0.01 & 0.03 & 0.02 \\ 0.01 & 0.05 & 0.02 \end{bmatrix},$$

$$\bar{N}_3 = \begin{bmatrix} 0.03 & 0.01 & 0.04 \\ 0.02 & 0.03 & 0.02 \\ 0.02 & 0.05 & 0.01 \end{bmatrix}.$$

Impulsive matrices take the following form:

$$D_1 = \begin{bmatrix} 1.05 & 0 & 0 \\ 0 & 1.02 & 0 \\ 0 & 0 & 1.04 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 1.02 & 0 & 0 \\ 0 & 1.01 & 0 \\ 0 & 0 & 1.06 \end{bmatrix}.$$

For given

$$p = (0.6, 0.2, 0.5)^{\top},$$

$S = 90$ s, $\varepsilon = 0$, $\omega = 1.25$, $b_1 = 1.5$, $b_2 = 90$, by solving conditions (4.2) to (4.9), we obtain $\delta_1 = 11.5$, $\delta_2 = 12.6$, and

$$\psi_1 = (0.5627824, 0.2660705, 0.4627824)^{\top},$$

$$\chi_1 = (0.5627821, 0.2695134, 0.4627821)^{\top},$$

$$\psi_2 = (0.5627823, 0.1852107, 0.4627823)^{\top},$$

$$\chi_2 = (0.5606936, 0.1855242, 0.4627820)^{\top},$$

$$\psi_3 = (0.5627822, 0.1627822, 0.4627822)^{\top},$$

$$\chi_3 = (0.5615130, 0.1625755, 0.4627819)^{\top}.$$

Therefore, by Theorem 4.1, system (4.1) is finite-time stable about (S, p, b_1, b_2).

The impulsive signal is in Figure 9, which shows the impulsive signal $p(t) = 1$ when $t = [30, 60]$, where $p(t) = 1$ indicates that the impulse jumps occur at these moments. Choosing the original value

$$z(0) = (1.1, 0.6, 1.3)^{\top},$$

the solution trajectory and system response from 0 to 90 s of system (4.1) are exhibited in Figures 10 and 11, respectively. What is more, there exist impulses for system (4.1), so the results in [25] are invalid. The solution and response states of system (4.1) are all discontinuous due to the impulse jumping effects.

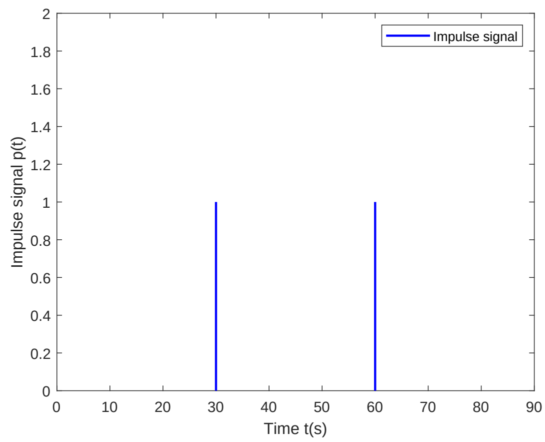

Figure 9. The impulsive signal.

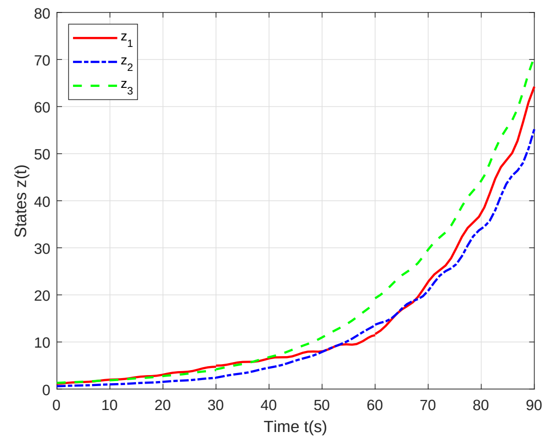

Figure 10. The solution trajectory of system (4.1).

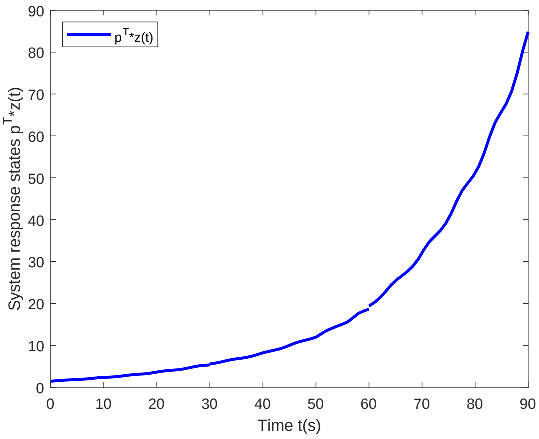

Figure 11. The response of system (4.1) from 0 to 90 s.

We present Algorithms 1 and 2 to verify Theorem 3.2. Then a similar algorithm can be easily obtained for Theorem 4.1 in Section 4.

Algorithm 1. The algorithm process of Theorem 3.2.

Step 1. Suppose that

$$\psi_1 = (z_1, z_2, z_3)^\top, \ \ \psi_2 = (z_4, z_5, z_6)^\top$$

and

$$\chi_1 = (z_7, z_8, z_9)^\top, \ \ \chi_2 = (z_{10}, z_{11}, z_{12})^\top.$$

Step 2. We substitute four vectors ψ_1, ψ_2, χ_1, χ_2 into the conditions of Theorem 3.2, and the corresponding parameters values satisfy the inequality condition. Then the linear inequalities are transformed into general inequalities.

Step 3. By using the linear programming algorithm and Lingo software, the objective function is min $=$ d; the constraint conditions are

$$\sum_{e=1}^{12} a_e z_e \leq d,$$

where $e = 1, 2, \cdots, 12$. Then, we can obtain the feasible solution $z_1 - z_{12}$.

Remark 5.1. It is noticeable that the recent results in [25] can not be used for Examples 5.1–5.3 due to the impulses. To some extent, the proposed results in this paper are more effective than previous ones.

Algorithm 2. The algorithm process of Theorem 4.1.

Step 1. Suppose that

$$\psi_1 = (z_1, z_2, z_3)^\top, \ \ \psi_2 = (z_4, z_5, z_6)^\top,$$

$$\psi_3 = (z_7, z_8, z_9)^\top, \ \ \chi_1 = (z_{10}, z_{11}, z_{12})^\top$$

and

$$\chi_2 = (z_{13}, z_{14}, z_{15})^\top, \ \ \chi_3 = (z_{16}, z_{17}, z_{18})^\top.$$

Step 2. We substitute six vectors ψ_1, ψ_2, ψ_3, χ_1, χ_2, ψ_3 into the conditions of Theorem 4.1, and the corresponding parameters values satisfy the inequality condition. Then the linear inequalities are transformed into general inequalities.

Step 3. By using the linear programming algorithm and Lingo software, the objective function is min $=$ d; the constraint conditions are

$$\sum_{e=1}^{18} a_e z_e \leq d,$$

where $e = 1, 2, \cdots, 18$. Then, we can obtain the feasible solution $z_1 - z_{18}$.

Remark 5.2. It should be emphasized that the designed linear programming process is relatively simple, which makes it easy to program and solve. By using Lingo software and setting the appropriate precision, we get a more accurate result.

Remark 5.3. For the convenience of simulations, we give small b_1 and large b_2, which can satisfy the inequality condition. In fact, the corresponding parameters of b_1 and b_2 are not unique; they are related to the parameters δ_1 and δ_2. Therefore, we can adjust the size of the parameters b_1 and b_2 to satisfy the condition, and the parameter b_2 is not sufficiently large in advance.

6. Conclusions

In this paper, we add the impulsive effects of PSLDS. Then, we provide FTS conditions for the systems with an appropriate Lyapunov functional. After that, we apply the main FTS criteria to general linear time-varying delayed systems with impulsive effects. At last, we provide three examples to verify the results, which include the specific algorithms progress. And our programming process is simple and easy to verify.

There are many limitations to our work. It is worth noting that the impulsive and switching instants may not be synchronous. We deal with the impulsive effects of PSLDS in this paper, which are synchronous with switching instants. However, the impulsive signals may not be instantaneous with switching signals. Then, we will gradually explore the case of FTS for the systems (1.2) and (4.1) with asynchronous switching impulsive signals and non-instantaneous impulses in the future. The method in this paper cannot be directly extended to the asynchronous impulses case. The main difficulty is how to deal with the Lyapunov functional value estimation under the coupling effects of asynchronous switches and state jumps, which will be left for our future study.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

References

1. L. Farina, S. Rinaldi, *Positive linear systems: theory and applications*, John Wiley & Sons, Inc., New York, 2000. https://doi.org/10.1002/9781118033029
2. P. De Leenheer, D. Aeyels, Stabilization of positive linear systems, *Syst. Control Lett.*, **44** (2001), 259–271. https://doi.org/10.1016/S0167-6911(01)00146-3
3. R. Shorten, F. Wirth, D. Leith, A positive systems model of TCP-like congestion control: asymptotic results, *IEEE/ACM Trans. Network*, **14** (2006), 616–629. https://doi.org/10.1109/TNET.2006.876178
4. D. Angeli, P. De Leenheer, E. D. Somgtag, Chemical networks with inflows and outflows: a positive linear differential inclusions approach, *Biotechnol. Progr.*, **25** (2009), 632–642. https://doi.org/10.1002/btpr.162
5. J. Lian, C. Li, B. Xia, Sampled-data control of switched linear systems with application to an F-18 aircraft, *IEEE Trans. Ind. Electron.*, **64** (2016), 1332–1340. https://doi.org/10.1109/TIE.2016.2618872
6. Y. Sun, Y. Tian, X. Xie, Stabilization of positive switched linear systems and its application in consensus of multiagent systems, *IEEE Trans. Autom. Control*, **62** (2017), 6608–6613. https://doi.org/10.1109/TAC.2017.2713951
7. M. Xiang, Z. Xiang, Stability, L_1-gain and control synthesis for positive switched systems with time-varying delay, *Nonlinear Anal. Hybrid Syst.*, **9** (2013), 9–17. https://doi.org/10.1016/j.nahs.2013.01.001
8. S. Liu, Z. Xiang, Exponential L_1 output tracking control for positive switched linear systems with time-varying delays, *Nonlinear Anal. Hybrid Syst.*, **11** (2014), 118–128. https://doi.org/10.1016/j.nahs.2013.07.002
9. X. Liu, Stability analysis of a class of nonlinear positive switched systems with delays, *Nonlinear Anal. Hybrid Syst.*, **16** (2015), 1–12. https://doi.org/10.1016/j.nahs.2014.12.002
10. Y. Sun, Stability analysis of positive switched systems via joint linear copositive Lyapunov functions, *Nonlinear Anal. Hybrid Syst.*, **19** (2016), 146–152. https://doi.org/10.1016/j.nahs.2015.09.001
11. W. Zhao, Y. Sun, Absolute exponential stability of switching Lurie systems with time-varying delay via dwell time switching, *J. Franklin Inst.*, **360** (2023), 11871–11891. https://doi.org/10.1016/j.jfranklin.2023.09.023
12. F. Amato, R. Ambrosino, M. Ariola, G. De Tommasi, Robust finite-time stability of impulsive dynamical linear systems subject to norm-bounded uncertainties, *Int. J. Robust Nonlinear Control*, **21** (2011), 1080–1092. https://doi.org/10.1002/rnc.1620

13. G. Garcia, S. Tarbouriech, J. Bernussou, Finite-time stabilization of linear time-varying continuous systems, *IEEE Trans. Autom. Control*, **54** (2009), 364–369. https://doi.org/10.1109/TAC.2008.2008325

14. S. Zhao, J. Sun, L. Liu, Finite-time stability of linear time-varying singular systems with impulsive effects, *Int. J. Control*, **81** (2008), 1824–1829. https://doi.org/10.1080/00207170801898893

15. K. Wang, E. Tian, S. Shen, L. Wei, J. Zhang, Input-output finite-time stability for networked control systems with memory event-triggered scheme, *J. Franklin Inst.*, **356** (2019), 8507–8520. https://doi.org/10.1016/j.jfranklin.2019.08.020

16. N. T. Thanh, P. Niamsup, V. N. Phat, Finite-time stability of singular nonlinear switched time-delay systems: a singular value decomposition approach, *J. Franklin Inst.*, **354** (2017), 3502–3518. https://doi.org/10.1016/j.jfranklin.2017.02.036

17. J. Wei, X. Zhang, H. Zhi, X. Zhu, New finite-time stability conditions of linear discrete switched singular systems with finite-time unstable subsystems, *J. Franklin Inst.*, **357** (2020), 279–293. https://doi.org/10.1016/j.jfranklin.2019.03.045

18. T. Zhang, F. Deng, W. Zhang, Finite-time stability and stabilization of linear discrete time-varying stochastic systems, *J. Franklin Inst.*, **356** (2019), 1247–1267. https://doi.org/10.1016/j.jfranklin.2018.10.026

19. L. Hou, C. Sun, H. Ren, Y. Wei, Finite-time stability of switched linear systems, *Proceedings of the 36th Chinese Control Conference*, 2017, 2338–2342. https://doi.org/10.23919/ChiCC.2017.8027707

20. W. Xiang, J. Xiao, Finite-time stability and stabilisation for switched linear systems, *Int. J. Syst. Sci.*, **44** (2013), 384–400. https://doi.org/10.1080/00207721.2011.604738

21. L. Liu, N. Xu, G. Zong, X. Zhao, New results on finite-time stability and stabilization of switched positive linear time-delay systems, *IEEE Access*, **8** (2020), 4418–4427. https://doi.org/10.1109/ACCESS.2019.2961683

22. G. Chen, Y. Yang, Finite-time stability of switched positive linear systems, *Int. J. Robust Nonlinear Control*, **24** (2012), 179–190. https://doi.org/10.1002/rnc.2870

23. N. Xu, Y. Chen, A. Xue, G. Zong, Finite-time stabilization of continuous-time switched positive delayed systems, *J. Franklin Inst.*, **359** (2022), 255–271. https://doi.org/10.1016/j.jfranklin.2021.04.022

24. M. Zhang, Q. Zhu, Finite-time input-to-state stability of switched stochastic time-varying nonlinear systems with time delays, *Chaos Solitons Fract.*, **162** (2022), 112391. https://doi.org/10.1016/j.chaos.2022.112391

25. T. Huang, Y. Sun, D. Tian, Finite-time stability of positive switched time-delay systems based on linear time-varying copositive Lyapunov functional, *J. Franklin Inst.*, **359** (2022), 2244–2258. https://doi.org/10.1016/j.jfranklin.2022.01.029

26. M. Hu, Y. Wang, J. Xiao, W. Yang, L_1-gain analysis and control of impulsive positive systems with interval uncertainty and time delay, *J. Franklin Inst.*, **356** (2019), 9180–9205. https://doi.org/10.1016/j.jfranklin.2019.08.010

27. T. Liu, B. Wu, L. Liu, Y. Wang, Asynchronously finite-time control of discrete impulsive switched positive time-delay systems, *J. Franklin Inst.*, **352** (2015), 4503–4514. https://doi.org/10.1016/j.jfranklin.2015.06.015

28. S. Peng, L. Yang, Global exponential stability of impulsive functional differential equations via Razumikhin technique, *Abstr. Appl. Anal.*, **2010** (2010), 987372. https://doi.org/10.1155/2010/987372

29. M. Hu, Y. Wang, J. Xiao, On finite-time stability and stabilization of positive systems with impulses, *Nonlinear Anal. Hybrid Syst.*, **31** (2019), 275–291. https://doi.org/10.1016/j.nahs.2018.10.004

30. M. Hu, J. Xiao, R. Xiao, W. Chen, Impulsive effects on the stability and stabilization of positive systems with delays, *J. Franklin Inst.*, **354** (2017), 4034–4054. https://doi.org/10.1016/j.jfranklin.2017.03.019

31. C. Briat, Dwell-time stability and stabilization conditions for linear positive impulsive and switched systems, *Nonlinear Anal. Hybrid Syst.*, **24** (2017), 198–226. https://doi.org/10.1016/j.nahs.2017.01.004

32. G. Chen, C. Fan, J. Sun, J. Xia, Mean square exponential stability analysis for itô stochastic systems with aperiodic sampling and multiple time-delays, *IEEE Trans. Autom. Control*, **67** (2022), 2473–2480. https://doi.org/10.1109/TAC.2021.3074848

33. G. Chen, J. Xia, Ju H. Park, H. Shen, G. Zhuang, Sampled-data synchronization of stochastic Markovian jump neural networks with time-varying delay, *IEEE Trans. Neural Networks Learn. Syst.*, **33** (2021), 3829–3841. https://doi.org/10.1109/TNNLS.2021.3054615

34. G. Chen, G. Du, J. Xia, X. Xie, J. H. Park, Controller synthesis of aperiodic sampled-data networked control system with application to interleaved flyback module integrated converter, *IEEE Trans. Circuits Syst. I*, **70** (2023), 4570–4580. https://doi.org/10.1109/TCSI.2023.3295940