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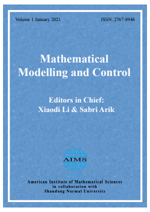

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Research article

A stochastic model with jumps for smoking incorporating media coverage

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Abstract: Media coverage is an important tool in the fight against smoking. So, in this paper we will incorporate media coverage in a deterministic SIRS model for smoking. Those who have studied this deterministic model have shown that by setting the constants of this model, we can control the tobacco epidemic. But this model is not very realistic: it does not take into account the action of media coverage and some other random factors. Thus, we incorporate the media coverage into this model and obtain a deterministic model with media coverage. Also, to take into account some randomness in the contact between individuals or sudden events that could disrupt the action of media coverage, we introduce in our deterministic model with media coverage white noise and jumps. We first prove the boundness of the solutions and the stability of the smoking-free equilibrium state of the deterministic model with media coverage. We prove that the solution of the stochastic differential equation with jumps of the stochastic model is unique, positive and global. Under certain conditions, we show that this solution oscillates respectively around each equilibrium state of our deterministic model. This allows us to consider conditions that lead to converge towards an extinction or persistence of smoking. The paper is ended by numerical simulations that corroborate our theoretical results.

Keywords: smoking model; media coverage; jump perturbation; extinction; numerical simulation

1. Introduction

This study is motivated by the desire to show an effective and very accessible way of fight against the spread of an epidemic : media coverage. It is a fairly easy tool to deploy compared to some others. Its diversity of forms (written press, radiophonic press, digital press or television press) allows it to reach places that seem inaccessible and to secure the loyalty of the public. For all these reasons, it is important for us to conduct an epidemic control study incorporating media coverage. In [1], A. Lahrouz et al. considered a deterministic model for smoking introduced in [2] by C. Castillo-Garsow et al. and improved in [3] by Sharomi and Gumel. The deterministic model explored by C. Castillo-Garsow et al. is composed of four compartments : potential smokers, i.e. people who do not smoke yet but might become smokers in the future (P), smokers (S) and smokers who have quit smoking temporarily(Q_T) and smokers who have quit smoking permanently (Q_p). The total population is supposed constant. For this, we can consider the density of individuals in each compartment instead of the numbers in these compartments. So $P(t), S(t), Q_T(t), Q_p(t)$ become, respectively, at time t, the proportions of the potential smokers, smokers, smokers who have quit smoking temporarily and smokers who have quit smoking permanently. Therefore we get $P(t)+S(t)+Q_T(t)+Q_p(t) = 1$. The nonlinear equation describing the dynamics of smoking is :

$$\begin{cases} dP(t) = [\mu - \mu P(t) - \beta P(t)S(t)]dt, \\ dS(t) = [-(\mu+\gamma)S(t) + \beta P(t)S(t) + \alpha Q_T(t)]dt, \\ dQ_T(t) = [-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)]dt, \\ dQ_p(t) = [-\mu Q_p(t) + \sigma\gamma S(t)]dt. \end{cases} \tag{1.1}$$

α, β, μ, γ and σ are real constants on [0,1]. Their meanings are as follows. α is the rate at which smokers who temporarily quit smoking revert to smoking, β is the contact rate (non-smokers can acquire smoking habits via effective contacts with smokers), γ is the rate at which smokers quit smoking forever, σ is the fraction of smokers who become smokers who have quit smoking permanently, $1-\sigma$ is the fraction of smokers who become smokers who have quit smoking temporarily and μ is the recruitment rate of the non-smokers and also the naturel death rate in each compartment. Since $P(t)+S(t)+Q_T(t)+Q_p(t)=1$, the system (1.1) will be restricted to the following three-dimensional system (1.2):

$$\begin{cases} dP(t) = [\mu - \mu P(t) - \beta P(t)S(t)]dt, \\ dS(t) = [-(\mu+\gamma)S(t) + \beta P(t)S(t) + \alpha Q_T(t)]dt, \\ dQ_T(t) = [-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)]dt. \end{cases} \quad (1.2)$$

In [3], O. Sharomi et al. study the deterministic aspect of this model. In [1], A. Lahrouz et al. continue by considering a stochastic model including a white noise. They prove the stochastic stability of the smoking-present equilibrium state. In [4], we add jumps in their model and study its asymptotic behavior around the different equilibrium states of the deterministic model. In the model (1.2), the transmission of the smoking is governed by the incidence rate $\beta P(t)S(t)$, and $\beta S(t)$ is called the infection force. In our case, media coverage influence the incidence rate directly. Continuous awareness against smoking in all media, schools, educational campaigns, social programs and the prohibition of smoking in public places could help reduce smoking and contact with smokers. As reported in the WHO Tobacco Report of 2017 in [5] : 4.7 billion people (63 percent of the world's population) are covered by at least one comprehensive tobacco control measure. In the latest WHO Tobacco Report from 2019 (see [6]), it is mentioned that in 2018, 24 percent of the world's popupaltion benefited from the action of the mass media in the fight against smoking. 52 percent of the world's population was warned about smoking in 2018.

For all these reasons, we introduce the effect of media coverage into the model (1.2) and redefine the incidence rate. We deal with some mathematical properties of the new model obtained. To take into account certain continuous or sudden random factors, we make the previous model a stochastic one including both white noise and jumps. For this new model, we study the extiction or persistence property. Our results are supported by simulations carried out with the MATLAB R2019b software.

2. Results

2.1. Some properties of the deterministic model

As in [7–9], we incorporate a nonlinear function of the number of Smokers in the transmission term to investigate the effects of media coverage on the transmission contact rate

$$f(S) = \beta_1 - \frac{\beta_2 S}{m+S}$$

where β_1 is the contact rate before the media actions. $\beta_1 \geq \beta_2$ and the expression $\frac{\beta_2 S}{m+S}$ is the reduction in the contact rate β_1 due to the effect of media coverage. The construction of the contact rate with media coverage is governed by the following realistic considerations. The presence of media coverage creates a desire among the population to protect themselves against the disease. This implies a reduction in the contact rate β_1 before media coverage. This reduction is represented by the expression $\frac{\beta_2 S}{m+S}$ where β_2 is the maximum reduction. Why the choice of this expression? In reality, the more the number of infected increases, the more individuals tend to protect themselves. This is why the reduction in the contact rate should be maximum if $S \to \infty$. In addition, the intensity of media coverage should be able to control this reduction. The strictly positive coefficient m symbolizes the non-response to media coverage. The larger is the media coverage, the smaller is m and the greater the reduction in the initial contact rate. All these reasons led us to choose this contact rate with media coverage : $\beta_1 - \frac{\beta_2 S}{m+S}$. Obviously, β_2 dominates this reduction. β_2 depends on the capacities of the existing media platform and on economic, environmental and human possibilities. The constant $m > 0$ allows the control of this reduction due to the media by measuring the impact of the media coverage in the reduction of the transmission rate. We have $\frac{\partial f}{\partial m}(S) = \frac{\beta_2 S}{(m+S)^2}$. So the higher is m, the lower is this impact and the higher is the effective contact rate. The Lower is m, the higher is this impact and the lower is the effective contact rate. The reduction on the contact rate also depends on the extent of smoking through the expression $\frac{S}{m+S}$. With these new conditions our model becomes:

$$\begin{cases} dP(t) = [\mu - \mu P(t) - (\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)})P(t)S(t)]dt, \\ dS(t) = [-(\mu+\gamma)S(t) + (\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)})P(t)S(t) \\ \qquad + \alpha Q_T(t)]dt, \\ dQ_T(t) = [-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)]dt. \end{cases} \tag{2.1}$$

We have proved that model (2.1) has a biological significance. Indeed, we have proved that for any initial state in $\Delta = \{(x_1, x_2, x_3) \in \mathbb{R}_+^{*3}; x_1 + x_2 + x_3 < 1\}$, the solution for model (2.1) are bounded in Δ (see the section Materials and methods, 4.1). We show that the basic smokers generation number of model system (2.1) is $R = \dfrac{\beta_1}{\mu(\mu+\alpha) + \gamma(\sigma\alpha + \mu)}$. The model described by (2.1) admits equilibrium states : the smoking-free equilibrium state $E_0 = (1, 0, 0)$ which always exists and the smoking-present equilibrium state $E^* = (P^*, S^*, Q_T^*)$ which exists when $R > 1$. It is important to consider the eradication of this epidemic. So for $R < 1$, we prove that the smoking-free equilibrium state $E_0 = (1, 0, 0)$ of the system (2.1) is globally asymptotically stable in Δ (see the section Materials and methods, 4.1).

2.2. Stochastic model with jumps and white noise

Here, as in [4, 10, 11] we, use mathematical notions to model random phenomena. Some ramdom factors can continuously influence the action of media coverage. For example, cigarette brand advertisements, celebrities admired by young people who smoke cigarettes publicly, ... These factors disturb the action of media coverage and can make the number of smokers grow up. That's why we decide to add a white noise in the previous model to symbolize such disturbences. At a time t, the white noise is proportional to the incidence rate after media alert, $(\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)})P(t)S(t)$. Unexpected events such as wars, putsches, earthquakes, hurricanes, terrorist attacks, ... can suddenly stop or slow the action of media coverage. So we also introduce jumps in the system (2.1) to represent these unusual and sudden changes.

We add randomly fluctuation by replacing the contact rate $\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)}$ in (2.1) by $(\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)}) - \sigma_0 \dfrac{dB(t)}{dt}$, where $\dfrac{dB(t)}{dt}$ is a white noise. B is a Brownian motion defined on a stochastic basis $(\Omega, \mathcal{F}, (\mathcal{F}_{t\geq 0}), P)$ and σ_0 is a positive constant which represents the intensity of $B(t)$. This random fluctuation represents continuous random factors. White noise is a special stationary process without "memory". The level of the series considered today has no bearing on its level tomorrow. Yesterday's level does not affect today's level. This is interesting for modeling a continuous random pertubabtion. The white noise is also interesting in our case because its response remains finite. Therefore the modelisation of any excitation by a white noise provides an approximation which grows in accurage as the damping of the system is lower. To consider some sudden factors that can influence the incidence rate we introduce the integral

$$\int_Z C(z)(\beta_1 - \frac{\beta_2 S(t-)}{m + S(t-)})P(t-)S(t-)\tilde{N}(\,dt, dz).$$

At the time t, this integral represents a random finite sum of small jumps of the incidence rate after a short time dt. This integral reflects a sudden disturbance of the contact rate. $X(t\text{-})$ means the left limit of $X(t)$, $N(dt, dz)$ is a Poisson counting measure with the stationary compensator $\Pi(dz)dt$. $\tilde{N}(dt, dz) = N(dt, dz) - \Pi(dz)dt$ and Π is defined on a measurable subset Z of $[0, \infty)$ with $\Pi(Z) < \infty$. $C > -1$ represents the intensity of the jumps. Finally we get the following stochastic system which describes our new model.

$$\begin{cases} dP(t) = [\mu - \mu P(t) - (\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)})P(t)S(t)]dt \\ \qquad + \sigma_0(\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)})P(t)S(t)dB(t) \\ \qquad - \displaystyle\int_Z C(z)(\beta_1 - \dfrac{\beta_2 S(t-)}{m + S(t-)}) \\ \qquad \times P(t-)S(t-)\tilde{N}(\,dt, dz), \\ dS(t) = [-(\mu+\gamma)S(t) + (\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)})P(t)S(t) \\ \qquad + \alpha Q_T(t)]dt \\ \qquad - \sigma_0(\beta_1 - \dfrac{\beta_2 S(t)}{m + S(t)})P(t)S(t)dB(t) \\ \qquad + \displaystyle\int_Z C(z)(\beta_1 - \dfrac{\beta_2 S(t-)}{m + S(t-)}) \\ \qquad \times P(t-)S(t-)\tilde{N}(\,dt, dz), \\ dQ_T(t) = [-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)]dt, \end{cases} \tag{2.2}$$

where B is a Brownian motion defined on a stochastic basis $(\Omega, \mathcal{F}, (\mathcal{F}_{t\geq 0}), P)$ and σ_0 is a positive constant.

$X(t\text{-})$ means the left limit of $X(t)$, $\tilde{N}(dt, dz)$ is a compensated Poisson counting measure with the stationary compensator $\Pi(dz)dt$ and Π is defined on a measurable subset Z of $[0, \infty)$ with $\Pi(Z) < \infty$ and $C > -1$. For practical and realistic reasons, we consider that at the initial state, each compartment of our model is not empty.

As in [4], for the jump diffusion coefficient we assume that for each $\varepsilon > 0$ there exists $L_\varepsilon > 0$ such that

- **(H1)** $\int_Z |H(x,z) - H(y,z)|^2 \Pi(\,dz) \leq L_\varepsilon |x-y|^2$, where $H(x,z) = C(z)(\beta_1 - \frac{\beta_2 S(x-)}{m + S(x-)})P(x-)S(x-)$, with $|x| \vee |y| \leq \varepsilon$
- **(H2)** $|\log(1 + C(z))| \leq K_1$, for $C(z) > -1$, where K_1 is a positive constant.

Let $\Delta = \{(x_1, x_2, x_3) \in \mathbb{R}_+^{*3}\,;\, x_1 + x_2 + x_3 < 1\}$. As in [11], we show that with these assumptions, jump processes can suppress the explosion. Using the assumptions **(H1)** and **(H2)**, we obtain that for any given initial value $(P(0), S(0), Q_T(0)) \in \Delta$, the equation (2.2) has a unique global solution $(P(t), S(t), Q_T(t))$ which lives almost surely in Δ for any $t \geq 0$. This classic result, justified as in [4], gives a biological significance to our model (2.2).

2.3. Towards an extinction of the smoking epidemic

In the same pedagogical approach as in [12], our study should allow the control of the epidemic. When $R < 1$, with conditions on some constants of the system, the solution of (2.2) oscillates more closely around the smoking-free equilibrium state as the parameter $[\gamma(1-\sigma)+\mu+\gamma-\beta_1]\beta_2$, the intensity of the noise and the jumps decrease. If the impact of the media coverage is high, the transmission rate becomes small. As we have seen in [4], this favors the extinction of the epidemic. The below theorem confirms this assertion.

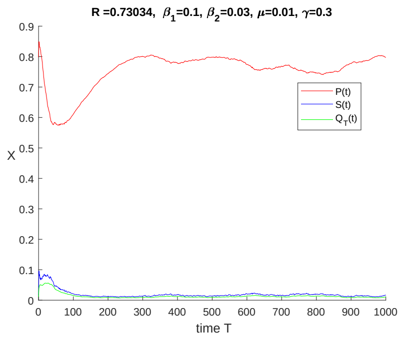
Theorem 2.1. *Let $(P(t), S(t), Q_T(t))$, be the solution of the system (2.2) with initial value $(P(0), S(0), Q_T(0)) \in \Delta$. If $R = \frac{\beta_1(\mu+\alpha)}{\mu(\mu+\alpha) + \gamma(\sigma\alpha + \mu)} < 1$, then there exists $K > 0$ such that*

$$\limsup_{t\to\infty} \tfrac{1}{t}\mathbb{E} \int_0^t [(P(\tau)-1)^2 + S^2(\tau) + Q_T(\tau)]\, d\tau$$

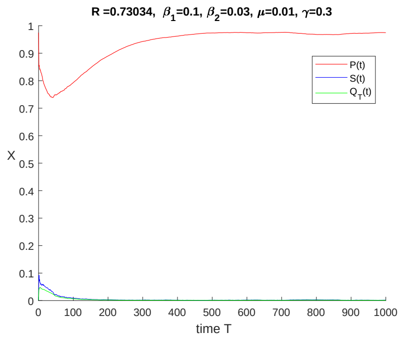
$$\leq \frac{K\xi + 2\sigma_0^2 + 2\int_Z 2(2C^2(z) + \frac{5}{2}|C(z)|)\Pi(\,dz)}{K'}$$

where $\xi = [\gamma(1-\sigma) + \mu + \gamma - \beta_1]\beta_2$ and $K' = min\{\mu\,;\, \frac{K}{2}[[\mu(\mu+\alpha) + \gamma(\mu+\sigma\alpha)](1-R) - \frac{2\alpha}{K}]\}$.

As an evidence this result, see the asymptotic Behavior around E_0 in 4.2 (section Materials and methods). Building on the algorithmic methods presented in [13, 14] , simulations carried out with MATLAB R2019b software confirm the above result. For the simulations, as in [4] we choose $\mu = 0.01$, $\gamma = 0.3$, $\alpha = 0.25$, and $\sigma = 0.4$, with $\beta_1 = 0.1$ and $\beta_2 = 0.03$. So $R < 1$. For low random disturbances, we take $\sigma_0 = 0.7$ and $C = 0.1$. The simulations following show that the solution oscillate around the smoking-free equilibrium state. With a higher impact of media coverage, the solution of the model (2.2) is much closer the Smoking-free Equilibrium State E_0. These simulations allow us to observe a trend towards the extinction of smoking. This situation is possible with a certain control of the constants of our model. For this, the desired values are imposed by some attitudes in real life.

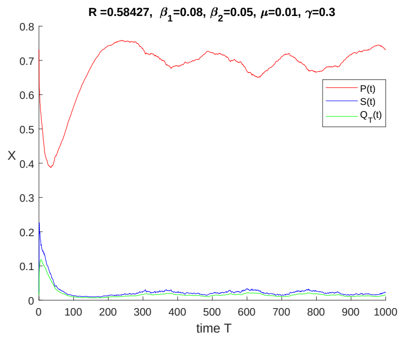


(a) Low impact of media coverage. $m = 10^2$**.**

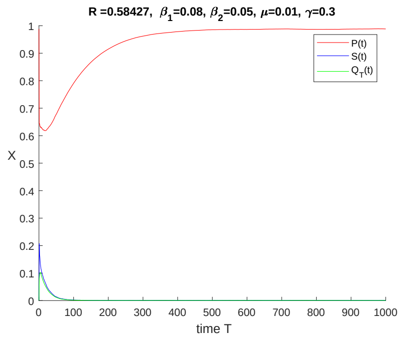


(b) High impact of media coverage. $m = 10^{-3}$**.**

Figure 1. Trajectory of the solution of the system (2.2) for $P(0) = 0.85955$, $S(0) = 0.111744$, $Q_T(0) = 0.025165$, $R < 1$.



(a) Low impact of media coverage. $m = 10^3$**.**



(b) High impact of media coverage. $m = 10^{-3}$**.**

Figure 2. Trajectory of the solution of the system (2.2) for $P(0) = 0.65000$, $S(0) = 0.26800$, $Q_T(0) = 0.04400$, $R < 1$.

2.4. *Persistence of the smoking epidemic*

When $R > 1$ and $\beta_1 < \mu+\gamma+\dfrac{\mu+\gamma\sigma}{2\mu+\gamma\sigma}\beta_1 S^*$ (This is true if $\beta_1 < \mu+\gamma$), the solution of the system (2.2) oscillate around the smoking-present equilibrium of the model (2.1) if β_2, the intensity of the noise and the jumps are getting closer to 0. Under these conditions on the parameters, the state of the epidemic can be stabilized around the smoking-present equilibrium of the model (2.1). The following theorem gives us the result.

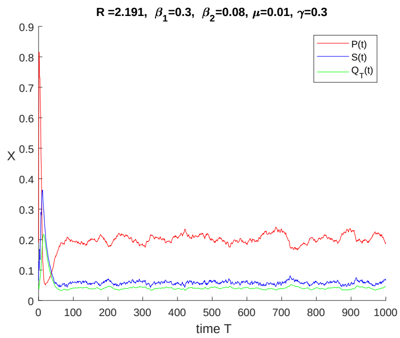
Theorem 2.2. *Let $(P(t), S(t), Q_T(t))$, be the solution of the system (2.2) with initial value $(P(0), S(0), Q_T(0)) \in \Delta$. If $R = \dfrac{\beta_1(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)} > 1$ and the condition $\beta_1 < \mu+\gamma+\dfrac{\mu+\gamma\sigma}{2\mu+\gamma\sigma}\beta_1 S^*$ is satisfied,*

then

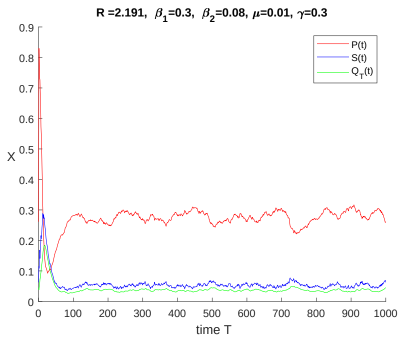
$$\limsup_{t\to\infty}\frac{1}{t}\mathbb{E}\int_0^t [(P(\tau)+Q_T(\tau)-P^*-Q_T^*)^2$$

$+(S(\tau)-S^*)^2+(Q_T(\tau)-Q_T^*)^2]\,\mathrm{d}\tau \le \frac{M}{M'}$, *where*
$M = 2|\sigma_0| + 2u\beta_2 + \frac{1}{2}u|\sigma_0| + \int_Z 2C^2(z) + \frac{1}{2}u\beta_1^2 C^2(z)\Pi(\,\mathrm{d}z)$
and $M' = min\{\mu\,,\ \mu+\gamma\sigma+u(\mu+\gamma-\beta)\,,\ (\mu+\alpha)v\}$
with $u = \frac{2\mu+\gamma\sigma}{\beta_1 S^*}$, $v = \frac{2\mu+\gamma\sigma}{\gamma(1-\sigma)}$.

To prove this result, see the asymptotic Behavior around E^* in 4.2 (section Materials and methods). Simulations confirm this result. As in [4], we use the previous values of the parameters, except the parameters below.

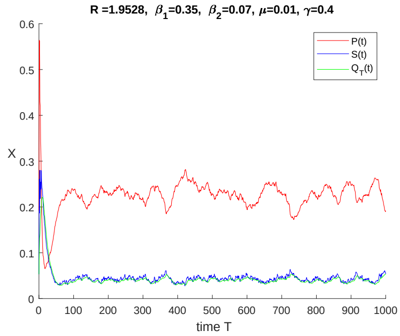


(a) Low impact of media coverage. $m = 10^7$.

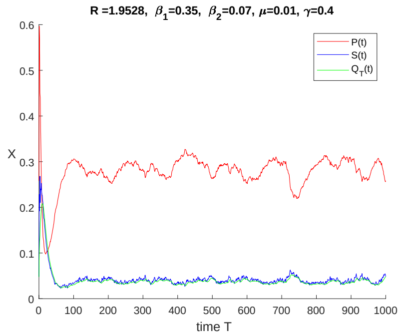


(b) High impact of media coverage. $m = 10^{-11}$.

Figure 3. Trajectory of the solution of the system (2.2) for $R_s > 1$, $\beta_1 < \mu+\gamma+\frac{\mu(\mu+\gamma\sigma)(R-1)}{2\mu+\gamma\sigma}$, $P(0) = 0.80301$, $S(0) = 0.10628$, and $Q_T(0) = 0.08260$. In these conditions with small β_2, strong media coverage slightly increases the rate of non-smokers.



(a) Low impact of media coverage. $m = 10^7$.



(b) High impact of media coverage. $m = 10^{-11}$.

Figure 4. Trajectory of the solution of the system (2.2) for $R > 1$, $\beta_1 < \mu+\gamma+\frac{\mu(\mu+\gamma\sigma)(R-1)}{2\mu+\gamma\sigma}$, $P(0) = 0.60000$, $S(0) = 0.20628$ and $Q_T(0) = 0.10000$. Strong media coverage slightly increases the rate of non-smokers.

We see that we can get a stabilization of smoking with the possibility of a slight improvement in the case of strong media coverage.

3. Discussion

It is known that media can play a big role in decreasing smoker's community. Our contribution while modeling the dynamics of smoking via media coverage consists in showing that effective media coverage can significantly slow the growth of the population of smokers and therefore make the measures set by public authorities to eradicate other epidemics more effective. While incorporating this media control in modeling the dynamics of smoking, we allow better sizing of media coverage and better forecasting of media action. The introduction of randomness with the theories of white noise and jumps allows our system to model reality by reducing the differences between the results it produces and those real. Thus, the jumps and the white noise allow a better control of the modeled epidemic. In our case, predictions could be made about smoking. The means to be implemented to eradicate smoking could be better defined.

In the continuity of [4], we designed a realistic model of the tobacco epidemic with media coverage, white noise and jumps. This model reflects the action of the media which could be hampered by certain continuous or sudden factors. Our contribution in modeling the dynamics of the tobacco epidemic with media coverage is to allow better control of the epidemic. The constants of the model have a biological significance and setting them in an efficient way allows the control of the epidemic according to the possibilities. We have proved that by properly setting the coefficients corresponding to the environment reflected by our model, in particular the transmission rate which could be controlled by media coverage, we can tend towards the end of this epidemic or a stabilization of smoking epidemic. This model could be used by the authorities of our countries for the calibration of the objectives to be reached in the fight against smoking. The parameters m and β_2 measure the quality of media coverage. Other parameters such as α, β_1, γ and σ could be sized by strict public measures such as bans on smoking in public places, heavy tobacco taxes ,... The parameter μ depends among other things, on the quality of public hygiene or health structures. Depending on thier means and the realities, the authorities can choose to stabilize or eradicate the epidemic. The calibration of the parameters have a cost which should be taken into account. For example, obtaining a small value of m requires an intense media campaign.

4. Materials and methods

4.1. The proofs of the results obtained with the deterministic model

In this sub-section we give the proofs of the results obtained in the sub-section 2.1.

- **Step 1 :** We prove that for any initial state in $\Delta = \{(x_1, x_2, x_3) \in \mathbb{R}_+^{*3};\ x_1 + x_2 + x_3 < 1\}$, the solution for model (2.1) is bounded in Δ.
First, we prove that the system (2.1) is strongly positive.
For $(P, S, Q_T) \in \mathbb{R}_+^{*3}$, we have
$\frac{dP(t)}{dt}|_{P=0} = \mu > 0$, $\frac{dS(t)}{dt}|_{S=0} = \alpha Q_T(t) > 0$ and $\frac{dQ_T(t)(t)}{dt}|_{Q_T=0} = \gamma(1-\sigma)S(t) > 0$.
Hence the solutions of system (2.1) initiating in positive octant $\mathbb{R}_+^{*3}$ remain there for all time. So all time, P, S and Q_T are strictly positive.
We will now show the invariance of Δ. Summing all the equations of the system (2.1); we get

$$\begin{aligned}&\frac{d[P(\tau)+S(\tau)+Q_p(\tau)]}{d\tau}\\&= \mu - \mu(P(\tau)+S(\tau)+Q_p(\tau)) - \gamma\sigma S(\tau)\\&\leq \mu - \mu(P(\tau)+S(\tau)+Q_p(\tau)).\end{aligned}$$

Integrating from 0 to t, we get
$P(t)+S(t)+Q_p(t) \leq 1-(1-(P(0)+S(0)+Q_p(0))e^{-\mu t} < 1$ because $(P(0)+S(0)+Q_p(0))$ lives in Δ and $1-(P(0)+S(0)+Q_p(0) < 1$. This implies our result.
- **Step 2 Calculation of R :** For calculating the generation number we use next generation matrix method as in [15]. We get two non-negative matrices F and V : $F = \begin{pmatrix}\beta_1 & 0\\ 0 & 0\end{pmatrix}$ and $V = \begin{pmatrix}(\mu+\gamma) & -\alpha\\ -\gamma(1-\sigma) & (\mu+\alpha)\end{pmatrix}$.
So, $R = \rho(FV^{-1}) = \frac{\beta_1(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)}$.
- **Step 3 Equilibria states :** Obviously, $E_0 = (1, 0, 0)$ is the smoking-free equilibrium state which always exists.
The smoking-present equilibrium state $E^* = (P^*, S^*, Q_T^*)$ must satisfy :
$\mu - \mu P^* - (\beta_1 - \frac{\beta_2 S^*}{m+S^*})P^*S^* = 0$,
$-(\mu+\gamma)S^* + (\beta_1 - \frac{\beta_2 S^*}{m+S^*})P^*S^* + \alpha Q_T^* = 0$ and
$-(\mu+\alpha)Q_T^* + \gamma(1-\sigma)S^* = 0$.
Therefore,
$P^* = \frac{\mu}{\mu + (\beta_1 - \frac{\beta_2 S^*}{m+S^*})S^*}$, $Q_T^* = \frac{\alpha\gamma(1-\sigma)S^*}{\mu+\alpha}$ and

$AS^{*2} + BS^* + C = 0$, where

$$A = -(\beta_1 - \beta_2)(\mu^2 + \mu\alpha + \mu\gamma + \alpha\gamma\sigma) < 0$$

and

$$\begin{aligned}
C &= \mu m[-(\mu+\gamma)(\mu+\alpha) + \alpha\gamma(1-\sigma) + (\mu+\alpha)\beta_1] \\
&= \mu m[-\mu(\mu+\alpha) - \gamma\mu - \gamma\alpha + \gamma\alpha - \alpha\gamma\sigma + (\mu+\alpha)\beta_1] \\
&= \mu m[-\mu(\mu+\alpha) - \gamma(\sigma\alpha+\mu) + (\mu+\alpha)\beta_1] \\
&= \mu m[\mu(\mu+\alpha) + \gamma(\sigma\alpha+\mu)] \\
&\quad \times [\frac{(\mu+\alpha)\beta_1}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)} - 1] \\
&= \mu m[\mu(\mu+\alpha) + \gamma(\sigma\alpha+\mu)](R-1) > 0.
\end{aligned}$$

$A < 0$ and $C > 0$. So, when $R > 1$, the model of the system (2.1) has a unique smoking-present equilibrium state $E^* = (P^*, S^*, Q_T^*)$.

- **Step 4 Global stability of** $E_0 = (1,0,0)$ **:** At first, we notice that if S is constantly zero, S = 0, by using the second equation of the system (2.1), we obtain that $dS(t) = \alpha Q_T(t)dt$. This implies that $Q_T = 0$. Finally the first equation of the system (2.1) gives $dP(t) = [\mu - \mu P(t)]dt$. The other compartments being empty and the population constant, so $P = 1$. In conclusion, $S = 0$ corresponds to the smoking-free equilibrium state $E_0 = (1,0,0)$. With Lasalle's Invariance Principle, we prove the Global stability of $E_0 = (1,0,0)$. For that, we use C^1-function V defined on the set of the states of (2.1) to $\mathbb{R}$, such that $\lim_{\|x\|\to+\infty} V(x) = +\infty$ and $V(E_0) = 0$ with $V(x) > 0$ for $x \neq E_0$. If $\dot{V}$ is equal to 0 for $x = E_0$ and negative everywhere else. Then E_0 is globally asymptotically stable. As in [3], we consider the following Lyapunov function defined in $\mathbb{R}^3_+$ by:
$V(x,y,z) = (\mu+\alpha)y + \alpha z$.
V is positive definite. We have for all t in $\mathbb{R}_+$,
$V(P(t),S(t),Q_T(t)) = (\mu+\alpha)S(t) + \alpha Q_T(t)$ and

$$\begin{aligned}
&\dot{V}(P(t),S(t),Q_T(t)) \\
=&\frac{dV(P(t),S(t),Q_T(t))}{dt} \\
=&(\mu+\alpha)\frac{dS(t)}{dt} + \alpha\frac{Q_T(t)}{dt} \\
=&(\mu+\alpha)[-(\mu+\gamma)S(t) + (\beta_1 - \frac{\beta_2 S(t)}{m+S(t)})P(t)S(t) \\
&+ \alpha Q_T(t)] + \alpha[-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)] \\
=&(\mu+\alpha)(\beta_1 - \frac{\beta_2 S(t)}{m+S(t)})P(t)S(t) - (\mu+\alpha)(\mu+\gamma)S(t) \\
&+ (\mu+\alpha)\alpha Q_T(t) + \alpha\gamma(1-\sigma)S(t) - \alpha(\mu+\alpha)Q_T(t) \\
=&(\mu+\alpha)(\beta_1 - \frac{\beta_2 S(t)}{m+S(t)})P(t)S(t) \\
&- (\mu+\alpha)(\mu+\gamma)S(t) + \alpha\gamma(1-\sigma)S(t) \\
=&\{(\mu+\alpha)(\beta_1 - \frac{\beta_2 S(t)}{m+S(t)})P(t) - (\mu+\alpha)(\mu+\gamma) \\
&+ \alpha\gamma(1-\sigma)\}S(t).
\end{aligned}$$

If $(P,S,Q_T) \neq E_0$, i.e. $S \neq 0$ we get

$$\begin{aligned}
&\dot{V}(P(t),S(t),Q_T(t)) \\
=&\{(\mu+\alpha)(\beta_1 - \frac{\beta_2 S(t)}{m+S(t)})P(t) - (\mu+\alpha)(\mu+\gamma) \\
&+ \alpha\gamma(1-\sigma)\}S(t) \\
\leq&\{\beta_1(\mu+\alpha) - (\mu+\alpha)(\mu+\gamma) + \alpha\gamma(1-\sigma)\} \\
=&\{\beta_1(\mu+\alpha) - [\mu(\mu+\alpha) + \gamma(\alpha\sigma+\mu)]\} \\
=&[\mu(\mu+\alpha) + \gamma(\sigma\alpha+\mu)] \times [\frac{\beta_1(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)} - 1] \\
\leq&[\mu(\mu+\alpha) + \gamma(\sigma\alpha+\mu)] \times (R-1) \\
<&0.
\end{aligned}$$

In addition, we have $\dot{V}(X) = 0 \Longleftrightarrow X = E_0$. Hence by the LaSalle's Invariance Principle, we get our result for $R < 1$.

4.2. *The proofs of the results obtained with the stochastic case*

The proofs of the theorems etablished in the sub-sections 2.3 and 2.4 are similar to those established in [4]. We use Lyapunov methods to show that the solution of model (2.2) is positive and global in Δ. With Lyapunov functions, we prove the asymptotic behavior around E_0 and E^*. So, we invite you to consult [4] for these proofs.

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Conflict of interest

This manuscript has not been published and is not under consideration for publication elsewhere. We have no conflicts of interest to disclose.

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