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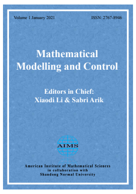

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Research article

Controllability of nonlinear ordinary differential equations with non-instantaneous impulses

Zhen Xin[1,2,*], **Yuhe Yang**[1,2] **and Qiaoxia Li**[1,2]

[1] School of Mathematics and Statistics, Yili Normal University, Yining 835000, China

[2] Institute of Applied Mathematics, Yili Normal University, Yining 835000, China

* **Correspondence:** E-mail: xz18394138767@163.com; Tel: 18394138767.

Abstract: In this paper, we consider controllability of the initial value problem with non-instantaneous impulse on ordered Banach spaces. We firstly give a solution expression for initial value problems with non-instantaneous impulses in ordered Banach Spaces by using Schauder fixed point theorem. Sufficient conditions for controllability results are obtained by Krasnoselskii's fixed point theorem in the infinite-dimensional spaces. An example is also given to illustrate the feasibility of our theoretical results.

Keywords: non-instantaneous impulses; controllability; Banach spaces

1. Introduction

The dynamics of many evolving processes are subject to abrupt changes, such as shocks, harvesting and natural disaster. These phenomena involve short term perturbations from continuous and smooth dynamics, whose duration is negligible in comparison with the duration of an entire evolution. Sometimes time abrupt changes may stay for time intervals such impulses are called non-instantaneous impulses. The theory of instantaneous impulsive differential equations is an important branch of differential equation theory, which has extensive physical, chemical, biological, engineering background and realistic mathematical model, and hence has been emerging as an important area of investigation in the last decades. Hernández and O'Regan [20] first studied the initial value problem for a new class of abstract evolution equations with non-instantaneous impulses in Banach spaces.

Control theory is an area of application-oriented mathematics which deals with basic principles underlying the analysis and design of control systems. It is well known that controllability plays a significant role in modern control theory and engineering since they are closely related to pole assignment, structural decomposition and quadratic optimal control. See functional analysis [1].

The notion of controllability means that it is possible to steer a dynamic system from an arbitrary initial state to an arbitrary final state using a set of admissible controls. There exist many criteria and definitions of controllability. They depend both on the constraints on the control signal and the state equation. It should be noticed that in infinite-dimensional spaces there exist linear subspaces which are not closed. We can distinguish two concepts of controllabilityin the case of infinite-dimensionalsystems. Exact controllability means that the system can be steered to an arbitrary final state. Approximate controllability enables us to steer the system to an arbitrary small neighborhood of the final state. It is self-evident that approximate controllability is essentially a weaker notion than exact controllability. As a result, the latter always implies the former, but the converse statement is not true in general. See the works of Bashirov and Kerimov (1997)[2], Bashirov and

Mahmudov (1999)[3], Benchohra and Ouahab (2005)[4]. Controllability of semilinear integrodifferential systems in Banach spaces was investigatedby Lasiecka and Triggiani (1991)[5].

For nonlinear systems, controllability results can be shown via contraction mapping principle, Schauder's fixed point theorem and Schaefer's fixed point theorem by constructing a suitable control function (see [6–9]). In particular, it is summarized that the sufficient conditions for approximate controllability of various types of dynamic systems using Schauder's fixed-point theorem in Hilbert spaces by Babiarz [10]. In 2013, Liu et al. [11] used the Krasnoselskii's fixed point theorem to studied controllabily of nonlinear fractional impulsive evolution systems. Of course, using Nussbaum's fixed point theorem can also be used to investigate the controllability of nonlinear systems (see [11–13,17–19]). In this paper, on the basis of previous research[14],[15], the controllability of the initial value problem is studied, and the fixed point theorem is applied to nonlinear ordinary differential equations with non-instantaneous impulses. Our work can be considered as a contribution to this nascent fields. The results obtained in this paper are a supplement to the existing literature and essentially extend some existing results in this area.

Next, we study the following controllability of nonlinear ordinary differential equation with non-instantaneous impulses by fixed point theorem in a Banach space E

$$\begin{cases} u'(t) = f(t, u(t)) + Bh(t), \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ u(t) = g_i(t, u(t)), \quad t \in (t_i, s_i],\ i = 1, 2, \cdots, m, \\ u(0) = u_0, \end{cases} \tag{1.1}$$

where $0 < t_1 < t_2 < \cdots < t_m < t_{m+1} := a$, $a > 0$ is a constant, $J = [0, a]$, $J' = J\backslash\{t_1, t_2, \cdots t_m\}$ and $s_0 = t_0 := 0$, $s_i \in (t_i, t_{i+1})$ for each $i = 1, 2, \cdots, m$. $f : [0, a] \times E \to E$ is a continuous nonlinear function, $g_i : (t_i, s_i] \times E \to E$, $i = 1, 2, \cdots, m$ is non-instantaneous impulsive function for all $i = 1, 2, \cdots, m$, and $u_0 \in E$. The control function $h(\cdot)$ is given in $L^2(J', U)$ with U as a Banach space and B is bounded linear operator from U into E.

2. Preliminaries

Let E be a Banach space with the norm $\|\cdot\|$, whose positive cone $P = \{x \in E \mid x \geq \theta\}$ is normal with normal constant N, where θ is the zero element in E. We denote by $C(J, E)$ the Banach space of all continuous functions from J into E endowed with the sup-norm

$$\|u\|_C = \sup_{t\in J} \|u(t)\|.$$

Then $C(J, E)$ is an ordered Banach space induced by the convex cone

$$P_C = \{u \in C(J, E) \mid u(t) \geq \theta, t \in J\}$$

and P_C is also a normal cone with normal constant N. Let

$$PC(J, E) = \{u : J \to E \mid u \text{ is continuous at } t \neq t_i \text{ , left}$$

$$\text{continuous at } t \doteq t_i \text{ and } u(t_i^+) \text{ exists for all } i = 1, 2, \cdots, m.\}$$

be a piecewise continuous function space. It is easy to see that $PC(J, E)$ is a Banach space endowed with the PC-norm

$$\|u\|_{PC} = \max\{\sup_{t\in J} \|u(t^+)\|,\ \sup_{t\in J} \|u(t^-)\|\},\ u \in PC(J, E).$$

For any finite number $r > 0$, let

$$\Omega_r = \{u \in PC(J, E) \mid \|u(t)\| \leq r, t \in J\}$$

be a bounded convex closed set.
Let $L^p(J, R)(1 \leq p \leq \infty)$ denote the Banach space of all Lebesgue measurable functions from J into E with $\|\varphi\|_{L^p(J,R)} := (\int_J |\varphi(t)|^p dt)^{\frac{1}{p}} < \infty$. And let $L^p(J, E)$ be the Banach space of functions $\varphi : J \to E$ which are Bochner integrable normed by $\|\varphi\|_{L^p(J,E)}$.

Definition 2.1. A function $u \in PC(J, E)$ is a solution of the system (1.1), it is equivalent to u satisfies that

$$\begin{cases} u(t) = u_0 + \int_0^t [f(s, u(s)) + Bh(s)]ds,\ t \in [0, t_1], \\ u(t) = g_i(t, u(t)), \quad t \in (t_i, s_i],\ i = 1, 2, \cdots, m, \\ u(t) = g_i(s_i, u(s_i)) + \int_{s_i}^t [f(s, u(s)) + Bh(s)]ds,\ t \in (s_i, t_{i+1},], \\ \qquad\qquad i = 1, 2, \cdots, m. \end{cases} \tag{2.1}$$

Lemma 2.2[10] (Schauder's theorem)(Kulmin, 2004). Every continuous operator that maps a closed convex subset of a

Banach space into a compact subset of itself has at least one fixed point.

Lemma 2.3[11] (Krasnoselskii's fixed point theorem). Let X be a Banach space, Ω a bounded closed and convex subset of X and F_1, F_2 maps Ω into X such that $F_1x + F_2y \in \Omega$ for every pair $x, y \in \Omega$. If F_1 is a contraction and F_2 is completely continuous, then the equation $F_1x + F_2x = x$ has a solution on Ω.

Definition 2.4. The system (1.1) is said to be completely controllable on $[0, t_f](t_f \in (0, T])$ if for every $u_0, u_{t_f} \in E$, there exist a control $h \in L^2(J', U)$ such that the solution $u(t)$ of the system (1.1) satisfies $u(t_f) = u_{t_f}$.

3. Main results

we need the following additional assumption conditions:

(H1) There are arbitray constant $p_0 > 0$, $p_1 \geq 0$, and $t \in J, u \in E$ such that

$$\|f(t, u(t))\| \leq p_1\|u\| + p_0;$$

(H2) There exist positive constant L_i $(i = 1, 2, \cdots, m)$, $\forall u, v \in E$ such that

$$\|g_i(t, u) - g_i(t, v)\| \leq \sum_{i=1}^{m} L_i\|u - v\|, \ \forall t \in (t_i, s_i], i = 1, 2 \cdots m;$$

(H3) The linear operator $B : L^2(J', U) \to L(J', U)$ is bounded, $W : L^2(J', U) \to E$ defined by

$$Wh = \int_0^{t_f} [f(s, u(s)) + Bh(s)]ds,$$

has an inverse operator W^{-1} which takes value in $L^2(J', U) \setminus \ker W$ and there exist two positive constants $D_1, D_2 > 0$ such that

$$\|B\| \leq D_1, \ \|W^{-1}\| \leq D_2.$$

Theorem 3.1. Assume that [H1]-[H3] and

$$\sum_{i=1}^{m} L_i + ap_1 < 1$$

hold, then system (1.1) has a solution $u \in PC(J, E)$ on $(0, T]$.

Proof. Define an operator $F : PC(J, E) \to PC(J, E)$ such that

$$(Fu)(t) = \begin{cases} u_0 + \int_0^t f(s, u(s))ds, \ t \in [0, t_1], \\ g_i(t, u(t)), \quad t \in (t_i, s_i], i = 1, 2, \cdots, m, \\ g_i(s_i, u(s_i)) + \int_{s_i}^t f(s, u(s))ds, \ t \in (s_i, t_{i+1}], \\ \qquad\qquad i = 1, 2, \cdots, m. \end{cases} \tag{3.1}$$

and we define the control function $h_u(t)$ by

$$h_u(t) = \begin{cases} W^{-1}[u_{t_f} - u_0 - \int_0^{t_f} f(s, u(s))ds], \ t \in [0, t_1], \\ W^{-1}[u_{t_f} - g_i(t, u(t))], \ t \in (t_i, s_i], i = 1, 2, \cdots, m, \\ W^{-1}[u_{t_f} - g_i(s_i, u(s_i)) - \int_{s_i}^{t_f} f(s, u(s))ds], \\ \qquad\qquad t \in (s_i, t_{i+1}], i = 1, 2, \cdots, m. \end{cases} \tag{3.2}$$

It is easy to see that the solution of the system (1.1) is equivalent to the fixed point of operator F defined by (3.1). Now, we will prove that there exists a constant $R > 0$, $\Omega_R = \{u \in PC(J, E) \mid \|u(t)\| \leq R, t \in J\}$ such that $F(\Omega_R) \subset \Omega_R$. If this is not ture, then for each $r > 0$, there would exist $u_r \in \Omega_r$, $t_r \in J$ such that

$$\|(Fu_r)(t_r)\| > r.$$

If $\forall t_r \in [0, t_1]$ then by (3.1), (H1) and (H3), we know that

$$\begin{aligned} \|(Fu_r)(t_r)\| &= \|u_0 + \int_0^{t_r} [f(s, u_r(s)) + B(s)h(s)]ds\| \\ &\leq \|u_0\| + t_r(p_1\|u_r\|_{PC} + p_0) + D_1D_2 \\ &\leq \|u_0\| + ap_1r + ap_0 + D_1D_2. \end{aligned} \tag{3.3}$$

If $\forall t_r \in [t_1, s_i]$, $i = 1, 2, \cdots, m$, then by (3.1) and (H2), we know that

$$\begin{aligned} \|(Fu_r)(t_r)\| &= \|g_i(t_r, u_r(t_r))\| \\ &\leq \sum_{i=1}^{m} L_i\|u_r(t_r)\| + \|g_i(t_r, \theta)\| \leq \sum_{i=1}^{m} L_ir + N, \end{aligned} \tag{3.4}$$

where $N = \max_{i=1,2,\cdots,m} \sup_{t \in J} \|g_i(t, \theta)\|$. If $\forall t_r \in (s_i, t_{i+1}]$, $i = 1, 2, \cdots, m$, then by (3.1) and (H1)-(H3), we know that

$$\begin{aligned} \|(Fu_r)(t_r)\| &= \|g_i(s_i, u_r(s_i)) + \int_{s_i}^{t_r} [f(s, u_r(s)) + B(s)h(s)]ds\| \\ &\leq \sum_{i=1}^{m} L_ir + N + ap_1r + ap_0 + D_1D_2. \end{aligned} \tag{3.5}$$

Combining (3.1)-(3.5) with the fact $\|(Fu_r)(t_r)\| > r$, we obtain

$$r < \|(Fu_r)(t_r)\| \leq \|u_0\| + \sum_{i=1}^{m} L_i r + N + ap_1 r + ap_0 + D_1 D_2. \tag{3.6}$$

Dividing both side of (3.6) by r and taking the lower limit as $r \to \infty$, we have

$$1 < ap_1 + \sum_{i=1}^{m} L_i. \tag{3.7}$$

Next, we prove that F is continuous in Ω_R.

Let $u_n \in \Omega_R$ be a sequence, such that $\lim_{n\to\infty} u_n = u$ in Ω_R. By the continuity of nonlinear term f with respect to the second variable, for each $s \in J$ we have

$$\lim_{n\to\infty} f(s, u_n(s)) = f(s, u(s)) \tag{3.8}$$

If $\forall s \in (t_i, t_{i+1}]$, $i = 1, 2, \cdots, m$, by (3.4), (3.8) and Lebesgue dominated convergence theorem, we obtain

$$\begin{aligned} \|(Fu_n)(t) - (Fu)(t)\| &= \| \int_{s_i}^{t} f(s, u_n(s))ds - \int_{s_i}^{t} f(s, u(s))ds\| \\ &\to 0, \quad (n \to \infty) \end{aligned}$$

Then we concluded that $\|Fu_n - Fu\|_{pc} \to 0$, $(n \to \infty)$. Which meas that F is continuous in Ω_R.

Now, we demonstrate that the operator $F : \Omega_R \to \Omega_R$ is equicontinuous. For any $u \in \Omega_R$ and $s_i \leq t' < t'' \leq t_{i+1}$, $i = 0, 1, 2, \cdots, m$, we can conclude that

$$\begin{aligned} \|(Fu)(t'') - (Fu)(t')\| &= \| \int_{s_i}^{t''} [f(s, u(s)) + B(s)h(s)]ds \\ &\quad - \int_{s_i}^{t'} [f(s, u(s)) + B(s)h(s)]ds\| \\ &= \| \int_{t'}^{t''} f(s, u(s))ds\| + \| \int_{t'}^{t''} B(s)h(s)\| \\ &\leq (t'' - t')(p_1\|u\| + p_0 + D_1 D_2). \end{aligned} \tag{3.9}$$

Therefore, when $t'' - t' \to 0$, then $\|(Fu)(t'') - (Fu)(t')\| \to 0$. As a result, the operator $F : \Omega_R \to \Omega_R$ is equicontinuous. It follows the Arzela-Ascoli theorem that $F\Omega_R$ is sequentially compact in $PC(J, E)$. By Schauder's fixed point theorem (Lemma 2.2) F has a fixed point in $F\Omega_R$. Therefore, u are a solutions of the system (1.1). This completes the proof of Theorem 3.1.

Theorem 3.2. Assume that [H2], [H3] and

$$\sum_{i=1}^{m} L_i < 1$$

hold, then system (1.1) is completely controllable on $[0, t_f]$ for some $t_f \in [0, T]$.

Proof. Let

$$(Fu)(t) = (F_1 u)(t) + (F_2 u)(t), \tag{3.10}$$

by

$$(F_1 u)(t) = \begin{cases} u_0, & t \in [0, t_1], \\ g_i(t, u(t)), & t \in (t_i, s_i], i = 1, 2, \cdots, m, \\ g_i(s_i, u(s_i)), & t \in (s_i, t_{i+1}], i = 1, 2, \cdots, m. \end{cases} \tag{3.11}$$

$$(F_2 u)(t) = \begin{cases} \int_0^t f(s, u(s))ds, & t \in [0, t_1], \\ 0, & t \in (t_i, s_i], \quad i = 1, 2, \cdots, m, \\ \int_{s_i}^t f(s, u(s))ds, & t \in (s_i, t_{i+1}], i = 1, 2, \cdots, m. \end{cases} \tag{3.12}$$

By (H3), for any $u \in PC(J, E)$, From the references [14], we know that operators $F_1 : \Omega_R \to \Omega_R$ is Lipschitz continuous and by the proof of Theorem 3.1, we can get that $F\Omega_R \subset \Omega_R$. According to the definition of completely continuous and the proof of Theorem 3.1, we get that F_2 is completely continuous Now, we prove that F_1 is contraction.

Let any $u, u' \in PC(J, E)$ and $t \in (s_i, t_{i+1}]$, $i = 0, 1, 2, \cdots, m$, we have

$$\begin{aligned} \|(F_1 u)(t) - (F_1 u')(t)\| &\leq \|g_i(t, u(t)) - g_i(t, u'(t))\| \\ &\leq \sum_{i=1}^{m} L_i \|u - u'\|_{PC}. \end{aligned} \tag{3.13}$$

We derive that $\sum_{i=1}^{m} L_i < 1$, which implies F_1 is contraction. According to the Lemma 3, F has a fixed point $u \in \Omega_R$. Thus, If $t_f \in [0, t_1]$, it is easy to check that $u(t_f) = u_{t_f}$. Similarly if $t_f \in (t_i, s_i]$, $t_f \in (s_i, t_{i+1}]$, $i = 1, 2, \cdots, m$, $u(t_f) = u_{t_f}$. Consequently, the system (1.1) is completely controllable on $[0, t_f]$.

4. An example

As an application of the abstract result, we consider the following controllability of nonlinear differential equations with non-instantaneous impulses

$$\begin{cases} u'(t) = \int_0^1 \frac{e^{(s-t)}}{10} u(s)ds + \frac{1}{50}u(t) + 1 + Bh(t), \ t \in [0, \frac{1}{3}] \bigcup (\frac{2}{3}, 1], \\ u(t) = \dfrac{t + |u(t)|}{1 + t + |u(t)|}, \quad t \in (\frac{1}{3}, \frac{2}{3}], \\ u(0) = u_0, \end{cases} \tag{4.1}$$

Where $E = C(I)$, $I = [0,1]$, $J = [0,1]$, $t_0 = s_0 = 0$, $t_1 = \frac{1}{3}$, $s_1 = \frac{2}{3}$, $t_2 = 1$. $u \in C(I \times J, \mathbb{R})$. Let $f(t,u(t)) = \int_0^1 \frac{e^{(s-t)}}{10} u(s)ds + \frac{1}{50}u(t) + 1$, $g_1(t,u(t)) = \frac{t+|u(t)|}{1+t+|u(t)|}$, $h(t) = \frac{1}{2}e^{-(t-\frac{1}{2})}$. And then we're going to prove that we satisfy our assumption conditions (H1)-(H3).

Let $t \in [0, \frac{1}{3}] \bigcup (\frac{2}{3}, 1]$, we get

$$\begin{aligned} \|f(t,u(t))\| &\le \|\frac{1}{10}\int_0^1 e^{(s-t)}u(s)ds\| + \|\frac{1}{50}u(t)\| + 1 \\ &\le (\frac{1}{10}\int_0^1 e^{(s-t)}ds + \frac{1}{50})\|u\|_{PC} + 1 \\ &\le (\frac{1}{10}e - \frac{2}{25})\|u\|_{PC} + 1. \end{aligned} \tag{4.2}$$

For any $u, v \in C(I \times J, \mathbb{R})$, $t \in (\frac{1}{3}, \frac{2}{3}]$, we have

$$\|g_1(t,u) - g_1(t,v)\| \le \|\frac{u-v}{1+t}\| < \frac{3}{4}\|u-v\|_{PC}. \tag{4.3}$$

Next, we define a continuous mapping $B : L^2(J', R) \to L^2(J', R)$, with domain

$$Bh(t) = \int_0^{t_f} \frac{1}{2}e^{-(t-\frac{1}{2})}dt. \tag{4.4}$$

A direct calculation gives

$$\|Bh\| \le -\frac{1}{2}e^{-(t_f-\frac{1}{2})} + \frac{1}{2}e^{\frac{1}{2}}.$$

To achieve $\|B\| \le \frac{11}{20}$. Let's calculate

$$\begin{aligned} \|Wh\| &= \|\int_0^{t_f}[f(s,u(s)) + Bh(s)]ds\| \\ &\le \|\int_0^{t_f}[\frac{1}{10}\int_0^1 e^{(s-t)}u(s)ds + \frac{1}{50}u(t) + 1]\|ds \\ &+ \frac{1}{2}\cdot\frac{11}{20} \le [(\frac{1}{10}e - \frac{2}{25})\|u\|_{PC} + 1]t_f + \frac{11}{40}. \end{aligned} \tag{4.5}$$

To achieve $\|W^{-1}\| \le \frac{40}{11}$. Thus, where $p_1 = \frac{1}{10}e - \frac{2}{25}$, $p_0 = 1$, $K = \frac{3}{4}$, $N = \frac{2}{5}$, $D_1 = \frac{11}{20}$, $D_2 = \frac{40}{11}$, the initial value problem (4.1) satisfies the conditions (H1)-(H3). According to Formula (4.2)-(4.5) is established. By Theorem 3.2, the system (4.1) is controllable on [0, 1].

5. Conclusions

In this paper, the existence of solutions of ordinary differential equations with non-instantaneous impulses in Banach space is studied by using Schauder fixed point theory, and sufficient conditions for controllability results in infinite dimensional space are obtained by using Krasnoselskii fixed point theorem and the definition of complete controllability. However, the necessity of controllability cannot be obtained under this condition. Subsequent studies can obtain the approximate controllability of the system (1.1) under the condition that the controllable function satisfies the presolution formula.

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Conflict of interest

The authors declare that there is no interest in this paper.

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