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Research article

The large-scale group consensus multi-attribute decision-making method based on probabilistic dual hesitant fuzzy sets

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Abstract: We proposed a novel decision-making method, the large-scale group consensus multi-attribute decision-making method based on probabilistic dual hesitant fuzzy sets, to address the challenge of large-scale group multi-attribute decision-making in fuzzy environments. This method concurrently accounted for the membership and non-membership degrees of decision-making experts in fuzzy environments and the corresponding probabilistic value to quantify expert decision information. Furthermore, it applied to complex scenarios involving groups of 20 or more decision-making experts. We delineated five major steps of the method, elaborating on the specific models and algorithms used in each phase. We began by constructing a probabilistic dual hesitant fuzzy information evaluation matrix and determining attribute weights. The following steps involved classifying large-scale decision-making expert groups and selecting the optimal classification scheme based on effectiveness assessment criteria. A global consensus degree threshold was established, followed by implementing a consensus-reaching model to synchronize opinions within the same class of expert groups. Decision information was integrated within and between classes using an information integration model, leading to a comprehensive decision matrix. Decision outcomes for the objects were then determined through a ranking method. The method's effectiveness and superiority were validated through a case study on urban emergency capability assessment, and its advantages were further emphasized in comparative analyses with other methods.

Keywords: probabilistic dual hesitant fuzzy sets; large-scale group; multi-attribute decision-making; group classification model; consensus-reaching model

Abbreviations: MADM: Multi-attribute decision-making; DM: Decision-making; GDM: Group

Decision-making; LSG-MADM: Large-scale group multi-attribute decision-making; LSG: Large-scale group; LSGC-MADM: Large-scale group consensus multi-attribute decision-making; LSGC: Large-scale group consensus; FSs: Fuzzy sets; IFSs: Intuitionistic fuzzy sets; PFSs: Pythagorean fuzzy sets; HFSs: Hesitant fuzzy sets; PHFSs: Probabilistic hesitant fuzzy sets; PDHFEs: Probabilistic dual hesitant fuzzy elements; PDHFI: Probabilistic dual hesitant fuzzy information; CDM: Consensus decision-making; the LSGC-MADM Method based on PDHFSs: the Large-Scale Group Consensus Multi-Attribute Decision-Making Method based on Probabilistic Dual Hesitant Fuzzy Sets; EPD: Equal probability distance

1. Introduction

Multi-attribute decision-making (MADM) is a process in which multiple people make decisions and consider multiple attributes. It is widely used in the decision-making (DM) of complex problems, such as emergency management [1], watershed management [2], and investment selection [3]. Building on decision-making (DM) research, scholars have developed a variety of models and algorithms for group decision-making (GDM) [4,5]. With the increase in the number of people participating in GDM, the problems related to MADM gradually develop into large-scale group multi-attribute decision-making (LSG-MADM) problems [6–8]. Large-scale group decision-making, frequently encountered in today's society, is advancing as a prominent subject within decision science. Compared with traditional MADM methods, LSG-MADM methods are more applied to scenarios with multi-domain intersection and complex problems [9]. A minimum of twenty decision-making experts is typically required for LSG-MADM processes [10]. Incorporating a consensus mechanism into LSG-MADM issues and setting up the large-scale group consensus multi-attribute decision-making (LSGC-MADM) method can help different viewpoints better fit together [11]. The major areas of study for LSGC-MADM are consensus mechanisms [12], group clusters [13], and cooperative behaviors [14].

Scholars have achieved some research results, such as Du introduced a decision support approach for tackling large-scale decision-making in social networks, merging constrained community detection with multi-stage multi-cost consensus models to address clustering and consensus complexities [15]. Yu et al. enhanced group decision-making with the Enhanced Minimum Cost Consensus Model (EMCC), leveraging explicit adjustment paths and coordination elasticity to prevent over-adjustment and enhance consensus efficiency and adaptability [16]. Chen et al. proposed an expertise-structure and risk-appetite-integrated two-tiered framework for collective opinion generation in large-scale group decision-making [17]. Although scholars have made progress in researching large-scale decision-making, considering the fuzziness of decision-makers' thought processes and the complexity of decision-making events, we address decision information using the form of probabilistic dual hesitant fuzzy sets, thereby better capturing the authenticity and completeness of decision information.

To express the ambiguity and uncertainty of human thinking, since Zadeh proposed the concept of fuzzy sets (FSs) [18], the research theory of fuzzy DM has become more and more abundant. In order to deal with more complex DM problems, Atanasso and Yager defined intuitionistic fuzzy sets (IFSs) [19] and Pythagorean fuzzy sets (PFSs) [20], respectively. Torra proposed hesitant Fuzzy Sets (HFSs) [21], which can better characterize decision-making experts' indecision in evaluating information. Scholars are progressively incorporating probabilistic information into decision-making processes. For instance, Wang introduced a novel approach utilizing the Probabilistic Language Term Set to address the Probabilistic Language Preference Relationship (PLPR) in decision-making scenarios [22]. Liu et al. introduced a group decision-making approach based on the incomplete probabilistic language term set

(InPLTS), effectively managing uncertain decision information through specialized categorization, a mathematical programming model for consistency and consensus, and a reliability-induced operator [23]. Xu and Zhou proposed probabilistic hesitant fuzzy sets (PHFSs) [24] based on HFSs, which can provide membership degrees and their corresponding probabilities. Scholars such as Hao et al. defined probabilistic dual hesitant fuzzy sets (PDHFSs) [25], which can collect membership and non-membership evaluation information and their corresponding probability information. Consequently, PDHFSs offer a more nuanced capability for depicting evaluation information compared to fuzzy sets like PFSs and HFSs.

In recent years, there has been a significant increase in the development of diverse fuzzy sets of DM methods [26,27]. However, it is essential to note that existing research has the following issues: 1) Most PDHFSs' DM methods are based on individual or small and medium-sized group DM. There needs to be more study or literature that discusses large-scale group decision-making based on PDHFSs. 2) When decision-making experts use fuzzy preference relationships to express evaluation information, some existing studies ignore individual consensus levels, which may lead to conflicting DM results, resulting in low consensus in group preference information aggregation.

The literature review highlights a significant research gap in using probabilistic dual hesitant fuzzy information for decision-making in large groups, especially given the recent development of probabilistic dual hesitant fuzzy sets which have not been widely studied worldwide. This study aims to fill this gap by applying these fuzzy sets to make consensus decision-making more accurate and reliable for large groups, addressing the challenges of ambiguity and uncertainty. To more effectively tackle the issues above and the obstacles, this study presents the Large-Scale Group Consensus Multi-Attribute Decision-Making Method based on Probabilistic Dual Hesitant Fuzzy Sets (the LSGC-MADM Method based on PDHFSs). This method is important for advancing the theoretical basis of fuzzy decision-making and offers a practical, scalable solution for various fields where reaching consensus is key. Consequently, this research significantly contributes to the decision-making literature, presenting an effective instrument for navigating complex decision-making scenarios. The decision-making method is specifically designed for use by decision-making experts in complex scenarios involving 20 or more participants. Initially, an evaluation matrix with probabilistic dual hesitant fuzzy information (PDHFI) is formed, drawing on expert preference information. The entropy method is then applied to ascertain the weights of attributes. Following this, group similarity is measured using the equal probability distance metric, and the scheme for expert group classification is determined based on the net-making classification method and the classified test criteria. Next, the consensus-reaching model is adopted to achieve consensus in decision-making opinions within each expert class. Ultimately, the decision-making objects are ranked using the ranking method after integrating decision-making information within and between classes. In contrast to other methods, this method examines experts' probability information, membership degree, and non-membership degree in group decision-making. However, it also looks at the consensus degree of these experts when making large-scale group decisions. Therefore, the method proposed in this study makes the DM results more objective, reasonable and reliable.

The main contributions of this paper are as follows: 1) A comprehensive large-scale group consensus decision-making method, named the Large-Scale Group Consensus Multi-Attribute Decision-Making Method based on Probabilistic Dual Hesitant Fuzzy Sets, is proposed, integrating multiple approaches. 2) A group similarity measurement method is constructed based on probabilistic dual hesitant fuzzy evaluation information, utilizing the equal probability distance method, and a net-making classification method is proposed to classify decision-making experts. 3) A global consensus threshold is established to build the consensus-reaching model, which judges and adjusts the evaluation information of experts within each class, achieving consensus on the decision-making information of experts. 4) A comprehensive expert weight, combining class weight and class deviation weight, is used to obtain a comprehensive information decision matrix, from which the final evaluation result of the decision object is derived. The LSGC-MADM Method based on PDHFSs advances the field of decision-making under uncertainty. This method facilitates the coordination of opinions across different expert categories and the integration of these insights to inform action, offering an effective, scalable solution for large-scale fuzzy decision-making challenges applicable in domains requiring sophisticated decision strategies.

This paper is structured as follows: Section 2 introduces PDHFSs and the related concepts of the LSGC-MADM problem. Section 3 proposes the research framework and this study's specific models and methods. Section 4 conducts a case study on applying the LSGC-MADM Method based on PDHFSs. This study is summarized and projected in Section 5.

2. Basic concepts

2.1. Probabilistic dual hesitant fuzzy sets (PDHFSs)

Definition 1 [28]**:** Let X be the domain, then $PD=\{\langle x,\tilde{h}(x),\tilde{g}(x)\rangle \mid x\in X\}$ is called a probabilistic dual hesitant fuzzy set on X. $\tilde{h}(x)=h(x)\mid p(x)$ and $\tilde{g}(x)=g(x)\mid q(x)$ respectively represent the degree of membership and non-membership and the corresponding probability distribution information, among which $h(x)\mid p(x)=(\gamma_1\mid p_1,\gamma_2\mid p_2,...,\gamma_{\#\tilde{h}(x)}\mid p_{\#\tilde{h}(x)})$ and $g(x)\mid q(x)=(\eta_1\mid q_1,\eta_2\mid q_2,...,\eta_{\#\tilde{g}(x)}\mid q_{\#\tilde{g}(x)})$. $\#\tilde{h}(x)$ and $\#\tilde{g}(x)$ respectively represent the number of corresponding elements in the membership and non-membership degree, and satisfy $\sum_{i=1}^{\#\tilde{h}(x)} p_i \le 1$, $\sum_{j=1}^{\#\tilde{g}(x)} q_j \le 1$, $\gamma_i \ge 0$, $\eta_i \ge 0$, $\gamma^* + \eta^* \le 1$, γ^* and η^* represent the maximum value of the membership degree and non-membership degree, respectively, where $i=1,2,...,\#\tilde{h}(x)$ and $j=1,2,...,\#\tilde{g}(x)$.

Definition 2 [29]**:** For a probabilistic dual hesitant fuzzy element (PDHFE) $pd=\langle \tilde{h}(x),\tilde{g}(x)\rangle$, abbreviated as $pd=\langle \tilde{h},\tilde{g}\rangle=\langle h\mid p,g\mid q\rangle$, its complement pd^c is expressed as Eq (1).

$$pd^c=\begin{cases}\langle \tilde{g},\tilde{h}\rangle=\langle g\mid q,h\mid p\rangle, & \tilde{h}\ne\varnothing,\tilde{g}\ne\varnothing\\ \langle \tilde{h},\varnothing\rangle=\langle 1-h\mid p,\rangle, & \tilde{h}\ne\varnothing,\tilde{g}\ne\varnothing\\ \langle \varnothing,\tilde{g}\rangle=\langle ,1-g\mid q\rangle, & \tilde{h}\ne\varnothing,\tilde{g}\ne\varnothing\end{cases}. \quad (1)$$

Suppose two PDHFEs are pd_1 and pd_2, respectively. The operation law is defined as follows:

$$pd_1\oplus pd_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2,\eta_1\in g_1,\eta_2\in g_2}^{n}\langle(\gamma_1+\gamma_2-\gamma_1\gamma_2)\mid p_1p_2,(\eta_1\eta_2)\mid q_1q_2\rangle, \quad (2)$$

$$pd_1\otimes pd_2=\bigcup_{\gamma_1\in h_1,\gamma_2\in h_2,\eta_1\in g_1,\eta_2\in g_2}\langle(\gamma_1\gamma_2)\mid p_1p_2,(\eta_1+\eta_2-\eta_1\eta_2)\mid q_1q_2\rangle, \quad (3)$$

$$\lambda pd_1 = \bigcup_{\gamma_1 \in h_1, \eta_1 \in g_1} \left\langle 1-(1-\gamma_1)^{\lambda} \mid p_1, \eta_1^{\lambda} \mid q_1 \right\rangle, \tag{4}$$

$$pd_1^{\lambda} = \bigcup_{\gamma_1 \in h_1, \eta_1 \in g_1} \left\langle \gamma_1^{\lambda} \mid p_1, 1-(1-\eta_1)^{\lambda} \mid q_1 \right\rangle, \tag{5}$$

where $\lambda \geq 0$.

Definition 3: Let $pd = \left\langle \tilde{h}, \tilde{g} \right\rangle = \left\langle h \mid p, g \mid q \right\rangle$ be a PDHFE, and the score function of it can be expressed as Eq (6).

$$S^*(pd) = \left[\sum_{i=1}^{\#\tilde{h}} \gamma_i \cdot p_i - \sqrt{\sum_{i=1}^{\#\tilde{h}} p_i \cdot (\gamma_i - \sum_{i=1}^{\#\tilde{h}} \gamma_i \cdot p_i)^2} \right] - \left[\sum_{j=1}^{\#\tilde{g}} \eta_j \cdot q_j - \sqrt{\sum_{j=1}^{\#\tilde{g}} q_j \cdot (\eta_j - \sum_{j=1}^{\#\tilde{g}} \eta_j \cdot q_j)^2} \right]. \tag{6}$$

Among them, $\gamma_i \in h$ and $p_i \in p$ represent the membership value and the corresponding probability of the membership part. $\eta_j \in g$ and $q_j \in q$ represent the non-membership value and the corresponding probability of the non-membership part, respectively.

Definition 4: The comparison between two PDHFEs pd_1 and pd_2 can be expressed as follows:

(I) If $S^*(pd_1) > S^*(pd_2)$ is considered, pd_1 is considered to be better than pd_2, recorded as $pd_1 > pd_2$.

(II) If $S^*(pd_1) = S^*(pd_2)$, it means that pd_1 and pd_2 are indistinguishable, denoted as $pd_1 \sim pd_2$.

Definition 5: Any PDHFE can be normalized. In the normalized PDHFE, the sum of all membership and non-membership probability values is 1, respectively. Let a PDHFE be $pd = \left\langle \tilde{h}, \tilde{g} \right\rangle = \left\langle h \mid p, g \mid q \right\rangle$, and then its normalized form is as Eq (7) [30].

$$pd^N = \left\langle \left\{ \gamma_1 \mid \frac{p_1}{\sum_{i=1}^{\#\tilde{h}} p_i}, \gamma_2 \mid \frac{p_2}{\sum_{i=1}^{\#\tilde{h}} p_i}, \ldots, \gamma_{\#\tilde{h}} \mid \frac{p_{\#\tilde{h}}}{\sum_{i=1}^{\#\tilde{h}} p_i} \right\}, \left\{ \eta_1 \mid \frac{q_1}{\sum_{j=1}^{\#\tilde{g}} q_j}, \eta_2 \mid \frac{q_2}{\sum_{j=1}^{\#\tilde{g}} q_j}, \ldots, \eta_{\#\tilde{g}} \mid \frac{q_{\#\tilde{g}}}{\sum_{j=1}^{\#\tilde{g}} q_j} \right\} \right\rangle. \tag{7}$$

2.2. Description of the large-scale group consensus multi-attribute decision-making (LSGC-MADM) problem

The LSGC-MADM problem is an interactive activity among many individuals in a social environment [31]. Scholars widely study group classification and consensus building as effective methods to solve LSGC-MADM problems. Consensus and selection are the two fundamental processes of the consensus-reaching model [32]. The consensus process includes the measurement of group consensus degree, the identification of disagreements, and the regulation of opinions. The difficulties of large-scale consensus decision problems include the following points: 1) Effective classification of large-scale group members. 2) Identify the individual with a low consensus contribution degree. 3) Measure the consensus level of the population. 4) Building an effective consensus guidance mechanism for the group can quickly reach a consensus. A consensus decision-making (CDM) solution can be obtained by DM members of a large-scale group (LSG) using the LSGC-MADM Method based on PDHFSs proposed in this study.

3. Main methods and models

This section primarily introduces the process and steps of the Large-Scale Group Consensus Multi-Attribute Decision-Making Method based on Probabilistic Dual Hesitant Fuzzy Sets (the LSGC-MADM Method based on PDHFSs). It also provides a detailed overview of the specific methods and models included in this study. The flowchart of the LSGC-MADM Method based on PDHFSs is illustrated in Figure 1.

3.1. The LSGC-MADM method based on PDHFSs

Suppose that in the LSGC-MADM Method based on PDHFSs process, the set of T decision-making experts is $E=\{E_k, k=1,2,...,T\}$, the set of decision-making objects is $A=\{A_i, i=1,2,...,M\}$, and the set of decision-making attributes is $C=\{C_j, j=1,2,...,N\}$. The specific steps of the LSGC-MADM Method based on PDHFSs proposed in this study are summarized as follows.

Step 1: Construct the probabilistic dual hesitant fuzzy information (PDHFI) evaluation matrix and calculate the attribute weights. Decision-making experts provide PDHFI for each attribute of the decision-making object. This process results in the formation of a comprehensive PDHFI evaluation matrix $PD^{(k)}$, encompassing inputs from all experts. The weights ω_j for each attribute C_j are determined using the entropy method. Detailed algorithms and formulas related to Step 1 are presented in Section 3.2.1.

Step 2: Classify decision-making experts into group classes and establish optimal classification. Based on the PDHFI evaluation matrix from all experts, we employ a group classification model to categorize the decision-making experts. The optimal classification result is determined according to the criteria I_p, which are used to test the effectiveness of the classification. Detailed steps of this group classification model are outlined in Section 3.2.2.

Step 3: Calculate and adjust expert opinions to achieve internal consensus within each class. By constructing the consensus-reaching model, the decision-making evaluation value of each class of experts are harmonized to reach internal consensus. For a detailed description of the steps involved in the consensus-reaching model, refer to Section 3.2.3.

Step 4: Utilize the decision-making information integration model to obtain the comprehensive decision matrix. Employ the decision-making information integration model to merge both intra-class and inter-class expert decision-making information, resulting in a comprehensive decision-making information matrix R_i^Z. The specific steps of this model are detailed in Section 3.2.4.

Step 5: Rank decision-making objects to identify the optimal decision. According to the comprehensive decision-making information matrix R_i^Z and the sum of squared deviations SR_i^2 of the comprehensive decision information, different decision-making objects $A_i, A_{i^*} (i, i^*=1,2,...,M)$ are ranked to determine the optimal decision-making result. The specific steps of the ranking method are shown in Section 3.2.5.

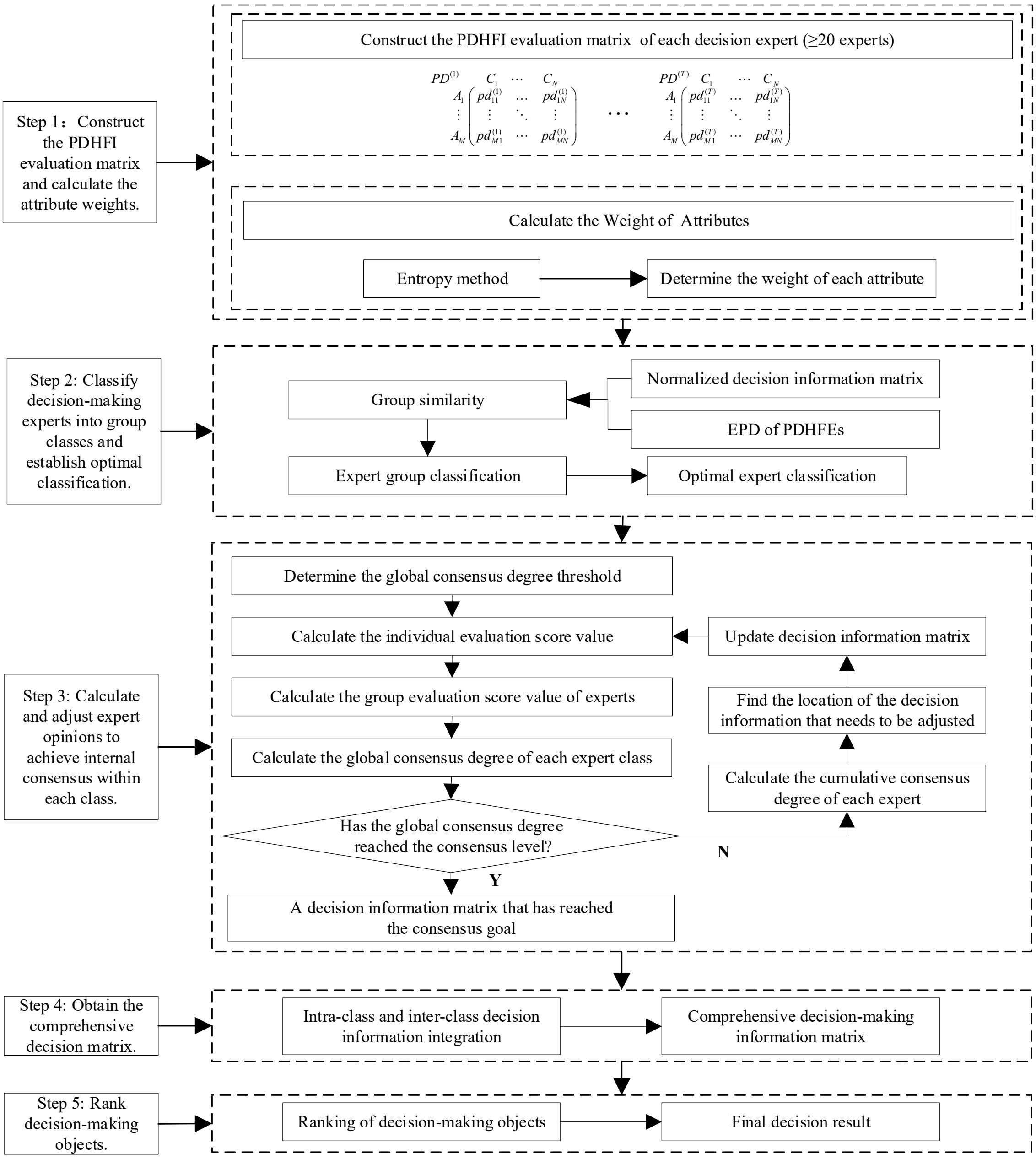

Figure 1. The flowchart of the LSGC-MADM Method based on PDHFSs.

3.2. Implementation steps of models and methods

3.2.1. The PDHFI evaluation matrix and the attribute weights

(I) The PDHFI evaluation matrix

$pd_{ij}^{(k)}$ represents the evaluation value of the kth expert E_k on the decision-making attribute C_j of the object A_i. $PD^{(k)}$ refers to the PDHFI evaluation matrix provided by the kth expert, as shown in Eq (8). $pd_{ij}^{(k)} = \left\langle \tilde{h}_{ij}, \tilde{g}_{ij} \right\rangle = \left\langle \tilde{h}_{ij} \mid p_{ij}, \tilde{g}_{ij} \mid q_{ij} \right\rangle$, where $i = 1,2,...M$, $j = 1,2,...,N$.

$$PD^{(k)} = \begin{bmatrix} pd_{11}^k & \cdots & pd_{1j}^k \\ \vdots & \ddots & \vdots \\ pd_{i1}^k & \cdots & pd_{ij}^k \end{bmatrix}_{(i \times j)} . \tag{8}$$

(II) The attribute weight

In this study, we calculate the attribute weight ω_j using the entropy method [33]. The entropy value of the decision-making attribute C_j, denoted as e_j, is used in the calculation of ω_j, which is detailed in Eq (9).

$$\omega_j = \frac{1-e_j}{\sum_{j=1}^{N}(1-e_j)} . \tag{9}$$

3.2.2. The group classification model

An expert group classification model is constructed to classify experts into groups according to their decision-making evaluation values. Experts within the same class possess relatively consistent decision-making evaluation value. Therefore, the weights of experts in the same class can be considered as equal values [34].

(I) Group similarity measurement

This section designs a similarity measurement between experts based on the equal probability distance (EPD) to measure the consistency of experts.

(i) Calculation of the equal probability distance (EPD). Assume that two PDHFEs are pd_1 and pd_2, then the EPD between pd_1 and pd_2 is denoted as $EPD(pd_1, pd_2)$, where $0 \le EPD(pd_1, pd_2) \le 1$. The calculation process of $EPD(pd_1, pd_2)$ is as follows:

1) pd_1 and pd_2 are denoted as pd_1^N and pd_2^N after normalization, and the normalized PDHFI evaluation matrix is denoted as $PD^{(k)N}$.

$$pd_1{}^N = \left\langle (\gamma_{11} \mid p_{11}^N, \gamma_{12} \mid p_{12}^N, \dots, \gamma_{1\#\tilde{h}_1} \mid p_{1\#\tilde{h}_1}^N), (\eta_{11} \mid q_{11}^N, \eta_{12} \mid q_{12}^N, \dots, \eta_{1\#\tilde{g}_1} \mid q_{1\#\tilde{g}_1}^N) \right\rangle , \tag{10}$$

$$pd_2{}^N = \left\langle (\gamma_{21} \mid p_{21}^N, \gamma_{22} \mid p_{22}^N, \dots, \gamma_{2\#\tilde{h}_1} \mid p_{2\#\tilde{h}_2}^N), (\eta_{21} \mid q_{21}^N, \eta_{22} \mid q_{22}^N, \dots, \eta_{2\#\tilde{g}_2} \mid q_{2\#\tilde{g}_2}^N) \right\rangle . \tag{11}$$

2) Let $NH_1 = (\gamma_{11} \mid p_{11}^N, \gamma_{12} \mid p_{12}^N, \dots, \gamma_{1\#\tilde{h}_1} \mid p_{1\#\tilde{h}_1}^N)$, $NH_2 = (\gamma_{21} \mid p_{21}^N, \gamma_{22} \mid p_{22}^N, \dots, \gamma_{2\#\tilde{h}_1} \mid p_{2\#\tilde{h}_2}^N)$ and $YH = 0$. Then, compare the probability values of two elements in the first position in NH_1 and NH_2.

a) If $p_{11}^N = p_{21}^N$, let $YH = YH + |\gamma_{11} - \gamma_{21}| p_{11}^N$. Then after deleting the two elements in the first position in NH_1 and NH_2, let $NH_1 = (\gamma_{12} \mid p_{12}^N, \dots, \gamma_{1\#\tilde{h}_1} \mid p_{1\#\tilde{h}_1}^N)$ and $NH_2 = (\gamma_{22} \mid p_{22}^N, \dots, \gamma_{2\#\tilde{h}_1} \mid p_{2\#\tilde{h}_2}^N)$.

b) If $p_{11}^N \succ p_{21}^N$, let $YH = YH + |\gamma_{11} - \gamma_{21}| p_{21}^N$. Then after deleting the element in the first position in NH_2, let $NH_2 = (\gamma_{22} \mid p_{22}^N, \dots, \gamma_{2\#\tilde{h}_2} \mid p_{2\#\tilde{h}_2}^N)$. After replacing the element in the first position in NH_1 with $\gamma_{11} \mid (p_{11}^N - p_{21}^N)$, let $NH_1 = (\gamma_{11} \mid (p_{11}^N - p_{21}^N), \gamma_{12} \mid p_{12}^N, \dots, \gamma_{1\#\tilde{h}_1} \mid p_{1\#\tilde{h}_1}^N)$ $\dots \gamma_{12} \mid p_{12}^N, \dots, \gamma_{1\#\tilde{h}_1} \mid p_{1\#\tilde{h}_1}^N)$.

c) If $p_{11}^N \prec p_{21}^N$, then let $YH = YH + |\gamma_{11} - \gamma_{21}| p_{11}^N$, then delete the element at the first position in NH_1, let $NH_1 = (\gamma_{12} \mid p_{12}^N, \dots, \gamma_{1\#\tilde{h}_1} \mid p_{1\#\tilde{h}_1}^N)$, replace the element at the first position in NH_2 with

$\gamma_{21} | (p_{21}^N - p_{11}^N)$ Let $NH_2 = (\gamma_{21} | (p_{21}^N - p_{11}^N), \gamma_{22} | p_{22}^N, ..., \gamma_{2\#\tilde{h}_2} | p_{2\#\tilde{h}_2}^N)$.

The probability values continue to be compared according to the above method until NH_1 and NH_2 are empty sets, and finally, the value of YH is obtained.

3) The non-membership part of pd_1 and pd_2 is processed according to the above method. YG is used to replace YH, and YG value can be obtained, and $EPD(pd_1, pd_2) = (YH + YG)/2$.

Example 1: Suppose the two probabilistic dual hesitant fuzzy elements are $pd_1 = \langle (0.6|0.3, 0.7|0.4, 0.8|0.3), (0.2|0.2, 0.3|0.8) \rangle$ and $pd_2 = \langle (0.7|0.2, 0.8|0.3, 0.9|0.5), (0.1|0.5, 0.2|0.3, 0.3|0.2) \rangle$. After the above calculation steps, we can get $YH = 0.13$ and $YG = 0.11$, so the equal probability distance (EPD) measure of pd_1 and pd_2 is $EPD(pd_1, pd_2) = (YH + YG)/2 = 0.12$.

(ii) Calculation of the group similarity. $pd_{ij}^{(k)}$ and $pd_{ij}^{(l)}$ represent the evaluation values of the kth and lth experts on the decision-making attribute C_j of the object A_i, where $0 \prec k \prec l \prec T$. The similarity $SM_{i,j}^{k,l}$ between $pd_{ij}^{(k)}$ and $pd_{ij}^{(l)}$ is defined as Eq (12).

$$SM_{i,j}^{k,l} = 1 - EPD(pd_{ij}^{(k)}, pd_{ij}^{(l)}) = 1 - (YH + YG)/2. \tag{12}$$

$SM^{k,l}$ represents the similarity between decision-making experts E_k and E_l, the calculation formula is shown in Eq (13).

$$SM^{k,l} = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{N} \omega_j SM_{i,j}^{k,l}. \tag{13}$$

(II) Classification of expert groups

(i) The net-making classification method based on the similarity matrix. The similarity between decision-making experts constitutes the expert similarity matrix SM_S. Since SM_S is a symmetric matrix, only the upper triangular needs to be calculated, as Eq (14).

$$SM_S = \begin{bmatrix} 1 & SM^{1,2} & \cdots & SM^{1,T} \\ & 1 & \cdots & SM^{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ & & & 1 \end{bmatrix}. \tag{14}$$

This section uses the similarity matrix-based net-making classification method to classify experts [35]. The specific steps are as follows:

1) Set the cut level, that is, the similarity threshold as $\alpha_e (0 \le \alpha_e \le 1)$.

2) Construct an upper triangular matrix P_r, and the rules for the values of matrix P_r are as follows:

a) If $SM^{k,l} \succ \alpha_e (k \ne l)$, let $P_r^{k,l} = 1$.

b) If $SM^{k,l} \le \alpha_e (k \ne l)$, let $P_r^{k,l} = 0$.

3) In the upper triangular matrix P_r, substitute the elements on the main diagonal with the corresponding numbers of the experts. Convert all "1" to "*" in the elements above the main diagonal within P_r, and remove any elements that possess a value of "0". Utilize "*" as the nodal

point for constructing warp and weft lines, thereby weaving the network. Experts E_k linked through this network are deemed to be in the same class. This approach effectively generates a preliminary classification scheme $\Omega_c (c = 1, 2, ...C)$ for the expert groups, with C representing the total number of identified classes.

(ii) Set the classification effect test criteria I_p.

In order to choose the best similarity threshold α_e, it is necessary to set the criteria for checking the classification effect. The larger the proportion of the sum of squares of the inter-class decision information, the better the classification effect of experts [36]. According to Eq (6), the score function $S^*(pd_{ij}^{(k)})$ of the expert E_k on the C_j of the A_i can be obtained. The average information of all experts in the Ω_c on the C_j of the A_i is $o_{ij}^{\Omega_c} = \frac{1}{|\Omega_c|}\sum_{k=1}^{|\Omega_c|} S^*(pd_{ij}^{(k)})(k \in \Omega_c)$, and the average information value is $o_{ij} = \frac{1}{T}\sum_{k=1}^{T} S^*(pd_{ij}^{(k)})$. The definition of the criteria I_p to test the expert classification effect is as Eq (15).

$$I_p = \frac{\sum_{c=1}^{C} |\Omega_c| (\sum_{i=1}^{M}\sum_{j=1}^{N} (o_{ij}^{\Omega_c} - o_{ij})^2)}{\sum_{c=1}^{C}\sum_{k \in \Omega_c} (\sum_{i=1}^{M}\sum_{j=1}^{N} (S^*(pd_{ij}^{(k)}) - o_{ij})^2)}. \tag{15}$$

Among them, $\sum_{c=1}^{C} |\Omega_c| (\sum_{i=1}^{M}\sum_{j=1}^{N} (o_{ij}^{\Omega_c} - o_{ij})^2)$ represents the squared deviation of the inter-class decision information, $|\Omega_c|$ represents the number of experts in class Ω_c, and the total squared deviation of all expert information is $\sum_{c=1}^{C}\sum_{k \in \Omega_c} (\sum_{i=1}^{M}\sum_{j=1}^{N} (S^*(pd_{ij}^{(k)}) - o_{ij})^2)$. When the number of decision expert classes is 1 or each expert constitutes their own class, the classification becomes meaningless. This section calculates the criteria I_p for testing the classification effect by continuously adjusting the similarity threshold $\alpha_e (0 \le \alpha_e \le 1)$. The optimal expert group classification scheme can be obtained when the I_p value is maximized.

3.2.3. The consensus-reaching model

In order to obtain a decision-making scheme that is satisfied by the experts in the same class, it is necessary to consider whether the decision-making opinions of the experts in the same class reach a certain level of consensus. The five steps that comprise the consensus-reaching model that is built in this section are as follows.

(i) Determine the global consensus degree threshold $\bar{C}^{\Omega_c}$. Set the initial value of the adjustment times t to "0", and the initial decision information score matrix to be $R_0^k = S^*(pd_{ij,0}^{(k)})_{M \times N} = S^*(pd_{ij}^{(k)})_{M \times N}$, where $k \in \Omega_c$.

(ii) Calculate the global consensus degree CL^{Ω_c} of the expert group in class Ω_c.

1) Calculate the evaluation score of the individual expert. According to the weighted summation of the expert E_k in the class Ω_c in the dimension of the attribute C_j, the individual evaluation score $\xi_i^k (i = 1, ..., M; k \in \Omega_c)$ of the expert E_k to the object A_i can be obtained, as Eq (16).

$$\xi_i^k = \sum_{j=1}^{N} \omega_j S^*(pd_{ij}^{(k)}). \tag{16}$$

2) Calculate the evaluation score value of the expert group. Since the weights of experts in the same class are treated as equal values, the group evaluation score $\xi_i^{\Omega_c}$ of the object A_i by the expert group in class Ω_c can be obtained by calculating the mean value, as Eq (17).

$$\xi_i^{\Omega_c} = \frac{\sum_{k\in\Omega_c} \xi_i^k}{|\Omega_c|}. \tag{17}$$

3) Calculate the consensus level of the expert class except the expert E_l. According to ξ_i^k and $\xi_i^{\Omega_c}$, CL_i^l is the consensus level of other experts except the expert E_l on the object A_i in the expert group of class Ω_c, as Eq (18). $I(\Omega_c \setminus E_l)$ represents the set of other experts except the expert E_l in the expert group of class Ω_c.

$$CL_i^l = \frac{\sum_{k\in I(\Omega_c \setminus E_l)} (1-|\xi_i^k - \xi_i^{\Omega_c}|)}{|\Omega_c \setminus E_l|}. \tag{18}$$

4) Calculate the consensus level of individual experts. CL_i is the consensus level of individual experts in class Ω_c on the object A_i, as Eq (19).

$$CL_i = \frac{\sum_{k\in I(\Omega_c)} (1-|\xi_i^k - \xi_i^{\Omega_c}|)}{|\Omega_c|}. \tag{19}$$

5) Calculate the global consensus degree of the expert group. CL^{Ω_c} is the global consensus degree of the expert group in class Ω_c, as Eq (20).

$$CL^{\Omega_c} = \frac{1}{M}\sum_{i=1}^{M} CL_i = \frac{1}{M}\frac{\sum_{k\in I(\Omega_c)} (1-|\xi_i^k - \xi_i^{\Omega_c}|)}{|\Omega_c|}. \tag{20}$$

If the value of CL^{Ω_c} is greater than the value of $\overline{C}^{\Omega_c}$, proceed to step 5. Otherwise, proceed to Step 3.

(iii) Determine which experts' opinions require modification and in which locations those opinions need to be modified.

1) Calculate the cumulative consensus degree CD^l of expert E_l, as Eq (21). CD^l reflects the contribution degree of expert E_l to the consensus of the group in the consensus-reaching process. If $CD^l \succ 0$, it means that the expert E_l plays a positive role in the process of group consensus-reaching.

$$CD^l = \sum_{i=1}^{M} (CL_i - CL_i^l). \tag{21}$$

2) Adjust the decision-making information that experts need to modify. Find the expert with the smallest value of CD^l and denote it as E_{ls}. Calculate the consensus contribution degree CD_{ij}^{ls} of

E_{ls} to A_i on the C_j. Denote the smallest value of CD_{ij}^{ls} as $\min(CD_{ij}^{ls})$. At this time, (i,j) is the position where the decision information needs to be adjusted, let (p,q) equal (i,j).

(iv) Adjust the decision information of the element in the position (p,q) to form a new decision-making information matrix.

1) Calculate the consensus contribution degree of individual experts. The consensus level CL_{ij} of all experts in class Ω_c to the A_i on the C_j is calculated as Eq (22). The consensus level of all experts in class Ω_c except the expert E_l to A_i on C_j is CL_{ij}^l, as Eq (23). The consensus contribution degree of E_l in Ω_c to A_i on C_j is CD_{ij}^l, as Eq (24).

$$CL_{ij} = \frac{1}{|\Omega_c|}\sum\nolimits_{k\in I(\Omega_c)}(1 - S^*(pd_{ij}^{(k)}) - \frac{\sum\nolimits_{k\in\Omega_c} S^*(pd_{ij}^{(k)})}{|\Omega_c|}), \tag{22}$$

$$CL_{ij}^l = \frac{1}{|\Omega_c \setminus E_l|}\sum\nolimits_{k'\in I(\Omega_c\setminus E_l)}\begin{pmatrix}(1 - S^*(pd_{ij}^{(k')}) \\ -\frac{\sum\nolimits_{k'\in\Omega_c} S^*(pd_{ij}^{(k')})}{|\Omega_c \setminus E_l|})\end{pmatrix}, \tag{23}$$

$$CD_{ij}^l = CL_{ij} - CL_{ij}^l. \tag{24}$$

Calculate the consensus contribution degree $CD_{pq}^k(k\in\Omega_c)$ of all experts in Ω_c to A_p on the C_q, and set the decision-making expert with the highest consensus contribution degree value as E_{lh}.

2) Adjust the decision-making information of the expert E_{ls} at the (p,q) position [37]. Let λ represent the tuning parameter, satisfying $0 \prec \lambda \prec 1$. Modify the decision information of the (p,q) position while maintaining the information of the other position elements intact, resulting in a new decision-making information score matrix denoted as $\bar{S}^*(pd_{ij,t+1}^{(k)})$, as Eq (25).

$$\bar{S}^*(pd_{ij,t+1}^{(k)}) = \begin{cases}\lambda S^*(pd_{pq}^{(ls)}) \oplus (1-\lambda)S^*(pd_{pq}^{(lh)}), i = p, j = q \\ S^*(pd_{ij,t}^{(k)}), i \neq p, j \neq q\end{cases}. \tag{25}$$

Then, let $t = t+1$, go to Step 2.

(v) After the iterative steps, the final consensus decision-making information matrix $\bar{R}_{ij}^{(k)}(\Omega_c)$ is obtained. Let $\bar{R}_{ij}^{(k)}(\Omega_c) = \bar{S}^*(pd_{ij,t}^{(k)})$ and $E_k \in \Omega_c$. At this juncture, a consensus has been reached by all experts.

3.2.4. The decision-making information integration model

(I) Intra-class decision-making information integration

Integrate the information of $\bar{R}_{ij}^{(k)}(\Omega_c)$ on the attribute dimension ω_j, and obtain the decision-making information matrix $\bar{R}_i^{(k)'}(\Omega_c)$ of each expert for the object A_i, as Eq (26), where $k\in\Omega_c$.

$$\overline{R}_i^{(k)'}(\Omega_c) = \sum_{j=1}^{N} \omega_j \overline{R}_{ij}^{(k)}(\Omega_c) = \sum_{j=1}^{N} \omega_j \overline{S}^*(pd_{ij,t}^{(k)}) . \tag{26}$$

Since the weights of experts within the same class are equal, the decision-making information matrix $\overline{R}_i^{\Omega_c}$ of the class Ω_c expert group for A_i is calculated as shown in Eq (27).

$$\overline{R}_i^{\Omega_c} = \sum_{k=1}^{|\Omega_c|} \overline{R}_i^{(k)'}(\Omega_c) / |\Omega_c| . \tag{27}$$

(II) Inter-class decision-making information integration

In this section, the class weight and class deviation weight are comprehensively considered to obtain the comprehensive decision-making information matrix.

(i) Calculate the class weight ω_n^c, as Eq (28). The class weight is determined by the ratio of the number of experts in this class to the total number of experts.

$$\omega_n^c = \frac{|\Omega_c|}{\sum_{c=1}^{C} |\Omega_c|} . \tag{28}$$

(ii) Calculate the class deviation weight ω_p^c. This section further determines the class deviation weight ω_p^c according to the deviation value between classes.

1) Calculate the distance D_c between the mean value of each class's decision information and all decision information, as Eq (29). The calculation of $o_{ij}^{\Omega_c}$ and o_{ij} is shown in Section 3.2.2.

$$D_c = \sum_{i=1}^{M} \sum_{j=1}^{N} \sqrt{(o_{ij}^{\Omega_c} - o_{ij})^2} . \tag{29}$$

2) Calculate the class deviation weight ω_p^c according to D_c, as Eq (30).

$$\omega_p^c = \frac{D_c}{\sum_{c=1}^{C} D_c} . \tag{30}$$

(iii) Calculate the final comprehensive expert class weight ω_z^c, as Eq (31). Set the preference coefficient $\tau(\tau \in [0,1])$. The class weight ω_n^c and the class deviation weight ω_p^c are comprehensively integrated, and the final comprehensive class weight is ω_z^c.

$$\omega_z^c = \tau \omega_n^c + (1-\tau) \omega_p^c . \tag{31}$$

Generally, when the decision-making result is focused on the opinions of the majority of experts, take $\tau \succ 0.5$. Unless otherwise specified, take $\tau = 0.5$.

(iv) Calculate the comprehensive decision-making information matrix R_i^Z, as Eq (32).

$$R_i^Z = \sum_{c=1}^{C} \omega_z^c \overline{R}_i^{\Omega_c} . \tag{32}$$

3.2.5. The ranking method

By comparing the value of the R_i^Z value, different decision-making objects A_i, A_{i^*} can be compared and ranked, which can be expressed as follows:

(i) If $R_i^Z \succ R_{i^*}^Z$, the ranking result of the object A_i is better than A_{i^*}, denoted as $A_i \succ A_{i^*} (i, i^* = 1,2,...,M)$.

(ii) If $R_i^Z = R_{i^*}^Z$, the squared deviation SR_i^2 of decision-making information needs to be further compared, as Eq (33).

$$SR_i^2 = \sum_{i=1}^{M} (R_i^Z - \frac{\sum_{i=1}^{M} R_i^Z}{M})^2, (i = 1,2,...,M) \cdot \tag{33}$$

1) If $SR_i^2 \succ SR_{i^*}^2$, the ranking result of the object A_i is considered to be better than A_{i^*}, denoted as $A_i \succ A_{i^*}$.

2) If $SR_i^2 = SR_{i^*}^2$, the ranking result of the object A_i and A_{i^*} are considered equal, denoted as $A_i \sim A_{i^*}$.

4. Case study

4.1. Case background

To conduct an in-depth evaluation of the emergency management capabilities of three Chinese cities $A_i (i = 1,2,3)$ in response to sudden incidents, the government decision-making department has initiated a comprehensive assessment project. This evaluation focuses on three core decision-making attributes $C_j (j = 1,2,3)$: emergency support capability (C_1), emergency early warning capability (C_2), and post-disaster recovery capability (C_3), aiming to fully understand the comprehensive strength of each city in crisis response. To ensure the scientific accuracy of the assessment, the government management department has taken into account the geographic location of the cities, historical disaster records, and existing emergency management facilities and resources, while setting preference coefficients to reflect the importance of different attributes. The government department carefully selected 20 decision-making experts $E_k (k = 1,2,...,20)$ with extensive experience and expertise in various fields of emergency management, including but not limited to natural disaster response, public safety, urban planning, health, and sanitation. These experts were asked to provide personalized assessment information based on their professional knowledge and practical experience. In this way, the evaluation aims to capture the unique insights of each expert, with the goal of developing a comprehensive and in-depth understanding of the cities' emergency management capabilities [38].

This case study not only promises to provide valuable insights into the cities' emergency preparedness and response capabilities for the government but also, through comparative analysis, can reveal the strengths and weaknesses of each city in terms of emergency management. This, in turn, can guide future policy making and resource allocation, enhancing the resilience of cities and their ability to respond to sudden public health emergencies.

4.2. Decision-making process

In this section, the LSGC-MADM Method based on PDHFSs is applied to evaluate the emergency management capabilities of three cities. The specific steps are as follows.

Step 1: PDHFI for each attribute of three cities is provided by decision-making experts. The resulting PDHFI evaluation matrix is presented in Table 1 (showing partial information). The weights of each attribute are calculated using the entropy method, as detailed in Table 2.

Step 2: The group classification model in Section 3.2.2 is used to classify all experts and generate classification results. The expert similarity $SM_{i,j}^{k,l}(k \neq l)$ is calculated according to Eq (12). The decision-making experts are classified according to the net-making classification method. It can be calculated that the minimum similarity between the twenty decision-making experts is 0.8247, and the maximum similarity is 0.9155. Classification becomes meaningful when the similarity threshold α_e falls within the interval of [0.8247, 0.9155). The relationship between the classification effect test criteria I_p and α_e value is shown in Figure 2. The experts' classification is optimal when the I_p value peaks at 8.8599. At this point, the experts are divided into four classes, as shown in Table 3.

Step 3: Set the global consensus degree threshold $\overline{C}^{\Omega_c} = 0.85$. As an example, the experts in class Ω_1 reached a consensus using the consensus-reaching model in Section 3.2.3. The consensus decision-making information matrix is obtained after class Ω_1 reaches consensus, as shown in Table 4.

Step 4: Using the decision-making information integration model in Section 3.2.4, the decision-making information matrix $\overline{R}_i^{\Omega_c}$ of the four expert groups for A_i can be obtained, as shown in Table 5. The calculation results of the weights for each class are shown in Table 6. The Eq (32) is used to get the comprehensive decision-making information matrix R_i^Z for the three evaluation cities, shown in Table 7.

Step 5: According to the decision-making score of A_i, the evaluation cities are ranked to obtain the optimal decision scheme. Since $R_1^Z \succ R_3^Z \succ R_2^Z$, the emergency management capability of the three cities is ranked as $A_1 \succ A_3 \succ A_2$.

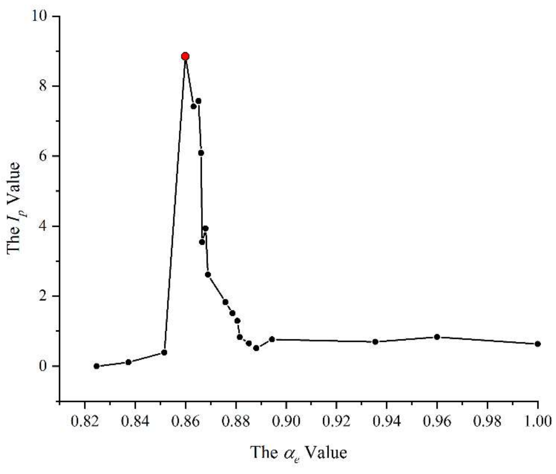

Figure 2. The relationship between I_p and α_e value.

Table 1. The PDHFI evaluation matrix of 20 decision-making experts.

<table>
<tr><th></th><th colspan="3">A_1</th><th colspan="3">A_2</th><th colspan="3">A_3</th></tr>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th><th>C_1</th><th>C_2</th><th>C_3</th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>E_1</td><td><(0.5|0.6,0.6|0.3,0.7|0.1),(0.2|0.3,0.3|0.3,0.4|0.4)></td><td><(0.6|0.5,0.7|0.2,0.8|0.3),(0.3|0.3,0.4|0.3,0.5|0.4)></td><td><(0.5|0.5,0.6|0.2,0.7|0.3),(0.3|0.3,0.4|0.3,0.5|0.4)></td><td><(0.5|0.3,0.6|0.3,0.7|0.3),(0.3|0.1,0.4|0.4,0.5|0.5)></td><td><(0.7|0.2,0.2|0.3,0.9|0.6),(0.1|0.3,0.2|0.3,0.3|0.4)></td><td><(0.2|0.5,0.3|0.2,0.4|0.3),(0.6|0.1,0.7|0.3,0.8|0.6)></td><td><(0.4|0.2,0.5|0.3,0.6|0.5),(0.4|0.3,0.5|0.3,0.6|0.4)></td><td><(0.3|0.5,0.4|0.2,0.5|0.3),(0.5|0.3,0.6|0.3,0.7|0.4)></td><td><(0.3|0.5,0.4|0.2,0.5|0.3),(0.5|0.1,0.4|0.3,0.7|0.6)></td></tr>
<tr><td>E_2</td><td><(0.6|0.4,0.8|0.6),(0.2|0.1,0.3|0.4,0.4|0.5)></td><td><(0.6|0.7,0.7|0.3),(0.3|0.6,0.4|0.4)></td><td><(0.3|0.6,0.4|0.3,0.5|0.1),(0.6|0.3,0.7|0.7)></td><td><(0.4|0.2,0.5|0.3,0.6|0.5),(0.4|0.3,0.5|0.3,0.6|0.4)></td><td><(0.1|0.1,0.8|0.4,0.3|0.6),(0.7|0.6,0.8|0.4)></td><td><(0.3|0.6,0.4|0.3,0.5|0.1),(0.5|0.3,0.6|0.4,0.7|0.3)></td><td><(0.5|0.3,0.6|0.3,0.7|0.3),(0.3|0.1,0.4|0.4,0.5|0.5)></td><td><(0.5|0.6,0.6|0.3,0.7|0.1),(0.3|0.6,0.4|0.4)></td><td><(0.2|0.6,0.3|0.3,0.4|0.1),(0.6|0.3,0.7|0.4,0.8|0.3)></td></tr>
<tr><td>E_3</td><td><(0.1|0.4,0.2|0.5,0.3|0.1),(0.7|0.1,0.8|0.7,0.9|0.2)></td><td><(0.1|0.3,0.2|0.4,0.3|0.3),(0.7|0.1,0.8|0.7,0.9|0.2)></td><td><(0.7|0.3,0.8|0.4,0.9|0.3),(0.2|0.5,0.3|0.4,0.4|0.1)></td><td><(0.3|0.4,0.4|0.3,0.5|0.1),(0.5|0.2,0.6|0.3,0.7|0.5)></td><td><(0.7|0.4,0.2|0.7,0.9|0.2),(0.1|0.1,0.2|0.7,0.3|0.2)></td><td><(0.1|0.3,0.2|0.4,0.3|0.3),(0.7|0.2,0.8|0.4,0.9|0.4)></td><td><(0.1|0.4,0.2|0.3,0.3|0.1),(0.8|0.3,0.9|0.7)></td><td><(0.3|0.1,0.4|0.4,0.5|0.5),(0.5|0.1,0.6|0.7,0.7|0.2)></td><td><(0.1|0.3,0.2|0.4,0.3|0.3),(0.7|0.2,0.3|0.4,0.9|0.4)></td></tr>
<tr><td>E_4</td><td><(0.6|0.3,0.7|0.4,0.8|0.3),(0.2|0.2,0.3|0.8)></td><td><(0.5|0.2,0.6|0.3,0.8|0.5),(0.2|0.2,0.3|0.8)></td><td><(0.3|0.2,0.4|0.3,0.5|0.5),(0.6|0.2,0.7|0.8)></td><td><(0.4|0.3,0.5|0.3,0.6|0.3),(0.4|0.5,0.5|0.2,0.6|0.3)></td><td><(0.3|0.2,0.6|0.8,0.5|0.5),(0.5|0.2,0.6|0.8)></td><td><(0.4|0.2,0.5|0.3,0.6|0.5),(0.4|0.4,0.5|0.625)></td><td><(0.5|0.3,0.6|0.3,0.7|0.3),(0.3|0.2,0.4|0.8)></td><td><(0.4|0.3,0.5|0.3,0.6|0.4),(0.4|0.2,0.5|0.8)></td><td><(0.1|0.2,0.2|0.3,0.3|0.5),(0.7|0.375,0.7|0.6)></td></tr>
<tr><td>E_5</td><td><(0.5|0.4,0.6|0.6),(0.3|0.1,0.4|0.7,0.5|0.2)></td><td><(0.3|0.4,0.5|0.6),(0.5|0.1,0.6|0.5,0.7|0.4)></td><td><(0.7|0.3,0.8|0.4,0.9|0.3),(0.2|0.3,0.3|0.5,0.4|0.2)></td><td><(0.6|0.4,0.7|0.3,0.8|0.1),(0.2|0.4,0.3|0.1,0.4|0.5)></td><td><(0.2|0.3,0.7|0.5,0.4|0.3),(0.6|0.1,0.7|0.5,0.8|0.4)></td><td><(0.3|0.3,0.4|0.1,0.5|0.6),(0.5|0.2,0.6|0.5,0.7|0.3)></td><td><(0.4|0.4,0.5|0.3,0.6|0.1),(0.4|0.2,0.5|0.7,0.6|0.1)></td><td><(0.7|0.2,0.8|0.4,0.9|0.4),(0.1|0.1,0.2|0.5,0.3|0.4)></td><td><(0.1|0.3,0.2|0.1,0.3|0.6),(0.7|0.2,0.3|0.5,0.9|0.3)></td></tr>
<tr><td>E_6</td><td><(0.7|0.2,0.8|0.3,0.9|0.5),(0.1|0.5,0.2|0.3,0.3|0.2)></td><td><(0.6|0.1,0.7|0.2,0.9|0.7),(0.3|0.5,0.4|0.5)></td><td><(0.3|0.1,0.4|0.2,0.5|0.7),(0.6|0.4,0.7|0.6)></td><td><(0.7|0.2,0.8|0.3,0.9|0.5),(0.1|0.1,0.2|0.3,0.3|0.6)></td><td><(0.1|0.2,0.8|0.5,0.3|0.6),(0.7|0.5,0.8|0.5)></td><td><(0.4|0.1,0.5|0.3,0.6|0.6),(0.4|0.4,0.5|0.5,0.6|0.1)></td><td><(0.6|0.2,0.7|0.3,0.8|0.5),(0.2|0.5,0.3|0.3,0.4|0.2)></td><td><(0.8|0.3,0.9|0.2,1|0.5),(0.1|0.5,0.2|0.5)></td><td><(0.1|0.1,0.2|0.3,0.3|0.6),(0.7|0.4,0.7|0.5,0.9|0.1)></td></tr>
<tr><td>E_7</td><td><(0.2|0.3,0.4|0.7),(0.5|0.4,0.6|0.2,0.7|0.4)></td><td><(0.2|0.4,0.3|0.6),(0.7|0.4,0.8|0.2,0.9|0.4)></td><td><(0.3|0.4,0.4|0.5,0.5|0.1),(0.6|0.2,0.7|0.2,0.8|0.6)></td><td><(0.2|0.5,0.3|0.3,0.4|0.4),(0.6|0.3,0.7|0.2,0.8|0.5)></td><td><(0.5|0.5,0.4|0.2),(0.3|0.4,0.4|0.2,0.5|0.4)></td><td><(0.2|0.4,0.3|0.2,0.4|0.4),(0.6|0.2,0.7|0.2,0.8|0.6)></td><td><(0.1|0.4,0.2|0.2),(0.7|0.4,0.8|0.2,0.9|0.4)></td><td><(0.5|0.2,0.6|0.5,0.7|0.3),(0.3|0.4,0.4|0.2,0.5|0.4)></td><td><(0.3|0.4,0.4|0.2,0.5|0.4),(0.5|0.2,0.7|0.2,0.7|0.6)></td></tr>
<tr><td>E_8</td><td><(0.5|0.4,0.6|0.6),(0.3|0.7,0.4|0.3)></td><td><(0.3|0.2,0.5|0.8),(0.3|0.7,0.4|0.3)></td><td><(0.6|0.2,0.7|0.7,0.8|0.1),(0.3|0.5,0.4|0.5)></td><td><(0.3|0.3,0.4|0.3,0.5|0.3),(0.5|0.5,0.6|0.3,0.7|0.2)></td><td><(0.4|0.2,0.5|0.3,0.6|0.1),(0.4|0.7,0.5|0.3)></td><td><(0.3|0.2,0.4|0.3,0.5|0.5),(0.5|0.3,0.6|0.3,0.7|0.4)></td><td><(0.4|0.3,0.5|0.3,0.6|0.3),(0.4|0.7,0.5|0.3)></td><td><(0.8|0.4,0.9|0.6),(0.2|0.7,0.3|0.3)></td><td><(0.7|0.2,0.8|0.3,0.9|0.5),(0.1|0.3,0.4|0.3,0.3|0.4)></td></tr>
<tr><td>E_9</td><td><(0.4|0.4,0.5|0.3,0.6|0.3),(0.3|0.8,0.4|0.2)></td><td><(0.2|0.3,0.3|0.2,0.4|0.5),(0.7|0.8,0.8|0.2)></td><td><(0.4|0.3,0.5|0.2,0.6|0.5),(0.2|0.8,0.3|0.2)></td><td><(0.4|0.2,0.5|0.3,0.6|0.5),(0.4|0.3,0.5|0.2,0.6|0.5)></td><td><(0.2|0.3,0.7|0.2,0.4|0.5),(0.6|0.8,0.7|0.2)></td><td><(0.2|0.3,0.3|0.2,0.4|0.5),(0.6|0.8,0.7|0.25)></td><td><(0.3|0.2,0.4|0.3,0.5|0.5),(0.5|0.8,0.6|0.2)></td><td><(0.8|0.5,0.9|0.5),(0.3|0.8,0.4|0.2)></td><td><(0.3|0.3,0.4|0.2,0.5|0.5),(0.5|0.75,0.3|0.3)></td></tr>
<tr><td>E_{10}</td><td><(0.7|0.4,0.8|0.6),(0.1|0.4,0.2|0.1,0.3|0.5)></td><td><(0.5|0.3,0.7|0.7),(0.3|0.4,0.4|0.1,0.5|0.5)></td><td><(0.8|0.3,0.9|0.7),(0.1|0.4,0.2|0.1,0.3|0.5)></td><td><(0.7|0.4,0.8|0.3,0.9|0.1),(0.1|0.5,0.2|0.1,0.3|0.4)></td><td><(0.3|0.4,0.6|0.1,0.5|0.2),(0.5|0.4,0.6|0.1,0.7|0.5)></td><td><(0.5|0.2,0.6|0.4,0.7|0.4),(0.3|0.4,0.4|0.1,0.5|0.5)></td><td><(0.6|0.4,0.7|0.3,0.8|0.1),(0.2|0.4,0.3|0.1,0.4|0.5)></td><td><(0.8|0.4,0.9|0.6),(0.2|0.4,0.3|0.1,0.4|0.5)></td><td><(0.8|0.2,0.9|0.4.4),(0.3|0.4,0.2|0.1,0.5|0.5)></td></tr>
<tr><td>E_{11}</td><td><(0.6|0.2,0.7|0.5,0.8|0.3),(0.2|0.2,0.3|0.3,0.4|0.5)></td><td><(0.6|0.1,0.7|0.4,0.8|0.5),(0.2|0.2,0.3|0.3,0.4|0.5)></td><td><(0.4|0.1,0.5|0.4,0.6|0.5),(0.1|0.2,0.2|0.3,0.3|0.5)></td><td><(0.4|0.4,0.5|0.3,0.6|0.3),(0.4|0.2,0.5|0.3,0.6|0.5)></td><td><(0.2|0.3,0.7|0.4,0.4|0.3),(0.6|0.2,0.7|0.4,0.8|0.4)></td><td><(0.6|0.1,0.7|0.4,0.8|0.5),(0.2|0.2,0.3|0.3,0.4|0.5)></td><td><(0.5|0.4,0.6|0.3,0.7|0.1),(0.3|0.2,0.4|0.3,0.5|0.5)></td><td><(0.3|0.4,0.4|0.4,0.5|0.2),(0.5|0.2,0.6|0.4,0.7|0.4)></td><td><(0.4|0.1,0.5|0.4,0.6|0.5),(0.4|0.2,0.2|0.3,0.6|0.5)></td></tr>
</table>

Continued on next page

<table>
<tr><th></th><th>A_1</th><th></th><th></th><th>A_2</th><th></th><th></th><th>A_3</th><th></th><th></th></tr>
<tr><th></th><th>C_1</th><th>C_2</th><th>C_3</th><th>C_1</th><th>C_2</th><th>C_3</th><th>C_1</th><th>C_2</th><th>C_3</th></tr>
<tr><td>E_{12}</td><td><(0.5|0.4,0.7|0.5,0.8|0.1),(0.3|0.3,0.4|0.2,0.5|0.5)></td><td><(0.5|0.3,0.6|0.4,0.8|0.3),(0.4|0.3,0.5|0.2,0.6|0.5)></td><td><(0.8|0.4,0.9|0.6),(0.1|0.3,0.2|0.2,0.4|0.5)></td><td><(0.3|0.4,0.4|0.3,0.5|0.1),(0.5|0.3,0.6|0.2,0.7|0.5)></td><td><(0.6|0.5,0.3|0.2,0.8|0.1),(0.2|0.3,0.3|0.2,0.4|0.5)></td><td><(0.3|0.3,0.4|0.4,0.5|0.3),(0.5|0.3,0.6|0.2,0.7|0.5)></td><td><(0.4|0.4,0.5|0.3,0.6|0.1),(0.4|0.3,0.5|0.2,0.6|0.5)></td><td><(0.2|0.2,0.3|0.4,0.4|0.4),(0.6|0.3,0.7|0.2,0.8|0.5)></td><td><(0.1|0.3,0.2|0.4,0.3|0.3),(0.7|0.3,0.2|0.2,0.9|0.5)></td></tr>
<tr><td>E_{13}</td><td><(0.7|0.3,0.9|0.7),(0.1|0.5,0.2|0.3,0.3|0.2)></td><td><(0.5|0.2,0.8|0.8),(0.2|0.5,0.3|0.3,0.4|0.2)></td><td><(0.4|0.1,0.5|0.5,0.6|0.4),(0.1|0.6,0.3|0.4)></td><td><(0.5|0.5,0.6|0.3,0.7|0.3),(0.3|0.5,0.4|0.3,0.5|0.2)></td><td><(0.7|0.3,0.2|0.3,0.9|0.2),(0.1|0.5,0.2|0.3,0.3|0.2)></td><td><(0.2|0.2,0.3|0.5,0.4|0.3),(0.6|0.5,0.7|0.3,0.8|0.2)></td><td><(0.6|0.5,0.7|0.3),(0.2|0.5,0.3|0.3,0.4|0.2)></td><td><(0.4|0.3,0.5|0.5,0.6|0.2),(0.4|0.5,0.5|0.3,0.6|0.2)></td><td><(0.7|0.2,0.8|0.5,0.9|0.3),(0.1|0.5,0.3|0.3,0.3|0.2)></td></tr>
<tr><td>E_{14}</td><td><(0.6|0.4,0.8|0.5,0.9|0.1),(0.2|0.3,0.3|0.2,0.4|0.5)></td><td><(0.6|0.3,0.7|0.4,0.9|0.3),(0.3|0.3,0.4|0.2,0.5|0.5)></td><td><(0.6|0.3,0.7|0.4,0.8|0.3),(0.3|0.3,0.4|0.2,0.5|0.5)></td><td><(0.3|0.4,0.4|0.3,0.5|0.1),(0.5|0.3,0.6|0.2,0.7|0.5)></td><td><(0.5|0.4,0.4|0.2,0.7|0.2),(0.3|0.3,0.4|0.2,0.5|0.5)></td><td><(0.6|0.3,0.7|0.4,0.8|0.3),(0.2|0.3,0.3|0.2,0.4|0.5)></td><td><(0.5|0.4,0.6|0.3,0.7|0.1),(0.3|0.3,0.4|0.2,0.5|0.5)></td><td><(0.2|0.3,0.3|0.4,0.4|0.3),(0.6|0.3,0.7|0.2,0.8|0.5)></td><td><(0.6|0.3,0.7|0.4,0.8|0.3),(0.2|0.3,0.4|0.2,0.4|0.5)></td></tr>
<tr><td>E_{15}</td><td><(0.2|0.3,0.3|0.4,0.4|0.3),(0.6|0.4,0.7|0.5,0.8|0.1)></td><td><(0.2|0.2,0.3|0.3,0.4|0.5),(0.6|0.4,0.7|0.5,0.8|0.1)></td><td><(0.4|0.2,0.5|0.3,0.6|0.5),(0.7|0.4,0.8|0.6)></td><td><(0.4|0.3,0.5|0.3,0.6|0.3),(0.4|0.4,0.5|0.5,0.6|0.1)></td><td><(0.4|0.7,0.8|0.5),(0.7|0.4,0.8|0.5,0.9|0.1)></td><td><(0.7|0.5,0.8|0.3,0.9|0.2),(0.1|0.4,0.2|0.5,0.3|0.1)></td><td><(0.1|0.3,0.2|0.3,0.3|0.3),(0.7|0.4,0.8|0.5,0.9|0.1)></td><td><(0.3|0.2,0.4|0.3,0.5|0.5),(0.5|0.4,0.6|0.5,0.7|0.1)></td><td><(0.4|0.5,0.5|0.3,0.6|0.2),(0.4|0.4,0.8|0.5,0.6|0.1)></td></tr>
<tr><td>E_{16}</td><td><(0.3|0.2,0.4|0.6,0.5|0.2),(0.6|0.8,0.7|0.2)></td><td><(0.1|0.1,0.3|0.5,0.4|0.4),(0.7|0.8,0.8|0.2)></td><td><(0.6|0.1,0.7|0.5,0.8|0.4),(0.3|0.8,0.4|0.2)></td><td><(0.2|0.5,0.3|0.3,0.4|0.4),(0.6|0.8,0.7|0.2)></td><td><(0.2|0.3,0.7|0.2,0.4|0.3),(0.6|0.8,0.7|0.2)></td><td><(0.2|0.3,0.3|0.5,0.4|0.2),(0.6|0.8,0.7|0.2)></td><td><(0.2|0.5,0.3|0.2),(0.6|0.8,0.7|0.2)></td><td><(0.6|0.1,0.7|0.5,0.8|0.4),(0.2|0.8,0.3|0.2)></td><td><(0.6|0.3,0.7|0.5,0.8|0.2),(0.2|0.8,0.4|0.2)></td></tr>
<tr><td>E_{17}</td><td><(0.6|0.3,0.7|0.7),(0.1|0.3,0.2|0.3,0.3|0.4)></td><td><(0.4|0.2,0.6|0.8),(0.4|0.3,0.5|0.3,0.6|0.4)></td><td><(0.4|0.1,0.5|0.5,0.6|0.4),(0.8|0.5,0.9|0.5)></td><td><(0.7|0.5,0.8|0.3,0.9|0.2),(0.1|0.3,0.2|0.3,0.3|0.4)></td><td><(0.3|0.1,0.6|0.3,0.5|0.4),(0.5|0.3,0.6|0.3,0.7|0.4)></td><td><(0.7|0.4,0.8|0.5,0.9|0.1),(0.1|0.3,0.2|0.3,0.3|0.4)></td><td><(0.5|0.5,0.6|0.2),(0.1|0.3,0.2|0.3,0.3|0.4)></td><td><(0.7|0.1,0.8|0.5,0.9|0.4),(0.1|0.3,0.2|0.3,0.3|0.4)></td><td><(0.3|0.4,0.4|0.5,0.5|0.1),(0.5|0.3,0.9|0.3,0.7|0.4)></td></tr>
<tr><td>E_{18}</td><td><(0.2|0.4,0.4|0.5,0.5|0.1),(0.6|0.1,0.7|0.4,0.8|0.5)></td><td><(0.2|0.3,0.3|0.4,0.5|0.3),(0.7|0.1,0.8|0.4,0.9|0.5)></td><td><(0.5|0.3,0.6|0.4,0.7|0.3),(0.4|0.2,0.5|0.8)></td><td><(0.4|0.4,0.5|0.3,0.6|0.1),(0.4|0.1,0.5|0.4,0.6|0.5)></td><td><(0.6|0.7,0.3|0.4),(0.2|0.1,0.3|0.4,0.4|0.5)></td><td><(0.6|0.6,0.7|0.4),(0.2|0.1,0.3|0.4,0.4|0.5)></td><td><(0.1|0.4,0.2|0.3,0.3|0.1),(0.7|0.1,0.8|0.4,0.9|0.5)></td><td><(0.4|0.3,0.5|0.4,0.6|0.3),(0.4|0.1,0.5|0.4,0.6|0.5)></td><td><(0.6|0.6,0.7|0.4),(0.2|0.1,0.5|0.4,0.4|0.5)></td></tr>
<tr><td>E_{19}</td><td><(0.2|0.2,0.3|0.3,0.4|0.5),(0.6|0.8,0.7|0.2)></td><td><(0.2|0.1,0.3|0.2,0.4|0.7),(0.7|0.8,0.8|0.2)></td><td><(0.8|0.8,0.9|0.2),(0.1|0.8,0.2|0.2)></td><td><(0.5|0.2,0.6|0.3,0.7|0.5),(0.3|0.8,0.4|0.2)></td><td><(0.3|0.2,0.6|0.2,0.5|0.6),(0.5|0.8,0.6|0.2)></td><td><(0.1|0.3,0.2|0.2,0.3|0.5),(0.7|0.8,0.8|0.2)></td><td><(0.1|0.2,0.2|0.3,0.3|0.5),(0.7|0.8,0.8|0.2)></td><td><(0.7|0.1,0.8|0.2,0.9|0.7),(0.1|0.8,0.2|0.2)></td><td><(0.2|0.3,0.3|0.2,0.4|0.5),(0.6|0.8,0.2|0.2)></td></tr>
<tr><td>E_{20}</td><td><(0.3|0.4,0.4|0.6),(0.5|0.2,0.6|0.3,0.7|0.5)></td><td><(0.1|0.4,0.3|0.6),(0.7|0.2,0.8|0.3,0.9|0.5)></td><td><(0.1|0.3,0.2|0.4,0.3|0.3),(0.7|0.4,0.8|0.6)></td><td><(0.2|0.4,0.3|0.3,0.4|0.1),(0.6|0.2,0.7|0.3,0.8|0.5)></td><td><(0.3|0.3,0.6|0.3,0.5|0.3),(0.5|0.2,0.6|0.3,0.7|0.5)></td><td><(0.4|0.1,0.5|0.4,0.6|0.5),(0.4|0.2,0.5|0.3,0.6|0.5)></td><td><(0.2|0.4,0.3|0.3,0.4|0.1),(0.6|0.2,0.7|0.3,0.8|0.5)></td><td><(0.7|0.3,0.8|0.4,0.9|0.3),(0.1|0.2,0.2|0.3,0.3|0.5)></td><td><(0.6|0.1,0.7|0.4,0.8|0.5),(0.2|0.2,0.8|0.3,0.4|0.5)></td></tr>
</table>

Table 2. The weight of each attribute.

	C_1	C_2	C_3
Weights	0.3578	0.4103	0.2319

Table 3. The classification results of the decision-making experts.

Expert classes	Experts
Ω_1	E_1, E_2, E_4, E_5, E_6, E_7, E_8
Ω_2	E_3, E_9, E_{10}, E_{11}, E_{12}, E_{14}
Ω_3	E_{13}, E_{15}, E_{18}
Ω_4	E_{16}, E_{17}, E_{19}, E_{20}

Table 4. The consensus decision-making information matrix.

Ω_1's experts	A_1			A_2			A_3		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
E_1	0.2560	0.2659	0.1659	0.2250	0.6331	-0.4901	0.0250	-0.2341	-0.2901
E_2	0.3484	0.2952	-0.3366	0.1502	-0.5081	-0.2396	0.0702	0.1919	-0.4396
E_4	0.3825	0.3151	-0.2881	-0.0337	-0.1881	0.0378	0.1039	-0.0131	-0.5622
E_5	0.1268	-0.2507	0.5025	0.1805	-0.4434	-0.2000	-0.1697	0.5792	-0.6000
E_6	0.6600	0.4200	-0.2122	0.5690	-0.5400	0.0769	0.4600	0.8328	-0.5231
E_7	-0.2622	-0.5047	-0.3540	-0.3968	0.1894	-0.4494	-0.6628	0.2294	-0.2494
E_8	0.2268	0.0882	0.3361	-0.2182	0.0520	-0.1750	-0.0250	0.6249	0.6250

Table 5. The decision-making information matrix.

	A_1	A_2	A_3
Ω_1	0.1196	-0.0705	0.0897
Ω_2	0.2133	-0.0389	-0.0825
Ω_3	-0.1514	0.1263	-0.0935
Ω_4	-0.1628	-0.1093	0.1397

Table 6. The class weight, the class deviation weight, and the comprehensive class weight.

	Ω_1	Ω_2	Ω_3	Ω_4
ω_n^c	0.3500	0.3000	0.1500	0.2000
ω_p^c	0.1766	0.2370	0.3052	0.2812
ω_z^c	0.2807	0.2748	0.2121	0.2325

Table 7. The comprehensive decision-making information matrix.

	A_1	A_2	A_3
Score	0.0222	-0.0291	0.0152

4.3. Comparison of different methods

This section presents a comparative analysis between the method proposed in this study and other existing decision-making methods, demonstrating the efficacy and superiority of the proposed method. Table 8 presents a comprehensive comparison of the pertinent attributes associated with the four distinct decision-making methods. Zhang et al. [39] suggested a hesitant fuzzy language adaptive consensus model based on individual cumulative consensus contributions in the previous study as a way to find emergency medical facilities. This is called "Method 1". "Method 2" was proposed by Garg and Kaur [40], who proposed a PDHFSs method based on the MSM operator to quantify the gesture information of patients with cerebral hemorrhage. Wu and Xu [41] developed a large-scale consensus decision-making model with hesitant fuzzy information and variable clusters, which is documented as "Method 3". The LSGC-MADM Method based on PDHFSs is proposed in this study called "Method 4". The case problem presented in Section 4.1 is then solved using four different decision-making methods, and the final decision-making results are displayed in Table 9.

The ranking of decision-making methods varies slightly as a result of the distinct characteristics and focal points inherent in each method. According to the findings presented in Table 9, it is evident that Methods 1, 3 and 4 collectively assert that city A_1 possesses the most effective emergency management capability. Conversely, Methods 2–4 collectively contend that city A_2 exhibits the weakest emergency management capability. This observation highlights the reliability and validity of Method 4, which is the decision-making method proposed in this study. The adaptive consensus model in Method 1 requires that experts' weights and decision-making information be changed all the time. This could mean that the final decisions are different from what was known at the start. The absence of a consensus-building mechanism in Method 2 may lead to errors in resolving complex group decision-making problems. While Method 3 and Method 4 yield identical decision-making outcomes, Method 3 fails to incorporate the non-membership and probability information of decision-making experts, thereby limiting its ability to comprehensively depict decision-making information. The LSGC-MADM Method based on PDHFSs effectively addresses the issue of incomplete decision-making information collection and exhibits a wider range of applicability.

Table 8. Comparison of different decision-making methods.

Decision methods	Whether to consider large scale groups	Whether to consider probability information	Whether to consider group clustering	Whether to consider consensus-reaching	Whether to consider non-membership
Method 1 [39]	×	×	×	√	×
Method 2 [40]	×	√	×	×	√
Method 3 [41]	√	×	√	√	×
Method 4*	√	√	√	√	√

* "Method 4" is the decision-making method proposed in this study.

Table 9. Decision-making results of four decision-making methods.

Decision methods	Utility value			The ranked scheme
	A_1	A_2	A_3	
Method 1	0.6716	0.6486	0.5054	$A_1 > A_2 > A_3$
Method 2	0.5127	0.4495	0.7937	$A_3 > A_1 > A_2$
Method 3	0.7479	0.5753	0.6381	$A_1 > A_3 > A_2$
Method 4*	0.0222	-0.0291	0.0152	$A_1 > A_3 > A_2$

* "Method 4" is the decision-making method proposed in this study.

5. Conclusions

In this study, we present a proposed solution to achieve consensus in decision-making processes involving large-scale groups in uncertain, fuzzy environments. The proposed method is called the Large-Scale Group Consensus Multi-Attribute Decision-Making Method based on Probabilistic Dual Hesitant Fuzzy Sets (the LSGC-MADM Method based on PDHFSs). The probabilistic dual hesitant fuzzy information evaluation matrix and attribute weights are initially obtained. Furthermore, the expert group is classified, and the effectiveness of the classification is assessed. Next, the consensus-reaching model is used for each class of experts. This model is designed to identify and modify the evaluation information for experts within the same class, aiming to achieve a consensus among them. Subsequently, the integration of information within and between classes is conducted by considering the decision-making evaluation value of all experts. Determining the prioritization of decision-making objects is achieved by utilizing the ranking method. Finally, the case study provides proof of the feasibility and effectiveness of the proposed decision-making method.

The LSGC-MADM Method based on PDHFSs presents a well-defined set of calculation procedures that effectively mitigate the potential bias introduced by subjective artificial weighting. This method successfully addresses the issues of a cumbersome computation procedure, poor dependability, and disorganized classification in an ambiguous, fuzzy environment. Moreover, it provides an innovative research perspective for improving decision-making methodologies in this field. This methodology can be employed in various domains, such as scheme evaluation, emergency management, big data analytics, and numerous other disciplines. The forthcoming research phase will concentrate on advancing and visualizing decision-making software systems designed to facilitate large-scale group decision-making in fuzzy and uncertain conditions.

While the method presented in this paper has made significant progress, there remains room for improvement in its application to large-scale group decision-making problems, especially in optimizing expert classification algorithms and consensus feedback mechanisms. Future research should focus on incorporating a wider range of real-world issues to refine these algorithms, thereby enhancing the efficiency and accuracy of the decision-making process. Additionally, the development of a large-scale group consensus decision-making software system, coupled with its visualization application through information technology, represents another critical research direction. This will not only improve the operability and user experience of the decision-making process but also facilitate the broader application of the method presented in this study, offering a more effective tool for solving complex decision-making problems.

Use of AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

References

1. Q. Ding, Y. M. Wang, M. Goh, TODIM Dynamic emergency decision-making method based on hybrid weighted distance under probabilistic hesitant fuzzy information, *Int. J. Fuzzy Syst.*, **23** (2021), 474–491. https://doi.org/10.1007/s40815-020-00978-8
2. R. Ikram, S. Meshram, M. Hasan, X. Cao, E. Alvandi, C. Meshram, et al., The application of multi-attribute decision making methods in integrated watershed management, *Stochastic Environ. Res. Risk Assess.*, (2023). https://doi.org/10.1007/s00477-023-02557-3
3. F. Jin, Y. Zhu, Y. Zhang, S. Guo, J. Liu, L. Zhou, Interval type-2 trapezoidal fuzzy multi-attribute decision-making method and its application to the corporate investment selection, *J. Intell. Fuzzy Syst.*, **45** (2023), 2319–2330. https://doi.org/10.3233/JIFS-230310
4. Y. Song, G. Li, A mathematical programming approach to manage group decision making with incomplete hesitant fuzzy linguistic preference relations, *Comput. Ind. Eng.*, **135** (2019), 467–475. https://doi.org/10.1016/j.cie.2019.06.036
5. Y. Song, G. Li, D. Ergu, N. Liu, An optimisation-based method to conduct consistency and consensus in group decision making under probabilistic uncertain linguistic preference relations, *J. Oper. Res. Soc.*, **73** (2022), 840–854. https://doi.org/10.1080/01605682.2021.1873079
6. Y. Shen, X. Ma, J. Zhan, A two-stage adaptive consensus reaching model by virtue of three-way clustering for large-scale group decision making, *Inf. Sci.*, **649** (2023), 119658. https://doi.org/10.1016/j.ins.2023.119658
7. B. Yu, Z. Zheng, Z. Xiao, Y. Fu, Z. Xu, A large-scale group decision-making method based on group-oriented rough dominance relation in scenic spot service improvement, *Expert Syst. Appl.*, **233** (2023), 120999. https://doi.org/10.1016/j.eswa.2023.120999
8. F. Meng, D. Zhao, X. Zhang, A fair consensus adjustment mechanism for large-scale group decision making in term of Gini coefficient, *Eng. Appl. Artif. Intell.*, **126** (2023), 106962. https://doi.org/10.1016/j.engappai.2023.106962
9. X. Gou, Z. Xu, Double hierarchy linguistic term set and its extensions: The state-of-the-art survey, *Int. J. Intell. Syst.*, **36** (2021), 832–865. https://doi.org/10.1002/int.22323
10. Y. Shen, X. Ma, J. Zhan, A two-stage adaptive consensus reaching model by virtue of three-way clustering for large-scale group decision making, *Inf. Sci.*, **649** (2023). https://doi.org/10.1016/j.ins.2023.119658
11. X. Zhou, S. Li, C. Wei, Consensus reaching process for group decision-making based on trust network and ordinal consensus measure, *Inf. Fusion*, **101** (2024). https://doi.org/10.1016/j.inffus.2023.101969
12. S. M. Yu, Z. J. Du, X. Y. Zhang, H. Y. Luo, X. D. Lin, Trust Cop-Kmeans clustering analysis and minimum-cost consensus model considering voluntary trust loss in social network large-scale decision-making, *IEEE Trans. Fuzzy Syst.*, **30** (2022), 2634–2648. https://doi.org/10.1109/TFUZZ.2021.3089745
13. Y. Li, Y. Ji, S. Qu, Consensus building for uncertain large-scale group decision-making based on the clustering algorithm and robust discrete optimization, *Group Decis. Negotiation*, **31** (2022), 453–489. https://doi.org/10.1007/s10726-022-09774-1

14. G. R. Yang, X. Wang, R. X. Ding, S. P. Lin, Q. H. Lou, E. Herrera-Viedma, Managing non-cooperative behaviors in large-scale group decision making based on trust relationships and confidence levels of decision makers, *Inf. Fusion*, **97** (2023), 101820. https://doi.org/10.1016/j.inffus.2023.101820
15. Z. Du, S. Yu, C. Cai, Constrained community detection and multistage multicost consensus in social network large-scale decision-making, *IEEE Trans. Comput. Social Syst.*, (2023). https://doi.org/10.1109/TCSS.2023.3265701
16. S. Yu, X. Zhang, Z. Du, Enhanced minimum-cost consensus: focusing on overadjustment and flexible consensus cost, *Inf. Fuusion*, **89** (2023), 336–354. https://doi.org/10.1016/j.inffus.2022.08.028
17. Z. Chen, X. Zhang, R. Rodriguez, W. Pedrycz, L. Martinez, M.J. Skibniewski, Expertise-structure and risk-appetite-integrated two-tiered collective opinion generation framework for large-scale group decision making, *IEEE Trans. Fuzzy Syst.*, **30** (2022), 5496–5510. https://doi.org/10.1109/TFUZZ.2022.3179594
18. L. A. Zadeh, Fuzzy logic equals computing with words, *IEEE Trans. Fuzzy Syst*., **4** (1996), 103–111. https://doi.org/10.1109/91.493904
19. K. Atanassov, Intuitionistic fuzzy-sets, *Fuzzy Sets Syst.*, **20** (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3
20. R. R. Yager, Pythagorean membership grades in multicriteria decision making, *IEEE Trans. Fuzzy Syst.*, **22** (2014), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989.
21. V. Torra, Hesitant fuzzy sets, *Int. J. Intell. Syst.*, **25** (2010), 529–539. https://doi.org/10.1002/int.20418
22. P. Wang, R. Dang, P. Liu, D. Pamucar, Attitude- and cost-driven consistency optimization model for decision-making with probabilistic linguistic preference relation, *Comput. Ind. Eng*., **186** (2023). https://doi.org/10.1016/j.cie.2023.109748
23. P. Liu, P. Wang, W. Pedrycz, Consistency-and consensus-based group decision-making method with incomplete probabilistic linguistic preference relations, *IEEE Trans. Fuzzy Syst*., **29** (2021), 2565–2579. https://doi.org/10.1109/TFUZZ.2020.3003501
24. Z. Xu, W. Zhou, Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment, *Fuzzy Optim. Dec. Making*, **16** (2017), 481–503. https://doi.org/10.1007/s10700-016-9257-5
25. Z. Hao, Z. Xu, H. Zhao, Z. Su, Probabilistic dual hesitant fuzzy set and its application in risk evaluation, *Knowledge-Based Syst.*, **127** (2017), 16–28. https://doi.org/10.1016/j.knosys.2017.02.033
26. Y. Zhu, W. Zhang, J. Hou, R. Zhang, Multi-attribute group decision making algorithm with probabilistic dual hesitant fuzzy sets and PROMETHEE method, *Comput. Eng. Appl.*, **58** (2022), 88–97. https://doi.org/10.3778/j.issn.1002-8331.2203-0483
27. B. Ning, H. Wang, G. Wei, C. Wei, Several similarity measures of probabilistic dual hesitant fuzzy sets and their applications to new energy vehicle charging station location, *Alexandria Eng. J.*, **71** (2023), 371–385. https://doi.org/10.1016/j.aej.2023.03.052
28. W. Zhang, Y. Zhu, The probabilistic dual hesitant fuzzy multi-attribute decision-making method based on cumulative prospect theory and its application, *Axioms*, **12** (2023). https://doi.org/10.3390/axioms12100925
29. Z. Ren, Z. Xu, H. Wang, The strategy selection problem on artificial intelligence with an integrated VIKOR and AHP method under probabilistic dual hesitant fuzzy information, *IEEE Access*, **7** (2019), 103979–103999. https://doi.org/10.1109/ACCESS.2019.2931405

30. X. Wang, H. Wang, Z. Xu, Z. Ren, Green supplier selection based on probabilistic dual hesitant fuzzy sets: A process integrating best worst method and superiority and inferiority ranking, *Appl. Intell.*, **52** (2022), 8279–8301. https://doi.org/10.1007/s10489-021-02821-5
31. H. Li, F. Li, J. Zuo, J. Sun, C. Yuan, L. Ji, et al., Emergency decision-making system for the large-scale infrastructure: a case study of the south-to-north water diversion project, *J. Infrastruct. Syst.*, **28** (2022). https://doi.org/10.1061/(ASCE)IS.1943-555X.0000659
32. X. Gou, Z. Xu, H. Liao, F. Herrera, Consensus model handling minority opinions and noncooperative behaviors in large-scale group decision-making under double hierarchy linguistic preference relations, *IEEE Trans. Cybern.*, **51** (2021), 283–296. https://doi.org/10.1109/TCYB.2020.2985069
33. A. Aydogdu, S. Gul, New entropy propositions for interval-valued spherical fuzzy sets and their usage in an extension of ARAS (ARAS-IVSFS), *Expert Syst.*, **39** (2022). https://doi.org/10.1111/exsy.12898
34. H. Garg, T. Mahmood, U. ur Rehman, Z. Ali, CHFS: Complex hesitant fuzzy sets-their applications to decision making with different and innovative distance measures, *CAAI Trans. Intell. Technol.*, **6** (2021), 93–122. https://doi.org/10.1049/cit2.12016
35. Z. Ma, J. Zhu, S. Zhang, H. Wang, X. Liu, Classification-based aggregation model on large scale group decision making with hesitant fuzzy linguistic information, *J. Control Decis.*, **34** (2019), 167–179.
36. L. Wang, H. Xue, Group decision-making method based on expert classification consensus information integration, *Symmetry*, **12** (2020), 1180. https://doi.org/10.3390/sym12071180
37. H. Hassani, R. Razavi-Far, M. Saif, E. Herrera-Viedma, Consensus-based decision support model and fusion architecture for dynamic decision making, *Inf. Sci.*, **597** (2022), 86–104. https://doi.org/10.1016/j.ins.2022.03.040
38. K. Qi, H. Chai, Q. Duan, Y. Du, Q. Wang, J. Sun, et al., A collaborative emergency decision making approach based on BWM and TODIM under interval 2-tuple linguistic environment, *Int. J. Mach. Learn. Cybern.*, **13** (2022), 383–405. https://doi.org/10.1007/s13042-021-01412-7
39. S. Zhang, X. Liu, J. Zhu, and Z. Wang, Adaptive consensus model with hesitant fuzzy linguistic information considering individual cumulative consensus contribution, *J. Control Decis.*, **36** (2021), 187–195.
40. H. Garg, G. Kaur, Quantifying gesture information in brain hemorrhage patients using probabilistic dual hesitant fuzzy sets with unknown probability information, *Comput. Ind. Eng.*, **140** (2020). https://doi.org/10.1016/j.cie.2019.106211
41. Z. Wu, J. Xu, A consensus model for large-scale group decision making with hesitant fuzzy information and changeable clusters, *Inf. Fusion*, **41** (2018), 217–231. https://doi.org/10.1016/j.inffus.2017.09.011