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Editorial

Preface to the Special Issue: Nonlinear PDEs and geometric analysis – Dedicated to Neil Trudinger on the occasion of his 80th birthday[†]

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Abstract: This contribution is the preface of the Special Issue: Nonlinear PDEs and geometric analysis – Dedicated to Neil Trudinger on the occasion of his 80th birthday.
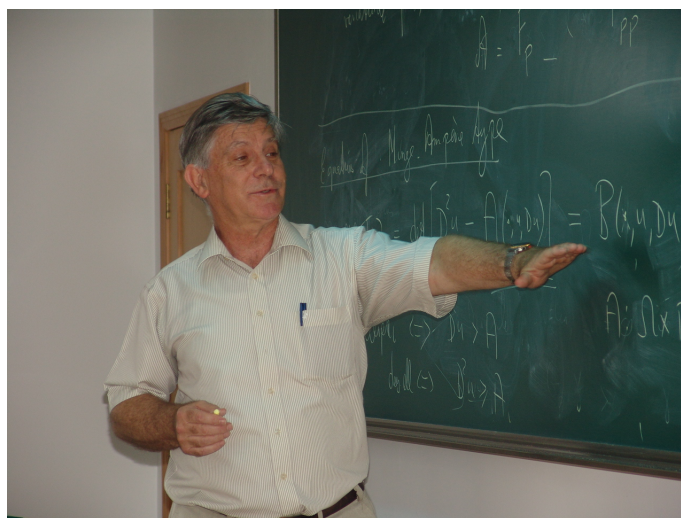
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This special issue is dedicated to Professor Neil Trudinger, celebrating his exceptional contributions to the field of nonlinear elliptic partial differential equations. Throughout his illustrious career, Professor Trudinger has achieved remarkable milestones, leaving an indelible mark on the mathematical community.

Born on 20 June 1942 in Ballarat, Victoria, Australia, Neil Trudinger received his education at Richmond High School, New South Wales. In 1962, he earned a B.Sc. from the University of New England in Armidale, New South Wales, Australia. Subsequently, he pursued graduate studies at Stanford University in the United States under the guidance of David Gilbarg. In 1966, Neil was awarded a Ph.D. and embarked on his academic journey as a Courant Instructor at the Courant Institute of Mathematical Sciences of New York University. He returned to Australia in 1967, where he held positions at Macquarie University, the University of Queensland (UQ), and in 1973, he joined the Australian National University (ANU) as Head of the Department of Pure Mathematics. Neil's leadership continued as he served as Director of the Commonwealth Special Research Centre for Mathematical Analysis from 1982 to 1990, followed by his directorship of the ensuing Centre for Mathematics and its Application (CMA) at ANU. From 1992 to 2000, he assumed the role of Dean of

the School of Mathematical Sciences at ANU. Currently, he is an Emeritus Professor at ANU as well as an Honorary Professor at UQ and the University of Wollongong.

Neil lectured at Zhejiang University.

Undoubtedly, Neil Trudinger stands among the foremost researchers in the field of partial differential equations (PDE). His early contributions focused on quasilinear elliptic PDE and related analysis and geometry including the influential "Moser-Trudinger inequality", as well as partial solution of the Yamabe problem and Harnack inequalities for quasilinear elliptic and parabolic equations. From the 1980s onward, Neil's ensuing work concentrated in the area of fully nonlinear elliptic equations, notably Hessian and curvature equations, including the Monge-Ampere equation. He innovated new techniques for Hessian equations and a viscosity approach to curvature equations. In collaboration with Xu-Jia Wang, he developed a theory of Hessian measures, successfully resolving the Chern and Calabi conjectures on affine maximal surfaces and subsequently the affine Plateau problem, with local regularity proved in two dimensional affine maximal surfaces. In optimal transportation, a final resolution of the famous Monge mass transfer problem was found. For general cost functions, a critical condition now known as "Ma-Trudinger-Wang condition" was discovered ensuring the regularity results, which has spawned a new and active international research line addressing the relationship with Riemannian geometry. More recently, Neil has initiated research on prescribed Jacobian equations, with applications in geometric optics and conformal geometry.

Neil Trudinger's exceptional achievements have garnered numerous distinctions and honors. He was elected as a Fellow of the Australian Academy of Science in 1978 and a Fellow of the Royal Society of London in 1997. In 1981, he received the first medal of the Australian Mathematical Society, and in 1996, he was awarded the inaugural Australian Academy of Science Hannan Medal for Pure Mathematics. Neil was awarded the Leroy P. Steele Prize for mathematical exposition by the American Mathematical Society in 2008 and was elected as an inaugural Fellow of the American Mathematical Society in 2012.

For over half a century, Neil has generously shared his knowledge, wisdom, and friendship with mathematicians worldwide. He is widely recognised among younger generations as a visionary mentor who exemplifies generosity and compassion. The influential book "Elliptic Partial Differential

Equations of Second Order" by Gilbarg and Trudinger has attained revered status within the field, serving as a fundamental resource for countless researchers and students and being extensively cited in its various domestic and foreign editions.

The book!

This special issue is dedicated to celebrating Neil's 80th birthday, a momentous occasion marking a lifetime of extraordinary contributions. We express our heartfelt gratitude to all the authors who have contributed high-quality articles [1–17] to this issue. We would also like to extend our sincere appreciation to Professor Enrico Valdinoci, the Editor-in-Chief of the journal Mathematics in Engineering, for his unwavering support and invaluable assistance.

In closing, we extend our gratitude to all those involved and wish Professor Neil Trudinger a very Happy Birthday!

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

Conflict of interest

The authors declare no conflict of interest.

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