Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/mbe

MBE, 21(3): 4440–4462.
DOI: 10.3934/mbe.2024196
Received: 17 December 2023
Revised: 18 January 2024
Accepted: 29 January 2024
Published: 27 February 2024

Research article

Prescribed-time cluster practical consensus for nonlinear multi-agent systems based on event-triggered mechanism

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Abstract: This paper investigates the prescribed-time event-triggered cluster practical consensus problem for a class of nonlinear multi-agent systems with external disturbances. To begin, to reach the prescribed-time cluster practical consensus, a new time-varying function is introduced and a novel distributed continuous algorithm is designed. Based on the Lyapunov stability theory and inequality techniques, some sufficient conditions are given, ensuring the prescribed-time cluster practical consensus. Moreover, to avoid different clusters' final states overlapping, a virtual leader is considered for each cluster. In this case, an event-triggered distributed protocol is further established and some related conditions are given for achieving prescribed-time cluster practical consensus. Additionally, it is proven that the Zeno behavior can be avioded by choosing parameters appropriately. Finally, some numerical examples are presented to show the effectiveness of the theoretical results.

Keywords: event-triggered; cluster consensus; multi-agent systems; prescribed-time

1. Introduction

Recently, with the development of unmanned aerial vehicle technology, formation control, and sensor networks [1–3], the multi-agent systems (MASs) consensus problem has attracted widespread research from many scholars. Up to the present, a series of consensus problems of MASs have been reported in the literatures [4–10]. From the perspective of dynamics, existing works can be divided into integrator systems [4], linear systems [5], and nonlinear systems [6]. Considering different control methods, the consensus problems of MASs were studied by using continuous control [7, 8] and event-triggered control [9, 10]. For the consensus states, there are complete consensus and bipartite consensus. Compared with complete consensus, bipartite consensus means all agents converge to the common state in modulus but with opposite signs. In [11], the authors studied the bipartite consensus for a class of nonlinear MASs under switching topologies. Furthermore, the distributed containment control based on the event-triggered mechanism was considered in [12], in which the containment control implies that all followers' states converge to the convex hull formed by multiple leaders.

It is important to note that achieving consensus through asymptotic manner is an idealized process in MASs. Hence, in the aforementioned results [4–12], the asymptotic convergence algorithms may not be undesirable in some practical situations because of the fact that many production tasks require being finished in a finite-time. In view of this reason, the finite-time control approach has been widely used for MASs. Since the finite-time consensus can provide a faster convergence rate and better robustness than the asymptotic consensus [13], several typical finite-time consensus results have been published for MASs including complete consensus [14–16], bipartite consensus [17, 18], and containment consensus [19]. However, one of the drawbacks of finite-time consensus is that the convergence time estimation of the system is related to the initial conditions.

In fact, because the initial state selection of the system may be arbitrary or impossible to be obtained, it is challenging to determine the settling time in some practical applications. To overcome this shortcoming, lots of results about fixed-time consensus have been published [20–24]. For instance, a novel distributed observer was proposed for each follower agent, then two types of controllers were designed to study the fixed-time consensus problem for a class of heterogeneous nonlinear MASs in [20]. Taking into account the control costs, a distributed algorithm based on the event-triggered control strategy was designed to study the fixed-time time-varying formation tracking problem for nonlinear MASs in [21]. The fixed-time bipartite consensus problems for integer-order and fractional-order MASs were investigated in [22, 23], respectively. Moreover, in [24], the fixed-time containment consensus problem was addressed for nonlinear MASs by using an event-triggered control protocol. Although the settling time estimation is independent of the initial values of the system in above results [20–24], the convergence time cannot be set arbitrarily in advance. Furthermore, in [25, 26], based on the fixed-time theory, an improved prescribed-time control method was proposed to solve the synchronization issue in networks of piecewise smooth systems. In addition, by introducing a time-varying function in the control protocol, some prescribed-time consensus issues have been studied in [27–29]. For example, under the framework of the event-triggered control mechanism, the prescribed-time practical consensus and bipartite consensus problems were discussed for first-order MASs in [28, 29], respectively.

In the above research, all agents were treated as one whole group. However, in some complex tasks, agents are usually divided into multiple subgroups to perform related tasks, and both the cooperative and competitive relationships exist among different agents, such as intelligent combat of drone swarms, multi-target encirclement, and so on. In this case, agents typically exhibit cluster consensus behaviors. Cluster consensus indicates that agents achieve consensus within the same cluster and they may not achieve consensus between different clusters. Recently, cluster consensus has received extensive attention from researchers. In [30] and [31], the authors considered the asymptotical cluster consensus of higher-order MASs. The asymptotical cluster consensus was studied for linear MASs under a directed graph by the event-triggered method, and the optimal selection of some parameters was also discussed in [32]. Furthermore, some novel finite-time control protocols were proposed to solve cluster consensus in [33, 34]. Based on the fixed-time stability theory, the fixed-time cluster consensus problems for MASs have been considered in [35, 36]. It's not difficult to find that all the above researches were focused on asymptotically consensus, finite-time consensus, and fixed-time consensus. To the best of our knowledge, few works have been paid to the prescribed-time cluster consensus for nonlinear MASs based on event-triggered control, which is motivation of this study.

Inspired by the aforementioned discussion, this paper investigates the prescribed-time event-triggered cluster practical consensus for nonlinear MASs with external disturbances. The substantial difficulties of this paper are summarized as two aspects. On the one hand, it is noted that additional waste of the limited control resource is not advisable in practical. In this case, how to design an event-triggered mechanism to enable intermittent updates of the control protocol is a problem worth solving. On the other hand, in existing researches [30–36], the settling time for achieving cluster consensus cannot be explicitly preselected. Therefore, it's essential to design a new control protocol to achieve the prescribed-time cluster consensus.

To solve these problems, this paper studies the prescribed-time cluster practical consensus for nonlinear MASs based on the event-triggered control method. The main contributions of this paper can be concluded as follows:

1) The cluster practical consensus is investigated for a class of nonlinear MASs with external disturbances, in which both the cooperative and competitive relationships are considered. When the MASs has only one cluster, the cluster consensus can degenerate into traditional consensus. Therefore, the problem considered in this paper can be regarded as an extension of the existing one.

2) Compared with asymptotic control algorithms [31, 32], finite-time control algorithms [33, 34], and fixed-time control algorithms [35, 36], a novel distributed control algorithm with time-varying function is proposed, which can ensure the prescribed-time convergence of the controlled MASs.

3) To further reduce the communication burden and the controller's update frequency, an improved prescribed-time control algorithm with an event-triggered communication mechanism is designed, in which a dynamic event triggering condition is employed.

The rest of this paper is structured as follows. Some preliminaries are given in Section 2. Section 3 provides two distributed prescribed-time control algorithms. Simulation results are given in Section 4. Section 5 gives a conclusion of this paper.

Notations. $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ represent the real matrices with $n \times 1$ and $n \times m$ dimensions, respectively. $\mathcal{R}$ is the set of real numbers. $\mathcal{R}_+$ denotes the positive real number set. For a symmetric matrix $\mathcal{M}$, $\lambda_{\max}(\mathcal{M})$ and $\lambda_2(\mathcal{M})$ represent the maximum and the second smallest eigenvalues of $\mathcal{M}$, respectively. $\mathcal{M}^T$ denotes the transpose of $\mathcal{M}$. For an arbitrary vector $x = [x_1, x_2, \ldots, x_N]^T$, $\text{sign}(x) \triangleq [\text{sign}(x_1), \text{sign}(x_2), \ldots, \text{sign}(x_N)]^T$, where $\text{sign}(\cdot)$ represents the sign function. $\|\cdot\|_p$ is represented as the p-norm for vectors or matrices. $\text{diag}(\cdot)$ represents the diagonal matrix and $\otimes$ stands for the Kronecker product. $\mathbf{1}_N$ denotes the N dimensional column vector with entries all being 1. $\emptyset$ is emptyset. $|\cdot|$ represents the absolute value.

2. Preliminaries

2.1. Graph theory

Let the communication topology among agents be modeled by an undirected connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. $\mathcal{V} = \{1, 2, \cdots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent the node set and the edge set, respectively. A communication link $(i, j) \in \mathcal{E}$, with $i \neq j$, indicates that there is a directed edge from node i to node j. $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$ is the weighted adjacency matrix with $a_{ij} \neq 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$

otherwise. The in-degree matrix is $\mathcal{D} = \text{diag}(d_1, d_2, \cdots, d_N)$ with $d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix of graph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A} = [l_{ij}] \in \mathcal{R}^{N\times N}$ with $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = \sum_{j=1}^{N} a_{ij}$. Denote $\mathcal{B} = \text{diag}(b_1, b_2, \cdots, b_N)$ with $b_i \in \{0, 1\}$, where $b_i = 1$ if agent i can receive information from the leader, otherwise $b_i = 0$. Furthermore, graph $\mathcal{G}$ is connected if there is a path between any pair of distinct nodes.

2.2. *Useful Lemma*

Before moving on, a time-varying function is constructed as follows [29]:

$$\nu(t) = \begin{cases} \frac{T^h}{(T+t_0-\theta t)^h}, & t \in [t_0, t_1), \\ 1, & t \in [t_1, \infty), \end{cases} \tag{1}$$

where $h > 0$ and $0 < \theta < 1$ are real numbers. $t_1 = t_0 + T$, where $T > 0$ is a given settling time. Suppose that the initial time is $t_0 = 0$, then $t_1 = T$. It can be deduced that $\nu(t)^{-\gamma}(\gamma > 0)$ is monotonically decreasing during the interval $[0, T)$, $\nu(0)^{-\gamma} = 1$ and $\lim_{t\to T^-} \nu(t)^{-\gamma} = 0$. $\dot{\nu}(T)$ is defined by using the righthand derivative of $\nu(t)$ at $t = T$.

Definition 1. Given the following system

$$\begin{cases} \dot{x}(t) = g(x(t), t), \\ x(0) = x_0, \end{cases} \tag{2}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $g(x(t), t) : \mathcal{R}^n \times \mathcal{R}_+ \to \mathcal{R}^n$ is a nonlinear function, and x_0 is the initial state, the origin of System (2) is globally prescribed-time practical stable if it satisfies that

$$\begin{cases} \lim\limits_{t\to T} \|x(t)\|_2 \leq \psi, \\ \|x(t)\|_2 \leq \psi, \quad \forall t \geq T, \\ \lim\limits_{t\to\infty} \|x(t)\|_2 = 0, \end{cases} \tag{3}$$

for any arbitrary initial values x_0, where ψ is a positive constant and the settling time T is a time-independent constant and can be preselected.

Lemma 1. Given System (2), if there exist two continuous and strictly increasing functions $\hbar_1(t), \hbar_2(t)$ satisfying $\hbar_i(t) : \mathcal{R}_+ \to \mathcal{R}_+$, with $\hbar_i(0) = 0$ and $\hbar_i(t) \to \infty$ as $t \to \infty (i = 1, 2)$, and exists a Lyapunov function $\mathcal{W}(x(t)) : \mathcal{R}^n \to \mathcal{R}_+$ such that

$$\hbar_1(t)\|x(t)\|_2 \leq \mathcal{W}(x(t)) \leq \hbar_2(t)\|x(t)\|_2, \tag{4}$$

$$\dot{\mathcal{W}}(x(t)) \leq -\alpha\frac{\dot{\nu}(t)}{\nu(t)}\mathcal{W}(x(t)) - \beta\mathcal{W}(x(t)), \quad t \in [0, T), \tag{5}$$

$$\dot{\mathcal{W}}(x(t)) \leq -\beta\mathcal{W}(x(t)), \quad t \in [T, \infty), \tag{6}$$

with $\alpha > 0, \beta > 0$, then the origin of System (2) is globally prescribed-time practical stable with the settling time T.

Proof. When $t \in [0, T)$, by multiplying $\nu^{\alpha}(t)$ on both sides of Eq (5), one has

$$\nu^{\alpha}(t)\dot{\mathcal{W}}(x(t)) \leq -\alpha\dot{\nu}(t)\nu(t)^{\alpha-1}\mathcal{W}(x(t)) - \beta\nu^{\alpha}(t)\mathcal{W}(x(t)).$$

It follows that

$$\frac{d(\nu^{\alpha}(t)\mathcal{W}(x(t)))}{dt} = \alpha\dot{\nu}(t)\nu(t)^{\alpha-1}\mathcal{W}(x(t)) + \nu^{\alpha}(t)\dot{\mathcal{W}}(x(t)) \leq -\beta\nu^{\alpha}(t)\mathcal{W}(x(t)). \tag{7}$$

Solving the differential Eq (7), one obtains

$$\nu^{\alpha}(t)\mathcal{W}(x(t)) \leq \nu^{\alpha}(0)\mathcal{W}(x(0))\exp(-\beta t).$$

and then

$$\mathcal{W}(x(t)) \leq \nu^{-\alpha}(t)\mathcal{W}(x(0))\exp(-\beta t).$$

By Eq (4), we can get that $\lim_{t\to T}\|x(t)\|_2 \leq \psi$, where $\frac{\mathcal{W}(x(0))\exp(-\beta t)}{\hbar_1(t)}(1-\theta)^{h\alpha}$.

Next, when $t \in [T,\infty)$, we have $\dot{\mathcal{W}}(x(t)) \leq -\beta\mathcal{W}(x(t))$. From the definition of $\mathcal{W}(x(t))$, we get that $\mathcal{W}(x(t))$ keeps monotonically decreasing and $\|x(t)\|_2 \leq \psi$ $(t \geq T)$. Combine with Eq (6), one has $\|x(t)\|_2 \leq \frac{\mathcal{W}(x(T))}{\hbar_1(t)}\exp(-\beta(t-T))$, which implies that $\lim_{t\to\infty}\|x(t)\|_2 = 0$. This completes the proof.

Lemma 2. (see [37]) For an undirected connected graph $\mathcal{G}$, the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ has a simple zero eigenvalue and $\mathbf{1}_N$ is the associated eigenvector. The eigenvalue of matrix $\mathcal{L}$ satisfies $0 < \lambda_2 \leq \ldots \leq \lambda_N$. Moreover, if $\mathbf{1}_N^T x = 0$ with $x = [x_1, x_2, \ldots, x_N]^T$, one has $x^T\mathcal{L}x \geq \lambda_2 x^T x$.

Lemma 3. (see [38]) Let $\kappa_1, \kappa_2, \cdots, \kappa_M \geq 0$ be nonnegative numbers, then

$$\left(\sum_{i=1}^{M}\kappa_i\right)^s \leq \sum_{i=1}^{M}\kappa_i^s \leq M^{1-s}\left(\sum_{i=1}^{M}\kappa_i\right)^s, \quad 0 < s \leq 1,$$
$$M^{1-s}\left(\sum_{i=1}^{M}\kappa_i\right)^s \leq \sum_{i=1}^{M}\kappa_i^s \leq \left(\sum_{i=1}^{M}\kappa_i\right)^s, \quad 1 < s < \infty.$$

2.3. Problem formulation

Suppose there is a nonlinear MAS with N agents. The dynamics of the agent i is given by

$$\dot{x}_i(t) = f(x_i(t),t) + w_i(x_i(t),t) + u_i(t), \quad i = 1,2,\cdots N, \tag{8}$$

where $x_i(t) \in \mathcal{R}^n$ is the state, $u_i(t) \in \mathcal{R}^n$ is the control input, and $f(\cdot,\cdot) : \mathcal{R}^n \times \mathcal{R}_+ \to \mathcal{R}^n$ is the nonlinear dynamic. $w_i(x_i(t),t)$ is the external disturbances and satisfies $\|w_i(x_i(t),t)\|_2 \leq \tilde{\omega}$, in which $\tilde{\omega}$ is a positive constant.

To investigate the cluster consensus problem, we assume the node set $\mathcal{V}$ is divided into m cluster or subgraphs, i.e., $\mathcal{G}_1 = (\mathcal{V}_1,\mathcal{E}_1,\mathcal{A}_1)$, $\mathcal{G}_2 = (\mathcal{V}_2,\mathcal{E}_2,\mathcal{A}_2)$, $\cdots$, $\mathcal{G}_m = (\mathcal{V}_m,\mathcal{E}_m,\mathcal{A}_m)$, such that $\mathcal{V} = \mathcal{V}_1\cup\mathcal{V}_2\cup\cdots\cup\mathcal{V}_m$, where $\mathcal{V}_1 = \{1,2,\cdots,r_1\}$, $\mathcal{V}_2 = \{r_1+1, r_1+2,\cdots,r_2\}$, $\cdots$, $\mathcal{V}_m = \{r_{m-1}+1, r_{m-1}+2,\cdots,r_m\}$, $r_0 = 0$, $r_m = N$, $\mathcal{V}_k \neq \emptyset$, $\mathcal{V}_k \cap \mathcal{V}_{k'} = \emptyset$ for $k \neq k'$ $(k,k' = 1,2,\cdots,m)$. The Laplacian matrix of $\mathcal{G}$ is defined as

$$\begin{bmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} & \cdots & \mathcal{L}_{1m} \\ \mathcal{L}_{21} & \mathcal{L}_{22} & \cdots & \mathcal{L}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{L}_{m1} & \mathcal{L}_{m2} & \cdots & \mathcal{L}_{mm} \end{bmatrix},$$

where $\mathcal{L}_{kk}$ and $\mathcal{L}_{kk'}$ represent the Laplacian matrix of intra-cluster and intercluster, $k \neq k'(k, k' = 1, 2, \cdots, m)$.

Assumption 1. The intercluster coupling strength a_{ij} satisfies that $\sum\limits_{j \in \mathcal{V}_{k'}} a_{ij} = 0$, $\forall i \in \mathcal{V}_k$, $k \neq k'$, $k, k' = 1, 2, \cdots, m$. Furthermore, the communication topological graph $\mathcal{G}_k$ in each cluster is connected and undirected.

Assumption 2. For any $x_i(t), x_j(t) \in \mathcal{R}^n$, the nonlinear dynamic $f(\cdot, \cdot)$ satisfies

$$\|f(x_i(t), t) - f(x_j(t), t)\|_2 \leq \rho \|x_i(t) - x_j(t)\|_2,$$

where ρ is a positive constant.

Definition 2. The System (8) reaches prescribed-time cluster practical consensus if there exists a controller $u_i(t)$ such that

$$\begin{cases} \lim\limits_{t \to T} \|x_i(t) - x_j(t)\|_2 \leq \psi, \\ \|x_i(t) - x_j(t)\|_2 \leq \psi, \quad \forall t \geq T, \\ \lim\limits_{t \to \infty} \|x_i(t) - x_j(t)\|_2 = 0, \end{cases} \tag{9}$$

and $x_i(t) \neq x_j(t)$, if $i \in \mathcal{V}_k$, $j \in \mathcal{V}_{k'}$, $k \neq k'$, $T > 0$ is a user-assignable finite positive real number.

Remark 1. In Assumption 1, the information exchange between agents should be balanced in any two subgraphs. This assumption is always used in the cluster network, which also can be found in existing publications [32, 34]. Assumption 2 is a Lipschitz condition. In practical applications, many practical models satisfy this assumption such as Hopfield neural networks, Chua's circuit system, and so on. Therefore, in this paper, we assume that $f(\cdot, \cdot)$ satisfies the Lipschitz condition.

3. Main result

In this section, we will consider the nonlinear MASs (8) to achieve prescribed-time cluster practical consensus. At first, a continuous prescribed-time control protocol is proposed for System (8). Moreover, in order to reduce the communication burden, a novel distributed event-triggered control strategy is adopted to solve the cluster consensus problem in a prescribed-time interval.

3.1. Continuous control protocol

In this part, a continuous control algorithm is developed to achieve prescribed-time cluster consensus. The control protocol for agent i is devised as follows

$$\begin{aligned} u_i(t) = &-\Big(\alpha + \beta \frac{\dot{\nu}(t)}{\nu(t)}\Big)\Big(\delta_1 \sum_{j \in \mathcal{V}_k} a_{ij}(x_i(t) - x_j(t)) + \delta_2 \sum_{k' \neq k} \sum_{j \in \mathcal{V}_{k'}} a_{ij}(x_i(t) - x_j(t))\Big) \\ &- \phi \sum_{j \in \mathcal{V}_k} a_{ij} \mathrm{sign}(x_i(t) - x_j(t)), \quad i \in \mathcal{V}_k, \quad k = 1, 2, \cdots, m, \end{aligned} \tag{10}$$

where $\alpha, \beta, \delta_1, \delta_2, \phi > 0$ are design parameters.

Remark 2. From control protocol (10), we can see that the terms $\sum_{j \in \mathcal{V}_k} a_{ij}(x_i(t) - x_j(t))$ and

$\sum_{k'\neq k}\sum_{j\in\mathcal{V}_{k'}} a_{ij}(x_i(t)-x_j(t))$ represent the information exchange between agents in the intra- and interclusters. When removing the term $\sum_{k'\neq k}\sum_{j\in\mathcal{V}_{k'}} a_{ij}(x_i(t)-x_j(t)))$, the protocol degenerates into dealing with a complete consensus problem.

The following gives the main theorems for the prescribed-time cluster practical consensus problem.

Theorem 1. For System (8), if Assumptions 1-2 hold and the control parameters $\beta > 0, \delta_2 > 0$ and α, δ_1, ϕ satisfy

$$\alpha > \frac{2\rho}{2\underline{\lambda}\delta_1 - \Phi\delta_2(N-\underline{N})}, \quad \delta_1 > \frac{\Phi\delta_2(N-\underline{N})}{2\underline{\lambda}}, \quad \phi \geq \frac{\tilde{\omega}\sqrt{2\overline{N}m}}{\underline{\lambda}'}, \tag{11}$$

then the prescribed-time cluster practical consensus problem can be achieved under the controller (10).

Proof. Let $\xi_i(t) = x_i(t) - \frac{1}{r_k - r_{k-1}}\sum_{j=r_{k-1}+1}^{r_k} x_j(t)$, $i\in\mathcal{V}_k$. Choose the Lyapunov function as follows

$$V(t) = \frac{1}{2}\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k} \xi_i^T(t)\xi_i(t). \tag{12}$$

The derivative of $V(t)$ is given by

$$\dot{V}(t) = \sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k} \xi_i^T(t)\dot{\xi}_i(t) = \sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k} \xi_i^T(t)\Big(\dot{x}_i(t) - \frac{1}{r_k - r_{k-1}}\sum_{j=r_{k-1}+1}^{r_k}\dot{x}_j(t)\Big). \tag{13}$$

Due to $\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\xi_i^T(t)\Big(\frac{1}{r_k-r_{k-1}}\sum_{j=r_{k-1}+1}^{r_k}\dot{x}_j(t)\Big) = 0$, Eq (13) can be rewritten as

$$\begin{aligned}
\dot{V}(t) =& \sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k} \xi_i^T(t)\dot{x}_i(t)\\
=& \sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k} \xi_i^T(t)\Big(f(x_i(t),t) + w_i(x_i(t),t) + u_i(t)\Big)\\
=& -\delta_1\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i,j=r_{k-1}+1}^{r_k} a_{ij}\xi_i^T(t)(x_i(t)-x_j(t))\\
&-\delta_2\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\sum_{k'\neq k}\sum_{j\in\mathcal{V}_{k'}} a_{ij}\xi_i^T(t)(x_i(t)-x_j(t))\\
&-\phi\sum_{k=1}^{m}\sum_{i,j=r_{k-1}+1}^{r_k} a_{ij}\xi_i^T(t)\mathrm{sign}(x_i(t)-x_j(t))\\
&+\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k} \xi_i^T(t)\Big(f(x_i(t),t) + w_i(x_i(t),t)\Big).
\end{aligned} \tag{14}$$

For the first item of Eq (14), we have the following results

$$-\delta_1\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i,j=r_{k-1}+1}^{r_k} a_{ij}\xi_i^T(t)(x_i(t)-x_j(t))$$

$$
\begin{aligned}
&= -\frac{1}{2}\delta_1\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i,j=r_{k-1}+1}^{r_k} a_{ij}(\xi_i(t) - \xi_j(t))^T(\xi_i(t) - \xi_j(t)) \\
&= -\delta_1\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\tilde{\xi}_k^T(t)(\mathcal{L}_{kk}\otimes I_n)\tilde{\xi}_k(t) \\
&\leq -\delta_1\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\lambda_2(\mathcal{L}_{kk})\tilde{\xi}_k^T(t)\tilde{\xi}_k(t) \\
&\leq -2\delta_1\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\underline{\lambda}V(t),
\end{aligned}
\tag{15}
$$

where $\tilde{\xi}_k(t) = [\xi^T_{r_{k-1}+1}(t), \xi^T_{r_{k-1}+2}(t), \cdots, \xi^T_{r_k}(t)]^T$ and $\underline{\lambda} = \min_{k=1,2,\cdots,m}\{\lambda_2(\mathcal{L}_{kk})\}$.

According to Assumption 1 and Young's inequality, from the second item of Eq (14), it can be obtained that

$$
\begin{aligned}
&-\delta_2\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\sum_{k'\neq k}\sum_{j\in\mathcal{V}_{k'}} a_{ij}\xi_i^T(t)(x_i(t) - x_j(t)) \\
=&\delta_2\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\sum_{k'\neq k}\sum_{j\in\mathcal{V}_{k'}} a_{ij}\xi_i^T(t)\xi_j(t) \\
\leq&\Phi\delta_2\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\sum_{k'\neq k}\sum_{j\in\mathcal{V}_{k'}}\sum_{s=1}^{n}|\xi_i^s(t)||\xi_j^s(t)| \\
\leq&\frac{1}{2}\Phi\delta_2\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{s=1}^{n}\Big(\sum_{i\in\mathcal{V}_1}\sum_{k'\neq 1}\sum_{j\in\mathcal{V}_{k'}}\Big(|\xi_i^s(t)|^2 + |\xi_j^s(t)|^2\Big) \\
&+\sum_{i\in\mathcal{V}_2}\sum_{k'\neq 2}\sum_{j\in\mathcal{V}_{k'}}\Big(|\xi_i^s(t)|^2 + |\xi_j^s(t)|^2\Big) + \cdots + \sum_{i\in\mathcal{V}_{m-1}}\sum_{j\in\mathcal{V}_m}\Big(|\xi_i^s(t)|^2 + |\xi_j^s(t)|^2\Big)\Big) \\
\leq&\frac{1}{2}\Phi\delta_2\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\sum_{s=1}^{n}|\xi_i^s(t)|^2\Big(N - (r_k - r_{k-1})\Big) \\
\leq&\Phi\delta_2\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)(N - \underline{N})V(t),
\end{aligned}
\tag{16}
$$

where $\Phi = \max_{i\in\mathcal{V}_k, j\in\mathcal{V}_{k'}, k\neq k'}\{|a_{ij}|\}$, $\underline{N} = \min_{k=1,2,\cdots,m}\{r_k - r_{k-1}\}$.

On the basis of Lemmas 2 and 3, one has

$$
\begin{aligned}
&-\phi\sum_{k=1}^{m}\sum_{i,j=r_{k-1}+1}^{r_k} a_{ij}\xi_i^T(t)\text{sign}(x_i(t) - x_j(t)) \\
&= -\frac{1}{2}\phi\sum_{k=1}^{m}\sum_{i,j=r_{k-1}+1}^{r_k} a_{ij}(\xi_i(t) - \xi_j(t))^T\text{sign}(\xi_i(t) - \xi_j(t)) \\
&\leq -\frac{1}{2}\phi\sum_{k=1}^{m}\Big(\sum_{i,j=r_{k-1}+1}^{r_k} a_{ij}^2\|\xi_i(t) - \xi_j(t)\|_2^2\Big)^{\frac{1}{2}}
\end{aligned}
$$

$$\leq -\frac{1}{2}\phi\sum_{k=1}^{m}\Big(2\lambda_2(\mathcal{L}_{kk}^2)\tilde{\xi}_k^T(t)\tilde{\xi}_k(t)\Big)^{\frac{1}{2}}$$
$$\leq -\phi\underline{\lambda}'V^{\frac{1}{2}}(t). \tag{17}$$

where $\underline{\lambda}' = \min_{k=1,2,\cdots,m}\{\lambda_2^{\frac{1}{2}}(\mathcal{L}_{kk}^2)\}$. Note that

$$\begin{aligned}\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\xi_i^T(t)w_i(x_i(t),t) &\leq \tilde{\omega}\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\|\xi_i(t)\|_2\\ &\leq \tilde{\omega}\sum_{k=1}^{m}(r_k-r_{k-1})^{\frac{1}{2}}\Big(\sum_{i=r_{k-1}+1}^{r_k}\|\xi_i(t)\|_2^2\Big)^{\frac{1}{2}}\\ &\leq \tilde{\omega}(2\overline{N}m)^{\frac{1}{2}}V^{\frac{1}{2}}(t),\end{aligned} \tag{18}$$

where $\overline{N} = \max_{k=1,2,\cdots,m}\{r_k - r_{k-1}\}$. According to Eqs (14)-(18) and Condition (11), the following results can be obtained

$$\begin{aligned}\dot{V}(t) \leq &-\Big(\Big(2\underline{\lambda}\delta_1 - \Phi\delta_2(N-\underline{N})\Big)\alpha - 2\rho\Big)V(t)\\ &-\Big(2\underline{\lambda}\delta_1 - \Phi\delta_2(N-\underline{N})\Big)\beta\frac{\dot{\nu}(t)}{\nu(t)}V(t) - (\phi\underline{\lambda}' - \tilde{\omega}(2\overline{N}m)^{\frac{1}{2}})V^{\frac{1}{2}}(t)\\ \leq &-c_1V(t) - c_2\frac{\dot{\nu}(t)}{\nu(t)}V(t),\end{aligned} \tag{19}$$

where $c_1 = \Big(2\underline{\lambda}\delta_1 - \Phi\delta_2(N-\underline{N})\Big)\alpha - 2\rho$, $c_2 = \Big(2\underline{\lambda}\delta_1 - \Phi\delta_2(N-\underline{N})\Big)\beta$.
Based on Lemma 1, from Eq (19) we obtain

$$V(t) \leq \nu^{-c_2}(t)\exp(-c_1t)V(0), \quad t\in[0,T). \tag{20}$$

Let $\xi(t) = (\xi_1^T(t), \xi_2^T(t), \cdots, \xi_N^T(t))^T$. Furthermore, we can get that

$$\|\xi(t)\|_2 \leq \sqrt{2V(0)}\nu^{-\frac{c_2}{2}}(t), \quad t\in[0,T), \tag{21}$$

and

$$\lim_{t\to T}\|\xi(t)\|_2 \leq \sqrt{2V(0)}(1-\theta)^{\frac{hc_2}{2}}. \tag{22}$$

For $t \geq T$, we have $\dot{V}(t) \leq -c_1V(t)$. It can be obtained that $\|\xi(t)\|_2 \leq \sqrt{2V(0)}\exp(-\frac{c_1}{2}(t-T))$, which indicates $\lim_{t\to\infty}\|\xi(t)\|_2 = 0$. Therefore, the prescribed-time cluster practical consensus can be achieved by the above analysis process.

Remark 3. In [27], a kind of time-varying scaling function $\mu(t)$ is utilized in the controller, in which $\mu(t) = \frac{T^h}{(T-t)^h}$ for $t \in [0,T)$. In this case, when the system state reaches consensus, the boundedness of the controller $u_i(t) = -(k + \frac{\dot{\mu}(t)}{\mu(t)})e_i(t)$ needs to be further verified. Because the function $\frac{\dot{\mu}(t)}{\mu(t)}$ grows to infinity when t approaches T, this may make the controller $u_i(t)$ unbounded. In this paper, a new time-varying function $\nu(t) = \frac{T^h}{(T-\theta t)^h}$ is introduced to avoid discussing the boundedness of the controller $u_i(t)$, as $\lim_{t\to T}\nu^{\frac{1}{h}}(t)$ is bounded. Specifically, when $\theta = 1$ in function $\nu(t)$, the function $\nu(t)$ becomes $\mu(t)$. Therefore, this time-varying function $\nu(t)$ can be viewed as an improved version of that in [27].

3.2. Event-triggered control protocol

From the protocol (10), we can clearly find out the controller requires continuous communication, which may lead to wasting a lot of resources and aggravating the communication burden. To overcome the above shortcomings, we will design an event-triggered control scheme. It is worth noting that different clusters may eventually overlap, so we design a leader $\dot{x}_{0k} = f(x_{0k}(t), t)$ $(k = 1, 2, \cdots, m)$ for each cluster.

With the above preparation, a novel prescribed-time control protocol is designed as follows

$$u_i(t) = -\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\Big(\delta_1 y_i(t_l^i) + \delta_2\tilde{y}_i(t_l^i)\Big) - \phi\mathrm{sign}(y_i(t_l^i)), \quad t \in [t_l^i, t_{l+1}^i), \quad i \in \mathcal{V}_k, \tag{23}$$

where

$$y_i(t) = \sum_{j\in\mathcal{V}_k} a_{ij}(x_i(t) - x_j(t)) + b_i(x_i(t) - x_{0k}(t)),$$

and

$$\tilde{y}_i(t) = \sum_{k'\neq k}\sum_{j\in\mathcal{V}_{k'}} a_{ij}(x_i(t) - x_j(t)),$$

α, β, ϕ are defined as that in Eq (10), and t_l^i $(t_0^i = 0)$ is the lth triggering time instant of agent i. The following notations are given: The communication graph composed of $r_k - r_{k-1}$ followers and one leader is denoted by $\tilde{\mathcal{G}}_k$. The communication subgraph among followers is defined as $\mathcal{G}_k$. Define $\mathcal{H}_{kk} = \mathcal{L}_{kk} + \mathcal{B}_k$, where $\mathcal{B}_k = \mathrm{diag}(b_i)$ and $i \in \mathcal{V}_k$.

Assumption 3. The graph $\tilde{\mathcal{G}}_k$ contains a directed spanning tree with the leader as the root node, where $k = 1, 2, \cdots, m$.

Next, the triggering mechanism is constructed as

$$t_{l+1}^i = \inf\{t > t_l^i : \|E_i(t)\|_2 > \eta_1\|y_i(t)\|_2 + \eta_2\varrho(t)\}, \tag{24}$$

where the measurement error is

$$\begin{aligned} E_i(t) =& \Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\Big(\delta_1 y_i(t_l^i) + \delta_2\tilde{y}_i(t_l^i)\Big) + \phi\mathrm{sign}(y_i(t_l^i)) \\ &- \Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\Big(\delta_1 y_i(t) + \delta_2\tilde{y}_i(t)\Big) - \phi\mathrm{sign}(y_i(t)), \end{aligned} \tag{25}$$

and $\varrho(t)$ is defined as $\varrho(t) = \nu^{-\varphi}(t)$ and $\varrho(t) = 0$ at $t \in [0, T)$ and $t \in [T, \infty)$, respectively, and η_1, η_2, φ are positive constants to be designed.

Theorem 2. For System (8), if Assumptions 1-3 hold and the control parameters $\beta > 0, \delta_2 > 0$, and α, δ_1, ϕ satisfy

$$\alpha > \frac{\frac{\rho N\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})} + \eta_1\lambda_{\max}(\mathcal{H})}{\delta_1\lambda_{\min}(\mathcal{H}) - 2\delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}}, \quad \delta_1 > \frac{\delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}}{\lambda_{\min}(\mathcal{H})}, \quad \phi \geq \eta_2 + \tilde{w}, \tag{26}$$

then the prescribed-time cluster practical consensus problem can be achieved under the distributed event-triggered control protocol (23). Additionally, Zeno behavior can be excluded.

Proof. Define $\tau_i(t) = x_i(t) - x_{0k}(t)$ and $\tilde{x}_k(t) = [\tau^T_{r_{k-1}+1}(t), \tau^T_{r_{k-1}+2}(t), \cdots, \tau^T_{r_k}(t)]^T$, $\tilde{x}(t) = [\tilde{x}^T_1(t), \tilde{x}^T_2(t), \cdots, \tilde{x}^T_m(t)]^T$. Consider the Lyapunov function as

$$V(t) = \frac{1}{2}\sum_{k=1}^{m} \tilde{x}^T_k(t)(\mathcal{H}_{kk} \otimes I_n)\tilde{x}_k(t). \tag{27}$$

The derivative of $V(t)$ is given by

$$\begin{aligned}\dot{V}(t) = &\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k} y^T_i(t)\Big(f(t, x_i(t)) + w_i(t, x_i(t)) - E_i(t) \\ &- \Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\Big(\delta_1 y_i(t) + \delta_2\tilde{y}_i(t)\Big) - \phi\text{sign}(y_i(t)) - f(t, x_{0k}(t))\Big).\end{aligned} \tag{28}$$

Since $\|\tau_i(t)\|_2 \le \|\tilde{x}(t)\|_2 \le \frac{\sqrt{\tilde{x}^T(t)(\mathcal{H}\otimes I_n)(\mathcal{H}\otimes I_n)\tilde{x}(t)}}{\lambda_{\min}(\mathcal{H})} = \frac{\sqrt{\sum_{i=1}^{N}\|y_i(t)\|_2^2}}{\lambda_{\min}(\mathcal{H})} \le \frac{\sum_{i=1}^{N}\|y_i(t)\|_2}{\lambda_{\min}(\mathcal{H})}$, where $\mathcal{H} = \text{diag}(\mathcal{H}_{11}, \mathcal{H}_{22}, \cdots, \mathcal{H}_{mm})$, we have

$$\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k} y^T_i(t)(f(t, x_i(t)) - f(t, x_0(t))) \le \frac{\rho N}{\lambda_{\min}(\mathcal{H})}\sum_{i=1}^{N}\|y_i(t)\|_2^2. \tag{29}$$

Based on triggering condition (24) and Eq (29), we have

$$\begin{aligned}\dot{V}(t) \le &\eta_1\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\|y_i\|_2^2 - \Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k} y^T_i(t)(\delta_1 y_i(t) + \delta_2\tilde{y}_i(t)) \\ &+ \frac{\rho N}{\lambda_{\min}(\mathcal{H})}\sum_{i=1}^{N}\|y_i(t)\|_2^2 - \Big(\phi - \eta_2\varrho(t) - \tilde{w}\Big)\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\|y_i\|_2.\end{aligned} \tag{30}$$

According to Lemma 3 and Assumption 1, we can get

$$\begin{aligned}\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k} y^T_i(t)\tilde{y}_i(t) \le &\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\sum_{s=1}^{n}|y^s_i(t)||\tilde{y}^s_i(t)| \\ \le &\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\sum_{s=1}^{n}|y^s_i(t)||\sum_{j\in N\setminus\mathcal{V}_k} a_{ij}x^s_j(t)| \\ \le &\Phi\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\sum_{s=1}^{n}|y^s_i(t)||\sum_{j\in N\setminus\mathcal{V}_k}\tau^s_j(t)| \\ \le &\Phi\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\|y_i(t)\|_2\|\tilde{x}(t)\|_1 \\ \le &2N\Phi\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}V(t),\end{aligned} \tag{31}$$

where $\Phi = \max\limits_{i\in\mathcal{V}_k, j\in\mathcal{V}_{k'}, k\neq k'}\{|a_{ij}|\}$. Since $2\lambda_{\min}(\mathcal{H})V(t) \leq \sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k} y_i^T(t)y_i(t) \leq 2\lambda_{\max}(\mathcal{H})V(t)$, it thus follows from Eqs (30) and (31) that

$$\begin{aligned}
\dot{V}(t) \leq & -\Big(\Big(\delta_1\lambda_{\min}(\mathcal{H}) - \delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}\Big)2\alpha - \frac{2\rho N\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})} - 2\eta_1\lambda_{\max}(\mathcal{H})\Big)V(t) \\
& - \Big(\delta_1\lambda_{\min}(\mathcal{H}) - \delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}\Big)2\beta\frac{\dot{\nu}(t)}{\nu(t)}V(t) - \Big(\phi - \eta_2\varrho(t) - \tilde{w}\Big)\sum_{k=1}^{m}\sum_{i\in\mathcal{V}_k}\|y_i\|_2 \\
\leq & -\Big(\Big(\delta_1\lambda_{\min}(\mathcal{H}) - \delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}\Big)2\alpha - \frac{2\rho N\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})} - 2\eta_1\lambda_{\max}(\mathcal{H})\Big)V(t) \\
& - \Big(\delta_1\lambda_{\min}(\mathcal{H}) - \delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}\Big)2\beta\frac{\dot{\nu}(t)}{\nu(t)}V(t) \\
= & -c_1V(t) - c_2\frac{\dot{\nu}(t)}{\nu(t)}V(t),
\end{aligned} \tag{32}$$

where $c_1 = -\Big(\Big(\delta_1\lambda_{\min}(\mathcal{H}) - \delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}\Big)2\alpha - \frac{2\rho N\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})} - 2\eta_1\lambda_{\max}(\mathcal{H})\Big)$, and $c_2 = -\Big(\delta_1\lambda_{\min}(\mathcal{H}) - \delta_2\Phi N\sqrt{\frac{n\lambda_{\max}(\mathcal{H})}{\lambda_{\min}(\mathcal{H})}}\Big)$.

The following proof can refer to Theorem 1, and the detailed proof is omitted here.

Now, we will prove that Zeno behavior does not occur. For the case of $t \in [0, T)$, under the definition of measurable errors $E_i(t)$, one gets

$$\begin{aligned}
\Big\|\dot{E}_i(t)\Big\|_2 = & \Big\| -\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)'\Big(\delta_1y_i(t) + \delta_2\tilde{y}_i(t)\Big) - \Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\Big(\delta_1y_i(t) + \delta_2\tilde{y}_i(t)\Big)'\Big\|_2 \\
\leq & \Big\|\frac{\beta h\theta^2}{(T-\theta t)^2}\Big(\delta_1y_i(t) + \delta_2\tilde{y}_i(t)\Big)\Big\|_2 + \Big\|\Big(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)}\Big)\Big(\delta_1\dot{y}_i(t) + \delta_2\dot{\tilde{y}}_i(t)\Big)\Big\|_2 \\
\leq & \frac{\delta_1\beta h\theta^2}{(T-\theta t)^2}\|y_i(t)\|_2 + \frac{\delta_2\beta h\theta^2}{(T-\theta t)^2}\|\tilde{y}_i(t)\|_2 \\
& + \delta_1(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)})\|\dot{y}_i(t)\|_2 + \delta_2(\alpha + \beta\frac{\dot{\nu}(t)}{\nu(t)})\|\dot{\tilde{y}}_i(t)\|_2.
\end{aligned} \tag{33}$$

According to Lemma 3, we have

$$\|y_i(t)\|_2 \leq \sum_{i=1}^{N}\|y_i(t)\|_2 \leq N^{\frac{1}{2}}\Big(\sum_{i=1}^{N}y_i^T(t)y_i(t)\Big)^{\frac{1}{2}} \leq N^{\frac{1}{2}}\Big(2\lambda_{\max}(\mathcal{H})V(0)\Big)^{\frac{1}{2}}, \tag{34}$$

and

$$\begin{aligned}
\|\tilde{y}_i(t)\|_2 = & \Big\|\sum_{j\in N\setminus\mathcal{V}_k}a_{ij}x_j(t)\Big\|_2 \\
= & \Big\|\sum_{j\in N\setminus\mathcal{V}_k}a_{ij}(x_j(t) - x_{0k}(t) + x_{0k}(t))\Big\|_2
\end{aligned}$$

$$\leq \Phi\sqrt{\frac{2Nn}{\lambda_{\min}(\mathcal{H})}V(0)}. \tag{35}$$

Furthermore, based on the definition of $y_i(t)$ and $\tilde{y}_i(t)$, one has

$$\begin{aligned}\|\dot{y}_i(t)\|_2 =&\Big\|\sum_{j\in\mathcal{V}_k} a_{ij}(\dot{x}_i(t)-\dot{x}_j(t))+b_i(\dot{x}_i(t)-\dot{x}_{0k}(t))\Big\|_2\\ \leq&\Big\|\sum_{j\in\mathcal{V}_k} h_{ij}^k u_j(t_{l_j}^j)\Big\|_2+\rho\sum_{j\in\mathcal{V}_k} a_{ij}\Big(\|\tau_i(t)\|_2+\|\tau_j(t)\|_2\Big)\\ &+\rho b_i\|\tau_i(t)\|_2+\tilde{w}\Big(2\sum_{j\in\mathcal{V}_k} a_{ij}+b_i\Big),\end{aligned} \tag{36}$$

where h_{ij}^k is the element of matrix $\mathcal{H}_{kk}$. Combining with Theorem 2, we can easily get that $\tau_i(t)$ is bounded over the time interval $[0, T)$, i.e., there exists a positive constant M such as $\|\tau_i(t)\|_2 \leq M$. Then we get

$$\|\dot{y}_i(t)\|_2 \leq \Big\|\sum_{j\in\mathcal{V}_k} h_{ij}^k u_j(t_{l_j}^j)\Big\|_2+(M\rho+\tilde{w})(2\sum_{j\in\mathcal{V}_k} a_{ij}+b_i). \tag{37}$$

Similar to the analysis of $\|\dot{y}_i(t)\|_2$, we can obtain

$$\|\dot{\tilde{y}}_i(t)\|_2 \leq \Big\|\sum_{j\in N\setminus\mathcal{V}_k} h_{ij}^k u_j(t_{l_j}^j)\Big\|_2+(2\rho M+\tilde{w})\Phi N. \tag{38}$$

Substituting Eqs (34), (35), (37), and Eq (38) into Eq (33) yields

$$\begin{aligned}\|\dot{E}_i(t)\|_2 \leq&\frac{\delta_1\beta h\theta^2}{(T-\theta t)^2}\|y_i(t)\|_2+\frac{\delta_2\beta h\theta^2}{(T-\theta t)^2}\|\tilde{y}_i(t)\|_2\\ &+\delta_1\Big(\alpha+\frac{\beta h\theta}{T}\nu^{\frac{1}{h}}(t)\Big)\|\dot{y}_i(t)\|_2+\delta_2\Big(\alpha+\frac{\beta h\theta}{T}\nu^{\frac{1}{h}}(t)\Big)\|\dot{\tilde{y}}_i(t)\|_2\\ \leq&\frac{\delta_1\beta h\theta^2}{(T-\theta t)^2}N^{\frac{1}{2}}\Big(2\lambda_{\max}(\mathcal{H})V(0)\Big)^{\frac{1}{2}}+\frac{\delta_2\beta h\theta^2}{(T-\theta t)^2}\Phi\sqrt{\frac{2Nn}{\lambda_{\min}(\mathcal{H})}V(0)}\\ &+\delta_1\Big(\alpha+\frac{\beta h\theta}{T}\nu^{\frac{1}{h}}(t)\Big)\Big(\Big\|\sum_{j\in\mathcal{V}_k} h_{ij}^k u_j(t_{l_j}^j)\Big\|_2+(M\rho+\tilde{w})(2\sum_{j\in\mathcal{V}_k} a_{ij}+b_i)\Big)\\ &+\delta_2\Big(\alpha+\frac{\beta h\theta}{T}\nu^{\frac{1}{h}}(t)\Big)\Big(\Big\|\sum_{j\in N\setminus\mathcal{V}_k} h_{ij}^k u_j(t_{l_j}^j)\Big\|_2+(2\rho M+\tilde{w})\Phi N\Big)\\ =&(\chi_1+\chi_2)\frac{\beta h\theta^2}{T^2}\nu^{\frac{2}{h}}(t)+(\chi_3+\chi_4)\Big(\alpha+\frac{\beta h\theta}{T}\nu^{\frac{1}{h}}(t)\Big),\end{aligned} \tag{39}$$

where $\chi_1 = \delta_1 N^{\frac{1}{2}}\Big(2\lambda_{\max}(\mathcal{H})V(0)\Big)^{\frac{1}{2}}$, $\chi_2 = \delta_2\Phi\sqrt{\frac{2Nn}{\lambda_{\min}(\mathcal{H})}V(0)}$, $\chi_3 = \delta_1\Big(\Big\|\sum_{j\in\mathcal{V}_k} h_{ij}^k u_j(t_{l_j}^j)\Big\|_2+(M\rho+\tilde{w})(2\sum_{j\in\mathcal{V}_k} a_{ij}+b_i)\Big)$, $\chi_4 = \delta_2\Big(\Big\|\sum_{j\in N\setminus\mathcal{V}_k} h_{ij}^k u_j(t_{l_j}^j)\Big\|_2+(2\rho M+\tilde{w})\Phi N\Big)$. Since $E_i(t_l^i) = 0$, it yields

$$\|E_i(t)\|_2 \leq \int_{t_l^i}^{t}\Big((\chi_1+\chi_2)\frac{\beta h\theta^2}{T^2}\nu^{\frac{2}{h}}(s)+(\chi_3+\chi_4)\Big(\alpha+\frac{\beta h\theta}{T}\nu^{\frac{1}{h}}(s)\Big)\Big)ds+\|E_i(t_l^i)\|_2$$

$$\leq \int_{t_l^i}^{t_{l+1}^i} \Big((\chi_1 + \chi_2) \frac{\beta h \theta^2}{T^2} \nu^{\frac{2}{h}}(s) + (\chi_3 + \chi_4)\Big(\alpha + \frac{\beta h \theta}{T} \nu^{\frac{1}{h}}(s)\Big)\Big) ds. \tag{40}$$

Since $\nu(t_{l+1}^i) \leq \frac{1}{(1-\theta)^h}$ holds, Eq (40) can be rewritten as

$$\|E_i(t)\|_2 \leq \pi(t_{l+1}^i - t_l^i), \tag{41}$$

where $\pi = \Big((\chi_1 + \chi_2)\frac{\beta h \theta^2}{T^2}\frac{1}{(1-\theta)^2} + (\chi_3 + \chi_4)\Big(\alpha + \frac{\beta h \theta}{T}\frac{1}{(1-\theta)}\Big)\Big) > 0$.

According to the triggering condition (24), the next triggering instant t_{l+1}^i must satisfy $\pi(t_{l+1}^i - t_l^i) \geq \eta_2 \varrho(t_{l+1}^i)$, i.e., $(t_{l+1}^i - t_l^i) \geq \frac{\eta_2 \varrho(t_{l+1}^i)}{\pi}$, where $\varrho(t_{l+1}^i) > 0$, then Zeno behavior does not occur for $t \in [0, T)$.

Next, for the case of $t \in [T, \infty)$, we have $\nu(t) = 1$, and the control protocol (23) turns into

$$u_i(t) = -\Big(\alpha + \beta\frac{h\theta}{T}\Big)\Big(\delta_1 y_i(t_l^i) + \delta_2 \tilde{y}_i(t_l^i)\Big) - \phi \mathrm{sign}(y_i(t_l^i)), \quad t \in [t_l^i, t_{l+1}^i), \quad i \in \mathcal{V}_k. \tag{42}$$

Similarly, the triggering mechanism (24) changes into

$$t_{l+1}^i = \inf\{t > t_l^i : \|E_i(t)\|_2 > \eta_1 \|y_i(t)\|_2\}. \tag{43}$$

For $t \in [T, \infty)$, according to Lemma 1, the prescribed-time event-triggered consensus problem transformed into the asymptotical consensus problem based on the event-triggered mechanism. Recalling literature [39], the authors analyze a necessary condition of Zeno behavior and further transform the exclusion problem of Zeno behavior into the nonexistent problem of proving some finite-time convergence. Exclusion methods for the Zeno phenomenon is similar to the analysis of Yu et al. [39] in [39] and is omitted here.

Remark 4. Different from the fixed-time cluster consensus result in literature [35], where a continuous algorithm is designed and the convergence time is related to other system parameters, we select that the control scheme in this paper has following distinguished features: 1) In order to reduce the communication burden, an event-triggered algorithm is proposed. The convergence time of the proposed algorithm is independent of the control parameters and can be preselected. 2) To avoid overlap of the finial states of different clusters, we design a leader for each cluster.

Remark 5. From the above analysis, it can be seen that when $t \geq T$, the consensus is achieved in an asymptotic manner. In order to further improve the convergence rate in our future work, we will improve the controller (10) to reach consensus in a finite-time convergence manner for $t \geq T$.

4. Numerical simulation

In this section, two numerical examples are conducted to test the performance of Theorems 1 and 2.

Example 1. (Continuous control algorithm) We consider System (8), where the nonlinear dynamics are designed by $f(x_i(t), t) = [0.5x_{i1}(t) + 0.8\sin(t), 0.5x_{i2}(t) + 0.8\sin(t)]^T$ and external disturbances are given as $w_i(x_i(t), t) = [0.1\cos(x_{i1}(t)), 0.1\cos(x_{i2}(t))]^T$, $i = 1, 2, \cdots, 9$. It is easy to get that $\tilde{w} = \sqrt{0.02}$. The initial states are chosen as $x_1(0) = [14, -24]^T$, $x_2(0) = [-13, 22]^T$, $x_3(0) = [20, 10]^T$, $x_4(0) = [15, -25]^T$, $x_5(0) = [-28, 20]^T$, $x_6(0) = [-25, -20]^T$, $x_7(0) = [-28, -26]^T$, $x_8(0) = [25, -19]^T$,

$x_9(0) = [23, 25]^T$. Consider that the MAS consists of nine agents, and the communication topology between agents is exhibited in Figure 1 with three clusters, $\mathcal{V}_1 = \{1, 2, 3\}$, $\mathcal{V}_2 = \{4, 5\}$, and $\mathcal{V}_3 = \{6, 7, 8, 9\}$. Furthermore, the Laplacian matrix of the coupling topologies are given as:

$$\mathcal{L} = \left[\begin{array}{c|c|c} \mathcal{L}_{11} & \mathcal{L}_{12} & \mathcal{L}_{13} \\ \hline \mathcal{L}_{21} & \mathcal{L}_{22} & \mathcal{L}_{23} \\ \hline \mathcal{L}_{31} & \mathcal{L}_{32} & \mathcal{L}_{33} \end{array}\right] = \left[\begin{array}{ccc|cc|cccc} 1 & 0 & -1 & 0.5 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & -0.1 & 0.1 \\ -1 & -1 & 2 & -0.5 & 0.5 & 0 & 0 & 0.1 & -0.1 \\ \hline 0.5 & 0 & -0.5 & 1 & -1 & -0.3 & 0.3 & 0 & 0 \\ -0.5 & 0 & 0.5 & -1 & 1 & 0.3 & -0.3 & 0 & 0 \\ \hline 0 & 0 & 0 & -0.3 & 0.3 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & -0.3 & -1 & 2 & -1 & 0 \\ 0 & -0.1 & 0.1 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0.1 & -0.1 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

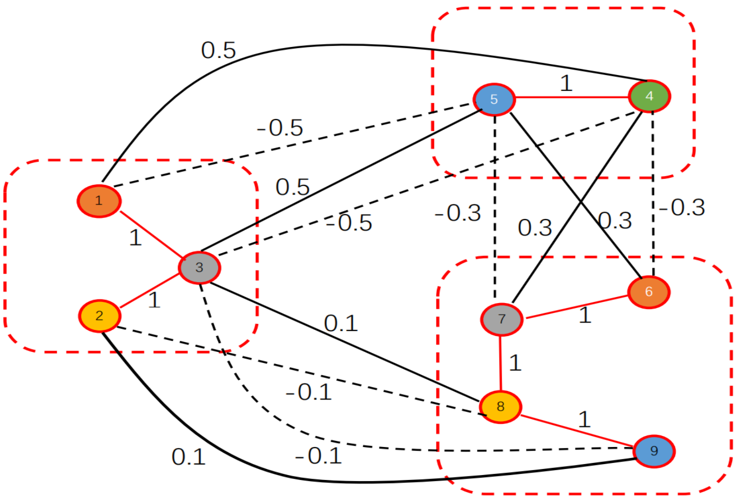

Figure 1. Communication topology.

By calculating, we can get $\underline{\lambda} = 0.3431$, $\Phi = 0.5$, $\underline{N} = 1$, $\overline{N} = 2$, $m = 3$. The other parameters are selected as $\alpha = 1.6, \beta = 2.6, h = 1.8, \phi = 1.1, \delta_1 = 1, \delta_2 = 0.1, \theta = 0.9, T = 0.5$. Based on the initial conditions and related parameters designed above, the observed result is shown in Figures 2 and 3. From Figures 2 and 3, we can get that the cluster consensus is obtained within the prescribed-time $T = 0.5$. To better illustrate the theoretical results, the responses of the system state are shown in Figures 4 and 5 under initial condition $x_1(0) = [-7, 5]^T$, $x_2(0) = [5, -8]^T$, $x_3(0) = [-5, -7]^T$, $x_4(0) = [-10, 7]^T$, $x_5(0) = [0, -4]^T$, $x_6(0) = [5, 9]^T$, $x_7(0) = [8, 2]^T$, $x_8(0) = [-5, 9]^T$, $x_9(0) = [-3, -5]^T$, and $x_1(0) = [0, 11]^T$, $x_2(0) = [-5, 12]^T$, $x_3(0) = [5, -9]^T$, $x_4(0) = [10, 5]^T$, $x_5(0) = [-6, -4]^T$, $x_6(0) = [15, -4]^T$, $x_7(0) = [-8, 2]^T$, $x_8(0) = [6, -2]^T$, $x_9(0) = [1, -15]^T$.

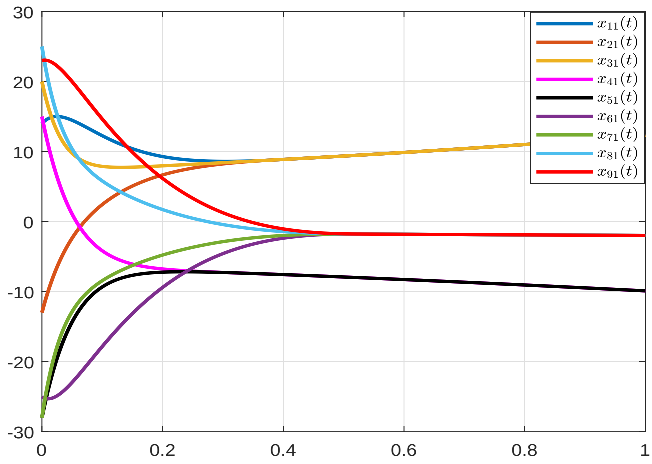

Figure 2. Trajectories of $x_{i1}(t)$.

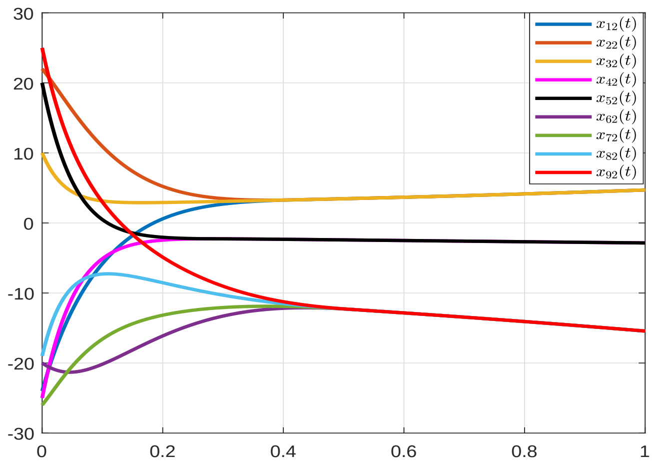

Figure 3. Trajectories of $x_{i2}(t)$.

Remark 6. In order to better verify the theoretical results, we consider the cluster consensus problem in the case of multiple initial values. It can be seen from Figures 4 and 5 that the final state values may be different under different initial value conditions, but the cluster consensus can be achieved within the prescribed-time $T = 0.5$.

Remark 7. As shown in the Figure 1, we can get that the weighted factors a_{ij} satisfy $a_{ij} \geq 0$ for any $i, j \in \mathcal{V}_k$, and $a_{ij} \in \mathcal{R}$ for any $i \in \mathcal{V}_k, j \in N \setminus \mathcal{V}_k$. Positive and negative weights mean that agents have cooperative and competitive relationships between intra- and interclusters. This can be a very realistic representation of a variety of real-life scenarios, such as basketball games and food chains in the animal kingdom.

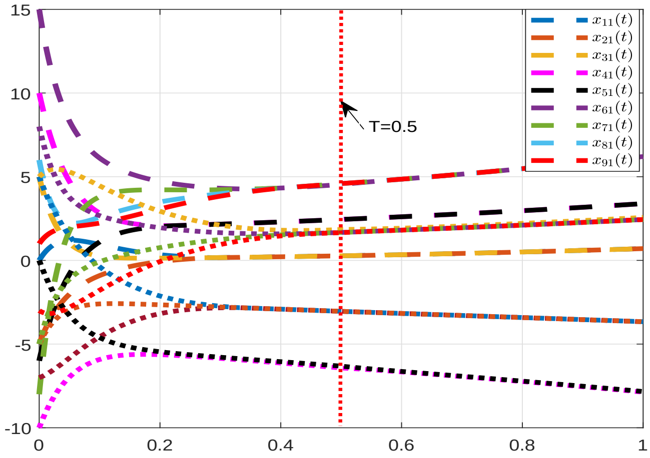

Figure 4. Trajectories of $x_{i1}(t)$.

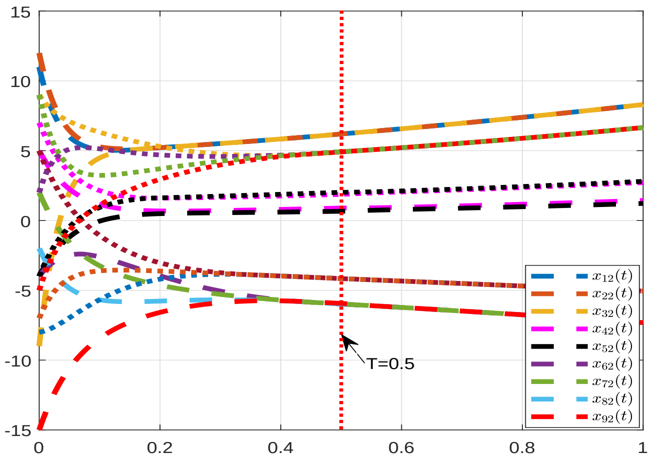

Figure 5. Trajectories of $x_{i2}(t)$.

Example 2. (Event-triggered control algorithm) Consider the MAS consisting of nine agents and three leaders. The communication topology among agents is described by Figure 6. The functions $f(x_i(t), t)$, and external disturbances $w_i(x_i(t), t)$ are chosen the same as those in Example 1. Choose the initial state as $x_1(0) = [13, -14]^T$, $x_2(0) = [-23, 22]^T$, $x_3(0) = [20, 20]^T$, $x_4(0) = [-18, -11]^T$, $x_5(0) = [-30, 15]^T$, $x_6(0) = [-25, -20]^T$, $x_7(0) = [-28, -26]^T$, $x_8(0) = [25, -19]^T$, $x_9(0) = [23, 25]^T$, $x_{01}(0) = [14, -2]^T$, $x_{02}(0) = [3, 22]^T$, $x_{03}(0) = [-4, 11]^T$. The parameters are selected as $\alpha = 5.71, \beta = 0.1, h = 1.3, \phi = 0.6, \delta_1 = 1, \delta_2 = 0.5, \theta = 0.9, \eta_1 = 14, \eta_2 = 0.1, T = 0.5$. From Figures 7 and 8, we can observe that the cluster consensus is obtained within the prescribed time $T = 0.5$. Figure 9 shows the event-triggering instants of nine agents, meaning no Zeno behavior occurs.

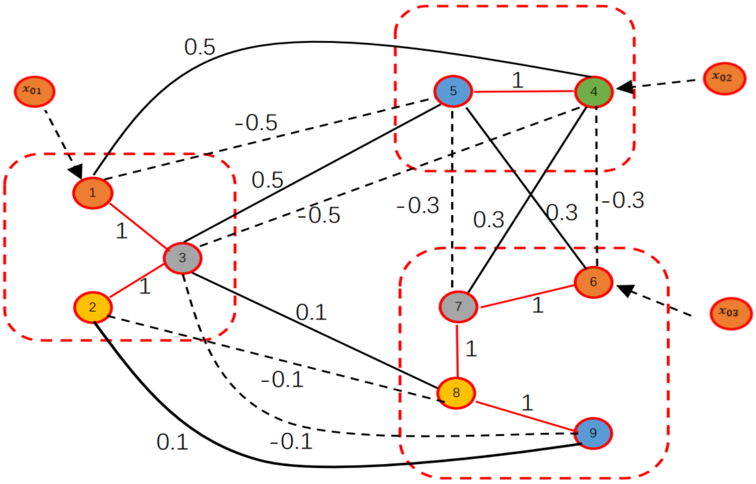

Figure 6. Communication topology.

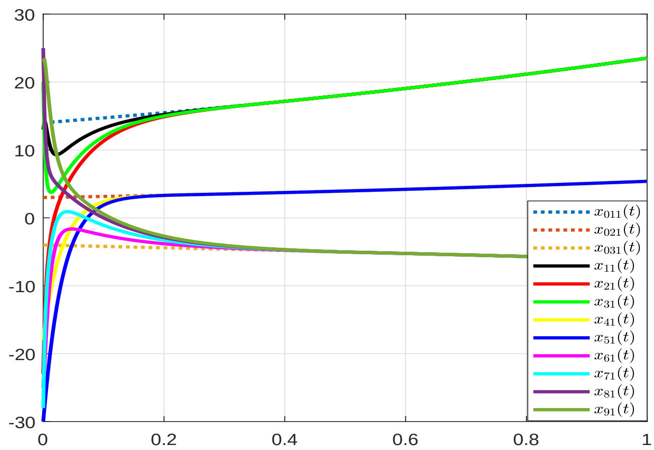

Figure 7. Trajectories of $x_{i1}(t)$.

It can be obtained from Figure 9 that when the settling time approaches 0.5, the number of triggering instants of each agent increases rapidly, which may make it difficult to rule out Zeno behavior. As shown in Figure 10, it can be concluded that the shortest event-triggering interval of nine agents is 0.002. However, the iteration step size in the numerical simulation is 0.001, which implies that Zeno behavior can be ruled out.

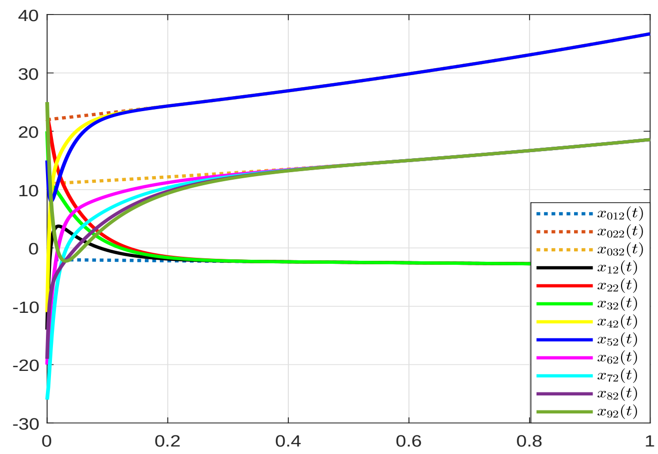

Figure 8. Trajectories of $x_{i2}(t)$.

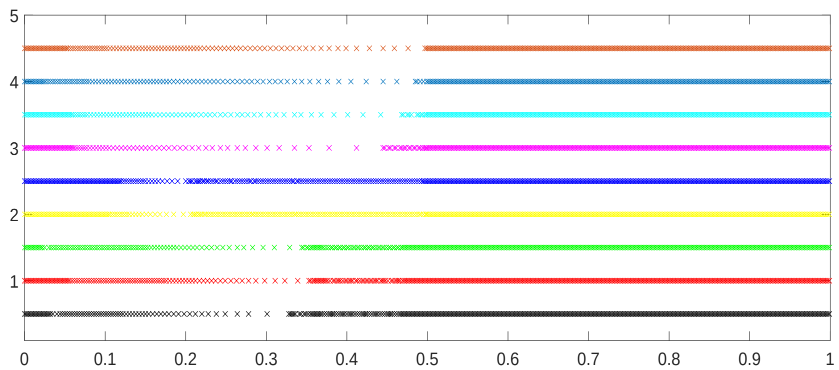

Figure 9. The triggering instants of nine agents.

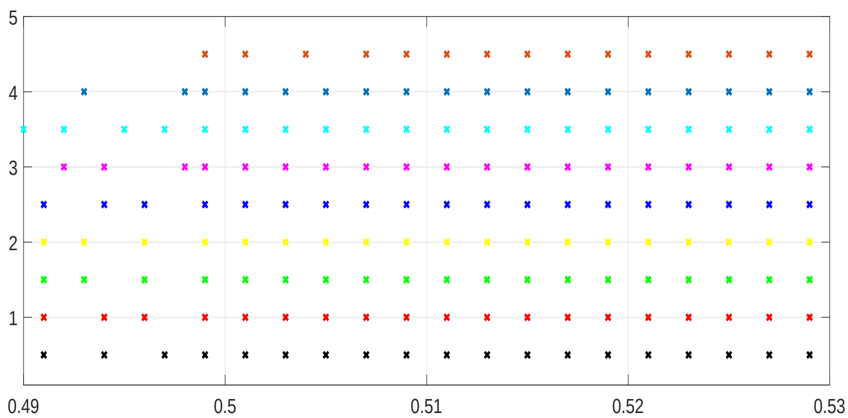

Figure 10. The triggering instants of agents over [0.49, 0.53].

5. Conclusions

In this paper, the prescribed-time cluster practical consensus problem has been investigated for nonlinear MASs with external disturbances. First, a continuous control algorithm was designed such that all agents in the same cluster reach consensus within the prescribed-time. Moreover, compared to the proposed continuous control algorithm, the event-triggered control algorithm was designed to address the prescribed-time cluster consensus issue while reducing the communication cost. Additionally, it has proven that Zeno behavior did not exhibit in the above consensus issue. In our future work, we will focus on the prescribed-time cluster consensus problem for nonlinear MASs with switching directed topologies.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (Grant Nos. 62363033, 62163035), in part by the Natural Science Foundation of Xinjiang Uygur Autonomous Region (Grant No. 2023D01C162), in part by the Special Project for Local Science and Technology Development Guided by the Central Government (Grant No. ZYYD2022A05), and in part by Tianshan Talent Program (Grant No. 2022TSYCLJ0004).

Conflict of interest

The authors declare there is no conflict of interest.

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