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Research article

Observer-based event-triggered impulsive control of delayed reaction-diffusion neural networks

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Abstract: In this paper, we present a novel design of an observer-based event-triggered impulsive control strategy for delayed reaction-diffusion neural networks subject to impulsive perturbation. The impulsive instants of impulsive control are determined in an event-triggered way, and the control strength is designed by the sampling output of an impulsive observer. Several criteria with Lyapunov conditions and linear matrix inequalities are established for the global exponential stability of delayed reaction-diffusion neural networks. It inherits the advantages of event-triggered impulsive control such as low triggering frequency and high efficiency, and is applicable for networks with unmeasurable states. Finally, the effectiveness of theoretical results is verified by a numerical example.

Keywords: event-triggered impulsive control; impulsive observer; neural networks; reaction-diffusion

1. Introduction

Due to their wide application in pattern recognition and machine learning, the dynamic behavior of neural networks has been extensively investigated, such as stability [1, 2], synchronization [3, 4], periodicity, and so forth [5, 6]. To achieve desirable dynamical performance, various kinds of control strategies have been proposed, such as feedback control, event-triggered control, sliding mode control, and sampled-data control, to name just a few.

In the transmission process of electrons in a non uniform electric field, the displacement of electrons sometimes occurs; consequently, the reaction-diffusion is involved in neural networks, which ignites the research interest of dynamic analysis of reaction-diffusion neural networks (RDNNs) including synchronization [7–9], passivity [10], control design [11], and stability analysis [12, 13]. For instance, the stabilization problem of fuzzy RDNNs was studied by a fuzzy adaptive event-triggered sampled-data control method in [11]. In [8], the unbounded parameter was considered, and a novel distributed adaptive controller was proposed to achieve the tracking synchronization of the coupled RDNNs. Furthermore, the lag $\mathcal{H}_\infty$ synchronization and stability of RDNNs with state coupling and spatial diffusion coupling were achieved by feedback control in [12].

Time delay is inevitable in the signal's transmission through neurons, so delayed reaction-diffusion neural networks (DRDNNs) were put forward, and their dynamical properties were widely investigated by researchers, including stability [14, 15], synchronization [16–18], and so on [19, 20]. For instance, the global exponential stability (GES) and synchronization of the DRDNNs with Dirichlet boundary conditions under impulsive control were discussed in [17]. The stability and synchronization of nonautonomous DRDNNs with general time-varying delays were investigated in [19]. Then a relaxed Lyapunov function method was introduced to design feedback control and adaptive control to achieve finite-time synchronization of DRDNNs in [20].

On the other hand, from the viewpoint of control design, event-triggered impulsive control (ETIC) is a classic kind of discontinuous control strategy with a low triggering frequency and high efficiency, which makes it an area of growing interest among scholars. Various kinds of properties for nonlinear systems and neural networks are achieved by ETIC, such as stability [21–23], input-to-state stability [24, 25], synchronization [26–28], and consensus [29, 30]. In [26], the ETIC with bounded triggering instants was proposed for the synchronization of continuous-time neural networks. Furthermore, a distributed ETIC strategy was designed to achieve the lead-following consensus problem of multi-agent systems in [29]. In [24], the ETIC was extended to solve the input-to-state stability problem of nonlinear systems.

When the system state is inaccessible, the control strategies have to be designed on the basis of the output of the system. One possible choice can be observer-based control, such as observer-based finite-time control [31, 32], observer-based impulsive control [33, 34], impulsive-observer-based control [35, 36], disturbance-observer-based control [37, 38]. In [33], feedback control based on an impulsive observer (IO) was designed for the stability of uncertain linear systems. Then, IO-based impulsive control was designed for the stabilization of time-delay systems in [35]. Furthermore, IO-based impulsive control was applied for synchronization of delayed neural networks with unmeasurable neural states in [36]. One may observe that most of existing work about the IO and impulsive control has concentrated on continuous neural networks, but impulsive control based on an IO for discontinuous neural networks, especially impulsive neural networks, have rarely been studied. The IO-based impulsive control for impulsive neural networks has to consider the influence of impulsive perturbation and connect the properties of impulsive perturbation and impulsive control gain.

Motivated by this discussion, a novel IO-based ETIC strategy was designed for the GES of DRDNNs with impulsive perturbation. The main contributions lie in three aspects: (i) even though the DRDNNs involve impulsive perturbation, the designed IO is free of impulsive perturbation and consequently generates the impulsive control input in the neural networks; (ii) the instants of impulsive control and impulsive sampling are determined in an event-triggered way, that is event-triggered IO-based impulsive control with low triggering frequency and high efficiency; and (iii) the ETIC strategy is designed on the basis of the sampled output of neural networks, so it is applicable for DRDNNs with unmeasurable states.

The rest of paper is arranged as follows: Section 2 introduces the DRDNNs, proposes the IO, and gives some basic definitions, assumptions and lemmas. Section 3 presents the ETIC strategy for the GES of the DRDNNs under Lyapunov conditions and linear matrix inequalities. Numerical simulations and conclusion are provided in Sections 4 and 5, respectively.

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of non-negative real numbers, $\mathbb{Z}_+$ the set of positive integer numbers, and $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ the n-dimensional and $n\times m$ dimensional real spaces equipped with the Euclidean norm $|\cdot|$, respectively. $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} \mid x \geq 0\}$. A^T and A^{-1} denote the transpose and the inverse of A. Let $\varsigma_1 \vee \varsigma_2$ and $\varsigma_1 \wedge \varsigma_2$ denote the maximum and minimum value of ς_1 and ς_2. I is the identity matrix with appropriate dimensions. $\Omega \subset \mathbb{R}^m$ is a bounded compact set with a smooth boundary $\partial\Omega$ and satisfies $|x_i| < l_i$, $i = 1, 2, \ldots, m$, for all $(x_1, x_2, \ldots, x_m)^T \in \Omega$. $\mathbb{L}^2(\Omega)$ is a Hilbert space with the inner product $\langle u_1, u_2\rangle = \int_\Omega u_1(x)u_2(x)dx$ and the norm $\|u\|^2 = \langle u, u\rangle$. $\mathbb{H} = \{z \in \mathbb{L}^2(\Omega) : (\partial u)/(\partial x_i), (\partial^2 u)/(\partial x_i\partial x_j) \in \mathbb{L}^2(\Omega),\ u(t, x)|_{x\in\partial\Omega} = 0,\ i, j = 1, 2, \cdots, m\}$. $PC_\tau(J)$ represents the space of functions $f : [-\tau, 0] \to J$ which have at most a finite number of jump discontinuities on $[-\tau, 0]$ and $f(t^+) = f(t)$ for $\forall t \in [-\tau, 0]$. $\mathbb{PC}^n_\tau = \{f | f \in PC_\tau((\mathbb{L}^2(\Omega))^n \cap \mathbb{H}^n)$ and f is bounded on $[-\tau, 0]\}$ with norm $\|f\|_{\mathbb{PC}^n_\tau} = \sup_{-\tau\leq t\leq 0} \|f(t)\|$.

2. Preliminaries

Consider the following DRDNNs with impulsive perturbation:

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} = D\Delta u(t, x) - Au(t, x) + Bf(u(t, x)) \\ \qquad\qquad +Hf(u(t - \tau, x)),\ t \in \mathbb{R}_{\geq 0} \setminus (T_1 \cup T_2), \\ u(t, x) = (I + M)u(t^-, x),\ t \in T_1, \\ u(t, x) = u(t^-, x) + E\hat{u}(t^-, x) + F(y(t^-, x) - \hat{y}(t^-, x)),\ t \in T_2, \\ y(t, x) = Cu(t, x),\ t \geq t_0, \\ u(t_0 + \theta, x) = \phi(\theta, x) \in \mathbb{PC}^n_\tau,\ -\tau \leq \theta \leq 0, \end{cases} \tag{2.1}$$

where $x \in \Omega$, $u = (u_1, u_2, \ldots, u_n)^T$ is the system state, $D = \text{diag}(d_1, d_2, \ldots, d_n)$, $\Delta u = (\sum_{j=1}^m \frac{\partial^2 u_1}{\partial x_j^2}, \sum_{j=1}^m \frac{\partial^2 u_2}{\partial x_j^2}, \ldots, \sum_{j=1}^m \frac{\partial^2 u_n}{\partial x_j^2})^T$, $d_i \geq 0$ represents the transmission diffusion coefficient, $A = \text{diag}(a_1, a_2, \ldots, a_n)$ represents the self feedback coefficient, B, H are connection weights between the neurons, f denotes the activation function, τ corresponds to the transmission delay along the neuron, $T_1 = \{t_k\}_{k\in\mathbb{N}_+}$ is the time sequence of impulsive perturbation satisfying $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $t_k \longrightarrow +\infty$ as $k \longrightarrow +\infty$ to prevent the occurrence of accumulation points, $T_2 = \{\hat{t}_k\}_{k\in\mathbb{N}_+}$ is the time sequence of impulsive control which will be determined by event-triggered mechanism (ETM), $\hat{u}$ is the state of the observer, $\hat{y}$ is the output of the observer, E, F are impulsive gain matrices, $y(t, x)$ is the output, and M, C are constant matrices with compatible dimensions. The Dirichlet boundary condition of DRDNNs (2.1) is expressed as follows:

$$u_i(t, x) = 0,\ (t, x) \in (-\tau, +\infty) \times \partial\Omega. \tag{2.2}$$

Assume that the function f satisfies the global Lipschitz condition and linear growth condition, in which the DRDNNs (2.1) admit a strong solution [39–41]. When the states of DRDNNs (2.1) are unmeasurable, the IO is proposed as follows:

$$\begin{cases} \frac{\partial \hat{u}(t,x)}{\partial t} = D\Delta\hat{u}(t,x) - A\hat{u}(t,x) + Bf(\hat{u}(t,x)) \\ \qquad\qquad +Hf(\hat{u}(t-\tau,x)), t \in \mathbb{R}_{\geq 0} \setminus T_2, \\ \hat{u}(t,x) = (I+\hat{E})\hat{u}(t^-,x) + \hat{F}(y(t^-,x) - \hat{y}(t^-,x)),\ t \in T_2, \\ \hat{y}(t,x) = C\hat{u}(t,x),\ t \geq t_0, \\ \hat{u}(t_0+\theta,x) = \hat{\phi}(\theta,x) \in \mathbb{PC}_\tau^n,\ -\tau \leq \theta \leq 0. \end{cases} \tag{2.3}$$

where $x^T \in \Omega$, $\hat{E}$, $\hat{F}$ are impulsive gain matrices. The Dirichlet boundary condition is given by

$$\hat{u}_i(t,x) = 0,\ (t,x) \in (-\tau,+\infty) \times \partial\Omega. \tag{2.4}$$

Obviously, the impulsive perturbation is omitted in the IO (2.3). Define the error between the DRDNNs (2.1) and IO (2.3) by $e(t,x) = \hat{u}(t,x) - u(t,x)$, and then the error dynamics can be expressed by

$$\begin{cases} \frac{\partial e(t,x)}{\partial t} = \nabla \cdot (D \circ \nabla e(t,x)) - Ae(t,x) + Bg(e(t,x)) \\ \qquad\qquad +Hg(e(t-\tau,x)),\ t \in \mathbb{R}_{\geq 0} \setminus (T_1 \cup T_2), \\ e(t,x) = (I+M)e(t^-,x) - M\hat{u}(t^-,x),\ t \in T_1, \\ e(t,x) = (\hat{E}-E)\hat{u}(t^-) + (I+FC-\hat{F}C)e(t^-,x),\ t \in T_2, \\ e(t_0+\theta,x) = \bar{\phi}(\theta,x) \triangleq \hat{\phi}(\theta,x) - \phi(\theta,x),\ -\tau \leq \theta \leq 0, \end{cases} \tag{2.5}$$

where $x^T \in \Omega$ and $g(e) = f(\hat{u}) - f(u)$. Combining (2.3) and (2.5), the coupled system of the error and observer is rewritten as

$$\begin{cases} \frac{\partial \xi(t,x)}{\partial t} = \nabla \cdot (\widetilde{D} \circ \nabla\xi(t,x)) + \widetilde{A}\xi(t,x) + \widetilde{B}G(\xi(t,x)) \\ \qquad\qquad +\widetilde{H}G(\xi(t-\tau,x)),\ t \in \mathbb{R}_{\geq 0} \setminus (T_1 \cup T_2), \\ \xi(t,x) = \widetilde{M}\xi(t^-,x),\ t \in T_1, \\ \xi(t,x) = \widetilde{N}\xi(t^-,x),\ t \in T_2, \\ \xi(t_0+\theta,x) = \tilde{\phi}(\theta,x) \in \mathbb{PC}_\tau^{2n},\ -\tau \leq \theta \leq 0, \end{cases} \tag{2.6}$$

where $x^T \in \Omega$ and

$$\xi = \begin{bmatrix} e \\ \hat{u} \end{bmatrix},\ \widetilde{D} = \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix},\ \widetilde{A} = \begin{bmatrix} -A & 0 \\ 0 & -A \end{bmatrix},\ \widetilde{B} = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix},\ \widetilde{H} = \begin{bmatrix} H & 0 \\ 0 & H \end{bmatrix},$$

$$G = \begin{bmatrix} g \\ f \end{bmatrix},\ \widetilde{M} = \begin{bmatrix} I+M & -M \\ 0 & I \end{bmatrix},\ \widetilde{N} = \begin{bmatrix} I+FC-\hat{F}C & \hat{E}-E \\ -\hat{F}C & I+\hat{E} \end{bmatrix},\ \tilde{\phi} = \begin{bmatrix} \bar{\phi} \\ \hat{\phi} \end{bmatrix}.$$

Definition 1 ([44]). The solution of DRDNNs (2.1) is said to be globally exponentially stable if there exist constants $\lambda > 0$ and $\mu \geq 1$ such that

$$\|u(t,x)\| \leq \mu e^{-\lambda(t-t_0)} \|\phi\|_{\mathbb{PC}_\tau^{2n}},\ t \geq t_0.$$

Obviously, the GES of DRDNNs (2.1) can be derived from the GES of the coupled system (2.6), since both the error and the observer state will be globally exponentially stable.

Definition 2 ([44]). Given a locally Lipschitz function $V : (\mathbb{L}^2(\Omega))^{2n} \to \mathbb{R}_+$, the upper right-hand Dini derivative of V along system is defined by

$$D^+V(\xi) = \limsup_{h\to 0^+} \frac{1}{h}[V(\xi + h\chi(\xi)) - V(\xi)],$$

where $\chi(\xi) = \nabla \cdot (\widetilde{D} \circ \nabla\xi(t, x)) + \widetilde{A}\xi(t, x) + \widetilde{B}G(\xi(t, x)) + \widetilde{H}G(\xi(t - \tau, x))$.

Assumption 1. The activation functions satisfy the global Lipschitz condition, i.e., there are positive constants l_j such that $\left|f_j(s_1) - f_j(s_2)\right| \le l_j |s_1 - s_2|$, $\forall s_1, s_2 \in \mathbb{R}, j = 1, 2, \ldots, n$, where $l_j > 0$ are Lipschitz constants.

Lemma 1 ([45]). *For any vectors $x, y \in \mathbb{R}^n$ and a positive definite matrix $Q \in \mathbb{R}^{n\times n}$, it holds that*

$$2x^T y \le x^T Qx + y^T Q^{-1}y.$$

Lemma 2 ([46]). *For any symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$, the following conditions are equivalent:*

(i) $S < 0$;
(ii) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;
(iii) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

3. Main results

In this section, the IO-based impulsive control is designed for the GES of DRDNNs (2.1), where the instants of impulsive control are determined by a non-Zeno ETM as follows:

$$\begin{aligned} \hat{t}_k &= \min\left\{\hat{t}_k^*,\ \hat{t}_{k-1} + T_{\max}\right\}, \\ \hat{t}_k^* &= \inf\left\{t \ge \hat{t}_{k-1} | V(\xi(t, x)) \ge e^a V(\xi(\hat{t}_{k-1}^+, x))\right\}, \end{aligned} \tag{3.1}$$

where $a, T_{\max} > 0$ are event-triggering parameters, $V(\xi)$ is the Lyapunov function, and $\hat{t}_0 \ge 0$ denotes the initial instant. From $\hat{t}_k = \min\left\{\hat{t}_k^*,\ \hat{t}_{k-1} + T_{\max}\right\}$, each instant of impulsive control is the minimum of $\hat{t}_k^*$ and $\hat{t}_{k-1} + T_{\max}$, where $\hat{t}_k^*$, called the event-triggering instant, is determined by the state of Lyapunov function by $\hat{t}_k^* = \inf\left\{t \ge \hat{t}_{k-1} | V(\xi(t, x)) \ge e^a V(\xi(\hat{t}_{k-1}^+, x))\right\}$, and $\hat{t}_{k-1} + T_{\max}$, called the force-triggering instant, determines the low bound $T_{\max}$ between two impulsive instants of impulsive control.

Theorem 1. *Suppose that there exist positive constants α_1, α_2, α, β, ϱ, $\tilde{\gamma} < 1$, $\rho_1 > 1$, $\rho_2 < 1$, and a locally Lipschitz function $V : \mathbb{R}^n \to \mathbb{R}_+$ such that*

(1) $\alpha_1 \|\xi\| \le V(\xi) \le \alpha_2 \|\xi\|$, $\forall \xi \in (\mathbb{L}^2(\Omega))^n$;

(2) $D^+V(\xi(t, x)) \le \alpha V(\xi(t, x)) + \beta V(\xi(t - \tau, x))$, *where $\xi(t, x)$ is the solution of system* (2.6) *through* $(t_0, \tilde{\phi})$ *and* $\tilde{\phi} \in \mathbb{PC}_\tau^{2n}$;

(3) $V(\xi(t, x)) \le \rho_1 V(\xi(t^-, x))$, $t \in T_1$; $V(\xi(t, x)) \le \rho_2 V(\xi(t^-, x))$, $t \in T_2$;

(4) $\alpha + \frac{\beta e^{\varrho l}}{\rho_2} + \frac{\ln \tilde{\gamma}}{T_{\max}} < 0$ *and* $\rho_1\rho_2 \le \tilde{\gamma}^2$.

Then the DRDNNs (2.1) *are globally exponentially stable via the IO-based impulsive control under the ETM* (3.1).

Proof. Let $\gamma = \frac{1}{\tilde{\gamma}} > 1$ and $\xi(t,x) = \xi(t,x;\hat{t}_0,\tilde{\phi})$ be the solution of the system (2.6) through $(\hat{t}_0,\tilde{\phi})$, where $\hat{t}_0 \in \mathbb{R}^+$ and $\tilde{\phi} \in \mathbb{PC}_\tau^{2n}$, and set

$$\Gamma(t) = \begin{cases} e^{\lambda(t-\hat{t}_0)} V(\xi(t,x)), & t \in [\hat{t}_0, \infty)\,, \\ V_0, & t \in [\hat{t}_0 - \tau, \hat{t}_0)\,. \end{cases} \tag{3.2}$$

To prove the feasibility of the proposed ETIC, the Zeno behavior is firstly excluded under the ETM (3.1). There are three cases about the impulsive instants: (i) the impulsive instants are fully generated by event-triggering instants $\{\hat{t}_k^*\}_{k=1}^\infty$; (ii) the impulsive instants are generated by the event-triggering instants $\{\hat{t}_k^*\}_{k=1}^\infty$ and the force-triggering instants $\{\hat{t}_{k-1} + T_{\max}\}_{k=1}^\infty$; and (iii) the impulsive instants are fully generated by force-triggering instants $\{\hat{t}_{k-1} + T_{\max}\}_{k=1}^\infty$. Since the force-triggering instants naturally exclude the Zeno behavior, we only need to consider Case i and Case ii.

Case i: in this case, it follows from the ETM (3.1) that

$$e^{\lambda(t-\hat{t}_0)} V(\xi(t,x)) < e^{\lambda(t-\hat{t}_0)} e^a V(\xi(\hat{t}_{k-1}^+, x)) = e^{\lambda(\hat{t}_{k-1}^+ - \hat{t}_0)} V(\xi(\hat{t}_{k-1}^+, x) e^{a+\lambda(t-\hat{t}_{k-1}^+)},$$

for $\forall t \in [\hat{t}_{k-1}, \hat{t}_k)$, which is equivalent to

$$\Gamma(t) < e^{a+\lambda(t-\hat{t}_{k-1}^+)} \Gamma(\hat{t}_{k-1}^+) = e^{a+\lambda(t-\hat{t}_{k-1})} \Gamma(\hat{t}_{k-1}).$$

Therefore, the ETM can be expressed as follows:

$$\hat{t}_k = \inf\left\{ t > \hat{t}_{k-1} | \Gamma(t) \geq e^{a+\lambda(t-\hat{t}_{k-1})} \Gamma(\hat{t}_{k-1}) \right\}. \tag{3.3}$$

Note that $\Gamma(\hat{t}_1^-) = e^{a+\lambda(\hat{t}_1-\hat{t}_0)} \Gamma(\hat{t}_0) = e^{a+\lambda(\hat{t}_1-\hat{t}_0)} V_0 > V_0$, and there exists $\underline{\hat{t}_1} = \sup\{t \in [\hat{t}_0, \hat{t}_1) \,|\Gamma(t) \leq V_0\}$ such that $\Gamma(\underline{\hat{t}_1}) = V_0$. Then we have that $V_0 < \Gamma(t) < \Gamma(\hat{t}_1^-)$, $\forall t \in \left[\underline{\hat{t}_1}, \hat{t}_1\right)$; consequently, it holds that

$$\Gamma(t-\tau) \leq \begin{cases} e^{a+\lambda(\hat{t}_1-\hat{t}_0)} \Gamma(t), & \text{if } t-\tau \in [\hat{t}_0, \hat{t}_1)\,, \\ \Gamma(t), & \text{if } t-\tau \in [\hat{t}_0 - \tau, \hat{t}_0)\,. \end{cases}$$

Similarly, we have $\Gamma(\hat{t}_2^-) = e^{a+\lambda(\hat{t}_2-\hat{t}_1)} \Gamma(\hat{t}_1) > \Gamma(\hat{t}_1)$ and there exists $\underline{\hat{t}_2} = \sup\{t \in [\hat{t}_1, \hat{t}_2) \,|\Gamma(t) \leq \Gamma(\hat{t}_1)\}$ such that $\Gamma(\underline{\hat{t}_2}) = \Gamma(\hat{t}_1)$. Then, we obtain $V_0 < \Gamma(\hat{t}_1) = \Gamma(\underline{\hat{t}_2}) \leq \Gamma(t) < \Gamma(\hat{t}_2^-)$ for $\forall t \in \left[\underline{\hat{t}_2}, \hat{t}_2\right)$; consequently, we derive

$$\Gamma(t-\tau) \leq \begin{cases} e^{a+\lambda(\hat{t}_2-\hat{t}_1)} \Gamma(t), & \text{if } t-\tau \in [\hat{t}_1, \hat{t}_2)\,, \\ e^{a+\lambda(\hat{t}_1-\hat{t}_0)} \Gamma(t), & \text{if } t-\tau \in [\hat{t}_0, \hat{t}_1)\,, \\ \Gamma(t), & \text{if } t-\tau \in [\hat{t}_0 - \tau, \hat{t}_0)\,. \end{cases}$$

Repeating the steps above, we have

$$\Gamma(\hat{t}_k^-) = e^{a+\lambda(\hat{t}_k-\hat{t}_{k-1})} \Gamma(\hat{t}_{k-1}) > \Gamma(\hat{t}_{k-1}),$$

and there exists $\underline{\hat{t}_k} = \sup\{t \in [\hat{t}_{k-1}, \hat{t}_k) \,|\Gamma(t) \leq \Gamma(\hat{t}_{k-1})\}$ such that $\Gamma(\underline{\hat{t}_k}) = \Gamma(\hat{t}_{k-1})$. We then find that $V_0 < \Gamma(\hat{t}_{k-1}) = \Gamma(\underline{\hat{t}_k}) \leq \Gamma(t) < \Gamma(\hat{t}_k^-)$ for $\forall t \in \left[\underline{\hat{t}_k}, \hat{t}_k\right)$; consequently, we get

$$\Gamma(t-\tau) \leq \begin{cases} e^{a+\lambda(\hat{t}_k-\hat{t}_{k-1})} \Gamma(t), & \text{if } t-\tau \in [\hat{t}_{k-1}, \hat{t}_k)\,, \\ e^{a+\lambda(\hat{t}_{k-1}-\hat{t}_{k-2})} \Gamma(t), & \text{if } t-\tau \in [\hat{t}_{k-2}, \hat{t}_{k-1})\,, \\ \dots & \\ e^{a+\lambda(\hat{t}_2-\hat{t}_1)} \Gamma(t), & \text{if } t-\tau \in [\hat{t}_1, \hat{t}_2)\,, \\ e^{a+\lambda(\hat{t}_1-\hat{t}_0)} \Gamma(t), & \text{if } t-\tau \in [\hat{t}_0, \hat{t}_1)\,, \\ V_0, & \text{if } t-\tau \in [\hat{t}_0 - \tau, \hat{t}_0)\,. \end{cases}$$

By Condition (4), we know that there exist $\lambda \in (0, \varrho]$ and $\varepsilon_0 > 0$ such that

$$\left[\alpha + \lambda + (\gamma + \varepsilon)\frac{\beta e^{\varrho\tau}}{\gamma\rho_2}\right] T_{\max} < \ln\gamma, \tag{3.4}$$

where $\hat{t}_k - \hat{t}_{k-1} < T_{\max}$. First, we prove that for any $\varepsilon \in (0, \varepsilon_0]$, we have

$$\Gamma(t) < (\gamma + \varepsilon)V_0,\ t \in [\hat{t}_0, \hat{t}_1].$$

Obviously, $\Gamma(\hat{t}_0) = V(\hat{t}_0) < (\gamma + \varepsilon)V_0$. Suppose that we choose $\hat{t}_0 \le \underline{t} < \bar{t} < \hat{t}_1$ such that

$$\Gamma(\bar{t}) = (\gamma + \varepsilon)V_0,\ \Gamma(\underline{t}) = V_0,\ V_0 \le \Gamma(t) \le (\gamma + \varepsilon)V_0,\ t \in \left[\underline{t}, \bar{t}\right],$$

and

$$V(t)\left\{e^{\lambda(t-\hat{t}_0)} \vee 1\right\} \le (\gamma + \varepsilon)V_0, t \in [\hat{t}_0 - \tau, \bar{t}].$$

According to Condition (2), for $t \in \left[\underline{t}, \bar{t}\right]$, we can derive

$$\begin{aligned}
D^+\Gamma(t) &\le \lambda e^{\lambda(t-\hat{t}_0)} V(\xi(t,x)) + e^{\lambda(t-\hat{t}_0)}\left[\alpha V(\xi(t,x)) + \beta V(\xi(t-\tau,x))\right] \\
&= (\lambda + \alpha)\Gamma(t) + \beta e^{\lambda(t-\tau-\hat{t}_0)+\lambda\tau} V(\xi(t-\tau,x)) \\
&\le (\lambda + \alpha)\Gamma(t) + \beta V(\xi(t-\tau,x))\left[e^{\lambda(t-\tau-\hat{t}_0)} \vee 1\right] e^{\lambda\tau} \\
&\le \Gamma(t)\left[\alpha + \lambda + \beta(\gamma + \varepsilon)e^{\varrho\tau}\right].
\end{aligned} \tag{3.5}$$

Integrating both sides of the inequality above from $\underline{t}$ to $\bar{t}$ leads to

$$\ln(\gamma + \varepsilon) \le \left[\alpha + \lambda + (\gamma + \varepsilon_0)\beta e^{\varrho\tau}\right] T_{\max}. \tag{3.6}$$

By Condition (4), we have $\frac{1}{\gamma\rho_2} \ge \gamma\rho_1 \ge 1$. The inequality (3.6) is a contradiction of (3.4). Thus, for any $t \in [\hat{t}_0, \hat{t}_1)$, $\varepsilon \in (0, \varepsilon_0]$, $\Gamma(t) \le (\gamma + \varepsilon)V_0$. According to the arbitrary value of ε, we have $\Gamma(t) \le \gamma V_0$. On the basis of the Condition (3), we get

$$\Gamma(\hat{t}_1) \le \rho_2 V(\hat{t}_1^-) e^{\lambda(\hat{t}_1-\hat{t}_0)} = \rho_2\Gamma(\hat{t}_1^-) \le \gamma\rho_2 V_0.$$

Similar to the discussion above, we can see that for any $t \in [\hat{t}_1, \hat{t}_2)$, $\Gamma(t) \le \gamma V_0$. Hence, it holds that $\Gamma(t) \le \gamma V_0,\ t \in [\hat{t}_0, \hat{t}_2]$. Then, similar to the discussion above, we obtain

$$\Gamma(t) \le \gamma V_0,\ t \in T_2.$$

We then derive

$$\Gamma(t-\tau) \le \begin{cases}
e^{a+\lambda(\hat{t}_k-\hat{t}_{k-1})}\gamma V_0, & \text{if } t-\tau \in [\hat{t}_{k-1}, \hat{t}_k), \\
e^{a+\lambda(\hat{t}_{k-1}-\hat{t}_{k-2})}\gamma V_0, & \text{if } t-\tau \in [\hat{t}_{k-2}, \hat{t}_{k-1}), \\
\cdots & \\
e^{a+\lambda(\hat{t}_2-\hat{t}_1)}\gamma V_0, & \text{if } t-\tau \in [\hat{t}_1, \hat{t}_2), \\
e^{a+\lambda(\hat{t}_1-\hat{t}_0)}\gamma V_0, & \text{if } t-\tau \in [\hat{t}_0, \hat{t}_1), \\
V_0, & \text{if } t-\tau \in [\hat{t}_0-\tau, \hat{t}_0).
\end{cases} \le e^{a+\lambda T_{\max}}\gamma\Gamma(t).$$

Therefore, whether we have $t-\tau<\hat{t}_0$ or $t-\tau\geq\hat{t}_0$, it always holds that

$$e^{\lambda(t-\tau-\hat{t}_0)}V(\xi(t-\tau,x))\leq e^{a+\lambda T_{\max}+\lambda(t-\hat{t}_0)}\gamma V(\xi(t,x)),\ \forall t\in\left[\underline{\hat{t}_k},\hat{t}_k\right).$$

which implies that

$$\frac{e^{-a-\lambda(T_{\max}+\tau)}}{\gamma}V(\xi(t-\tau,x))\leq V(\xi(t,x)),\ \forall t\in\left[\underline{\hat{t}_k},\hat{t}_k\right),$$

On the basis of (3.5), we can derive

$$\begin{aligned}D^{+}\Gamma(t)&\leq(\lambda+\alpha)\Gamma(t)+\beta e^{\lambda(t-\tau-\hat{t}_0)+\lambda\tau}V(\xi(t-\tau,x))\\&=(\lambda+\alpha)\Gamma(t)+\beta e^{\lambda\tau}\Gamma(t-\tau)\\&\leq(\lambda+\alpha)\Gamma(t)+\gamma\beta e^{\lambda\tau+a+\lambda T_{\max}}\Gamma(t)\\&=\left[\lambda+\alpha+\gamma\beta e^{a+\lambda(\tau+T_{\max})}\right]\Gamma(t).\end{aligned}\tag{3.7}$$

Integrating both sides of (3.7) from $\underline{\hat{t}_k}$ to $\hat{t}_k$ leads to

$$e^{a+\lambda(\hat{t}_k-\hat{t}_{k-1})}\Gamma(\underline{\hat{t}_k})=\Gamma(\hat{t}_k^-)\leq e^{\left[\lambda+\alpha+\gamma\beta e^{a+\lambda(\tau+T_{\max})}\right](\hat{t}_k-\underline{\hat{t}_k})}\Gamma(\underline{\hat{t}_k}),$$

which indicates that

$$\hat{t}_k-\hat{t}_{k-1}>\hat{t}_k-\underline{\hat{t}_k}\geq\frac{a}{\alpha+\gamma\beta e^{a+\lambda(\tau+T_{\max})}}>0,\ k=1,2,\ldots$$

Therefore, it means that $\hat{t}_k\to\infty$ as $k\to\infty$ and the Zeno behavior is excluded.

Case ii: the impulsive instants are determined by the event-triggering instants $\left\{\hat{t}_k^*\right\}_{k=1}^{\infty}$ and the force-triggering times $\{\hat{t}_{k-1}+T_{\max}\}_{k=1}^{\infty}$. In this case, we consider $[\hat{t}_0,T]$ as the finite time interval exhibiting Zeno behavior, where T represents the Zeno time. In this case, there are infinitely many impulsive instants with $\inf\{\hat{t}_k-\hat{t}_{k-1}|k\in\mathbb{Z}_+\}=T_{\max}>0$ in the interval $\left[T-\frac{T_{\max}}{2},T\right]$. Let $\{\hat{t}_{N_0+i}\}_{i=1}^{\infty}\subset\left[T-\frac{T_{\max}}{2},T\right]$ be the subsequence of the impulsive sequence $\{\hat{t}_k\}_{k=1}^{\infty}$ such that $\hat{t}_{N_0+j}\to T$ as $j\to\infty$, where $N_0\in\mathbb{N}$, if there exists $\tau_l\in\{\hat{t}_{N_0+i}\}_{i=1}^{\infty}$ for some $l\in\mathbb{Z}_+$, then, by the definition of $T_{\max}$, there must be one and only one $\tau_l\in\{\hat{t}_{N_0+i}\}_{i=1}^{\infty}$. This indicates that for $\hat{t}_{N_0+i}=\hat{t}^*_{N_0+i}$, the impulsive instants are when $N_0+i\geq l+1$. Subsequently, by employing arguments similar to those in Theorem 1, together with Condition (2) and ETM (3.1), for $t\in\left(\hat{t}_{j-1},\hat{t}_j\right]$, and $j-1\geq l+1$, we find that

$$V(\xi(\hat{t}_j,x))=e^{a}V(\xi(\hat{t}^+_{j-1},x))\leq e^{\delta(\hat{t}_j-\hat{t}_{j-1})}V(\xi(\hat{t}^+_{j-1},x)),$$

which implies that $\hat{t}_j-\hat{t}_{j-1}\geq\frac{a}{\delta}$. It then can be deduced that

$$\hat{t}_{j+m}\geq\frac{am}{\delta}+\hat{t}_j.$$

Obviously, we can see that $\hat{t}_{j+m}\to\infty$ as $m\to\infty$, which is a contradiction of the definition of Zeno time T. Hence, the Zeno behavior is excluded.

Next, we prove the GES of DRDNNs (2.1). By Case (i), we can obtain

$$\Gamma(t)\leq\gamma V_0,\ t\in T_2,$$

that is

$$V(\xi(t,x)) \le \gamma V_0 e^{-\lambda(t-\hat{t}_0)}.$$

Since $\alpha_1 \|\xi\| \le V(\xi) \le \alpha_2 \|\xi\|$, we conclude that

$$\alpha_1 \|\xi\| \le \gamma\alpha_2 \left\|\tilde{\phi}\right\| e^{-\lambda(t-\hat{t}_0)},\ t \in T_2,$$

which implies that

$$\|\xi\| \le e^{-\lambda(t-\hat{t}_0)} \left\|\tilde{\phi}\right\| \frac{\gamma\alpha_2}{\alpha_1}.$$

Thus, the coupled system (2.6) is globally exponentially stable; consequently, the DRDNNs (2.1) are globally exponentially stable, since both the error and the observer state will be globally exponentially stable. The proof is completed.

Remark 1. Note that the instants of impulsive control input and sampling of the output of DRDNNs are determined by ETM (3.1), so the designed control scheme has a low triggering frequency and high efficiency compared with traditional time-triggered control methods [33, 35, 36]. The impulsive instants via ETM (3.1) comprise the event-triggering instants $\left\{\hat{t}_k^*\right\}_{k=1}^{\infty}$ and the force-triggering instants $\{\hat{t}_{k-1} + T_{\max}\}_{k=1}^{\infty}$. The event-triggering instants depend on the dynamics of the systems at the last triggering time and are independent of the time of the impulsive perturbation. Consequently, the designed observer is free of impulsive perturbation and can generate the impulsive control input in the neural networks. The force-triggering instants determine the frequency of the impulsive control, which is necessary to achieve the GES of the DRDNNs.

Remark 2. Compared with the existing control strategies of DRDNNs [7, 42, 43], the proposed impulsive control approach is designed on the basis of the sampled output of neural networks instead of the network state. The network state is observed by the IO (2.3) whose sampling time is determined by the ETM (3.1). In addition, the IO (2.3) is tolerant of perturbation existing in the DRDNNs (2.1). These characteristics of the proposed control approach result in low triggering frequency, high efficiency, and applicability to networks with unmeasurable states.

Theorem 2. *For given positive constants α, β, ϱ, $\tilde{\gamma} = \frac{1}{\gamma} < 1$, $T_{\max}$, $\rho_1 > 1$, and $\rho_2 < 1$, if there exist $2n \times 2n$ positive definite matrices P, Q_1, Q_2, such that $Q_2 < \beta P$, $\alpha + \frac{\beta e^{\varrho l}}{\rho_2} + \frac{\ln\tilde{\gamma}}{T_{\max}} < 0$, $\rho_1\rho_2 \le \tilde{\gamma}^2$, and the following matrix inequalities hold:*

$$\begin{bmatrix} \Pi_1 & P\widetilde{B} & P\widetilde{H} \\ * & -Q_1 & 0 \\ * & * & -Q_2 \end{bmatrix} < 0, \tag{3.8}$$

$$\begin{bmatrix} -\rho_1 P & \widetilde{M}^T P \\ * & -P \end{bmatrix} < 0, \tag{3.9}$$

$$\begin{bmatrix} -\rho_2 P & \widetilde{N}^T P \\ * & -P \end{bmatrix} < 0, \tag{3.10}$$

where $\Pi_1 = -2\delta^2\hat{D}P + P\widetilde{A}^T + \widetilde{A}P + \widetilde{L}^T Q_1\widetilde{L} - \alpha P$, $\widetilde{L} = diag\{L, L\}$ *with* $L = diag\{l_1, l_2, \dots, l_n\}$*, and then the DRDNNs* (2.1) *are globally exponentially stable via impulsive control under the following ETM:*

$$\hat{t}_k = \min\{\hat{t}_k^*, \hat{t}_{k-1} + T_{\max}\},$$
$$\hat{t}_k^* = \inf\left\{t \geq \hat{t}_{k-1} | \int_\Omega \xi^T(t,x)P\xi(t,x)dx \geq e^a \int_\Omega \xi^T(\hat{t}_{k-1}^+, x)P\xi(\hat{t}_{k-1}^+, x)dx\right\}.$$

Proof. Choose the Lyapunov function via $V(\xi(t,x)) = \int_\Omega \xi^T(t,x)P\xi(t,x)dx$. For $t \in [t_{k-1}, t_k)$, $k \in \mathbb{Z}_+$, it follows from (3.8) that

$$\begin{aligned}
D^+V(\xi(t,x)) &= 2\int_\Omega \xi^T(t,x)P\dot{\xi}(t,x)dx \\
&\leq \int_\Omega \xi^T(t,x)(-2\delta^2\hat{D}P + P\widetilde{A}^T + \widetilde{A}P + \widetilde{L}^T Q_1\widetilde{L} + P\widetilde{B}Q_1^{-1}\widetilde{B}^T P)\xi(t,x)dx \\
&\quad + \int_\Omega \xi^T(t-\tau,x)(P\widetilde{H}Q_2^{-1}\widetilde{H}^T P + \widetilde{L}^T Q_2\widetilde{L})\xi(t-\tau,x)dx \\
&\leq \alpha\int_\Omega \xi^T(t,x)P\xi(t,x)dx + \beta\int_\Omega \xi^T(t-\tau,x)P\xi(t-\tau,x)dx \\
&= \alpha V(\xi(t,x)) + \beta V(\xi(t-\tau,x)).
\end{aligned}$$

When $t = t_k, k \in \mathbb{Z}_+$, on the basis of (3.9) and Lemma 2, we can derive

$$\begin{aligned}
V(\xi(t,x)) &= \int_\Omega \xi^T(t,x)P\xi(t,x)dx = \int_\Omega \xi^T(t_k^-,x)\widetilde{M}^T P\widetilde{M}\xi(t_k^-,x)dx \\
&= \int_\Omega \xi^T(t_k^-,x)\widetilde{M}^T PP^{-1}P\widetilde{M}\xi(t_k^-,x)dx \leq \rho_1\int_\Omega \xi^T(t_k^-,x)P\xi(t_k^-,x)dx \\
&= \rho_1 V(\xi(t_k^-,x)).
\end{aligned}$$

When $t = \hat{t}_k, k \in \mathbb{Z}_+$, using the inequality (3.10) and Lemma 2, we obtain

$$\begin{aligned}
V(\xi(t,x)) &= \int_\Omega \xi^T(t,x)P\xi(t,x)dx = \int_\Omega \xi^T(\hat{t}_k^-,x)\widetilde{N}^T P\widetilde{N}\xi(\hat{t}_k^-,x)dx \\
&= \int_\Omega \xi^T(\hat{t}_k^-,x)\widetilde{N}^T PP^{-1}P\widetilde{N}\xi(\hat{t}_k^-,x)dx \leq \rho_2\int_\Omega \xi^T(\hat{t}_k^-,x)P\xi(\hat{t}_k^-,x)dx \\
&= \rho_2 V(\xi(\hat{t}_k^-,x)).
\end{aligned}$$

Therefore, all the conditions in Theorem 1 are satisfied, so the DRDNNs (2.1) are globally exponentially stable. The proof is completed.

Theorem 3. *For the given positive constants* α, β, ϱ, $\tilde{\gamma} = \frac{1}{\gamma} < 1$, $T_{\max} > 0$, $\mathcal{N} \in \mathbb{Z}_+$, $\rho_1 > 1$, *and* $\rho_2 < 1$, *if there exist* $2n \times 2n$ *positive definite matrices* $\bar{P}$, $\bar{Q}_1$, $\bar{Q}_2$, *such that* $\bar{Q}_2 < \beta\bar{P}$, $\alpha + \frac{\beta e^{\varrho l}}{\rho_{\mathcal{N}}} + \frac{\ln\tilde{\gamma}}{T_{\max}} < 0$, $\rho_1\rho_2 \leq \tilde{\gamma}^2$, *and the following linear matrix inequalities (LMIs) hold:*

$$\begin{bmatrix} \Pi_2 & \widetilde{B}\bar{P} & \widetilde{H}\bar{P} \\ * & -\bar{Q}_1 & 0 \\ * & * & -\bar{Q}_2 \end{bmatrix} < 0, \tag{3.11}$$

$$\begin{bmatrix} -\rho_1\bar{P} & \bar{P}\widetilde{M}^T \\ * & -\bar{P} \end{bmatrix} < 0, \tag{3.12}$$

$$\begin{bmatrix} -\rho_2\bar{P} & K \\ * & -\bar{P} \end{bmatrix} < 0, \tag{3.13}$$

where

$$\bar{P} = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix}^{-1}, K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix},$$

$\Pi_2 = -2\delta^2\hat{D}\bar{P} + \widetilde{A}^T\bar{P} + \bar{P}\widetilde{A} + \widetilde{L}^T\bar{Q}_1\widetilde{L} - \alpha\bar{P}$, *and* $\widetilde{L}$ *is given in Theorem 2, then the DRDNNs* (2.1) *are globally exponentially stable via impulsive control with the control gain* $\hat{E} = \Xi_1 - I_1,\ E = \Xi_1 - \Xi_3 - I_2$, $\hat{F} = -\Xi_2 C^T(CC^T)^{-1}$, *and* $F = (\Xi_4 - \Xi_2 - I_3)C^T(CC^T)^{-1}$ *under the following ETM:*

$$\begin{aligned} \hat{t}_k &= \min\{\hat{t}_k^*, \hat{t}_{k-1} + T_{\max}\}, \\ \hat{t}_k^* &= \inf\left\{ t \geq \hat{t}_{k-1} | \int_\Omega \xi^T(t,x)P\xi(t,x)dx \geq e^a \int_\Omega \xi^T(\hat{t}_{k-1}^+, x)P\xi(\hat{t}_{k-1}^+, x)dx \right\}. \end{aligned} \tag{3.14}$$

where $\Xi_1 = (T_3K_{12} + T_4K_{22})^T$, $\Xi_2 = (T_1K_{12} + T_2K_{22})^T$, $\Xi_3 = (T_3K_{11} + T_4K_{21})^T$, *and* $\Xi_4 = (T_1K_{11} + T_2K_{21})^T$.

Proof. Assume that $P = \bar{P}^{-1}$, $Q_1 = \bar{P}^{-1}\bar{Q}_1\bar{P}^{-1}$, $Q_2 = \bar{P}^{-1}\bar{Q}_2\bar{P}^{-1}$, $\hat{P} = diag\{P, P, P\}$, and $\check{P} = diag\{P, P\}$. First, by pre- and post-multiplying both sides of (3.11) by $\hat{P}$, we obtain

$$\hat{P}\begin{bmatrix} \Pi_2 & \widetilde{B}\bar{P} & \widetilde{H}\bar{P} \\ * & -\bar{Q}_1 & 0 \\ * & * & -\bar{Q}_2 \end{bmatrix}\hat{P} < 0,$$

which is equivalent to

$$\begin{bmatrix} -2\delta^2\hat{D} + P\widetilde{A}^T\bar{P} + \widetilde{A} + P\widetilde{L}^T\bar{Q}_1\widetilde{L} - \alpha & P\widetilde{B}\bar{P} & P\widetilde{H}\bar{P} \\ \widetilde{B}^T & -P\bar{Q}_1 & 0 \\ \widetilde{H}^T & 0 & -P\bar{Q}_2 \end{bmatrix}\hat{P} < 0.$$

We then have

$$\begin{bmatrix} -2\delta^2\hat{D}P + P\widetilde{A}^T + \widetilde{A}P + \widetilde{L}^T P\bar{Q}_1 P\widetilde{L} - \alpha P & P\widetilde{B} & P\widetilde{H} \\ * & -P\bar{Q}_1 P & 0 \\ * & * & -P\bar{Q}_2 P \end{bmatrix} < 0,$$

that is (3.8). Second, by pre- and post-multiplying both sides of (3.12) by $\check{P}$, we find that

$$\check{P}\begin{bmatrix} -\rho_1\bar{P} & \bar{P}\widetilde{M}^T \\ * & -\bar{P} \end{bmatrix}\check{P} < 0,$$

which is equivalent to

$$\begin{bmatrix} -\rho_1 I & \widetilde{M}^T \\ P\widetilde{M}\bar{P} & -I \end{bmatrix}\check{P} < 0.$$

We then have have

$$\begin{bmatrix} -\rho_1 P & \widetilde{M}^T P \\ P\widetilde{M} & -P \end{bmatrix} < 0,$$

that is (3.9). Similarly, we can obtain (3.10) from (3.13). Hence, all conditions in Theorem 1 are satisfied. Futhermore, it follows from $\widetilde{N}^T P = PKP$ that

$$\begin{bmatrix} I + FC - \hat{F}C & \hat{E} - E \\ -\hat{F}C & I + \hat{E} \end{bmatrix}^T = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix},$$

which implies that the gain matrices can be calculated by the following equations:

$$\begin{aligned} &I + FC - \hat{F}C = (T_1 K_{11} + T_2 K_{21})^T,\ \hat{E} - E = (T_3 K_{11} + T_4 K_{21})^T, \\ &-\hat{F}C = (T_1 K_{12} + T_2 K_{22})^T,\ I + \hat{E} = (T_3 K_{12} + T_4 K_{22})^T. \end{aligned}$$

The proof is completed.

Remark 3. Theorem 3 presents the sufficient conditions in terms of LMIs for the GES of the coupled system (2.6). The continuous dynamics and impulsive perturbation are determined by the LMIs (3.11) and (3.12), from which we can see that the continuous dynamics are unstable and that impulsive perturbation has a negative effect on the stability, since $\alpha > 0$ and $\rho_1 > 1$. To stabilize the system, the ETIC is input into the system whose control strength is determined by the LMI (3.13). So the designed ETIC not only stabilizes the unstable continuous dynamics but also regulates the impulsive perturbation.

Remark 4. Note that the DRDNNs (2.1) have unmeasurable states. The control strategies based on the feedback of the system are invalid, such as [20, 21, 47]. In this paper, the ETIC strategy is designed on the basis of the sampled output of the DRDNNs (2.1) through an impulsive observer (2.3). It does not require the network state and the continuous output of the network.

4. Numerical examples

In this section, the effectiveness of theoretical results is verified by the numerical simulations of DRDNNs whose global exponential stability is achieved by observer-based ETIC.

Example 1. Considering the two-dimensional (2D) DRDNNs (2.1) with impulsive perturbation whose parameters are given by

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ A = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},\ B = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, \tag{4.1}$$

$$H = \begin{bmatrix} 0.1 & 0.15 \\ -0.01 & 0.1 \end{bmatrix}, M = \begin{bmatrix} 0.18 & 0 \\ 0 & 0.18 \end{bmatrix},\ C = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \tag{4.2}$$

$$f(s) = \tanh(s),\ \Omega = [-1, 1]. \tag{4.3}$$

The initial conditions of such DRDNNs are chosen as $\phi_1(\theta, x) = \cos(\frac{\pi x}{2})$, $\phi_2(\theta, x) = \sin(\pi x)$ for $-\tau \leq \theta \leq 0$ and $x \in \Omega$. Figure 1 presents the spatiotemporal evolution of DRDNNs diverging on the temporal scale.

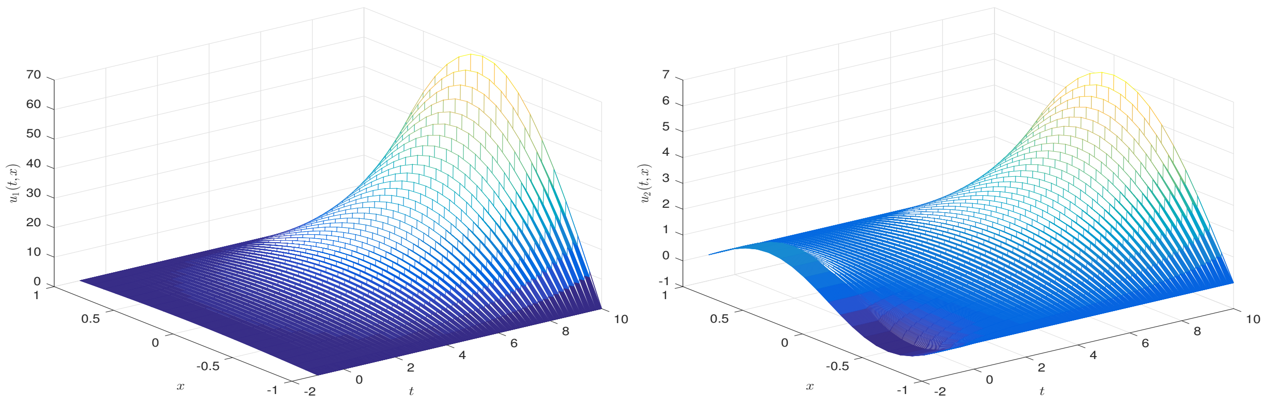

Figure 1. Spatiotemporal evolution of DRDNNs (2.1) without control.

To achieve the stability of DRDNNs, the impulsive observer (2.3) is designed to obtain the network state, and the IO-based impulsive control is input into the DRDNNs (2.1) and IO (2.3). To further determine the impulsive instants of IO and IO-based impulsive control, the parameters of the ETM (3.1) are chosen as $T_{\max} = 0.1$, $a = \ln 11 - \ln 10$, and $\bar{P}$ is the solution of the LMIs (3.11)–(3.13). In addition, if we choose $\alpha = 1$, $\beta = 1.3$, $\rho_1 = 4.8$, and $\rho_2 = 0.6$, the LMIs (3.11)–(3.13) admit a feasible solution as follows:

$$\bar{P} = \begin{bmatrix} 0.1411 & 0.0001 & -0.0005 & 0 \\ 0.0001 & 0.1440 & 0 & -0.0005 \\ -0.0005 & 0 & 0.1408 & 0.0001 \\ 0 & -0.0005 & 0.0001 & 0.1437 \end{bmatrix}.$$

In this case, the impulsive control gains are given by

$$E = \begin{bmatrix} -0.8021 & 0.0009 \\ 0.0009 & -0.7959 \end{bmatrix}, \ FC = \begin{bmatrix} -0.4004 & -0.4004 \\ -0.3973 & -0.3973 \end{bmatrix},$$
$$\hat{E} = \begin{bmatrix} -0.8030 & 0.0009 \\ 0.0009 & -0.7968 \end{bmatrix}, \ \hat{F}C = \begin{bmatrix} 0.0004 & 0.0004 \\ 0.0004 & 0.0004 \end{bmatrix},$$

According to Theorem 3, the DRDNNs (2.1) are globally exponentially stable via the IO-based ETIC. Figure 2 illustrates the spatiotemporal evolution of the observer error and DRDNNs (2.1) under control where the initial condition of the IO is given by $\phi_1(\theta, x) = 2\cos(\frac{\pi x}{2})$ and $\phi_2(\theta, x) = 2\sin(\pi x)$ for $-\tau \leq \theta \leq 0$ and $x \in \Omega$. It shows the stability of DRDNNs under the proposed IO-based ETIC strategy. To demonstrate the impulsive strength of ETIC and show the control effect, the trajectories of the Lyapunov function without control and with control is given in Figure 3. Obviously, the Lyapunov function without control diverges from the equilibrium due to the existence of impulsive perturbation, whereas the Lyapunov function decreases towards zero under the proposed ETIC method, which verifies the validity of the theoretical results.

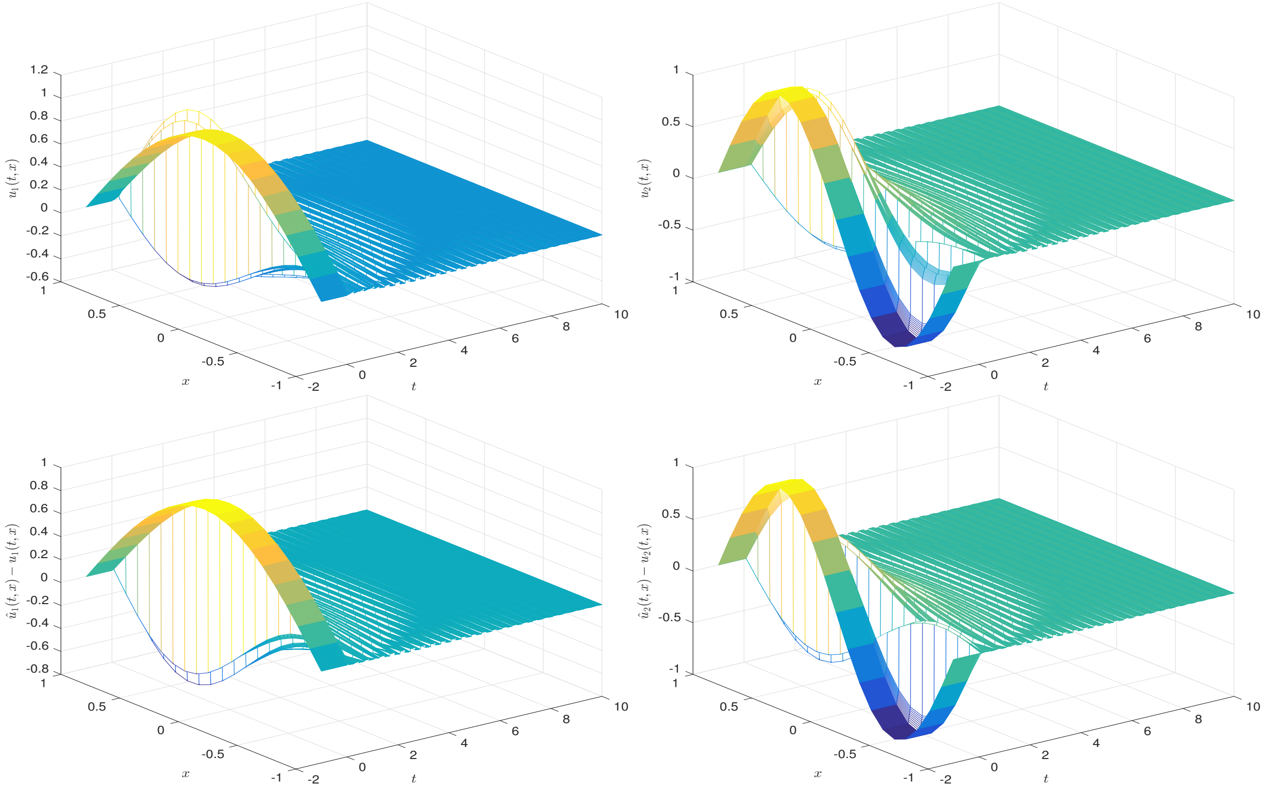

Figure 2. Spatiotemporal evolution of DRDNNs (2.1) (top) and the observer error (bottom) under the IO-based ETIC.

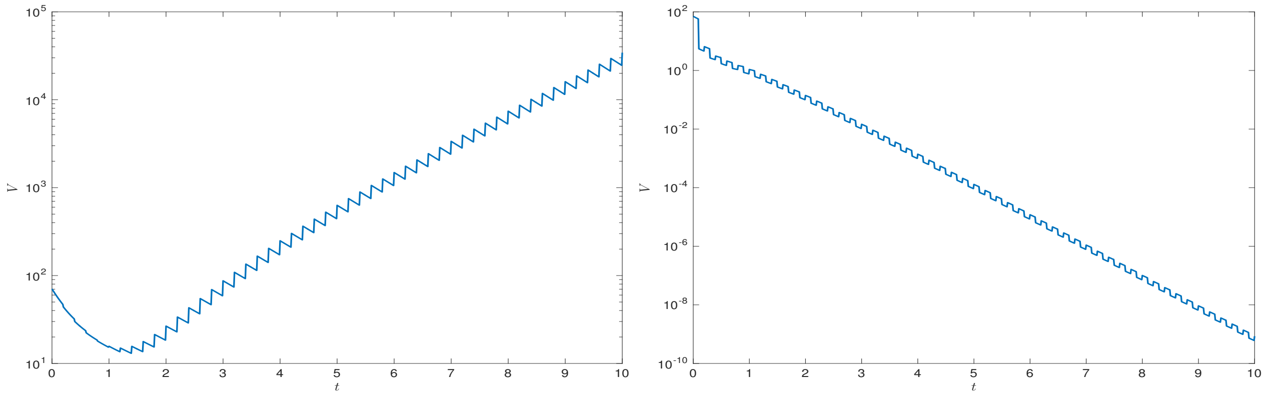

Figure 3. The trajectories of the Lyapunov function associated with DRDNNs without control (left) and with control (right).

5. Conclusions

This paper presents several criteria with Lyapunov conditions and linear matrix inequalities for the global exponential stability of delayed reaction-diffusion neural networks via observer-based ETIC. The designed control strategy is not only applicable to neural networks with unmeasurable states but also can deal with the impulsive perturbation existing in networks. It has the advantages such as low triggering frequency and high efficiency, since the impulsive control is determined by the sampling output in an event-triggered manner. Note that the ETIC approach is designed on the basis of the impulsive stabilization of unstable flow by a Lyapunov function so that the impulsive strength is constrained by an upper bound. To relax the upper bound of impulsive strength and derive more general results, the ETIC can be designed on the basis of the theory of impulsive systems with unstable jumps and unstable flow [48, 49], which will be our future work.

Author Contributions

Luyao Li: writing-original draft. Licheng Fang: writing-review and editing. Huan Liang: writing-original draft. Tengda Wei: writing-review and editing, supervision, and validation.

Use of AI tools declaration

The authors declare they have not used artificial intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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