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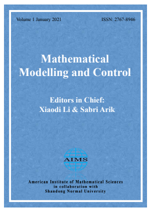

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Research article

A fractional order mathematical model of teenage pregnancy problems and rehabilitation in Nigeria

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Abstract: Teenage pregnancy is a social problem in Nigeria, whereby girls between the ages of 10-14 become pregnant by sexual intercourse after ovulation or first menstrual period. This article involves the fractional order mathematical model formulation describing the societal problem of teenage pregnancy in the sense of Caputo. The positivity, existence and uniqueness results of the model were established, and the two equilibria, which are the teenage pregnancy-free and teenage pregnancy-present equilibrium solutions of the model are presented. The graphical illustrations showing the behavior of the model variables when the basic reproduction number R_{pr} is less and greater than unity are displayed, using the numerical technique of Fractional Multi-Stage Differential Transform Method (FMSDTM) in comparison with the Runge-Kutta fourth order method (RK4) via the maple computational software. In addition, simulations involving the effect of rehabilitation is observed not to lessen R_{pr} below unity, which shows that further mitigation measures are needed to halt teenage pregnancy problems in Nigeria.

Keywords: basic reproduction number R_{pr}; teenage pregnancy; FMSDTM; Runge-Kutta fourth order (RK4)

1. Introduction

The rate of teenage pregnancy with its attendant consequences in the world calls for concern. The World Health Organization (WHO), reported that approximately 12 million teenage girls in age range of 15-19 years and below 777, 000 adolescent girls under 15 years give birth annually, where about 10 million unwanted pregnancies occurs annually in developing nations [1]. Complications due to pregnancy, childbirth and abortion are the leading cause of teenage girls mortality, while approximately 5.6 million adolescent girls go for abortions each year which results to lasting health issues, maternal mortality, morbidity, economic loss, school drop out rate, crime, etc. Nigeria is one of the nations in the world affected by the problem of adolescent pregnancies yearly [2].

Okonofua [3] performed a univariate analysis using a logistic regression model to determine the risk factors associated with adolescent pregnancy in a rural community of Gbogan in south western Nigeria. Their results showed that both pregnant and non-pregnant adolescents had a poor knowledge of, and attitude towards contraception and only a small percentage of them have ever used contraceptives. Also, publication from this is African news [4], revealed that Family, Life and HIV Education Curriculum (FLHE) should be taken into consideration in Nigeria to reduce the problem of teenage pregnancy, while data on birth and death rates in Nigerian population is seen in [5, 6].

Reports by the Demographic Health Survey (DHS) and fact sheet regarding teenage pregnacy in Nigeria, revealed that only three out of ten women have had sexual intercourse at the age of 20, while 54 % was said to have had sex before turning 18, while another 24 % indicated that they had not even been 15 years yet [7]. The DHS also reported that just 2% of sexually active girls between 15 and 19 years of age use contraceptives. An important reason is that they do not have access to contraceptives. Large numbers of girls become pregnant because of voluntary early sex and peer pressure, while others are sexually abused or forced to marry early. This occurs because they lack proper sex education and information on contraceptives [33].

Mathematical models are used to depict real life social problems [8–12]. Mokaya *et al.,* [13] formulated mathematical model sub-divided into classes of susceptible humans with no corrupt morals, humans with corrupt morals and treated humans incorporating the use of contraception and education using Kenyan data. In their results, $R_0 > 1$ is sufficient to increase corruption of morals unless controls of contraception and education is increased, while Danford, Kimathi and Mirau [14] derived a model to analyze corruption dynamics with intervention using Tanzanian statistics. They showed in the results that, the combined effect of mass education and religious teaching proves effective in the elimination of bad morals among Tanzanian adolescents. Also, Binuyo and Akinola [15] and Egudam, Oguntolu and Ashezua [16] modeled the dynamics of social menace of corruption, thereby suggesting probable ways to minimize it. Also, Mathematical model techniques are applied to model the spread of the novel COVID-19 and dengue disease co-infection in human host population [17–20].

Fractional derivatives and integrals are important parts of Mathematics applied to diverse fields in social, physical and biological systems [21–23], because it explains better the nature of models compared to the classical case. The reason for the efficient nature of fractional dynamics of complex phenomena is due to the liberty in choosing any arbitrary order of fractional operators, which is not applicable to classical derivatives and effective memory of past information due to its nonlocal nature in predicting the physical behavior of the system. Also, several semi-analytical methods have been used to obtain approximate solutions to several physical models based on ordinary and partial differential equations, Agarwal *et al.,* [24]. The method of interest in this paper is the Differential Transform Method (DTM), which was first proposed by Zhou [25], in obtaining approximate solutions to linear and nonlinear problems in electric circuit analysis. The merit of this method is that it reduces the volume of computation, requires no discretization and forms a good approximation in a small domain. Several extensions and modification of DTM in solving fractional order models of linear and nonlinear problems can be seen in [26–30].

The reason for the use of fractional order mathematical model using integrals and derivatives of non-integer order is due to the fact that exponential laws are based on classical approach to analyzed the dynamics of population densities, but there are more complex dynamics which are faster or slower than exponential laws, which in such cases are best described by fractional order functions due to memory effect. Fractional calculus is used to illustrate real world problems modeled with non-integer order derivatives. Several examples of these are seen in Engineering, Biology, Physics, Economics etc. Fractional derivative operators based on nonlinear differential equations can be said to be non-local. The advantage of Caputo fractional operator in this work is that it allows the traditional initial values to be included in the problem derivation and takes into account, the interaction with past information and non-local properties [31, 32].

The chief motivation behind this work is that, previous literature only considered the effect of moral corruption in adolescents using classical derivatives. Due to the efficient nature of the Caputo fractional operator, this work considers a new six compartmental fractional order Caputo model describing sexual interactions and negative peer influence among sexually active males and female teenagers sub-divided into female teenagers susceptible to early pregnancy, pregnant teenagers, pregnant teenagers who practice abortion, pregnant teenagers who dropped out of school and rehabilitated teenagers. The main of advantage of the study is that it helps to describe and predict the level of effect of teenage pregnancy burden in Nigeria and to guide policy makers on how best to use medical and social controls to minimize the menace caused by teenage pregnancy in the nation. To the best of the author's knowledge, this has not been considered. The paper is placed into sections, Section 2 presents the model formulation from classical to non-classical order case in the sense of Caputo. Section 3 establishes the qualitative results of the fractional order model, while Section 4 involves obtaining the equilibrium solutions, while the computation of R_{pr} and numerical solution of the fractional order model variables using FMSDTM are discussed in Section 5.

Finally, Section 6 deals with the numerical simulations and discussion of results and Section 7 presents the conclusion of the work.

1.1. Mathematical preambles

Definition 1.1. *[32, 22] The Riemann - Lioville integral of order $\iota > 0$ of function $f(\alpha)$ is defined by the integral* $D^{\iota}_{0,\alpha}[f(\alpha)] = \frac{1}{\Gamma(\iota)\int_0^{\alpha}(\alpha-m)^{\iota-1}f(m)dm,}$ $\alpha > 0$.

Definition 1.2. *[26, 22] Given a well-defined continuous function $f(\alpha) \in C^n[0,T]$ with $\iota > 0$, the Caputo fractional derivative of $f(\alpha)$ is defined by* ${}^{C}D^{\iota}_{0,\alpha}[f(\alpha)] = \frac{1}{\Gamma(n-\iota)\int_0^{\alpha}(\alpha-m)^{n-\iota-1}f^{(n)}mdm,}$ *where $n-1 < \iota \leq n$, $n \in N$, such that if $\iota \to 1$, then ${}^{C}D^{\iota}_{0,\alpha}f(\alpha) \to f'(\alpha)$. If $\alpha \in (0,1)$, then one obtains* ${}^{C}D^{\iota}_{0,\alpha}f(\alpha) = \frac{1}{\Gamma(1-\iota)}\int_0^{\alpha}\frac{f'(m)}{(\iota-m)^{\alpha}}dm$.

2. The mathematical model

Here, the total human fertile male population M_s is given by $N_2(t)$ and the total population of female teenagers denoted by $N_1(t) = T_a(t) + T_p(t) + A_b(t) + T_d(t) + R_e(t)$, where T_a denotes the teenage female susceptible to early pregnancy, T_p denotes the pregnant teenagers, A_b denotes pregnant teenagers who practice abortion, while T_d denotes the pregnant teenagers who dropped out of school and R_e denotes the rehabilitated teenage females at time $t > 0$, which gives rise to the classical model given by

$$
\begin{cases}
\frac{dT_a}{dt} = \Pi_A - (\beta_1 M_s + \beta_2 T_d + \beta_3 A_b)T_a - \mu T_a,\\
\frac{dM_s}{dt} = \Pi_m - (\beta_4 A_b + \beta_5 T_d)M_s - \mu M_s,\\
\frac{dT_p}{dt} = \beta_1 T_a M_s - (\mu + \delta + \sigma)T_p - \rho_1 T_p,\\
\frac{dA_b}{dt} = (\beta_3 T_a + \beta_4 M_s)A_b + \sigma T_p - (\mu + \xi + \gamma)A_b - \rho_2 A_b,\\
\frac{dT_d}{dt} = (\beta_2 T_a + \beta_5 M_s)T_d + \gamma A_b - (\mu + \rho_3)T_d,\\
\frac{dR_e}{dt} = \rho_1 T_p + \rho_2 A_b + \rho_3 T_d - \mu R_e.
\end{cases}
\tag{2.1}
$$

In (2.1), Π_A and Π_m denotes the recruitment rates of female teenagers and sexually active males respectively, while β_1 denotes the sexual interactions between susceptible teenage girls and sexually active males and $\beta_2, \beta_3, \beta_4$ and β_5 represents the negative peer influence incidence rates among the classes of human compartments. μ, δ and ξ denotes the natural death, death due to pregnancy complication and death due to abortion complication rates respectively. Also, σ and γ denotes the progression rates of pregnant teenagers to abortion and school drop out compartments, while ρ_1, ρ_2 and ρ_3 denotes rehabilitation rates respectively. Transforming (2.1) into a fractional order model under the Caputo sense yields

$$
\begin{cases}
{}^{C}D^{\iota}_{0,t}[T_a] = \Pi_A - (\beta_1 M_s + \beta_2 T_d + \beta_3 A_b)T_a - \mu T_a,\\
{}^{C}D^{\iota}_{0,t}[M_s] = \Pi_m - (\beta_4 A_b + \beta_5 T_d)M_s - \mu M_s,\\
{}^{C}D^{\iota}_{0,t}[T_p] = \beta_1 T_a M_s - (\mu + \delta + \sigma)T_p - \rho_1 T_p,\\
{}^{C}D^{\iota}_{0,t}[A_b] = (\beta_3 T_a + \beta_4 M_s)A_b + \sigma T_p\\
\qquad -(\mu + \xi + \gamma)A_b - \rho_2 A_b,\\
{}^{C}D^{\iota}_{0,t}[T_d] = (\beta_2 T_a + \beta_5 M_s)T_d + \gamma A_b - (\mu + \rho_3)T_d,\\
{}^{C}D^{\iota}_{0,t}[R_e] = \rho_1 T_p + \rho_2 A_b + \rho_3 T_d - \mu R_e.
\end{cases}
\tag{2.2}
$$

Subject to the initial conditions $T_a \geq 0, M_s \geq 0, T_p \geq 0, A_b \geq 0, T_d \geq 0$, and $R_e \geq 0$. Hereafter, model system (2.2) shall be referred to.

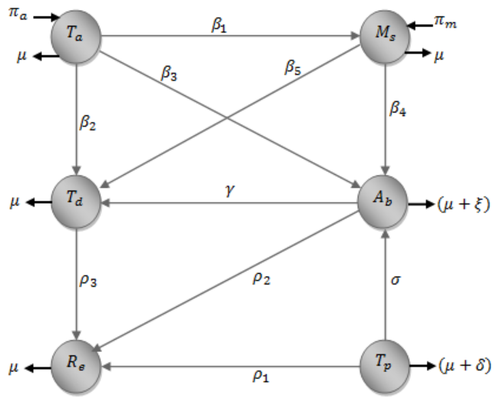


Figure 1. Block diagram of the human interactions leading to teenage pregnancy.

Parameters and variables describing the social menace of teenage pregnancy in Nigeria are tabulated below as obtained from existing literature.

Table 1. Parameters Descriptions

Variables/Parameters	Descriptions	Values/Per Month	Sources
Π_a	Recruitment rate of susceptible teenage girls	3.2192	[23, 25]
Π_m	Recruitment rate of sexually active males	1.801	[23, 25]
β_1	Sexual interaction rate between fertile males and susceptible teenage girls	0.411	[11, 23]
β_2	Negative peer influence between susceptible teenage girls and dropout teenage girls	0.320	[20, 22]
β_3	Negative peer influence between teenage girls and teenage girls who aborted pregnancies	0.532	[20, 22]
β_4	Negative Peer influence between fertile males and teenage girls who aborted pregnancies	0.0877	[20, 22]
β_5	Negative Peer influence between susceptible fertile males and drop-out teenage girls	0.201	[20, 22]
μ	Natural mortality rate	0.2863	[26]
δ	Death induced by pregnancy complications	0.426	[20, 26]
σ	Transition rate of pregnant teenage girls to Abortion class	0.370	[20, 22]
$\rho_i(i = 1-3)$	Transition rates to rehabilitation	0 - 1	[20, 22]
ξ	Death induced by abortion complications	0.370	[3, 20, 22]

3. Qualitative analysis of the fractional order model

3.1. Existence and uniqueness results

Here, the existence and the uniqueness of the model via the Caputo operator is established. Consider a real valued and continuous function denoted by a function B(Z) which is a Banach space on $Z[0,b]$ with norm$\|T_a, M_s, T_p, A_b, T_d, R_e\| = \|T_a\| + \|M_s\| + \|T_p\| + \|A_b\| + \|T_d\| + \|R_e\|$, where $\|T_a\| = sup_{t\in Z}|T_a(t)|, \|M_s\| = sup_{t\in Z}|M_s(t)|, \|T_p\| = sup_{t\in Z}|T_p(t)|, \|A_b\| = sup_{t\in Z}|A_b(t)|, \|T_d\| = sup_{t\in Z}|T_d(t)|$ and $\|R_e\| = sup_{t\in Z}|R_e(t)|$. Using the Caputo integral operator on the model system (2.2) yields

$$\begin{cases} T_a(t) - T_a(0) = ^C D_{0,t}^t \{\Pi_A - (\beta_1 M_s + \beta_2 T_d + \beta_3 A_b)T_a - \mu T_a\}, \\ M_s(t) - M_s(0) = ^C D_{0,t}^t \{\Pi_m - (\beta_4 A_b + \beta_5 T_d)M_s - \mu M_s\}, \\ T_p(t) - T_p(0) = ^C D_{0,t}^t \left\{\beta_1 T_a M_s - (\mu + \delta + \sigma)T_p - \rho_1 T_p\right\}, \\ A_b(t) - A_b(0) = ^C D_{0,t}^t \left\{(\beta_3 T_a + \beta_4 M_s)A_b + \sigma T_p \right. \\ \qquad \left. -(\mu + \xi + \gamma)A_b - \rho_2 A_b\right\}, \\ T_d(t) - T_d(0) = ^C D_{0,t}^t \{(\beta_2 T_a + \beta_5 M_s)T_d + \gamma A_b - (\mu + \rho_3)T_d\}, \\ R_e(t) - R_e(0) = ^C D_{0,t}^t \left\{\rho_1 T_p + \rho_2 A_b + \rho_3 T_d - \mu R_e\right\}. \end{cases} \tag{3.1}$$

The model expression in (3.1) refers to:

$$\begin{cases} T_a(t) - T_a(0) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_1(\upsilon, \kappa, T_a(\kappa))d\kappa, \\ M_s(t) - M_s(0) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_2(\upsilon, \kappa, M_s(\kappa))d\kappa, \\ T_p(t) - T_p(0) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_3(\upsilon, \kappa, T_p(\kappa))d\kappa, \\ A_b(t) - A_b(0) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_4(\upsilon, \kappa, A_b(\kappa))d\kappa, \\ T_d(t) - T_d(0) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_5(\upsilon, \kappa, T_D(\kappa))d\kappa, \\ R_e(t) - R_e(0) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_6(\upsilon, \kappa, R_e(\kappa))d\kappa. \end{cases} \tag{3.2}$$

And the kernels are defined as follows

$$\begin{cases} U_1(\upsilon, t, T_a(t)) = \Pi_A - (\beta_1 M_s + \beta_2 T_d + \beta_3 A_b)T_a - \mu T_a, \\ U_2(\upsilon, t, M_s(t)) = \Pi_m - (\beta_4 A_b + \beta_5 T_d)M_s - \mu M_s, \\ U_3(\upsilon, t, T_p(t)) = \beta_1 T_a M_s - (\mu + \delta + \sigma)T_p - \rho_1 T_p, \\ U_4(\upsilon, t, A_b(t)) = (\beta_3 T_a + \beta_4 M_s)A_b + \sigma T_p \\ \qquad -(\mu + \xi + \gamma)A_b - \rho_2 A_b, \\ U_5(\upsilon, t, T_d(t)) = (\beta_2 T_a + \beta_5 M_s)T_d + \gamma A_b - (\mu + \rho_3)T_d, \\ U_6(\upsilon, t, R_e(t)) = \rho_1 T_p + \rho_2 A_b + \rho_3 T_d - \mu R_e. \end{cases} \tag{3.3}$$

Now, $U_i (i = 1-6)$ must guarantee the Lipschitz condition's validity with $T_a(t), T_p(t), A_b(t), T_D(t)$, and $R_e(t)$ as upper bounds. Taking into considerations the functions $T_a(t)$ and $T_a^*(t)$, then we can write

$$\begin{aligned} &\|U_1(\upsilon, t, T_a(t)) - U_1(\upsilon, t, T_a^*(t))\| \\ =&\| - ((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)(T_a(t) - T_a^*(t))\|. \end{aligned} \tag{3.4}$$

Assuming that $\lambda_1^* = \| - ((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)\|$, one obtains

$$\|U_1(\upsilon, t, T_a(t)) - U_1(\upsilon, t, T_a^*(t))\| = \lambda_1^*(T_a(t) - T_a^*(t))\|. \tag{3.5}$$

Following the same process in (3.4) - (3.5), one obtains

$$\begin{cases} \|U_2(\upsilon, t, M_s(t)) - U_2(\upsilon, t, M_s^*(t))\| \le \lambda_2^*(M_s(t) - T_s^*(t))\|, \\ \|U_3(\upsilon, t, T_p(t)) - U_3(\upsilon, t, T_p^*(t))\| \le \lambda_3^*(T_p(t) - T_p^*(t))\|, \\ \|U_4(\upsilon, t, A_b(t)) - U_4(\upsilon, t, A_b^*(t))\| \le \lambda_4^*(A_b(t) - A_b^*(t))\|, \\ \|U_5(\upsilon, t, T_d(t)) - U_5(\upsilon, t, T_d^*(t))\| \le \lambda_5^*(T_d(t) - T_d^*(t))\|, \\ \|U_6(\upsilon, t, R_e(t)) - U_6(\upsilon, t, R_e^*(t))\| \le \lambda_6^*(R_e(t) - R_e^*(t))\|. \end{cases} \tag{3.6}$$

Hence, the Lipschitz conditions for the kernels is established. Furthermore, (3.6) can be expressed recursively as

$$\begin{cases} T_a(t) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_1(\upsilon, \kappa, T_{an-1}(\kappa))d\kappa, \\ M_s(t) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_2(\upsilon, \kappa, M_{sn-1}(\kappa))d\kappa, \\ T_p(t) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_3(\upsilon, \kappa, T_{pn-1}(\kappa))d\kappa, \\ A_b(t) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_4(\upsilon, \kappa, A_{bn-1}(\kappa))d\kappa, \\ T_d(t) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_5(\upsilon, \kappa, T_{dn-1}(\kappa))d\kappa, \\ R_e(t) = V(\upsilon) \int_0^t (t-\kappa)^{-\upsilon} U_6(\upsilon, \kappa, R_{en-1}(\kappa))d\kappa. \end{cases} \tag{3.7}$$

Together with the initial conditions $T_a \ge 0, M_s \ge 0, T_p \ge 0, A_b \ge 0, T_d \ge 0$, and $R_e \ge 0$, so that we obtain

$$\begin{cases} \zeta_{T_{a,n}(t)} = T_a(t) - T_{an-1}(t) = V(\upsilon)\int_0^t (t-\kappa)^{-\upsilon} \\ (U_1(\upsilon,\kappa,T_{an-1}(\kappa)) - U_1(\upsilon,\kappa,T_{an-2}(\kappa)))d\kappa, \\ \zeta_{M_{s,n}(t)} = M_s(t) - M_{sn-1}(t) = V(\upsilon)\int_0^t (t-\kappa)^{-\upsilon} \\ (U_2(\upsilon,\kappa,M_{sn-1}(\kappa)) - U_2(\upsilon,\kappa,M_{sn-2}(\kappa)))d\kappa, \\ \zeta_{T_{p,n}(t)} = T_p(t) - T_{pn-1}(t) = V(\upsilon)\int_0^t (t-\kappa)^{-\upsilon} \\ (U_3(\upsilon,\kappa,T_{pn-1}(\kappa)) - U_3(\upsilon,\kappa,T_{pn-2}(\kappa)))d\kappa, \\ \zeta_{A_{b,n}(t)} = A_b(t) - A_{bn-1}(t) = V(\upsilon)\int_0^t (t-\kappa)^{-\upsilon} \\ (U_4(\upsilon,\kappa,A_{bn-1}(\kappa)) - U_4(\upsilon,\kappa,A_{bn-2}(\kappa)))d\kappa, \\ \zeta_{T_{d,n}(t)} = T_d(t) - T_{dn-1}(t) = V(\upsilon)\int_0^t (t-\kappa)^{-\upsilon} \\ (U_5(\upsilon,\kappa,T_{dn-1}(\kappa)) - U_5(\upsilon,\kappa,T_{dn-1}(\kappa)))d\kappa, \\ \zeta_{R_{e,n}(t)} = R_e(t) - R_{en-1}(t) = V(\upsilon)\int_0^t (t-\kappa)^{-\upsilon} \\ (U_6(\upsilon,\kappa,R_{en-1}(\kappa)) - U_6(\upsilon,\kappa,R_{en-1}(\kappa)))d\kappa. \end{cases} \tag{3.8}$$

It is pertinent to consider $T_a(t) = \sum_{i=0}^n \zeta_{T_{a,\ i}}(t)$, $M_s(t) = \sum_{i=0}^n \zeta_{M_{s,\ i}}(t)$, $T_p(t) = \sum_{i=0}^n \zeta_{T_{p,\ i}}(t)$, $A_b(t) = \sum_{i=0}^n \zeta_{A_{b,\ i}}(t)$, $T_d(t) = \sum_{i=0}^n \zeta_{T_{d,\ i}}(t)$ and $R_e(t) = \sum_{i=0}^n \zeta_{R_{e,\ i}}(t)$. Moreover, from (3.3) and (3.4) and supposing that $\zeta_{T_a,\ n-1}(t) = T_{an-1}(t) - T_{an-2}(t), \zeta_{M_s,\ n-1}(t) = M_{sn-1}(t) - M_{sn-2}(t), \zeta_{T_p,\ n-1}(t) = T_{pn-1}(t) - T_{pn-2}(t), \zeta_{T_a,\ n-1}(t) = A_{bn-1}(t) - A_{bn-2}(t), \zeta_{T_d,\ n-1}(t) = T_{dn-1}(t) - T_{Dn-2}(t)$ and $\zeta_{R_e,\ n-1}(t) = R_{en-1}(t) - R_{en-2}(t)$, one obtains

$$\begin{cases} \|\zeta_{T_{a,n}(t)}\| \le V(\upsilon)\varpi_1 \int_0^t (t-\kappa)^{-\upsilon}\|\zeta_{T_a,n-1}(\kappa)\|d\kappa, \\ \|\zeta_{M_{s,n}(t)}\| \le V(\upsilon)\varpi_2 \int_0^t (t-\kappa)^{-\upsilon}\|\zeta_{M_s,n-1}(\kappa)\|d\kappa, \\ \|\zeta_{T_{p,n}(t)}\| \le V(\upsilon)\varpi_3 \int_0^t (t-\kappa)^{-\upsilon}\|\zeta_{T_p,n-1}(\kappa)\|d\kappa, \\ \|\zeta_{A_{b,n}(t)}\| \le V(\upsilon)\varpi_4 \int_0^t (t-\kappa)^{-\upsilon}\|\zeta_{A_b,n-1}(\kappa)\|d\kappa, \\ \|\zeta_{T_{d,n}(t)}\| \le V(\upsilon)\varpi_5 \int_0^t (t-\kappa)^{-\upsilon}\|\zeta_{T_d,n-1}(\kappa)\|d\kappa, \\ \|\zeta_{R_{e,n}(t)}\| \le V(\upsilon)\varpi_6 \int_0^t (t-\kappa)^{-\upsilon}\|\zeta_{R_e,n-1}(\kappa)\|d\kappa. \end{cases} \tag{3.9}$$

Theorem 3.1. *Assume that $\frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_i < 1, i = 1, \cdots, 6$, then the governing model possess a unique solution for $t \in [0, b]$.*

Proof. The boundedness and existence of $(T_a(t), M_s(t), T_p(t), A_b(t), T_d(t), R_e(t)$ have been established. In addition, (3.4) and (3.5) are Lipschitz. Then, combining (3.9) with a recursive hypothesis yields

$$\begin{cases} \|\zeta_{T_{an}}(t)\| \le \|T_{ao}(t)\|\left(\frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_1\right)^n, \\ \|\zeta_{M_{sn}}(t)\| \le \|M_{so}(t)\|\left(\frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_2\right)^n, \\ \|\zeta_{T_{pn}}(t)\| \le \|T_{po}(t)\|\left(\frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_3\right)^n, \\ \|\zeta_{A_{bn}}(t)\| \le \|A_{bo}(t)\|\left(\frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_4\right)^n, \\ \|\zeta_{T_{dn}}(t)\| \le \|T_{do}(t)\|\left(\frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_5\right)^n, \\ \|\zeta_{R_{en}}(t)\| \le \|T_{eo}(t)\|\left(\frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_6\right)^n. \end{cases} \tag{3.10}$$

Thus, $\|\zeta_{T_{p,n}}\| \to 0, \|\zeta_{M_{s,n}}\| \to 0, \|\zeta_{T_{p,n}}\| \to 0, \|\zeta_{A_{b,n}}\| \to 0, \|\zeta_{T_{d,n}}\| \to 0, \|\zeta_{R_{e,n}}\| \to 0$ as $n \to \infty$. Moreover, from (3.10) and imposing the triangular inequality for any k, one obtains

$$\begin{cases} \|T_{an+k}(t) - T_{an}(t)\| \le \sum_{w=n+1}^{n+k} r_1^i = \frac{r_1^{n+1} - r_1^{n+k+1}}{1-r_1}, \\ \|M_{sn+k}(t) - M_{sn}(t)\| \le \sum_{w=n+1}^{n+k} r_2^i = \frac{r_2^{n+1} - r_2^{n+k+1}}{1-r_2}, \\ \|T_{pn+k}(t) - T_{pn}(t)\| \le \sum_{w=n+1}^{n+k} r_3^i = \frac{r_1^{n+1} - r_3^{n+k+1}}{1-r_3}, \\ \|A_{bn+k}(t) - A_{bn}(t)\| \le \sum_{w=n+1}^{n+k} r_4^i = \frac{r_1^{n+1} - r_4^{n+k+1}}{1-r_4}, \\ \|T_{dn+k}(t) - T_{dn}(t)\| \le \sum_{w=n+1}^{n+k} r_5^i = \frac{r_1^{n+1} - r_5^{n+k+1}}{1-r_5}, \\ \|R_{en+k}(t) - R_{en}(t)\| \le \sum_{w=n+1}^{n+k} r_6^i = \frac{r_1^{n+1} - r_6^{n+k+1}}{1-r_6}. \end{cases} \tag{3.11}$$

Hypothetically, it can be deduced that $q_i = \frac{V(\upsilon)}{\upsilon} b^{\upsilon}\varpi_i < 1$. Therefore, $T_{an}, M_{sn}, T_{pn}, A_{bn}, T_{dn}$ and R_{en} are known as the Cauchy sequences in B(Z) and are uniformly convergent. Using the proposition on limit in (3.6) as $n \to \infty$ shows that the model (2.2) is unique. Hence the existence of a unique solution is established

3.2. Invariant region

Firstly, the fractional order model (2.2), is analyzed in a feasible domain, such that the model is considered in two parts, $N_1(t) = T_a(t) + M_s(t) + T_p(t) + A_b(t) + T_d(t) + R_e(t)$ for the total female host population and $N_2(t) = M_s(t)$ for the male host population.

Theorem 3.2. *The region $\Gamma = \Gamma_1 \times \Gamma_2$, where $\Gamma_1 = \left\{(T_a(t), T_p(t), A_b(t), T_d(t), R_e(t)) \in \Re^{+5} : 0 \le N_1 \le \frac{\Pi_A}{\mu}\right\}$ and $\Gamma_2 = \left\{(M_s(t)) \in \Re^{+} : 0 \le N_1 \le \frac{\Pi_m}{\mu}\right\}$ is positively invariant.*

Proof. The total female teenage girls is considered such that in the absence of death due to pregnancy and abortion complications yields

$${}^{C}D_{0,t}^{t} N(t) = \Pi_A - \mu N_1 - \delta T_p - \xi A_b \le \Pi_A - \mu N_1 \tag{3.12}$$

so that

$$\frac{d}{dt}(N_1 e^{\mu t^{\iota}}) = \Pi_A \tag{3.13}$$

and

$$N_1(t) = \frac{\Pi_A}{\mu}\Big[1 - e^{-\mu t^{\iota}}\Big]. \tag{3.14}$$

Similarly for the male population, one obtains

$$N_2(t) = \frac{\Pi_m}{\mu}\Big[1 - e^{-\mu t^{\iota}}\Big]. \tag{3.15}$$

As $t \to \infty$ in (3.14) and (3.15), the total female teenage girls and fertile male population start and stays in the domains

$$\Gamma_1 = \left\{(T_a(t), T_p(t), A_b(t), T_d(t), R_e(t)) \in \Re^{+5} : 0 \le N_1 \le \frac{\Pi_A}{\mu}\right\} \tag{3.16}$$

and

$$\Gamma_2 = \left\{(M_s(t)) \in \Re^{+} : 0 \le N_1 \le \frac{\Pi_m}{\mu}\right\}. \tag{3.17}$$

Therefore, (3.16) and (3.17) shows that $T_a(t), M_s(t), T_p(t), A_b(t), T_d(t)$ and $R_e(t)$ are bounded for all $t > 0$ and are not capable of leaving Γ which implies that the fractional order model (2.2) is positively invariant.

Theorem 3.3. *The fractional order model solutions of (2,2) and initial conditions are non-negative for time $t > 0$.*

Proof. Form the first mathematical expression in (2,2),

$$\begin{aligned} {}^{C}D^{\iota}_{0,t}[T_a(t)] &= \Pi_A - ((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)T_a \\ &\ge -((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)T_a, \end{aligned} \tag{3.18}$$

and

$${}^{C}D^{\iota}_{0,t}[T_a(t)] \ge -((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)dt, \tag{3.19}$$

so that

$$\int {}^{C}D^{\iota}_{0,t}[T_a(t)] \ge -\int ((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)dt \tag{3.20}$$

becomes

$${}^{C}D^{\iota}_{0,t}T_a(t) \ge T_a(0)e^{((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)t} > 0. \tag{3.21}$$

In a similar approach to the remaining sub-equations in (2.2), one obtains

$$\begin{cases} {}^{C}D^{\iota}_{0,t}T_a(t) \ge T_a(0)e^{((\beta_1 M_s + \beta_2 T_d + \beta_3 A_b) - \mu)t} > 0, \\ {}^{C}D^{\iota}_{0,t}M_s(t) \ge M_s(0)e^{((\beta_4 A_b + \beta_5 T_d) - \mu)t} > 0, \\ {}^{C}D^{\iota}_{0,t}A_b(t) \ge A_b(0)e^{((\mu + \xi + \gamma) - \rho_2)t} > 0, \\ {}^{C}D^{\iota}_{0,t}T_d(t) \ge T_d(0)e^{(\mu + \rho_3)t} > 0, \\ {}^{C}D^{\iota}_{0,t}R_e(t) \ge R_e(0)e^{(\mu)t} > 0. \end{cases} \tag{3.22}$$

Hence the solutions of model (2.2) are positive.

4. Equilibrium solutions and computation of R_{pr}

The model system in (2.2) has two equilibria, which are the teenage pregnancy - free and teenage pregnancy - present equilibrium solutions. The equilibrium solutions are obtained by fixing the left hand side of (2.2) to zero, to yield the teenage pregnancy - free equilibrium solution given by

$$E_k^o = (T_a, M_s, T_p, A_b, T_d, R_e) = \Big(\frac{\Pi_A}{\mu}, \frac{\Pi_m}{\mu}, 0, 0, 0, 0\Big). \tag{4.1}$$

Also, the teenage pregnancy - present equilibrium solution given by

$$\begin{cases} E_k^* = (T_a^*, M_s^*, T_p^*, A_b^*, T_d^*, R_e^*) = \\ T_a^* = \frac{\Pi_A - (\beta_1 M_s^* + \beta_2 + \beta_3 A_b^*)T_d^*}{\mu}, \\ M_s^* = \frac{\Pi_m}{\beta_4 A_b^* + \beta_5 T_d^* + \mu}, \\ T_p^* = \frac{\beta_1 M_s^* T_d^*}{(\mu + \delta + \sigma + \rho_1)}, \\ A_b^* = \frac{\sigma T_p^*}{(\gamma + \mu + \rho_2 + \xi) - \beta_3 T_a^* - \beta_4 M_s^*}, \\ T_d^* = \frac{\gamma A_b^*}{\beta_2 T_a^* + \beta_5 M_s^* - (\mu + \rho_3)}, \\ R_e^* = \frac{\rho_1 T_p^* + \rho_2 A_b^* + \rho_3 T_d^*}{\mu}. \end{cases} \tag{4.2}$$

The basic reproduction number R_{pr} in this work denotes the average rate at which new cases of teenage pregnancies occur due to the introduction of a fertile male into a naive susceptible teenage girls population during their course of sexual interactions. The next generation matrix method, used by [10], is employed to obtain the R_{pr} of model (2.2). The R_{pr} of model (2.2) is given by

$$R_{pr} = \frac{\beta_1 \Pi_A (\Pi_A \beta_2 + \Pi_m \beta_5)(\Pi_A \beta_3 + \Pi_m \beta_4)}{\mu^3 (\mu + \rho_3)(\mu + \delta + \sigma + \rho_1)(\mu + \xi + \gamma + \rho_2)}. \tag{4.3}$$

The threshold in (4.3) means that, when $R_{pr} < 1$, the menace of teenage pregnancy goes to extinction and when $R_{pr} > 1$, teenage pregnancy becomes prevalent in the host community.

5. Numerical technique of model solution

In order to obtain the approximate solution of the fractional order model (2.2) the differential transform method and its modification, called the Fractional Multi-Stage Differential Transform Method (FMSDTM) is considered [27–31]. Consider a system of fractional ordinary differential equations given by

$$\begin{cases} {}^{C}D_{0,t_1}^{\phi} x_1(t) = f_1(t, x_1, x_2, \cdots, x_n), \\ {}^{C}D_{0,t_2}^{\phi} x_1(t) = f_1(t, x_1, x_2, \cdots, x_n), \\ \vdots \\ {}^{C}D_{0,tn}^{\phi} x_1(t) = f_1(t, x_1, x_2, \cdots, x_n). \end{cases} \tag{5.1}$$

Together with initial conditions $x_i(t_o) = k_i, i = 1, 2, \cdots, n$, where ${}^{C}D_{0,t}^{\phi_i}$ is a Caputo derivative of order ϕ_i, where $0 < \phi_i \leq 1$, for $i = 1, 2, \cdots, n$. Let $[t_o, T]$ be the interval where the solution of (5.1) is to be determined. The k^{th} order approximate solution of the (5.1) is given by the finite series of the form

$$x_i(t) = \sum_{i=0}^{K} X_i(k)(t - t_o)^{k\phi_i}, t \in [t_o, T], \tag{5.2}$$

where $X_i(k)$ satisfies the recurrence relation;

$$\frac{\Gamma((k+1)\phi_i + 1)}{\Gamma(k\phi_i + 1)} X_i(k+1) = F_i(k, X_1, X_2, \cdots, X_n). \tag{5.3}$$

In (5.3), $X_i(0) = c_i$ and $F_i(k, X_1, X_2, \cdots, X_n)$ are the initial conditions and differential transforms of functions $f_i(t, x_1, x_2, \cdots, x_n)$ for $i = 1, 2, \cdots, n$. Furthermore, assume that the interval $[t_o, T]$ is partitioned into P sub-intervals $[t_{p-1}, t_p], p = 1, 2, \cdots, P$ of equal step length $h = (T - t_o)/P$, by the use of the nodes $t_p = t_o + ph$.

In order to perform the numerical implementation, firstly, the differential transform method is applied to (2.2) to give

$$\begin{cases} T_a(k+1) = \frac{\Gamma(\phi_1 k+1)}{\Gamma(\phi_1(k+1)+1)}\Big(\Pi_A - (\beta_1 M_s(k-l) + \beta_2 T_d(k-l) \\ \qquad +\beta_3 A_b(k-l))T_a(k) - \mu T_a(k)\Big), \\ M_s(k+1) = \frac{\Gamma(\phi_2 k+1)}{\Gamma(\phi_2(k+1)+1)}\Big(\Pi_m - (\beta_4 A_b(k-l) \\ \qquad +\beta_5 T_d(k-l))M_s(k) - \mu M_s(k)\Big), \\ T_p(k+1) = \frac{\Gamma(\phi_3 k+1)}{\Gamma(\phi_3(k+1)+1)}\Big(\beta_1 T_a(k) M_s(k-l) \\ \qquad -(\mu + \delta + \sigma)T_p(k) - \rho_1 T_p(k)\Big), \\ A_b(k+1) = \frac{\Gamma(\phi_4 k+1)}{\Gamma(\phi_4(k+1)+1)}\Big((\beta_3 T_a(k-l) + \beta_4 M_s(k-l))A_b(k) \\ \qquad +\sigma T_p(k) - (\mu + \xi + \gamma)A_b(k) - \rho_2 A_b(k)\Big), \\ T_d(k+1) = \frac{\Gamma(\phi_5 k+1)}{\Gamma(\phi_5(k+1)+1)}\Big((\beta_2 T_a(k-l) + \beta_5 M_s(k-l))T_d(k) \\ \qquad +\gamma A_b(k) - (\mu + \rho_3)T_d(k)\Big), \\ R_e(k+1) = \frac{\Gamma(\phi_6 k+1)}{\Gamma(\phi_6(k+1)+1)}\Big(\rho_1 T_p(k) + \rho_2 A_b(k) + \rho_3 T_d(k) \\ \qquad -\mu R_e(k)\Big). \end{cases} \tag{5.4}$$

where $T_a(k), M_s(k), T_p(k), A_b(k), T_d(k)$ and $R_e(k)$ with initial conditions $T_a \geq 0, M_s \geq 0, T_p \geq 0, A_b \geq 0, T_d \geq 0$, and $R_e \geq 0$ are the differential transforms of $T_a(t), M_s(t), T_p(t), A_b(t), T_d(t)$ and $R_e(t)$ respectively. In view of the differential inverse transform, the differential transform series solution for (5.4) is obtained as

$$\begin{cases} t_a(t) = \sum_{n=0}^{N} T_a(n) t^{\phi_1 n}, \\ m_s(t) = \sum_{n=0}^{N} M_s(n) t^{\phi_2 n}, \\ t_p(t) = \sum_{n=0}^{N} T_p(n) t^{\phi_3 n}, \\ a_b(t) = \sum_{n=0}^{N} A_b(n) t^{\phi_4 n}, \\ t_d(t) = \sum_{n=0}^{N} T_d(n) t^{\phi_5 n}, \\ r_e(t) = \sum_{n=0}^{N} R_e(n) t^{\phi_6 n}. \end{cases} \tag{5.5}$$

Using the Fractional Multi-Step Differential Transform Method (FMSDTM), (5.4)-(5.5) becomes

$$t_a(t) = \begin{cases} \sum_{n=0}^{K} T_{a1}(n) t^{\phi_1 n}, & t \in [0, t_1], \\ \sum_{n=0}^{K} T_{a2}(n)(t - t_1)^{\phi_1 n}, & t \in [t_1, t_2], \\ \vdots \\ \sum_{n=0}^{K} T_{aP}(n)(t - t_{P-1})^{\phi_1 n}, & t \in [t_{P-1}, t_P]. \end{cases} \tag{5.6}$$

$$m_s(t) = \begin{cases} \sum_{n=0}^{K} M_{s1}(n) t^{\phi_2 n}, & t \in [0, t_1], \\ \sum_{n=0}^{K} M_{s2}(n)(t - t_1)^{\phi_2 n}, & t \in [t_1, t_2], \\ \vdots \\ \sum_{n=0}^{K} M_{sP}(n)(t - t_{P-1})^{\phi_2 n}, & t \in [t_{P-1}, t_P]. \end{cases} \tag{5.7}$$

$$t_p(t) = \begin{cases} \sum_{n=0}^{K} T_{p1}(n) t^{\phi_3 n}, & t \in [0, t_1], \\ \sum_{n=0}^{K} T_{p2}(n)(t - t_1)^{\phi_3 n}, & t \in [t_1, t_2], \\ \vdots \\ \sum_{n=0}^{K} T_{pP}(n)(t - t_{P-1})^{\phi_3 n}, & t \in [t_{P-1}, t_P]. \end{cases} \tag{5.8}$$

$$a_b(t) = \begin{cases} \sum_{n=0}^{K} A_{b1}(n) t^{\phi_4 n}, & t \in [0, t_1], \\ \sum_{n=0}^{K} A_{b2}(n)(t - t_1)^{\phi_4 n}, & t \in [t_1, t_2], \\ \vdots \\ \sum_{n=0}^{K} A_{bP}(n)(t - t_{P-1})^{\phi_4 n}, & t \in [t_{P-1}, t_P]. \end{cases} \tag{5.9}$$

$$t_d(t) = \begin{cases} \sum_{n=0}^{K} T_{d1}(n) t^{\phi_5 n}, & t \in [0, t_1], \\ \sum_{n=0}^{K} T_{d2}(n)(t - t_1)^{\phi_5 n}, & t \in [t_1, t_2], \\ \vdots \\ \sum_{n=0}^{K} T_{dP}(n)(t - t_{P-1})^{\phi_5 n}, & t \in [t_{P-1}, t_P]. \end{cases} \tag{5.10}$$

and

$$r_e(t) = \begin{cases} \sum_{n=0}^{K} R_{e1}(n) t^{\phi_6 n}, & t \in [0, t_1], \\ \sum_{n=0}^{K} R_{e2}(n)(t - t_1)^{\phi_6 n}, & t \in [t_1, t_2], \\ \vdots \\ \sum_{n=0}^{K} R_{eP}(n)(t - t_{P-1})^{\phi_6 n}, & t \in [t_{P-1}, t_P]. \end{cases} \tag{5.11}$$

where $T_{a_i}(n), M_{s_i}(n), T_{p_i}(n), A_{b_i}(n), T_{d_i}(n)$ and $R_{e_i}(n)$ satisfy the following recurrence relations given by

$$\begin{cases} T_{a_i}(k+1) = \frac{\Gamma(\phi_1 k+1)}{\Gamma(\phi_1(k+1)+1)}\Big(\Pi_A - (\beta_1 M_{s_i}(k-l) + \beta_2 T_{d_i}(k-l) \\ \qquad +\beta_3 A_{b_i}(k-l))T_{a_i}(k) - \mu T_{a_i}(k)\Big), \\ M_{s_i}(k+1) = \frac{\Gamma(\phi_2 k+1)}{\Gamma(\phi_2(k+1)+1)}\Big(\Pi_m - (\beta_4 A_{b_i}(k-l) \\ \qquad +\beta_5 T_{d_i}(k-l))M_{s_i}(k) - \mu M_{s_i}(k)\Big), \\ T_{p_i}(k+1) = \frac{\Gamma(\phi_3 k+1)}{\Gamma(\phi_3(k+1)+1)}\Big(\beta_1 T_{a_i}(k) M_{s_i}(k-l) \\ \qquad -(\mu+\delta+\sigma)T_{p_i}(k) - \rho_1 T_{p_i}(k)\Big), \\ A_{b_i}(k+1) = \frac{\Gamma(\phi_4 k+1)}{\Gamma(\phi_4(k+1)+1)}\Big((\beta_3 T_{a_i}(k-l) + \beta_4 M_{s_i}(k-l))A_{b_i}(k) \\ \qquad +\sigma T_{p_i}(k) - (\mu+\xi+\gamma)A_{b_i}(k) - \rho_2 A_{b_i}(k)\Big), \\ T_{d_i}(k+1) = \frac{\Gamma(\phi_5 k+1)}{\Gamma(\phi_5(k+1)+1)}\Big((\beta_2 T_{a_i}(k-l) + \beta_5 M_{s_i}(k-l))T_{d_i}(k) \\ \qquad +\gamma A_{b_i}(k) - (\mu+\rho_3)T_{d_i}(k)\Big), \\ R_{e_i}(k+1) = \frac{\Gamma(\phi_6 k+1)}{\Gamma(\phi_6(k+1)+1)}\Big(\rho_1 T_{p_i}(k) + \rho_2 A_{b_i}(k) + \rho_3 T_{d_i}(k) \\ \qquad -\mu R_{e_i}(k)\Big). \end{cases} \tag{5.12}$$

Such that $T_{ai}(0) = t_{ai}(t_{i-1}) = t_{ai-1}(t_{i-1}), M_{si}(0) = m_{si}(t_{i-1}) = m_{si-1}(t_{i-1}), T_{pi}(0) = t_{pi}(t_{i-1}) = t_{pi-1}(t_{i-1}), A_{bi}(0) = a_{bi}(t_{i-1}) = a_{bi-1}(t_{i-1}), T_{di}(0) = t_{di}(t_{i-1}) = t_{di-1}(t_{i-1})$ and $R_{ei}(0) = r_{ei}(t_{i-1}) = r_{ei-1}(t_{i-1})$.

6. Numerical simulations

The numerical simulations of the fractional order model (2.2) are carried out using the FMSDTM scheme for the model in comparison with the RK4 method via maple computational software using the parameter values in Table 1. The initial values of the model variables are assumed to be $T_a(0) = 0.105000, M_s(0) = 0.72000, T_p(0) = 0.61000, A_b(0) = 0.34300, T_d(0) = 0.23000$ and $R_c(0) = 0.14500$. Figures 2(a)–2(f) shows the behavior of the fractional order model (2.2) variables, which converges to the teenage pregnancy - free equilibrium when $R_{pr} < 1$ and teenage pregnancy - present equilibrium when $R_{pr} > 1$. Figure 2(a) shows that teenage girls susceptible to early pregnancy increases and move out of the class to be influenced into having sex with males or negatively influenced by already pregnant females who dropped out of school and practice abortion as time increases. Also, Figure 2(b) shows the increasing rate of sexually active males who look out for teenagers for sexual interactions, while the decline implies that more sexually active males negatively influence school drop out teenage girls or teenage girls into practicing abortion overtime. Figure 2(c) shows the rate at which pregnant teenagers increase within 2 months before gradually decreasing. This occurs due to the increase in the rate of teenage females who practice abortion as time increases as shown in Figure 2(d). The effect of early sexual debut, pregnancy and abortion also results to gradual increase in school drop-out rate among female teenagers as time increases in Figure 2(e). Rehabilitation of pregnant teenagers, teenagers who aborted and teenage female school drop-out is shown to be effective in Figure 2(f). As time increases, the curve peaks in the first month but flattens within the third to the twelfth month which shows that rehabilitation is effective in curtailing the menace of teenage pregnancy.

It is shown in Figure 3(a), that as sexually active males involves in sexual interactions with susceptible female

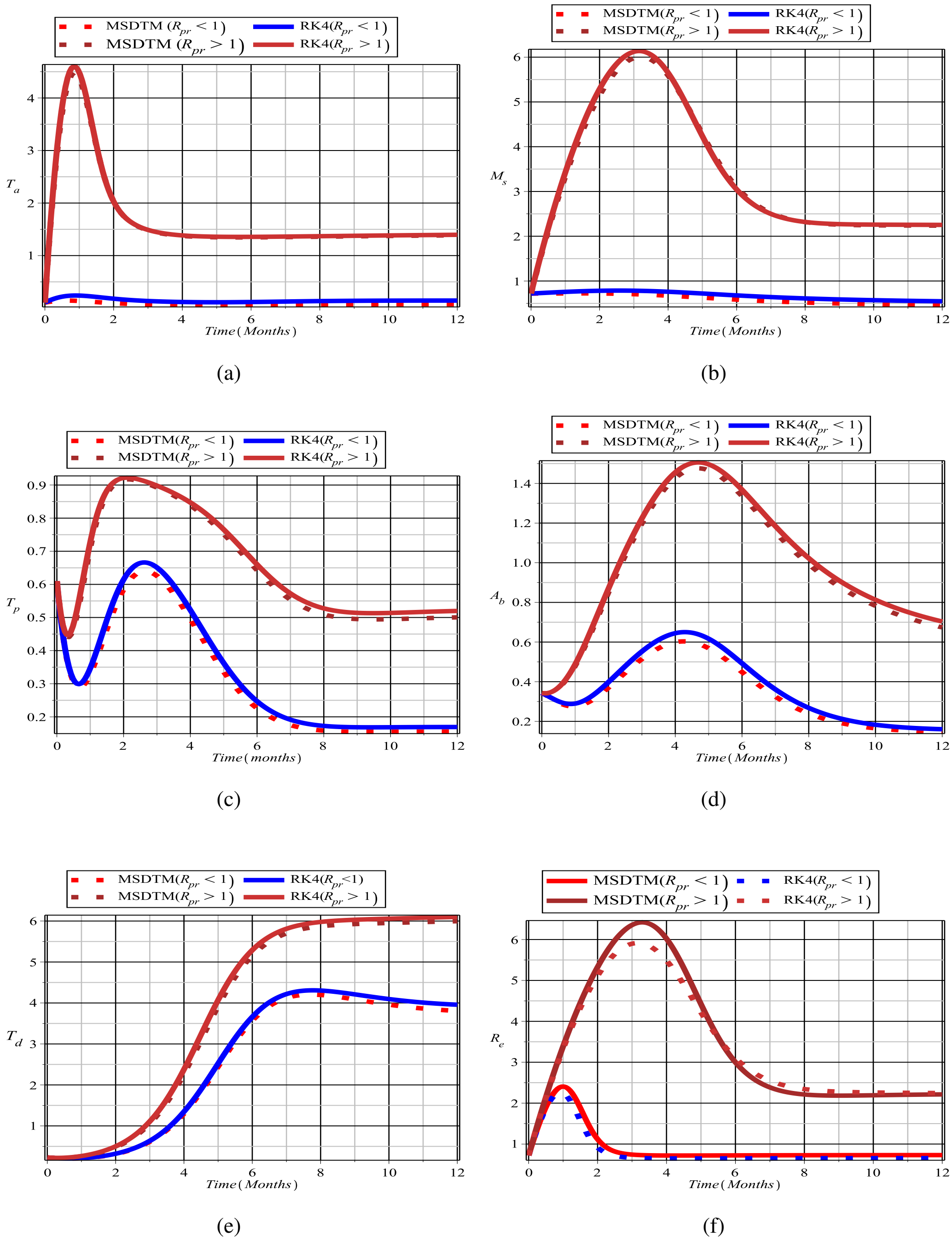


Figure 2. Behavior of the model variables using FMSDTM and RK4 when $R_{pr} < 1$ and $R_{pr} > 1$.

teenagers, more teenagers become more exposed to early pregnancy, and abortion as time increases, while the effect of negative peer influence or pressure on susceptible female teenagers, pregnant teenagers and female teenagers who engage in abortion practice is shown in Figures 3(b)–3(e). As these rates ($\beta_i, i = 2, \cdots, 5$) increases gradually, more female teenagers engage in these social ills as time increases unless controls are applied to curtail the menace of teenage pregnancy and its attendant consequences. It is observed in Figures 4(a)–4(c) that more pregnant teenage females involved in abortion and are school drop-outs exhibit positive behavior as they are rehabilitated at different

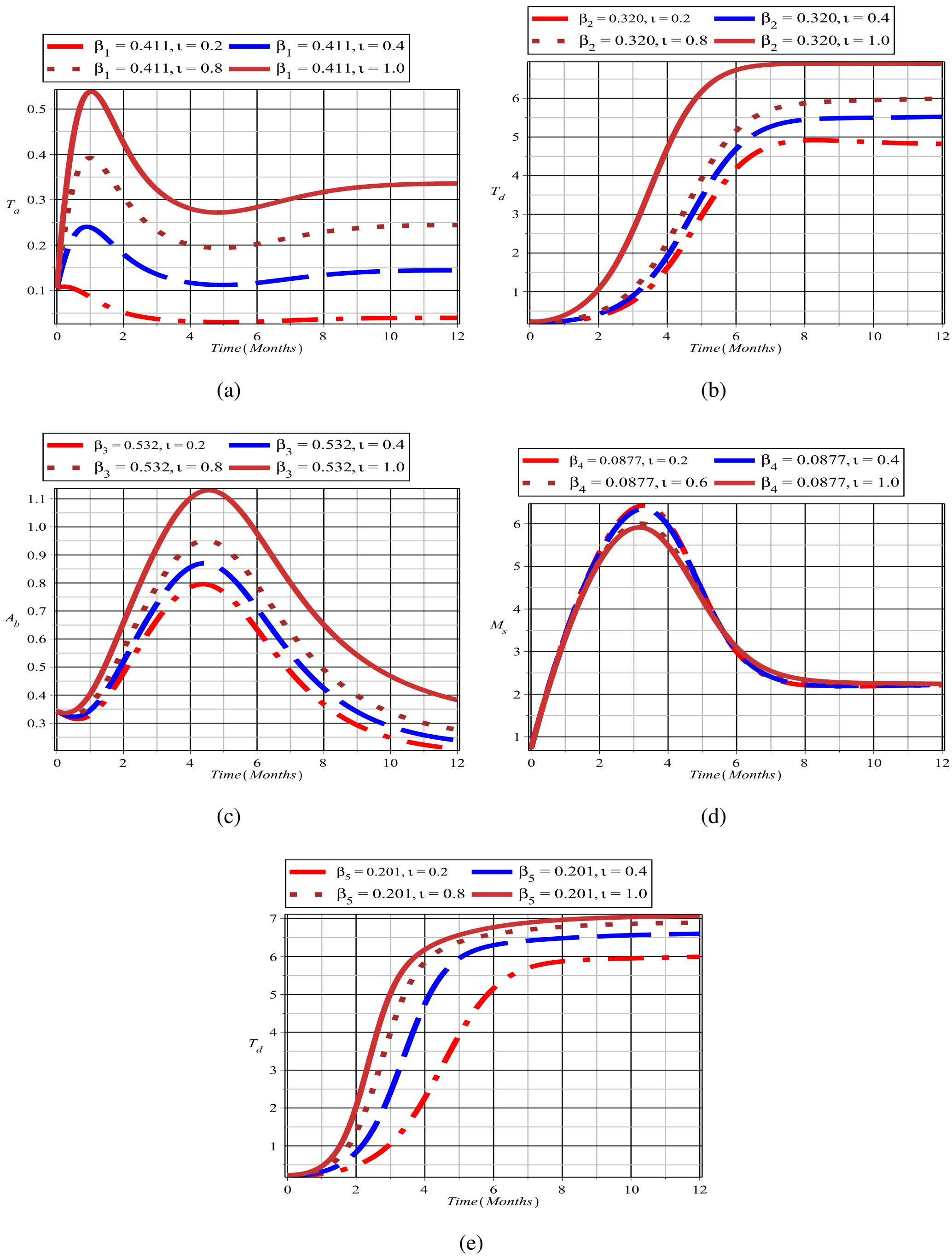


Figure 3. Simulations of sexual contact rate β_1 and negative peer influence rates $\beta_2, \beta_3, \beta_4$ and β_5 at their fixed values and different fractional orders $\iota = 0.2, 0.4, 0.8$ and integer order value 1.

fractional order values and integer order. This shows that rehabilitation must be scaled up to eradicate this menace in Nigeria.

6.1. The effect of rehabilitation rates ($\rho_i, i = 1, 2, 3$) on R_{pr}

Figures 5(a)–5(c) shows the effect of rehabilitation rates on R_{pr} threshold. It is observed that rehabilitation of pregnant teenagers is still low since $R_{pr} > 1$, while the rehabilitation of teenage girls who practice abortion and dropped out increases and lessens R_{pr}, but not below unity, which shows that rehabilitation level is low in order to eradicate teenage pregnancy.

6.2. Effects of $\beta_i, i = 1, \cdots, 6$ and ξ on the estimation of R_{pr}

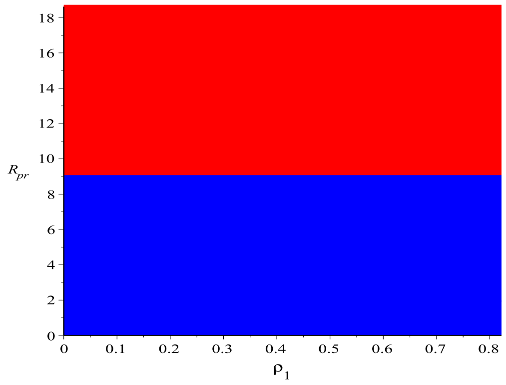
The effect of varying the sexual contact rate β_1 between sexually actives males and teenage girls is observed in Figure 6(a) to increase R_{pr}. Also, the effect of negative peer influence among classes of human compartments as they interact in Figures 6(b)-6(e) is shown to increase R_{pr}, while death due to abortion complications also have a fatal effect on R_{pr}. Therefore, Figures 5 and 6 shows that in order to eradicate teenage pregnancy menace in Nigeria, the level of rehabilitation must be increased and additional controls added to curtail the social problem.

7. Conclusions

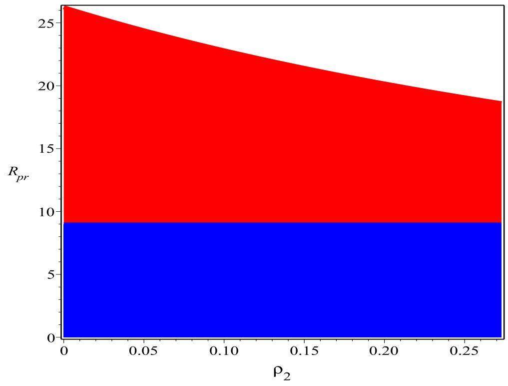
A fractional model illustrating the dynamics of social menace of early pregnancy in female teenagers in the sense of Caputo is formulated and analyzed. The existence and uniqueness criteria of the fractional order model is established, while the model is found to be positive and bounded. The basic reproduction number R_{pr} of the model is computed using the next generation matrix technique. The numerical FMSDTM in comparison with fractional RK4 method via maple computational software is used to obtain the approximate solution of the fractional order model variables, which showed the convergence of the methods when R_{pr} is less and greater than unity. Furthermore, simulations of the model parameters and the effect of rehabilitation, sexual contact and negative peer influence rates on R_{pr} is established. The behavior of R_{pr} as to the effect of these parameters show that the level of rehabilitation must be increased while further controls of condom and contraceptive usage and media education must be imposed on the model system to minimize and eradicate the menace teenage pregnancy in Nigeria.

Conflict of interest

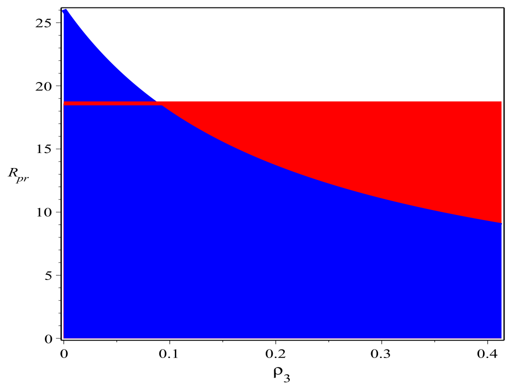
All authors declare no conflicts of interest in this paper.



(a)



(b)



(c)

Figure 4. Effects of rehabilitation parameters ρ_1, ρ_2 and ρ_3 on R_{pr}.

References

1. Adolescent pregnancy, World Health Organization. Available from: `https://www.who.int/news-room/fact-sheets/detail/adolescent-pregnancy`.

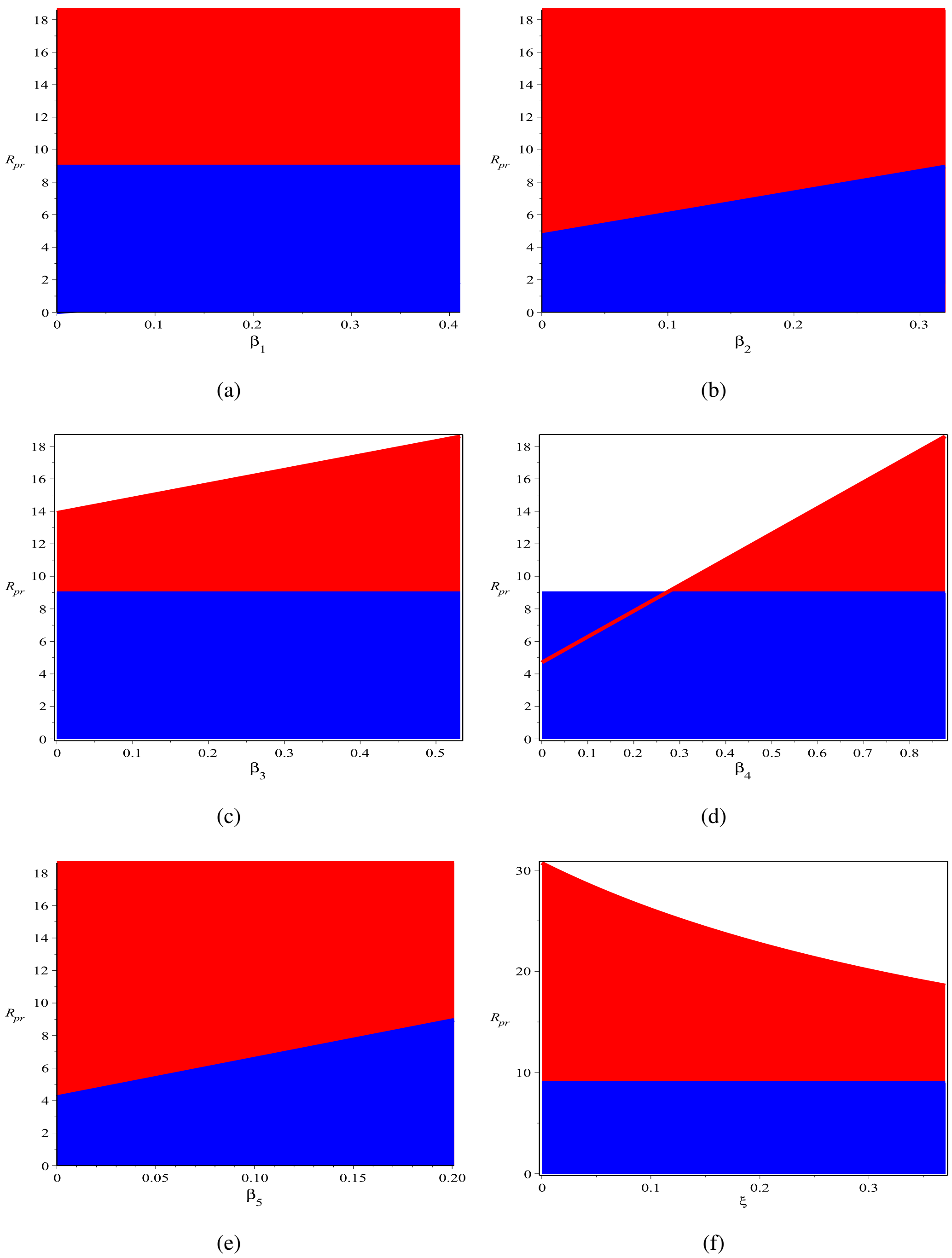


Figure 5. The effects of sexual contact rate β_1 and negative peer influence rates $\beta_i, i = 2 - 5$ and death due to abortion complications ξ on R_{pr}.

2. O. T. Alabi, I. O. Oni, Teenage Pregnancy in Nigeria: Causes, Effect and Control, *International Journal of Academic Research in Business and Social Sciences,* **7** (2017), 17–32.

3. F. E. Okonofua, Factors associated with adolescent pregnancy in rural Nigeria, *J. Youth Adolescence*, **24** (1995), 419–438. https://doi.org/10.1007/BF01537189

4. Teenage pregnancy and challenges to the realisation of sexual and reproductive rights in Nigeria, 2015. Available from: `https://thisisafrica.me/politi`

cs-and-society/teenage-pregnancy-challenges-realisation-sexual-reproductive-rights-nigeria/

5. Nigeria Birth Rate 1950-2022, MacroTrends. Available from: https://www.macrotrends.net/countries/NGA/nigeria/birth-rate

6. Nigeria Death Rate 1950-2022, MacroTrends. Available from: https://www.macrotrends.net/countries/NGA/death-rate

7. Factsheet: Understanding Nigeria's teenage pregnancy burden, 2021. Available from: https://dhsprogram.com/Who-We-Are/News-Room/Teenage-Pregnancy-in-Nigeria-Facts-and-Truth.cfm

8. J. O. Akanni, F. O. Akinpelu, S. Olaniyi, A. T. Oladipo, A. W. Ogunsola, Modeling financial crime population dynamics: optimal control and cost-effectiveness analysis, *International Journal of Dynamics and Control,* **8** (2020), 531–544. https://doi.org/10.1007/s40435-019-00572-3

9. H. T. Alemneh, Mathematical modeling, analysis, and optimal control of corruption dynamics, *J. Appl. Math.,* **13** (2020), 5109841. https://doi.org/10.1155/2020/5109841

10. C. Castillo-Chavez, Z. Feng, W. Huang, On the computation of R_0 and its role on global stability, *Mathematical Approaches for Emerging and Reemerging Infectious Diseases: An Introduction*, Springer, **1** (2002), 229–250. https://doi.org/10.1007/978-1-4757-3667-0_13

11. J. A. Feijo, The mathematics of sexual attraction, *J. Biol.*, **9** (2010), 1–5. https://doi.org/10.1186/jbiol233

12. H. Singh, D. Baleanu, J. Singh, H. Dutta, Computational study of fractional order smoking model, *Chaos, Solitons and Fractals*, **142** (2021), 110–440. https://doi.org/10.1016/j.chaos.2020.110440

13. N. O. Mokaya, H. T. Alemmeh, C. G. Ngari, G. Gakii Muthuri, Mathematical Modeling and Analysis of Corruption of Morals amongst Adolescents with Control Measures in Kenya, *Discrete Dyn. Nat. Soc.*, **1** (2021). https://doi.org/10.1155/2021/6662185

14. O. Danford, M. kimathi, S. Mirau, Mathematical modeling and analysis of corruption dynamics with control measures in Tanzania, *Journal of Mathematics and Informatics*, **19** (2020), 57–79. http://dx.doi.org/10.22457/jmi.v19a07179

15. A. O. Binuyo, V. O. Akinsola, Stability analysis of the corruption free equilibrium of the mathematical model of corruption in Nigeria, *Mathematical Journal of Interdisciplinary Sciences*, **8** (2020), 61–68. https://doi.org/10.15415/mjis.2020.82008

16. F. Y. Egudam, F. Oguntolu, T. Ashezua, Understanding the dynamics of corruption using mathematical modeling approach, *International Journal of Innovative Science, Engineering and Technology*, **4** (2017), 2348–7968.

17. S. M. E. K. Chowdhury, M. Forkan, S. F. Ahmed, P. Agarwal, A. B. M. Showkat Ali, S. M. Muyeen, Modeling the SARS-COV-2 parallel transmission dynamics: Asymptomatic and symptomatic pathways, *Comput. Biol. Med.*, **143** (2022), 105264. https://doi.org/10.1016/j.compbiomed.2022.105264

18. A. Rehman, R. Singh, P. Agarwal, Modeling, analysis and prediction of new variants of COVID-19 and dengue co-infection in complex network, *Chaos Solitons and Fractals*, **150** (2021), 111008. https://doi.org/10.1016/j.chaos.2021.111008

19. P. Agrawal, J. J. Nieto, M. Ruhansky, D. F. M. Torres, *Analysis of infectious disease problems (COVID-19) and their global impact*, Springer, 2021. https://doi.org/10.1007/978-981-16-2450-6

20. S. M. E. K. Chowdhury, J. T. Chowdhury, S. F. Ahmed, P. Agarwal, I. A. Badruddin, S. Kamangar, Mathematical modeling of COVID-19 disease dynamics:interaction between immune system and SARS-COV-2 within host, *AIMS Mathematics*, **7** (2022), 2018–2033. https://doi.org/10.3934/math.2022147

21. O. M. Ogunmiloro, S. E. Fadugba, E. O. Titiloye, On the existence, uniqueness and computational analysis of a fractional order spatial model for the squirrel population dynamics under the Atangana-Baleanu-Caputo operator, *Mathematical Modeling and Computing*, **8** (2021), 432–443. https://doi.org/10.23939/mmc2021.03.432

22. O. M. Ogunmiloro, Mathematical analysis and approximate solution of a fractional order Caputo fascioliasis disease model, *Chaos, Solitons and Fractals*, **146** (2021), 110851. https://doi.org/10.1016/j.chaos.2021.110851

23. O. M. Ogunmiloro, A. S. Idowu, T. O. Ogunlade, R. O. Akindutire, On the Mathematical Modeling of Measles Disease Dynamics with Encephalitis and Relapse Under the Atangana-Baleanu-Caputo Fractional Operator and Real Measles Data of Nigeria, *Int. J. Appl. Comput. Math.*, **7** (2021), 1–20. https://doi.org/10.1007/s40819-021-01122-2

24. P. Agarwal, S. Denis, S. Jain, A. A. Alderremy, S. Ally, A new analysis of partial differential equations arising in biology and population genetics via semi-analytical techniques, *Physica A*, **542** (2020), 122769. https://doi.org/10.1016/j.physa.2019.122769

25. J. Zhou, *Differential Transformation and Its Applications for Electrical Circuits*, Huazhong University Press, Wuhan, China, 1986.

26. Z. Alkhudhari, S. Al-Sheikh, S. Al-Tuwairqi, Global dynamics of a mathematical model on smoking, *Appl. Math.,* **1**(2014), 847075. https://doi.org/10.1155/2014/847075

27. E. Bonyah, A. Freihat, M. A. Khan, A. Khan, S. Islam, Application of the multi-step differential transform method to solve system of nonlinear fractional differential algebraic equations, *J. Appl. Environ. Biol. Sci.,* **6** (2016), 83–95.

28. A. Hytham, A. Ahmad, I. Ismail, Multi-step fractional differential transform method for the solution of fractional order stiff systems, *Ain Shams Eng. J.*, **12** (2021), 4223–4231. https://doi.org/10.1016/j.asej.2017.03.017

29. Z. Odibat, S. Momani, V. S. Erturk, Generalized differential transform method: Application to differential equations of fractional order, *Appl. Math. Comput.*, **197** (2008), 467–477. https://doi.org/10.1016/j.amc.2007.07.068

30. S. Abuasad, A. Yildirim, I. Hashim, S. Ariffin Abdul Karim, J. F. Gomez-Aguilar, Fractional multi-step differential transform method for approximating a fractional stochastic SIS epidemic model with imperfect vaccination, *Int. J. Environ. Res. Public Health.,* **16** (2019), 973. https://doi.org/10.3390/ijerph16060973

31. A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, North-Holland mathematical studies, Vol. 204, North-Holland-Amsterdam: Elsevier Science Publishers, 2006.

32. C. P. Li, Y. T. Ma, Fractional dynamical system and its linearization theorem, *Nonlinear Dynam.*, **71** (2013), 621-633. https://doi.org/10.1007/s11071-012-0601-1

33. N. C. Okafor, I. Oyakhiromen, Nigeria and Child Marriage:Legal Issues, Complications, Implications, Prospects and Solutions, *Journal of Law, Policy and Globalization.*, **29** (2014). ISSN 2224-3240

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