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Research article

Dynamic event-triggered consensus control for nonlinear multi-agent systems under DoS attacks

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Abstract: In this paper, we investigated leader-following consensus control for nonlinear multi-agent systems (MASs) experiencing denial-of-service (DoS) attacks. We proposed a distributed control strategy incorporating an adaptive scheme and a state feedback control gain to eliminate the effects of system nonlinear dynamics and uncertainties. In addition, we introduced a dynamic event-triggered control (DETC) to minimize the utilization of communication resources. Finally, we provided simulation results to show the validity of the proposed approach.

Keywords: nonlinear multi-agent systems; DoS attacks; dynamic event-triggered mechanism

1. Introduction

In recent years, multi-agent consensus control has been recognized as a vital element of distributed collaborative control for applications such as distributed computing, unmanned aerial vehicle formation, and intelligent transportation systems. Researchers have shown significant interest in this area, and a wide range of control mechanisms have been explored in the past few years. These control mechanisms include adaptive [1–4], fault-tolerant [5], impulse [6, 7], and sliding mode [8] methods.

In practical systems, the stability of the system state is susceptible to disruption from unexpected factors, including nonlinear dynamics and system uncertainties. Existing research has primarily focused on continuous-time systems, where the state information of intelligent agents is continuously transmitted between nodes, leading to significant network usage and energy consumption. However, the development of event-triggered control solves this problem by avoiding constant communication. Earlier studies on event-triggered control that can be found have investigated centralized, distributed, and self-triggered event-triggered control techniques [9–11]. One researcher proposed an adaptive event-triggered control scheme for strongly connected networks that dynamically adjusted the triggering time interval on the basis of sampled data [12]. Another mechanism using a dynamic event-triggering mechanism was proposed to reduce communication resource wastage compared with traditional event-triggering mechanisms [13]. Others have assumed that system parameters such as the efficiency factor of the executor, external disturbances, and precursor control input signals are all unknown and introduced a fault-tolerant control to obtain sufficient conditions for consistent tracking [14]. However, these findings primarily investigate traditional event triggering mechanisms. In [15, 16], researchers proposed a dynamic event-triggered mechanism, which can significantly reduce the number of triggers and conserve communication resources. researchers respectively proposed centralized and distributed dynamic event-triggered mechanisms in [15, 16], while scholars suggested both centralized and distributed mechanisms in [17], verifying their superiority.

However, the rapid growth of network information technologies has also led to a rise in cyber attacks. Among them, denial-of-service (DoS) attacks are the most common, being relatively easy to execute in the attack space. These attacks usually target the controller or exhaust the resources of the target system directly, resulting in the system being unable to provide normal services or communication. In some cases, these attacks cause the system to crash. Therefore, countering DoS attacks has received significant research attention. Researchers studied the multi-agent systems(MASs) under DoS attacks in given attack frequency and upper bounds on attack duration [18, 19]. Compared with linear systems, nonlinear MASs are more widely used in real life. Among these, a secure controller based on event triggering was proposed to solve the lead-following consensus problem of second-order nonlinear systems [20]. This is more common than linear systems. Another proposal was for an event-triggered adaptive fault-tolerant control strategy, which reduced the computational cost of heterogeneity [21]. For nonperiodic DoS attacks, the upper bounds of network attacks, actuator failures, attack duration, and frequency are obtained. Another method uses a security mechanism employing a prediction-based switching observer scheme to address the issue of invalidation in event-triggered mechanisms during attack intervals [22]. A novel framework for observer-based event-triggered containment control, taking into account the occurrence of DoS attacks, has also been introduced [23]. This framework establishes a resilient event-triggered controller, using a specially designed observer. The goal is to achieve consistent control of MASs in the presence of DoS attacks.

Based on these observations, we aim to explore the security consensus problem of nonlinear MAS with external disturbances under DoS attacks in this paper. Our contributions are as follows.

1) In this paper, a nonlinear system with external disturbances is considered, and the effects of the nonlinear dynamics and uncertainty of the system are eliminated by designing an adaptive scheme and state-feedback control gains by updating the laws of the adaptive parameters online.

2) Compared with [22, 23], a dynamic variable is introduced to adjust the triggering instances under DoS attacks. Therefore, the event-triggered mechanism proposed in this paper is more flexible and can effectively save communication resources. In addition, continuous communication between agents is not required to determine whether a trigger condition satisfies the trigger condition.

Notation $\mathbb{R}$ is the set of real numbers, and $\mathbb{R}^{N\times N}$ is the set of $N \times N$ real matrix. $\|\cdot\|$ represents a Euclidean norm of vectors or matrices. The superscripts A^{-1} and A^T represent the inverse and transpose of matrix A. $\lambda_{max}(A)$ is the maximum eigenvalue, and $\lambda_{min}(A)$ is the minimum eigenvalue of matrix A. $D^{+}(\cdot)$ denotes the righthand derivative of a function, and $\bigotimes$ is Kronecker product.

$diag\{A_1, \cdots, A_n\}$ is the diagonal matrix. $\cap$ is the intersection of sets, and $\cup$ denotes the union of sets.

2. Problem statement and preliminaries

2.1. Communication graphs

For a given MAS, the digraph $\mathcal{G}$ is $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ is the set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the edge set of followers. The information exchange between each node can be described by the adjacency matrix A and the Laplacian matrix L. $A = \left[a_{ij}\right] \in \mathbb{R}^{N\times N}$ if agents i and j communicate with one another, $a_{ij} = 1$; otherwise $a_{ij} = 0$ and $L = \left[l_{ij}\right] \in \mathbb{R}^{n\times n}$ where $L = D - A$, The degree matrix $D = diag(d_i)$ with $d_i = \sum_{i=1}^{N} a_{ij}$. In this paper, we assume that the agents are linked by a balancing topology, i.e., $a_{ij} = a_{ji}$. If the agent i communicates with the leader, then $b_i = 1$; otherwise, $b_i = 0$.

2.2. MAS modeling

For a leader-following system, the dynamics of the leader are described by the equation

$$\dot{x}_0 = Ax_0 + f(t, x_0(t)). \tag{2.1}$$

The ith follower system is

$$\dot{x}_i = Ax_i + Bu_i(t) + f(t, x_i(t)) + w_i. \tag{2.2}$$

In the preceding, $x(t) \in \mathbb{R}^n$ are positions of the agent, $u_i(t) \in \mathbb{R}$ is the control input, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times p}$ are system matrices, $f(x)$ is a nonlinear function, and w_i is the uncertainty input satisfying

$$w_i \leq \varsigma_{i1} |u| + \varsigma_{i2} |x| + \gamma_i, \tag{2.3}$$

where $\varsigma_{i1} < 1$,ς_{i2} and γ_i are unknown constants.

Lemma 1. If the nonlinear function $f(t, x_i(t)), i = 1, 2, ...,$ is continuously differentiable in a region $S \in R^2$ and $x_i(t_0) \in S$, then for any $x_i(t_0) \in S$, the following formula is satisfied:

$$f(x_i(t), t) - f(x_i(t_0), t) = \frac{\partial f(\cdot)}{\partial x_i} \times (x_i(t) - x_i(t_0)), \tag{2.4}$$

where $f(\cdot) = f(x_i(t_0)) + \Delta(x_i(t) - x_i(t_0))$, $0 < \Delta < 1$.

Assumption 1. If there is a continuously differentiable function $f(t, x_i(t))$ and the highest order $s \in \{1, 2, \cdots N\}$, there exist bounded positive scalars δ_{ix}, such that

$$\left|\frac{\partial f(z_i)}{\partial x_i}\right| \leq \delta_{ix} x_{is}, \tag{2.5}$$

where $x_{is} = |x_i|^s + |x_i|^{s-1} + \cdots + 1$, $s \geq 1$. We also need some assumptions to ensure that the purpose is achieved.

Assumption 2. A, B can be stabilized, and the digraph $\mathcal{G}$ is strongly connected.

Next, we define the position errors $e_i(t)$:

$$e_i(t) = \sum_{j=0, j\neq i}^{N} a_{ij}(x_i(t) - x_j(t)) + b_i(x_i(t) - x_0(t)). \tag{2.6}$$

According to **Definition 1**, we have

$$
\begin{aligned}
\dot{e}_i(t) = &\sum_{j=0, j\neq i}^{N} a_{ij}(\dot{x}_i(t) - \dot{x}_j(t)) + b_i(\dot{x}_i(t) - \dot{x}_0(t)) \\
= &\sum_{j=0, j\neq i}^{N} a_{ij}(A(x_i - x_j) + B(u_i - u_j) + \\
&[f(t, x_i(t)) - f(t, x_j(t))] + (w_i - w_j)) + \\
&b_i(A(x_i - x_0) + Bu_i) + w_i + \\
&b_i[f(t, x_i(t)) - f(t, x_0(t))] \\
= &\sum_{j=0, j\neq i}^{N} Aa_{ij}((x_i - x_j) + b_i(x_i - x_0)) + \\
&Ba_{ij}(u_i - u_j) + a_{ij}\frac{\partial f(z_i)}{\partial x_i}(x_i(t) - x_j(t_0)) + \\
&a_{ij}(w_i - w_j)) + b_i(Bu_i + w_i)) + \\
&b_i[\frac{\partial f(z_i)}{\partial x_i}(x_i(t) - x_0(t))] \\
= &\sum_{j=0, j\neq i}^{N} A(a_{ij}(x_i - x_j) + b_i(x_i - x_0)) + \\
&\frac{\partial f(z_i)}{\partial x_i}[a_{ij}(x_i(t) - x_j(t_0)) + b_i(x_i(t) - x_0(t))] + \\
&a_{ij}(B(u_i - u_j)) + (w_i - w_j)) + b_i(Bu_i + w_i))
\end{aligned}
\quad . \tag{2.7}
$$

We also have

$$\dot{e}(t) = (L + L_0)(Bu(t) + w(t)) + (A + \frac{\partial \boldsymbol{f}(\boldsymbol{z_i})}{\partial \boldsymbol{x}})e(t), \tag{2.8}$$

where $e(t) = [e_1(t), e_2(t), \cdots, e_i(t)]$, $\frac{\partial \boldsymbol{f}(\boldsymbol{z_i})}{\partial \boldsymbol{x}} = diag_{i=1}^{N}[\frac{\partial f(z_i)}{\partial x_i}]$, $L_0 = diag_{i=1}^{N}[b_i]$. L is defined as

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{N} a_{ik}, & j = i \\ -a_{ij}, & j \neq i \end{cases} \quad . \tag{2.9}$$

,then Eq (2.8) can be expressed as

$$\dot{e}(t) = (A + \Delta_A)e(t) + \bar{L} \otimes (Bu(t) + w(t)), \tag{2.10}$$

where $A = \begin{bmatrix} I_{N\times N} & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{L} = \begin{bmatrix} 0 \\ L + L_0 \end{bmatrix}$, $\Delta_A = \begin{bmatrix} 0 & 0 \\ 0 & \frac{\partial \boldsymbol{f}(\boldsymbol{z_i})}{\partial \boldsymbol{x}} \end{bmatrix}$.

Definition 1. MAS (2.1) and (2.2) are said to have consensus if each agent's position state in the system satisfies

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, i = 1, 2, \cdots, N. \tag{2.11}$$

2.3. Controller design

The distributed adaptive control input is

$$u_i(t) = k_i e_i(t) - \bar{e}_i \Psi_i(t), \tag{2.12}$$

where $\bar{e}_i = \sum_{j=1}^{N} a_{ji} e_{ji}, a_{ii} = a_{i0} + \sum_{j=1,j\neq i}^{N} a_{ij}$. k_i is the control gain determined based on the linear matrix inequality(LMI)

$$(A + \bar{L} \otimes BK)^T P + P(A + \bar{L} \otimes BK) \leq 0, \tag{2.13}$$

where $P = \begin{bmatrix} P_0 & 0_{N\times N} \\ 0_{N\times N} & I_{N\times N} \end{bmatrix} > 0, K = diag[k_i], andP_0$ is a positive definition matrix. $\Psi_i(t)$ is defined as

$$\Psi_i = \begin{cases} \frac{1}{1-\varsigma_{i1}}(\frac{\varsigma_{i1}|k_i e_i| + \varsigma_{i2}|x_{is}| + \gamma_i}{|e_i|} + \frac{2\hat{\delta}_{ix}|e|^2 x_{is}}{|e_i|^2}), & e_i \neq 0 \\ 0, & e_i = 0 \end{cases} \tag{2.14}$$

where $\hat{\delta}_{ix}$ is the estimate of unknown parameters δ_{ix}. The following describes the updated laws for the adaptive parameters:

$$\frac{\mathrm{d}\hat{\delta}_i}{\mathrm{d}t} = |e_i|^2 x_{is}. \tag{2.15}$$

Since δ_i is an unknown constant, defined as $\tilde{\delta}_i(t) = \hat{\delta}_i(t) - \delta_i$, the adaptive error systems are described by

$$\frac{\mathrm{d}\tilde{\delta}_i}{\mathrm{d}t} = \frac{\mathrm{d}\hat{\delta}_i}{\mathrm{d}t}. \tag{2.16}$$

It follows from Eq (2.12) that

$$u(t) = ke(t) - \bar{e}\Psi(t), \tag{2.17}$$

where $k(t) = [k_1(t), k_2(t), \cdots]^T$ and $\bar{e} = [\bar{e}_1, \bar{e}_2 \cdots \bar{e}_n]$. According to Eqs (2.11) and (2.15), we have

$$\dot{e}(t) = (A + \bar{L} \otimes BK)e(t) + \Delta_A e(t) - \bar{L}(B\bar{e}_v \Psi(t)) - \bar{L}w(t). \tag{2.18}$$

Next, we define the event trigger time series as $\left\{t_k^j\right\}$ for the jth agent. Therefore, the next triggering time t_{k+1}^i for the ith agent can be expressed as

$$t_{k+1}^i = \inf\left\{t > t_k^i | H_i(t) \geq 0\right\}. \tag{2.19}$$

The function $H_i(\cdot)$ is given by

$$H_i(\cdot) = -\theta_i \chi_i(t) + \alpha_i \|q_i(t)\|^2 - \eta_i \|e_i(t)\|^2, \tag{2.20}$$

where $\theta_i > 0$,$\alpha_i, \eta_i \in \mathbb{R}^n > 0$. $q_i(t)$ is defined as the measurement error according to Eq (2.6):

$$q_i(t) = e_i(t_k^i) - e_i(t). \tag{2.21}$$

$\chi_i(t)$ satisfies

$$\dot{\chi}_i(t) = -\beta_i \chi_i(t) + \eta_i \|e_i(t)\|^2 - \alpha_i \|q_i(t)\|^2, \tag{2.22}$$

where $\beta_i > 0$, initial value $\chi_i(0) > 0$ could be randomly selected, and $t_0^i = 0$.

Remark 1: The internal dynamic variable updates according to internal variables such as self-feedback, measurement error, and neighborhood error. In comparison with the conventional static triggering strategy [22, 23], the dynamic event triggered control protocol we proposed can more effectively reduce network communication and save resources.

3. Cyber attack model

A DoS attack aims to block the communication channels so the targeted system cannot exchange information normally. Communication channels are not the only things affected by DoS attacks because the attacks can damage communication equipment along with hindering data transmission, measurement, and control channels simultaneously. DoS attacks are extremely destructive to the system, but their energy consumption requires attackers to replenish energy supplies after the attack is over, which takes time. Therefore, the time series can be split into two sections based on whether a DoS assault was launched. In the absence of the DoS attack, the system functions and communicates properly. However, in the presence of a DoS attack, communication is cut off, and the controller stops functioning. Here, we assume that the time interval of DoS attacks is $\{t_m\}_{m\in\mathbb{N}}$, where t_m is the moment of the DoS attack, and $[t_m, t_m + \Delta_m]$ is the mth DoS time interval, and Δ_m is the time duration of the mth attack. The DoS attack interval is the same for all multi-agents. Thus, the set time instants where communication is blocked (the interval of the DoS attack) are

$$\Xi_a(t_0, t) = \left\{ \bigcup_{m\in\mathbb{N}} [t_m, t_m + \Delta_m] \right\} \cap [t_0, t]\,. \tag{3.1}$$

Similarly, the sequence of time intervals without attacks is given by

$$\Xi_s(t_0, t) = [t_0, t] \setminus \Xi_a(t_0, t). \tag{3.2}$$

Because of the recovery mechanism, the MAS cannot immediately restore communication after the end of a DoS attack, and due to the event-triggering mechanism, there is an upper bound for the time when the two events occur consecutively. We assume that they can exist at the same time. Therefore, the actual DoS attack lasts longer, and consequently, the mth DoS attack's actual time frame may be described as $\left[t_m, t_m + \bar{\Delta}_m\right]$. The new time period of the DoS attack is as follows:

$$\tilde{\Xi}_a(t_0, t) = \left\{ \bigcup_{m\in\mathbb{N}} \left[t_m, t_m + \bar{\Delta}_m\right] \right\} \cap [t_0, t] \tag{3.3}$$

$$\tilde{\Xi}_s(t_0, t) = [t_0, t] \setminus \tilde{\Xi}_a(t_0, t) \tag{3.4}$$

Assumption 2. Define $n_a(t_0, t)$ as the number of attacks in the period $[t_0, t]$, so the attack frequency $F_a(t_0, t) > 0$ satisfies

$$F_a(t_0, t) = \frac{n_a(t_0, t)}{t - t_0}. \tag{3.5}$$

Assumption 3. Define $N_a(t_0, t)$ as the total time interval of the DoS attack in the period $[t_0, t]$. The constants $T_0 \geq 0, F_0 \geq 0, 0 < \frac{1}{T_1} < 1, 0 < \frac{1}{F_1} < 1$ are such that

$$|\Xi_a(t_0, t)| \leq \Xi_0 + \frac{t - t_0}{T_1} \text{ and} \tag{3.6}$$

$$N_a(t_0, t) \leq F_0 + \frac{t - t_0}{F_1}, \tag{3.7}$$

where $\frac{1}{T_1}$ is the attack strength.

Lemma 1. Previous research considers Eq (2.1) and this DoS attack model under Assumptions 2 and 3 [18]. If the Lyapunov function $V_1(t), V_2(t)$ satisfies

$$\begin{cases} \dot{V}_1(t) \leq -l_0 V(t) + \tau_0 & t \in \tilde{\Xi}_s \\ \dot{V}_2(t) \leq l_1 V(t) + \tau_1 & t \in \tilde{\Xi}_a \end{cases}, \tag{3.8}$$

where l_0, l_1, τ_0, τ_1 are positive constants. T_1, F_1 defined in Assumption 3 satisfies

$$\begin{aligned} \frac{1}{T_1} &< \frac{l_0 - \eta^*}{l_0 + l_1} \\ \frac{1}{F_1} &< \frac{\eta^*}{2\ln\kappa + (l_0 + l_1)\rho} \end{aligned}, \tag{3.9}$$

where $0 < \eta^* < l_0$ is the time to restore communication. $\rho > 0$,$\kappa \geq 1$ is a constant satisfying

$$\begin{cases} \kappa V_2((t_m + \bar{\Delta}_m)^-) - V_1(t_m + \bar{\Delta}_m) \geq 0 \\ \kappa V_1(t_m^-) - V_2(t_{m+1}) \geq 0 \end{cases}. \tag{3.10}$$

Thus, we say that $V(t)$ are bounded.

Remark 2: Lemma 1 gives an upper bound on DoS attack frequency and duration, ensuring that the Lyapunov function remains stable over the entire time span [18].

Remark 3: The DoS attack considered in this paper mainly attacks the communication channels between agents. Thus, when the DoS attack comes, there is no information interaction between neighboring agents, and the event-triggering control is not triggered. In addition, we consider a DETC. Compared with the traditional event-triggering control, we introduce a dynamic variable that uses communication resources more effectively. In the simulation section below, we compare our method with the traditional event-triggering mechanism.

4. Stability analysis

In this section, we prove system stability. Our presentation has two sections: the stability study of the MAS (2.1) and (2.2) under a DoS attack and the proof of non-Zeno behavior.

Theorem 1. For the MAS (2.1) and (2.2) under DoS attacks, we consider Assumption 1 and the controller (2.12). If the LMI (2.13) satisfies $(A + \bar{L} \otimes BK)^T P + P(A + \bar{L} \otimes BK) \leq \xi_i P$, where $\xi_i \in \mathbb{R}^n = \sigma_i \eta_i, \sigma_i > 1$, then a feasible solution exists and the MAS is said to achieve leader-following consensus.

Proof of Theorem 1. The system stability proof is also divided into two parts. The communication of the system is damaged under a DoS attack, but the system is not always in an impassable state. The proof is divided between DoS attacks and non-DoS attacks, as per the prior section. When there are non-DoS attacks in the system, we consider the Lyapunov function

$$W(t) = V(t) + \sum_{i=1}^{N} \chi_i(t) = e^T(t) P e(t) + \sum_{i=1}^{N} \kappa_i^{-1} \tilde{\delta}_i^2 + \sum_{i=1}^{N} \chi_i(t). \tag{4.1}$$

It follows from Eqs (2.20)–(2.22) that

$$\dot{\chi}_i = -\beta_i \chi_i - \theta_i \chi_i. \tag{4.2}$$

The preceding implies that

$$\chi_i(t) \geq \chi_i(0)e^{-(\beta_i+\theta_i)t} > 0, \tag{4.3}$$

which leads to $W(t) > 0$.

The derivative of W is

$$\begin{aligned}\dot{W}(t) = e^T[(A + \bar{L} \otimes BK)^T P + P(A + \bar{L} \otimes BK)]e+ \\ 2e^T P\Delta_A e - 2e^T P\bar{L}\bar{e}\Psi(t) + 2e^T P\bar{L}w(t)+ \\ \sum_{i}^{N} 2\kappa_i^{-1}\dot{\tilde{\delta}}_i\tilde{\delta}_i + \sum_{i=1}^{N}\dot{\chi}_i(t).\end{aligned} \tag{4.4}$$

According to Eq (2.3), the condition in Assumption 1, and the control protocol in Eq (2.12), we have

$$\begin{aligned}\dot{W}(t) \leq\ & e^T[(A + \bar{L} \otimes BK)^T P + P(A + \bar{L} \otimes BK)]e+ \\ & 2\sum_{i=1}^{N}|e|^2\frac{\partial f(z_i)}{\partial x_i} - 2\sum_{i=1}^{N}|e|^2\Psi_i+ \\ & 2\sum_{i=1}^{N}|e|(\varsigma_{i1}|u| + \varsigma_{i2}|x| + \delta_i)+ \\ & \sum_{i}^{N} 2\kappa_i^{-1}\dot{\tilde{\delta}}_i\tilde{\delta}_i + \sum_{i=1}^{N}\dot{\chi}_i(t) \\ & \leq e^T[(A + \bar{L} \otimes BK)^T P + P(A + \bar{L} \otimes BK)]e+ \\ & 2\sum_{i=1}^{N}\delta_{ix}|e|^2 x_{is} - 2\sum_{i=1}^{N}|e|^2(1-\varsigma_{i1})\Psi_i+ \\ & 2\sum_{i=1}^{N}|e|(\varsigma_{i1}|k_i e_i| + \varsigma_{i2}|x| + \delta_i) \\ & + \sum_{i}^{N} 2\kappa_i^{-1}\dot{\tilde{\delta}}_i\tilde{\delta}_i + \sum_{i=1}^{N}\dot{\chi}_i(t).\end{aligned} \tag{4.5}$$

Choosing $\Psi_i(t)$ as in Eq (2.15), we obtain

$$\begin{aligned}\dot{W}(t) \leq\ & e^T[(A + \bar{L} \otimes BK)^T P + P(A + \bar{L} \otimes BK)]e- \\ & 2\sum_{i=1}^{N}\delta_{ix}|e|^2 x_{is} + \sum_{i}^{N} 2\kappa_i^{-1}\dot{\tilde{\delta}}_i\tilde{\delta}_i + \sum_{i=1}^{N}\dot{\chi}_i(t) \\ & \leq e^T[(A + \bar{L} \otimes BK)^T P + P(A + \bar{L} \otimes BK)]e+ \\ & \sum_{i=1}^{N}\dot{\chi}_i(t)\end{aligned}. \tag{4.6}$$

On the other hand

$$\sum_{i=1}^{N}\xi_i e_i^T(t)Pe_i(t) \leq \lambda_{max}(P)\sum_{i=1}^{N}\xi_i\|e_i(t)\|^2. \tag{4.7}$$

Based on the condition in Theorem 1, Eq (2.22), and Eq (4.6), we have

$$\begin{aligned}\dot{W}(t) &\leq -\sum_{i=1}^{N}(\xi_i-\eta_i)\left\|e_i(t)\right\|^2-\sum_{i=1}^{N}\alpha_i\left\|q_i(t)\right\|^2-\sum_{i=1}^{N}\beta_i\chi_i \\ &\leq -\sum_{i=1}^{N}\eta_i(\sigma_i-1)\left\|e_i(t)\right\|^2-\sum_{i=1}^{N}\beta_i\chi_i,\end{aligned} \tag{4.8}$$

then

$$\begin{aligned}\dot{W}(t) &\leq -(\sigma_M-1)\sum_{i=1}^{N}\xi_i\left\|e_i(t)\right\|^2-\sum_{i=1}^{N}\beta_i\chi_i, \\ &\leq -l_0W(t)+\tau_0\end{aligned} \tag{4.9}$$

where $l_0 = \min([(\sigma_M - 1)/\lambda_{\max}(P)], 1, \beta_m) > 0, \sigma_M = \max[\sigma_i], \beta_m = \min[\beta_i], \tau_0 = \sum_{i=1}^{N}\kappa_i^{-1}\tilde{\delta}_i^2, l_0, and \tau_0$ are positive constants.

When there are DoS attacks in the system, then communication and control channel blockages exist. In this case, the control input becomes 0, $u_i(t) = 0$, so the Lyapunov function can be expressed as

$$V(t) = e^T(t)Pe(t) + \sum_{i=1}^{N}\kappa_i^{-1}\tilde{\delta}_i^2. \tag{4.10}$$

Similar to (4.4), (4.10) can be written as

$$\begin{aligned}\dot{V}(t) &\leq V(t) + 2\sum_{i=1}^{N}\delta_{ix}\left|e\right|^2 x_{is} + 2\sum_{i=1}^{N}\left|e\right|(\varsigma_{i2}\left|x\right| + \delta_i) \\ &\leq l_1V(t) + \tau_1,\end{aligned} \tag{4.11}$$

where $l_1 = 1$, and $\tau_1 = 2\sum_{i=1}^{N}\delta_{ix}\left|e\right|^2 x_{is} + 2\sum_{i=1}^{N}\left|e\right|(\varsigma_{i2}\left|x\right| + \delta_i)$. According to the conditions of (3) and Assumption 1, we know that τ_1 has an upper bound. From Lemma 1, we know that the system stabilizes in a limited time under a DoS attack. The proof is completed.

Next is the proof of no Zeno behavior. We assume that there is a positive constant T_0 such that $\lim_{k\to\infty} t_k^i = T_0$. Based on the property of limit, we know that for any $\varepsilon_0 > 0$, there exists $N(\varepsilon_0)$ such that $t_k^i \in (T_0 - \varepsilon_0, T_0 + \varepsilon_0), \forall k \geq N(\varepsilon_0)$. This means $t^i_{N(\varepsilon_0+1)} - t^i_{N(\varepsilon_0)} < 2\varepsilon_0$.

According to (4.11), $W(t)$ gradually decreases to 0, Then $\xi_m\lambda_{\min}(P)\left\|e_i(t)\right\|^2 \leq V(t) < W(t)$. Therefore, we have

$$\left\|e_i(t)\right\| \leq \sqrt{\frac{W_0}{\xi_m\lambda_{\min}(P)}} = \varpi_0. \tag{4.12}$$

Because $\|e_i(t)\|$ and $\|q_i(t)\|$ are bounded, the Dini derivative of $\|q_i(t)\|$ is

$$\begin{aligned} D^+ \|q_i(t)\| &\le \|\dot{q}_i(t)\| \\ &= \left\| -\sum_{j=1}^{N} a_{ij}(\dot{x}_i(t) - \dot{x}_j(t)) + b_i(\dot{x}_i(t) - \dot{x}_0(t)) \right\| \\ &\le \|A + \Delta_A\| \|e_i(t)\| + \|\bar{L}\| \|B\| \left\| \sum_{j=1}^{N} (u_i(t)) \right\| + , \\ &\|\bar{L}\| \|w_i(t)\| \\ &\le \|\bar{A}\| \varpi_0 + \|\bar{L}\| \|B\| M_1 + \|L\| M_2 = \hat{W}_0 \end{aligned} \tag{4.13}$$

where $\bar{A} = A + \Delta_A$. According to Eqs (2.3), (2.12), and (2.14), we obtain $u_i(t)$. $w_i(t)$ has an upper bound, and M_1, M_2 is their upper bound.

Since only the trigger condition in Eq (2.19) is met and the event is triggered when $\|q_i(t)\|$ is reset to 0, then $\|q_i(t)\| \ge \sqrt{\frac{\eta_i}{\alpha_i} \|e_i(t)\|^2 + \frac{\theta_i}{\alpha_i}\chi_i} \ge \sqrt{\frac{\theta_i}{\alpha_i}\chi_i}, t_k^{i-}, k = 1, 2 \cdots$, which implies that

$$\left\|q_i(t_k^{i-})\right\| \ge \sqrt{\frac{\theta_i}{\alpha_i}\chi_i(t_k^{i-})} = \sqrt{\frac{\theta_i}{\alpha_i}\chi_i(0)e^{-\frac{\beta_i+\theta_i}{2}t_k^{i-}}}, \tag{4.14}$$

then, we can obtain

$$t^i_{N(\varepsilon_0+1)} - t^i_{N(\varepsilon_0)} \ge \frac{1}{\hat{W}_0} \sqrt{\frac{\theta_i}{\alpha_i}\chi_i(0)e^{-\frac{\beta_i+\theta_1}{2}t^{i-}_{N(\varepsilon_0+1)}}}. \tag{4.15}$$

If $\varepsilon_0 > 0$ is a solution of

$$\frac{1}{\hat{W}_0} \sqrt{\frac{\theta_i}{\alpha_i}\chi_i(0)e^{-\frac{\beta_i+\theta_i}{2}T_O}} = 2\varepsilon_0 e^{\frac{\beta_i+\theta_i}{2}\varepsilon_0}, \tag{4.16}$$

then

$$t^i_{N(\varepsilon_0+1)} - t^i_{N(\varepsilon_0)} \ge \frac{1}{\hat{W}_0} \sqrt{\frac{\theta_i}{\alpha_i}\chi_i(0)e^{-\frac{\beta_i+\theta_i}{2}(T_0+\varepsilon_0)}} = 2\varepsilon_0. \tag{4.17}$$

As a result, the aforementioned assumption is false, concluding the evidence that the agent i does not have Zeno behavior. □

5. Simulation

To show the efficacy of the proposed control strategy, we present a simulation example in this section. Our simulation uses MASs composed of six agents as shown in Figure 1, where agent 1 is the leader, and others are followers. The system is

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + (-sin(x_i(t)) + 1.5cos(2.5 * t)) + w_i$$

The system parameters are set as

$$A = \begin{bmatrix} 0 & I_3 \\ A_1 & A_2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ I_3 \end{bmatrix}$$

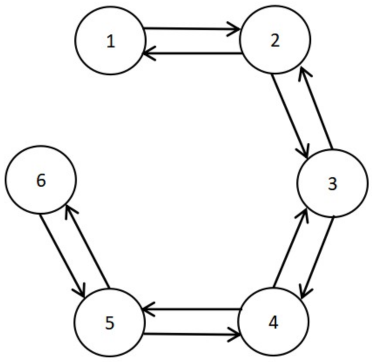

Figure 1. Graph $\mathcal{G}$ in the example.

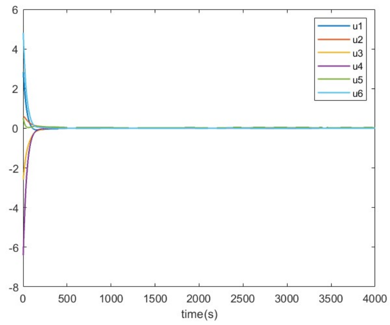

Figure 2. The control input's response curves.

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\phi^2 & 0 \\ 0 & 0 & -\phi^2 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 2\phi & 0 \\ -2\phi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\varsigma_{i1} = 0.1 \times \sqrt{1 + i^2}, \varsigma_{i2} = 0.5 \times \sqrt{1 + i^2}$$

In this example, we consider the flight of an aircraft, $\phi = 0.002$ is the angular velocity of the aircraft, and I_3 represents the identity matrix of 3×3.

$$\alpha_i = 87.5, \beta_i = 0.004, \theta_i = 3.5$$

$$\eta_i = \begin{bmatrix} 0.21 & 0.105 & 0.105 & 0.21 & 0.21 & 0.105 \end{bmatrix}$$

Figures 2 and 3 show the response curves and consistency errors of the system state for all agents. They show that the followers' states converge toward those of the leader as time progresses. Figures 3 and 4 show the control input curves and event trigger time instant for all agents. There are four times DoS attacks, with $T_0 = 3, F_0 = 4$. The duration of the DoS attack is $|\Xi_a(0, 40)| = 3.5$. In Table 1, we can see that the dynamic event-triggered mechanism proposed in this paper has far fewer triggering instances in the same time than the other two literatures [22, 23], which can effectively save communication resources. In addition, continuous communication between agents is not required to determine whether a trigger condition satisfies a trigger condition. Considering the static

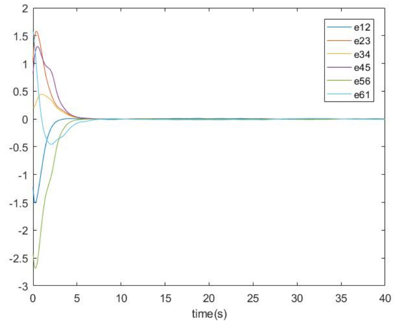

Figure 3. The consensus errors' response curves.

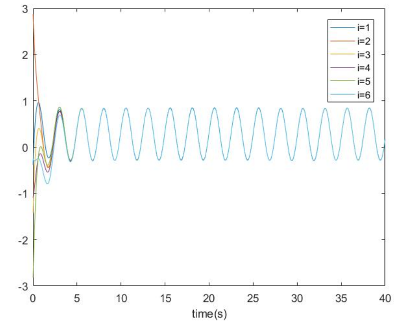

Figure 4. Position's response curves.

event-triggered control protocol, we have

$$t_{k+1}^{i} = \inf\left\{t > t_k^i |\, \|q_i(t)\|^2 - \rho\, \|e_i(t)\|^2 \geq 0\right\}$$

$$t_{k+1}^{i} = \inf\left\{t > t_k^i |\, \|q_i(t)\| - \varrho\, \|e_i(t)\| \geq 0\right\}$$

where ρ and ϱ are positive constants. Our DETC effectively reduces communication frequency.

6. Conclusions

In this paper, we propose a dynamic event-triggered adaptive control approach to address the leader-following consensus problem for nonlinear MASs experiencing DoS attacks. We have presented a

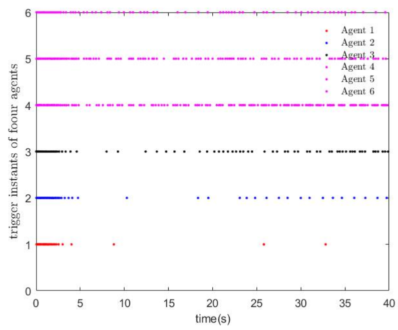

Figure 5. Event trigger time instant for all agents.

Table 1. Compared with the traditional triggering protocols in [0,40 s].

Agent i	1	2	3	4	5	6
[22]	260	1897	1914	1704	1635	1861
[23]	1632	1899	1917	1694	1633	1869
Our DETC	50	65	70	106	42	6

distributed control strategy and adaptive update laws to ensure system stability in the presence of uncertainties. The Lyapunov stability theory is used to derive conditions for achieving consensus. The DoS attacks considered here mainly target the MASs' communication channels. In reality, there are other types, scales, and levels of DoS attacks. Formulating mathematical models of these other types of DoS attacks and solving these models is the direction of our future research.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

References

1. X. Wang, G. H. Yang, Distributed fault-tolerant control for a class of cooperative uncertain systems with actuator failures and switching topologies, *Inform. Sci.*, **370** (2016), 650–666. https://doi.org/10.1016/j.ins.2015.11.002

2. S. Chen, DWC. Ho, L. Li, M. Liu, Fault-Tolerant Consensus of Multi-Agent System With Distributed Adaptive Protocol, *IEEE Trans. Cybern.*, **45** (2014), 2142–2155. https://doi.org/10.1109/TCYB.2014.2366204

3. X. Jin, X. Zhao, J. Yu, X. Wu, J. Chi, Adaptive fault-tolerant consensus for a class of leader-following systems using neural network learning strategy, *Neural Networks*, **121** (2020), 474–483. https://doi.org/10.1016/j.neunet.2019.09.028

4. M. Khalili, X. Zhang, M. M. Polycarpou, T. Parisini, Y. Cao, Distributed adaptive fault-tolerant control of uncertain multi-agent systems, *Automatica*, **87** (2018), 142–151. https://doi.org/10.1016/j.automatica.2017.09.002

5. H. Yang, M. Staroswiecki, B. Jiang, J. Liu, Fault tolerant cooperative control for a class of nonlinear multi-agent systems, *Syst. Control Letters*, **60** (2011), 271–277. https://doi.org/10.1016/j.sysconle.2011.02.004

6. W. He, F. Qian, J. Lam, G. Chen, Q. L. Han, J. Kurths, Quasi-synchronization of heterogeneous dynamic networks via distributed impulsive control: Error estimation, optimization and design, *Automatica*, **62** (2015), 249–262. https://doi.org/10.1016/j.automatica.2015.09.028

7. W. He, G. Chen, Q. L. Han, F. Qian, Network-based leader-following consensus of nonlinear multi-agent systems via distributed impulsive control, *Inform. Sci.*, **380** (2017), 145–158. https://doi.org/10.1016/j.ins.2015.06.005

8. H. Hou, Q. Zhang, Finite-time synchronization for second-order nonlinear multi-agent system via pinning exponent sliding mode control, *ISA Trans.*, **65** (2016), 96–108. https://doi.org/10.1016/j.isatra.2016.07.004

9. D. V. Dimarogonas, E. Frazzoli, Distributed event-triggered control strategies for multi-agent systems, in *2009 47th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, (2009), 906–910. https://doi.org/10.1109/ALLERTON.2009.5394897

10. D. V. Dimarogonas, E. Frazzoli, Distributed event-triggered control for multi-agent systems, *IEEE Trans. Autom. Control*, **57** (2011), 1291–1297. 10.1109/TAC.2011.2174666

11. G. S. Seyboth, D. V. Dimarogonas, K. H. Johansson, Control of multi-agent systems via event-based communication, *IFAC Proceed Volumes*, **44** (2011), 10086–10091. https://doi.org/10.3182/20110828-6-IT-1002.01496

12. D. Liu, A. Hu, H. Shao, Adaptive event-triggered control for consensus of multi-agent systems, *Comput. Eng. Appl.*, **53** (2017), 44–48. https://doi.org/10.3778/j.issn.1002-8331.1503-0235

13. D. Liu, G. H. Yang, A dynamic event-triggered control approach to leader-following consensus for linear multiagent systems, *IEEE Trans. Syst. Man Cybern. Syst.*, **51** (2021), 6271–6279. https://doi.org/10.1109/TSMC.2019.2960062

14. X. Wang, G. Yang, Distributed fault-tolerant control for a class of cooperative uncertain systems with actuator failures and switching topologies, *Inform. Sci.*, **370–371** (2016), 650–666. https://doi.org/10.1016/j.ins.2015.11.002

15. K. Zhang, Z. Hu, F. Song, X. Yang, Y. Liu, Consensus of input constrained multi-agent systems by dynamic time-varying event-triggered strategy with a designable minimal inter-event time, in *IEEE Transactions on Circuits and Systems II: Express Briefs*, (2023), 1. https://doi.org/10.1109/TCSII.2023.3332593

16. M. Li, Y. Long, T. Li, H. Liang, C. L. P. Chen, Dynamic event-triggered consensus control for input constrained multi-agent systems with a designable minimum inter-event time, *IEEE J. Autom. Sin.*, (2023), 1–12. https://doi.org/10.1109/JAS.2023.123582

17. H. Zhang, A. Wang, W. Ji, J. Qiu, H. Yan, Optimal consensus control for continuous-time linear multiagent systems: A dynamic event-triggered approach, *IEEE Trans. Neural Networks Learn. Syst.*, (2023), 1–9. https://doi.org/10.1109/TNNLS.2023.3279137

18. F. Zhi, G. Hu, Distributed secure average consensus for linear multi-agent systems

under DoS attacks, in *2017 American control conference (ACC)*, (2017), 2261–2266. https://doi.org/10.23919/ACC.2017.7963289

19. F. Zhi, G. Hu, Distributed secure leader-following consensus of multi-agent systems under DoS attacks and directed topology, in *2017 IEEE International Conference on Information and Automation (ICIA)*, (2017), 73–79. https://doi.org/10.1109/ICInfA.2017.8078885
20. T. Dong, Y. A. Gong, Leader-following secure consensus for second-order multi-agent systems with nonlinear dynamics and event-triggered control strategy under DoS attack, *Neurocomputing*, **416** (2020), 95–102. https://doi.org/10.1016/j.neucom.2019.01.113
21. X. G. Guo, P. M. Liu, J. L. Wang, C. K. Ahn, Event-triggered adaptive fault-tolerant pinning control for cluster consensus of heterogeneous nonlinear multi-agent systems under aperiodic DoS attacks, *IEEE Trans. Network Sci. Eng.*, **8** (2021), 1941–1956. https://doi.org/10.1109/TNSE.2021.3077766
22. Y. C. Sun, G. H. Yang, Event-triggered distributed state estimation for multiagent systems under DoS attacks, *IEEE Trans. Cybern.*, **52** (2020), 6901–6910. https://doi.org/10.1109/TCYB.2020.3034456
23. Y. Ma, W. Che, C. Deng, Z. Wu, Observer-based event-triggered containment control for MASs under DoS attacks, *IEEE Trans. Cybern.*, **52** (2021), 13156–13167. https://doi.org/10.1109/TCYB.2021.3104178
24. T. Hao, Y. J. Wu, X. Z. Jin, Robust adaptive leader-following control of a class of multi-agent systems, in *2020 Chinese Control And Decision Conference (CCDC)*, (2020), 2967–2972. https://doi.org/10.1109/CCDC49329.2020.9164753
25. Z. Liang, G. H. Yang, Adaptive fault-tolerant control for nonlinear multi-agent systems with DoS attacks, *Inform. Sci.*, **526** (2020), 39–53. https://doi.org/10.1016/j.ins.2020.03.083