Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/mbe

MBE, 21(1): 984–1016.
DOI: 10.3934/mbe.2024041
Received: 18 November 2023
Revised: 09 December 2023
Accepted: 13 December 2023
Published: 22 December 2023

Research article

A Padé approximation and intelligent population shrinkage chicken swarm optimization algorithm for solving global optimization and engineering problems

Tianbao Liu*, Yue Li and Xiwen Qin*

School of Mathematics and Statistics, Changchun University of Technology, Changchun 130012, Jilin, China

* **Correspondence:** Email: liutianbao@ccut.edu.cn, qinxiwen@ccut.edu.cn.

Abstract: Bio-inspired optimization algorithms are competitive solutions for engineering design problems. Chicken swarm optimization (CSO) combines the advantages of differential evolution and particle swarm optimization, drawing inspiration from the foraging behavior of chickens. However, the CSO algorithm may perform poorly in the face of complex optimization problems because it has a high risk of falling into a local optimum. To address these challenges, a new CSO called chicken swarm optimization combining Padé approximate, random learning and population reduction techniques (PRPCSO) was proposed in this work. First, a Padé approximate strategy was combined to help agents converge to the approximate real solution area quickly. Padé approximate was grounded in a rational function aligning with the power series expansion of the approximated function within a defined number of terms. The fitting function used in this strategy employs the above rational function and the extreme points are calculated mathematically, which can significantly improve the accuracy of the solution. Second, the random learning mechanism encouraged agents to learn from other good agents, resulting in better local exploitation capability compared to traditional CSO. This mechanism has a special idea that when it comes to selecting random individuals, it selects from the same type of high-performing agents, rather than selecting them completely at random. Third, a new intelligent population size shrinking strategy was designed to dynamically adjust the population size to prevent premature convergence. It considers fitness function calls and variations in recent optimal solutions creatively. To validate the algorithm's efficacy, PRPCSO was rigorously tested across 23 standard test functions and six kinds of practical engineering problems. We then compared PRPCSO with several mainstream algorithms, and the results unequivocally established PRPCSO's superior performance in most instances, highlighting its substantial practical utility in real engineering applications.

Keywords: chicken swarm optimization; Padé approximate; random learning mechanism; intelligent population size shrinkage; engineering optimization

1. Introduction

1.1. Background

Meta heuristic algorithms mainly seek optimal solutions by simulating natural phenomena. This kind of algorithm is often called the intelligent optimization algorithm. At present, algorithms of this type are needed in a large number of fields, such as engineering design problems [1–3], machine learning and theoretical analysis [4–7] and energy saving and environmental protection [8–10]. Sometimes people need to know what size industrial parts to choose to minimize cost, how to place trash cans in the city to provide maximum convenience for pedestrians, and so on. These problems can be solved quickly with the help of optimization algorithms. Depending on the source of inspiration, some researchers divide them into four categories: Population-based methods, methods inspired by physical phenomena, evolution-based strategies and methods inspired by human activities [11]. Population-based meta-heuristic algorithms explore the search space through a population containing multiple individuals. Each individual represents a potential solution, and these individuals generate new solutions through different update ways. Through several iterations, the agents in the population gradually converge to the vicinity of the true solution. Inspired by natural selection, evolution-based meta-heuristic algorithms search for optimal solutions by simulating the evolutionary and genetic mechanisms of living organisms. The algorithm updates the individual through genetic operations, produces new solutions and selects better offspring. Physics-based metaheuristics update individuals by simulating physical laws and gradually approaching the true solution. Human-based metaheuristics are updating agents by imitating human behavior. Researchers have proposed many metaheuristic algorithms like this one by looking to nature for inspiration. They have the advantage of fast convergence and simple computation. However, excessive pursuit of convergence speed may lead to insufficient population diversity and premature convergence. Of course, these challenges are also the motivation for researchers to continuously explore and improve metaheuristic algorithms.

1.2. Literature review

The chicken swarm optimization (CSO) algorithm was initially proposed by Meng et al. [12] in 2014. Since then, numerous scholars have made enhancements and extensions to the algorithm, enabling its application across a broader range of disciplines. Inspired by the behavior of chickens, the CSO algorithm incorporates three different roles in the swarm: Roosters, hens and chicks. The rooster assumes an active role in food search within the flock while hens follow suit; meanwhile, chicks explore their surroundings for sustenance. Each colony can be divided into groups comprising one rooster alongside multiple hens and chicks. Different roosters adhere to varying locomotion rules with competition arising among them based on a specific hierarchical order. Aiming at the problem that the original CSO algorithm is easy to converge prematurely and fall into local optimum, many people have improved the update method of agents. To address this issue, Wu et al. [13] introduced a new CSO that incorporates a portion of roosters' learning into the chicks' position update equation and introduces inertia weight and learning factor. As the most numerous individual in the population, the performance of CSO is greatly affected by how the hens update. Therefore, Chen et al. [14] improved the update equation of hens.

The process of chicks following hens to find food is blind. Due to the lack of autonomy, the chicks

are easily linked by the hens and, thus, fall into local optima. To solve this problem, Wang et al. [15] proposed to introduce a mutation strategy for chicks to enhance the population diversity and help them get rid of the dilemma of falling into local optimum. The hens follow the lead rooster, while the chicks follow their mother for food. When the roosters get trapped in local optima, the hens and chicks will converge too early, reducing the effect of global optimization. Based on this problem, Verma et al. [16] introduced Levy's Flight strategy to solve the problems of local optimal and premature convergence. Ahmed et al. [17] improved the search capability of CSO by applying logistic and tending to chaotic maps to help the CSO swarm to better explore the search space. Many researchers prefer to combine universal strategies such as Levy's flight and chaotic map with different swarm intelligence algorithms to improve the search effect of the algorithm. Other researchers have set their sights on improving these strategies. Recently, Yang et al. [18] studied how to improve the probabilistic selection of chaotic operators based on the maximum Lyapunov exponent (MLE), and added this new multiple chaotic local operator to the slime mold algorithm (SMA) to achieve surprising results.

Aiming at the problem that the convergence speed of CSO algorithm is not satisfactory enough and difficulty in obtaining the global optimal solution of CSO, Wang et al. [19] proposed an adaptive CSO algorithm with fuzzy strategy (FCSO). In the FCSO algorithm, the fuzzy system and cosine function are integrated into the CSO algorithm simultaneously. The fusion of optimization algorithms with machine learning techniques represents a widely explored area of research. Moldovan [20], for instance, leveraged the CSO algorithm to enhance the performance of the support vector machine (SVM) through hyperparameter optimization. This approach aims to achieve improved classification accuracy and showcases the growing synergy between optimization techniques and machine learning methods. In 2018, Mohamed [21] mapped real number vectors to integer space and used modified CSO to improve the performance of the greedy algorithm, which achieved good results. In addition to, some scholars have combined CSO with other swarm intelligence optimization algorithms or evolutionary algorithms to make up for some shortcomings of CSO. Abbas et al. [22] combined CSO and the bacterial foraging optimization (BFA) algorithm, and this hybrid technique effectively reduced the computational cost and load. Torabi and Safi-Esfahani [23] combined CSO with the raven roosting optimization (RRO) algorithm to achieve a better transition between local exploitation and global exploration. Deb and Gao [24] combined CSO and the ant lion optimization algorithm (ALO) [25] to deal with the charger distribution problem, and the combination of CSO and ALO can improve the accuracy of CSO, thus preventing it from falling into local optimum. Deb et al. [26] also proposed the synergy of CSO with a new teaching-learning-based optimization (TLBO) algorithm, and proposed a new constraint handling mechanism to improve CSO. In 2019, Zouache et al. [27] successfully applied CSO to the field of multi-objective optimization. Since then, some scholars [28] have continuously improved multi-objective CSO to improve algorithm performance and applied it to a lot of fields.

As previously mentioned in the CSO algorithm, roosters exhibit the highest search capability and are succeeded by hens, the largest agent class in the population. Chicks rely on their mothers for foraging and lack independence. Roosters handle global exploration, while hens and chicks manage local exploitation. However, the algorithm's search accuracy is unsatisfactory, prone to falling into local optima. Two reasons contribute to these issues: The hen's search method is not optimal, leading to low accuracy and premature convergence, and chicks' reliance on their mothers for food results in a limited learning scope and insufficient autonomous search ability. Consequently, chicks prematurely converge under maternal influence, leading to local optima challenges. Addressing this urgent concern, the pa-

per is dedicated to refining the search methods for hens and chicks. It introduces a new CSO algorithm called chicken swarm optimization combining Pad*é* approximation, random learning and population reduction techniques (PRPCSO). The primary contributions of this study are outlined as follows:

1) A novel and efficient approach based on Pad*é* approximate technique is proposed to enhance the ability of the CSO algorithm in solving optimization problems. The fitting function used in this strategy is different from other linear approximation or quadratic approximation methods, but with the help of a complex nonlinear rational function, the extreme point is calculated from the mathematical point of view to approximate, which can significantly improve the solution accuracy and development ability of the CSO algorithm, and avoid local convergence.

2) Aiming at the problem that chicks are easy to fall into local optimum under the influence of their mother hens, a random learning mechanism is introduced. This mechanism has a special way of thinking when it comes to selecting random individuals. It selects from the same type of high-performing agents (i.e., chicks), rather than selecting them completely at random. This mechanism encourages chicks to follow their mother while learning from other good peers, preventing them from falling into local optima.

3) This study has developed a novel intelligent population size shrinking strategy. It not only considers adjusting the population size based on the number of fitness function calls but also creatively takes into account the variations in the obtained optimal solutions over the recent iterations. The strategy can eliminate bad and preserve good, prevent premature convergence, and reduce the amount of computation.

The rest of this paper is organized as follows. In Section 2, the CSO algorithm is briefly reviewed. Section 3 describes the main content and framework of PRPCSO. In Section 4, our proposed algorithm is tested with the other four famous, excellent algorithms on 23 test functions, and the experimental results are compared and analyzed. To investigate the application of PRPCSO to real problems, in Section 5, six real engineering design problems are described and analyzed experimentally. Finally, Section 6 contains the conclusions and perspectives of this work.

2. Mathematical model and basic knowledge

2.1. Principle of CSO algorithm

CSO is inspired by the foraging behavior of the flock and maps the behavior of the flock to mathematics. For simplicity, the behavior of the chicken is idealized according to the following rules.

1) A chicken swarm is divided into small groups. Each group has a rooster, many hens and chicks.

2) The role identification of chickens and the division of groups are realized according to their fitness values. Several chickens with the best fitness values are selected as roosters, and each rooster is used as the leader of a group. Except for a few chicks with the worst fitness values, the rest were identified as hens. The hens randomly select a group and randomly associates with some chicks in the group.

3) The hierarchy, dominance and mother-child relationships in the population are updated every few (G) iterations.

4) The rooster acts as a leader in the search for food. Suppose chickens steal food randomly from each other. Chicks search for food near their mother (hens).

Suppose $rNum$, $hNum$, $cNum$ and $mNum$ correspond to the number of roosters, chickens, chickens

and mother hens, respectively. For the problem of optimizing minimization, *rNum* chickens with the lowest fitness value are assumed to be roosters, the worst *cNum* chicken is considered to be chicks and the rest of the chickens are considered hens. *mNum* chickens are randomly selected as mother hens in the hens swarm. All N chickens are represented by their positions $x_{i,j}^t$, ($i \in [1, ..., N]$, $j \in [1, ..., D]$) at time t, looking for food in a search space.

2.2. *The mathematics of CSO*

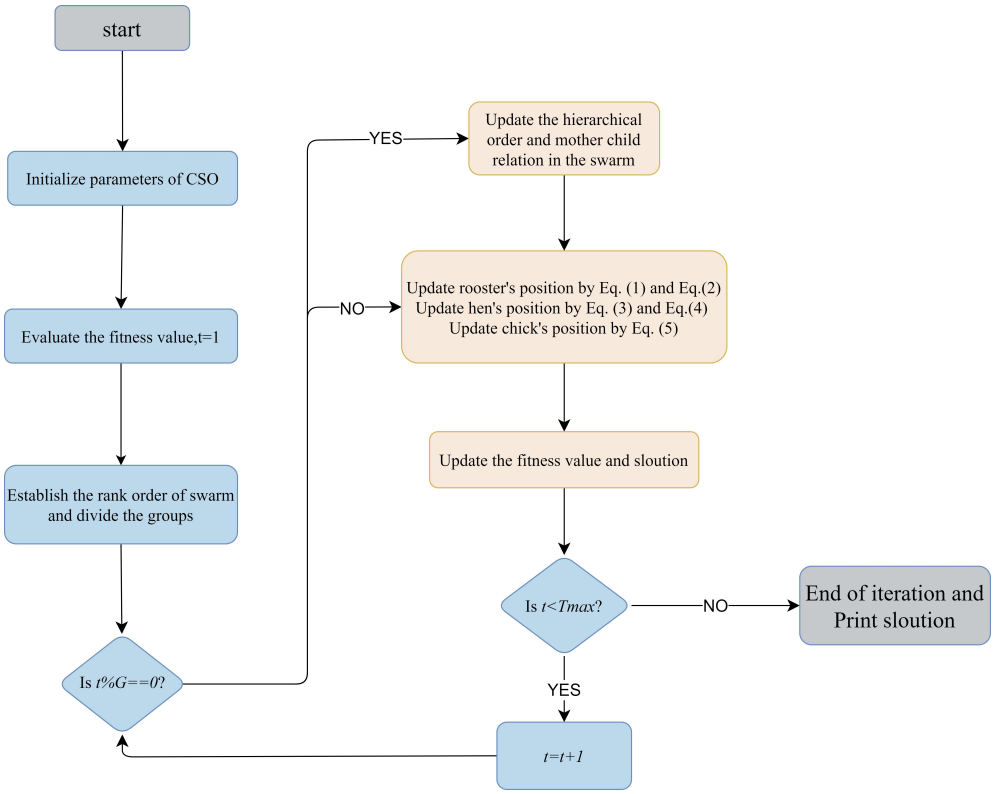


Figure 1. The framework of CSO.

Different members of the swarm move differently. Roosters search in the following way:

$$x_{i,j}^{t+1} = x_{i,j}^t(1 + Randn(0, \sigma^2)), \tag{2.1}$$

$$\sigma^2 = \begin{cases} 1, & if f_i \le f_k \\ exp(\frac{f_k - f_i}{|f_i| + \epsilon}), & otherwise \end{cases}, k \in [1, RN], k \neq i. \tag{2.2}$$

Here $Randn(0, \sigma^2)$ denotes a random number generated from a Gaussian distribution with mean zero and variance σ^2. k is the index of a rooster selected at random from the roosters and f_k is its corresponding fitness value. ϵ is the smallest constant added to avoid a zero denominator.

Hens also compete with other chickens for food while following their mate roosters for food:

$$x_{i,j}^{t+1} = x_{i,j}^t + C_1 Rand(x_{r1,j}^t - x_{i,j}^t) + C_2 Rand(x_{r2,j}^t - x_{i,j}^t), \tag{2.3}$$

$$\begin{cases} C_1 = exp(\frac{f_i - f_{r1}}{|f_i| + \epsilon}) \\ C_2 = exp(f_{r2} - f_i) \end{cases}, \tag{2.4}$$

where *Rand* is a random number generated from a uniform distribution obeying [0,1], $r1$ is the index of the mate of this hen in the group and its corresponding fitness value is f_{r1}. In addition, an individual randomly selected from the group of roosters and hens is $r2$, whose fitness value is f_{r2}. C_1 and C_2 represent the influence factors of the hen's mate and other hens on the focal hen.

The chicks are entirely dependent on the influence of their mother for locating food:

$$x_{i,j}^{t+1} = x_{i,j}^{t} + FL(x_{m,j}^{t} - x_{i,j}^{t}), \tag{2.5}$$

where m is the index of the mother hen that the chick follows and $x_{m,j}^{t}$ is the position of the mother hen of the ith chick (i.e., the mth hen) in the jth dimension. FL is a parameter chosen from [0,2] that represents the influence factor of the mother hen's position on the position of this chick.

3. The proposed optimization algorithm

3.1. Padé approximate strategy

In this section, the Pad*é* approximate technique is introduced in order to construct an efficient intelligent optimization algorithm. This is a local search strategy that swiftly identifies optimal points in the proximity of several agents through solving for extreme points in a rational function fitting manner. This strategy contributes to enhancing the precision and efficiency of the local search carried out by hens. The Pad*é* approximation approximates $f(x)$ by using the following rational function:

$$T(x) \simeq T_{[M,N]}(x) = \frac{P(x)}{Q(x)} = \frac{\sum_{i=0}^{M} p_i (x - x_0)^i}{\sum_{i=0}^{N} q_i (x - x_0)^i}, \tag{3.1}$$

which the mathematical expression is a kind of Pad*é* approximation. The coefficients p_i and q_i are unique up to constant and are generally normalized as $q_0 = 1$. Pad*é* approximate technique is a powerful approximate method for approximating rational functions. One of the most important advantages is that it is not restricted to the radius of convergence of the Taylor expansion [29]. Specifically, Pad*é* approximate has the best convergence performance when the numerator and denominator have equal or almost equal orders. Rational fractions can provide better approximations either within or outside the radius of convergence. This method is the backbone of asymptotic waveform evaluation [30, 31] and has also been widely used by researchers to calculate frequency responses in recent years due to its extremely high efficiency and accuracy. This is the inspiration for our use of rational functions instead of standard polynomials. Therefore, in this paper, Pad*é* approximate technique is used to help hens to locate excellent solutions more quickly and accurately, which improves the development ability and solution accuracy of the CSO algorithm. We consider using the following [2,1] Pad*é* approximation:

$$T(x) = \frac{a_1 + a_2 x^2}{1 + a_3 x}, \tag{3.2}$$

where a_1, a_2 and a_3 are real parameters. Let $T(x)$ be the interpolation rational function of $f(x)$ at abscissae x_k, x_{k+1}, x_{k+2}. That is,

$$\begin{cases} T(x_k) = \dfrac{a_1 + a_2 x_k^2}{1 + a_3 x_k} = f(x_k) \\ T(x_{k+1}) = \dfrac{a_1 + a_2 x_{k+1}^2}{1 + a_3 x_{k+1}} = f(x_{k+1}) \\ T(x_{k+2}) = \dfrac{a_1 + a_2 x_{k+2}^2}{1 + a_3 x_{k+2}} = f(x_{k+2}). \end{cases} \tag{3.3}$$

According to Eq (3.3), it can be calculated that:

$$a_1 = \frac{(x_{k+2} - x_{k+1}) f(x_{k+1}) f(x_{k+2}) x_k^2 + (x_k - x_{k+2}) f(x_k) f(x_{k+2}) x_{k+1}^2 + (x_{k+1} - x_k) f(x_k) f(x_{k+1}) x_{k+2}^2}{(f(x_{k+2}) x_{k+2} - f(x_{k+1}) x_{k+1}) x_k^2 + (f(x_k) x_k - f(x_{k+2}) x_{k+2}) x_{k+1}^2 + (f(x_{k+1}) x_{k+1} - f(x_k) x_k) x_{k+2}^2}, \tag{3.4}$$

$$a_2 = \frac{(x_k - x_{k+1}) f(x_k) f(x_{k+1}) + (x_{k+2} - x_k) f(x_k) f(x_{k+2}) + (x_{k+1} - x_{k+2}) f(x_{k+1}) f(x_{k+2})}{(x_k^2 - x_{k+1}^2) f(x_{k+2}) x_{k+2} + (x_{k+2}^2 - x_k^2) f(x_{k+1}) x_{k+1} + (x_{k+1}^2 - x_{k+2}^2) f(x_k) x_k}, \tag{3.5}$$

$$a_3 = \frac{(x_{k+2}^2 - x_{k+1}^2) f(x_k) + (x_k^2 - x_{k+2}^2) f(x_{k+1}) + (x_{k+1}^2 - x_k^2) f(x_{k+2})}{(x_{k+1}^2 - x_{k+2}^2) f(x_k) x_k + (x_{k+2}^2 - x_k^2) f(x_{k+1}) x_{k+1} + (x_k^2 - x_{k+1}^2) f(x_{k+2}) x_{k+2}}. \tag{3.6}$$

Combining the above formulas, the minimal of approximate curve $T(x)$ can be attained as follows:

$$x^* = -\frac{1}{a_3} \pm \sqrt{abs(\frac{1}{a_3^2} + \frac{a_1}{a_2})}. \tag{3.7}$$

The Pad*é* approximate operator is a local search operator that uses a curve to fit the structure of a rational function to reach the extreme point of the curve. In the Eq (3.7), there are two solutions of x^*, and here, the greedy idea is used to compare these two x^* with the fitness value of the original hen to select the best one among the three kinds of hens. The whole process is embodied in Algorithm 1, where the hen is first updated to get x^{t+1} in the original CSO way with Eqs (2.3) and (2.4). Next, the Pad*é* approximate strategy is performed and three neighboring hens x_k, x_{k+1}, x_{k+2} in the hen group are selected and the parameters a_1, a_2 and a_3 are calculated based on the positions and fitness values of these three hens, shown in Eqs (3.4)–(3.6). x^* is split into two solutions $H(k)$ and $h(k)$. Finally, the best one among the three ($H(k)$, $h(k)$ and x^{t+1}) is selected as the output solution.

The foraging capability of hens plays a crucial role in determining the search efficiency of the algorithm and directly influences the foraging efficiency of their offspring. Hence, integrating the Pad*é* approximation strategy can enhance the hen's food-finding ability, ultimately maximizing the overall performance of the algorithm.

Algorithm 1 The second degree Padé approximate strategy

Require: $x_{i,j}^{t}$
Ensure: $x_{i,j}^{t+1}$
Generate random number *Rand*
for each $i \in [rNum + 1, rNum + hNum]$ **do**
    Calculate $x_{i,j}^{t+1}$ and $fit(i)$ according to the Eqs (2.3) and (2.4)
end for
for each $i \in [rNum + 1, rNum + hNum - 2]$ **do**
    Select two agents around x_i:x_{i+1},x_{i+2}
    Calculate a_1,a_2,a_3 according to the Eqs (3.4)–(3.6)
    Calculate $H(i) = -\frac{1}{a_3} + \sqrt{abs(\frac{1}{a_3^2} + \frac{a_1}{a_2})}$, $h(i) = -\frac{1}{a_3} - \sqrt{abs(\frac{1}{a_3^2} + \frac{a_1}{a_2})}$
    Let $tag1 = fit(H(i))$, $tag2 = fit(h(i))$
    if $tag1 < tag2$ **then**
        $tag = tag1$; $H(i) = H(i)$
    else
        $tag = tag2$; $H(i) = h(i)$
    end if
end for
if $tag < fit(i + 2)$ **then**
    $x_{i+2,j}^{t+1} = H(i)$; $fit(i) = tag$
end if
    return $x_{i,j}^{t+1}$

3.2. Random learning strategy

Considering that the chicks' search for food was completely affected by its mother, it was difficult to escape the local optimal situation when the mother hens are in a local optimal, so this work incorporates a random learning strategy for the process of finding food for chicks. The objective of this strategy is to decrease the reliance of chicks on their mothers and enhance the diversity of the population. As chicks follow mother hens for food, they also get help from well-performing agents within the same class. This not only assists chicks in steering clear of local optima, but also ensures a progressive behavior in chicks.

$$k_1, k_2, k_3 = randperm(pop - (rNum + hNum + 1), 3) + rNum + hNum + 1, \tag{3.8}$$

$$[\sim, k] = min(fit(k_1), fit(k_2), fit(k_3)), \tag{3.9}$$

$$x_{i,j}^{t+1} = x_{i,j}^{t} + rand(x_{m,j}^{t} - x_{k,j}^{t}), \tag{3.10}$$

where k_1, k_2 and k_3 are three different random indexes. $\sim$ is a placeholder for the output parameter, meaning that the minimum value of $fit(k_i)$ is ignored; we only need to record the index k of the minimum value. Compare the fitness values of the chickens corresponding to the three indexes and select the optimal chicken x_k. The specific process can be seen in Algorithm 2.

Algorithm 2 Random learning strategy

Input: $x^t_{m,j}$, $x^t_{i,j}$
Output: $x^{t+1}_{i,j}$

Generate random number *Rand*
$k = randperm(pop - (rNum + hNum + 1), 3)$
$k = k + rNum + hNum + 1$
$[\sim, k] = min(fit(k_1), fit(k_2), fit(k_3))$
for each $i \in [rNum + hNum + 1, pop]$ **do**
    Calculate $x^{t+1}_{i,j}$ and $fit(i)$ according to the Eq (3.10)
end for
    return $x^{t+1}_{i,j}$

The random learning strategy can reduce the dependence of the chicks on their mothers. If the mother chickens are unfortunate enough to fall into the local optimum, this mechanism ensures that chicks still retain the ability to escape from local extrema. Choosing the most suitable individual from three random candidates for learning rather than following a purely random approach ensures that the chicks' actions progress effectively. In general, incorporating the random learning strategy can increase the diversity of the population and overcome premature convergence.

3.3. Intelligent population size shrinkage strategy

Several swarm intelligence optimization algorithms have a common problem; that is, in the later stage of iteration, the algorithm tends to converge, thus a large number of individuals participating in optimization is redundant for the algorithm. Many researchers consider dynamically adjusting the population size. For example, Yang et al. [32] proposed a new search method called a three-phase search approach with dynamic population size (TPSDP) and added the idea of dynamic population size, which depends on the number of iterations. Based on this, we propose a new intelligent population size shrinkage strategy. It takes into account both the number of fitness function calls and the optimal solution over r iterations. It endeavors to sustain a sizable population in the initial iterations, particularly when the algorithm's obtained optimal solution exhibits substantial fluctuations. Conversely, it diminishes the population size during later iterations when the algorithm's optimal solution shows minimal variability. This concept aims to safeguard the algorithm's optimization effectiveness while alleviating computational demands. The population size gradually decreases according to Eq (3.11).

$$pop = \begin{cases} p_{max} - round[\frac{(p_{max}-p_{min})NFE}{MAXNFE*\Phi^{\iota}(F)+\epsilon}], if \quad NFE < MAXNFE * \Phi^{\iota}(F) \\ max(pop, p_{min}), else \end{cases} \tag{3.11}$$

$$\Phi(F) = \frac{max(F) - mean(F)}{max(F) - min(F)}. \tag{3.12}$$

Here p_{max} and p_{min} are the preset population maximum and minimum population sizes, and pop is the number of agents within the current population. F is the set of optimal solutions obtained by the algorithm in the last r iterations, $max(F)$ is the maximum value in the set F, $min(F)$ is the minimum value in the set F and $mean(F)$ is the average value of F. $\Phi(F)$ evaluates the performance of the last r

iterations by looking at the information provided by the maximum, minimum and average value of F and $\Phi(F)$ is a number belonging to the range between zero and one. The value of $\Phi(F)$ is small when the convergence of the last r iterations is fast, the value of $\Phi(F)$ converges to one when the algorithm's optimization process tends to converge and ι is a parameter. $MAXNFE$ is the maximum number of fitness function evaluations and NFE is the number of fitness function evaluations within the current number of iterations. ϵ is the smallest constant added to avoid a zero denominator. Clearly, at the early stage of the population iteration, the algorithm converges faster, the NFE is smaller and the population size is reduced slowly, which is exactly what this study wants to achieve. When the algorithm reaches the late iteration, the NFE value gets closer to $MAXNFE$ and the convergence rate of the algorithm slows down so that the population size approaches p_{min}. This mechanism of intelligently adjusting the population size helps the algorithm to make a smooth transition between global exploration and local exploitation. Premature convergence is avoided and the computational burden is reduced. See Algorithm 3 for details.

Algorithm 3 Intelligent population size shrinkage strategy

Input: t, M,NFE,$MAXNFE$,G

Output: pop,New population after updating the number and population relationship of chickens

if $mod(t+1, G) == 1$ **then**
  $F = fmin(t-5, t-1)$
  Calculate $\Phi(F)$ according to the Eq (3.12)
  if $NFE < MAXNFE * \Phi^{\iota}(F)$ **then**
    Calculate pop according to the Eq (3.11)
  else
    $pop = max(pop, p_{min})$
  end if
end if
Update the number and population relationship of three types of chickens in a new population
return pop

Considering the unique population relationship updating mechanism of the CSO algorithm; that is, the population is disrupted and the order is restablished every G generations, it is possible to have an error while the number of chickens decreases but the relationships between them do not change. The population size shrinkage strategy is combined with the population relationship updating mechanism, then the population size is updated every G generations. After that, the chicken families are redistributed to the new group to avoid the mismatch between the number of chickens and the relationship. Since the last r optimal solutions are needed in the population size reduction strategy, the population size reduction strategy and the population relationship updating mechanism are applied when $mod(t+1, G) == 1$ after the optimal solution is found in the tth iteration.

Because the identity of the chickens and the relationship between them in the CSO algorithm are judged by the fitness value, the worst individuals can be naturally deleted after the implementation of the intelligent population size shrinkage strategy so that the excellent individuals can be retained. Reducing the population size helps to avoid getting stuck in a local optimum as well as reduce the computational effort of the algorithm. This process does not affect the accuracy of the algorithm

because the poor performing chickens are removed.

How to balance exploration and exploitation is a very important research content for swarm intelligence algorithms (SIA). Most SIA focus on global exploration in the early stage and focus on local exploitation in the later stage, but CSO is different from it. As mentioned earlier, in CSO, roosters are responsible for global exploration, while hens and chicks are exploited in the vicinity of their respective leaders, roosters and mother hens, respectively. In each iteration, these three types of chickens are searching for the optimal solution at the same time, so the global exploration and local exploitation of CSO are carried out simultaneously. The population size shrinkage strategy and the population relationship update mechanism are carried out simultaneously, so when the population size changes, the roles of chickens in the population will be reconstructed. In the new population, agents with strong search ability can still be selected to be responsible for global exploration, and agents with weak search ability are responsible for local exploitation. When the algorithm enters the later stage, the number of agents with strong ability (roosters) is greatly reduced, and the number of agents with weak search ability (hens and chicks) is also reduced, but the hens are still the most numerous roles in all agents. The proportion of agents responsible for global exploration and local exploitation has not changed, so the balance between exploitation and exploration can be maintained even when the number of agents decreases.

3.4. The framework of PRPCSO

The improvement in CSO is mainly reflected in three aspects. First, the Pad*é* approximate operator helps the hens seek the optimal solution faster. It helps the agents to quickly converge near the approximate true solution, so as to improve the accuracy and exploitation ability of the algorithm. Meanwhile, due to the unique connection between hens and chicks, it also partially helps chicks in the foraging process. Second, combination of random learning strategy can help the population to be more diverse and help chicks avoid falling into local optima. Finally, a new intelligent population size shrinkage strategy is proposed. When the algorithm shows a convergence trend in nearly r iterations, the poor individuals in the population were deleted. These three strategies are combined to construct the PRPCSO of this work.

The implementation process of PRPCSO is described in Algorithm 4. Initially, the population is initialized, then the ranks and individual relationships within the population are updated in the first iteration. The largest number of chickens in the population, referred to as "hens", is then updated using the Pad*é* approximate strategy outlined in Algorithm 1. Because the chicks completely rely on their mother hens to find food, their position update method is combined with a random learning strategy to avoid falling into local optimum, as shown in Algorithm 2. After updating the positions of all three types of chickens, fitness values for each individual are evaluated. Once a preset number of G iterations has been completed, the population undergoes shuffling and poor-performing chickens are removed based on the population size shrinkage strategy mentioned in Algorithm 3. This process creates a new order and individual relationships within the population. The iteration continues until termination conditions are met and outputs global optimal solution.

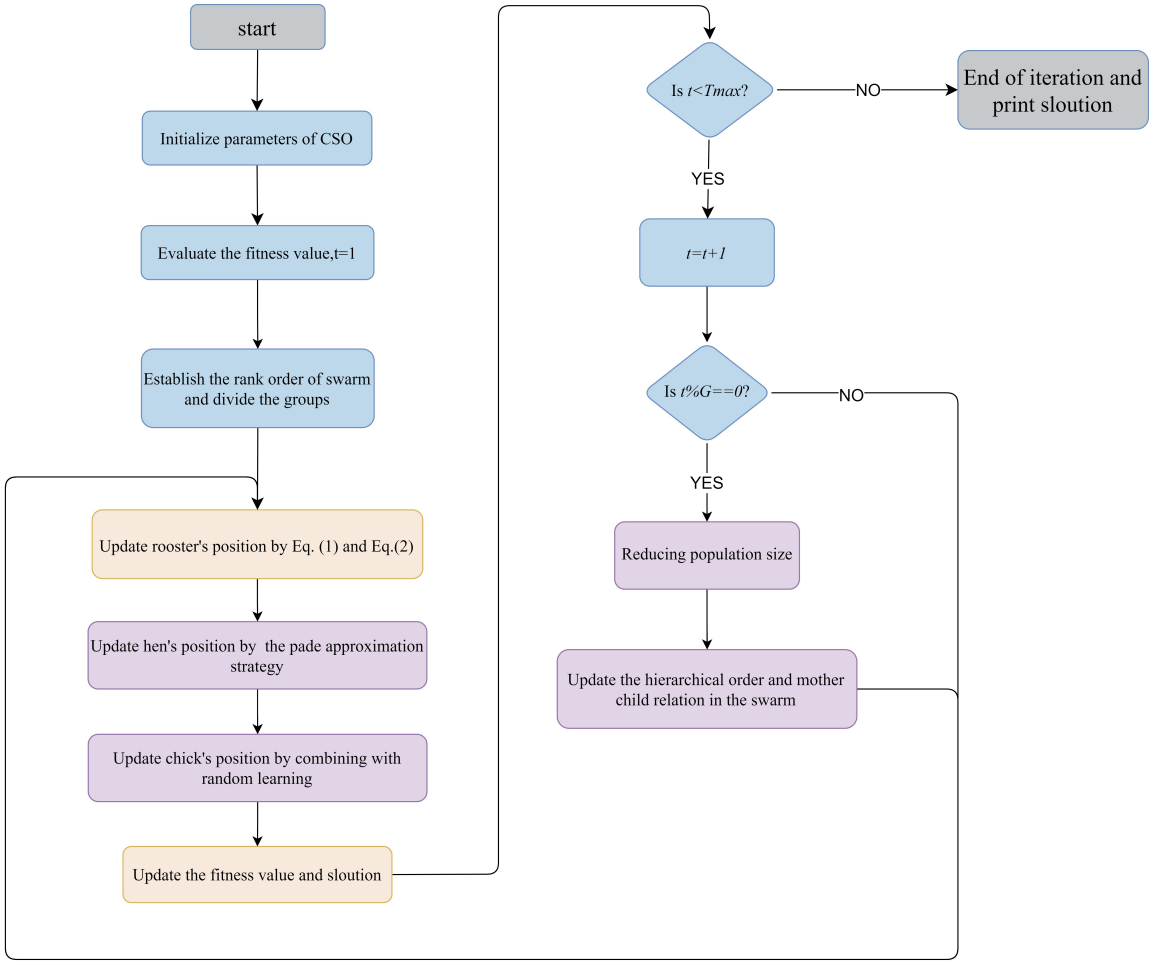


Figure 2. The framework of PRPCSO.

4. Experimental Study I: Solving benchmark functions

4.1. Experimental preparation

To evaluate the robustness of PRPCSO, it was compared with six excellent algorithms on test functions and some engineering problems, they are CSO, grey wolf optimizer (GWO) [33], sine cosine algorithm (SCA) [34], ALO [25], salp swarm algorithm (SSA) [35], moth-flame optimization (MFO) [36]. All experiments were run on MATLAB (R2021b). The portable computer used in this work is 12th Gen Intel (R) Core (trademark) i7-12700 (20 central processing units) with @2.10 Giga Hertz. Windows 10 operating system. All algorithms are compared after running 30 times with an initial population of 50 and 1000 iterations. All experiments were performed with the same settings. Finally, the performance metrics of 30 runs on the benchmark functions are obtained, which are the mean, standard deviation (std), minimum and maximum values of various benchmark functions, which are given in Tables 6–8, respectively. In the table, the bold font is the optimal solution, and the underlined font represents the sub-optimal solution.

Algorithm 4 Pseudo code of the PRPCSO algorithm

Input: $FitFunc$, M, pop, p_{min}, dim, lb, ub, $rPercent$, $hPercent$, $mPercent$
Output: Global best solution, Respective position vector

```
Initialize pop chickens and enter the relevant parameters;
All chickens were evaluated;
while t < M do
    if t == 1 then
        Rank according to fitness values to establish order as well as individual relationships within the group;
    end if
    for each i ∈ [1, pop] do
        if i == rooster then
            Update its location using Eqs (2.1) and (2.2);
        else
            if i == hen then
                Update its location by Algorithm 1;
            else
                Update its location by Algorithm 2;
            end if
        end if
        Evaluate the new solution;
    end for
    if mod(t + 1, G) == 1 then
        Generate new populations and update chickens' hierarchal order;
    end if
end while
    return Global best solution, Respective position vector
```

4.2. Complexity analysis

As mentioned before, pop stands for the number of agents, dim is the number of dimensions and M is the maximum number of iterations. The computational complexity of the metaheuristic can be defined as: $O(pop \cdot dim \cdot M)$. The computational complexity of PRPCSO is $O(pop \cdot dim + (2pop \cdot dim + pop + 2) \cdot M + pop \cdot dim/G) \approx O(pop \cdot dim \cdot M)$. Because it is shown in Algorithm 4, it has only one inner loop, and the algorithm needs additional fitness evaluation for $H(i)$ and $h(i)$ only in the Padé approximate part. To ensure fair and unbiased experiments, pop, dim and M are kept the same for all algorithms in this paper.

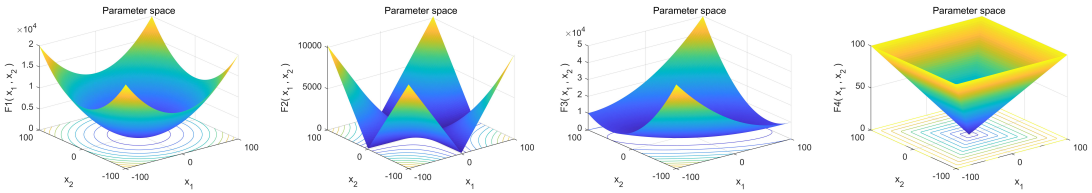
4.3. Benchmark functions

This section verifies the search capability of PRPCSO through 23 classical test functions. PRPCSO is used to find the extrema of these 23 functions. These functions are classical problems used to test the robustness of the algorithm and they are of various types, which facilitates researchers to investigate the algorithm from different aspects.

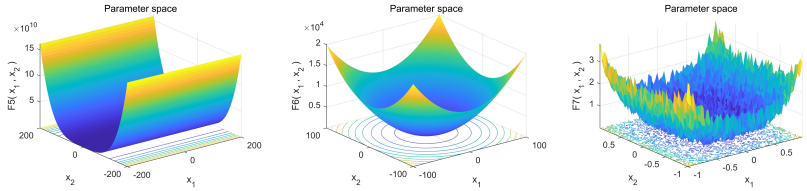
Tables 1–3 show the details of the 23 functions, where F1 to F13 are high-dimensional problems and F14 to F23 are mixed multi-modal problems. Functions F1 to F7 are unimodal, F6 is a step function with a minimum and is discontinuous, and F7 is a quadratic function containing noise. The multi-modal functions are F8 to F13, whose number of locally optimal solutions grows exponentially with the dimensionality of the problem, and it has been pointed out in [38] that this type of problem seems to be the most difficult class of optimization algorithms. The last class is F14 to F23, which are low-dimensional and contain few locally optimal solutions.

Table 1. Information on the seven unimodal benchmark functions.

Functions	Dim	Range	f_{min}
$f_1(x) = \sum_{i=1}^{n} x_i^2$	30	$[-100, 100]^n$	0
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	$[-10, 10]^n$	0
$f_3(x) = \sum_{i=1}^{n} (\sum_{j-1}^{i} x_j)^2$	30	$[-100, 100]^n$	0
$f_4(x) = max_i\{\lvert x_i \rvert, 1 \leqslant i \leqslant n\}$	30	$[-100, 100]^n$	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	$[-30, 30]^n$	0
$f_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	30	$[-100, 100]^n$	0
$f_7(x) = \sum_{i=1}^{n} ix_i^4 + random[0, 1)$	30	$[-1.28, 1.28]^n$	0

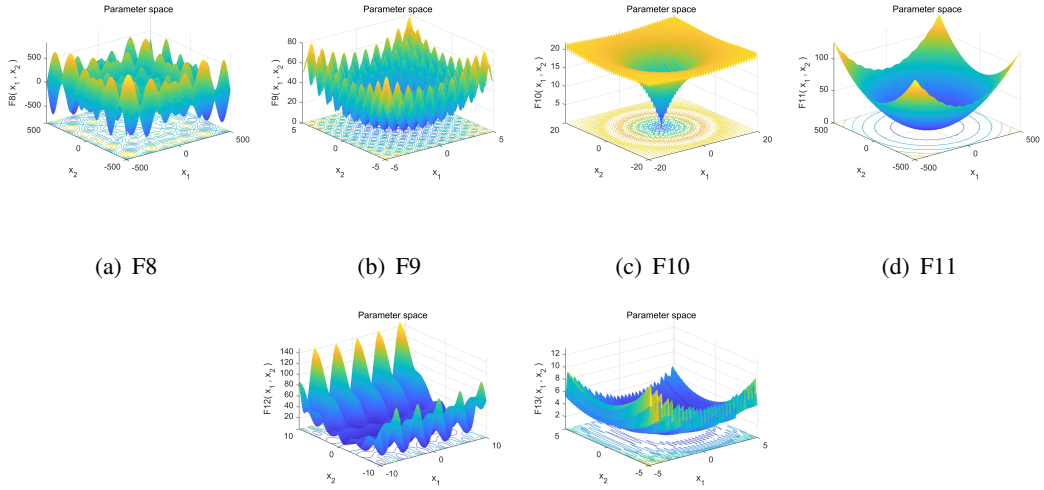


(a) F1 (b) F2 (c) F3 (d) F4



(e) F5 (f) F6 (g) F7

Figure 3. Unimodal benchmark functions.



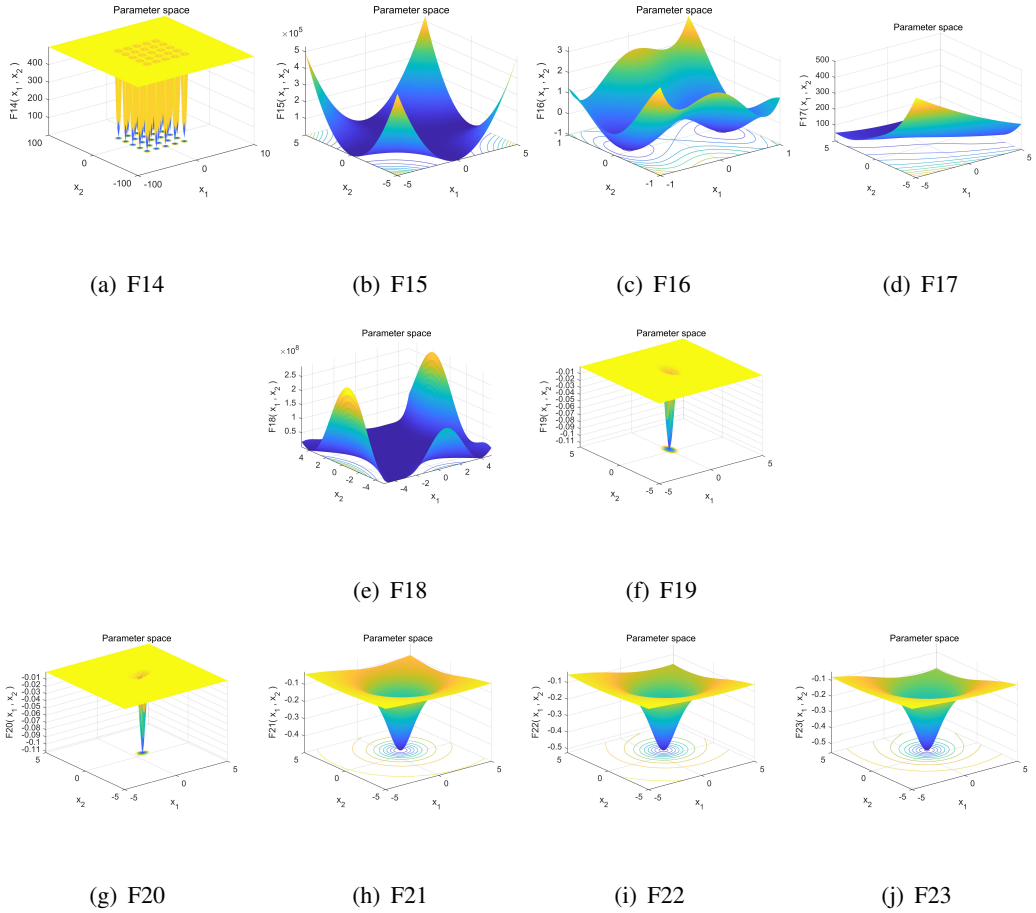
(a) F8 (b) F9 (c) F10 (d) F11

(e) F12 (f) F13

Figure 4. Multi-modal benchmark functions.

Table 2. Information on six multimodal functions.

Functions	Dim	Range	f_{min}
$f_8(x) = \sum_{i=1}^{n} -x_i sin(\sqrt{\lvert x_i \rvert})$	30	$[-500, 500]^n$	-12569.5
$f_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	30	$[-5.12, 5.12]^n$	0
$f_{10}(x) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2})$ $-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)) + 20 + e$	30	$[-32, 32]^n$	0
$f_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{(i)}}) + 1$	30	$[-600, 600]^n$	0
$f_{12}(x) = \frac{\pi}{n}\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})]$ $+(y_n - 1)^2\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i+1}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	30	$[-50, 50]^n$	0
$f_{13}(x) = 0.1\{\sin^2(3\pi x_i) + \sum_{i=1}^{n} x_i - 1)^2[1 + \sin^2(3\pi x_i + 1)]$ $+(x_n - 1)^2[1 + \sin^2(2\pi x_n)]\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30	$[-50, 50]^n$	0



(a) F14 (b) F15 (c) F16 (d) F17

(e) F18 (f) F19

(g) F20 (h) F21 (i) F22 (j) F23

Figure 5. Fixed-dimension Multi-modal benchmark functions.

Table 1 corresponds to the mathematical representation of seven unimodal benchmark functions including function calculation formulas, dimension, the scope of the search space and the true optimal solution. Figure 3 is a schematic diagram on its 3D space, and it can be seen that the 3D plot of each function is concave and unimodal. Table 2 and Figure 4 correspond to the mathematical representation and 3D image presentation of the six multi-modal benchmark functions, respectively. It is obvious that the mathematical formulation of the multi-modal function is more complex than the mathematical formulation of the unimodel, and its multi-modal nature can be clearly seen from Figure 4. Finally, Table 3 and Figure 5 show fixed-dimension multi-modal benchmark functions, which have smaller dimension and search range than the other two, and the true optimal solution is also different from them.

Table 3. Fixed-dimension Multi-modal benchmark functions.

Functions	Dim	Range	f_{min}
$f_{14}(x) = (\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j+\sum_{i=1}^{2}(x_i - a_{ij})^6})^{-1}$	2	$[-65.536, 65.536]^n$	1
$f_{15}(x) = \sum_{i=1}^{11}[a_i - \frac{x_1(b_i^2+b_1x_2)}{b_i^2+b_1x_3+x_4}]^2$	4	$[-5, 5]^n$	0.0003075
$f_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_3^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	$[-5, 5]^n$	-1.0316285
$f_{17}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2$ $+10(1 - \frac{1}{8\pi})\cos x_1 + 10$	2	$[-5, 5]^n$	0.398
$f_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1$ $+3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2$ $\times(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	$[-2, 2]^n$	3
$f_{19}(x) = \sum_{i=1}^{4} c_i \exp(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2)$	4	$[1, 3]^n$	-3.86
$f_{20}(x) = \sum_{i=1}^{4} c_i \exp(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2)$	6	$[0, 1]^n$	-3.32
$f_{21}(x) = \sum_{i=1}^{5}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]^n$	-10
$f_{22}(x) = \sum_{i=1}^{7}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]^n$	-10
$f_{23}(x) = \sum_{i=1}^{10}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	$[0, 10]^n$	-10

4.4. Parameter setting

This section describes the parameter settings involved in the experiments. In order to ensure the fairness of the experiment, we only set the common parameters of all algorithms (such as population size, dimension of decision variable, maximum iteration number, etc.) without modifying the original code of the comparison algorithm. Other unique parameters will be configured in accordance with the settings outlined in the respective references of the algorithm. Moreover, specific parameters (ι, p_{min}, r) associated with PRPCSO require testing to ascertain their optimal values for the final configuration. For the parameter $MAXNFE$, it is set to 200,000 in order to meet the requirements of the number of iterations; the other parameters of PRPCSO are kept consistent with the CSO algorithm.

During the experiment, four levels were considered for each parameter to be tested separately. Following the principles of the Taguchi method [37], 16 distinct value schemes were delimited. Considering that these three parameters all serve the population shrinkage strategy in order to reduce the running pressure of the algorithm, the average time of each run after running PRPCSO 30 times is taken as the response variable (consider the F1 test function) , and the experimental results are detailed in Table 4.

Table 4. Parameter matrix and response variable values.

Index	ι	p_{min}	r	response variable values
1	0.05	26	3	0.2488
2	0.05	28	5	0.2508
3	0.05	30	7	0.2517
4	0.05	32	9	0.3016
5	0.10	26	5	0.2416
6	0.10	28	3	0.2493
7	0.10	30	9	0.2500
8	0.10	32	7	0.2537
9	0.15	26	7	0.2458
10	0.15	30	3	0.2467
11	0.15	28	9	0.2474
12	0.15	32	5	0.2528
13	0.20	26	9	0.2377
14	0.20	32	3	0.2512
15	0.20	28	7	0.2444
16	0.20	30	5	0.2503

The average response variable values (ARV) corresponding to different levels of each parameter are shown in Table 5. The results indicate that the algorithm achieves optimal running speed when ι is set to 0.2, p_{min} is set to 26 and r is set to five.

Table 5. ARV for different parameter values.

ι	ARV	p_{min}	ARV	r	ARV
0.05	0.2633	**26**	**0.2435**	3	0.2490
0.10	0.2487	28	0.2480	**5**	**0.2488**
0.15	0.2482	30	0.2497	7	0.2489
0.20	**0.2459**	32	0.2649	9	0.2592

4.5. Performance assessment

The test results of the PRPCSO algorithm and the other six algorithms on 23 benchmark functions are evaluated in this section. See Tables 6–8. The performance of each algorithm on these seven unimodal are provided in Table 6. The table results indicate GWO's strong competitiveness in addressing unimodal problems. However, PRPCSO still demonstrates commendable performance across five functions, excelling notably in functions F4 and F5, while securing the second-best performance in the remaining three functions (F1, F2, F7). The optimization problems in the F8 to F13 class are considered to be the most challenging. According to the performance analysis of these six problems in Table 7, it is not difficult to find that the performance of our proposed algorithm is significantly better than the other six comparison algorithms on F9–F12. In addition, the ranking of our proposed algorithm on F13 also shows strong competitiveness, which proves the superiority of PRPCSO in solving multimodal optimization problems. F14 to F23 are fixed-dimension multi-modal optimization problems. Table 8 shows that PRPCSO performs satisfactorily on 10 benchmark functions. Although other comparison algorithms also achieve similar optimal solutions, PRPCSO achieves lower std values with

similar solutions. This proves that the superiority of PRPCSO is not only reflected in the solution accuracy but also in the solution robustness. In addition, PRPCSO shows better search ability than CSO on all test functions, which indicates that our proposed algorithm is an effective improvement.

Unimodal functions can be used to evaluate the exploitation ability of an algorithm, and multi-modal functions can be used to evaluate the exploration ability of an algorithm. The ability of the algorithm to avoid local optima can be evaluated by using a multi-modal function. The experimental performance of the PRPCSO algorithm on 23 functions shows that it is an excellent combination of local exploitation and global exploration.

Table 6. Results of unimodal benchmark functions.

function		CSO	GWO	SCA	ALO	SSA	MFO	PRPCSO
f_1	mean	3.52E-45	**1.38E-69**	2.27E-03	5.85E-07	8.83E-09	1333.3334	2.08E-46
	std	1.58E-44	**3.79E-69**	6.04E-03	4.17E-07	2.13E-09	3457.4590	4.40E-46
	min	9.09E-55	**1.03E-72**	8.13E-08	9.86E-08	5.07E-09	6.46E-06	9.23E-53
	max	8.52E-44	**1.69E-68**	2.45E-02	1.73E-06	1.40E-08	10000	2.12E-45
f_2	mean	8.00E-39	**5.70E-41**	1.00E-05	3.81E-01	0.5653	38.0002	2.69E-39
	std	1.75E-38	**5.50E-41**	1.86E-05	4.52E+01	0.8731	18.4578	4.69E-39
	min	1.70E-42	**3.05E-42**	4.97E-08	1.44E-02	0.0022	10.0001	5.84E-45
	max	7.89E-38	**1.86E-40**	7.65E-05	1.43E+02	3.8700	80.0000	1.95E-38
f_3	mean	2035.5631	**3.72E-19**	2860.8623	2.75E+02	41.8160	13459.6012	790.1403
	std	996.3636	**1.54E-18**	3010.3132	1.00E+02	29.3666	11102.5381	653.0734
	min	265.2085	**2.32E-25**	167.8583	8.25E+01	5.2595	142.0716	14.7509
	max	4489.5070	**8.47E-18**	11581.3715	5.24E+02	119.9813	35006.2993	2.73e+03
f_4	mean	8.8512	1.34E-17	13.21308	10.6803	4.2649	52.3683	**2.98e-244**
	std	8.6687	1.50E-17	7.2976	4.7471	3.2599	10.0618	**0**
	min	0.0053	1.11E-18	1.7985	4.0835	0.3896	25.0419	**0**
	max	26.3844	7.05E-17	29.5624	27.6545	12.2401	72.0400	**8.94e-243**
f_5	mean	34.2925	26.4411	86.190379	95.1932	131.0338	24484.8673	**25.7341**
	std	34.8687	0.6502	161.1930	189.8797	169.8673	40232.0860	**0.6217**
	min	26.9533	25.1891	28.0691	25.6544	24.9240	17.7747	**24.6656**
	max	218.8825	27.7481	647.0520	1044.0978	665.3988	90079.5694	**27.2786**
f_6	mean	3.0110	0.4261	4.291645	5.00E-07	**8.98E-09**	330.0084	0.0021
	std	0.5119	0.2772	0.3205	4.50E-07	**1.49E-09**	1807.5301	0.0015
	min	1.9506	0.000008	3.5429	7.13E-08	**5.92E-09**	9.19E-07	2.40e-04
	max	3.9455	1.253018	5.2019	1.72E-06	**1.186E-08**	9900.2500	0.0071
f_7	mean	0.016057	**0.0003**	0.0196	0.0437	0.0527	6.2370	0.0058
	std	0.0334	**0.0002**	0.0215	0.0189	0.0154	10.2140	0.0032
	min	0.0008	**0.0000**	0.0018	0.0120	0.0181	0.0262	0.0021
	max	0.1413	**0.0008**	0.1199	0.0891	0.0820	34.9651	0.0158

Table 7. Results of multi-modal benchmark functions.

function		CSO	GWO	SCA	ALO	SSA	MFO	PRPCSO
f_8	mean	-7654.4666	-6382.8472	-3926.4987	-5585.4462	-7754.1925	**-8814.4881**	-7424.1018
	std	562.7150	695.4723	**266.1356**	555.6855	587.7483	705.6383	706.8838
	min	-8659.9485	-8284.6623	-4434.8448	-8502.9480	-8891.6091	**-10590.9578**	-8525.7853
	max	-6868.3753	-4839.1327	-3424.6095	-5417.6748	-6783.2155	**-7340.6252**	-6075.9664
f_9	mean	0.7205	0.6116	19.4212	67.7566	47.7580	158.6928	**0**
	std	3.9465	1.9175	25.5091	19.8224	12.4763	33.1943	**0**
	min	0	0	3.00E-6	38.8033	18.9042	100.4905	**0**
	max	21.6160	7.2840	116.6615	117.4048	78.6015	238.0782	**0**
f_{10}	mean	6.93E-15	1.31E-14	10.6479	1.9030	1.8327	12.3331	**6.09E-15**
	std	1.86E-15	2.41E-15	9.3909	0.5052	0.7939	8.5879	**1.80E-15**
	min	4.44E-15	7.99E-15	0.0001	0.9329	2.15E-05	0.0010	**4.44E-15**
	max	7.99E-15	1.51E-14	20.3164	3.4621	3.2844	19.9591	**7.99E-15**
f_{11}	mean	0.0076	0.0014	0.1945	0.0121	0.0102	15.0656	**0.0007**
	std	0.0270	0.0055	0.2568	0.0123	0.0113	34.2321	**0.0029**
	min	0	0	2.00E-6	3.91E-05	1.82E-08	1.06E-05	**0**
	max	0.1156	0.0229	0.8175	0.0495	0.0394	90.7379	**0.0136**
f_{12}	mean	1.0635	0.0215	0.5964	8.8174	3.7450	8533334.23	**0.0003**
	std	2.3907	0.0131	0.2191	5.5645	2.5036	46738994.7573	**0.0002**
	min	0.1084	0.0057	0.3888	3.9331	0.1169	**1.01E-05**	3.79E-05
	max	9.0288	0.0654	1.4304	31.8536	12.9931	256000017.7553	**0.0011**
f_{13}	mean	3.7145	0.2961	6.6460	0.0147	**0.0090**	27337516.8500	0.0810
	std	11.3556	0.2001	9.4774	0.0268	**0.0134**	104036254.2859	0.0753
	min	0.9714	0.000016	2.0163	7.45E-08	**4.06E-10**	9.02E-06	0.0032
	max	63.8140	0.8582	40.0867	0.1316	**0.0548**	410062760.1113	0.3172

Table 8. Results of fixed-dimension multi-modal benchmark functions.

function		CSO	GWO	SCA	ALO	SSA	MFO	PRPCSO
f_{14}	mean	1.0641	1.0311	1.3949	1.2958	0.9980	1.4612	**0.9980**
	std	0.3622	4.0562	0.8071	0.6457	2.45E-16	0.7701	**0**
	min	0.9980	0.9980	0.9980	0.9980	0.9980	0.9980	**0.9980**
	max	2.9821	12.6705	2.9821	3.9683	0.9980	2.9821	**0.9980**
f_{15}	mean	0.0007	0.0050	0.0009	0.0048	0.0009	0.0015	**0.0003**
	std	0.0001	0.0085	0.0004	0.0125	0.0003	0.0036	**0.0001**
	min	0.0004	0.0003	0.0003	0005	0.0003	0.0006	**0.0003**
	max	0.0014	0.0203	0.0014	0.0632	0.0013	0.0204	**0.0007**
f_{16}	mean	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	**-1.0316**
	std	1.12E-10	2.98E-09	1.46E-05	4.00E-14	6.97E-15	6.77E-16	**9.89E-11**
	min	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	**-1.0316**
	max	-1.0316	-1.0316	-1.0315	-1.0316	-1.0316	-1.0316	**-1.0316**
f_{17}	mean	0.3978	0.3978	0.3987	0.3979	0.3979	0.3979	**0.3979**
	std	1.09e-07	5.43E-05	1.25E-03	4.67E-15	2.16E-15	0	**0**
	min	0.3978	0.3978	0.3979	0.3979	0.3979	0.3979	**0.3979**
	max	0.3978	0.3981	0.4029	0.3979	0.3979	0.3979	**0.3979**
f_{18}	mean	3	3.000005	3.000009	3	3	3	**3**
	std	2.17E-07	5.90E-06	1.84E-05	2.12E-13	7.82E-14	1.61E-15	**9.99E-16**
	min	3	3	3	3	3	3	**3**
	max	3	3	3.0001	3	3	3	**3**
f_{19}	mean	-3.8626	-3.8617	-3.8564	-3.8628	-3.8628	-3.8628	**-3.8628**
	std	6.25E-04	2.64E-03	3.12E-03	8.22E-15	8.42E-15	2.71E-15	**2.71E-15**
	min	-3.8627	-3.8627	-3.8625	-3.8628	-3.8628	-3.8628	**-3.8628**
	max	-3.8593	-3.8549	-3.8539	-3.8628	-3.8628	-3.8628	**-3.8628**
f_{20}	mean	-3.2680	-3.2679	-2.8998	-3.2507	-3.2382	-3.2203	**-3.2780**
	std	0.0660	0.0710	0.3329	0.0592	0.0558	0.0553	**0.0543**
	min	-3.3219	-3.3219	-3.2762	-3.3220	-3.3220	-3.3220	**-3.3220**
	max	-3.1459	-3.0866	-1.9181	-3.2031	-3.2008	-3.1376	**-3.2031**
f_{21}	mean	-9.8521	-9.3093	-2.8795	-7.3794	-7.9735	-7.1354	**-10.1328**
	std	0.9466	1.9185	2.2051	3.1235	3.2141	3.1857	**0.0819**
	min	-10.1531	-10.1531	-7.2573	-10.1532	-10.1532	-10.1532	**-10.1532**
	max	-5.0551	-5.0551	-0.497295	-2.6305	-2.6305	-2.6305	**-9.7140**
f_{22}	mean	-9.6777	-10.2254	-4.3896	-8.1360	-9.0111	-8.2851	**-10.0877**
	std	2.0425	0.9703	1.7626	3.0961	2.6190	3.3173	**0.0824**
	min	-10.4029	-10.4028	-6.8765	-10.4029	-10.4029	-10.4029	**-10.4029**
	max	-2.7658	-5.0876	-0.9079	-2.7519	-2.7519	-2.7659	**-9.9515**
f_{23}	mean	-10.2385	-10.2656	-4.881734	-7.4026	-9.3903	-8.3420	**-10.0627**
	std	2.7342	1.4814	1.6310	3.2671	2.6545	3.1846	**1.4805**
	min	-10.5364	-10.5363	-8.9339	-10.5364	-10.5364	-10.5364	**-10.5364**
	max	-2.8659	-2.4217	-0.9432	-2.8066	-2.4773	-2.4273	**-3.8354**

4.6. Statistical analysis

The nonparametric Friedman test [39] was used to evaluate algorithms and calculate the probability values (p-values). This statistical test is used for multiple comparisons to determine significant differences between algorithms, and the number of trials in this paper is 30. The Friedman statistic can be computed using the following formula:

$$\chi_F^2 = \frac{12N}{k(k+1)}[\sum_{j=1}^{k} r_j^2 - \frac{k(k+1)^2}{4}]. \tag{4.1}$$

In this work, N and k represent the number of datasets and algorithms, respectively, which are 23 and five; r_j denotes the average ranking of j_{th} algorithm. If the p-value is below a preset significance level (0.05 in this work), the null hypothesis is rejected and a statistically noteworthy distinction in the performance of each algorithm can be confirmed.

Table 9 displays the outcomes of the test, where our proposed algorithm PRPCSO has the highest average rank on 14 test functions and, in addition, achieves sub-optimal average rank on four functions (F1, F7, F22, F23). The p-values were all ≤ 0.05 and the null hypothesis was rejected. The advantages of PRPCSO over other algorithms are proved. In addition, the Friedman test is performed on the ranking of these seven algorithms over 23 functions, and the overall mean ranks obtained are shown in Figure 6. It is easy to find that PRPCSO ranks first, followed by GWO, SSA, CSO, MFO, ALO and SCA. PRPCSO demonstrates a strong performance, while SSA and GWO also exhibit notable competitiveness in the results.

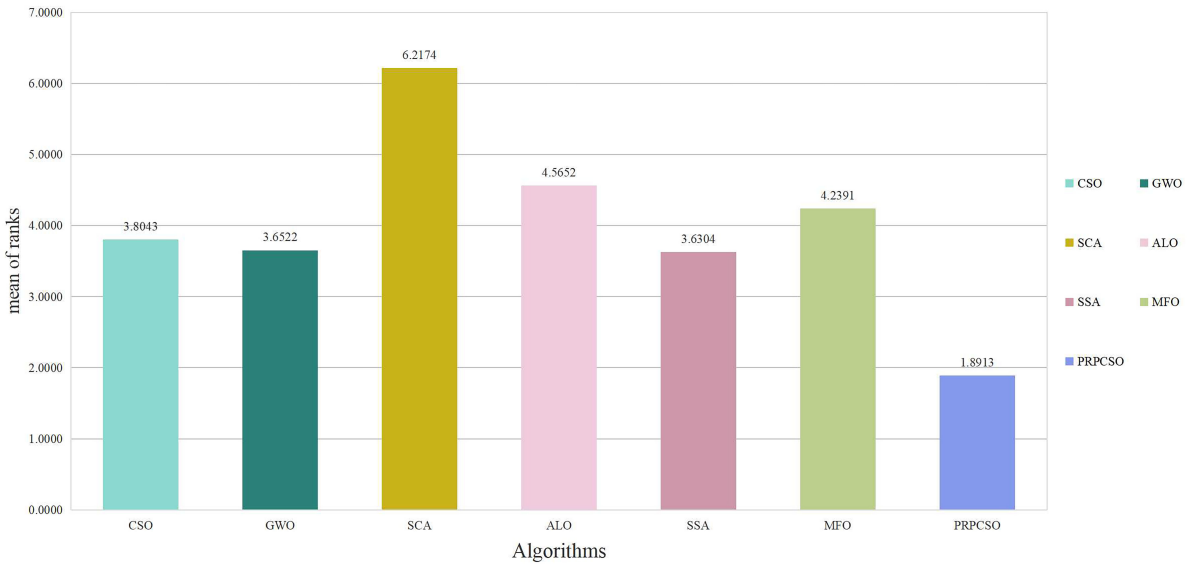


Figure 6. Rank of multiple algorithms on 23 functions.

Table 9. Mean ranks of tested algorithms and p-value from Friedman test for 23 benchmark function.

function	CSO	GWO	SCA	ALO	SSA	MFO	PRPCSO	p-value
f_1	2.6667	**1**	6.5667	5.0667	4	6.3667	2.3333	1.14E-34
f_2	2.3333	**1.2667**	4	6.1667	5.2000	6.6333	2.4000	2.66E-33
f_3	5.5667	**1**	5.3667	3.2667	2.0300	6.7667	4.0000	1.26E-32
f_4	4.4333	2	5.2667	4.6667	3.6333	7	**1**	3.00E-31
f_5	4.0000	2.6667	5.2333	4.5333	4.0667	6.2000	**1.3000**	1.52E-19
f_6	5.9667	4.5667	6.7000	2	**1**	3.8000	3.9667	1.88E-31
f_7	2.9667	**1**	3.8000	5.7333	5.5333	6.2000	2.7667	8.66E-28
f_8	2.5667	4.6000	6.9667	5.7667	2.5000	**1.2667**	4.3333	5.65E-31
f_9	1.9833	2.0667	4.2000	5.6667	5.1667	6.9333	**1.9833**	3.77E-34
f_{10}	1.5500	2.9500	5.9333	5.3000	4.9000	5.8667	**1.5000**	6.91E-30
f_{11}	2.5000	2.3833	5.5667	5	4.9000	5.7000	**1.9500**	1.48E-21
f_{12}	4.8333	2.4000	4.4667	6.6667	5.7000	2.5667	**1.3667**	1.18E-28
f_{13}	6.1667	4.7667	6.7667	2.2667	**1.7333**	2.6667	3.6333	2.26E-29
f_{14}	3.2000	6.1667	5.9000	4.4167	3.3667	3.3667	**1.5833**	2.06E-20
f_{15}	4.0333	3.0333	5.3667	4.3333	4.8333	5.3333	**1.0667**	3.11E-17
f_{16}	2.2667	5.9667	7	4.7000	4.1667	1.9500	**1.9500**	3.24E-34
f_{17}	2.5333	6.0333	6.9667	3.5833	3.7167	2.5333	**2.5333**	7.98E-32
f_{18}	2.2333	6.4667	6.5333	4.5667	4.3333	2.1833	**1.6833**	7.23E-33
f_{19}	5.1333	5.6333	6.9333	3.5833	3.6167	1.5500	**1.5500**	9.02E-33
f_{20}	3.7000	3.6667	7	3.1000	5.0667	3.4833	**1.9833**	2.72E-19
f_{21}	4.5667	3.7000	6.1333	4.7333	3.1333	3.4833	**2.2500**	2.10E-11
f_{22}	5.3333	4.2667	6.3667	4	**2.5000**	2.8000	2.7333	1.43E-15
f_{23}	5.3333	4.4667	6.2333	4.2333	3.3333	**2.0667**	2.3333	2.90E-17

After the above experimental analysis of 23 test functions, it can be concluded that compared with CSO, PRPCSO has greatly improved the solution accuracy and robustness in single-modal, multi-modal and mixed-dimensional multi-modal problems. This is due to the integration of multiple strategies: First, the Pad*é* approximation strategy helps hens to locate the optimal solution more quickly and accurately, and also improves the solving ability of chicks. This strategy helps the algorithm to exploit better locally. Second, the random learning mechanism is also very effective to help the chicks, which largely avoids the agents from falling into the dilemma of local optimum. In addition, the population size shrinkage strategy not only improves the running speed, but also ensures the balance between global exploration and local exploitation.

5. Experimental Study II: Solving engineering problems

Benchmark function tests underscore the superior performance of PRPCSO in various straightforward problems. Nevertheless, real-world optimization challenges in diverse fields tend to be more intricate than benchmark functions, often involving complex computational methods and constraints.

Consequently, there is a need to assess the effectiveness of PRPCSO in addressing real-world problems. Six practical engineering problems are first introduced (schematics from [40,41]) in this section. PRPCSO is tested and compared with the other six excellent comparison algorithms (CSO, GWO, SCA, ALO, SSA and MFO) on these practical engineering design problems. These are the questions: Three-bar truss design, vessel design, rolling element bearing design, tension/compression spring design, cantilever beam design and gear train design. The engineering optimization problems pose significant challenges due to the presence of numerous intricate constraints in the solving process.

5.1. Three bar truss design problem

The first question concerns the design of the three-bar truss. The weight of the truss is minimized by adjusting the element parameters s_1 and s_2. While the objective function is straightforward, the complexity arises from three types of constraints, namely, G_1, G_2 and G_3, corresponding to stress, deflection and buckling constraints, respectively. The mathematical formulation of the question is provided below and Figure 7 illustrates the schematic diagram:

$$minf(s) = (2\sqrt{2}s_1 + s_2) \times V, \tag{5.1}$$

$$\begin{aligned} s.t. \quad G_1 &= \frac{\sqrt{2}s_1 + s_2}{\sqrt{2}s_1^2 + 2s_1s_2}Q - \sigma \le 0, \\ G_2 &= \frac{z_2}{\sqrt{2}s_1^2 + 2s_1s_2}Q - \sigma \le 0, \\ G_3 &= \frac{1}{\sqrt{2}s_2 + s_1}Q - \sigma \le 0, \end{aligned} \tag{5.2}$$

where $s_1 \in [0,1], s_2 \in [0,1], V = 100cm, Q = 2KN/cm^2, \sigma = 2KN/cm^2$.

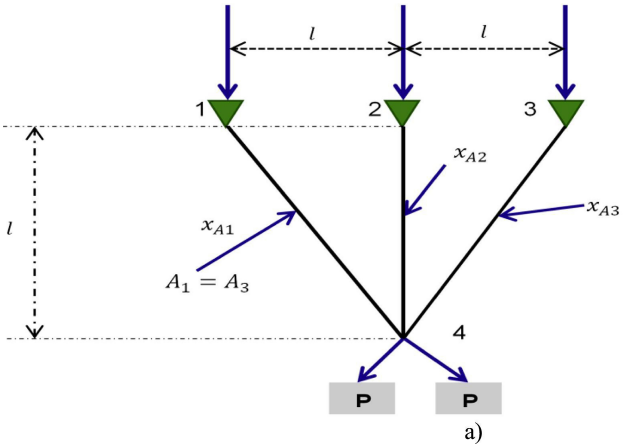


Figure 7. Three-bar truss design.

5.2. Pressure vessel design problem

The second problem wants to obtain the pressure vessel with the smallest possible manufacturing cost. The object function involves four variables (s_1, s_2, s_3, s_4), and the optimization process is subject

to four constraints (G_1, G_2, G_3, G_4). The representation of the problem is depicted in Figure 8:

$$min f(s) = 0.6224 s_1 s_3 s_4 + 1.7781 s_2 s_3^3 + 3.1661 s_1^2 s_4 + 19.84 s_1^2 s_3, \tag{5.3}$$

$$\begin{aligned} s.t. \quad & G_1 = -s_1 + 0.0193 s_3 \le 0, \\ & G_2 = -s_2 + 0.00954 s_3 \le 0, \\ & G_3 = -\pi s_3^2 s_4 - \frac{4}{3}\pi s_3^3 + 1,296,000 \le 0, \\ & G_4 = s_4 - 240 \le 0, \end{aligned} \tag{5.4}$$

where $s_1 \in [0, 99]$; $s_2 \in [0, 99]$; $s_3 \in [0, 200]$; $s_4 \in [0, 200]$.

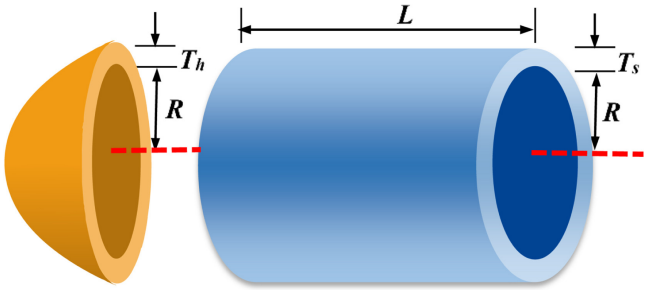


Figure 8. Pressure vessel design.

5.3. Rolling element bearing design problem

The objective of the third engineering case is to maximize the dynamic load carrying capacity of rolling element bearings. For computational convenience, the original objective function is transformed into a minimization optimization problem by $\times(-1)$ during the experimentation in this paper. The representation of this problem is shown in Figure 9:

$$\begin{aligned} & Variable \quad x = [d_m, d_b, Z, f_i, f_0, K_{d_{min}}, K_{d_{max}}, \epsilon, e, \xi], \\ & min f(x) = \begin{cases} f_c \cdot Z^{\frac{2}{3}} \cdot d_b^{1.8} & \text{if} \quad d_b \le 25.4mm \\ 3.647 f_c \cdot Z^{\frac{2}{3}} \cdot d_b^1.4 & \text{else} \end{cases}, \end{aligned} \tag{5.5}$$

$$\begin{aligned} s.t., \quad & G_1(x) = \frac{\Phi_0}{2\sin^{-1}(\frac{d_b}{d_m})} - Z + 1 \ge 0, \\ & G_2(x) = 2d_b - K_{d_{min}}(d - D) \ge 0, \\ & G_3(x) = K_{d_{max}}(d - D) - 2d_b \ge 0, \\ & G_4(x) = d_m - (0.5 - e)(d + D) \ge 0, \\ & G_5(x) = (0.5 + e)(d + D) - d_m \ge 0, \\ & G_6(x) = d_m - 0.5(d + D) \ge 0, \\ & G_7(x) = 0.5(d - d_m - d_b) - \epsilon d_b \ge 0, \end{aligned}$$

$$
\begin{aligned}
G_8(x) &= \zeta B_w - d_b \le 0,\\
G_9(x) &= f_i \ge 0.515,\\
G_{10}(x) &= f_0 \ge 0.515,
\end{aligned} \tag{5.6}
$$

where,

$$
f_c = 37.911 + [1.04(\frac{1-\gamma}{1+\gamma})^{1.72}(\frac{f_i(2f_0-1)}{f_0(2f_i-1)})^{0.4}]^{\frac{10}{3}^{-0.3}} \times (\frac{\gamma^{0.3}(1-\gamma)^{1.39}}{f_0(1+\gamma)^{\frac{1}{3}}})(\frac{2f_i}{2f_i-1})^{0.41},
$$

$$
\gamma = \frac{d_b}{d_m}, \quad f_i = \frac{r_i}{d_b}, \quad f_0 = \frac{r_0}{d_b},
$$

$$
\Phi_0 = 2\pi - 2\cos^{-1}\frac{(\frac{d-D}{2} - \frac{3T}{4})^2 + (\frac{d}{2} - \frac{T}{4} - d_b)^2 - (\frac{D}{2} + \frac{T}{4})^2}{2(\frac{d-D}{2} - \frac{3T}{4})(\frac{d}{2} - \frac{T}{4} - d_b)},
$$

$$
T = d - D - 2d_b, \quad d = 160, \quad D = 90, \quad B_w = 30, \quad r_i = r_0 = 11.033, \tag{5.7}
$$

with variable range

$$
\begin{aligned}
&0.5(d+D) \le d_m \le 0.6(d+D),\\
&0.15(d-D) \le d_b \le 0.45(d-D),\\
&4 \le Z \le 50,\\
&0.515 \le f_i \le 0.60,\\
&0.515 \le f_0 \le 0.60,\\
&0.40 \le K_{d_{Min}} \le 0.50,\\
&0.60 \le K_{d_{Max}} \le 0.70,\\
&0.30 \le \epsilon \le 0.40,\\
&0.02 \le e \le 0.10,\\
&0.60 \le \zeta \le 0.85.
\end{aligned} \tag{5.8}
$$

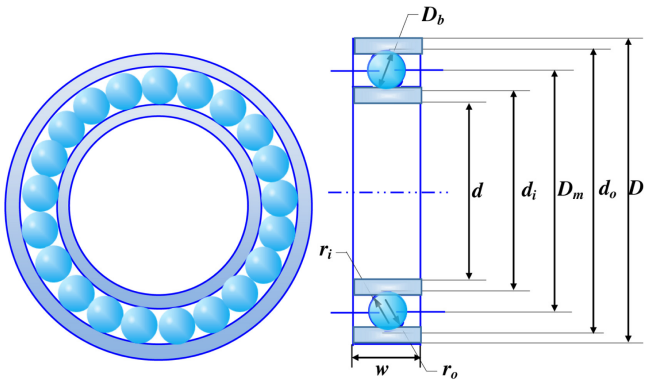


Figure 9. Rolling element bearing design.

5.4. Tension or compression spring design problem

This problem focuses on minimizing the coil weight by adjusting the variables x_1, x_2, x_3, while ensuring satisfaction of four constraints. The problem is formulated as follows and the representation is shown in Figure 10:

$$minf(X) = (x_3 + 2)x_2x_1^2, \tag{5.9}$$

$$X = [x_1 \quad x_2 \quad x_3] = [d \quad D \quad N], \tag{5.10}$$

$$\begin{aligned} s.t. \quad & G_1(x) = 1 - \frac{x_2^3 x_3}{71785x_1^4} \leq 0, \\ & G_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} \leq 0, \\ & G_3(x) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0, \\ & G_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0, \end{aligned} \tag{5.11}$$

with variable range

$$\begin{aligned} 0.05 \leq x_1 \leq 2.00, \\ 0.25 \leq x_2 \leq 1.30, \\ 2.00 \leq x_3 \leq 15.00. \end{aligned} \tag{5.12}$$

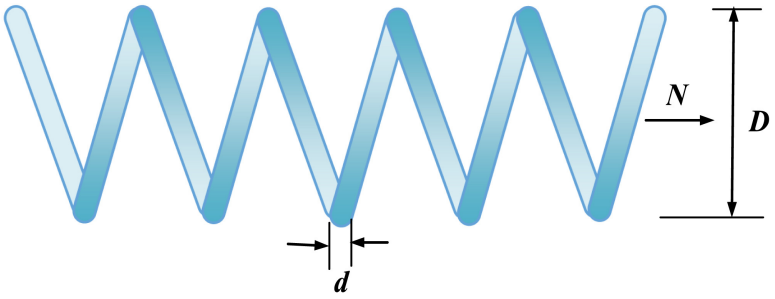


Figure 10. Tension or compression spring design.

5.5. Cantilever beam design problem

The objective is to minimize the weight of the cantilever while adhering to constraints imposed by five distinct block lengths. The problem is formulated as follows and the representation is shown in Figure 11:

$$minf(X) = 0.0624 \times (x_1 + x_2 + x_3 + x_4 + x_5) \times L, \tag{5.13}$$

$$s.t., \quad g_1(X) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0, \tag{5.14}$$

with variable range

$$0.01 \leq x_i \leq 100, i = (1, 2, ..., 5). \tag{5.15}$$

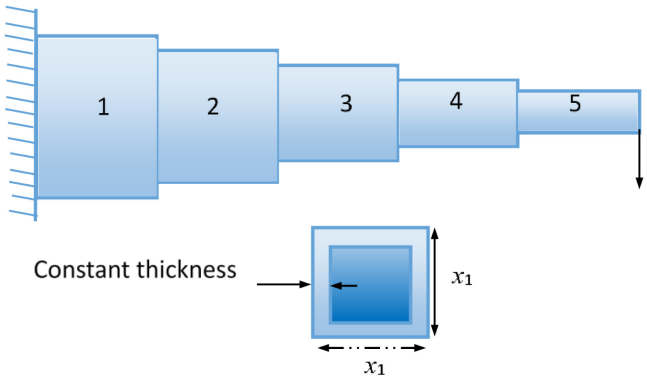


Figure 11. Cantilever beam design.

5.6. Gear train design problem

This question focuses on designing a gear train with the goal of minimizing the cost associated with the gear ratios. The problem has four integer variables where T_a, T_b, T_d and T_f denote the number of teeth of different gears. $\frac{T_b}{T_a} \times \frac{T_d}{T_f}$ is ratio of transmission. The equation of the problem is given below, as shown in Figure 12:

$$minf(X) = (\frac{1}{6.931} - \frac{x_3 x_2}{x_1 x_4})^2, \tag{5.16}$$

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4] = [T_a \quad T_b \quad T_d \quad T_f], \tag{5.17}$$

with variable range

$$12 \leq x_1, x_2, x_3, x_4 \leq 60. \tag{5.18}$$

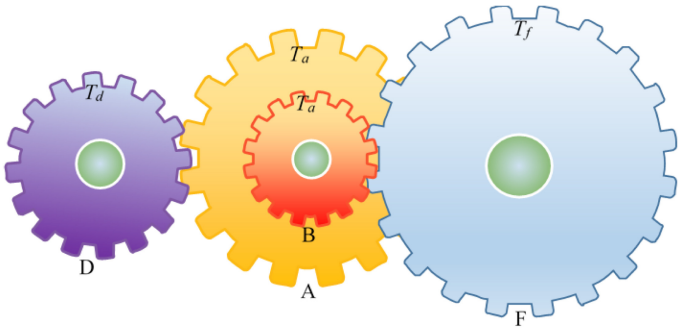


Figure 12. Gear train design.

Table 10 presents the optimization results of seven swarm intelligence optimization algorithms applied to six real engineering problems. The findings demonstrate that our proposed algorithm, PRPCSO, exhibits superior performance across most engineering problems reflected in its consistently lowest average fitness values. While GWO outperforms PRPCSO in specific scenarios such as the pressure vessel design and rolling bearing element design problems, the proposed algorithm consistently secures the second-best solution with enhanced robustness. Notably, real-world optimization challenges often involve intricate constraints and diverse computational methods. The demonstrated accuracy and robust performance of PRPCSO in the aforementioned tests gives us greater confidence in it's ability to address complex optimization problems across various real-world domains.

Table 10. Comparison of results for classical engineering problems.

function		CSO	GWO	SCA	ALO	SSA	MFO	PRPCSO
Three-bar truss design	mean	263.929	263.898	264.625	263.896	263.895	263.916	**263.896**
	std	0.064	0.003	3.441	0.0003	0.001	0.039	**6.09E-07**
	min	263.896	263.896	263.902	263.896	263.896	263.896	**263.896**
	max	264.115	263.909	282.843	263.898	263.901	264.048	**263.896**
Pressure vessel design	mean	6685.481	**6122.625**	6866.114	484761.218	6607.671	6318.142	6369.648
	std	506.396	479.805	543.753	337653.799	604.644	528.172	**330.952**
	min	5939.799	5888.117	6194.835	36746.398	5919.239	**5885.333**	5936.837
	max	7339.426	7298.334	8531.817	1785286.904	7996.973	7319.001	**7249.370**
Rolling element bearing	mean	-83350.373	**-85416.062**	-81429.349	1.63E+21	-84461.8	-85180.2	-85413.231
design	std	2949.252	**88.296**	2546.690	1.15E+21	2090.336	610.748	219.026
	min	-85483.173	-85525.858	-84298.256	9.69E+18	-85538.930	-85549.239	**-85549.239**
	max	-71839.949	**-85142.181**	-74060.956	4.30E+21	-76489.035	-83233.128	-84521.638
Tension/compression	mean	0.013	0.013	0.013	1.73E+19	0.013	0.013	**0.012**
spring design	std	**1.05E-07**	5.87E-05	1.5E-04	2.79E+19	0.0001	0.001	0.0005
	min	1.267E-02	0.013	0.013	0.020	0.013	0.013	**0.012**
	max	0.018	0.013	0.013	9.67E+19	**0.012**	0.017	0.015
Cantilever beam design	mean	1.373	1.340	1.378	7.592	1.340	1.340	**1.340**
	std	0.029	3.54E-05	0.016	1.614	**1.33E-05**	0.0002	3.66E-05
	min	1.341	1.340	1.349	4.332	1.340	1.340	**1.340**
	max	1.434	1.340	1.406	11.055	1.340	1.341	**1.340**
Gear train design	mean	8.22E-11	1.25E-10	1.149E-09	0.001	1.63E-09	4.31E-09	**6.31E-11**
	std	2.38E-10	2.96E-10	1.03E-09	0.003	3.28E-09	6.63E-09	**1.65E-10**
	min	2.70E-12	2.70E-12	2.70E-12	1.48E-06	2.70E-12	9.93E-11	**2.70E-12**
	max	9.75E-10	9.92E-10	4.50E-09	0.013	1.83E-08	2.73E-08	**8.89E-10**

6. Summary and discussion

This paper proposed a new CSO algorithm (PRPCSO). First, a new approximate technique and the addition of the Pad*é* approximate operator effectively improved the accuracy of the algorithm. The random learning operator helped to reduce the possibility of chicks falling into local optima under the influence of their mother hens. Finally, the intelligent population size shrinkage strategy was used to eliminate some chickens with poor search capability in the later iteration of the algorithm, so as to boost the running speed of the algorithm and prevent premature convergence. At the same time, it ensured the balance between global exploration and local exploitation of the algorithm.

In order to evaluate the optimization ability of the algorithm, the PRPCSO algorithm was tested on two different experiments: Test functions and engineering problems. In Experiment I, 23 different types of functions were used to test the local exploitation and global exploration capabilities of the

algorithm. Experimental results showed that PRPCSO algorithm has higher accuracy and stronger stability. The second set of experiments applied PRPCSO to practical engineering design, including six practical engineering design problems such as three-bar truss design, ship design, rolling bearing design, tension/compression spring design, cantilever beam design and gear train design. In addition, the high performance of PRPCSO in all experiments was verified by comparing six advanced optimization algorithms. The numerical results showed that the PRPCSO algorithm exhibits better stability and solution accuracy than CSO on all test problems. Among the seven algorithms, PRPCSO performed no worse than the other algorithms. In most cases, the results of PRPCSO were satisfactory. Therefore, PRPCSO can be considered as an effective improvement over the CSO algorithm. However, the experiments also showed some shortcomings of PRPCSO, mainly reflected in the processing of single-modal problems. GWO is a very competitive algorithm, which performed well on single-modal problems, while PRPCSO was at the second best level on partial problems. This is also one of the problems we need to solve in the future.

Although our improved algorithm performs well on 23 benchmark functions and six real-world engineering problems, there is no free lunch [42], and no single algorithm performs good for all types of functions. Our proposed algorithm still has the potential to advance, so more research directions may be mined. Combining the strategy proposed in this work with other advanced metaheuristics algorithms (such as GWO, dung beetle optimizer etc.) may provide more satisfactory results. In the future, PRPCSO can find application in a broader spectrum of fields, including tasks like hyperparameter optimization for extreme learning machines or aiding in addressing issues within recommendation systems. Furthermore, there is potential for the development of multi-objective and many-objective versions of PRPCSO to tackle more intricate optimization problems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

This work has been supported by the Education Department of Jilin Province project (JJKH20220662KJ), National Natural Science Foundation of China (12026430) and Department of Science and Technology of Jilin Province project (20210101149JC, 20200403182SF).

Conflict of interests

The authors declare there is no conflict of interest.

References

1. X. Hu, R. C. Eberhart, Y. Shi, Engineering optimization with particle swarm, in *Proceedings of the 2003 IEEE Swarm Intelligence Symposium. SIS'03 (Cat. No.03EX706)*, (2003), 53–57. https://doi.org/10.1109/SIS.2003.1202247

2. B. Gross, P. Roosen, Total process optimization in chemical engineering with evolutionary algorithms, *Comput. Chem. Eng.*, **22** (1998), S229–S236. https://doi.org/10.1016/S0098-1354(98)00059-3

3. D. D. Salam, I. Gunardi, A. Yasutra, Production optimization strategy using hybrid genetic algorithm, in *Abu Dhabi International Petroleum Exhibition and Conference*, 2015. https://doi.org/10.2118/177442-MS

4. X. Zeng, G. Luo, Progressive sampling-based Bayesian optimization for efficient and automatic machine learning model selection, *Health Inf. Sci. Syst.*, **5** (2017), 2. https://doi.org/10.1007/s13755-017-0023-z

5. G. Xu, An adaptive parameter tuning of particle swarm optimization algorithm, *Appl. Math. Comput.*, **219** (2013), 4560–4569. https://doi.org/10.1016/j.amc.2012.10.067

6. B. Ramakrishnan, S. S. Rao, A general loss function based optimization procedure for robust design, *Eng. Optim.*, **25** (1996), 255–276. https://doi.org/10.1080/03052159608941266

7. Y. Shu, T. Jin, Stability in measure and asymptotic stability of uncertain nonlinear switched systems with a practical application, *Int. J. Control*, **96** (2023), 2917–2927. https://doi.org/10.1080/00207179.2022.2117649

8. E. W. Davis, J. H. Patterson, A comparison of heuristic and optimum solutions in resource-constrained project scheduling, *Manage. Sci.*, **21** (1975), 944–955. https://doi.org/10.1287/mnsc.21.8.944

9. C. Miao, G. Chen, C. Yan, Y. Wu, Path planning optimization of indoor mobile robot based on adaptive ant colony algorithm, *Comput. Ind. Eng.*, **156** (2021), 107230. https://doi.org/10.1016/j.cie.2021.107230

10. L. Li, Y. He, H. Zhang, J. C. H. Fung, A. K. H. Lau, Enhancing IAQ, thermal comfort, and energy efficiency through an adaptive multi-objective particle swarm optimizer-grey wolf optimization algorithm for smart environmental control, *Build. Environ.*, **235** (2023), 110235. https://doi.org/10.1016/j.buildenv.2023.110235

11. M. Abdel-Basset, R. Mohamed, S. A. A. Azeem, M. Jameel, M. Abouhawwash, Kepler optimization algorithm: a new metaheuristic algorithm inspired by Kepler's laws of planetary motion, *Knowledge-Based Syst.*, **268** (2023), 110454. https://doi.org/10.1016/j.knosys.2023.110454

12. X. Meng, Y. Liu, X. Gao, H. Zhang, A new bio-inspired algorithm: chicken swarm optimization, in *Advances in Swarm Intelligence*, (2014), 86–94. https://doi.org/10.1007/978-3-319-11857-4_10

13. D. Wu, F. Kong, W. Gao, Y. Shen, Z. Ji, Improved chicken swarm optimization, in *2015 IEEE International Conference on Cyber Technology in Automation, Control, and Intelligent Systems (CYBER)*, (2015), 681–686. https://doi.org/10.1109/CYBER.2015.7288023

14. Y. L. Chen, P. L. He, Y. H. Zhang, Combining penalty function with modified chicken swarm optimization for constrained optimization, in *Proceedings of the First International Conference on Information Sciences, Machinery, Materials and Energy*, (2015), 1884–1892. https://doi.org/10.2991/icismme-15.2015.386

15. K. Wang, Z. Li, H. Cheng, K. Zhang, Mutation chicken swarm optimization based on nonlinear inertia weight, in *2017 3rd IEEE International Conference on Computer and Communications (ICCC)*, (2017), 2206–2211. https://doi.org/10.1109/CompComm.2017.8322928

16. S. Verma, S. P. Sahu, T. P. Sahu, MCSO: Levy's flight guided modified chicken swarm optimization, *IETE J. Res.*, (2023), 1–15. https://doi.org/10.1080/03772063.2023.2194265

17. K. Ahmed, A. E. Hassanien, S. Bhattacharyya, A novel chaotic chicken swarm optimization algorithm for feature selection, in *2017 Third International Conference on Research in Computational Intelligence and Communication Networks (ICRCICN)*, (2017), 259–264. https://doi.org/10.1109/ICRCICN.2017.8234517

18. J. Yang, Y. Zhang, T. Jin, Z. Lei, Y. Todo, S. Gao, Maximum lyapunov exponent-based multiple chaotic slime mold algorithm for real-world optimization, *IEEE Congr. Evol. Comput.*, 2023. https://doi.org/10.21203/rs.3.rs-2505598/v1

19. Z. Wang, C. Qin, B. Wan, W. W. Song, G. Yan, An adaptive fuzzy chicken swarm optimization algorithm, *Math. Probl. Eng.*, **2021** (2021), 8896794. https://doi.org/10.1155/2021/8896794

20. D. Moldovan, Cervical cancer diagnosis using a chicken swarm optimization based machine learning method, in *2020 International Conference on e-Health and Bioengineering (EHB)*, (2020), 1–4. https://doi.org/10.1109/EHB50910.2020.9280215

21. T. M. Mohamed, Enhancing the performance of the greedy algorithm using chicken swarm optimization: an application to exam scheduling problem, *Egypt. Comput. Sci. J.*, **42** (2018).

22. Z. Abbas, N. Javaid, A. J. Khan, M. H. A. Rehman, J. Sahi, A. Saboor, Demand side energy management using hybrid chicken swarm and bacterial foraging optimization techniques, in *2018 IEEE 32nd International Conference on Advanced Information Networking and Applications (AINA)*, (2018), 445–456. https://doi.org/10.1109/AINA.2018.00073

23. S. Torabi, F. Safi-Esfahani, A hybrid algorithm based on chicken swarm and improved raven roosting optimization, *Soft Comput.*, **23** (2019), 10129–10171. https://doi.org/10.1007/s00500-018-3570-6

24. S. Deb, X. Gao, A hybrid ant lion optimization chicken swarm optimization algorithm for charger placement problem, *Complex Intell. Syst.*, **8** (2022), 2791–2808. https://doi.org/10.1007/s40747-021-00510-x

25. S. Mirjalili, The ant lion optimizer, *Adv. Eng. Software*, **83** (2015), 80–98. https://doi.org/10.1016/j.advengsoft.2015.01.010

26. S. Deb, X. Gao, K. Tammi, K. Kalita, P. Mahanta, A new teaching–learning-based chicken swarm optimization algorithm, *Soft Comput.*, **24** (2020), 5313–5331. https://doi.org/10.1007/s00500-019-04280-0

27. D. Zouache, Y. O. Arby, F. Nouioua, F. B. Abdelaziz, Multi-objective chicken swarm optimization: a novel algorithm for solving multi-objective optimization problems, *Comput. Ind. Eng.*, **129** (2019), 377–391. https://doi.org/10.1016/j.cie.2019.01.055

28. Z. Wang, W. Zhang, Y. Guo, M. Han, B. Wan, S. Liang, A multi-objective chicken swarm optimization algorithm based on dual external archive with various elites, *Appl. Soft Comput.*, **133** (2023), 109920. https://doi.org/10.1016/j.asoc.2022.109920

29. Y. Honshuku, H. Isakari, A topology optimisation of acoustic devices based on the frequency response estimation with the Padé approximation, *Appl. Math. Modell.*, **110** (2022), 819–840. https://doi.org/10.1016/j.apm.2022.06.020

30. J. Zhang, J. Jin, Preliminary study of AWE method for FEM analysis of scattering problems, *Microwave Opt. Technol. Lett.*, **17** (1998), 7–12. https://doi.org/10.1002/(SICI)1098-2760(199801)17:1<7::AID-MOP2>3.0.CO;2-O

31. J. Gong, J. L. Volakis, AWE implementation for electromagnetic FEM analysis, *Electron. Lett.*, **32** (1996), 2216–2217. https://doi.org/10.1049/el:19961487

32. X. Yang, Z. Cai, T. Jin, Z. Tang, S. Gao, A three-phase search approach with dynamic population size for solving the maximally diverse grouping problem, *Eur. J. Oper. Res.*, **302** (2022), 925–953. https://doi.org/10.1016/j.ejor.2022.02.003

33. S. Mirjalili, S. M. Mirjalili, A. Lewis, Grey wolf optimizer, *Adv. Eng. Software*, **69** (2014), 46–61. https://doi.org/10.1016/j.advengsoft.2013.12.007

34. S. Mirjalili, SCA: a sine cosine algorithm for solving optimization problems, *Knowledge-Based Syst.*, **96** (2016), 120–133. https://doi.org/10.1016/j.knosys.2015.12.022

35. S. Mirjalili, A. H. Gandomi, S. Z. Mirjalili, S. Saremi, H. Faris, S. M. Mirjalili, Salp Swarm Algorithm: a bio-inspired optimizer for engineering design problems, *Adv. Eng. Software*, **114** (2017), 163–191. https://doi.org/10.1016/j.advengsoft.2017.07.002

36. S. Mirjalili, Moth-flame optimization algorithm: a novel nature-inspired heuristic paradigm, *Knowledge-Based Syst.*, **89** (2015), 228–249. https://doi.org/10.1016/j.knosys.2015.07.006

37. L. Zhu, Y. Zhou, S. Sun, Q. Su, A discrete squirrel search algorithm for the surgical cases assignment problem, *Appl. Soft Comput.*, **121** (2022), 108753. https://doi.org/10.1016/j.asoc.2022.108753

38. X. Yao, Y. Liu, G. Lin, Evolutionary programming made faster, *IEEE Trans. Evol. Comput.*, **3** (1999), 82–102. https://doi.org/10.1109/4235.771163

39. M. Friedman, The use of ranks to avoid the assumption of normality implicit in the analysis of variance, *J. Am. Stat. Assoc.*, **32** (1937), 675–701. https://doi.org/10.1080/01621459.1937.10503522

40. L. Wang, Q. Cao, Z. Zhang, S. Mirjalili, W. Zhao, Artificial rabbits optimization: a new bio-inspired meta-heuristic algorithm for solving engineering optimization problems, *Eng. Appl. Artif. Intell.*, **114** (2022), 105082. https://doi.org/10.1016/j.engappai.2022.105082

41. J. Xue, B. Shen, Dung beetle optimizer: a new meta-heuristic algorithm for global optimization, *J. Supercomput.*, **79** (2023), 7305–7336. https://doi.org/10.1007/s11227-022-04959-6

42. D. H. Wolpert, W. G. Macready, No free lunch theorems for optimization, *IEEE Trans. Evol. Comput.*, **1** (1997), 67–82. https://doi.org/10.1109/4235.585893

AIMS Press