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Research article

Average-delay impulsive control for synchronization of uncertain chaotic neural networks with variable delay impulses

Biwen Li and Yujie Liu*

Huangshi Key Laboratory of Metaverse and Virtual Simulation, School of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, China

* **Correspondence:** Email: liuyujie@stu.hbnu.edu.cn.

Abstract: This paper investigated the synchronization issue of uncertain chaotic neural networks (CNNs) using a delayed impulsive control approach. To address the disturbances caused by parameter uncertainty and the flexibility of impulsive delays, the concept of average impulsive delay (AID) and average impulsive interval (AII) were utilized to handle the delays as a whole. Under the condition that the norms of uncertain parameters are bounded, the synchronization criteria for uncertain CNNs were derived based on linear matrix inequalities (LMIs). Specifically, we relaxed the constraints on the delay in the impulsive control inputs, thus allowing it to flexibly vary without being bound by some conditions, which provides a broader applicability compared to most existing results. Additionally, the results show that delayed impulses can facilitate the synchronization of uncertain CNNs. Finally, the validity of the theoretical results was verified through a numerical example.

Keywords: delayed impulsive control; uncertain chaotic neural networks; synchronization; average impulsive delay (AID); average impulsive interval (AII)

1. Introduction

In recent decades, neural networks (NNs) have received much attention for their ability to simulate complex dynamics and are widely applied in fields such as nonlinear programming and image processing [1–7]. Some NNs exhibit a chaotic behavior, thus adding to the complexity of the system. Chaotic attractors have been discovered in three-dimensional autonomous networks [1, 2], and research indicates that chaotic behavior in the mammalian brain is related to associative memory and neuronal synchronization [3, 4]. Neuronal synchronization is crucial for information processing. Additionally, the chaotic characteristics also provide new avenues for information encryption [5], with their unpredictability and sensitivity making them effective tools for secure communication. As technology advances, the research on chaotic neural networks (CNNs) will continue to deepen and expand.

The study of chaotic synchronization has received a lot of attention due to its important applications in encryption, financial prediction, and secure communication, and other fields [5, 8–11]. Over the past two decades, impulsive control (IC) has been widely used as an effective method for system stabilization and chaotic synchronization [12–15]. It has been shown that CNNs can achieve synchronization under IC. The advantage is that discrete control inputs are only applied at specific moments, thus simplifying the controller structure and reducing communication stress and control costs. In addition, impulsive control has the several significant advantages. First, it enables fast system synchronization with less control energy, thus improving the control efficiency. Second, this control strategy is highly adaptable to the dynamic characteristics of the system and can better cope with uncertainties (such as random noise and unknown parameters) and nonlinear factors in complex systems. This efficient and economical strategy has been extensively applied in complex systems such as signal processing, financial market analyses, and neural networks [16–19]. With the technological advancements, the application prospects of impulsive control in chaotic synchronization are expected to become more broader.

In practical systems, the input of impulsive signals and the sampling of system states are usually affected by time delays. In recent years, the research on time-delay impulsive differential systems has gradually gained attention, with a continuous emergence of relevant results [16, 20–24]. For instance, researchers have explored the existence and uniqueness of solutions for time-delay impulsive systems [25, 26], stability criteria based on the Lyapunov method [20, 21], and chaos synchronization under time-delay impulsive control [14, 21, 22]. However, there are still two aspects that need further investigation in the context of time-delay impulsive synchronization control. First, the effect of impulse the control input on the synchronization performance has not been fully clarified [21]. Second, most existing studies assume that the impulse delay is either a fixed constant or subject to strict constraints, which may lead to conservatism in the research outcomes [20, 24]. In previous studies, the authors typically treated the time delay as a fixed constant or assumed that the delay varied within a specific range but did not exceed a predefined upper bound [27]. While this approach offers some conveniences in the theoretical analysis, the dynamic nature of impulsive delays (i.e., delays that vary with the timing of the impulsive) often makes such assumptions overly simplistic in practical applications. This might restrict the scope of application of the research outcomes in the real world. In reality, impulsive delays are not static but dynamically change with different impulsive triggering moments. This dynamic characteristic makes traditional fixed-delay approaches inadequate for the effective handling of impulsive delays. Therefore, it would be helpful to more effectively address the impulsive delay problem if it were treated as a whole. Recently, a new concept called the average impulsive delay (AID) has emerged. By using the idea of an AID, we can effectively mitigate the impact of varying delays at different impulsive moments and handle impulsive delays from a holistic perspective.

On the other hand, the system parameters are often affected by factors such as modeling inaccuracies, external disturbances, and measurement errors during the actual operation, thereby exhibiting uncertainty. When the model is subjected to disturbances in real-world scenarios (including nonlinear coefficients, external environmental interferences, periodic fluctuations, stochastic noise, and time-varying uncertainties), synchronization errors may fail to converge and even diverge, thus leading to system instability. Higher levels of uncertainty may trigger bifurcations (such as Hopf bifurcations), which can disrupt the synchronized state. Moreover, measurement errors and noise can further amplify these errors, thus causing the system to deviate from its intended trajectory. Therefore, designing appropriate control strategies to suppress uncertainties is crucial to ensure that the system remains stably synchronized in the presence of parameter perturbations. In recent years, the issue of parameter uncertainty has gradually become a research hotspot, which has attracted the attention of many scholars [28–30]. For example, in [28], the author proposed a method based on an improved Lyapunov-Krasovskii functional combined with linear matrix inequalities (LMIs) techniques, which established a new stability criteria for systems with parameter uncertainties. In previous studies [31], the author investigated the robust stability of nonlinear systems with mixed delays and parameter uncertainties. Moreover, due to the high sensitivity of chaotic dynamic systems to parameter changes, even slight uncertainties can have a significant effect on the overall behavior of the system. Therefore, achieving synchronization of chaotic systems under parameter uncertainties remains a challenging problem.

Based on the brief summary and discussion above, the purpose of this paper is to study the synchronization problem of CNNs with parameter uncertainties using the delayed impulse control method. The following are the main contributions of this paper:

(1) A delayed impulse control method based on an AID and an average impulsive interval (AII) is proposed to address the synchronization problem of uncertain CNNs. This method effectively relaxes the constraints on the delay of the impulse control inputs, thus making it more flexible than those in most existing studies.

(2) Several sufficient conditions for reaching the synchronization of CNNs with parametric uncertainties are derived using LMIs. In comparison to existing research, the obtained synchronization conditions are more general and significantly reduce conservatism.

(3) The effectiveness of the proposed method is verified through a theoretical analysis and numerical simulations. The results show that delayed impulses can significantly promote the synchronization of uncertain CNNs, which provide additional insights for research in this area.

Notations: Let $\Re^n$ represent the n-dimensional Euclidean space, and $\Re^{n\times m}$ represent the $n \times m$-dimensional Euclidean space. The notation $\mathfrak{Z}_+$ denotes the set of positive integers, and $\Re$ stands for the set of real numbers. The notation $\mathcal{Y} > 0$ indicates that the matrix $\mathcal{Y}$ is positive definite. The symbol $\|\cdot\|$ denotes the Euclidean norm for matrices or vectors. For any matrix Λ, Λ^T denotes its transpose and Λ^{-1} denotes its inverse. The symbol $\star$ denotes a symmetric matrix block. Matrix $\mathcal{I}$ is the identity matrix.

2. Preliminaries

2.1. Model description

Let the following uncertain CNNs serve as the drive system:

$$\dot{Q}(t) = -(\mathcal{F} + \nabla\mathcal{F}(t))\,Q(t) + (\mathcal{E} + \nabla\mathcal{E}(t))\,\varphi(Q(t)) + \mathcal{J}, \tag{2.1}$$

where $Q(t) = (Q_1(t), ..., Q_n(t))^T$ denotes the state variable of the drive system, $\dot{Q}(t)$ stands for the derivative of the state variable, $\mathcal{E}$ denotes the neuron connection weight matrix, $\mathcal{F} = \text{diag}(f_1, ..., f_n)$ denotes a diagonal matrix, where $f_i > 0, i \in \mathfrak{Z}_+$, $\nabla\mathcal{E}(t)$ and $\nabla\mathcal{F}(t)$ denote time-varying parameter uncertain matrices, $\mathcal{J}$ stands for the external input, and φ is a nonlinear activation function.

Consider the following system as the response system for uncertain CNNs (2.1) with parameter uncertainties:

$$\begin{cases}\dot{\mathcal{R}}(t) = -(\mathcal{F} + \nabla\mathcal{F}(t))\mathcal{R}(t) + (\mathcal{E} + \nabla\mathcal{E}(t))\varphi(\mathcal{R}(t)) + \mathcal{J}, & t \neq t_k,\\ \nabla\mathcal{R}(t) = \mathcal{U}(t), & t = t_k,\end{cases} \tag{2.2}$$

where $\mathcal{R}(t) = (\mathcal{R}_1(t), ...\mathcal{R}_n(t))$ stands for the state variable of system (2.2), $\nabla\mathcal{R}(t) = \mathcal{R}(t_k) - \mathcal{R}(t_k^-)$, and the impulsive sequence $\{t_k\}_{k\in\mathfrak{Z}_+}$ is strictly increasing, which satisfies $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = +\infty$. We assume that system (2.2) is right continuous at every instant $t = t_k$, (i.e., $\mathcal{R}(t_k^+) = \mathcal{R}(t)$). In this paper, we design $\mathcal{U}(t)$ as follows:

$$\mathcal{U}(t_k) = \Psi\varsigma((t_k - \delta_k)^-) - \varsigma(t_k^-), \ \ k \in \mathfrak{Z}_+, \tag{2.3}$$

where $\Psi \in \Re^{n\times n}$ is the impulsive gain matrix, and δ_k is the time delay in impulsive. $\varsigma(t) = \mathcal{R}(t) - \mathcal{Q}(t)$ is defined as the synchronization error of systems (2.1) and (2.2). Then,

$$\begin{aligned}\varsigma(t_k) &= \mathcal{R}(t_k) - \mathcal{Q}(t_k)\\ &= \mathcal{R}(t_k^-) + \nabla\mathcal{R}(t_k) - \mathcal{Q}(t_k)\\ &= \mathcal{R}(t_k^-) + \Psi\varsigma((t_k - \delta_k)^-) - \varsigma(t_k^-) - \mathcal{Q}(t_k)\\ &= \mathcal{R}(t_k^-) - \mathcal{Q}(t_k^-) + \Psi\varsigma((t_k - \delta_k)^-) - \varsigma(t_k^-)\\ &= \varsigma(t_k^-) + \Psi\varsigma((t_k - \delta_k)^-) - \varsigma(t_k^-)\\ &= \Psi\varsigma((t_k - \delta_k)^-).\end{aligned} \tag{2.4}$$

Then, the error system is obtained by subtracting system (2.1) from system (2.2) as follows:

$$\begin{cases}\dot{\varsigma}(t) &= -(\mathcal{F} + \nabla\mathcal{F}(t))\varsigma(t) + (\mathcal{E} + \nabla\mathcal{E}(t))\varphi(\varsigma(t)), \ t \neq t_k,\\ \varsigma(t_k) &= \Psi\varsigma((t_k - \delta_k)^-),\end{cases} \tag{2.5}$$

where $\varphi(\varsigma(t)) = \varphi(\mathcal{R}(t)) - \varphi(\mathcal{Q}(t))$, and we assume that the delayed impulsive δ_k satisfies $t_k < \delta_k < t_{k+1}$.

2.2. Assumptions, definitions, and lemmas

The following are some necessary assumptions, definitions, and lemmas presented in this paper.

Assumption 1 ([11]). *Suppose that the activation function φ satisfies the Lipschitz condition, that is, there exists a positive constant ι, such that*

$$\left|\varphi_j(z) - \varphi_j(r)\right| \leq \iota\,|z - r|, \tag{2.6}$$

where $z, r \in \Re$, $j = 1, ..., n$.

Definition 1 ([6]). *The method for calculating the AII $\mathcal{I}_\theta$ between two consecutive impulsive signals is defined as follows:*

$$\mathcal{I}_\theta = \limsup_{t\to+\infty} \frac{t - t_0}{\mathcal{M}_\xi(t, t_0)}, \tag{2.7}$$

where $\mathcal{M}_\xi(t, t_0)$ represents the count of impulses within the impulse sequence $\{t_k\}$ that occur in the interval (t_0, t).

Definition 2 ([7]). *The AID $\bar{\delta}$ for the impulse delay sequence $\{\delta_k\}_{k\in\mathfrak{Z}_+}$ is defined as follows:*

$$\bar{\delta} = \liminf_{t\to+\infty} \frac{\delta_1 + \delta_2 + ... + \delta_{\mathcal{M}_\xi(t,t_0)}}{\mathcal{M}_\xi(t, t_0)}, \tag{2.8}$$

where $\mathcal{M}_\xi(t, t_0)$ represents the count of impulses within the impulse sequence $\{t_k\}$ that occur in the interval (t_0, t).

Definition 3 ([22]). *Generally, if uncertain CNNs* (2.1) *and* (2.2) *satisfy the condition* $\lim_{t\to+\infty} \|\mathcal{R}(t) - Q(t)\| = 0$*, then they are considered synchronized, where $\mathcal{R}(t)$ and $Q(t)$ represent the solutions to systems* (2.1) *and* (2.2)*, respectively. Consequently, by the definition of synchronization, if the system error $\varsigma(t) = \mathcal{R}(t) - Q(t)$ satisfies the condition*

$$\lim_{t\to+\infty} \|\varsigma(t)\| = 0,$$

then, we can conclude that systems (2.1) *and* (2.2) *have achieved synchronization.*

Lemma 1 ([11]). *Let $\varepsilon > 0$; for any matrix $\mathcal{G}_1, \mathcal{G}_2 \in \Re^{n\times n}$, there exist a positive define matrix $\mathcal{D} > 0$, such that*

$$\mathcal{G}_1^T\mathcal{G}_2 + \mathcal{G}_2^T\mathcal{G}_1 \le \varepsilon\mathcal{G}_1^T\mathcal{D}\mathcal{G}_1 + \frac{1}{\varepsilon}\mathcal{G}_2^T\mathcal{D}^{-1}\mathcal{G}_2.$$

Lemma 2 ([32]). *Let u_1 and u_2 be real vectors with appropriate dimensions, and $\mathcal{S}$ and $\nabla\mathcal{A}(t)$ be real matrices with the appropriate dimensions, $\|\nabla\mathcal{A}(t)\| \le \gamma$; for $\forall\alpha > 0$, the following inequality holds:*

$$2u_1^T\mathcal{S}^T\nabla\mathcal{A}(t)u_2 \le \alpha u_1^T\mathcal{S}^T\mathcal{S}u_1 + \frac{\gamma^2}{\alpha}u_2^Tu_2.$$

Lemma 3 ([33]). *Give the following matrix:*

$$\mathrm{E} = \begin{pmatrix} \mathrm{E}_{11} & \mathrm{E}_{12} \\ \mathrm{E}_{21} & \mathrm{E}_{22} \end{pmatrix},$$

where $\mathrm{E}_{11}^T = \mathrm{E}_{11}, \mathrm{E}_{12}^T = \mathrm{E}_{21}, \mathrm{E}_{22}^T = \mathrm{E}_{22}$; then, $\mathrm{E} < 0$ is equivalent to any of the following conditions:

(1) $\mathrm{E}_{11} < 0$, *and* $\mathrm{E}_{22} - \mathrm{E}_{12}^T\mathrm{E}_{11}^{-1}\mathrm{E}_{12} < 0$.
(2) $\mathrm{E}_{22} < 0$, *and* $\mathrm{E}_{11} - \mathrm{E}_{12}\mathrm{E}_{22}^{-1}\mathrm{E}_{12}^T < 0$.

Remark 1. *Definition 1 and [34] define the AII in different ways. Definition 1 in [6] uses the upper limit $\mathcal{I}_\theta = \limsup_{t\to+\infty} \frac{t-t_0}{\mathcal{M}_\xi(t_0,t)}$ to describe the long-term average behavior of the impulsive sequence. It is widely applicable, especially for analyzing non-periodic or highly stochastic impulsive sequences. In contrast, reference [34] constrains the number of impulses in any time interval using the upper and lower bounds $\frac{t_2-t_1}{\mathcal{I}_\theta} - \mathcal{M}_0 \le \mathcal{M}_\xi(t_2, t_1) \le \frac{t_2-t_1}{\mathcal{I}_\theta} + \mathcal{M}_0$. Although this provides a more precise local control, it is more conservative and has a narrower scope of application. Definition 1 is less conservative because it imposes weaker constraints on the impulsive sequence, thus making it applicable for a broader range of scenarios while maintaining the universality of the theoretical analysis. It is particularly well-suited to study the long-term behavior and global stability of the systems.*

Remark 2. *Definitions 1 and 2 describe the characteristics of the impulsive sequence from the different perspectives of impulse intervals and impulse delays, respectively. Note that since both AII and AID are related to $\mathcal{M}_{\xi}(t, t_0)$, we can derive the following relationship:*

$$\liminf_{t\to+\infty} \frac{\delta_1 + \delta_2 + \cdots + \delta_{\mathcal{M}_{\xi}(t,t_0)}}{t - t_0} = \frac{\bar{\delta}}{\mathcal{I}_{\theta}}.$$

Remark 3. *We define two sets of time sequences $\Im$ and $\aleph$, which correspond to $\{t_k\}_{k\in\mathfrak{Z}_+}$ and $\{\delta_k\}_{k\in\mathfrak{Z}_+}$ that satisfy* (2.7) *and* (2.8)*, respectively. To simplify the expression, we denote the collection of sequences that satisfy $\{t_k\}_{k\in\mathfrak{Z}_+} \in \Im$ and $\{\delta_k\}_{k\in\mathfrak{Z}_+} \in \aleph$ as $\mathcal{H}(\{t_k\}, \{\delta_k\})_{k\in\mathfrak{Z}_+}$. We denote $(\Psi, \mathcal{H})$ as an impulsive controller aimed at synchronizing the uncertain CNNs* (2.1) *and* (2.2)*. Additionally, we term this control strategy as the average-delay impulsive control.*

3. Main results

In this section, the average-delay impulsive control method will be used to study the chaotic synchronization between systems (2.1) and (2.2). By the definitions of AII and AID , we have relaxed the restrictions on impulse delay and have derived the synchronization criteria between systems (2.1) and (2.2) based on LMIs. The results show that impulsive delay promotes the synchronization of the two uncertain CNNs.

Theorem 1. *Assume that Assumption 1 holds and under the average-delayed impulse controller $(\Psi, \mathcal{H})$. If there exists positive constants $\pi > 0$, $\lambda > 0$, and positive definite matrices $\mathcal{N} > 0, \mathcal{D} > 0$ such that*

$$\begin{pmatrix} -\mathcal{F}^T\mathcal{N} - \mathcal{N}\mathcal{F} + (\alpha_1 + \alpha_2)\mathcal{N}^2 + \frac{\gamma_1^2}{\alpha_1}\mathcal{I}_n + \frac{\gamma_2^2}{\alpha_2}\mathcal{L}^2 + \mathcal{L}\mathcal{D}\mathcal{L} + \mathcal{N}\mathcal{E}\mathcal{D}^{-1}\mathcal{E}^T\mathcal{N} - \pi\mathcal{N} & \mathcal{E}^T\mathcal{N} \\ \star & -\mathcal{D} \end{pmatrix} < 0, \tag{3.1}$$

$$\begin{pmatrix} -\lambda\mathcal{N} & \Psi^T\mathcal{N} \\ \star & -\mathcal{N} \end{pmatrix} < 0, \tag{3.2}$$

and

$$\ln\lambda + \pi(\mathcal{I}_{\theta} - \bar{\delta}) < 0, \tag{3.3}$$

where $\mathcal{L} = diag(\iota_1, ..., \iota_n)$, then, uncertain CNNs (2.1) *and* (2.2) *can reach synchronization.*

Proof. We choose a nonnegative Lyapunov function for the error system (2.5):

$$\mathcal{V}(t) = \varsigma^T(t)\mathcal{N}\varsigma(t), \tag{3.4}$$

where $\mathcal{N} > 0$; then, based on Lemmas 1 and 2, for $t \in [t_{k-1}, t_k)$, $k \in \mathfrak{Z}_+$, the right hand derivative of the Lyapunov function $\mathcal{V}(t)$ along the error system (2.5) is calculated as follows:

$$\begin{aligned} \dot{\mathcal{V}}(t) &= \dot{\varsigma}^T(t)\mathcal{N}\varsigma(t) + \varsigma^T(t)\mathcal{N}\dot{\varsigma}(t) \\ &= \left[-(\mathcal{F} + \nabla\mathcal{F}(t))\varsigma(t) + (\mathcal{E} + \nabla\mathcal{E}(t))\varphi(\varsigma(t))\right]^T\mathcal{N}\varsigma(t) \end{aligned}$$

$$
\begin{aligned}
&+\varsigma^{T}(t)\mathcal{N}\left[-(\mathcal{F}+\nabla\mathcal{F}(t))\varsigma(t)+(\mathcal{E}+\nabla\mathcal{E}(t))\varphi(\varsigma(t))\right]\\
&=\varsigma^{T}(t)\left[-\mathcal{F}^{T}\mathcal{N}-\mathcal{N}\mathcal{F}\right]\varsigma(t)+\varsigma^{T}(t)\left[-(\nabla\mathcal{F}(t))^{T}\mathcal{N}-\mathcal{N}\nabla\mathcal{F}(t)\right]\varsigma(t)+\varphi^{T}(\varsigma(t))\mathcal{E}^{T}\mathcal{N}\varsigma(t)\\
&+\varsigma^{T}(t)\mathcal{N}\mathcal{E}\varphi(\varsigma(t))+\varphi^{T}(\varsigma(t))(\nabla\mathcal{E}(t))^{T}\mathcal{N}\varsigma(t)+\varsigma^{T}(t)\mathcal{N}\nabla\mathcal{E}(t)\varphi(\varsigma(t)).
\end{aligned}
\tag{3.5}
$$

According to (2.6), Lemmas 1 and 2, we can derive the following:

$$
\varsigma^{T}(t)\left[-(\nabla\mathcal{F}(t))^{T}\mathcal{N}-\mathcal{N}(\nabla\mathcal{F}(t))\right]\varsigma(t)=-2\varsigma^{T}(t)\mathcal{N}\nabla\mathcal{F}(t)\varsigma(t)\leq\alpha_{1}\varsigma^{T}(t)\mathcal{N}^{2}\varsigma(t)+\frac{\gamma_{1}^{2}}{\alpha_{1}}\varsigma^{T}(t)\varsigma(t),
$$

$$
\begin{aligned}
\varphi^{T}(\varsigma(t))\mathcal{E}^{T}\mathcal{N}\varsigma(t)+\varsigma^{T}(t)\mathcal{N}\mathcal{E}\varphi(\varsigma(t))&\leq\varphi^{T}(\varsigma(t))\mathcal{D}\varphi(\varsigma(t))+\varsigma^{T}(t)\mathcal{N}\mathcal{E}\mathcal{D}^{-1}\mathcal{E}^{T}\mathcal{N}\varsigma(t)\\
&\leq\varsigma^{T}(t)\mathcal{L}\mathcal{D}\mathcal{L}\varsigma(t)+\varsigma^{T}(t)\mathcal{N}\mathcal{E}\mathcal{D}^{-1}\mathcal{E}^{T}\mathcal{N}\varsigma(t),
\end{aligned}
$$

$$
\begin{aligned}
\varphi^{T}(\varsigma(t))(\nabla\mathcal{E}(t))^{T}\mathcal{N}\varsigma(t)+\varsigma^{T}(t)\mathcal{N}\nabla\mathcal{E}(t)\varphi(\varsigma(t))&\leq\alpha_{2}\varsigma^{T}(t)\mathcal{N}^{2}\varsigma(t)\\
&+\frac{1}{\alpha_{2}}\varphi^{T}(\varsigma(t))(\nabla\mathcal{E}(t))^{T}(\nabla\mathcal{E}(t))\varphi(\varsigma(t))\\
&\leq\alpha_{2}\varsigma^{T}(t)\mathcal{N}^{2}\varsigma(t)+\frac{\gamma_{2}^{2}}{\alpha_{2}}\varsigma^{T}(t)\mathcal{L}^{2}\varsigma(t).
\end{aligned}
$$

From Lemma 3 and (3.1), one can obtain the following:

$$
-\mathcal{F}^{T}\mathcal{N}-\mathcal{N}\mathcal{F}+(\alpha_{1}+\alpha_{2})\mathcal{N}^{2}+\frac{\gamma_{1}^{2}}{\alpha_{1}}\mathcal{I}_{n}+\frac{\gamma_{2}^{2}}{\alpha_{2}}\mathcal{L}^{2}+\mathcal{L}\mathcal{D}\mathcal{L}+\mathcal{N}\mathcal{E}\mathcal{D}^{-1}\mathcal{E}^{T}\mathcal{N}-\pi\mathcal{N}<0. \tag{3.6}
$$

Then, by substituting the result of the simplification above into Eq (3.5), one can obtain the following:

$$
\begin{aligned}
\dot{\mathcal{V}}(t)&\leq\varsigma^{T}(t)\left(-\mathcal{F}^{T}\mathcal{N}-\mathcal{N}\mathcal{F}+(\alpha_{1}+\alpha_{2})\mathcal{N}^{2}+\frac{\gamma_{1}^{2}}{\alpha_{1}}\mathcal{I}_{n}+\frac{\gamma_{2}^{2}}{\alpha_{2}}\mathcal{L}^{2}+\mathcal{L}\mathcal{D}\mathcal{L}+\mathcal{N}\mathcal{E}\mathcal{D}^{-1}\mathcal{E}^{T}\mathcal{N}-\pi\mathcal{N}\right)\varsigma(t)\\
&\leq\varsigma^{T}(t)\pi\mathcal{N}\varsigma(t)\\
&\leq\pi\mathcal{V}(t).
\end{aligned}
\tag{3.7}
$$

From (3.7), we derive the following:

$$
\mathcal{V}(t)\leq\mathcal{V}(t_{k-1})e^{\pi(t-t_{k-1})}. \tag{3.8}
$$

Additionally, from (3.2), it can be derived that at the impulsive instant $t=t_k$, the following equation holds:

$$
\begin{aligned}
\mathcal{V}(t_k)&=\varsigma^{T}(t_k)\mathcal{N}\varsigma(t_k)\\
&\leq\varsigma^{T}\left((t_k-\delta_k)^{-}\right)\Psi^{T}\mathcal{N}\Psi\varsigma\left((t_k-\delta_k)^{-}\right)\\
&=\varsigma^{T}\left((t_k-\delta_k)^{-}\right)\Psi^{T}\mathcal{N}\mathcal{N}^{-1}\mathcal{N}\Psi\varsigma\left((t_k-\delta_k)^{-}\right)\\
&<\varsigma^{T}\left((t_k-\delta_k)^{-}\right)\lambda\mathcal{N}\varsigma\left((t_k-\delta_k)^{-}\right)\\
&=\lambda\mathcal{V}\left((t_k-\delta_k)^{-}\right).
\end{aligned}
\tag{3.9}
$$

The following analysis is based on (3.8) and (3.9).

Next, we will prove that the following inequality holds, for $\forall p \in \mathfrak{Z}_{+}, t \in \left[t_{p-1}, t_p\right)$:

$$\mathcal{V}(t) \leq \lambda^{p-1} \exp\left[-\pi \sum_{j=0}^{p-1} \delta_j\right] e^{\pi(t-t_0)} \mathcal{V}_0, \tag{3.10}$$

where $\mathcal{V}_0 = \varsigma(t_0)^T \mathcal{N} \varsigma(t_0)$, and $\varsigma(t) = \varsigma(t, t_0, \varsigma(t_0))$ represents the solution of the error system (2.5) through the initial value $\varsigma(t_0)$. Note that since δ_0 stands for the impulsive delay at the impulsive moment $t = 0$, there is no control input at $t = 0$. Therefore, when $j = 0$, $\delta_0 = 0$.

For $t \in [t_0, t_1)$, the following is derived from (3.8),

$$\mathcal{V}(t) \leq e^{\pi(t-t_0)} \mathcal{V}_0. \tag{3.11}$$

Therefore, it can be verified that (3.10) holds when $p = 1$. Next, we assume that (3.10) also holds for $p = q$, i.e., $\forall q \in \mathfrak{Z}_{+}, \forall t \in \left[t_{q-1}, t_q\right)$,

$$\mathcal{V}(t) \leq \lambda^{q-1} \exp\left[-\pi \sum_{j=0}^{q-1} \delta_j\right] e^{\pi(t-t_0)} \mathcal{V}_0, \tag{3.12}$$

when $p = q + 1$, from $t_{k-1} < \delta_k < t_k, k \in \mathfrak{Z}_{+}$; then, $t_q - \delta_q \in \left[t_{q-1}, t_q\right)$, and according to (3.9) and (3.10),

$$\begin{aligned} \mathcal{V}\left(t_q\right) &= \lambda \mathcal{V}\left(\left(t_q - \delta_q\right)^{-}\right) \\ &\leq \lambda \cdot \lambda^{q-1} \exp\left[-\pi \sum_{j=0}^{q-1} \delta_j\right] e^{\pi\left(t_q - \delta_q - t_0\right)} \mathcal{V}_0 \\ &= \lambda^{q} \exp\left[-\pi \sum_{j=0}^{q} \delta_j\right] e^{\pi\left(t_q - t_0\right)} \mathcal{V}_0. \end{aligned} \tag{3.13}$$

For $t \in \left[t_q, t_{q+1}\right)$, according to (3.8), we can get that $\mathcal{V}(t) \leq \mathcal{V}\left(t_q\right) e^{\pi\left(t-t_q\right)}$; substituting (3.13) into it, we have the following:

$$\begin{aligned} \mathcal{V}(t) &\leq \lambda^{q} \exp\left[-\pi \sum_{j=0}^{q} \delta_j\right] e^{\pi\left(t_q - t_0\right)} \mathcal{V}_0 \cdot e^{\alpha\left(t-t_q\right)} \\ &= \lambda^{q} \exp\left[-\pi \sum_{j=0}^{q} \delta_j\right] e^{\pi(t-t_0)} \mathcal{V}_0, \end{aligned} \tag{3.14}$$

i.e., when $p = q + 1$, (3.10) also holds. Therefore, through mathematical induction, we can deduce that (3.10) holds, for $\forall p \in \mathfrak{Z}_{+}$. Furthermore, since the condition $\{t_k\}_{k \in \mathfrak{Z}_{+}} \in \mathfrak{I}$ and $\{\delta_k\}_{k \in \mathfrak{Z}_{+}} \in \aleph$, it follows that

$$\mathcal{V}(t) \leq \lambda^{\mathcal{M}_{\xi}(t,t_0)} \exp\left[-\pi \sum_{j=0}^{\mathcal{M}_{\xi}(t,t_0)} \delta_j\right] e^{\pi(t-t_0)} \mathcal{V}_0$$

$$= \exp\left[\mathcal{M}_\xi(t,t_0)\ln\lambda - \pi\sum_{j=0}^{\mathcal{M}_\xi(t,t_0)}\delta_j\right]e^{\pi(t-t_0)}\mathcal{V}_0$$
$$= \exp\left[\left(\frac{\mathcal{M}_\xi(t,t_0)\ln\lambda - \pi\sum_{j=0}^{\mathcal{M}_\xi(t,t_0)}\delta_j}{\mathcal{M}_\xi(t,t_0)}\Big/\frac{t-t_0}{\mathcal{M}_\xi(t,t_0)}\right)\times(t-t_0)\right]e^{\pi(t-t_0)}\mathcal{V}_0$$
$$= \exp\left[\left(\frac{\mathcal{M}_\xi(t,t_0)\ln\lambda - \pi\sum_{j=0}^{\mathcal{M}_\xi(t,t_0)}\delta_j}{\mathcal{M}_\xi(t,t_0)}\Big/\frac{t-t_0}{\mathcal{M}_\xi(t,t_0)}+\pi\right)\times(t-t_0)\right]\mathcal{V}_0. \tag{3.15}$$

Then, based on the definitions of the AII, AID, and (3.3), we can obtain the following:

$$\limsup_{t\to+\infty}\left[\left(\frac{\mathcal{M}_\xi(t,t_0)\ln\lambda - \pi\sum_{j=0}^{\mathcal{M}_\xi(t,t_0)}\delta_j}{\mathcal{M}_\xi(t,t_0)}\Big/\frac{t-t_0}{\mathcal{M}_\xi(t,t_0)}+\pi\right)\times(t-t_0)\right]\mathcal{V}_0$$
$$\leq \frac{\ln\lambda - \pi\bar{\delta} + \pi\mathcal{I}_\theta}{\mathcal{I}_\theta} < 0. \tag{3.16}$$

Then, it follows from (3.15) and (3.16) that

$$\lim_{t\to+\infty}\mathcal{V}(t) = 0.$$

Taking (3.4) into account, it follows that $\lim_{t\to+\infty}\varsigma(t)^T\mathcal{N}\varsigma(t) = 0$. As a result, the uncertain CNNs systems (2.1) and (2.2) achieves synchronization. The proof is complete.

Remark 4. *In previous studies (such as references [14, 21, 22]), the time delay of the impulsive signal (i.e., signal delay) is usually supposed that the impulse delay is a fixed constant or subject to strict constraints. This assumption makes the existing research criteria relatively conservative in dealing with impulsive time delay problems. However, in practical systems, the time delay of the impulsive signal is not always fixed or strictly limited; on the contrary, it can vary flexibly. This flexibility is common in practical applications, though previous studies have not fully considered this aspect. Based on this, this paper employs the concept of an AID. This means that the time delay in the impulse control input can flexibly exist between two consecutive impulse moments. Compared with the AID concept in reference [7], this paper considers a more generalized lower-bound form of the AID concept. Additionally, this paper has employed the LMIs method to verify and analyze the effectiveness of the proposed controller.*

Remark 5. *To achieve synchronization between uncertain CNNs* (2.1) *and* (2.2)*, impulsive control inputs need to be continuously implemented. This means that to ensure these two systems can synchronize, $\mathcal{I}_\theta$ should not be set too large. According to condition* (3.3)*, we know that $\mathcal{I}_\theta < \bar{\delta} - \frac{\ln\lambda}{\pi}$. Because of the time delay in the impulse control inputs, we find that $\mathcal{I}_\theta$ can be larger than the value allowed without delay (i.e., $\mathcal{I}_\theta < -\frac{\ln\lambda}{\pi}$). Intuitively, it is observed that within each impulse interval (t_{k-1}, t_k), the error $\varsigma(t)$ may increase due to the absence of impulsive control inputs. This implies that under the condition $t_{k-1} < t_k - \delta_k \leq t_k$, the inequality $\|\varsigma((t_k-\delta_k)^-)\| \leq \|\varsigma(t_k^-)\|$ might be satisfied. Therefore, taking this into account, we can infer that the time delay of the impulse control input may positively affect the synchronization of systems* (2.1) *and* (2.2) *(this will be verified in the numerical examples section).*

In particular, if both the impulse interval and the delay of the impulse control input are set to constant values, (i.e., $t_k - t_{k-1} = \mathcal{I}^{\triangleright}$ and $\delta_k = \delta^{\triangleright}$, $\forall k \in \mathfrak{Z}_+$), then the following corollary can be drawn.

Corollary 1. *Suppose Assumption 1 is valid. Set the impulse interval* $\mathcal{I}^{\triangleright} > 0$*, the impulse delay* $\delta^{\triangleright} \geq 0$*, and the constants* $\pi > 0, \lambda > 0$*; assuming there exists positive definite matrices* $\mathcal{N} > 0$ *and* $\mathcal{D} > 0$*, which satisfy*

$$\Psi^T \mathcal{N} \Psi - e^{-\pi(\mathcal{I}^{\triangleright} - \delta^{\triangleright})} \mathcal{N} < 0,$$

and (3.1) *and* (3.2) *hold, then*

$$\Psi^T \mathcal{N} \mathcal{N}^{-1} \mathcal{N} \Psi - \lambda \mathcal{N} < 0,$$

and

$$-\mathcal{F}^T \mathcal{N} - \mathcal{N}\mathcal{F} + (\alpha_1 + \alpha_2) \mathcal{N}^2 + \frac{\gamma_1^2}{\alpha_1} \mathcal{I}_n + \frac{\gamma_2^2}{\alpha_2} \mathcal{L}^2 + \mathcal{L}\mathcal{D}\mathcal{L} + \mathcal{N}\mathcal{E}\mathcal{D}^{-1}\mathcal{E}^T \mathcal{N} - \pi \mathcal{N} < 0,$$

will hold. Then uncertain CNNs (2.1) *and* (2.2) *can reach synchronization under the impulse controller* $(\Psi, \{\mathcal{I}^{\triangleright}, \delta^{\triangleright}\})$.

4. Numerical example

In this section, the validity of the theoretical results under the proposed average-delay impulsive control strategy is verified by providing an example. We select the three-neuron uncertain CNNs (4.1) as the drive system and (4.2) as the corresponding response system. Their forms are as follows:

$$\dot{Q}(t) = -(\mathcal{F} + \nabla\mathcal{F}(t)) Q(t) + (\mathcal{E} + \nabla\mathcal{E}(t)) \varphi(Q(t)) + \mathcal{J}. \tag{4.1}$$

The corresponding response system is as follows:

$$\begin{cases} \dot{\mathcal{R}}(t) = -(\mathcal{F} + \nabla\mathcal{F}(t)) \mathcal{R}(t) + (\mathcal{E} + \nabla\mathcal{E}(t)) \varphi(\mathcal{R}(t)) + \mathcal{J}, & t \neq t_k, \\ \nabla\mathcal{R}(t) = \mathcal{U}(t), & t = t_k, \end{cases} \tag{4.2}$$

where the uncertain CNNs (4.1) has the initial condition $Q_0 = (-0.75, -1.5, 0.3)$. The behaviors of the uncertain CNNs (4.1) with the initial value Q_0 is shown in Figure 1, and the evolution of error norm $\|\varsigma(t)\|$ without control is shown in Figure 2.

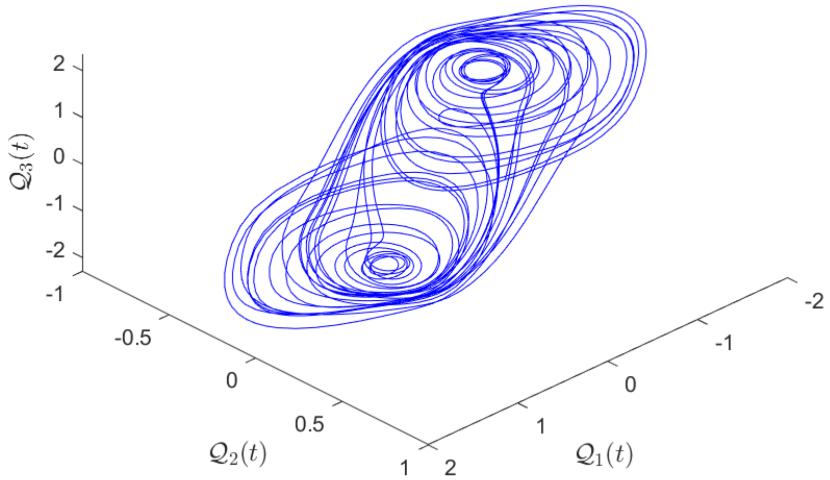

Figure 1. The behaviors of the uncertain CNNs (4.1).

The response system (4.2) has the initial condition $\mathcal{R}_0 = (-2.5, 2.1, 2.6)$, and the nonlinear activation function

$$\varphi(Q(t)) = [\varphi_1(Q_1(t)), \varphi_2(Q_2(t)), \varphi_3(Q_3(t))]^T,\ Q(t) \in \Re^3,$$

takes the form

$$\varphi_i(Q_i(t)) = \frac{1}{2}(|Q_i(t) + 1| - |Q_i(t) - 1|),\quad i = 1, 2, 3.$$

Moreover,

$$\mathcal{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \mathcal{E} = \begin{bmatrix} 1.2 & -1.6 & 0 \\ 1.2 & 1 & 0.9 \\ 0 & 2.2 & 1.5 \end{bmatrix},$$

$$\nabla\mathcal{F}(t) = 0.13\cos(t)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \nabla\mathcal{E}(t) = 0.1\cos(t)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},\ \mathcal{J} = 0.$$

It is evident that, the norm of the time-varying parameter uncertainty matrices $\nabla\mathcal{F}(t)$ and $\nabla\mathcal{E}(t)$ are bounded, under the above conditions, (i.e., $\|\nabla\mathcal{F}(t)\| \le 0.13$, $\|\nabla\mathcal{E}(t)\| \le 0.2$). Additionally, we can calculate to obtain $\mathcal{L} = \mathrm{diag}(\iota_1, \iota_2, \iota_3) = \mathcal{I}$. Select the parameters $\pi = 3.9, \lambda = 0.4$, and the impulsive gain matrix $\Psi = 0.6\mathcal{I}$. Moreover, with the help of MATLAB's Toolbox LMI, it can be verified that LMIs (3.1) and (3.2) in Theorem 1 are satisfied. Additionally, the following are the relevant feasible solutions:

$$\mathcal{N} = \begin{bmatrix} 0.3075 & 0.0070 & 0.0472 \\ 0.0070 & 0.3326 & -0.0206 \\ 0.0472 & -0.0206 & 0.2626 \end{bmatrix},$$

$$\mathcal{D} = \begin{bmatrix} 0.8178 & 0.0065 & 0.1997 \\ 0.0065 & 0.9436 & -0.0333 \\ 0.1997 & -0.0333 & 0.6372 \end{bmatrix}.$$

In this simulation example, we set the impulse sequence $t_k = 5k$, $k \in \mathfrak{Z}_+$. When there is a delay in the impulsive control input, the delay $\bar{\delta}$ needs to satisfy the condition $\bar{\delta} > \mathcal{I}_\theta + (\ln \lambda)/\pi \approx 4.77$. Under this condition, the drive-response systems (4.1) and (4.2) can achieve a synchronized state, which can be observed from the evolutionary trajectory of the error system in Figure 3, and the red solid line in Figure 4.

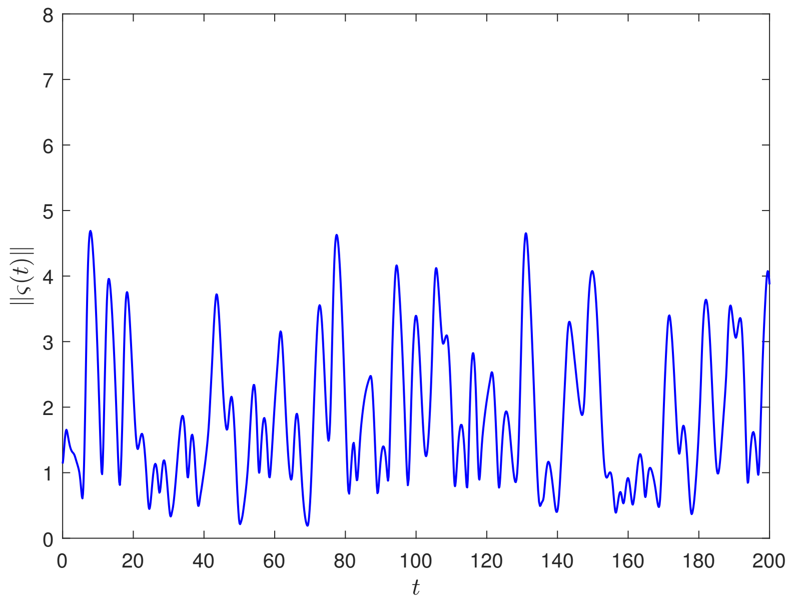

Figure 2. Evolution of the error norm $\|\varsigma(t)\|$ without control.

Specifically, the designed delayed impulsive sequence is $\delta_k = 4.85 + ((-1)^k)/20 + 0.04|\cos(k^2)|$, and it can be verified that $\bar{\delta} \geq 4.85$, which satisfies the condition (3.3) in Theorem 1. However, when there is no delay in the impulsive sequence, that is, $\delta_k = 0$, the uncertain CNNs (4.1) and (4.2) cannot achieve synchronization, which can be observed from the blue dashed line in Figure 4. Under this condition, we can calculate that $\pi(\mathcal{I}_\theta - \bar{\delta}) + \ln \lambda \nless 0$ does not satisfy the condition (3.3) in Theorem 1, which further confirms the positive effect of a delayed impulsive on achieving synchronization.

Moreover, if other parameters remain constant, then increasing the impulse control intensity results in a faster convergence rate of the system error. Similarly, if the impulse delay is increased while still satisfying (3.3) in Theorem 1, then the rate of stabilization of the error system will also be faster, as shown in Figure 5 ($\hat{\Psi} = \text{diag}\{0.4, 0.4, 0.4\}, \hat{\delta}_k = 4.86 + ((-1)^k)/20 + 0.04|\cos(k^2)|$). On the contrary, reducing the impulse gain and decreasing the impulse delay may lead to an instability of the error system (i.e., divergence), as shown in Figure 6 ($\check{\Psi} = \text{diag}\{0.67, 0.6, 0.6\}, \check{\delta}_k = 3 + ((-1)^k)/20 + 0.04|\cos(k^2)|$). This demonstrates that the control strategy designed in this paper is effective.

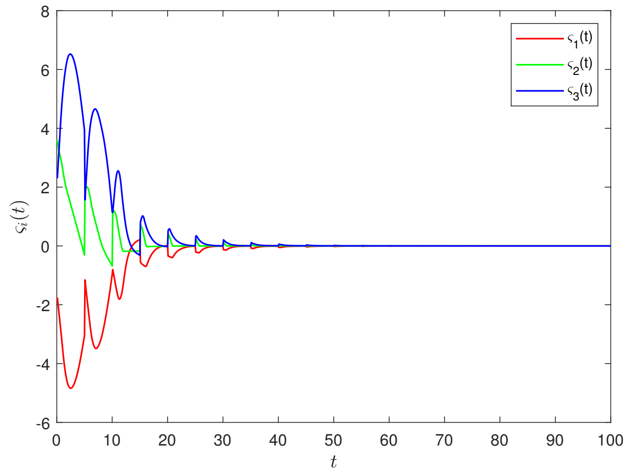

Figure 3. Evolution of the error $\varsigma_i(t)$ under average-delay impulsive control.

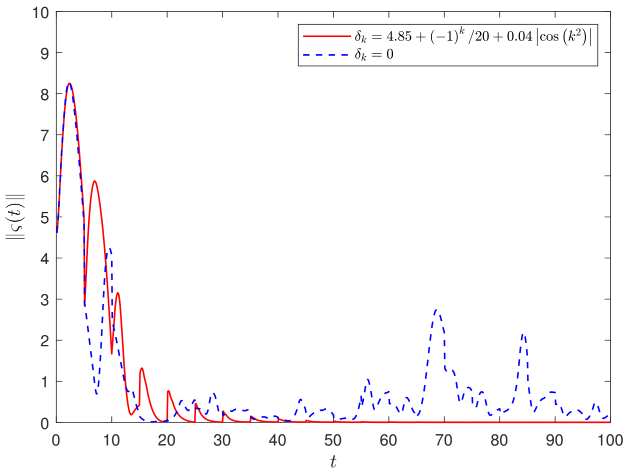

Figure 4. Evolution of the error norm $||\varsigma(t)||$ under different δ_k.

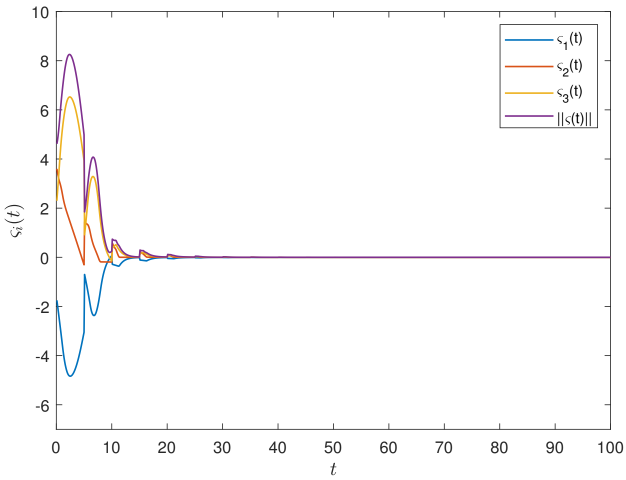

Figure 5. Evolution of the error under the control parameter $\hat{\Psi}$ and $\hat{\delta}_k$.

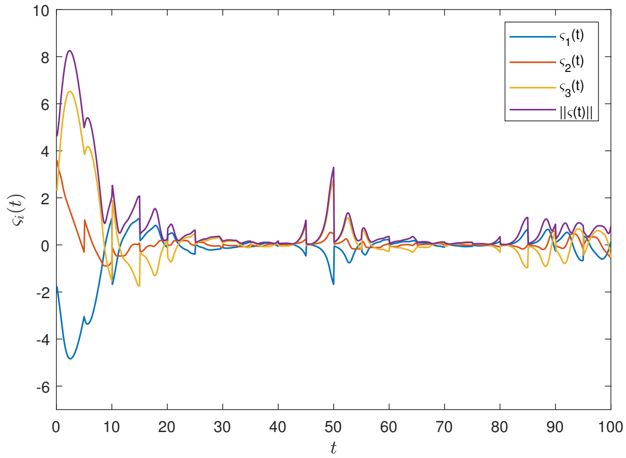

Figure 6. Evolution of the error under the control parameter $\check{\Psi}$ and $\check{\delta}_k$.

5. Conclusions

In this paper, the synchronization problem of uncertain CNNs was investigated via the delayed impulsive control technology. To address the variable delays in the impulsive control inputs, an average-delay impulsive control strategy was utilized. By utilizing the concepts of AII and AID, some sufficient conditions were derived based on LMIs for the synchronization of uncertain CNNs. The results of the study show that the presence of impulsive delays can effectively prompt the stabilization of the error system. Future research will consider the synchronization problem of delayed uncertain CNNs under the delayed impulse control.

Authors contribution

Biwen Li: Sriting–review, supervision, validation and formal analysis; Yujie Liu: Writing–review, writing–original draft, methodology, editing and software. Both authors have read and approved the final version of the manuscript for publication.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare there is no conflict of interest.

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