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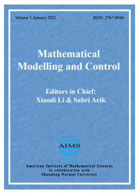

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Research article

Optimal reinsurance design under the VaR risk measure and asymmetric information

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Abstract: This paper analyzes a monopoly reinsurance market in the presence of asymmetric information. Insurers use Value-at-Risk measures to quantify their risks and have different risk exposures and risk preferences, but the type of each insurer is hidden information to the reinsurer. The reinsurer maximizes the expected profit under the constraint of incentive compatibility and individual rationality. We deduce the optimal reinsurance menu under the assumption that a type of insurer thinks he is at greater risks. Some comparative analyses are given for two strategies of separating equilibrium and pooling equilibrium.

Keywords: optimal insurance; value-at-risk; asymmetric information; separating equilibrium; pooling equilibrium

1. Introduction

Reinsurance is an effective risk management tool in which the insurer transfers part of the underwriting risk to the reinsurer to reduce the underwriting risk. It is characterized by an indemnity function f and a premium π, where f means the amount paid by the reinsurer when the insurer suffers losses and π is the reinsurance premium.

Optimal insurance policies have been investigated extensively in the literature. The optimal criteria commonly used are risk minimization, expected utility maximization, or some combination of them. The classic result of the common expected utility maximization criterion can be found in Borch [1], Arrow [2] and Raviv [3]. The results of minimizing insurer risk were measured by variance, Value-at-Risk(VaR) and general distortion risk measures can be found in Cai et al. [4], Assa [5], Zhuang et al. [6], Lo [7] and the references therein.

In almost all of the articles mentioned above, one implicit assumption is that the information between the insurer and the reinsurer is symmetrical. Such, the reinsurer knows clearly the risk that insurer may face, and can be targeted to design the policy to maximize their own interests. But in practice, the reinsurer gets only partial information from the insurer. The reinsurer cannot identify the risk distribution and the risk preference of the insurer. In the case of information asymmetry, the situation where the reinsurer dominates the policy is no longer valid, the insurer may benefit strictly from the transaction by imitating the information of others.

Most of the literature on information asymmetry in insurance market mainly focuses on adverse selection. Groundbreaking works such as Rothschild and Stiglitz [8] and Stiglitz [9] laid the foundation for information asymmetry modeling. They proposed a principal-agent model with only two types of insureds and a monopolistic insurer where the risks or utility functions of the two types of insurers were different. This model was extended along many ways in the past few decades. We refer the interested readers to, for example, Young and Browne [10], Ryan and Vaithianathan [11], Jeleva and Villeneuve [12], Chade and Schlee [13]. In recent years, Cheung et al. [14] considered the adverse selection reinsurance design problem when the insurers adopted the VaR measures to quantify their risks. Cheung et al. [15] extended this model further to some concave distortion risk measure. Another closely related study is Boonen and Zhang [18] which considered an information asymmetry reinsurance model without assuming the parametric form of the indemnity function. In this article, we continue using the classical model to study the optimal stop-loss reinsurance policies under asymmetric information. We assume that the insurers adopt VaR measures but the reinsurer can't know ahead of time which risk distribution and risk preference the insurer will be using. The optimization problem is solved under the individual rationality and incentive compatibility constraints.

The rest of this article is organized as follows. Section 2 states some pertinent definitions and notions. Section 3 and 4 study the optimal solutions for two policy design strategies of separating equilibrium and pooling equilibrium, respectively. Some numerical examples are given in Section 5 to further illustrate our results. Section 6 concludes the paper and puts forward the research direction in the future.

2. Preliminaries and problem setup

2.1. Preliminaries

Throughout the paper, all random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let X denote the non-negative total potential loss for which the insurer seeks reinsurance coverage. Assume that the variable X is supposed to realizations on $[0, \infty)$ and which has cumulative distribution function F_X and survival function S_X.

For simplicity, we consider that there are only two types of insurers and one monopolistic reinsurer in the market .The insurer often decides to reduce the risk exposure through purchasing a reinsurance contract (f, π) where f is known as the indemnity function and π is the non-negative premium. To partially exclude the moral hazard, we consider the following admissible set of ceded loss functions

$\mathcal{F} = \{f : [0, \infty) \to [0, \infty) \mid f(0) = 0, 0 \leq f(x) - f(y) \leq x - y, 0 \leq y \leq x\}$.

This setting is common in relevant literature, seen for instance Huberman et al. [16], Denuit and Vermandele [17], Boonen et al. [18]. Any ceded loss function f in $\mathcal{F}$ satisfies the incentive-compatible condition, which guarantees the non-negative of the ceded risk and less than the loss itself. Furthermore, growth rate in ceded loss function is lower than the incurred loss which further reduces moral hazard.

Based on the points discussed above, we set the reinsurer provides a stop-loss policy for two types of insurers. The insurers are confronted with different risks, denoted by X_1 and X_2, respectively. And then we can define the ceded loss function by $f_i = (X_i - d_i)_+$, for $i = 1, 2$.

In today's financial world, VaR has become the most widely used risk measure. Its importance is uncontroversial since regulators accept this model as the basis for setting capital requirements for market risk exposure. Accordingly, we assume that both of the insurers adopt VaR measure to evaluate their risk position. Given a risk X and a confidence level $\alpha \in (0, 1)$, the corresponding VaR at level $1-\alpha$, denoted by $VaR_\alpha(X)$, is defined as

$$VaR_\alpha(X) = \inf\{x \geq 0 : P(X > x) \leq \alpha\} = S_X^{-1}(\alpha).$$

The popularity of VaR ascribes to its nice properties, some of them are useful to us are illustrated below.

1. Translation invariance: for a random variable X and a constant c, $VaR_\alpha(X + c) = VaR_\alpha(X) + c$.
2. Comonotonic additivity: for any two comonotonic random variables X and Y, $VaR_\alpha(X + Y) = VaR_\alpha(X) + VaR_\alpha(Y)$.
3. Monotonicity: for two random variables X and Y if $P(X \leq Y) = 1$ holds, $VaR_\alpha(X) \leq VaR_\alpha(Y)$.
4. For any increasing left-continuous function f and a random variable X, $VaR_\alpha(f(X)) = f(VaR_\alpha(X))$.

In the next section, we present the setup for the optimal reinsurance problem under the asymmetric information.

2.2. Problem setup

In this section, we give the optimal stop-loss policies offered by a reinsurer when the insurers adopt VaR measures for the risk assessment. We assume that two types of insurers have different confidence level, which are denoted by α and β, respectively.

On the condition of information asymmetry, the type of each insurer is hidden information to the reinsurer. In other words, the reinsurer can't know ahead of time which risk distribution and risk preference the insurer will be using. But the reinsurer still knows that the proportions of the first and the second types of insurers in the market are p and $1 - p$ respectively. More precisely, the reinsurer knows that the probability of the insurer adopting VaR_α measure is p, while the probability of adopting VaR_β measure is $1 - p$.

In this paper, the reinsurer is assumed to be monopoly and risk-neutral. In order to maximize its expected profit, the reinsurer offers a reinsurance menu to the insurers which is given by $\{(f_1, \pi_1); (f_2, \pi_2)\}$ and consists of two stop-loss policies. In this menu, f_i is the indemnity function for the i-th type of the insurer and π_i is the corresponding premium charged from the i-th type of the insurer, $i = 1$ or 2. We can design this menu which makes insurers can freely choose the one that minimizes their risk exposures. Such a design makes the reinsurer knows the identity of the insurers when the policy is chosen by the insurers. Then the expected profit of the reinsurer is formulated as follows

$$p\Big(\pi_1 - (1+\theta)E[f_1(X_1)]) + (1-p)(\pi_2 - (1+\theta)E[f_2(X_2)]\Big), \tag{2.1}$$

where $\theta \geq 0$ denotes the risk safety loading.

In order to better address the problem of maximizing the expected profit of the reinsurer, we follow the standard arguments in Principal–Agent models to impose further restraint on a feasible policy $\{(f_1, \pi_1); (f_2, \pi_2)\}$ by using individual rationality (IR) constraints and incentive compatibility (IC). More specifically, the constraint (IR) ensures that the insurers who are no worse off for buying the designated reinsurance contract. The constraint (IC) ensure that the insurance contract are tailor-made for specific insureds. Under this restriction, the insurers will follow the suggestion of the reinsurer such that the insurer of type 1 may just choose the policy (f_1, π_1) rather than choose the policy (f_2, π_2) which is designed for the type 2 insurer, and vice versa. On the basis of the above-mentioned analysis, the reinsurer's wealth optimization problem is formalized as follows.

Problem 2.1.

$$\max_{\{(f_1,\pi_1);(f_2,\pi_2)\}} p\Big(\pi_1 - (1+\theta)E\left[f_1(X_1)\right]\Big) + (1-p)\Big(\pi_2 - (1+\theta)E\left[f_2(X_2)\right]\Big), \tag{2.2}$$

subject to the following constraints:

$$\text{IR1: } VaR_\alpha(X_1 - f_1(X_1) + \pi_1) \leq VaR_\alpha(X_1), \tag{2.3a}$$

$$\text{IR2: } VaR_\beta(X_2 - f_2(X_2) + \pi_2) \leq VaR_\beta(X_2), \tag{2.3b}$$

$$\text{IC1: } VaR_\alpha(X_1 - f_1(X_1) + \pi_1) \leq VaR_\alpha(X_1 - f_2(X_1) + \pi_2), \tag{2.3c}$$

$$\text{IC2: } VaR_\beta(X_2 - f_2(X_2) + \pi_2) \leq VaR_\beta(X_2 - f_1(X_2) + \pi_1). \tag{2.3d}$$

For the further improvement, we have the following assumptions. When IC1 holds with an equality, the insurer of type 1 would select the policy (f_1, π_1); when IC2 holds with an equality, the insurer of type 2 would select the policy (f_2, π_2). This is a standard assumption in asymmetric information models, for example, Landsberger and Meilijson [19] and Laffont and Martimort [20].

We can easily see that the reinsurer provides different contracts for the different insurers in Problem 2.1, this strategy is known as separating equilibrium. In the next section, we propose a strategy to solve Problem 2.1.

3. The optimal reinsurance contracts

In this section, we study Problem 2.1 that yields the optimal reinsurance contracts. Firstly, we try to further simplify this model by some reasonable assumptions.

Assumption 1. *(i).* $(1+\theta)E(X) \leq VaR_\kappa(X), \kappa \in \{\alpha, \beta\}$.

(ii). $VaR_\alpha(X_1) \leq VaR_\beta(X_2)$.

The first hypothesis ensured the cost of reinsurance is lower than the risk measure of the loss for the insurer so that purchases reinsurance for the loss is effective. What needs to be emphasized is that the asymmetric information model we studied is a common case where the two types of insurers have different risk preferences and they are not divided into high or low risk types. Because there must be an order between $VaR_\alpha(X_1)$ and $VaR_\beta(X_2)$, we assume $VaR_\alpha(X_1) \leq VaR_\beta(X_2)$ without losing generality. The second hypothesis means that the type 2 insurer regard their losses riskier than do the type 1 insurer.

By observing the Problem 2.1, we find that the four constraints (2.3a)~(2.3d) can be simplified further by using the comonotonic additivity and translational invariance of the VaR measure. Then, Problem 2.1 reduces to the following problem.

Problem 3.1. *Maximizing the objective function in (2.2) subject to the following constraints:*

$$\text{IR1: } \pi_1 \le VaR_\alpha(f_1(X_1)), \tag{3.1a}$$

$$\text{IR2: } \pi_2 \le VaR_\beta(f_2(X_2)), \tag{3.1b}$$

$$\text{IC1: } \pi_1 - VaR_\alpha(f_1(X_1)) \le \pi_2 - VaR_\alpha(f_2(X_1)), \tag{3.1c}$$

$$\text{IC2: } \pi_2 - VaR_\beta(f_2(X_2)) \le \pi_1 - VaR_\beta(f_1(X_2)). \tag{3.1d}$$

Remark 3.1. *We assume here that the premiums as constants because that will make it easier to simplify the IC and IR constraints. If we treat premiums as functions of the deductibles governed by some premium principle, rather than as independent decision variables. Then the decision variables of the objective function are changed from four to two, i.e d_1 and d_2. Then this optimization problem may become infeasible because it has four constraints.*

We can show by inspecting Problem 3.1 that the objective function is increasing with respect to π_1 and π_2. Obviously to maximize the expected profit, both π_1 and π_2 need to be taken the maximum within the limit of reasonable values.

Through IR1 and IC1, possible values of π_1 are described below

$$\pi_1 \le \min\left\{VaR_\alpha(f_1(X_1)), \pi_2 - VaR_\alpha(f_2(X_1)) + VaR_\alpha(f_1(X_1))\right\}.$$

Through IR2 and IC2, possible values of π_2 are described below

$$\pi_2 \le \min\left\{VaR_\beta(f_2(X_2)), \pi_1 - VaR_\beta(f_1(X_2)) + VaR_\beta(f_2(X_2))\right\}.$$

Therefore, in order to maximize the objective function, π_1 must reach either $VaR_\alpha(f_1(X_1))$ or $\pi_2 - VaR_\alpha(f_2(X_1)) + VaR_\alpha(f_1(X_1))$ and π_2 must reach either $VaR_\beta(f_2(X_2))$ or $\pi_1 - VaR_\beta(f_1(X_2)) + VaR_\beta(f_2(X_2))$. We can consider four possible scenarios.

Case 1: IR1 combines with IR2.

$$\pi_1 = VaR_\alpha(f_1(X_1)), \pi_2 = VaR_\beta(f_2(X_2)).$$

Case 2: IR1 combines with IC2.

$$\pi_1 = VaR_\alpha(f_1(X_1)),$$
$$\pi_2 = \pi_1 - VaR_\beta(f_1(X_2)) + VaR_\beta(f_2(X_2)).$$

Case 3: IC1 combines with IR2.

$$\pi_1 = \pi_2 - VaR_\alpha(f_2(X_1)) + VaR_\alpha(f_1(X_1)),$$
$$\pi_2 = VaR_\beta(f_2(X_2)).$$

Case 4: IC1 combines with IC2.

$$\pi_1 = \pi_2 - VaR_\alpha(f_2(X_1)) + VaR_\alpha(f_1(X_1)),$$
$$\pi_2 = \pi_1 - VaR_\beta(f_1(X_2)) + VaR_\beta(f_2(X_2)).$$

The upper bounds of π_1 and π_2 are not given in Case 4, their values are interacted on each other. Actually, the reinsurer could still increase π_1 and π_2 in the pursuit of greater profits until one of them reaches up to the maximum premium which is accepted by the relevant insurer. In other words, the premium will be increasing until reaches the upper bound of the IR constraint. Based on the above analysis, Case 4 could be considered as a special case of Case 1~3. Now, we can break down Problem 3.1 into the following three sub-problems.

Problem A. (Case 1)

$$\max_{\{f_1,f_2\}} p\Big(VaR_\alpha(f_1(X_1)) - (1+\theta)E[f_1(X_1)]\Big) + (1-p)\Big(VaR_\beta(f_2(X_2)) - (1+\theta)E[f_2(X_2)]\Big),$$

subject to the following constraints:

$$\text{IC1: } VaR_\alpha(f_2(X_1)) \le VaR_\beta(f_2(X_2)),$$
$$\text{IC2: } VaR_\beta(f_1(X_2)) \le VaR_\alpha(f_1(X_1)).$$

Problem B. (Case 2)

$$\max_{\{f_1,f_2\}} p\Big(VaR_\alpha(f_1(X_1)) - (1+\theta)E[f_1(X_1)]\Big) + (1-p)\Big(VaR_\alpha(f_1(X_1)) + VaR_\beta(f_2(X_2)) - VaR_\beta(f_1(X_2)) - (1+\theta)E[f_2(X_2)]\Big),$$

subject to the following constraints:

$$\text{IR2: } VaR_\alpha(f_1(X_1)) \le VaR_\beta(f_1(X_2)),$$
$$\text{IC1: } VaR_\alpha(f_2(X_1)) - VaR_\beta(f_2(X_2)) \le VaR_\alpha(f_1(X_1)) - VaR_\beta(f_1(X_2)).$$

Problem C. (Case 3)

$$\max_{\{f_1,f_2\}} \; p\Big(VaR_\alpha\left(f_1\left(X_1\right)\right) + VaR_\beta\left(f_2\left(X_2\right)\right) \\ - VaR_\alpha\left(f_2\left(X_1\right)\right) - (1+\theta)E[f_1\left(X_1\right)]\Big) \\ + (1-p)\Big(VaR_\beta\left(f_2\left(X_2\right)\right) - (1+\theta)E[f_2\left(X_2\right)]\Big),$$

subject to the following constraints:

$$\text{IR1: } VaR_\beta\left(f_2\left(X_2\right)\right) \le VaR_\alpha\left(f_2\left(X_1\right)\right),$$
$$\text{IC2: } VaR_\alpha\left(f_2\left(X_1\right)\right) - VaR_\beta\left(f_2\left(X_2\right)\right) \\ \le VaR_\alpha\left(f_1\left(X_1\right)\right) - VaR_\beta\left(f_1\left(X_2\right)\right).$$

Then we will analyze the above three sub-problems, obviously the solution to the maximum of the three objective functions is also the solution to Problem 3.1. The next theorem gives the optimal premium scheme for Problem 3.1.

Theorem 3.1. *Under Assumption 1, any optimal solution of Problem 3.1 should satisfy*

(i). $\pi_1 = VaR_\alpha\left(f_1\left(X_1\right)\right),$

(ii). $\pi_2 = \pi_1 - VaR_\beta\left(f_1\left(X_2\right)\right) + VaR_\beta\left(f_2\left(X_2\right)\right).$

Proof. First, we should simplify these three sub-problems. Because $f(x) = (x-d)_+$ is a monotonous increasing function, based on Assumption 1(ii) and property 4 of the VaR we can get

$$VaR_\alpha\left(f_i\left(X_1\right)\right) \le VaR_\beta\left(f_i\left(X_2\right)\right),\ i = 1 \text{ or } 2.$$

From the analysis above, constraints in Problem A are equivalent to

$$VaR_\beta\left(f_1\left(X_2\right)\right) = VaR_\alpha\left(f_1\left(X_1\right)\right),$$

constraints in Problem B are equivalent to

$$VaR_\alpha\left(f_2\left(X_1\right)\right) - VaR_\beta\left(f_2\left(X_2\right)\right) \\ \le VaR_\alpha\left(f_1\left(X_1\right)\right) - VaR_\beta\left(f_1\left(X_2\right)\right) \\ \le 0,$$

constraints in Problem C are equivalent to

$$\begin{cases} VaR_\alpha\left(f_2\left(X_1\right)\right) = VaR_\beta\left(f_2\left(X_2\right)\right), \\ VaR_\alpha\left(f_1\left(X_1\right)\right) = VaR_\beta\left(f_1\left(X_2\right)\right). \end{cases}$$

Then Problem A~C could be rewritten as follows.

Problem A$'$.

$$\max_{\{f_1,f_2\}} p\Big(VaR_\alpha\left(f_1\left(X_1\right)\right) - (1+\theta)E\left[f_1\left(X_1\right)\right]\Big) \\ + (1-p)\Big(VaR_\beta\left(f_2\left(X_2\right)\right) - (1+\theta)E\left[f_2\left(X_2\right)\right]\Big),$$

$$\text{s.t.} \quad VaR_\beta\left(f_1\left(X_2\right)\right) = VaR_\alpha\left(f_1\left(X_1\right)\right). \tag{3.2}$$

Problem B$'$.

$$\max_{\{f_1,f_2\}} VaR_\alpha\left(f_1\left(X_1\right)\right) - \Big[p(1+\theta)E[f_1\left(X_1\right)] \\ + (1-p)VaR_\beta\left(f_1\left(X_2\right)\right)\Big] \\ + (1-p)\Big(VaR_\beta\left(f_2\left(X_2\right)\right) - (1+\theta)E[f_2\left(X_2\right)]\Big),$$

$$\text{s.t.} \quad VaR_\alpha\left(f_2\left(X_1\right)\right) - VaR_\beta\left(f_2\left(X_2\right)\right) \\ \le VaR_\alpha\left(f_1\left(X_1\right)\right) - VaR_\beta\left(f_1\left(X_2\right)\right) \le 0. \tag{3.3}$$

Problem C$'$.

$$\max_{\{f_1,f_2\}} p\Big(VaR_\alpha\left(f_1\left(X_1\right)\right) - (1+\theta)E\left[f_1\left(X_1\right)\right]\Big) \\ + (1-p)\Big(VaR_\beta\left(f_2\left(X_2\right)\right) - (1+\theta)E\left[f_2\left(X_2\right)\right]\Big),$$

$$\text{s.t.} \quad VaR_\alpha\left(f_2\left(X_1\right)\right) = VaR_\beta\left(f_2\left(X_2\right)\right), \\ VaR_\alpha\left(f_1\left(X_1\right)\right) = VaR_\beta\left(f_1\left(X_2\right)\right). \tag{3.4}$$

Next, we will compare the optimal values of the objective functions in Problems A$'$ ~ C$'$, and show that the solution corresponding to the Problem B$'$ is also the solution to Problem 3.1.

Now use V_A,V_B and V_C to denote the optimal values of the objective functions in Problem A$'$, B$'$ and C$'$, respectively. By comparing the Problem A$'$ with Problem C$'$, we notice that these two problems have the same objective function. Nonetheless, the constraints of (3.2) are looser than the constraints of (3.4). So the solution of Problem C$'$ is by no means better than the solution of Problem A$'$. That is, $V_C \le V_A$.

Under Assumption 1(ii), when $VaR_\alpha(f_1(X_1)) - VaR_\beta(f_1(X_2)) = 0$, we have $VaR_\alpha(f_2(X_1)) - VaR_\beta(f_2(X_2)) \le VaR_\alpha(f_1(X_1)) - VaR_\beta(f_1(X_2)) \le 0$ but not vice-versa. So the constraints of (3.3) are looser than the constraints of (3.2).

Note that we can get $VaR_\alpha(f_1(X_1)) - VaR_\beta(f_1(X_2)) = 0$ when tighten the constraint of (3.3), then the objective function of Problem B$'$ could be rewritten as

$$p\left(VaR_{\alpha}\left(f_1\left(X_1\right)\right)-(1+\theta)E\left[f_1\left(X_1\right)\right]\right)$$
$$+(1-p)\left(VaR_{\beta}\left(f_2\left(X_2\right)\right)-(1+\theta)E\left[f_2\left(X_2\right)\right]\right),$$

that is precisely the objective function of Problem A$'$. Therefore, Problem A$'$ could be considered as a special case of Problem B$'$. That is, $V_A \le V_B$.

To sum up,$V_C \le V_A \le V_B$. □

Remark 3.2. *According to Theorem 3.1 and Assumption 1(i) , we have* $\pi_1 = VaR_{\alpha}(f_1(X_1)) \ge E[f_1(X_1)]$. *It shows that the premium* π_1 *satisfies the non-negative safety loading principle. However, it is possible that*

$$\pi_2 = VaR_{\alpha}\left(f_1\left(X_1\right)\right) - VaR_{\beta}\left(f_1\left(X_2\right)\right) + VaR_{\beta}\left(f_2\left(X_2\right)\right)$$
$$\le E[f_2\left(X_2\right)].$$

It suggests that the premium π_2 *may violate the principle of the non-negative safety loading. We notice that under the complete information conditions, the optimal premium designed for the type 2 insurer is* $\pi_2' = VaR_{\beta}\left(f_2\left(X_2\right)\right)$. *Compare* π_2 *and* π_2', *we have* $\pi_2' - \pi_2 = VaR_{\beta}\left(f_1\left(X_2\right)\right) - VaR_{\alpha}\left(f_1\left(X_1\right)\right) \ge 0$. *It shows that the reinsurer pays higher costs to attract the type 2 insurer under asymmetric information, which can be explained as the cost of information.*

Based on the Theorem 3.1, we have a new problem formulation of the original Problem 3.1.

Problem 3.2.

$$\max_{\{f_1,f_2\}} \Big[VaR_{\alpha}\left(f_1\left(X_1\right)\right) - p(1+\theta)E[f_1\left(X_1\right)]$$
$$-(1-p)VaR_{\beta}\left(f_1\left(X_2\right)\right)\Big]$$
$$+(1-p)\left(VaR_{\beta}\left(f_2\left(X_2\right)\right)-(1+\theta)E[f_2\left(X_2\right)]\right),$$
$$s.t.\quad VaR_{\beta}\left(f_2\left(X_2\right)\right) - VaR_{\alpha}\left(f_2\left(X_1\right)\right)$$
$$\ge VaR_{\beta}\left(f_1\left(X_2\right)\right) - VaR_{\alpha}\left(f_1\left(X_1\right)\right).$$

For the sake of simplicity, we denote $S_{X_i}^{-1}(\alpha)$ by a_i and denote $S_{X_i}^{-1}(\beta)$ by b_i. So the Assumption 1(ii) is redescribed as $a_1 \le b_2$, and $VaR_{\alpha}(f_j(X_i)) = f_j(a_i) = (a_i - d_j)_+$, $VaR_{\beta}(f_j(X_i)) = f_j(b_i) = (b_i - d_j)_+$, $i,\ j \in \{1,2\}$. Then the Problem 3.2 can be rewritten as

Problem 3.3.

$$\max_{\{d_1,d_2\}} \Big[(a_1-d_1)_+ - p(1+\theta)E\left[(X_1-d_1)_+\right]$$
$$-(1-p)\left(b_2-d_1\right)_+\Big]$$
$$+(1-p)\left((b_2-d_2)_+ - (1+\theta)E\left[(X_2-d_2)_+\right]\right),$$
$$s.t.\ (b_2-d_2)_+ - (a_1-d_2)_+ \ge (b_2-d_1)_+ - (a_1-d_1)_+\,.$$

There $d_i \in [0,\infty]$,$i \in \{1{,}2\}$, when $d_i = +\infty$, then $f_i = 0$ that is null policy,. In fact, the range of d_i can be reduced. We find that the reinsurer will provide a null policy to the corresponding insurer when d_i exceeds a certain size.

Theorem 3.2. *Given that* $(d_1^*, \pi_1^*; d_2^*, \pi_2^*)$ *is the optimal reinsurance menu. If* $d_1 \in (a_1,\infty]$, *then* $(\infty, 0; d_2^*, \pi_2^*)$ *is optimal; similarly, if* $d_2 \in (b_2,\infty]$, *then* $(d_1^*, \pi_1^*; \infty, 0)$ *is optimal.*

Proof. Given that $(d_1^*, \pi_1^*; d_2^*, \pi_2^*)$ is optimal, then it satisfies all the constraints in original Problem 3.1. When $d_1 > a_1$ such that $d_1^* > a_1$, by the IR1 constraint, we have

$$\pi_1^* \le (a_1 - d_1^*)_+ = 0 \quad \Rightarrow \quad \pi_1^* = 0.$$

Then, $0 = \pi_1^* \le (a_1 - d_1^*)_+ = 0 = (a_1 - \infty)_+$, so $(\infty, 0; d_2^*, \pi_2^*)$ satisfies the IR1 and IR2 constraints.

It is easy to find that

$$0 - (a_1 - \infty)_+ = 0 = \pi_1^* - (a_1 - d_1^*)_+ \le \pi_2^* - (a_1 - d_2^*)_+\,,$$

and

$$\pi_2^* - (b_2 - d_2^*)_+ \le \pi_1^* - (b_2 - d_1^*)_+$$
$$= -\,(b_2 - d_1^*)_+$$
$$\le -\,(b_2 - \infty)_+\,.$$

So $(\infty, 0; d_2^*, \pi_2^*)$ satisfies the IC1 and IC2 constraints. Evidenced by the same token, $(d_1^*, \pi_1^*; \infty, 0)$ satisfies all the constraints when $d_2 \in (b_2,\infty]$. We can prove that the objective function of Problem 3.3 is an increasing function of d_i over a given range, then the value can only get better if we replace d_i^* by ∞, where $i = 1$ or 2. □

Now, we rule out the null policy and only deal with the region where $d_1 \in [0, a_1]$ and $d_2 \in [0, b_2]$.

With this condition, we have

$$(b_2 - d_2)_+ - (a_1 - d_2)_+ \geq (b_2 - d_1)_+ - (a_1 - d_1)_+$$
$$\Leftrightarrow (b_2 - d_2) - (a_1 - d_2)_+ \geq b_2 - a_1.$$

Moreover, with $(b_2 - d_2) - (a_1 - d_2)_+ \geq b_2 - a_1$, we have either (i) $d_2 \geq a_1$ then $b_2 - d_2 \geq b_2 - a_1 \Rightarrow d_2 \leq a_1$; or (ii) $d_2 \leq a_1$ then $(b_2 - d_2) - (a_1 - d_2)_+ \geq b_2 - a_1$ always holds. With $d_2 \leq a_1$, we have
$(b_2 - d_2) - (a_1 - d_2)_+ = (b_2 - d_2) - (a_1 - d_2) \geq b_2 - a_1$.

Hence, $(b_2 - d_2) - (a_1 - d_2)_+ \geq b_2 - a_1$ is equivalent to $d_2 \leq a_1$, and problem 3.3 can be simplified as follows.

Problem 3.4.

$$\max_{\{d_1,d_2\}} p\left(a_1 - d_1 - (1+\theta)E\left[(X_1 - d_1)_+\right]\right) - (1-p)(b_2 - a_1)$$
$$+ (1-p)\left(b_2 - d_2 - (1+\theta)E\left[(X_2 - d_2)_+\right]\right),$$
$$s.t. \quad 0 \leq d_i \leq a_1,\ i = 1 \ or\ 2.$$

Let us define θ_1^* is the solution of the equation $(1+\theta)S_{X_1}(\theta_1^*) = 1$ and θ_2^* is the solution of the equation $(1+\theta)S_{X_2}(\theta_2^*) = 1$.We are now able to present the complete solution to Problem 3.4 in the next theorem.

Theorem 3.3. *Let $(d_1^*, \pi_1^*; d_2^*, \pi_2^*)$ be optimal for Problem 3.4. Then, the optimal solution to Problem 3.4 is summarized as follows.*

(i). If $\theta_2^* \leq a_1$, then $(d_1^*, \pi_1^*; d_2^*, \pi_2^*) = (\theta_1^*, a_1 - \theta_1^*; \theta_2^*, a_1 - \theta_2^*)$.

(ii). If $\theta_2^* > a_1$, then $(d_1^*, \pi_1^*; d_2^*, \pi_2^*) = (\theta_1^*, a_1 - \theta_1^*; a_1, 0)$.

Proof. Let $t_1(d_1) = a_1 - d_1 - (1+\theta)E\left[(X_1 - d_1)_+\right]$, we have $t_1'(d_1) = -1 + (1+\theta)S_{X_1}(d_1)$. Obviously $t_1(d_1)$ is a concave function on $[0, a_1]$, and there must exist a unique $\theta_1^* \in [0, a_1]$ such that $(1+\theta)S_{X_1}(\theta_1^*) = 1$. It can be seen that $t_1'(d_1) > 0$ for $d_1 \in [0, \theta_1^*)$ and $t_1'(d_1) < 0$ for $d_1 \in (\theta_1^*, a_1]$. Therefore, $t_1(d_1)$ attains its maximum at $d_1^* = \theta_1^*$.

Let $t_2(d_2) = b_2 - d_2 - (1+\theta)E\left[(X_2 - d_2)_+\right]$, the same argument can be applied to this case. There must exist a unique $\theta_2^* \in [0, b_2]$ such that $(1+\theta)S_{X_2}(\theta_2^*) = 1$, then $t_2(d_2)$ attains its maximum at $d_2^* = \theta_2^*$.

Note that solving Problem 3.4 is equivalent to solving the following two sub-problems

$$\begin{cases} \max & t_1(d_1), \\ \text{s.t.} & 0 \leq d_1 \leq a_1, \end{cases} \quad \begin{cases} \max & t_2(d_2), \\ \text{s.t.} & 0 \leq d_2 \leq a_1. \end{cases}$$

For the first sub-problem, clearly there $d_1^* = \theta_1^*$ such that $t_1(d_1^*) \geq t_1(d_1)$ for all $d_1 \in [0, \theta_1^*]$.

For the second sub-problem, we consider the following two cases

Case 1: If $\theta_2^* \leq a_1$, then $t_2(d_2)$ is increasing for $d_2 \in [0, \theta_2^*)$ and is decreasing for $d_2 \in (\theta_2^*, a_1]$. Therefore, $t_2(d_2)$ attains its maximum at $d_2^* = \theta_2^*$ for all $d_2 \in [0, a_1]$.

Case 2: If $\theta_2^* > a_1$, then $t_2(d_2)$ is increasing for $d_2 \in [0, a_1]$. Therefore, $t_2(d_2)$ attains its maximum at $d_2^* = a_1$ for all $d_2 \in [0, a_1]$.

Now, we summarize the results as follows.

(i). When $\theta_2^* \leq a_1$, then $d_1^* = \theta_1^*$, $d_2^* = \theta_2^*$, $\pi_1^* = a_1 - d_1^* = a_1 - \theta_1^*$, $\pi_2^* = (a_1 - d_1^*) + (b_2 - d_2^*) - (b_2 - d_1^*) = a_1 - \theta_2^*$.

(ii). When $\theta_2^* > a_1$, then $d_1^* = \theta_1^*$, $d_2^* = a_1$, $\pi_1^* = a_1 - d_1^* = a_1 - \theta_1^*$, $\pi_2^* = a_1 - d_2^* = 0$.

Hence, we obtain the desired results. □

Now we give the expected net profit of the reinsurer in the optimal reinsurance menu. For convenience, we relabel it as T_S and the definition of expected net profit can be seen in (2.1). Based on Theorem 3.2, if $\theta_2^* \leq a_1$ we have

$$T_S = p\left(a_1 - \theta_1^* - (1+\theta)E[(X_1 - \theta_1^*)_+]\right) + (1-p)\left(a_1 - \theta_2^* - (1+\theta)E[(X_2 - \theta_2^*)_+]\right). \tag{3.5}$$

and if $\theta_2^* > a_1$ we have

$$T_S = p\left(a_1 - \theta_1^* - (1+\theta)E[(X_1 - \theta_1^*)_+]\right) - (1-p)(1+\theta)E[(X_2 - \theta_2^*)_+]. \tag{3.6}$$

Next, we discuss the welfare gain for two types of insurers from the optimal reinsurance menu and relabel it as W. For the type 1 insurer, we have

$$\begin{aligned} W_1 &= VaR_\alpha(X_1) - VaR_\alpha(X_1 - f_1(X_1) + \pi_1^*) \\ &= VaR_\alpha(X_1) - VaR_\alpha(X_1) + (a_1 - d_1^*)_+ - \pi_1^* \\ &= (a_1 - \theta_1^*) - (a_1 - \theta_1^*) = 0. \end{aligned}$$

For the type 2 insurer, if $\theta_2^* \leq a_1$,

$$W_2 = VaR_\beta(X_2) - VaR_\beta(X_2 - f_2(X_2) + \pi_2^*)$$

$$= VaR_\beta(X_2) - VaR_\beta(X_2) + (b_2 - d_2^*)_+ - \pi_2^*$$
$$= (b_2 - \theta_2^*)_+ - (a_1 - \theta_2^*) \geq 0.$$

and if $\theta_2^* > a_1$,

$$W_2 = (b_2 - d_2^*)_+ - \pi_2^* = (b_2 - a_1)_+ - 0 \geq 0.$$

We found that the type 2 insurer may strictly benefit from reinsurance transactions. Conversely, the type 1 insurer seems insouciant about buying reinsurance or not buying reinsurance. This finding implies asymmetric information benefits the type 2 insurer. Maybe they can mimic to be of type 1 and appears to indifferent about insurance transactions to gain a larger benefit.

4. Pooling equilibrium contracts

In this section, the reinsurer can design only one policy that maximize his expected profit. More specifically, the reinsurer always offers the same contract (π, f) regardless of the identity of the insurer. This strategy we defined as pooling equilibrium.

We again assume that Assumption 1 holds and continue with the symbolic settings from section 3. According to the different markets' demands, we summarize three possible scenarios which are formalized as follows.

Problem 4.1. (*Close the type 2 insurer market*)

$$\max_{(d,\pi)\in[0,+\infty]\times[0,+\infty)} P\left(\pi - (1+\theta)E\left[(X_1 - d)_+\right]\right), \tag{4.1}$$
$$s.t. \quad (b_2 - d)_+ < \pi \leq (a_1 - d)_+ . \tag{4.2}$$

Problem 4.2. (*Close the type 1 insurer market*)

$$\max_{(d,\pi)\in[0,+\infty]\times[0,+\infty)} (1-p)\left(\pi - (1+\theta)E\left[(X_2 - d)_+\right]\right), \tag{4.3}$$
$$s.t. \quad (a_1 - d)_+ < \pi \leq (b_2 - d)_+ . \tag{4.4}$$

Problem 4.3. (*Open all markets*)

$$\begin{aligned} \max_{(d,\pi)\in[0,+\infty]\times[0,+\infty)} & p\left(\pi - (1+\theta)E\left[(X_1 - d)_+\right]\right) \\ & +(1-p)\left(\pi - (1+\theta)E\left[(X_2 - d)_+\right]\right), \end{aligned} \tag{4.5}$$
$$s.t. \quad \pi \leq \min\left\{(a_1 - d)_+, (b_2 - d)_+\right\}. \tag{4.6}$$

It is pretty easy to find that IC constraints are invalid by designing only one contract. The disappearance of IC constraints makes the design of insurance policies more flexible. If the proportion of one type of insurer in the market is too high, the reinsurer will often close the business of another type of insurer in pursuit of greater profits, which can be achieved by limiting the range of premiums.

Under the Assumption 1(ii), i.e.$a_1 \leq b_2$, we can conclude that the constraint (4.2) in Problem 4.1 cannot hold. This result agrees with the facts of the situation. The largest premium that the type 2 insurer can accept is higher than that of the type 1 insurer because they believe they are facing higher risk.

Similar to separating equilibrium, in Problem 4.2 and Problem 4.3, we only deal with nontrivial solution for $d \in [0, b_2]$, $d \in [0, a_1]$ respectively. So (4.4) is equivalent to $\pi = b_2 - d$ and (4.6) is equivalent to $\pi = a_1 - d$. After all, the reinsurer always pursue high premiums.

Before giving the solution of Problem 4.2 and Problem 4.3, we define θ_3^* is the solution of the equation

$$(1+\theta)S_{X_1}(d) + (1-v)(1+\theta)S_{X_2}(d) - 1 = 0.$$

Theorem 4.1. *Let (d^*, π^*) is optimal, then $(\theta_2^*, b_2 - \theta_2^*)$ is optimal for Problem 4.2 and $(\theta_3^*, a_1 - \theta_3^*)$ is optimal for Problem 4.3.*

Proof. In Problem 4.2, if $\pi = b_2 - d$, let $t_2(d) = b_2 - d_2 - (1+\theta)E\left[(X_2 - d_2)_+\right]$, and from the proof of Theorem 3.2 we know that $t_2(d^*)$ attains its maximum at $d^* = \theta_2^*$. Then $\pi^* = b_2 - d^* = b_2 - \theta_2^*$.

In Problem 4.3, if $\pi = a_1 - d$, the objective function (4.5) can be reduced to $t_3(d) = (a_1 - d) - p(1+\theta)E(X_1 - d)_+ - (1-p)(1+\theta)E(X_2 - d)_+$. and its derivative is $t_3'(d) = -1 + p(1+\theta)S_{X_1}(d) + (1-p)(1+\theta)S_{X_2}(d)$. Obviously $t_3(d)$ is a concave function on $[0, a_1]$ and there must exist a unique $\theta_3^* \in [0, a_1]$ such that $t_3'(d) > 0$ for $d_1 \in [0, \theta_3^*)$ and $t_3'(d) < 0$ for $d_1 \in (\theta_3^*, a_1]$. Therefore, $t_3(d)$ attains its maximum at $d^* = \theta_3^*$. Then $\pi^* = a_1 - d^* = a_1 - \theta_3^*$. □

Now we give the expected net profit of the reinsurer in the optimal reinsurance policy and relabel it as T_P. In Problem 4.2, we have

$$T_{P_1} = (1-p)\left(b_2 - \theta_2^* - (1+\theta)E[(X_2 - \theta_2^*)_+]\right). \tag{4.7}$$

and in Problem 4.3, we have

$$\begin{aligned} T_{P_2} =& p\left(a_1 - \theta_3^* - (1+\theta)E[(X_1 - \theta_3^*)_+]\right) \\ &+ (1-p)\left(a_1 - \theta_3^* - (1+\theta)E[(X_2 - \theta_3^*)_+]\right). \end{aligned} \tag{4.8}$$

Moreover, under the Problem 4.2, the welfare gains of the type 2 is given by $W_2 = (b_2 - \theta_2^*) - (b_2 - \theta_2^*) = 0$. Under the Problem 4.3, the welfare gains of the type 1 and type 2 insurer are given by $W_1 = (a_1 - \theta_3^*) - (a_1 - \theta_3^*) = 0$ and $W_2 = (b_2 - \theta_3^*) - (a_1 - \theta_3^*) \geq 0$, respectively. We get a similar result to the section 3.

5. Numerical examples

This section presents some examples to analyze the conclusions of sections 3 and 4. We shall discuss and compare the optimal expected profit when the reinsurer adopts separating equilibrium or pooling equilibrium strategy.

Example 5.1. *Both types of insurers apply VaR risk measures. Suppose that $\alpha = 0.01$, $\beta = 0.05$, and $\theta = 0.3$. The random losses X_1 and X_2 follow the Pareto distribution where*

$$S_{X_1}(x) = \left(\frac{20}{x}\right)^5 \quad \text{and} \quad S_{X_2}(x) = \left(\frac{20}{x}\right)^3.$$

(i). Given $p = 0.5$. Then $a_1 = 50.2377$, $b_2 = 54.2883$, $\theta_1^* = 21.0775$, $\theta_2^* = 21.8279$, $\theta_3^* = 21.3675$. Here $\theta_2^* \leq a_1$. Through the Eqs (3.5), (4.7) and (4.8), we have $T_S = 20.6934$, $T_{p_1} = 10.7733$, $T_{p_2} = 10.7733$.

(ii). If set $p = 0.1$, we have $T_S = 18.1354$, $T_{p_1} = 19.3919$, $T_{p_2} = 18.1301$.

We can observe that if the two risks X_1 and X_2 are not significantly different (i.e. $\theta_2^* \leq a_1$) and the difference of the market share of two types of insurers are not obvious, we have $T_s > T_{P_2} > T_{P_1}$. At this point, the reinsurer's profit under separating equilibrium is maximum and it is unwise to design policy only for the second type of insurer.

If we increase the weight of the type 2 insurer, we have $T_{P_1} > T_s > T_{P_2}$. In this extreme case the type 2 insurer market is crucial, it is optimal for reinsurer to abandon the type 1 insurer and only provides policy to the type 2 insurer. We need to stress that the pooling equilibrium strategy is never better than the separating equilibrium strategy under the premise of considering both insurer markets.

Example 5.2. *Suppose that $\alpha = 0.01$, $\beta = 0.05$, and $\theta = 0.3$, X_1 and X_2 follow the Pareto distribution where*

$$S_{X_1}(x) = \left(\frac{20}{x}\right)^5 \quad \text{and} \quad S_{X_2}(x) = \left(\frac{50}{x}\right)^3.$$

(i). Given $p = 0.6$. Then $a_1 = 50.2377$, $b_2 = 135.72$, $\theta_1^* = 21.0775$, $\theta_2^* = 54.5696$, $\theta_3^* = 40.5183$. Here $a_1 < \theta_2^*$. Through the Eqs (3.6), (4.7) and (4.8), we have $T_S = 1.4572$, $T_{p_1} = 21.5466$, $T_{p_2} = -10.3082$.

(ii). If set $p = 0.8$, we have $T_S = 12.6741$, $T_{p_1} = 10.7733$, $T_{p_2} = 1.6148$.

In Example 5.2, the risk of type 2 is significantly higher than that of type 1 such that $a_1 < \theta_2^*$. Therefore, the two types of insurers are divided into high-risk and low-risk types. When the proportion of low-risk type has no obvious advantage, it is unreasonable to design the same policy for two types of insurers because the benefits of lowering premiums to attract high-risk insurers are not enough to offset the risks. In the market of asymmetric information, it is optimal to directly reduce the coverage ratio for the low-risk insurers or even to open the market only to high-risk insurers in order to prevent high-risk insurer from imitating low-risk insurer. However, separating equilibrium strategy is optimal when low-risk insurer dominate the market.

6. Conclusions

In this article, we study the optimal reinsurance problem between the monopoly reinsurer and two types of insurers who adopt the VaR as the risk assessment tools under asymmetric information. We consider and analyze two strategies, separating equilibrium and pooling equilibrium. In general, separating equilibrium strategy is optimal in an unrestricted market. But when insurers can be divided into high-risk and low-risk, reinsurer tends to design policies only for high-risk insurer if low-risk insurer does not dominate the market. The design of optimal policies depends mainly on the composition of the market and the difference in risk between the two groups.

For convenience, this article focuses on VaR. However, due to the defects of VaR that discourage the practitioners from applying it for risk assessment. One possible research direction is to study optimal policies under TVaR or distortion risk measures, we decide to leave such problem for future research.

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Conflict of interest

The author declares that there is no conflicts of interest in this paper.

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