



Research article

A study of integrated pest management models with instantaneous and non-instantaneous impulse effects

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Abstract: The occurrence of pests and diseases during agricultural production affects the quality and quantity of agricultural products. It is important to evaluate the impact of various factors on pests to achieve optimal results of integrated pest management (IPM) during its implementation. In this paper, we considered the transient and non-transient effects of chemical control on pests and the effects on natural enemies at different times, and developed a corresponding pest control model. Detailed studies and comparisons were conducted for spraying pesticides either more or less frequently as compared to strategies for releasing natural enemies. The threshold conditions for global asymptotic stabilization of the pest extinction period solution was obtained. Using two-parameter and sensitivity analysis techniques, the parameters affecting the variation of the threshold were discussed. By comparing these two pest control strategies, we found the existence of optimal application and release frequencies. Finally, in order to control pests below the economic threshold level, the state-dependent pest model was numerically investigated. The results show that the presence or absence of chemical control of pests can depend on the values taken for the parameters in the model. Based on this information, pest control experts can make decisions about the best spraying time and the best release rate.

Keywords: integrated pest management; pesticide residue effects; sensitivity analysis; state-dependence pest model

1. Introduction

Crop pests have many characteristics, a great influence, and a strong explosiveness. In China, there are more than 1000 types of pests recorded on important crops. When pests erupt, they can affect crop yields and, in severe cases, even extinction, thus resulting in economic losses and being the biggest barrier to agricultural production [1–3]. For example, an outbreak of the grass-craving nightshade moth

in 2019 caused considerable economic losses in many parts of southern China [3]. Therefore, the management of pests to achieve the most favorable conditions for humans is an important research topic.

Chemical and biological control are the most popular approaches of pest management. Chemical control involves the use of pesticides and other chemicals to deal with pests, which has the advantages of convenience and a high efficiency. The effect on pests can be observed in a brief term; however, when pesticides are applied for a prolonged term, the pests will be resistant to the chemicals and cause pollution to the environment, thus affecting the health of the human body [4–6]. Biological control is the use of artificially released natural enemies and host natural enemies to control pests. It not only controls pests but also minimizes environmental pollution. However, natural enemies often require artificial cultivation and the expense is high [7–10].

Therefore, chemical and biological control are often combined (i.e., integrated pest management (IPM) is employed), which is a prolonged control strategy. Considering the population dynamics of pests and their environment, the pest population is controlled below the level of economic harm by using all appropriate methods and techniques [8, 11–14]. Economic threshold (ET) and economic injury level (EIL) are two essential concepts in the IPM process. EIL is defined as the lowest pest density that can cause economic losses, and the ET is determined as the pest density at which control measures should be applied to prevent the pest from reaching the EIL [12, 15]. In 2015, Xiang et al. [14] studied the application of comprehensive intervention strategies of pest control in the pest-natural enemy model, and discussed the situation where the frequency of pesticide application is greater than the frequency of natural enemy release. This information can help pest control experts come up with new ideas.

Spraying pesticides or releasing natural enemies during pest control can be described as a split-second act. To model this process, we can build either continuous or discrete impulse differential equations. Scholars have studied the transient effects of pesticides on pests using periodic pulse differential equations [16–18] and state-dependent pulse differential equations [19–21]. However, the impacts produced by pesticides on both pests and natural enemies occur not only in a single moment, but also on both organisms over the following timescale, which can be characterized by either a continuous or segmented continuous periodic function of time [22–24]. For example, Tang et al. [24] incorporated the non-transient effects of chemical control on pests and natural enemies in an integrated pest management model. Threshold conditions for the pest extinction period solutions were acquired, and the impacts of factors such as application frequency, killing efficiency, and the application period on the threshold conditions were analyzed.

Holling II predator-prey systems are thought to play an important role in characterizing the relationship between pest and natural enemy populations. Therefore, it is of a great biological importance to study models of Holling II [14, 25–27]. For instance, in 2018, Yang and Tan [25] investigated the effect of pesticide dosage on a predation model with a Holling II predation and showed complex dynamics in the proposed model. In 2018, Páez Chávez et al. [26] conducted a comparative study on integrated pest management strategies based on impulsive control. Compared with other functions, the modeling with the Holling II function was more effective in controlling pest populations. Additionally, Wei et al. [27] examined the classical Holling II predation model with the addition of multiple factors.

Because pesticide spraying affects natural enemies, their predation and conversion rates on pests can differ before and after pesticide spraying. Switching systems (or the Filippov system) can represent this process very well. Recently, the switching system has been extensively studied by scholars [28–33].

For instance, Gao et al. [33] established a corresponding Huanglong disease switching model which took the different removal and susceptibility rates of citrus trees with seasonal changes into account; additionally they investigated the effect of the switching control scheme on the model dynamics. In 2023, Liu et al. [32] investigated a pest control switching model with a transient and non-transient pulse impact and discussed the impact of applying a switching control scheme on the model dynamics when the pesticide spraying frequency was higher than the release frequency of natural enemies. Moreover, numerical studies of pest control switching models with state-dependent switching were performed. We all know that spraying pesticides has an impact on pests and natural enemies, and the conversion and predation rate of natural enemies to pests before and after spraying pesticides will also be different. However, none of the models studied so far simultaneously considered them in a predator-prey switching system of the Holling II. Therefore, in this paper, we establish the corresponding switching model while taking these factors into account.

In this paper, based on the above discussion and [21, 32], we study a model with the Holling II using IPM intervention as a pulse control strategy, to study the impact of different control strategies on pest management and population changes. We construct the corresponding model in Section 2. In Section 3, we study the situation where the frequency of pesticide spraying is higher than the frequency of natural enemy release, and analyze threshold conditions for pest extinction. The estimation of the partial correlation coefficient (PRCC) is carried out using the Latin hypercube sampling (LHS) method to give the effect of each parameter on the threshold. Moreover, in Section 4, we examine the situation where the frequency of natural enemy release is higher than the frequency of pesticide sprays, thereby comparing the effect of the parameters on the pest extinction thresholds in the two cases. In view of the fact that the ideal goal of the IPM is to control the density of pests below the ET, we establish a pest model with state-dependent modeling in Section 5, and analyze the number of pesticide sprays required when the parameters are varied. Finally, we analyze the biological significance they represent.

2. Model formulation

Many studies have been conducted in the previous literature for the Holling II pest management model, though the effects of pesticide residues have rarely been considered. Therefore, we consider the effects of pesticide residues on pests and natural enemies in this paper, thus leading to a better IPM. From reference [21], we have the following:

$$\begin{cases} \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - \frac{ax(t)y(t)}{1 + dx(t)}, \\ \frac{dy(t)}{dt} = \frac{kax(t)y(t)}{1 + dx(t)} - cy(t), \end{cases} \quad (2.1)$$

where $x(t)$ and $y(t)$ denote the pest and natural enemy densities, respectively, K is its carrying capacity, r is the intrinsic growth rate of the pest population, $(ax(t))/(1 + dx(t))$ is the Holling II function response, which is a saturating function of the number of pests present, and c is the mortality rate of the predator population.

Referring to references [24] and [32], the following exponential function is introduced to represent the impact of pesticide residues on pests and natural enemies:

$$b_i(t) = m_i e^{-\delta_i(t-nT)}, nT \leq t < (n+1)T,$$

where $m_i \geq 0 (i = 1, 2)$ denotes the non-instantaneous kill rate of the pesticide on pests and natural enemies, respectively, T represents for pulse period, and $\delta_i > 0 (i = 1, 2)$ denotes the positive decay rate of the pesticide on pests and natural enemies, respectively.

In the following third and fourth parts, we will study the frequency of spraying pests higher and lower than the frequency of releasing natural enemies. We have established the following two systems. Case 1: Applying pesticides more frequently than releasing natural enemies.

Considering the differences in capture and transformation rates of pests by natural enemies during the time of pesticide action and inaction, alongside the instantaneous and non-instantaneous impacts of insecticide spraying on pest and natural enemy populations, the following switching model for integrated pest control is established:

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - \frac{a_1 x(t)y(t)}{1 + dx(t)}, \\ \frac{dy(t)}{dt} = \frac{k_1 a_1 x(t)y(t)}{1 + dx(t)} - cy(t), \end{array} \right\} t \in (hT_N, (h+l)T_N],$$

$$\left. \begin{array}{l} x(t^+) = (1 - p_1)x(t), \\ y(t^+) = (1 - p_2)y(t), \end{array} \right\} t = (h+l)T_N + nT_k, n = 0, 1, 2, \dots, k-1,$$

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - \frac{a_2 x(t)y(t)}{1 + dx(t)} - b_1(t)x(t), \\ \frac{dy(t)}{dt} = \frac{k_2 a_2 x(t)y(t)}{1 + dx(t)} - cy(t) - b_2(t)y(t), \end{array} \right\} t \in ((h+l)T_N, (h+1)T_N], t \neq (h+l)T_N + nT_k,$$

$$\left. \begin{array}{l} x(t^+) = x(t), \\ y(t^+) = y(t) + \tau, \end{array} \right\} t = (h+1)T_N, h \in N,$$
(2.2)

$b_i(t) = m_i e^{-\delta_i(t-(h+l)T_N-nT_k)}$, $(h+l)T_N + nT_k \leq t < (h+l)T_N + (n+1)T_k$, where T_N is the pulse period, $0 < l < 1$ denotes the starting time of pesticide spraying, $0 < a_1, a_2 < 1$ and $0 < k_1, k_2 < 1$ represent the predation rate and transformation rate of the natural enemies on the pests in the corresponding time period, respectively, $\tau > 0$ is the release of the natural enemies at the moment $t = hT_N$, and $0 \leq p_1, p_2 < 1$ denote the momentary killing efficiency of the pesticide on pests and natural enemies at the moment $t = (h+l)T_N$, respectively. During the time T_N of releasing the natural enemy, the pesticide is periodically sprayed k times, $t = (h+l)T_N + nT_k$, ($n = 0, 1, 2, \dots, k-1$), and the period of the number of pesticide sprays is set to T_k , which is $(1-l)T_N = kT_k$. The other parameters are the same as in model (2.1).

For system (2.2), we have the following interpretation. In a large pulse period T_N , pests and natural enemies undergo several processes of change. At time $t \in (hT_N, (h+l)T_N)$, the density changes of pests and natural enemies follow their own development rules, which are expressed by the first two equations of system (2.2). When the time is $t = (h+l)T_N$, we spray pesticides. During this moment, pesticides have instantaneous effects on pests and natural enemies at the same time, and the action equations are the third and fourth equations of system (2.2). At time $t \in ((h+l)T_N, (h+l)T_N + T_k)$, followed by time $t = (h+l)T_N$, (i.e., after spraying pesticides), we consider the residual effects of pesticides on pests and natural enemies, (i.e., on the fifth and sixth equations). At time $t \in ((h+l)T_N, (h+1)T_N)$ in the

middle of the large pulse T_N , the pesticides are sprayed k times, indicating that the instantaneous and non-instantaneous effects of pesticides on pests and natural enemies are also carried out k times. At time $t = (h + 1)T_N$, the natural enemies are released, as expressed by the seventh and eighth equations. This is a complete cycle. The detailed process is given in the third section.

Case 2: Natural enemies are released more frequently than pesticide applications.

In a period where pesticide spraying is followed by the release of natural enemies, we have the following:

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - \frac{a_2x(t)y(t)}{1 + dx(t)} - b_1(t)x(t), \\ \frac{dy(t)}{dt} = \frac{k_2a_2x(t)y(t)}{1 + dx(t)} - cy(t) - b_2(t)y(t), \\ x(t^+) = x(t), \\ y(t^+) = y(t) + \tau, \end{array} \right\} t \neq hT_z, t \neq (h + l)T_z + kT_p, k = 0, 1, 2, \dots, p - 1,$$

$$\left. \begin{array}{l} x(t^+) = x(t), \\ y(t^+) = y(t) + \tau, \end{array} \right\} t = (h + l)T_z + kT_p, k = 0, 1, 2, \dots, p - 1,$$

$$\left. \begin{array}{l} x(t^+) = (1 - p_1)x(t), \\ y(t^+) = (1 - p_2)y(t), \end{array} \right\} t = (h + 1)T_z, h \in N,$$

$$b_i(t) = m_i e^{-\delta_i(t-hT_z)}, hT_z \leq t < (h + 1)T_z, i = 1, 2, \quad (2.3)$$

parameter meaning reference system (2.2). For system (2.3), when the frequency of releasing natural enemies is greater than that of spraying pesticides, we assume that the pesticides are sprayed first (i.e., pesticides are sprayed on pests at each large pulse period point $t = hT_z$). At time $t \in (hT_z, (h + l)T_z]$, pests and natural enemies will be affected by pesticides, as expressed as the first two equations of system (2.3). At time $t = (h + l)T_z$, we release natural enemies, and the action equations are the third and fourth equations of system (2.3). Similar to system (2.2), in system (2.3), we assume that the natural enemy is released p times within the time $t \in ((h + l)T_z, (h + 1)T_z)$ in the middle of the large pulse T_z . At time $t = (h + 1)T_z$, it is sprayed with pesticides, as expressed as the fifth and sixth equations of system (2.3). Thus, a pulse period T_z is formed. The detailed process is given in Section 4.

3. The study of system (2.2)

3.1. Threshold conditions for pest eradication

To begin with, we study the following subsystems of system (2.2):

$$\left\{ \begin{array}{l} \frac{dy(t)}{dt} = -cy(t), t \in (hT_N, (h + l)T_N], \\ y(t^+) = (1 - p_2)y(t), t = (h + l)T_N + nT_k, n = 0, 1, 2, \dots, k - 1, \\ \frac{dy(t)}{dt} = -cy(t) - b_2(t)y(t), t \in ((h + l)T_N, (h + 1)T_N], t \neq (h + l)T_N + nT_k, \\ y(t^+) = y(t) + \tau, t = (h + 1)T_N, h \in N. \end{array} \right. \quad (3.1)$$

Theorem 3.1. *The system (3.1) has a unique, globally asymptotically stable, positive periodic solution*

$y^*(t)$ and $\lim_{t \rightarrow \infty} |y(t) - y^{N^*}(t)| = 0$, where

$$y^{N^*}(t) = \begin{cases} y_1^{N^*}(t), t \in (hT_N, (h+l)T_N], \\ y_j^{N^*}(t), t \in ((h+l)T_N + (j-2)T_k, (h+l)T_N + (j-1)T_k], \\ j = 2, 3, \dots, k+1, \end{cases} \quad (3.2)$$

$$y_1^{N^*}(t) = y^{N^*} \exp(-c(t - hT_N)),$$

$$y_j^{N^*}(t) = y^{N^*} (1 - p_2)^{j-1} \exp(-c(t - hT_N) - (j-2) \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k}) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 (t - (h+l)T_N - (j-2)T_k)})),$$

$$y^{N^*} = \frac{\tau}{1 - (1 - p_2)^k \exp(-cT_N - k \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k}))}. \quad (3.3)$$

Proof. Solving the equation of system (3.1) on interval $[hT_N, (h+l)T_N)$ yields the following:

$$y^N(t) = y^N(hT_N^+) \exp(-c(t - hT_N)).$$

At the first spraying of pesticides in a period T_N , there are

$$y^N((h+l)T_N^+) = (1 - p_2) y^N(hT_N^+) \exp(-clT_N).$$

In one period T_N , after the first spraying of pesticides, i.e., $t \in ((h+l)T_N, (h+l)T_N + T_k)$, then

$$y^N(t) = (1 - p_2) y^N(hT_N^+) \exp(-c(t - hT_N) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 (t - (h+l)T_N)})).$$

On the second spraying of pesticides during the cycle, there are

$$y^N(((h+l)T_N + T_k)^+) = (1 - p_2)^2 y^N(hT_N^+) \exp(-c(lT_N + T_k) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k})).$$

After the second spraying of pesticides during the period, i.e., at $t \in ((h+l)T_N + T_k, (h+l)T_N + 2T_k)$, there are

$$y^N(t) = (1 - p_2)^2 y^N(hT_N^+) \exp(-c(t - hT_N) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k}) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 (t - (h+l)T_N - T_k)})).$$

Repeating the previous process, after the k th spraying of pesticides in a period T_N , i.e., $t \in ((h+l)T_N + (k-1)T_k, (h+1)T_N]$, we have the following:

$$y^N(t) = (1 - p_2)^k y^N(hT_N^+) \exp(-c(t - hT_N) - (k-1) \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k}) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 (t - (h+l)T_N - (k-1)T_k)})).$$

When $t = (h+1)T_N$, for which the first natural enemy is dropped, there are

$$y^N((h+1)T_N^+) = (1 - p_2)^k y^N(hT_N^+) \exp(-c(lT_N + kT_k) - k \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k})) + \tau.$$

Let $y_h^N = y^N(hT_N^+)$; then, we have the following equation:

$$y_{h+1}^N = (1 - p_2)^k y_h^N \exp(-c(IT_N + kT_k) - P \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k})) + \tau \triangleq G(y_h^N). \quad (3.4)$$

From the above equation, the only possible immovable point is the following:

$$y^{N*} = \frac{\tau}{1 - (1 - p_2)^k \exp(-cT_N - k \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k}))}.$$

Because (3.4) is a linear system and $\left| \frac{dG(y_h^N)}{y_h^N} \right| = 1 - (1 - p_2)^k \exp(-cT_N - k \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_k})) < 1$, it follows from the theory of differential equations that it is a positive equilibrium point of global asymptotic stability of the differential equations. Thus, system (2.2) exists as a single globally asymptotically stable periodic solution $y^{N*}(t)$. An arbitrary solution $y(t)$ of system (2.2) converges to $y^{N*}(t)$, as denoted by (3.2) when $t \rightarrow \infty$. The proof is accomplished.

Thus, system (2.2) has a pest extinction period solution $(0, y^{N*}(t))$.

Theorem 3.2. *If $R_0^{TN} < 1$ is established, the pest extinction period solution $(0, y^{N*}(t))$ of system (2.2) is globally asymptotically stable.*

Proof. In the first step, we show that the extermination period solution $(0, y^{N*}(t))$ of system (2.2) is locally asymptotically stable, which is determined by considering a small amplitude perturbation $(u(t), v(t))$ of the solution. Defining $x(t) = u(t)$, $y(t) = y^{N*}(t) + v(t)$, where $(u(t), v(t))$ is a small perturbation and can be written as follows:

$$\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \end{pmatrix}, t \in (0, T_N],$$

where $\Phi(t) = \prod_{j=1}^{k+1} \Phi_j(t)$, and $\Phi_j(t)$ fulfill

$$\frac{d\Phi_1(t)}{dt} = \begin{pmatrix} r - a_1 y_1^{N*}(t) & 0 \\ k_1 a_1 y_1^{N*}(t) & -c \end{pmatrix} \Phi_1(t), t \in (0, IT_N],$$

$$\frac{d\Phi_j(t)}{dt} = \begin{pmatrix} r - b_1(t) - a_2 y_j^{N*}(t) & 0 \\ k_2 a_2 y_j^{N*}(t) & -c - b_2(t) \end{pmatrix} \Phi_j(t),$$

where

$$t \in (IT_N + (j - 2)T_k, IT_N + (j - 1)T_k], j = 2, 3, \dots, k + 1.$$

Hence,

$$\Phi_1(t) = \begin{pmatrix} \exp(\int_0^t (r - a_1 y_1^{N*}(s)) ds) & 0 \\ * & \exp(-ct) \end{pmatrix},$$

$$\Phi_j(t) = \begin{pmatrix} V_1 & 0 \\ * & V_2 \end{pmatrix},$$

where

$$V_1 = \exp\left(\int_{lT_N+(j-2)T_k}^t (r - b_1(s) - a_2y_j^{N*}(s))ds\right),$$

$$V_2 = \exp\left(-c(t - lT_N - (j-2)T_k) - \frac{m_2}{\delta_2}(1 - e^{-\delta_2(t-lT_N-(j-2)T_k)})\right).$$

It is not necessary to compute the precise form of (*) because it is not needed in the following theories. By the Floquet theory, if the modes of the two eigenvalues of a single-valued matrix

$$U = \Phi_1(lT_N) \prod_{j=2}^{k+1} \begin{pmatrix} 1 - p_1 & 0 \\ 0 & 1 - p_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Phi_j(lT_N + (j-1)T_k),$$

are less than 1, the pest extinction period solution is locally asymptotically stable. Indeed, the eigenvalues are as follows:

$$\lambda_1^N = (1 - p_1)^k \exp\left(\int_0^{lT_N} (r - a_1y_1^{N*}(s))ds + \sum_{n=2}^{k+1} \int_{lT_N+(n-2)T_k}^{lT_N+(n-1)T_k} (r - b_1(s) - a_2y_n^{N*}(s))ds\right),$$

$$\lambda_2^N = (1 - p_2)^k \exp\left(-cT_N - k \frac{m_2}{\delta_2}(1 - e^{-\delta_2T_k})\right) < 1.$$

Thus, when $|\lambda_1^N| < 1$, i.e., $\frac{rT_N}{\ln\left(\frac{1}{(1-p_1)^k}\right) + A + B + \sum_{n=2}^{k+1} \int_{lT_N+(n-2)T_k}^{lT_N+(n-1)T_k} (a_2y_k^{N*}(s))ds} < 1$, where

$$A = \frac{a_1}{c}y^{N*}(1 - e^{-clT_N}), B = k \frac{m_1}{\delta_1}(1 - e^{-\delta_1T_k}).$$

Let

$$R_0^{T_N} = \frac{rT_N}{\ln\left(\frac{1}{(1-p_1)^k}\right) + A + B + \sum_{n=2}^{k+1} \int_{lT_N+(n-2)T_k}^{lT_N+(n-1)T_k} (a_2y_k^{N*}(s))ds},$$

that is, when $R_0^{T_N} < 1$, the periodic solution of system (2.2) for pest extinction is locally asymptotically stable. The first step of the proof is complete.

In the second step, we prove that the pest extinction period solution $(0, y^{N*}(t))$ of the system (2.2) is globally attractive. Choose a $\varepsilon(\varepsilon > 0)$ such that

$$\gamma \triangleq (1 - p_1)^k \cdot \exp\left(\int_{hT_N}^{(h+l)T_N} \left(r - \frac{a_1(y_1^{N*}(s) - \varepsilon)}{1 + d(K + \xi)}\right)ds\right) \cdot \exp\left(\sum_{n=2}^{k+1} \int_{(h+l)T_N+(n-2)T_k}^{(h+l)T_N+(n-1)T_k} \left(r - b_1(s) - \frac{a_2(y_n^{N*}(s) - \varepsilon)}{1 + d(K + \xi)}\right)ds\right) < 1.$$

Based on the second and sixth formulas of model (2.2), it follows that

$$\frac{dy(t)}{dt} > -cy(t), t \in (hT_N, (h+l)T_N],$$

$$\frac{dy(t)}{dt} > -cy(t) - b_2(t)y(t), t \in ((h+l)T_N, (h+1)T_N].$$

Consider the following comparison equation:

$$\begin{cases} \frac{dz(t)}{dt} = -cz(t), t \in (hT_N, (h+l)T_N], \\ z(t^+) = (1-p_2)z(t), t = (h+l)T_N + nT_k, n = 0, 1, 2, \dots, k-1, \\ \frac{dz(t)}{dt} = -cz(t) - b_2(t)z(t), t \in ((h+l)T_N, (h+1)T_N], t \neq (h+1)T_N + nT_k, \\ z(t^+) = z(t) + \tau, t = (h+1)T_N, \end{cases}$$

where $y(t) \geq z(t)$, and $z(t) \rightarrow y^{N^*}(t)$ as $t \rightarrow \infty$. Thus, this gives us $y(t) \geq z(t) > y^{N^*}(t) - \varepsilon$ for a sufficiently large t and a sufficiently small ε . For the sake of simplicity, it can be assumed that $y(t) \geq z(t) > y^{N^*}(t) - \varepsilon$ for all $t \geq 0$.

By the first equation of model (2.2), we obtain the following:

$$\frac{dx(t)}{dt} \leq rx(t)\left(1 - \frac{x(t)}{K}\right).$$

Consider the following comparison equation:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right),$$

where $x(t) \leq N(t)$ and $N(t) \rightarrow K$ as $t \rightarrow \infty$. Thus, there exists a $\xi > 0$ making $x(t) \leq K + \xi$ for a large enough t . We assume $x(t) \leq K + \xi$ for all $t > 0$ without a loss of generality. By system (2.2), it follows that

$$\begin{cases} \frac{dx(t)}{dt} \leq x(t)\left(r - \frac{a_1(y_1^{N^*}(s) - \varepsilon)}{1 + d(K + \xi)}\right), t \in (hT_N, (h+l)T_N], \\ x(t^+) = (1-p_1)x(t), t = (h+l)T_N + nT_k, n = 0, 1, 2, \dots, k-1, \\ \frac{dx(t)}{dt} \leq x(t)\left(r - b_1(s) - \frac{a_2(y_2^{N^*}(s) - \varepsilon)}{1 + d(K + \xi)}\right), t \in ((h+l)T_N, (h+1)T_N], t \neq (h+1)T_N + nT_k, \\ x(t^+) = x(t), t = (h+1)T_N. \end{cases}$$

According to the comparison theorem for impulse differential equations, for $t \in (hT_N, (h+1)T_N]$, then

$$\begin{aligned} x((h+l)T_N) &\leq x(hT_N^+) \exp\left(\int_{hT_N}^{(h+l)T_N} \left(r - \frac{a_1(y_1^{N^*}(s) - \varepsilon)}{1 + d(K + \xi)}\right) ds\right), \\ x((h+l)T_N + T_k) &\leq (1-p_1)x(hT_N^+) \cdot \exp\left(\int_{hT_N}^{(h+l)T_N} \left(r - \frac{a_1(y_1^{N^*}(s) - \varepsilon)}{1 + d(K + \xi)}\right) ds\right) \\ &\quad \cdot \exp\left(\int_{(h+l)T_N}^{(h+l)T_N + T_k} \left(r - b_1(s) - \frac{a_2(y_2^{N^*}(s) - \varepsilon)}{1 + d(K + \xi)}\right) ds\right), \\ &\quad \vdots \end{aligned}$$

$$\begin{aligned}
 x((h+1)T_N) &\leq (1-p_1)^k x(hT_N^+) \cdot \exp\left(\int_{hT_N}^{(h+l)T_N} \left(r - \frac{a_1(y_1^{N*}(s) - \varepsilon)}{1+d(K+\xi)}\right) ds\right) \\
 &\cdot \exp\left(\sum_{n=2}^{k+1} \int_{(h+l)T_N+(n-2)T_k}^{(h+l)T_N+(n-1)T_k} \left(r - \frac{a_2(y_n^{N*}(s) - \varepsilon)}{1+d(K+\xi)} - b_1(s)\right) ds\right) \\
 &\triangleq x(hT_N^+) \gamma.
 \end{aligned}$$

Since $\gamma < 1$, we obtain $x(hT_N^+) \leq x(0^+) \gamma^h$ and $x(hT_N^+) \rightarrow 0$ as $t \rightarrow \infty$. Thus $x(t) \rightarrow 0$ as $t \rightarrow \infty$.

Since $\lim_{t \rightarrow \infty} x(t) \rightarrow 0$, for a small enough $\varepsilon_1 > 0$, there exists a $t_1 > 0$ such that $0 < x(t) < \varepsilon_1$ for $t > t_1$.

Clearly, for a sufficiently small $\varepsilon_2 > 0$, there exists a $t_2 > t_1$, making $\frac{a_1 x(t) y(t)}{1+d x(t)} < \varepsilon_2$ for $t > t_2$.

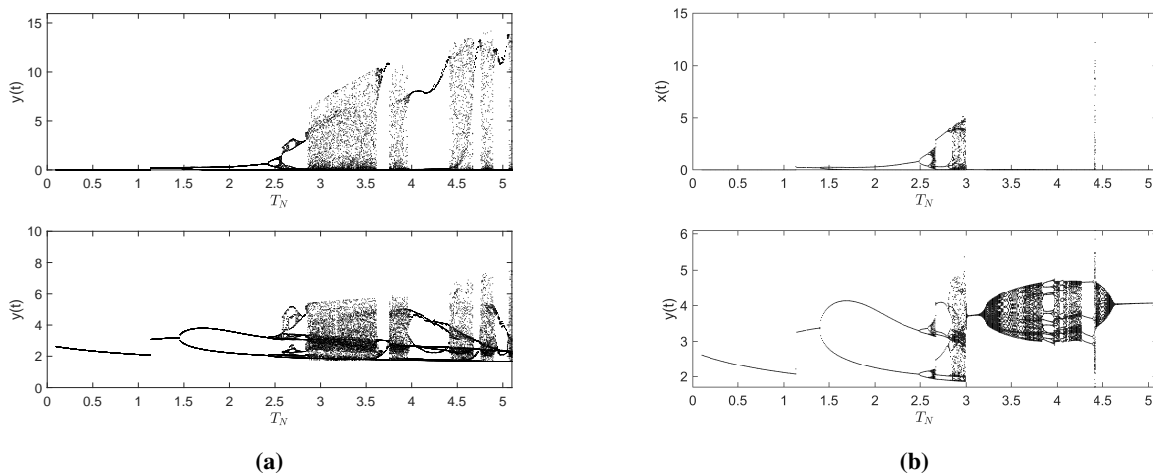


Figure 1. Bifurcation diagram of the system (2.2) with respect to the impulse period T_N .

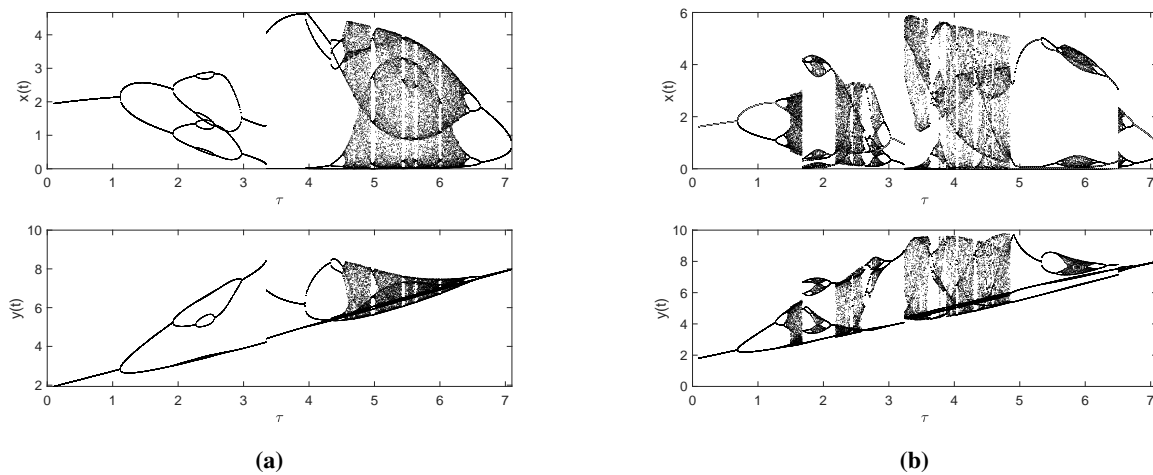


Figure 2. Bifurcation diagram of system (2.2) on the number of released natural enemy τ .

In the following discussion, we will show that $y(t) \rightarrow y^{N*}(t)$ as $t \rightarrow \infty$. From system (2.2), we obtain the following:

$$\begin{aligned}
 -cy(t) &\leq \frac{dy(t)}{dt} \leq \varepsilon_2 - cy(t), t \in (hT_N, (h+l)T_N], \\
 -cy(t) &\leq \frac{dy(t)}{dt} \leq \varepsilon_2 - cy(t) - b_2(t)y(t), t \in ((h+l)T_N, (h+1)T_N].
 \end{aligned}$$

We get $y(t) \geq z(t)$ and $z(t) \rightarrow y^{N^*}(t)$ as $t \rightarrow \infty$ by the left side of the two inequalities listed above. With respect to the right side of the two inequalities mentioned above, we study the following equation:

$$\begin{cases} \frac{d\omega(t)}{dt} = \varepsilon_2 - c\omega(t), t \in (hT_N, (h+l)T_N], \\ \omega(t^+) = (1-p_2)\omega(t), t = (h+l)T_N + nT_k, n = 0, 1, 2, \dots, k-1, \\ \frac{d\omega(t)}{dt} = \varepsilon_2 - c\omega(t) - b_2(t)\omega(t), t \in ((h+l)T_N, (h+1)T_N], t \neq (h+1)T_N + nT_k, \\ \omega(t^+) = \omega(t) + \tau, t = (h+1)T_N. \end{cases}$$

From Theorem 3.1, the details of which can be found in Appendix, we obtain the following

$$\omega^*(t) = \begin{cases} \omega_1^*(t), t \in (hT_N, (h+l)T_N], \\ \omega_j^*(t), t \in ((h+l)T_N + (j-2)T_k, (h+l)T_N + (j-1)T_k], \\ j = 2, 3, \dots, k+1, \end{cases}$$

$$\omega_1^*(t) = \omega^* \exp(-c(t-hT_N)) + \frac{\varepsilon_2}{c}(1 - \exp(-c(t-hT_N))),$$

$$\omega_j^*(t) = (1-p_2)^{j-1} \omega^* \exp(-c(t-hT_N) - A_{j-2} - B_{j-2}) + \frac{\varepsilon_2(1-p_2)^{j-1}}{c} \cdot (1 - \exp(-c(t-hT_N)))$$

$$\cdot \exp(-C_0 - A_{j-2} - B_{j-2}) + \frac{\varepsilon_2(1-p_2)}{c + m_2 \exp(-\delta_2 T_k)} \cdot \sum_{i=1}^{j-2} (1-p_2)^{i-1} \cdot \exp(-C_{j-1-i} - A_{i-1} - B_{j-2})$$

$$- \frac{\varepsilon_2}{c + m_2} \cdot \sum_{i=0}^{j-2} (1-p_2)^i \cdot \exp(-C_{j-2-i} - A_i - B_{j-2})$$

$$+ \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2(t - (h+l)T_N - (j-2)T_k))},$$

$$\omega^* = \frac{E}{(1 - (1-p_2)^k \exp(-cT_N - A_k))},$$

where

$$E = \frac{\varepsilon_2(1-p_2)^k}{c} \cdot (1 - \exp(-c(t-hT_N))) \cdot \exp(-ckT_k - A_k)$$

$$+ \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2 T_k)} \cdot \sum_{i=1}^k [(1-p_2)^{i-1} \exp(-c(i-1)T_k - A_{i-1})]$$

$$- \frac{\varepsilon_2}{c + m_2} \cdot \sum_{i=0}^{k-1} [(1-p_2)^i \exp(-c(i+1)T_k - A_{i+1})] + \tau$$

with

$$A_i = \frac{im_2}{\delta_2}(1 - e^{-\delta_2 T_k}), A_0 = 0,$$

$$B_i = \frac{m_2}{\delta_2}(1 - e^{-\delta_2(t - (h+l)T_N - iT_k)}), B_0 = \frac{m_2}{\delta_2}(1 - e^{-\delta_2(t - (h+l)T_N)}),$$

$$C_i = c(t - (h+l)T_N - iT_k), C_0 = c(t - (h+l)T_N), i = 0, 1, \dots, k.$$

Thus, for any $\varepsilon_3 > 0$, there exists a $t_3 > 0$ such that

$$y^{N^*}(t) - \varepsilon_3 < y(t) < \omega^*(t) + \varepsilon_3.$$

Let $\varepsilon, \varepsilon_1, \varepsilon_2 \rightarrow 0$; we obtain $y^{N^*}(t) - \varepsilon_3 < y(t) < y^{N^*}(t) + \varepsilon_3$ for a sufficiently large t , which this means that $y(t) \rightarrow y^{N^*}(t)$ as $t \rightarrow \infty$.

As a consequence, then the pest extinction cycle solution of model (2.2) is globally asymptotically stable if $R_0^{T_N} < 1$ stands. The proof ends here.

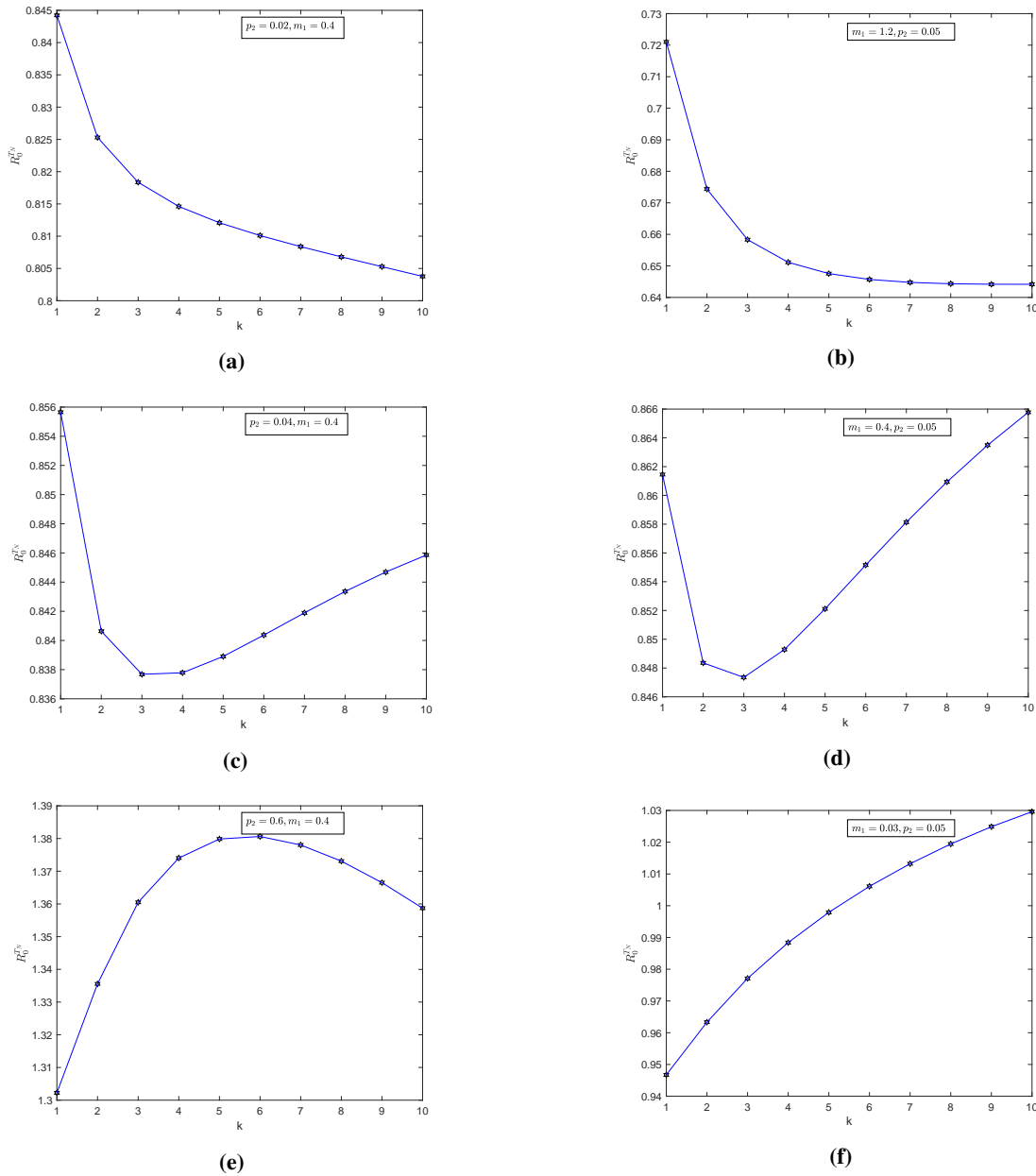


Figure 3. The effects of the spraying frequency k on the threshold $R_0^{T_N}$, other parameters are $r = 1.5, a_1 = 0.4, a_2 = 0.3, c = 0.2, k_1 = 0.8, k_2 = 0.6, p_1 = 0.1, l = 0.2, m_2 = 0.03, \delta_1 = 0.2, \delta_2 = 0.4, \tau = 9, T_N = 9$.

3.2. Dynamic complexity analysis of system (2.2)

Up to now, we have only theoretically analyzed system (2.2). By theoretical studies, sufficient conditions are obtained for the global asymptotic stability of the pest extinction period solution $(0, y^{N^*}(t))$.

Below, we investigate the complexity of the dynamics of system (2.2) by numerical modeling. We take $r = 8, K = 20, a_1 = 2.1, c = 0.5, k_1 = 0.82, p_1 = 0.4, p_2 = 0.2, d = 0.15, a_2 = 2, k_2 = 0.4, l = 0.2, \delta_1 = 0.3, \delta_2 = 0.1, m_1 = 0.4, m_2 = 0.13, \tau = 1.6$, and $(x(0), y(0)) = (2, 2.5)$. By fixing these parameters, we simulate the bifurcation diagram of system (2.2) with respect to the impulse period T_N in Figure 1. Figure 1(a) shows the complex dynamic behavior of system (2.2) as T_N increases from 0.1 to 5.1. The results show that the pest extinction period solution $(0, y^{N^*}(t))$ of system (2.2) has global asymptotic stability when the parameter a is increased from 0.1 to 1.1. With an increase of the impulse period T_N , system (2.2) exhibits a variety of complex dynamic behaviors, such as periodic, multiperiodic bifurcation, and chaotic phenomena. In Figure 1(b), we let $a_1 = 1.9$, and $d = 0.12$; as the parameter a increases from 3.1 to 4.7, we observe that the natural enemy population undergoes a complex dynamic behavior. Moreover, there is a coexistence of multiple attractors, which finally stabilizes.

Chaos refers to the unpredictable and random motion of a deterministic dynamic system because of its sensitivity to initial values [34, 35]. In Figure 2(a), letting $r = 8, K = 8, a_1 = 1.9, c = 0.5, k_1 = 0.82, p_1 = 0.4, p_2 = 0.2, d = 0.12, a_2 = 2, k_2 = 0.4, l = 0.2, \delta_1 = 0.3, \delta_2 = 0.1, m_1 = 0.3, m_2 = 0.13, T_N = 2.7$, and $(x(0), y(0)) = (2, 2.5)$, and fixing these parameters, we simulate a bifurcation diagram of system (2.2) with respect to the number of natural enemy releases τ , this showing the complex dynamic behavior of system (2.2) as τ increases from 0.1 to 7.1. The system (2.2) undergoes period \rightarrow doubling bifurcation \rightarrow chaos \rightarrow half-period bifurcation \rightarrow chaos \rightarrow inverse multiplicative cycle bifurcation \rightarrow stabilization as the parameter τ increases from 0.1 to 7.1. Letting $(x(0), y(0)) = (2, 2.3)$, we obtain the complex dynamic behavior of system (2.2), including switching transients and chaotic phenomena, as shown in Figure 2(b).

3.3. Effect of parameters on the threshold

Next, we study the influence of the key parameters in system (2.2) on the threshold condition ($R_0^{T_N} < 1$). For this reason, we first chose the instantaneous killing efficiency p_2 of pesticides on natural enemies, the non-transient death efficiency m_1 of pesticides on pests, and the frequency k of pesticide spraying as the bifurcation parameters while the others were fixed, as shown in Figure 3. The results show that the spraying frequency k affects $R_0^{T_N}$ differently when parameters p_2 and m_1 are varied, and that an increased frequency is not necessarily better for pest control. In Figure 3(a),(c),(e), we investigate the effects of the instantaneous killing efficiency p_2 of pesticides on natural enemies and the frequency k of pesticide spraying on the threshold $R_0^{T_N}$. The findings show that if the pesticide kills natural enemies at a lower killing rate p_2 (i.e., $p_2 = 0.02$), then the threshold $R_0^{T_N}$ is a monotonically decreasing function of k (as shown in Figure 3(a)). If the instantaneous killing efficiency p_2 of the pesticide on natural enemies increases, then the threshold $R_0^{T_N}$ is not monotonic with the number of applications k , as shown in Figure 3(c), in which case there is an optimal number of applications during the period T_N . If the killing rate p_2 for natural enemies increases from 0.04 to 0.6, then the value of the threshold $R_0^{T_N}$ initially increases and then decreases with an increase of application times, that is, there is a maximum point, as shown in Figure 3(e). This suggests that fewer applications should be made if insecticides have a significant impact on natural enemies. Similar effects of the non-transient death efficiency m_1 of pesticides against pests and the number of pesticide applications k on the threshold $R_0^{T_N}$ are shown in Figure 3(b),(d),(f). The larger the non-transient death efficiency m_1 of pesticides against pests, the smaller the threshold. The above findings suggest that the frequency of pesticide spraying during the release of natural enemies must be carefully considered, and that an increased frequency of

spraying is not better for pest control.

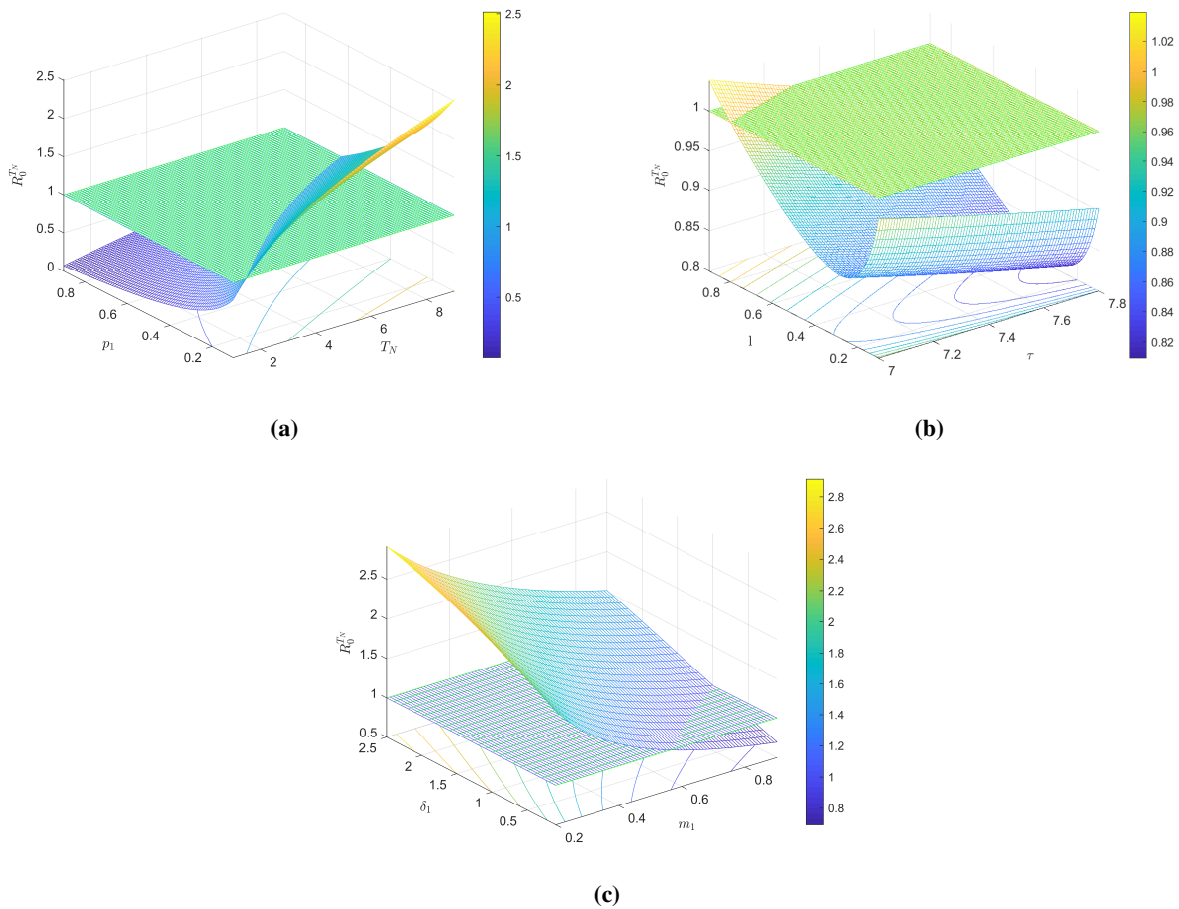


Figure 4. (a) p_1 and T_N on the $R_0^{T_N}$, $l = 0.2$, $\tau = 0.4$, $\delta_1 = 0.2$, and $m_1 = 0.2$; (b) l and τ on the $R_0^{T_N}$, $\delta_1 = 0.1$, $T_N = 16$, $p_1 = 0.1$, and $m_1 = 0.2$; (c) m_1 and δ_1 on the $R_0^{T_N}$, $T_N = 5$, $l = 0.3$, $\tau = 0.4$, and $p_1 = 0.1$. Other parameters are $r = 0.5$, $a_1 = 0.7$, $a_2 = 0.3$, $c = 0.693$, $k_1 = 0.6$, $k_2 = 0.5$, $p_2 = 0.05$, $m_2 = 0.1$, $\delta_2 = 0.5$.

In addition, we can perform a two-parameter bifurcation analysis on the threshold $R_0^{T_N}$, as shown in Figure 4, where we observe that the parameter m_1 has a significant effect on the threshold $R_0^{T_N}$ relative to the other parameters. Increasing the number of natural enemies released τ , increasing the instantaneous kill rate p_1 and the non-instantaneous kill rate m_1 of the pesticide against the pests are favorable for pest control, and neither the release period T_N nor the increase in the decay rate δ_1 of the pests are favorable for pest control. Moreover, we note that $R_0^{T_N}$ is not a monotonic function with respect to l . Therefore, we should be careful in choosing the timing of pesticide spraying.

3.4. Sensitivity analyses

In this subsection, we assess the PRCC of various input parameters by evaluating them against the threshold condition $R_0^{T_N}$ and then identifying the most important parameters that affect the threshold. PRCC enables the measurement of the effect of uncertainty in the estimated input parameter values on the imprecision of the predicted output variable values, and investigates the sensitivity of the parameters to thresholds. In our analysis, we used the LHS method to perform uncertainty and sensitivity analyses

of all parameters in system (2.2) on 5000 samples. For the PRCC values in Figure 5, a parameter is positively correlated with the threshold if the sign of the PRCC for that parameter is positive, (i.e., $R_0^{T_N}$ will increase as the parameter increases and the reverse is also true). Conversely, they are negatively correlated if the sign of the parameter is negative, (i.e., $R_0^{T_N}$ decreases as the parameter increases and the reverse is also true). If the absolute value of its PRCC is larger than 0.4, then it means that the parameter has a strong correlation with $R_0^{T_N}$ and has a large influence on $R_0^{T_N}$. If the absolute value of its PRCC is between 0.2 and 0.4, then it demonstrates that the parameter has a moderate influence on $R_0^{T_N}$. If the absolute value of its PRCC is less than 0.2, then it indicates that the parameter is weakly correlated with $R_0^{T_N}$ and has little influence on $R_0^{T_N}$.

As seen in Figure 5, the absolute values of the PRCC for the parameters $r, a_1, a_2, p_1, p_2, \tau$, and T_N are greater than or close to 0.4, thus demonstrating that these parameters have a large influence on $R_0^{T_N}$ and are important control parameters. By increasing the values of the parameters r, p_2, m_2 , and T_N , the value of $R_0^{T_N}$ also increases; thus, an increase in all of these parameters may lead to pest explosion. Besides, a_1, p_1, a_2 , and τ will decrease the value of $R_0^{T_N}$; thus, an increase in all these parameters will favor pest control.

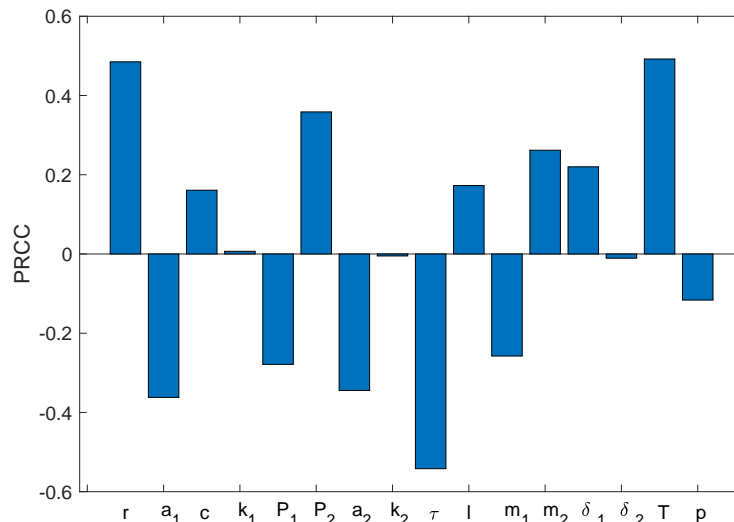


Figure 5. PRCC results for threshold $R_0^{T_N}$ with baseline parameter values: $r = 3, a_1 = 0.6, a_2 = 0.5, c = 0.35, k_1 = 0.8, k_2 = 0.6, p_1 = 0.3, p_2 = 0.2, l = 0.7, m_1 = 0.5, m_2 = 0.1, \delta_1 = 0.5, \delta_2 = 0.3, \tau = 6, T_N = 8, p = 5$.

4. The study of system (2.3)

4.1. Threshold conditions for pest eradication

To start, we study the following subsystems of system (2.3):

$$\begin{cases} \frac{dy(t)}{dt} = -cy(t) - b_2(t)y(t), t \neq hT_z, t \neq (h+l)T_z + kT_p, k = 0, 1, 2, \dots, p-1, \\ y(t^+) = y(t) + \tau, t = (h+l)T_z + kT_p, k = 0, 1, 2, \dots, p-1, \\ y(t^+) = (1-p_2)y(t), t = (h+1)T_z, \\ b_2 = m_2 e^{-\delta_2(t-hT_z)}, hT_z \leq t < (h+1)T_z. \end{cases} \quad (4.1)$$

Theorem 4.1. System (4.1) has a unique, globally asymptotically stable, positive, periodic solution $y^*(t)$ and $\lim_{t \rightarrow \infty} |y(t) - y^{Z^*}(t)| = 0$, where

$$y^{Z^*}(t) = \begin{cases} y_1^{Z^*}(t), & t \in (hT_z, (h+l)T_z], \\ y_i^{Z^*}(t), & t \in ((h+l)T_z + (i-2)T_p, (h+l)T_z + (i-1)T_p], \\ & j = 2, 3, \dots, p+1, \end{cases} \quad (4.2)$$

$$y_1^{Z^*}(t) = y^{Z^*} \exp\left(-c(t - hT_z) - \frac{m_2}{\delta_2} (1 - \exp(-\delta_2(t - hT_z)))\right),$$

$$y_j^{Z^*}(t) = y^{Z^*} \exp\left(-c(t - hT_z) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 t T_z}) - M_{j-2} - N_{j-2}\right) + \tau \sum_{i=0}^{j-2} \exp(-Q_i - M_{j-2-i} - N_{j-2}),$$

$$y^{Z^*} = \frac{(1 - p_2) \cdot \tau \cdot \sum_{i=1}^p \exp(-ciT_p - M_i)}{1 - (1 - p_2) \exp\left(-cT_z - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_z}) - M_p\right)}.$$

Note

$$Q_i = c(t - (h+l)T_z - iT_p), C_0 = c(t - (h+l)T_z), M_i = \frac{im_2}{\delta_2} (1 - e^{-\delta_2 T_p}),$$

$$M_0 = 0, N_i = \frac{m_2}{\delta_2} (1 - e^{-\delta_2(t - (h+l)T_z - iT_p)}), N_0 = \frac{m_2}{\delta_2} (1 - e^{-\delta_2(t - (h+l)T_z)}), i = 0, 1, \dots, p.$$

Proof. Solving the equations of system (4.1) on the interval $[hT_z, (h+l)T_z)$ yields the following:

$$y^Z(t) = y^Z(hT_z^+) \exp\left(-c(t - hT_z) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2(t - hT_z)})\right).$$

When $t = (h+l)T_z$, we have the following:

$$y^Z((h+l)T_z^+) = y^Z(hT_z^+) \exp\left(-cT_z - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_z})\right) + \tau.$$

When $t \in ((h+l)T_z, (h+l)T_z + T_p)$, we have the following:

$$y^Z(t) = y^Z(hT_z^+) \exp\left(-c(t - hT_z) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_z}) - N_0\right) + \tau \cdot \exp(-Q_0 - N_0).$$

At the second release of natural enemies during cycle T_z , we have the following:

$$y^Z\left(\left((h+l)T_z + T_p\right)^+\right) = y^Z(hT_z^+) \exp\left(-c(lT_z + T_p) - \frac{m_2}{\delta_2} (1 - e^{-\delta_2 T_z}) - M_1\right) + \tau \cdot \exp(-cT_p - M_1) + \tau.$$

In $t \in ((h + l)T_Z + T_p, (h + l)T_Z + 2T_p)$, we have the following:

$$y^Z(t) = y^Z(hT_Z^+) \exp\left(-c(t - hT_Z) - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 l T_Z}) - M_1 - N_1\right) + \tau \cdot \exp(-Q_0 - M_1 - N_1) + \tau \cdot \exp(-Q_1 - N_1).$$

Repeating the previous process, after the p th release of the natural enemy in a cycle T_Z , i.e., $t \in ((h + l)T_Z + (k - 1)T_p, (h + 1)T_Z]$, we have the following:

$$y^Z(t) = y^Z(hT_Z^+) \exp\left(-c(t - hT_Z) - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 l T_Z}) - M_{p-1} - N_{p-1}\right) + \tau \cdot \sum_{i=0}^{p-1} \exp(-Q_i - M_{p-1-i} - N_{p-1}).$$

When $t = (h + 1)T_Z$, and pesticides are sprayed, then we have the following:

$$y^Z((h+1)T_Z^+) = (1 - p_2)y^Z(hT_Z^+) \exp\left(-cT_Z - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 l T_Z}) - M_p\right) + (1 - p_2) \cdot \tau \cdot \sum_{i=1}^p \exp(-ciT_p - M_i).$$

Let $y_h^Z = y^Z(hT_Z^+)$; thus, we have the following:

$$y_{h+1}^Z = (1 - p_2)y_h^Z \exp\left(-cT_Z - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 l T_Z}) - M_p\right) + (1 - p_2) \cdot \tau \cdot \sum_{i=1}^p \exp(-ciT_p - M_i) \triangleq F(y_h^N).$$

From the above equation, the only possible immovable point is as follows:

$$y^{Z*} = \frac{(1 - p_2) \cdot \tau \cdot \sum_{i=1}^p \exp(-ciT_p - M_i)}{1 - (1 - p_2) \exp\left(-cT_Z - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 l T_Z}) - M_p\right)}.$$

Because (3.4) is a linear system and $\left|\frac{dF(y_h^N)}{dy_h^N}\right| = 1 - (1 - p_2) \exp\left(-cT_Z - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 l T_Z}) - M_p\right) < 1$, it follows from the theory of differential equations that it is a positive equilibrium point of global asymptotic stability of the differential equations. Thus, system (2.3) exists as a single globally asymptotically stable periodic solution $y^{Z*}(t)$. An arbitrary solution $y(t)$ of system (2.3) converges to $y^{Z*}(t)$, as denoted by (4.2) when $t \rightarrow \infty$. The proof is accomplished.

Thus, system (2.3) exists as a pest extinction period solution $(0, y^{Z*}(t))$.

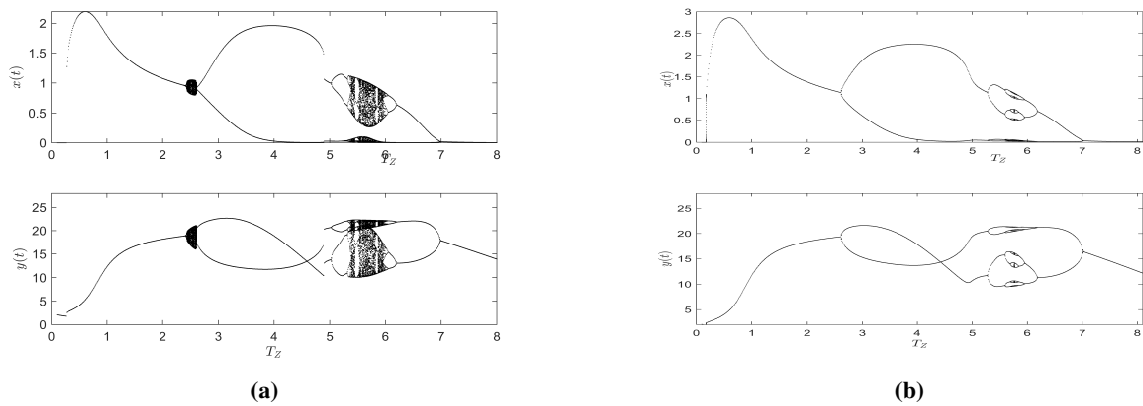


Figure 6. Bifurcation diagram of the system (2.3) with respect to the impulse period T_Z .

Theorem 4.2. *If $R_0^{T_z} < 1$ stands, then the pest extinction period solution for system (2.3) is globally asymptotically stable.*

Proof. In the first step, we show that the extermination period solution $(0, y^{Z^*}(t))$ of system (2.3) is locally asymptotically stable, which is determined by considering a small amplitude perturbation $(\alpha(t), \beta(t))$ of the solution. Defining $x(t) = \alpha(t), y(t) = y^{Z^*}(t) + \beta(t)$, where $\alpha(t), \beta(t)$ is a small perturbation, which can be written as follows:

$$\begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = \varphi(t) \begin{pmatrix} \alpha(0) \\ \beta(0) \end{pmatrix}, t \in (0, T_Z],$$

where $\varphi(t) = \prod_{j=1}^{p+1} \varphi_j(t)$, and $\varphi_j(t)$ fulfill

$$\frac{d\varphi_1(t)}{dt} = \begin{pmatrix} r - b_1(t) - a_2 y_1^{Z^*}(t) & 0 \\ k_2 a_2 y_1^{Z^*}(t) & -c \quad -c - b_2(t) \end{pmatrix} \varphi_1(t), t \in (0, lT_Z],$$

$$\frac{d\varphi_j(t)}{dt} = \begin{pmatrix} r - b_1(t) - a_2 y_j^{Z^*}(t) & 0 \\ k_2 a_2 y_j^{Z^*}(t) & -c - b_2(t) \end{pmatrix} \varphi_j(t),$$

where

$$t \in (lT_Z + (j - 2)T_p, lT_Z + (j - 1)T_p], j = 2, 3, \dots, p + 1.$$

Hence,

$$\varphi_1(t) = \begin{pmatrix} \exp\left(\int_0^t (r - b_1(s) - a_2 y_1^{Z^*}(s)) ds\right) & 0 \\ * & \exp\left(-ct - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 t})\right) \end{pmatrix},$$

$$\varphi_j(t) = \begin{pmatrix} V_3 & 0 \\ * & V_4 \end{pmatrix},$$

where

$$V_3 = \exp\left(\int_{lT_Z+(j-2)T_p}^t (r - b_1(s) - a_2y_j^{Z*}(s))ds\right),$$

$$V_4 = \exp(-c(t - lT_Z - (j - 2)T_p) - \frac{m_2}{\delta_2}(1 - e^{-\delta_2(t-lT_Z-(j-2)T_p)})).$$

It is not necessary to compute the precise form of (*) because it is not needed in the following theories. By the Floquet theory, if the modes of the two eigenvalues of a single-valued matrix

$$V = \begin{pmatrix} 1 - p_1 & 0 \\ 0 & 1 - p_2 \end{pmatrix} \varphi_1(lT_Z) \prod_{j=2}^{p+1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \varphi_j(lT_Z + (j - 1)T_p),$$

are less than 1, then the pest extinction period solution is locally asymptotically stable. Indeed, the eigenvalues are as follows:

$$\lambda_1^Z = (1 - p_1) \exp\left(\int_0^{lT_Z} (r - b_1(s) - a_2y_1^{Z*}(s))ds\right) + \sum_{j=2}^{p+1} \int_{lT_Z+(j-2)T_p}^{lT_Z+(j-1)T_p} (r - b_1(s) - a_2y_j^{Z*}(s))ds,$$

$$\lambda_2^Z = (1 - p_2) \exp\left(-cT_Z - (p + 1)\frac{m_2}{\delta_2}(1 - e^{-\delta_2T_p})\right) < 1.$$

Thus, when $|\lambda_1^Z| < 1$, i.e., $\frac{rT_z}{\ln\left(\frac{1}{1-p_1}\right) + C + D + \sum_{k=2}^{p+1} \int_{lT_z+(k-2)T_p}^{lT_z+(k-1)T_p} (ay_k^{z*}(s))ds} < 1$, where

$$C = \frac{m_1}{\delta_1}(1 - e^{-\delta_1 lT_Z}) + \frac{ay^{Z*}}{c + m_2} - \frac{ay^{Z*} \exp(-clT_Z - \frac{m_2}{\delta_2}(1 - e^{-\delta_2 lT_Z}))}{c + m_2 e^{-\delta_2 lT_Z}},$$

$$D = p \frac{m_1}{\delta_1} e^{-\delta_1 lT_Z} (1 - e^{-\delta_1 T_p}).$$

Let

$$R_0^{T_z} = \frac{rT_z}{\ln\left(\frac{1}{1-p_1}\right) + C + D + \sum_{k=2}^{p+1} \int_{lT_z+(k-2)T_p}^{lT_z+(k-1)T_p} (ay_k^{z*}(s))ds},$$

that is, when $R_0^{T_z} < 1$, the periodic solution of system (2.3) for pest extinction is locally asymptotically stable.

Similar to the global attractiveness proof of Theorem 3.2, we can also conclude that the periodic solution $(0, y^{Z*}(t))$ of system (2.3) is globally attractive.

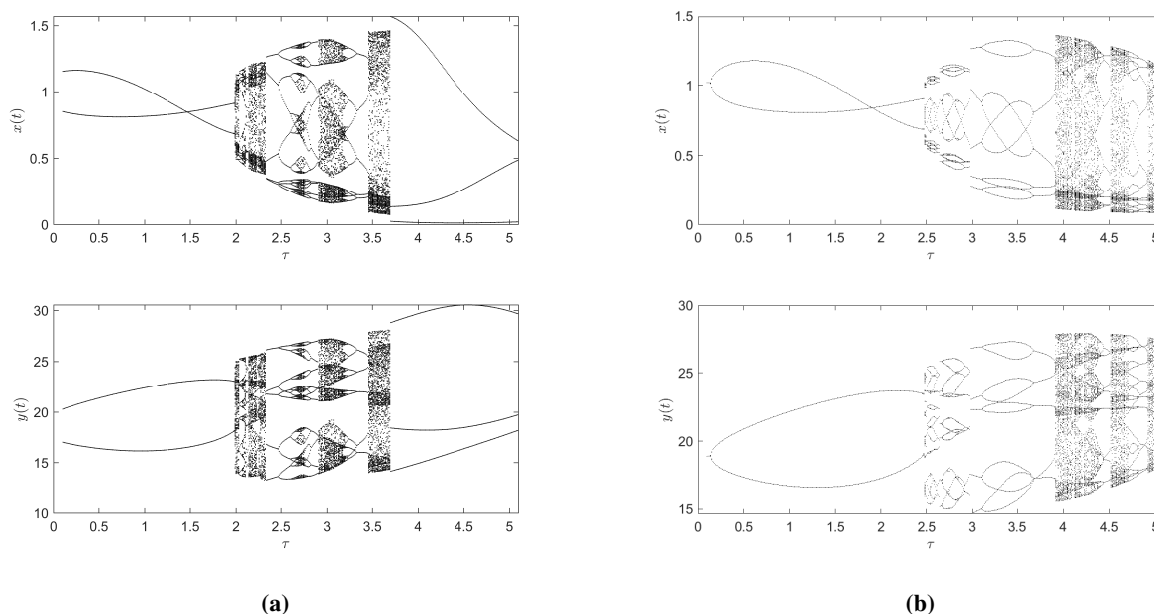


Figure 7. Bifurcation diagram of system (2.3) on the number of released natural enemy τ .

4.2. Dynamic complexity analysis of system (2.3)

For the case where the frequency of natural enemy releases is higher than the frequency of pesticide spraying, we consider a bifurcation diagram of system (2.3) with respect to the pulse period and the number of natural enemy releases.

In Figure 6(a), we take $r = 6, K = 7, c = 0.4, p_1 = 0.6, p_2 = 0.3, d = 0.5, a_2 = 0.35, k_2 = 1.8, l = 0.8, \delta_1 = 0.4, \delta_2 = 0.2, m_1 = 0.3, m_2 = 0.1, \tau = 0.2$, and $(x(0), y(0)) = (2, 2.5)$; fixing these parameters, we simulate the bifurcation diagram of system (2.3) with respect to the impulse period T_Z . Observing Figure 6(a), as T_Z increases from 0.1 to 8.1, system (2.3) undergoes complex dynamical behaviors, including cycles, tangent bifurcations (from cycles directly into chaos), chaos, multiplicative bifurcations, half-cycle bifurcations, and stabilization. The value of the parameter c is varied so that it is $c = 0.45$, as shown in Figure 6(b), which is known as the mouse diagram.

In Figure 7, we model the bifurcation diagram of system (2.3) with respect to the number of releases τ by natural enemies. Let $r = 6, K = 7, c = 0.4, p_1 = 0.6, p_2 = 0.3, d = 0.5, a_2 = 0.35, k_2 = 1.8, l = 0.8, \delta_1 = 0.3, \delta_2 = 0.1, m_1 = 0.3, m_2 = 0.13, T_Z = 2.7$, and $(x(0), y(0)) = (2, 2.5)$ to obtain Figure 7(a). The dynamic behavior consists of a series of periodically doubled chaotic direct, reverse cascade, and non-unique dynamics as τ increases from 0.1 to 5.1. We can vary the value of the parameter c so that $c = 0.42$, to obtain Figure 7(b), where there is a clear coexistence of multiple attractors.

4.3. Making comparisons of the effect of parameters on the two thresholds

Here, we fix all other parameters and make $k(p)$ change; for various periods $T_N = T_Z, R_0^{T_N}$ is compared with $R_0^{T_Z}$, as shown in Figure 8. First, we observe that the threshold $R_0^{T_N}$ is a monotonically increasing function with regard to the number of sprays k in the shorter period T_N , which indicates that the control measures are not ideal (as shown in Figure 8(a)). If the period T_N increases, then the relationship between the threshold and the number of sprays k is not monotonic, and the threshold $R_0^{T_N}$

has minimal values between the number of sprays (4-5). If the period T_N increases from 9 to 14, then the threshold $R_0^{T_N}$ is monotonically decreasing as a function of the number of sprays k , at which point more sprays are better. However, we must note that the value of the threshold $R_0^{T_N}$ in the short period T_N is always smaller than the value of the threshold in the long period T_N , thus we should choose the number of sprays carefully in a given period.

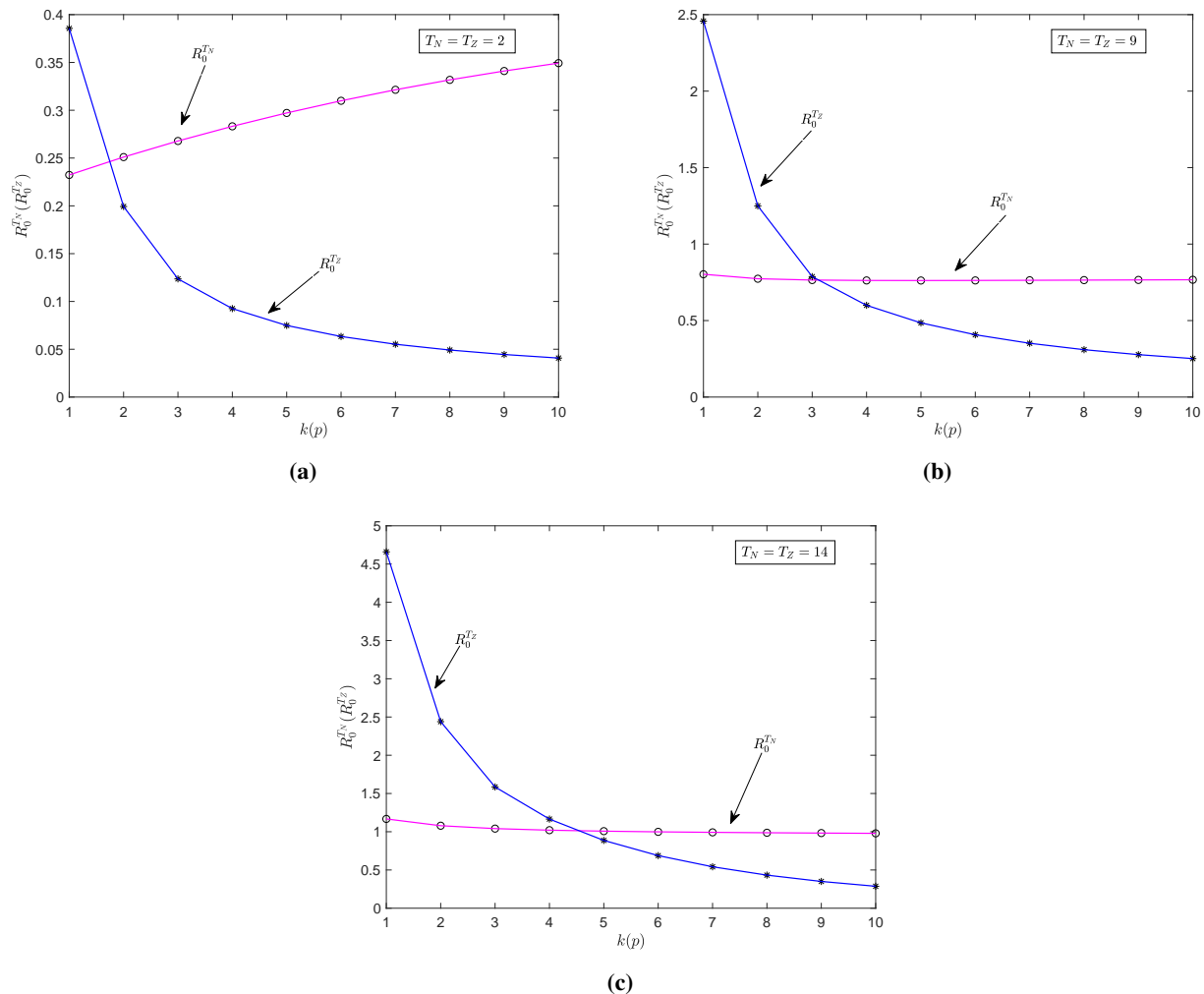


Figure 8. Comparison of parameter sets on thresholds $R_0^{T_N}$ and $R_0^{T_Z}$, with baseline parameter values: $r = 1.5$, $a_1 = 0.4$, $a_2 = 0.3$, $c = 0.2$, $k_1 = 0.8$, $k_2 = 0.6$, $p_1 = 0.1$, $p_2 = 0.05$, $l = 0.2$, $m_1 = 0.7$, $m_2 = 0.03$, $\delta_1 = 0.2$, $\delta_2 = 0.4$, $\tau = 9$.

Observing Figure 8, under the same period, the threshold $R_0^{T_Z}$ is always a monotonic decreasing function with respect to the parameter p , thus indicating that the frequent release of natural enemies is beneficial to the prevention of pests, regardless of the length of the period. Comparing the variation of the thresholds $R_0^{T_N}$ and $R_0^{T_Z}$, we also conclude that the thresholds $R_0^{T_N}$ and $R_0^{T_Z}$ intersect at a point within each given period $T_N = T_Z$. Before this intersection point, the value of the threshold $R_0^{T_Z}$ is greater than the value of the threshold $R_0^{T_N}$. This indicates that when we have a low release frequency or spraying frequency, applying a spraying pesticide frequency greater than the release of natural enemies frequency measure is better for controlling pests. The above findings indicate that the long-term use of

chemical or biological controls is not ideal, and that we should combine these two measures to design the best pest control strategy.

The more prominent control parameters affecting the threshold $R_0^{T_Z}$ when the frequency of release of natural enemies is higher than the frequency of pesticide application are the instantaneous killing efficiency of pesticides against pests p_1 , the non-instantaneous killing efficiencies of pesticides against both pests and natural enemies m_1 and m_2 , and the period of time to implement a pest control strategy T_Z (shown in Figure 9).

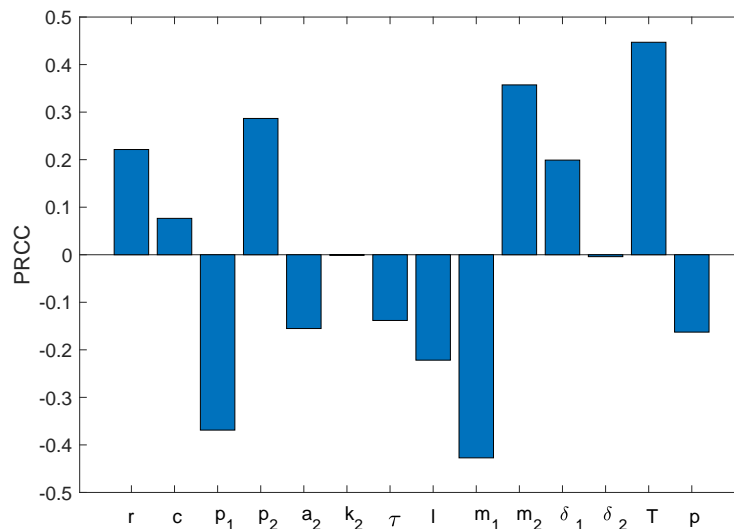


Figure 9. PRCC results for threshold $R_0^{T_Z}$ with baseline parameter values: $r = 4$, $a_2 = 0.3$, $c = 0.3$, $k_2 = 0.4$, $p_1 = 0.4$, $p_2 = 0.2$, $l = 0.6$, $m_1 = 0.5$, $m_2 = 0.3$, $\delta_1 = 0.2$, $\delta_2 = 0.4$, $\tau = 5$, $T_N = 2$, $p = 3$.

5. State feedback strategy

Here, we study system (2.2) in more depth. In Parts III and IV, we are more interested in adopting effective measures to eliminate pests. However, from an ecological and economic point of view, the strategy of pest extermination is not ideal. This is because frequent spraying of pesticides causes severe environmental pollution, the cost of captive breeding of natural enemies is high, and more importantly, the right amount of pests is conducive to maintaining the balance of natural ecosystems. Therefore, we only need to take control measures when the pest density reaches the ET to prevent the pest density from increasing to the EIL.

In this section, it is hypothesized that the IPM is employed when the pest density reaches ET; if the appropriate time series are $\lambda_1, \lambda_2, \dots$, and $\lambda_1 < \lambda_2 < \dots < \lambda_j < \lambda_{j+1} < \dots$, then $x(\lambda_j) = ET$ for $j = 1, 2, \dots$. Following these assumptions, the following state-dependent IPM model with an ET is established.

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - \frac{a_1x(t)y(t)}{1 + dx(t)}, \\ \frac{dy(t)}{dt} = \frac{k_1a_1x(t)y(t)}{1 + dx(t)} - cy(t), \\ x(t^+) = (1 - p_1)x(t), \\ y(t^+) = (1 - p_2)y(t), \end{array} \right\} t \in (0, \lambda_1], \quad (5.1)$$

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - \frac{a_2x(t)y(t)}{1 + dx(t)} - b_1(t)x(t), \\ \frac{dy(t)}{dt} = \frac{k_2a_2x(t)y(t)}{1 + dx(t)} - cy(t) - b_2(t)y(t), \\ x(t^+) = x(t), \\ y(t^+) = y(t) + \tau, \end{array} \right\} t \neq \lambda_j, t > \lambda_j,$$

$$\left\{ \begin{array}{l} x(t^+) = x(t), \\ y(t^+) = y(t) + \tau, \end{array} \right\} t = hT_N, h \in N,$$

where $b_i(t) = m_i e^{-\delta_i(t-\lambda_j)}$, $i = 1, 2, \lambda_j \leq t < \lambda_{j+1}, j \in N^+$. The starting values are $x(0) = x_0 < ET, y(0) = y_0$, and the other parameters have the same meaning as in model (2.2).

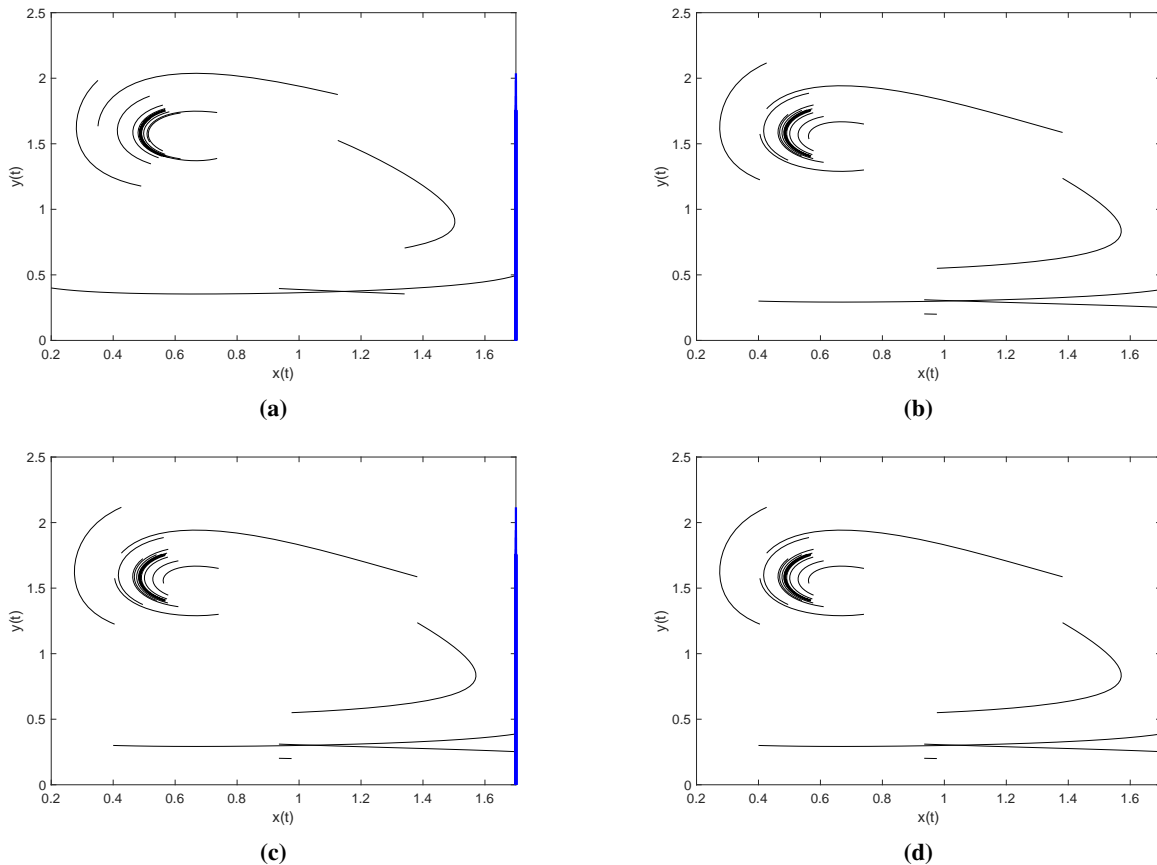


Figure 10. Comparison of parameter sets on thresholds R_0^{TN} and R_0^{TZ} , with baseline parameter values: $r = 1.5, a_1 = 0.4, a_2 = 0.3, c = 0.2, k_1 = 0.8, k_2 = 0.6, p_1 = 0.1, p_2 = 0.05, l = 0.2, m_1 = 0.7, m_2 = 0.03, \delta_1 = 0.2, \delta_2 = 0.4, \tau = 9$.

It is very difficult to theoretically study system (5.1). This is because we are unable to determine the exact time and frequency of insecticide spraying. In the following, we investigated the effect of the

starting values $(x(0), y(0))$, the intrinsic growth rate r , the non-instantaneous kill rate of pesticides on pests m_1 , the natural enemy release τ , and the impulse release period T_N on the number of pesticide applications for controlling pests up to the ET in a given time by numerical simulations.

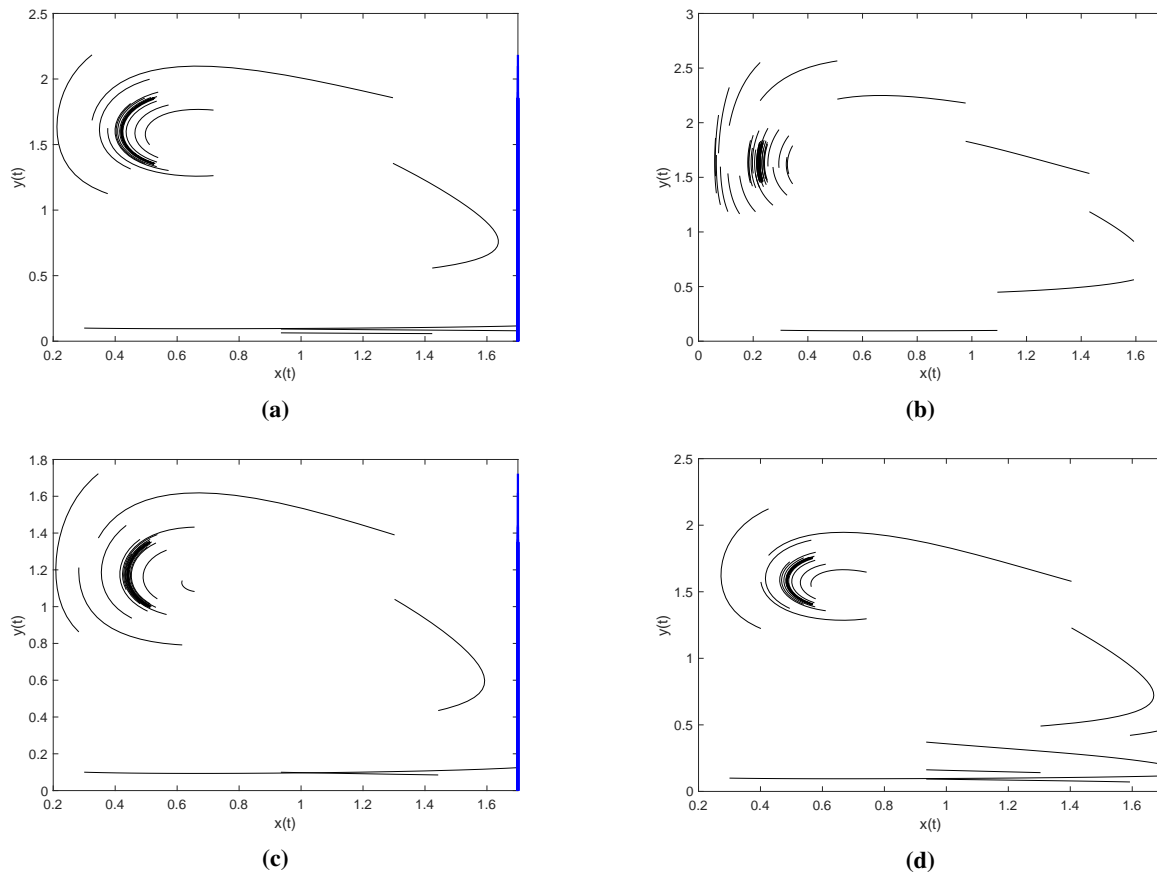


Figure 11. Comparison of parameter sets on thresholds $R_0^{T_N}$ and $R_0^{T_Z}$, with baseline parameter values: $r = 1.5, a_1 = 0.4, a_2 = 0.3, c = 0.2, k_1 = 0.8, k_2 = 0.6, p_1 = 0.1, p_2 = 0.05, l = 0.2, m_1 = 0.7, m_2 = 0.03, \delta_1 = 0.2, \delta_2 = 0.4, \tau = 9$.

We take $r = 2.45, K = 2.2, a_1 = 1.5, a_2 = 1.2, c = 0.45, k_1 = 0.6, k_2 = 0.4, p_1 = 0.5, p_2 = 0.08, d = 0.5, l = 0.8, m_1 = 0.2, m_2 = 0.1, \delta_1 = 0.2, \delta_2 = 0.4$, and $ET = 1.7$. We fix $T_N = 2.7$ and $\tau = 0.35$ for a fixed time $t \in [0, 140]$. We fix the values of other parameters and change the initial values in Figure 10. In Figure 10(a), let $(x(0), y(0)) = (0.2, 0.4)$ be the original densities of the pest and natural enemy populations, and the density of the pests reaches the ET only once, so we only need to apply insecticides to them once.

If its initial density is changed, then the number of times chemical control is required changes. In Figure 10(b)–(d), we set the original densities as $(0.4, 0.3)$, $(0.2, 0.1)$, and $(0.3, 0.1)$, respectively. We observe that two, four, and five pesticide sprays are required to control the pests below the ET. Setting the same initial value as Figure 10(d), increasing the release of natural enemy τ to 0.5, as compared with $\tau = 0.35$, the need for chemical control will be reduced. What would have required five sprays of pesticide to bring the density of its pests below ET now requires only two sprays (see Figure 11(a)).

In Figure 11(b), we change the value of T_N to reduce it from 2.7 to 0.8, and the other parameters are

the same as in Figure 11(a). From Figure 11(b), we can see that the density of the pests never got to 1.7, which is the value of the threshold ET. This indicates that with the appropriate parameter values, the density of the pests can be controlled under the ET in the absence of insecticide spraying. When we reduce the parameter r from 2.45 to 1.8, then chemical control only needs to be implemented once compared to Figure 11(a), see Figure 11(c). Similarly, with other parameters similar to those in Figure 11(a), the number of pesticide sprays changed from five to three when $m_1 = 0.6$ (see Figure 11(d)). The upper results indicate that the intrinsic growth rate, initial density, number of released natural enemies, non-instantaneous kill rate of pesticides on pests, and the pulse release period influence the frequency of pesticide application. Importantly, appropriate values of the parameters (i.e., adjusting the control strategy) can eliminate the need for chemical control for a given time or maintain the density of the pest population below the ET after a finite insecticide application.

6. Conclusions

The use of pesticides has an impact not only on pests, but also on natural enemies. In addition, the effects of pesticide spraying on organisms not only occur instantaneously, but are present for the following period of time. Therefore, in this article, we take these factors into account and propose a model that is more consistent with the interrelationship between pests and natural enemies. In this model, the interaction between predator and prey is determined by the nutritional function of Holling II, which can control the pest population more effectively and is more in line with the law of biological development in nature.

First, we considered two scenarios with different frequencies of pest control: pesticide application at a higher or lower frequency than the release of natural enemies. Bifurcation analyses were performed for models (2.2) and (2.3), and the system was found to have complex dynamical behaviors, such as a period-doubling bifurcation, a half-period bifurcation, chaos, the coexistence of multiple attractors, and non-unique dynamics. For the bifurcation diagrams of models (2.2) and (2.3), we can observe that with an increase in pulse period and the number of released natural enemies, the pest and natural enemy populations show almost the same change trend, which is in line with the natural biological development trend. For both cases, we also discuss conditions for local stability, global attractiveness, and global stability of pest extinction period solutions. For the threshold of global asymptotic stability, we investigate the effect of parameters on the threshold conditions. For the case where the frequency of pesticide spraying is higher than the release of natural enemies, as based on the analysis, we can formulate the optimal number of insecticide applications to minimize the threshold. For the case where the frequency of release of natural enemies is higher than the frequency of pesticide spraying, the optimal number of applications of natural enemies can be formulated.

Second, we compared the effect of the spraying frequency on the thresholds in both cases as the impulse period varied, and concluded that there was an intersection of the two thresholds as the frequency increased. It is better to apply a more frequent strategy of natural enemy release before that intersection point is better for controlling pests and to apply pesticide spraying more frequently after the intersection point. By using two-parameter and LHS/PRCC uncertainty and sensitivity analysis techniques to determine the most important parameters of the threshold conditions, we obtained that the key parameters affecting the threshold are the intrinsic growth rate r , the natural enemy release τ , the attenuation rate δ_1 , the instantaneous kill rates p_1 and p_2 , the impulse control period T_N and T_Z , and

the non-instantaneous kill rates m_1 and m_2 . Additionally, we found that applying pesticides too early or too late is not conducive to preventing pests, there is an optimal application time, and increasing the release of natural enemies is conducive to controlling the number of pests. All of these results can help us design appropriate control strategies.

Third, according to the definition of the IPM, the ideal control goal is to maintain its pest population density below the ET rather than eradicating it. Therefore, based on system (2.2), we studied the pest control model with a state-dependent correlation in Section 4. It has been observed that pests and their natural enemies coexist in ecosystems and that an appropriate number of pests is conducive to maintaining the balance of natural ecosystems. You can change the value of each parameter and observe its effect on pest control. Through numerical simulations, we found that varying the values of the initial density, the intrinsic growth rate, the number of natural enemies released, the non-instantaneous kill rate, and the impulse release period all affect the number of pesticides that need to be sprayed in order to control pest densities below the ET. Moreover, it can be seen that at appropriate parameter values, we do not need to chemically control them because pest densities are consistently below the ET. In addition, the effectiveness of increasing the release of natural enemies to control pests was also obtained, which follows the laws of biological change in nature.

Finally, in this paper, we only considered the effects of pesticides on pests and natural enemies and the effects of different application frequencies and measures to control pests. We did not consider the effects of pest resistance following repeated applications of pesticides. In future research, we will set out to investigate how to integrate pesticide resistance with the transient and non-transient effects found in this paper.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

Data availability

The data used to support the findings of this study are included within the article.

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Appendix

$$\begin{cases} \frac{d\omega(t)}{dt} = \varepsilon_2 - c\omega(t), t \in (hT_N, (h+l)T_N], \\ \omega(t^+) = (1 - p_1)\omega(t), t = (h+l)T_N + nT_k, n = 0, 1, 2, \dots, k-1, \\ \frac{d\omega(t)}{dt} = \varepsilon_2 - c\omega(t) - b_2(t)\omega(t), t \in ((h+l)T_N, (h+1)T_N], t \neq (h+1)T_N + nT_k \\ \omega(t^+) = \omega(t), t = (h+1)T_N. \end{cases}$$

When $t \in [hT_N, (h+l)T_N)$, we have the following:

$$\omega(t) = \omega(hT_N^+) \exp(-c(t - hT_N)) + \frac{\varepsilon_2}{c}(1 - \exp(-c(t - hT_N))).$$

With $t = (h+l)T_N$, there is the following:

$$\omega((h+l)T_N^+) = (1 - p_2)\omega(hT_N^+) \exp(-clT_N) + \frac{\varepsilon_2(1 - p_2)}{c}(1 - \exp(-clT_N)).$$

When $t \in ((h+l)T_N, (h+l)T_N + T_k)$, we have the following:

$$\begin{aligned} \omega(t) = & (1-p_2)\omega(hT_N^+) \exp(-c(t-hT_N) - B_0) + \frac{\varepsilon_2(1-p_2)}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-C_0 - B_0) \\ & - \frac{\varepsilon_2}{c+m_2} \exp(-C_0 - B_0) + \frac{\varepsilon_2}{c+m_2 \exp(-\delta_2(t-(h+l)T_N))}. \end{aligned}$$

When $t = (h+l)T_N + T_k$, we have the following:

$$\begin{aligned} \omega(((h+l)T_N + T_k)^+) = & (1-p_2)^2\omega(hT_N^+) \exp(-c(lT_N + T_k) - A_1) \\ & + \frac{\varepsilon_2(1-p_2)^2}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-cT_k - A_1) \\ & + \frac{\varepsilon_2(1-p_2)}{c+m_2 \exp(-\delta_2T_k)} - \frac{\varepsilon_2(1-p_2)}{c+m_2} \exp(-cT_k - A_1), \end{aligned}$$

With $t \in ((h+l)T_N + T_k, (h+l)T_N + 2T_k)$, there is the following:

$$\begin{aligned} \omega(t) = & (1-p_2)^2\omega(hT_N^+) \exp(-c(t-hT_N) - A_1 - B_1) + \frac{\varepsilon_2(1-p_2)^2}{c} \cdot (1 - \exp(-clT_N)) \\ & \cdot \exp(-C_0 - A_1 - B_1) + \frac{\varepsilon_2(1-p_2)}{c+m_2 \exp(-\delta_2T_k)} \cdot \exp(-C_1 - B_1) \\ & - \frac{\varepsilon_2}{c+m_2} [(1-p_2) \cdot \exp(-C_0 - A_1 - B_1) + \exp(-A_1 - B_1)] \\ & + \frac{\varepsilon_2}{c+m_2 \exp(-\delta_2(t-(h+l)T_N - T_k))}, \end{aligned}$$

At $t = (h+l)T_N + 2T_k$, there is the following:

$$\begin{aligned} \omega(((h+l)T_N + 2T_k)^+) = & (1-p_2)^3\omega(hT_N^+) \exp(-c(lT_N + 2T_k) - A_2) \\ & + \frac{\varepsilon_2(1-p_2)^3}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-2cT_k - A_2) \\ & + \frac{\varepsilon_2(1-p_2)}{c+m_2 \exp(-\delta_2T_k)} \cdot [(1-p_2) \exp(-cT_k - A_1) + 1] \\ & + \frac{\varepsilon_2(1-p_2)}{c+m_2} \cdot [(1-p_2) \cdot \exp(-2cT_k - A_2) + \exp(-cT_k - A_1)]. \end{aligned}$$

When $t \in ((h+l)T_N + 2T_k, (h+l)T_N + 3T_k)$, we have the following:

$$\begin{aligned} l\omega(t) = & (1-p_2)^3\omega(hT_N^+) \exp(-c(t-hT_N) - A_2 - B_2) + \frac{\varepsilon_2(1-p_2)^3}{c} \cdot (1 - \exp(-clT_N)) \\ & \cdot \exp(-C_0 - A_2 - B_2) + \frac{\varepsilon_2(1-p_2)}{c+m_2 \exp(-\delta_2T_k)} \cdot [(1-p_2) \cdot \exp(-C_1 - A_1 - B_2) + \exp(-C_2 - B_2)] \\ & - \frac{\varepsilon_2}{c+m_2} \cdot [(1-p_2)^2 \cdot \exp(-C_0 - A_2 - B_2) + (1-p_2) \exp(-C_1 - A_1 - B_2) + \exp(-C_2 - B_2)] \\ & + \frac{\varepsilon_2}{c+m_2 \exp(-\delta_2(t-(h+l)T_N - 2T_k))}. \end{aligned}$$

When $t = (h + l)T_N + 3T_k$, we have the following:

$$\begin{aligned} \omega(((h + l)T_N + 3T_k)^+) &= (1 - p_2)^4 \omega(hT_N^+) \exp(-c(lT_N + 3T_k) - A_3) \\ &+ \frac{\varepsilon_2(1 - p_2)^4}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-3cT_k - A_3) + \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2 T_k)} \\ &\cdot \left[(1 - p_2)^3 \exp(-2cT_k - A_2) + (1 - p_2)^2 \exp(-cT_k - A_1) + (1 - p_2) \right] \\ &- \frac{\varepsilon_2}{c + m_2} \cdot \left((1 - p_2)^3 \cdot \exp(-3cT_k - A_3) + (1 - p_2)^2 \right. \\ &\cdot \exp(-2cT_k - A_2) + (1 - p_2) \cdot \exp(-cT_k - A_1) \left. \right). \end{aligned}$$

Therefore, when $t \in ((h + l)T_N + (k - 1)T_k, (h + 1)T_N)$, we have the following:

$$\begin{aligned} \omega(t) &= (1 - p_2)^k \omega(hT_N^+) \exp(-c(t - hT_N) - A_{k-1} - B_{k-1}) \\ &+ \frac{\varepsilon_2(1 - p_2)^k}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-C_0 - A_{k-1} - B_{k-1}) \\ &+ \frac{\varepsilon_2(1 - p_2)}{c + m_2 \exp(-\delta_2 T_k)} \cdot \sum_{i=1}^{k-1} (1 - p_2)^{i-1} \cdot \exp(-C_{k-i} - A_{i-1} - B_{k-1}) \\ &- \frac{\varepsilon_2}{c + m_2} \cdot \sum_{i=0}^{k-1} (1 - p_2)^i \cdot \exp(-C_{k-1-i} - A_i - B_{k-1}) \\ &+ \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2(t - (h + l)T_N - (k - 1)T_k))}. \end{aligned}$$

When $t = (h + 1)T_N$, we have the following:

$$\begin{aligned} \omega(((h + 1)T_N)^+) &= (1 - p_2)^k \omega(hT_N^+) \exp(-cT_N - A_k) + \frac{\varepsilon_2(1 - p_2)^k}{c} \\ &\cdot (1 - \exp(-clT_N)) \cdot \exp(-kcT_k - A_k) \\ &+ \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2 T_k)} \cdot \sum_{i=1}^k \left[(1 - p_2)^{i-1} \exp(-c(i - 1)T_k - A_{i-1}) \right] \\ &- \frac{\varepsilon_2}{c + m_2} \cdot \sum_{i=0}^{k-1} \left[(1 - p_2)^i \exp(-c(i + 1)T_k - A_{i+1}) \right] + \tau. \end{aligned}$$

Let $\omega_h = \omega(hT_N^+)$, we have the following equation:

$$\begin{aligned} \omega_{h+1} &= (1 - p_2)^k \omega_h \exp(-cT_N - A_k) + \frac{\varepsilon_2(1 - p_2)^k}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-ckT_k - A_k) \\ &+ \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2 T_k)} \cdot \sum_{i=1}^k \left[(1 - p_2)^{i-1} \exp(-c(i - 1)T_k - A_{i-1}) \right] \\ &- \frac{\varepsilon_2}{c + m_2} \cdot \sum_{i=0}^{k-1} \left[(1 - p_2)^i \exp(-c(i + 1)T_k - A_{i+1}) \right] + \tau \triangleq G(\omega_h). \end{aligned}$$

By investigating the above equation, we get that the equation has a unique immovable point

$$\omega^* = \frac{E}{\left(1 - (1 - p_2)^k \exp(-cT_N - A_k)\right)},$$

where

$$E = \frac{\varepsilon_2(1 - p_2)^k}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-ckT_k - A_k) + \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2 T_k)} \\ \cdot \sum_{i=1}^k \left[(1 - p_2)^{i-1} \exp(-c(i-1)T_k - A_{i-1}) \right] - \frac{\varepsilon_2}{c + m_2} \cdot \sum_{i=0}^{k-1} \left[(1 - p_2)^i \exp(-c(i+1)T_k - A_{i+1}) \right] + \tau,$$

$$A_i = \frac{im_2}{\delta_2} (1 - e^{-\delta_2 T_k}), A_0 = 0,$$

$$B_i = \frac{m_2}{\delta_2} (1 - e^{-\delta_2(t - (h+l)T_N - iT_k)}), B_0 = \frac{m_2}{\delta_2} (1 - e^{-\delta_2(t - (h+l)T_N)}),$$

$$C_i = c(t - (h+l)T_N - iT_k), C_0 = c(t - (h+l)T_N), i = 0, 1, \dots, k.$$

Therefore, we have the following:

$$\omega^*(t) = \begin{cases} \omega_1^*(t), t \in (hT_N, (h+l)T_N], \\ \omega_j^*(t), t \in ((h+l)T_N + (j-2)T_k, (h+l)T_N + (j-1)T_k], \\ j = 2, 3, \dots, k+1. \end{cases}$$

$$\omega_1^*(t) = \omega^* \exp(-c(t - hT_N)) + \frac{\varepsilon_2}{c} (1 - \exp(-c(t - hT_N))),$$

$$\omega_j^*(t) = (1 - p_2)^{j-1} \omega^* \exp(-c(t - hT_N) - A_{j-2} - B_{j-2}) \\ + \frac{\varepsilon_2(1 - p_2)^{j-1}}{c} \cdot (1 - \exp(-clT_N)) \cdot \exp(-C_0 - A_{j-2} - B_{j-2}) \\ + \frac{\varepsilon_2(1 - p_2)}{c + m_2 \exp(-\delta_2 T_k)} \cdot \sum_{i=1}^{j-2} (1 - p_2)^{i-1} \cdot \exp(-C_{j-1-i} - A_{i-1} - B_{j-2}) \\ - \frac{\varepsilon_2}{c + m_2} \cdot \sum_{i=0}^{j-2} (1 - p_2)^i \cdot \exp(-C_{j-2-i} - A_i - B_{j-2}) \\ + \frac{\varepsilon_2}{c + m_2 \exp(-\delta_2(t - (h+l)T_N - (j-2)T_k))},$$

$$\omega^* = \frac{E}{\left(1 - (1 - p_2)^k \exp(-cT_N - A_k)\right)}.$$



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