



*Theory article*

## **An improved sampled-data control for a nonlinear dynamic positioning ship with Takagi-Sugeno fuzzy model**

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**Abstract:** This article considered the sampled-data control issue for a dynamic positioning ship (DPS) with the Takagi-Sugeno (T-S) fuzzy model. By introducing new useful terms such as second-order term of time, an improved Lyapunov-Krasovskii function (LKF) was constructed. Additionally, the reciprocally convex method is introduced to bound the derivative of LKF. According to the constructed LKF, the sampling information during the whole sampling period was fully utilized, and less conservatism was obtained. Then, the stability condition, robust performance, mode uncertainty and sampled-data controller design were analyzed by means of the linear matrix inequality (LMI). Finally, an example was given to demonstrate the effectiveness of the proposed method.

**Keywords:** dynamic positioning ship; sampled-data control; Lyapunov-Krasovskii functional; T-S fuzzy model

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### **1. Introduction**

Ship dynamic positioning is a system used in ships and offshore vessels to automatically maintain the ship's positions and heading by thrusters and propulsion systems [1]. Eliminating the need for traditional anchoring, dynamic positioning ship (DPS) is equipped with a sophisticated system that allows it to stay in a specific location or follow a set track. And DPS has been used in various maritime applications, including offshore oil and gas exploration, subsea construction, cable laying, and so on. Recently, there is a wealth of research in the academic literature on the control for DPS ([2–7]), and the control algorithms and strategies include that proportional integral derivative (PID) control, adaptive control, intelligent control, etc. [2]. The research direction includes sensor fusion and position

reference systems [3]; robust control [4]; fault tolerance control [5]; cooperative control and networked system [6]; simulation and validation [7] etc. For example, in [3], the technology of sensor fusion and the selection and optimization of position reference systems for DPS are discussed, which emphasizes the requirements of system accuracy and reliability. Reference [4] uses  $H_\infty$  robust control techniques for DPS to obtain a robust controller with better performance than traditional dynamic positioning systems. Trajectory preservation is achieved through appropriate weighting functions. Reference [5] discusses the asymptotic stability conditions for DPS with partial actuator failure. The saturation proportional differential controller has been designed, and the asymptotic stability conditions are given by using the LKF approaches. In [6], a new event triggered collaborative DP control algorithm is proposed for multiple surface vessels. This algorithm adopts an undirected interactive topology to construct a communication network between surface vessels, so that all followers can obtain leader information. Reference [7] proposes a multi physics modeling method for a comprehensive electric propulsion system used for DPS, which can run in real-time and connect to control hardware.

In recent years, due to the advantages of discreteness, high efficiency, and low communication costs, the sampled-data control system has received widespread attention and it has been widely applied in various systems, such as the hypersonic vehicle system [8,9], Lur'e system [10–12], neural networks system [13–17], Markovian jump system [18,19] and so on [20,21]. It can be applied to practical system such as fast train [22], unmanned aerial vehicle [23], large cruise liner [24,25], small unmanned ships [26], etc. Nowadays, the Takagi-Sugeno fuzzy system (TSFS) has received widespread attention due to their significant effectiveness in describing the complex nonlinear system, and considerable sampled-data control methods are reported for TSFS [27,28]. For example, in [29], the sampled-data control issue for TSFS with random time delay and actuator faults is studied using the input-delay method; In [30], the stability issue for nonlinear sampled-data TSFS is studied using the LKF method, which is validated by a single link robotic arm with an electric motor model. In [31], the stability characteristics of TSFS under sampling-data control are studied. The system is considered as a variable time delay system based on the input delay method. An appropriate LKF candidates and quadratic delay product terms are provided. Reference [32] studies the nonfragile sampling control problem for TSFS with parameter uncertainty. A new augmented LKF with sufficient sample data information has been constructed, which results in less conservative results.

Recently, the research on sampled-data control systems for DPS has attracted widespread attention. In [33], A semi-globally practically asymptotically stable controller is designed using the integration and backstepping technique of the Euler approximation models. In [34], based on the Euler approximation model and nonlinear sampling control theory, a state feedback controller and an output feedback controller based on a reduced order observer are designed for DPS to stabilize the tracking error. In [35], the sampling control system was applied to the dynamic positioning ship, and the input delay method was combined with the Wirtinger integral inequality to obtain results with less conservatism. In [36], the control problem of a sampling system for DPS with actuator failure is discussed, and a sampling controller is designed to ensure the exponential stability of the system and meet the  $H_\infty$  performance index. In [37], the fuzzy robust stability criterion of sampled-control systems for DPS is studied, and a fuzzy sampling controller is designed using an input time delay method to ensure that the system remains stable under external disturbances. In [38], the reliable tracking control problem for DPS based on nonperiodic measurement information is studied and the mean square exponential stability criterion is obtained.

However, there is still much room for further study for the sampled-data DPS. Firstly, some

important characteristics are still overlooked, which include the features of sampling pattern and the effective characteristics of the constructed LKF. Secondly, the positivity limit of the Lyapunov matrix is strict, which will result in the conservatism of the system.

Inspired by the above discussions, we aim to study the stability condition and controller design for sampled-data DPS with the T-S fuzzy model. In addition, we will construct an improved LKF by adding the new useful terms and introducing the reciprocally convex method. The main advantages of this article are listed that:

(1) To capture the features of sampling pattern fully, some novel terms, like  $(t_{k+1} - t) \int_{t_k}^t e^{2\alpha s} \begin{bmatrix} x(s) \\ x(t_k) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \begin{bmatrix} x(s) \\ x(t_k) \end{bmatrix} ds$  are introduced in the constructed LKF.

(2) The tighter technique of reciprocally convex combination is introduced during the differentiation of LKF. Hence, the conservatism of stability criteria can be greatly reduced.

(3) A second-order term  $(t_{k+1} - t)(t - t_k) \begin{bmatrix} x^T(t_k) N_1 x(t_k) \end{bmatrix}$  related to time  $t$  is added during the construction of  $V(t)$ , thereby a less conservative result is obtained.

Notations:  $\mathbb{R}^m$  denotes  $m$ -dimensional Euclidean space, and “ $T$ ” represents the matrix transposition. “ $*$ ” is a symmetric block matrix.

## 2. Problem formulation

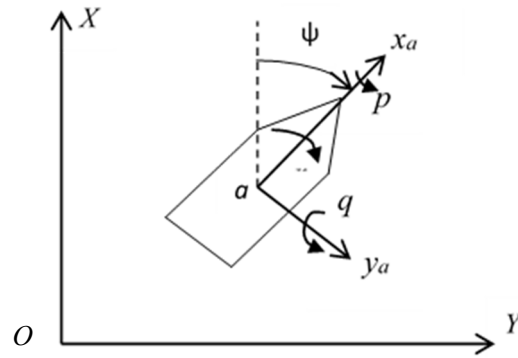
Consider the dynamic equations for DPS as follows:

$$\begin{aligned} M\dot{v}(t) + Dv(t) + G\eta(t) &= u(t) + w(t), \\ \dot{\eta}(t) &= J(\psi(t))v(t), \end{aligned} \quad (1)$$

where

$$\begin{aligned} M &= \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}, D = \begin{bmatrix} n_{11} & 0 & 0 \\ 0 & n_{22} & n_{23} \\ 0 & n_{32} & n_{33} \end{bmatrix}, G = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix}, \\ J(\psi(t)) &= \begin{bmatrix} \cos(\psi(t)) & -\sin(\psi(t)) & 0 \\ \sin(\psi(t)) & \cos(\psi(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

where  $\eta(t) = [x_a(t) \ y_a(t) \ \psi(t)]^T$  is the positions and heading angle in the northeast coordinate system respectively.  $v(t) = [p(t) \ v(t) \ r(t)]^T$  represents the surge, sway, and yaw velocity vector in the attached coordinate system (see Figure 1).  $J(\psi)$  is the coordinate rotation matrix.  $u(t)$  is the control force vector;  $M$  and  $D$  denote the inertial matrix and damping matrix, respectively.  $G$  is the mooring force matrix, and  $w(t) = [w_1(t), w_2(t), w_3(t)]$  is the environmental disturbance including wind, waves, and currents.



**Figure 1.** Body-fixed coordinate systems.

Remark 1. During the modelling of the ship, it can be considered that the interaction forces between the various points of action of the ship are ignored, that is, the ship is considered as a rigid body. Hence, the combination of forward, lateral, and turning motions with heave, roll, and pitch motions is neglected, and we only consider the three degrees of freedom plane motion of the DPS.

Define

$$\begin{aligned} x(t) &= [\eta(t) \quad v(t)]^T \\ &= [x_p(t) \quad y_p(t) \quad \psi(t) \quad p(t) \quad q(t) \quad r(t)]^T, \end{aligned} \quad (2)$$

Let

$$-M^{-1}D = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}, M^{-1} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}, -M^{-1}G = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{bmatrix}. \quad (3)$$

Substitute (2) and (3) into (1) such that

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t), \quad (4)$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & \cos(\psi(t)) & -\sin(\psi(t)) & 0 \\ 0 & 0 & 0 & \sin(\psi(t)) & \cos(\psi(t)) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ b_{11} & 0 & 0 & a_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\ 0 & b_{32} & b_{33} & 0 & a_{32} & a_{33} \end{bmatrix}, B = B_w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix},$$

where  $a_{11}, a_{22}, a_{23}, a_{32}, a_{33}, d_{11}, d_{22}, d_{23}, d_{32}, d_{33}, b_{11}, b_{22}, b_{23}, b_{32}, b_{33}$  are constant.

The global fuzzy models can be described as follows:

Mode Rule  $i$ : IF  $z_1(t)$  is  $f_{i1}, \dots, z_n(t)$  is  $f_{in}$ , THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) + B_{wi} w(t), \quad i=1,2,\dots,n \quad (5)$$

where  $z_1(t), z_2(t), \dots, z_n(t)$  represent the premise variables,  $f_{ij}$  is the fuzzy set, and  $n$  is the rules number.

Assume the yaw angle satisfies  $\psi(t) = [-\pi/2, \pi/2]$ , then the controller rules are considered such that:

Model Rule 1:

IF  $\psi(t) = 0$ , THEN

$$\dot{x}(t) = A_1 x(t) + B_1 u(t) + B_{w1} w(t), \quad (6)$$

Model Rule 2:

IF  $\psi(t) = \frac{\pi}{2}$ , THEN

$$\dot{x}(t) = A_2 x(t) + B_2 u(t) + B_{w2} w(t), \quad (7)$$

Model Rule 3:

IF  $\psi(t) = -\frac{\pi}{2}$ , THEN

$$\dot{x}(t) = A_3 x(t) + B_3 u(t) + B_{w3} w(t), \quad (8)$$

where

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & -\alpha & 0 \\ 0 & 0 & 0 & \alpha & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ b_{11} & 0 & 0 & a_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\ 0 & b_{32} & b_{33} & 0 & a_{32} & a_{33} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 0 & \beta & -1 & 0 \\ 0 & 0 & 0 & 1 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ b_{11} & 0 & 0 & a_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\ 0 & b_{32} & b_{33} & 0 & a_{32} & a_{33} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & \beta & 1 & 0 \\ 0 & 0 & 0 & -1 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ b_{11} & 0 & 0 & a_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} & 0 & a_{22} & a_{23} \\ 0 & b_{32} & b_{33} & 0 & a_{32} & a_{33} \end{bmatrix}, B_i = B_{wi} = \begin{bmatrix} 0 & 0 & 0 \\ d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}, \quad i=1,2,3.$$

where  $\alpha = \sin(2^\circ)$ ,  $\beta = \cos(88^\circ)$ . The global fuzzy models are inferred such that:

$$\dot{x}(t) = \sum_{i=1}^3 \mu_i(z(t)) [A_i x(t) + B_i u(t) + B_{wi} w(t)], \quad (9)$$

where

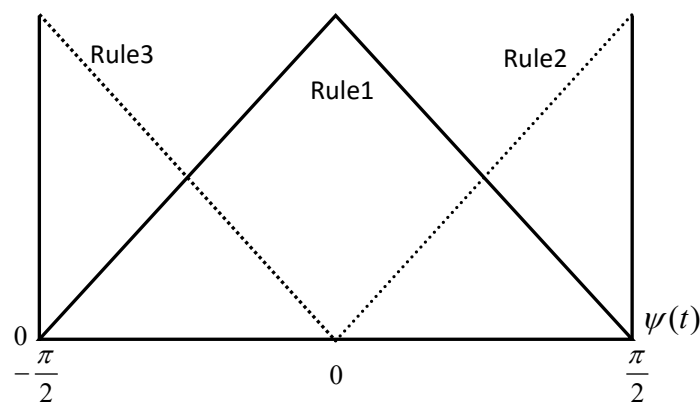
$$\mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^3 \omega_i(z(t))} \geq 0, \quad i = 1, 2, 3,$$

$$\omega_i(z(t)) = \prod_{j=1}^n f_{ij}(z_j(t)),$$

$$\sum_{i=1}^3 \mu_i(z(t)) = 1,$$

$$z(t) = [z_1(t), z_2(t), \dots, z_n(t)],$$

The membership function of  $\psi(t)$  is shown in Figure 2.

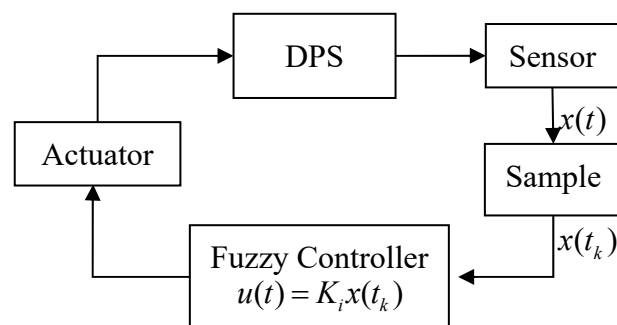


**Figure 2.** Membership function of  $\psi(t)$ .

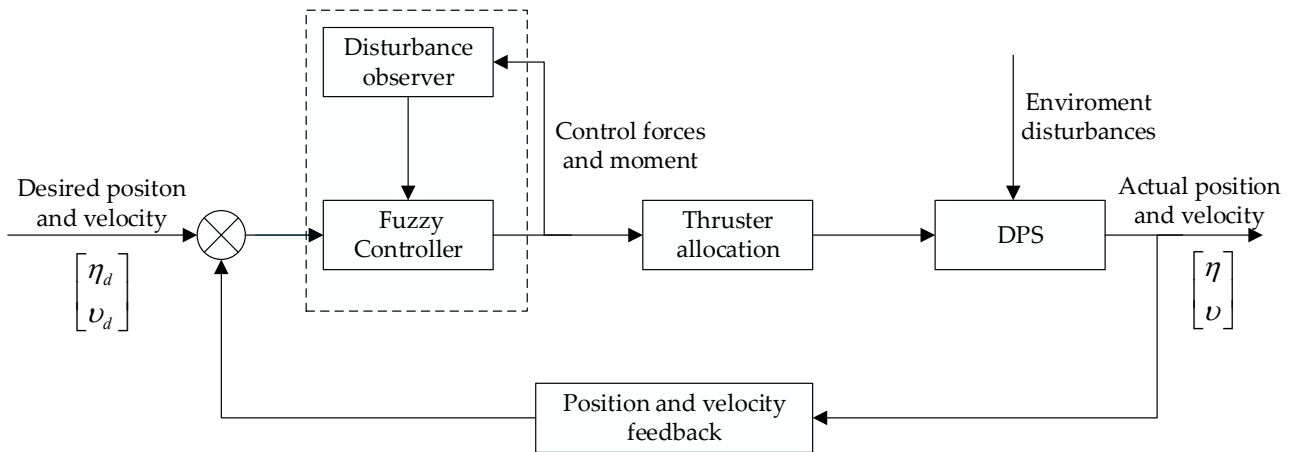
Assuming that the states of the DPS are obtained at each sampling instant  $t_k$ , the sampling time is assumed to be

$$0 = t_0 < t_1 < \dots < t_k < \dots < \lim_{k \rightarrow \infty} t_k = +\infty.$$

The structure of the system is depicted in Figure 3.



**Figure 3.** The structure of sampled-data DPS system.



**Figure 4.** The working principle of the DPS system.

The system stability process is shown in Figure 4. It can be seen that the DPS fuzzy controller calculates the ship's control force and moment based on the deviation between the desired and actual position and velocity of DPS. Then, the thrust allocation distributes control force and moment for DPS to generate the actual position and velocity under the environmental disturbances, which achieves the stability target of the system.

Let  $d$  represent the upper bound of the sampling interval, i.e.,

$$0 < t_{k+1} - t_k \leq d, \forall k \geq 0, d > 0,$$

then we consider the following sampled-data controller

$$u(t) = Kx(t_k), \quad t_k \leq t < t_{k+1}, \quad (10)$$

where  $K$  represents the controller matrix. Then the T-S fuzzy controller can be described that:

Controller Rule 1:

IF  $\psi(t) = 0$ , THEN

$$u(t) = K_1 x(t_k). \quad (11)$$

Controller Rule 2:

IF  $\psi(t) = \frac{\pi}{2}$ , THEN

$$u(t) = K_2 x(t_k). \quad (12)$$

Controller Rule 3:

IF  $\psi(t) = -\frac{\pi}{2}$ , THEN

$$u(t) = K_3 x(t_k). \quad (13)$$

Hence, the overall fuzzy controller is inferred to be:

$$u(t) = \sum_{j=1}^3 \mu_j(z(t)) K_j x(t_k), t_k \leq t < t_{k+1}, k = 0, 1, 2, \dots \quad (14)$$

Substitute (14) into (11)

$$\dot{x}(t) = \sum_{i=1}^3 \sum_{j=1}^3 \mu_i(z(t)) \mu_j(z(t)) \left[ A_i x(t) + B_i K_j x(t_k) + B_{wi} w(t) \right] \quad (15)$$

When the system has an uncertainty parameter, the system is converted into

$$\dot{x}(t) = \sum_{i=1}^3 \sum_{j=1}^3 \mu_i(z(t)) \mu_j(z(t)) \left[ (A_i + \Delta A) x(t) + (B_i + \Delta B) K_j x(t_k) + B_{wi} w(t) \right] \quad (16)$$

where  $\Delta A$  and  $\Delta B$  are the uncertainty parameter matrix, and satisfy

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = DF(t) \begin{bmatrix} E_a & E_b \end{bmatrix} \quad (17)$$

$D, E_a, E_b$  are known dimensional constant matrices, and  $F(t)$  is unknown time-varying matrices with Lebesgue measurable elements that satisfy

$$F^T(t)F(t) \leq I, \forall t \quad (18)$$

For the further deriving process, the two lemmas are given.

**Lemma 1** [39] For any matrix  $H > 0$ , and a vector with appropriate dimension  $\mu(s)$ , the inequality satisfies

$$-\int_{t_1}^{t_2} \mu^T(s) H \mu(s) ds \leq (t_2 - t_1) \xi_t^T F^T H^{-1} F \xi_t + 2 \xi_t^T H^T \int_{t_1}^{t_2} \mu(s) ds \quad (19)$$

where, matrix  $F$  and vector  $\xi$  are not dependent on integral variables and have any dimensions.

**Lemma 2** [40] For any matrix  $J, R = R^T > 0$ , constant  $d_1 \leq d(t) \leq d_2$ , and vector function  $\dot{x}: [d_2, d_1] \rightarrow \mathbb{R}^n$ , the following integral inequality holds:

$$(d_1 - d_2) \int_{t-d_2}^{t-d_1} \dot{x}^T(s) R \dot{x}(s) ds \leq -\mathcal{G}^T(t) \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix}^T W \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \mathcal{G}(t) \quad (20)$$

where

$$\mathcal{G}^T(t) = \begin{bmatrix} x^T(t-d_1) & x^T(t-d(t)) & x^T(t-d_2) \end{bmatrix}, \quad W = \begin{bmatrix} -R & J \\ * & -R \end{bmatrix} \leq 0$$

**Lemma 3** [41] Given matrices  $\Omega^T = \Omega, \Gamma, X$ , for any  $F$  satisfying  $F^T F \leq L$ , the inequality is established

$$\Omega + HFE + E^T F^T H^T < 0 \quad (21)$$

and only if there is a scalar  $\varepsilon > 0$ , then

$$\Omega + \varepsilon HH^T + \varepsilon^{-1} E^T L E < 0 \quad (22)$$



### 3. Main results

#### 3.1. Stability analysis

In the section, a novel LKF is constructed, and the stabilization condition of system (15) is proposed. To begin, the notation is defined as follows:

$$\xi^T(t) = \left[ x^T(t) \quad \dot{x}^T(t) \quad x^T(t_k) \quad x^T(t-d) \quad \int_{t_k}^t x^T(t_k) \right]$$

Theorem 1: The system (15) with  $w(t) = 0$  is asymptotically stable, if there exist positive-definite matrices  $X = [X_1^T \quad X_2^T \quad X_3^T \quad X_4^T \quad X_5^T]^T$ ,  $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$ ,  $Q, Z, U, S, R, N_1, N_2, M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix}$ , and given scale  $d > 0$ , such that

$$\begin{bmatrix} Z & U \\ * & Z \end{bmatrix} > 0 \quad (23)$$

$$\Psi_1 + d\Psi_3 < 0 \quad (24)$$

$$\begin{bmatrix} \Psi_1 + d\Psi_2 & dX \\ * & -dM_{11}^{-1} \end{bmatrix} < 0 \quad (25)$$

where

$$\Psi_1 = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ * & \Xi_{22} & \Xi_{23} & 0 & P_2 \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} \\ * & * & * & \Xi_{44} & 0 \\ * & * & * & * & 0 \end{bmatrix}$$

$$\Psi_2 = -I_3^T M_{22} I_3 - I_3^T N_1 I_3$$

$$\Psi_3 = I_{23}^T M I_{23} + I_3^T N_1 I_3$$

$$\Xi_{11} = G_1 A_i + A_i^T G_1^T + P_2 + P_2^T + Q - Z + X_1$$

$$\Xi_{12} = -G_1 + S R^T + A_i^T G_2 + P_1 + X_2^T$$

$$\Xi_{13} = G_1 B_i K_j - M_{12}^T + Z - U + X_3^T - X_1$$

$$\Xi_{14} = U + X_4^T$$

$$\Xi_{15} = P_3^T + X_5^T$$

$$\Xi_{22} = -G_2 - G_2^T + d^2 Z$$

$$\Xi_{23} = G_2 B_i K_j - X_2$$

$$\Xi_{33} = M_{12} - 2Z + U^T + U - X_3$$

$$\Xi_{34} = Z - U - X_4^T$$

$$\begin{aligned}\Xi_{35} &= -X_5^T - P_3^T \\ \Xi_{44} &= -Q - Z \\ I_{23} &= \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}, \\ I_3 &= [0 \quad 0 \quad I \quad 0 \quad 0 \quad 0], \\ L &= \text{diag} \{l_1, l_2, \dots, l_n\}\end{aligned}$$

Proof. Consider the following LKF:

$$V(t) = \sum_{i=1}^5 V_i(t), \quad t \in [t_k, t_{k+1}) \quad (26)$$

$$\begin{aligned}V_1(t) &= \begin{bmatrix} x(t) \\ \int_{t_k}^t x(s) ds \end{bmatrix}^T \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} \begin{bmatrix} x(t) \\ \int_{t_k}^t x(s) ds \end{bmatrix} \\ V_2(t) &= \int_{t-d}^t x(s)^T Q x(s) ds \\ V_3(t) &= (d - (t - t_k)) \int_{t_k}^t \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix}^T M \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix} ds \\ V_4(t) &= (t_{k+1} - t)(t - t_k) [x^T(t_k) N_1 x(t_k)] \\ V_5(t) &= d \int_{-d}^0 \int_{t+\theta}^t \dot{x}^T(s) Z \dot{x}(s) ds d\theta\end{aligned}$$

Take the derivative of  $V(t)$ , which yields

$$\begin{aligned}\dot{V}_1(t) &= 2 \begin{bmatrix} x(t) \\ \int_{t_k}^t x(s) ds \end{bmatrix}^T \begin{bmatrix} P_1 + SR^T & P_2 \\ P_2^T & P_3 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t_k) \end{bmatrix} \\ &= 2x^T(t) P_1 \dot{x}(t) + 2 \left( \int_{t_k}^t x(s) ds \right)^T P_2^T \dot{x}(t) + 2x^T(t) P_2 x(t) + 2 \left( \int_{t_k}^t x(s) ds \right)^T P_3 x(t) \\ &\quad + 2x^T(t) SR^T \dot{x}(t) - 2 \left( \int_{t_k}^t x(s) ds \right)^T P_3 x(t_k)\end{aligned} \quad (27)$$

$$\dot{V}_2(t) = x(t)^T Q x(t) - x(t-d)^T Q x(t-d) \quad (28)$$

$$\begin{aligned}\dot{V}_3(t) &= - \int_{t_k}^t \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix}^T M \begin{bmatrix} \dot{x}(s) \\ x(t_k) \end{bmatrix} ds + (d - (t - t_k)) \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix}^T M \begin{bmatrix} \dot{x}(t) \\ x(t_k) \end{bmatrix} \\ &= - \int_{t_k}^t \dot{x}(s) M_{11} \dot{x}(s) ds - 2x(t_k) M_{12} [x(t) - x(t_k)] \\ &\quad - (t - t_k) x(t_k) M_{22} x(t_k) + (d - (t - t_k)) \zeta(t) M \zeta^T(t)\end{aligned} \quad (29)$$

$$\begin{aligned}\dot{V}_4(t) &= (d_k - (t - t_k)) [x^T(t_k) N_1 x(t_k)] - (t - t_k) [x^T(t_k) N_1 x(t_k)] \\ &\leq (d - (t - t_k)) [x^T(t_k) N_1 x(t_k)] - (t - t_k) [x^T(t_k) N_1 x(t_k)]\end{aligned} \quad (30)$$

$$\dot{V}_5(t) = d^2 \dot{x}^T(t) Z \dot{x}(t) - d \int_{t-d}^t \dot{x}^T(s) Z \dot{x}(s) ds \quad (31)$$

By Lemma 1, it can be obtained that

$$-\int_{t_k}^t \dot{x}(s) M_{11} \dot{x}(s) ds \leq (t - t_k) \xi^T(t) X M_{11}^{-1} X^T \xi(t) + 2 \xi^T(t) X [x(t) - x(t_k)] \quad (32)$$

Then

$$\begin{aligned} \dot{V}_3(t) &\leq (t - t_k) \xi^T(t) V M_{11}^{-1} V^T \xi(t) + 2 \xi^T(t) V [x(t) - x(t_k)] - 2x(t_k) M_{12} [x(t) - x(t_k)] \\ &\quad - (t - t_k) x(t_k) M_{22} x(t_k) + (d - (t - t_k)) \zeta(t) M \zeta^T(t) \end{aligned}$$

By Lemma 2, we have

$$-d \int_{t-d}^t \dot{x}^T(s) Z \dot{x}(s) ds \leq \mathcal{G}^T(t) \begin{bmatrix} -Z & Z - U & U \\ * & -2Z + U^T + U & Z - U \\ * & * & -Z \end{bmatrix} \mathcal{G}(t) \quad (33)$$

where

$$\mathcal{G}(t) = [x^T(t) \quad x^T(t_k) \quad x^T(t-d)]^T$$

then

$$\dot{V}_5(t) \leq d^2 \dot{x}^T(t) Z \dot{x}(t) - \mathcal{G}^T(t) \begin{bmatrix} Z & -Z + U & -U \\ * & 2Z - U^T + U & -Z + U \\ * & * & Z \end{bmatrix} \mathcal{G}(t) \quad (34)$$

For any matrixes  $G_1, G_2$ , we have

$$2[x^T(t)G_1 + \dot{x}^T(t)G_2] \times \left[ -\dot{x}(t) + \sum_{i=1}^r \sum_{j=1}^r \mu_i(\theta(t)) \mu_j(\theta(t)) [A_i x(t) + B_i K_j x(t_k)] \right] = 0 \quad (35)$$

For any diagonal matrix  $D > 0$ , it can be obtained that

$$-2[Lx(t_k)]^T D [-Lx(t_k)] \geq 0 \quad (36)$$

Then, from (27) to (36), we have

$$\dot{V}(t) \leq \sum_{i=1}^3 \sum_{j=1}^3 \mu_i(\theta(t)) \mu_j(\theta(t)) [\xi^T(t) \Psi(\tau(t)) \xi(t)] \quad (37)$$

where

$$\tau(t) = t - t_k$$

$$\xi^T(t) = \left[ x^T(t) \quad \dot{x}^T(t) \quad x^T(t_k) \quad x^T(t-d) \quad \int_{t_k}^t x^T(t_k) \right]$$

$$\Psi(\tau(t)) = \Psi(t - t_k) = \Psi_1 + (t - t_k)(\Psi_2 + V^T \bar{M}_{11}^{-1} V) + (d - (t - t_k)) \Psi_3$$

$\Psi_1, \Psi_2, \Psi_3$  is defined in Theorem 1.

By Lemma 2, we can derive a linear convex combination about  $\tau(t)$ , for all  $\tau(t) \in [0, d]$ , if

$$\Psi(0) = \Psi_1 + d\Psi_3 < 0 \quad (38)$$

$$\Psi(d) = \Psi_1 + d(\Psi_2 + V^T \bar{M}_1^{-1} V) < 0 \quad (39)$$

Then,  $\Psi(\tau(t))$  is negative definite.

Following the Schur complement, (24) and (25) imply (38) and (39), which means that  $\dot{V}(t) < 0$ . This proof is completed.

**Remark 1.** In the constructed LKF,  $V_1(t)$  and  $V_2(t)$  are the general energy function of the system state, and it represents the energy changes of the system state and derivative from  $t-d$  to  $t$ . It is the foundation of the entire LKF.  $V_5(t)$  symbolizes the relationship between state derivatives and time delay, and it is the foundation of time delay dependent stability standards. Additionally,  $V_3(t)$  and  $V_4(t)$  include the characteristic of time delay, and they play an important role in obtaining the final solution.

**Remark 2.** Compared with literature [42], two  $(t_k, t_{k+1})$  dependent terms like  $(t_{k+1} - t) \int_{t_k}^t e^{2\alpha s} \begin{bmatrix} x(s) \\ x(t_k) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \begin{bmatrix} x(s) \\ x(t_k) \end{bmatrix} ds$  and  $(t_{k+1} - t)(t - t_k) [x^T(t_k) N_1 x(t_k)]$  are added in the LKF, which means that the available characteristic of sampling patterns are fully captured, and the LKF has a more general form.

**Remark 3.** In some previously published papers (such as [43,44]), they used overly relaxed delimitation techniques when estimating some integral terms such as  $-d \int_{t-d}^{t_k} \dot{x}^T(s) Z \dot{x}(s) ds$ . However, this paper directly uses a tighter technique by reciprocally convex combination in Lemma 2. Besides, free matrices  $G$  and  $G_1$  are proposed during the derivation procedure of LKF, which improves the design flexibility. Therefore, the conservatism of stability criteria can be greatly reduced.

### 3.2. $H_\infty$ performance analysis

To prove the robustness of the system, the following  $H_\infty$  performance function is proposed

$$J_{yw} = \int_0^t [y^T(s)y(s) - \gamma^2 w^T(s)w(s)] ds, \quad \gamma > 0 \quad (40)$$

where  $w(t) \neq 0$  and  $w(t) \in L_2[0, \infty)$ , and  $y(t)$  denotes the system output, and satisfies that

$$y(t) = \sum_{i=1}^3 \sum_{j=1}^3 \mu_i(z(t)) \mu_j(z(t)) C_i x(t), \quad (41)$$

Then, based on the Theorem 2, the system (15) can be proved to satisfy the robust performance with disturbances.

Theorem 2: the system (15) is asymptotically stable, if there exist positive-definite matrices  $X = [X_1^T \ X_2^T \ X_3^T \ X_4^T \ X_5^T]^T$ ,  $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}$ ,  $Q, Z, U, S, R, N_1, N_2, M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix}$ , and given scale  $d > 0$ , such that

$$\begin{bmatrix} Z & U \\ * & Z \end{bmatrix} > 0 \quad (42)$$

$$\Psi_1 + d\Psi_3 < 0 \quad (43)$$

$$\begin{bmatrix} \Psi_1 + d\Psi_2 & dX \\ * & -dM_{11}^{-1} \end{bmatrix} < 0 \quad (44)$$

where

$$\tilde{\Psi}_1 = \begin{bmatrix} \tilde{\Xi}_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & 0 \\ * & \Xi_{22} & \Xi_{23} & 0 & P_2 & 0 \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & 0 \\ * & * & * & \Xi_{44} & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\tilde{\Xi}_{11} = G_1 A_i + A_i^T G_1^T + P_2 + P_2^T + Q - Z + X_1 + C^T C$$

$\Xi_{12}, \Xi_{13}, \Xi_{14}, \Xi_{15}, \Xi_{22}, \Xi_{23}, \Xi_{33}, \Xi_{34}, \Xi_{35}, \Xi_{44}$  are defined in Theorem 1.

Prove:

$$\begin{aligned} J_{yw} &= \int_0^t [y^T(s)y(s) - \gamma^2 w^T(s)w(s)] ds, \\ &= \int_0^t [y^T(s)y(s) - \gamma^2 w^T(s)w(s) + \dot{V}(s)] ds - V(t) \\ &\leq \int_0^t [y^T(s)y(s) - \gamma^2 w^T(s)w(s) + \dot{V}(s)] ds \\ &\leq \sum_{i=1}^3 \sum_{j=1}^3 \mu_i(\theta(t)) \mu_j(\theta(t)) [\zeta^T(t) \Theta(\tau(t)) \zeta(t)] \end{aligned} \quad (45)$$

where

$$\begin{aligned} \tau(t) &= t - t_k \\ \zeta^T(t) &= \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t_k) & x^T(t-d) & \int_{t_k}^t x^T(t_k) & w^T(t) \end{bmatrix} \\ \Psi(\tau(t)) &= \Psi(t - t_k) = \tilde{\Psi}_1 + (t - t_k)(\Psi_2 + V^T \bar{M}_{11}^{-1} V) + (d - (t - t_k)) \Psi_3 \end{aligned}$$

The proof is finished.

### 3.3. Mode uncertainty analysis

Next, Theorem 3 provides the stability conditions for systems with norm bounded uncertainty.

Theorem 3: the system (15) is asymptotically stable, if there exist positive-definite matrices

$$X = \begin{bmatrix} X_1^T & X_2^T & X_3^T & X_4^T & X_5^T \end{bmatrix}^T, \quad P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}, \quad Q, Z, U, S, R, N_1, N_2, M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix}, \quad \text{and given}$$

scale  $d > 0, \varepsilon > 0$ , such that

$$\begin{bmatrix} Z & U \\ * & Z \end{bmatrix} > 0 \quad (46)$$

$$\Psi_1 + d\Psi_3 < 0 \quad (47)$$

$$\begin{bmatrix} \Psi_1 + d\Psi_2 & dX \\ * & -dM_{11}^{-1} \end{bmatrix} < 0 \quad (48)$$

where

$$\hat{\Psi}_1 = \begin{bmatrix} \Xi_{11} + \varepsilon E_a^T E_a & \Xi_{12} & \Xi_{13} + \varepsilon E_a^T E_b & \Xi_{14} & \Xi_{15} & G_1 D \\ * & \Xi_{22} & \Xi_{23} & 0 & P_2 & G_2 D \\ * & * & \Xi_{33} + \varepsilon E_b^T E_b & \Xi_{34} & \Xi_{35} & 0 \\ * & * & * & \Xi_{44} & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix}$$

$\Xi_{12}, \Xi_{13}, \Xi_{14}, \Xi_{15}, \Xi_{22}, \Xi_{23}, \Xi_{33}, \Xi_{34}, \Xi_{35}, \Xi_{44}$  are defined in Theorem 1.

Prove: replace  $A$  and  $A_d$  in (25) with  $A + DF(t)E_a$  and  $B + DF(t)E_b$ , respectively. It can be obtained that:

$$\Psi + \Gamma F(t)\Omega + \Omega^T F^T(t)\Gamma^T < 0 \quad (49)$$

where

$$\Gamma = [G_1 D \quad G_2 D \quad 0 \quad 0 \quad 0], \Omega = [E_a \quad 0 \quad E_b \quad 0 \quad 0]$$

According to Lemma 3, if there exist a scale  $\varepsilon > 0$ , the following inequality holds

$$\Psi + \varepsilon \Omega^T \Omega + \varepsilon^{-1} \Gamma \Gamma^T < 0 \quad (50)$$

According to Schu's complement, Eq (48) is equivalent to Eq (50). The proof is completed.

### 3.4. Controller design

Then, the controller method can be obtained according to Theorem 4.

Theorem 4: for a given scale  $d > 0$ , if there exist positive-definite matrices

$$X = [X_1^T \quad X_2^T \quad X_3^T \quad X_4^T \quad X_5^T]^T, P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}, Q, Z, U, S, R, N_1, N_2, M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix}, \text{ such that}$$

$$\begin{bmatrix} Z & U \\ * & Z \end{bmatrix} > 0 \quad (51)$$

$$\bar{\Psi}_1 + d\bar{\Psi}_3 < 0 \quad (52)$$

$$\begin{bmatrix} \bar{\Psi}_1 + d\bar{\Psi}_2 & d\bar{X} \\ * & -d\bar{M}_{11}^{-1} \end{bmatrix} < 0 \quad (53)$$

where

$$\bar{\Psi}_1 = \begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} & \bar{\Xi}_{13} & \bar{\Xi}_{14} & \bar{\Xi}_{15} \\ * & \bar{\Xi}_{22} & \bar{\Xi}_{23} & 0 & \bar{P}_2 \\ * & * & \bar{\Xi}_{33} & \bar{\Xi}_{34} & \bar{\Xi}_{35} \\ * & * & * & \bar{\Xi}_{44} & 0 \\ * & * & * & * & 0 \end{bmatrix}$$

$$\bar{\Psi}_2 = -I_3^T \bar{M}_{22} I_3 - I_3^T \bar{N}_1 I_3$$

$$\bar{\Psi}_3 = I_{23}^T \bar{M} I_{23} + I_3^T \bar{N}_1 I_3$$

$$\bar{\Xi}_{11} = A_i G + G^T A_i^T + \bar{P}_2 + \bar{P}_2^T + \bar{Q} - Z + \bar{X}_1$$

$$\bar{\Xi}_{12} = -G + \varepsilon G A_i^T + S \bar{R}^T + \bar{P}_1 + \bar{X}_2^T$$

$$\bar{\Xi}_{13} = B_i \bar{K}_j - \bar{M}_{12}^T + \bar{Z} - \bar{U} + \bar{X}_3^T - \bar{X}_1$$

$$\bar{\Xi}_{14} = \bar{U} E + \bar{X}_4^T$$

$$\bar{\Xi}_{15} = \bar{P}_3^T + \bar{X}_5^T$$

$$\bar{\Xi}_{22} = -\varepsilon G - \varepsilon G^T + d^2 \bar{Z}$$

$$\bar{\Xi}_{23} = \varepsilon B_i \bar{K}_j - \bar{X}_2$$

$$\bar{\Xi}_{33} = \bar{M}_{12} - 2\bar{Z} + \bar{U}^T + \bar{U} - \bar{X}_3$$

$$\bar{\Xi}_{34} = Z - U - \bar{X}_4^T$$

$$\bar{\Xi}_{35} = -\bar{X}_5^T - \bar{P}_3^T$$

$$\bar{\Xi}_{44} = -\bar{Q} - \bar{Z}$$

$$I_{23} = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}$$

$$L = \text{diag}\{l_1, l_2, \dots, l_n\}$$

$$\xi^T(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t_k) & x^T(t-d) & \int_{t_k}^t x^T(t_k) \end{bmatrix}$$

then the system (15) with  $w(t) = 0$  is asymptotically stable. Moreover, the controller gain is

$$K_j = \bar{K}_j G^{-1} \quad (54)$$

**Proof:** Define

$$G_1 = G^{-1}, G_2 = \varepsilon G^{-1}, \bar{K} = KG, \bar{P} = \text{diag}\{G, G\} P \text{diag}\{G, G\}, \bar{M} = \text{diag}\{G, G, G\} M \text{diag}\{G, G, G\}, \\ \bar{D} = GDG, \bar{Q} = GQG, \bar{N}_1 = GN_1G, \bar{N}_2 = GN_2G, \bar{R} = GRG, \bar{U} = GUG, \bar{Z} = GZG.$$

Let  $\text{diag}\{G, G, G, G, G, G\}^T$  and  $\text{diag}\{G, G, G, G, G, G\}$ . By pre- and post-multiplying (24)

respectively, (52) can be obtained. Let  $\text{diag}\{G, G, G, G, G, G, G\}^T$  and  $\text{diag}\{G, G, G, G, G, G, G\}$ , pre- and post-multiplying (25) respectively, (53) can be obtained. This completes the proof.

**Remark 4.** The system (15) can obtain less conservativeness than that in [45], which is mainly due to two results. First, during the construction of  $V(t)$ , a second-order term  $(t_{k+1}-t)(t-t_k)[x^T(t_k)N_1x(t_k)]$  related to time  $t$  is also added, which can reduce the conservatism of the results to some extent. Second, during the derivative of the LKF, the reciprocally convex combination method is used, which can obtain a more stringent derivative upper limit than Jensen's inequality, thereby less conservativeness will be achieved.

#### 4. Numerical examples

To demonstrate the merits of the method, a practical example is carried out. The  $M$  and  $D$  is considered as follows [46].

$$M = \begin{bmatrix} 1.0852 & 0 & 0 \\ 0 & 2.0575 & -0.4087 \\ 0 & -0.4087 & 0.2153 \end{bmatrix}, D = \begin{bmatrix} 0.0865 & 0 & 0 \\ 0 & 0.0762 & 0.1510 \\ 0 & 0.0151 & 0.0031 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.0389 & 0 & 0 \\ 0 & 0.0266 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Let  $\alpha = \sin 2^\circ$  and  $\beta = \cos 88^\circ$ , then

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1.0000 & -0.0349 & 0 \\ 0 & 0 & 0 & 0.0349 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \\ -0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\ 0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\ 0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0.0349 & -1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 & 0.0349 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \\ -0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\ 0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\ 0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 0.0349 & 1.0000 & 0 \\ 0 & 0 & 0 & -1.0000 & 0.0349 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \\ -0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\ 0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\ 0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \end{bmatrix},$$



$$B_i = B_{wi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.9215 & 0 & 0 \\ 0 & 0.7802 & 1.4811 \\ 0 & 1.4811 & 7.4562 \end{bmatrix}, \quad i=1,2,3.$$

The allowable maximum sampling interval is an important control performance index of the sampled-data system. A longer sampling period indicates a lower communication burden and fewer time consumption of the actuator, which helps to save energy, time and so on. Table 1 lists the results which uses different methods to obtain the maximum sampling interval. From Table 1, it can be seen that the sampling interval in Theorem1 improves [36–38] about 189.2%, 173.86%, and 35.9%, respectively, which means that the designed fuzzy controller is beneficial to obtain a larger maximum sampling interval.

**Table 1.** Maximum sampling interval for different methods.

Method	[36]	[37]	[38]	Theorem 1
$d$	0.25	0.264	0.532	0.723

Assume the initial value of the system that  $x_s(t) = [15m \ 15m \ 0.2^\circ \ 0m/s \ 0m/s \ 0^\circ/s]$ .

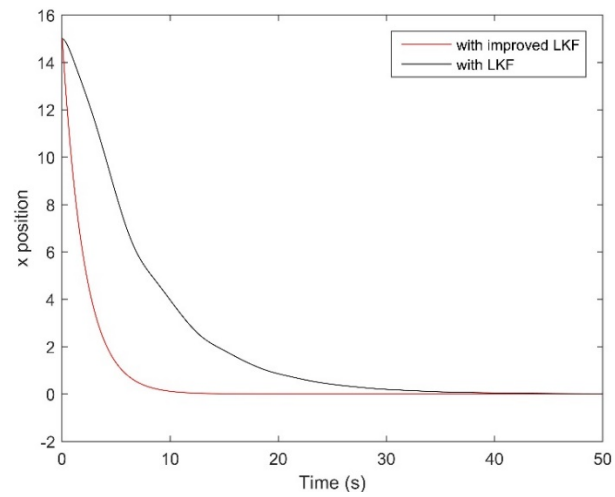
Then, the gain can be computed such that

$$K_1 = \begin{bmatrix} -0.0281 & -0.0444 & -0.0339 & -0.7195 & -0.0002 & -0.0003 \\ 0.0708 & -0.0085 & 0.0457 & 0.0004 & -1.3877 & 0.2656 \\ -0.0142 & 0.0074 & 0.0088 & -0.0001 & 0.2795 & -0.1371 \end{bmatrix}$$

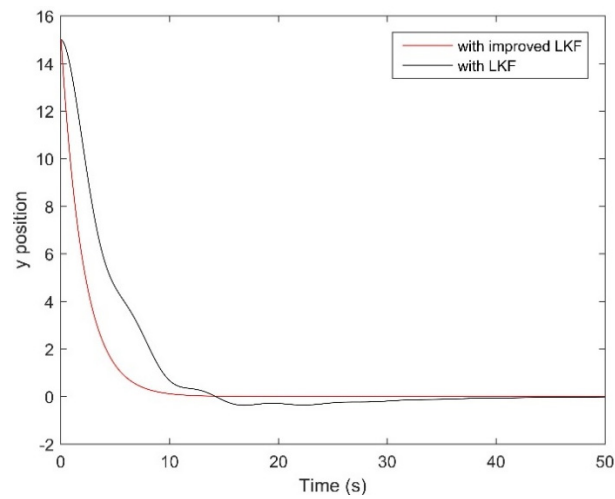
$$K_2 = \begin{bmatrix} -0.0123 & 0.0166 & 0.0170 & -0.7473 & -0.0002 & 0.0000 \\ -0.0228 & 0.0281 & 0.0790 & 0.0002 & -1.4402 & 0.2745 \\ 0.0047 & 0.0002 & 0.0029 & -0.0001 & 0.2900 & -0.1421 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -0.0123 & 0.0166 & 0.0170 & -0.7473 & -0.0002 & 0.0000 \\ -0.0228 & 0.0281 & 0.0790 & 0.0002 & -1.4402 & 0.2745 \\ 0.0047 & 0.0002 & 0.0029 & -0.0001 & 0.2900 & -0.1421 \end{bmatrix}$$

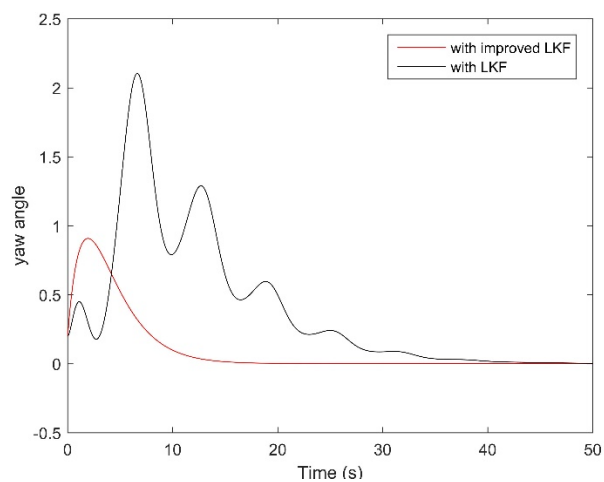
To demonstrate the control performance of the proposed method, Figures 5–10 present the comparison results with LKF in [36]. From Figures 5–10, it illustrates that the system state  $x_s(t)$  reaches the expected values within approximately 10 seconds, while [36] achieves the desired target and requires a longer time. This indicates that the response speed in the proposed method is faster. Besides, the system's oscillation amplitude is smaller, which indicates that our result has good anti-interference ability under environment disturbance. In addition, Figures 5–10 further illustrate that the proposed method has more preferable control performance.



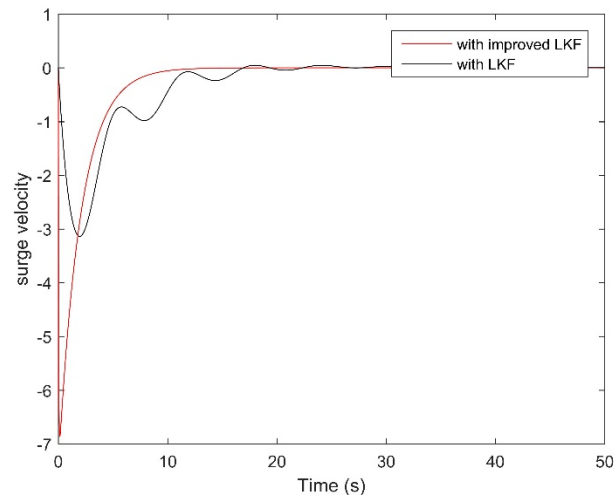
**Figure 5.** Time response of the  $x$  position.



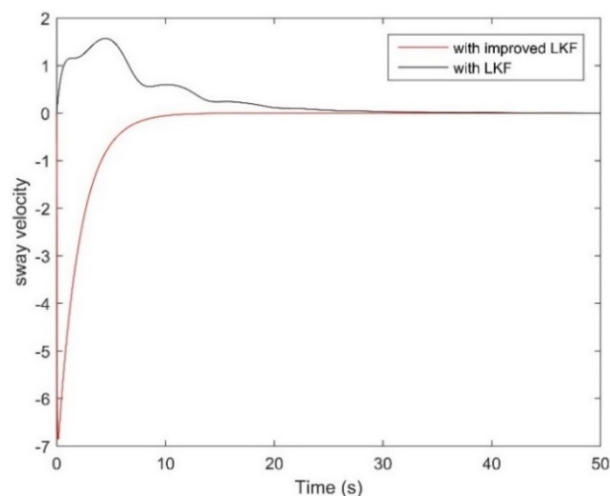
**Figure 6.** Time response of the  $y$  position.



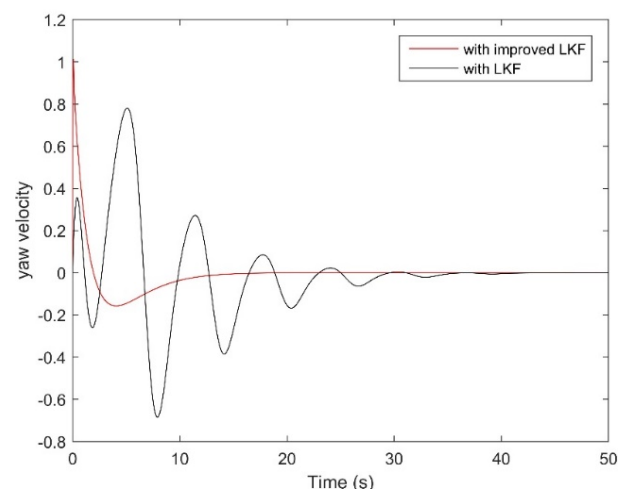
**Figure 7.** Time response of the yaw angle.



**Figure 8.** Time response of the surge velocity.



**Figure 9.** Time response of the sway velocity.



**Figure 10.** Time response of the yaw velocity.

## 5. Conclusions

The stabilization issue for fuzzy DPS with sampled-data is studied. To reduce the conservative and improve the sampling interval, a novel LKF is constructed by adding some integral terms, which can utilize the information of actual sampling modes fully. Then a fuzzy sampling control scheme is proposed. By introducing free matrices and reciprocally convex approach, the conservatism of the system is reduced. Simulation research demonstrates that the proposed method can achieve better control performance. However, the proposed methodologies didn't consider the lower bound of the sampling interval, and some matrix in the LKF are still required to be positive definite. Additionally, how to avoid the emergence of hierarchical phenomena in the system is an urgent problem to be solved. Furthermore, we will improve the LKF and discuss the aperiodic sampled-data control problem for DPS, which both considers the lower and upper bound of the sampling interval.

### Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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### Conflict of interest

The authors declare there is no conflict of interest.

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