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Research article

Developing trust among players in a vendor-managed inventory model for random demand under environmental impact

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Abstract: Retailers play a vital role in supply chain management because they deal directly with consumers. Occasionally, retailers may cover the entire system's statistics and not disclose these data to the manufacturer. Therefore, asymmetry is generated in the data throughout the system. The main motive of this research was to prevent unreliability throughout the system using a vendor-managed inventory policy. This research shows that by applying a cap and trade policy, the total carbon emitted from the production and transportation sectors can be controlled in the atmosphere. Finally, numerical and sensitivity analyses, along with pictorial representations of various parameters, are performed to examine the optimal results of this study. In addition, the retailer's lead time demand for items is assumed to be random rather than fixed and follows uniform and normal distribution functions. Under these two distribution functions, the optimal retailer lot size, service provided by the retailer to customers, and retailer reorder points are assessed. Furthermore, an evaluation of the total carbon released from an environmental viewpoint is illustrated using numerical findings. The numerical results show that this research is 50.24% more economically beneficial than the methods used in previous studies, whereas the mean value of demand follows a uniform distribution.

Keywords: supply chain management; vendor-managed inventory; reverse logistics; cap and trade policy; carbon tax; information asymmetry

1. Introduction

Supply chain management (SCM) operates in certain functional sectors to grow businesses and increase consumer value. These functions include controlling inventories, managing material and product quality, packaging, warehousing, logistics, and recycling. In the past, the manufacturers have been the ones who have designed the supply chain. Retailers transfer all relevant data and money flows to upward SCM. However, retailers are sometimes unreliable in terms of their behavior. They hide consumer information from manufacturers. Thus, data asymmetry issues often arise, rendering the entire system unreliable. Hence, data safety is a major factor in eliminating unreliability.

Although retailers are major players in SCM because they have the opportunity to deal directly with consumers, they are sometimes the main reason for system unreliability. However, retailers may also provide additional services to customers to increase their profits. Furthermore, retailers may keep this information confidential from the manufacturers. Consequently, asymmetric information flows throughout the system. Therefore, the manufacturer is willing to implement a vendor-managed inventory (VMI) policy. The manufacturer maintains the inventory level for each retailer. The VMI policy discusses the sharing of retailers carrying costs.

The manufacturer sends finished products to all the retailers at distinct locations using forward logistics. The manufacturer accumulates all items utilized by each retailer through reverse logistics. The cost of reverse logistics varies with the rate of accumulated used goods. Realistically, it is impossible to gather all sold products, implying a 100% accumulation rate. Both forward and backward logistics emit carbon into the atmosphere. Industries are taking the initiative to reduce carbon emissions to protect the environment. Industries are charged a certain amount for every ton of carbon emissions they produce, a carbon tax. The cap-and-trade (CAPT) policy sets a permitted higher emissions limit, and a cap on production factories. Moreover, if an unconsumed emissions limit exists after production, the industry sells that extra emissions limit to another industry (trade). This supports the tracking of total emissions.

A distinct literature related to the VMI supply chain highlights factors such as information asymmetry, carbon emissions reduction, and forward and backward logistics. Currently, demand is a major aspect of marketing. Most studies have dealt with fuzziness, lead time, advertisement, a distribution-free approach, and all the factors discussed earlier. However, random lead time demand with unreliability, carbon tax, CAPT policy, and forward and reverse logistics in a two-echelon VMI supply chain have not yet been discussed. This is the largest research gap concerning random lead time demand in an unreliable supply chain, together with the carbon tax, CAPT policy, VMI contracts, forward and reverse logistics, reorder points, and safety stock. From this point of view, this research gap has motivated us to conduct this research.

1.1. Contribution of the study

The proposed contribution fills the research gap in the literature by recommending random lead time demand with two types of distribution functions that incorporate a carbon tax, CAPT policy from an environmental perspective, consideration of safety stock, reorder points, and VMI system into an unreliable supply chain model.

The other goals of this research are as follows. First, to discard information asymmetry from SCM. Second, to incorporate a VMI contract for SCM coordination. Third, to control random demand to fulfill supply chain demand. Fourth, to adapt a CAPT policy in consideration of the environment. Finally, to improve economic growth and SCM collaboration for an everlasting business association. The significant contributions of this study are as follows:

(a) This study is conducted to discard unreliability factors from the supply chain system.

(b) VMI policy is highlighted to eliminate this unreliable matter from the supply chain.

(c) Random lead time demand under two distribution functions, uniform and normal, is considered to handle uncertainty.

(d) Environmental characteristics are observed through a carbon tax, CAPT policy, and accumulated utilized items.

(e) An improved economic benefit is provided for the industry compared to previous literature.

1.2. Structure of the study

The remainder of this paper is structured as follows. Section 3 elaborates on the problem definition, notation, and hypothesis. Section 4 represents mathematical modelling. Section 5 describes a numerical example with a comparative graphical presentation. Section 6 presents the sensitivity analyses with diagrammatic illustrations. In Section 7, important managerial insights from this study are discussed. Section 8 presents this study's conclusions, limitations, findings, and recommendations. Finally, some references utilized throughout the model are provided.

2. Literature review

This section presents a detailed literature review. The discussion is based on the keywords used in this research.

2.1. Unreliable retailers and information asymmetry

As retailers are the last members of upward SCM, they receive major data regarding their customer requirements, information on their investments in customer satisfaction, and knowledge about upcoming market trends. However, in some cases, the retailers are unreliable. In such cases, information asymmetry occurs throughout the system. This asymmetry of retailer data creates unfaithful circumstances within SCM. This directly impacts inventory and can effect profits. Avinadav et al. [1] studied the idea of an ex-post voluntary declaration under information asymmetry, along with a revenue-contributing technique. They found that communicating with a risk-averse retailer is much more profitable for the manufacturer than communicating with a risk-neutral retailer under hidden superiority. Guchhait and Sarkar [2] formulated a mathematical model to reduce system unreliability. By contrast, the manufacturer used a VMI policy for several unreliable retailers. Fallahpour et al. [3] examined an integrated system to develop sustainability and Industry 4.0, the principles of supplier selection management. Their study assumed the fuzzy best-worst method (FBWM) and a two-stage fuzzy inference system (FIS) to determine supplier selection. Xu and Xu [4] presented a two-echelon supply chain model for production and distribution enterprises. They discussed a signal game model in which manufacturing enterprises select channel encroachment without information. They showed that while the direct selling cost was maximum, encroachment benefited the distribution enterprise.

2.2. *Vendor-managed inventory (VMI)*

With the help of this policy, the manufacturer can eliminate the system's unreliability, as it has the power to control supply chain management. Based on the agreement of the VMI policy, the manufacturer is responsible for taking care of the retailer's holding costs and shares the revenue of the total profit. This policy ensures inventory shrinkage and maintains symmetrical information in supply chain management. First, the VMI policy reduces the system's unreliability. However, this policy supports retailers in reducing their total costs and achieving profits. By adopting the concept of environmental pollution along with green emissions, Karampour et al. [5] presented a two-echelon supply chain model to offer a green backorder for VMI and carbon emission reduction. Their model justified optimality by inserting a non-dominated sorting genetic algorithm (NSGA-II), multi-objective Keshtel algorithm, and multi-objective deer algorithm. Taleizadeh et al. [6] applied a VMI policy to a system with a single vendor and two retailers. They differentiated between the (R, T) and (r, Q) replenishment methods to determine which method was more cost-effective under partial backlogging. Bertazzi et al. [7] addressed a long–haul transportation method in which full container load deliveries on one side and less than container load or air freight shipments were integrated under the VMI policy. Asadkhani et al. [8] considered a vendor-buyer supply chain model with vendor-managed inventory with a consignment stock (VMI-CS) contract. The vendor carried the buyer's financial holding costs according to the contract. In addition, each shipment included a random portion of repairable products that needed to be removed from the inventory system.

2.3. *Cap and trade (CAPT) policy and carbon tax*

The amount of carbon emitted into the air is higher during forward and backward logistics. The CAPT policy can control carbon emission levels. The manufacturer pays an amount for each ton of carbon emitted (carbon tax). It is uncertain how much carbon will be emitted by any industry. The CAPT policy sets the authorized maximum emission limit (i.e., the cap on industries). The industry must purchase additional limits if the maximum emission level is achieved during manufacturing. The industry with an extra unutilized limit sells that limit to other factories according to their requirements, that is, trade. Ma et al. [9] developed a mathematical model to determine the effect of a carbon tax. A supplier evaluation technique was used to select the suppliers to fulfill the manufacturer's random demand. Later, Fathollahi-Fard et al. [10] described a truck-scheduling model using a cross-docking system. Their model was used to formulate three new social engineering optimizers to solve the truck scheduling problem. They also determined the optimal circumstances for receiving and transferring truck sequences. Manupati et al. [11] investigated various inventory models in the multi-echelon supply chain with strict carbon capping, carbon tax, and CAPT policies under the consideration of lead time. Zhou et al. [12] analyzed the research advancements in SCM in the carbon tax field by observing a collaboration between qualitative and quantitative techniques. Sarkar and Guchhait [13] expanded the earlier literature with a hybrid closed-loop SCM in cooperation, in which the carbon cap restricted emissions from the vendor's entire hybrid manufacturing system. Certain elements were considered in their study, such as service provided by the retailer and product collection by the retailer. Their model analyzed the situation under random market demand. Their study highlighted the techniques vendors and other supply chain players use to contribute to reproduction responsibility. Table 1 provides a comparable study to the earlier literature.

Table 1. Authors' contributions table.

Author(s)	Unreliability factor	Contract variety	Carbon emissions control	Demand follows
Guchhait and Sarkar [2]	Information asymmetry	VMI	Carbon tax and CAPT	Distribution free approach
Asadkhan et al. [8]	–	VMI-CS	–	–
Turki et al. [14]	–	–	CAPT	–
Babai et al. [15]	–	–	–	Lead time function
Xu et al. [16]	Information asymmetry	–	Carbon tax	–
Wang et al. [17]	Information asymmetry	VMI	–	–
This model	Information asymmetry	VMI	Carbon tax and CAPT	Normal and uniform distributions

VMI: vendor-managed inventory; VMI-CS: vendor-managed inventory-consignment stock; CAPT: cap and trade; –: not applicable.

3. Problem definition, notation, and hypothesis

This section briefly discusses this study's limitations. This section presents all associated notation and hypotheses for the proposed model.

3.1. Problem definition

This study presents a two-echelon SCM model with a single manufacturer and multiple unreliable retailers for one product type. Each retailer within this SCM is independent of other retailers. As these retailers are unreliable, asymmetry occurs in the system data. The manufacturer utilizes the VMI technique to control inventories to overcome this unreliability. Under VMI policy, the manufacturer handles the inventory from each retailer. Although retailers order products, manufacturers decide how much time to replenish and schedule products. Furthermore, the manufacturer pays each retailer the inventory cost. The entire management system is under the CAPT policy, and a carbon tax is charged to the manufacturer for carbon emissions. This study considers both forward and backward logistics. All utilized products are acquired from each retailer and returned to the manufacturer during reverse logistics. The return rate of the products is β. The retailer's lead time demand for items follows uniform and normal distribution function.

3.2. Notation

The following notation is used to generate the proposed model:

Index	
r	retailer r, $r = 1, 2, ..., n$
Decision	*variables*

s_r	service provided by the retailer r to customers
l_r	retailer r's lot size (units/cycle)
q_r	safety stock of the retailer r
N_r	number of times the manufacturer produces q_r quantity (integer)
Dependent	*variable*
Q_m	manufacturer's lot size (units/cycle), $Q_m = \sum_{r=1}^{n} l_r$
R_r	reorder point of retailer r (units/cycle)
Parameters	
L_r	retailer r's lead time (week)
d_r	retailer r's demand (unit/unit time)
D	manufacturer's demand $D = \sum_{r}^{n} d_r$ (unit/unit time)
P	constant production rate of the manufacturer (unit/unit time)
y_r	retailer r's per unit selling price (\$/unit)
C_m	cost of manufacturing per unit product (\$/unit)
C_p	purchasing cost of retailer r from the manufacturer per unit product (wholesale price of the manufacturer) (\$/unit)
C_{s_r}	cost of stockout of retailer r per unit (\$/unit)
$C_{s_r}^0$	marginal profit due to partial backorder shortage of retailer r per unit (\$/unit)
C_{h_m}	manufacturer's holding cost per unit per unit time (\$/unit/unit time)
C_{h_r}	retailer r's carrying cost (\$/unit/unit time)
A_r	cost for ordering of retailer r (\$/order)
I_r	retailer r's service investment (\$/cycle)
X_r	retailer r's random lead time demand having cumulative distribution function (c.d.f.) F
$M_r L_r$	mean of lead time demand X_r
$\sigma_r \sqrt{L_r}$	standard deviation of lead time demand X_r
a_r, b_r	minimum and maximum parametric value of the interval ($X_r \sim U(a_r, b_r)$)
g_1	emitted carbon from production (gallon/unit)
g_2	emitted carbon from transportation (gallon/km)
f	carbon tax per unit item (\$/unit)
M_c	carbon cap limit of the manufacturer (gallon/cycle)
P_c	cost of purchasing carbon cap (\$/unit)
T_c	trading price of non-utilized carbon cap limit (\$/unit)
J_{mr}	distance from manufacturer to rth retailer in forward logistics (km)
J_{rm}	distance from rth retailer to manufacturer in reverse logistics (km)
K_v	manufacturer's variable transportation cost (\$/km)
K_f	manufacturer's fixed transportation cost (\$/shipment)
l_c	customer satisfaction rate (> 0)
l_{c_r}	cost related to customer satisfaction ($0 < l_{c_r} < 1$)
β	utilized product's return rate (> 0)
ϕ	backorder rate ($0 < \phi < 1$)
Others	

W	total distance for transporting of products (km)
T_e	total emitted carbon (gallon/cycle)
T_{CE}	total cost of emitted carbon ($/gallon)
$E[\cdot]$	expected value of demand
R_c	manufacturer's revenue gain from CAPT ($/cycle)
EP_r	retailers expected total profit ($/cycle)
EP_m	manufacturer's expected total profit ($/cycle)
EP_c	expected total profit of SCM ($/cycle)

3.3. Hypothesis

The following hypotheses are taken to originate the proposed research.

1) In this research, a two-echelon SCM is illustrated in which a single manufacturer and multiple retailers are there. The manufacturer produces a single type of product and transfers it to several retailers. Because there is only one manufacturer, the wholesale price per unit product, that is, C_p, remains fixed for all retailers. In addition, n retailers are positioned in various locations, and therefore selling price of per unit product, i.e., y_r fluctuates [14–16].
2) In that two-echelon SCM model, retailers are assumed as unreliable. For example, retailers provide useful services to their customers, however, the manufacturer is unaware of this information. Retailers may hide some valuable data from the manufacturer. This results in information asymmetry throughout the system. The manufacturer applies VMI policy to control this situation and clear unreliability from the entire system. By the contract of this policy, the manufacturer shares the carrying cost of retailers [17, 18].
3) In this model, the market demand for products is fixed. Retailer r's lead time demand for products follows some distribution functions X_r, which are uniform and normal distribution functions [19]. Retailer r has a backorder rate ϕ_r ($0 < \phi_r < 1$) due to the random lead time demand.
4) The entire system emits carbon at the time of production and transportation from manufacturer to retailers. The manufacturer pays a carbon tax to the government for carbon emissions. The CAPT policy restricts the limit of emitted carbon for the manufacturer. The manufacturer cannot emit carbon higher than the carbon emissions limit (cap). If the production system emits lower carbon than the emissions limit, the manufacturer sells the new extra emissions limit to other industries (trade) [20].
5) While the retailer's inventory level touches the reorder point, the retailer orders products from the manufacturer. The reorder point (R_r) of retailer r is $R_r = \sum_{r=1}^{n} \left(M_r L_r + q_r \sigma_r \sqrt{L_r} \right)$, where $M_r L_r$ is the expected value and $\sigma_r \sqrt{L_r}$ is the standard deviation of the random variable X_r. $q_r \sigma_r \sqrt{L_r}$ is the safety stock and q_r is the safety factor of the retailer r. Here, the safety factor q_r is considered a decision variable instead of reorder point R_r [21, 22].
6) This research analyzes two types of logistics during the transfer of finished products from the manufacturer to each retailer. Through forward logistics, the manufacturer transport products to several retailers, and by reverse logistics, all the utilized products are accumulated from all retailers to the manufacturer. The cost during reverse logistics varies with the acquired used items [23].

4. Mathematical modelling

A single manufacturer develops a single type of product, and after that, those products are sent to n number of retailers. r retailers are having their demand of products $D = \sum_{r=1}^{n} d_r$ and are ordering products of $Q_m = \sum_{r=1}^{n} l_r$. VMI is utilized to maintain the inventory by the manufacturer. The manufacturer accumulates the used items. The manufacturer analyzes the emitted carbon from the production system. The manufacturer pays for the carbon emissions, and after that, bound the emitted carbon in the atmosphere, the CAPT policy is taken. Retailer's lead time demand is observed as random and follows two distinct distribution functions, normal and uniform. Safety stock, lead time, and reorder point are adequately examined here. Further, the manufacturer & retailer's model and associated costs are clarified in the following section.

4.1. Manufacturer's model

All the related revenue (wholesale price and CAPT policy for emitted carbon) and costs of the manufacturer (manufacturing cost, transportation cost, holding cost, and carbon tax for emitted carbon) are discussed below. The manufacturer produces $N_r l_r$ quantity for every order l_r and sends it to l_r over N_r times. Subsequently, the cycle length of the manufacturer is $\sum_{r=1}^{n} \frac{N_r l_r}{d_r} = \frac{NQ_m}{D}$.

4.1.1. Manufacturing cost

The manufacturer produces products as per the ordering amount. The manufacturer produces $NQ_m = \sum_{r=1}^{n} N_r l_r$ products for n retailers. If the unit cost of manufacturing is C_m, then the entire cost of manufacturing Q_m items per cycle is $\frac{(C_m NQ_m)D}{NQ_m} = C_m D$.

4.1.2. Wholesale price

The manufacturer sells the product to each retailer with a fixed wholesale price C_p. Once the manufacturer has received the total order of items Q_m from all retailers, they gain revenue. This revenue per cycle is analyzed as $\frac{(C_p NQ_m)D}{NQ_m} = C_p D$.

4.1.3. Carbon tax for emitted carbon

The manufacturer pays carbon tax for emissions from production and transportation. Each unit required energy in inventory processing performance. $W = \sum_{r=1}^{n} J_{mr} + \sum_{r=1}^{n} J_{rm}$ is the entire transportation distance for forward and reverse logistics for delivering and collecting products. g_1 is the per unit carbon emissions from the production section, and g_2 is the per kilometer emitted carbon from the transportation sector. Subsequently, the total emitted carbon from two sectors, i.e., from the production and transportation sectors, is

$$T_e = \frac{[g_1 NQ_m + g_2 W]\, D}{NQ_m}.$$

In the aforementioned equation, the first term represents the carbon emissions from the manufacturing sector, and the second term stands for the emitted carbon from the transportation section. If f is per unit carbon tax paid by the manufacturer, then the entire carbon tax for emissions per cycle, that is,

T_{CE} is

$$T_{CE} = fT_e = f\left[g_1 D + \frac{g_2 W D}{N Q_m}\right].$$

4.1.4. CAPT policy for emitted carbon

Using the CAPT policy, the manufacturer reduces unnecessary emitted carbon from the production sector. The manufacturer purchases several carbon cap (M_c) from any reputed agency or the government with some price P_c. Hence, the manufacturer can emit carbon (T_e) up to the cap limit. Two scenarios may emerge during production time. In the first case, if a limit exists on some unused emissions, the manufacturer sells this limit to other industries at a given price T_c. In the second case, if the carbon emissions limit M_c is used up, the manufacturer will purchase another carbon limit from an external source. Subsequently, related expressions of the CAPT policy for emitted carbon are defined as

$$M_c - T_e = M_c - \frac{[g_1 N Q_m + g_2 W]\, D}{N Q_m},\ T_e - M_c = \frac{[g_1 N Q_m + g_2 W]\, D}{N Q_m} - M_c.$$

In the aforementioned equation, the first equation stands for the non-utilized capability of carbon emissions. The second equation is the shortage quantity of carbon emissions limit while the manufacturer emits maximum carbon rather than the cap limit. $g_1 N Q_m$ is the emissions from the production section and $g_2 W$ is the emissions from transportation. Hence, the revenue observed from the CAPT policy is $R_c = T_c(M_c - T_e)^+ - P_c(T_e - M_c)^+ = T_c(M_c - T_e) + (T_c - P_c)(T_e - M_c)^+$. Applying the shortage amount of emitted carbon, the revenue is

$$R_c \le T_c\left(M_c - \frac{[g_1 N Q_m + g_2 W]\, D}{N Q_m}\right) + \frac{T_c - P_c}{2}\left(\sqrt{\left(\frac{[g_1 N Q_m + g_2 W]\, D}{N Q_m} - M_c\right)^2}\right.$$
$$\left. + \frac{[g_1 N Q_m + g_2 W]\, D}{N Q_m} - M_c\right),\ \text{[using Cauchy–Schwarz inequality]}.$$

4.1.5. Transportation cost

The manufacturer considers several transportation costs when sending products to retailers and acquiring utilized products from retailers. Now, two types of transportation costs are taken in this model. First is a fixed cost, the constant base cost for the transport system, that is, K_f. That fixed transportation cost K_f is the lowest cost while transporting items, whatever the products and distance. Second is variable transportation cost K_v, which varies with the ordering items and distance between all retailers and the manufacturer. During forward logistics, the distance from the manufacturer to the retailer r is J_{mr}. Subsequently, the total transportation cost per cycle during forward logistics is

$$\frac{\left(K_f + K_v \sum_{r=1}^{n} N_r l_r J_{mr}\right) D}{N Q_m}.$$

In addition, for reverse logistics, the distance from the manufacturer to the retailer r is J_{rm}. The accumulation rate of utilized products is β. The accumulated products for transportation is $\beta \sum_{r=1}^{n} N_r l_r J_{rm}$. The total transportation cost during reverse logistics is

$$\frac{\left(K_f + K_v \beta \sum_{r=1}^{n} N_r l_r J_{rm}\right) D}{N Q_m}.$$

Hence, the entire cost of transportation is

$$\frac{2K_f D}{NQ_m} + K_v \sum_{r=1}^{n} d_r \left(J_{mr} + \beta J_{rm}\right).$$

4.1.6. Holding cost

The manufacturer receives Q_m ordered products from r number of retailers and produces NQ_m quantity products at a rate P unit. The manufacturer delivers Q_m products to all retailers over N times. Each retailer receives l_r quantity over N_r times, that is, $N = \sum_{r=1}^{n} N_r$. $D_m = \sum_{r=1}^{n} d_r$. Subsequently, the total inventory is

$$\sum_{r=1}^{n} \frac{l_r}{2}\left[\frac{2d_r}{P} + N_r\left(1 - \frac{d_r}{P}\right) - 1\right].$$

Subsequently, the holding cost of the manufacturer is

$$C_{h_m} \sum_{r=1}^{n} \frac{l_r}{2}\left[\frac{2d_r}{P} + N_r\left(1 - \frac{d_r}{P}\right) - 1\right].$$

In addition, the manufacturer pays the holding cost for retailers too, as per the VMI contract.

4.1.7. Total profit of the manufacturer

Equation (4.1) provides the expected total profit of the manufacturer per cycle, which is described as

$$\begin{aligned} EP_m(N_r) = {} & (C_p - C_m)\sum_{r=1}^{n} d_r - C_{h_m}\sum_{r=1}^{n}\frac{l_r}{2}\left[\frac{2d_r}{P} + N_r\left(1 - \frac{d_r}{P}\right) - 1\right] - f\Bigg[g_1\sum_{r=1}^{n} d_r + g_2\sum_{r=1}^{n}(J_{mr} + J_{rm}) \\ & \frac{d_r}{N_r l_r}\Bigg] - 2K_f\sum_{r=1}^{n}\frac{d_r}{N_r l_r} - K_v\sum_{r=1}^{n} d_r\left(J_{mr} + \beta J_{rm}\right) - \sum_{r=1}^{n} C_{h_r}\Bigg[\frac{l_r}{2} + q_r\sigma_r\sqrt{L_r} + (1 - \phi_r)\frac{\sigma_r\sqrt{L_r}}{2} \\ & \left(\sqrt{1 + q_r^2} - q_r\right)\Bigg] + T_c\left[M_c - \left(g_1\sum_{r=1}^{n} d_r + g_2 W\sum_{r=1}^{n}\frac{d_r}{N_r l_r}\right)\right] + \frac{T_c - P_c}{2} \\ & \left(\sqrt{\left(g_1\sum_{r=1}^{n} d_r + g_2 W\sum_{r=1}^{n}\frac{d_r}{N_r l_r} - M_c\right)^2} + g_1\sum_{r=1}^{n} d_r + g_2 W\sum_{r=1}^{n}\frac{d_r}{N_r l_r} - M_c\right). \end{aligned} \tag{4.1}$$

4.2. Retailer's model

It is considered that multiple retailers are with a single manufacturer in a two-echelon SCM. Retailers of the supply chain are untrustable. Retailers hide data from the manufacturer. In addition, retailers furnish some services to their customers; however, the manufacturer is unaware of this data. Consequently, information instability originates in the system. From the settlement of the VMI policy, the manufacturer bears the retailer's holding cost. The retailer r receives ordered quantity l_r with a cycle time $\frac{d_r}{l_r}$.

In the next section, retailers' associated revenue (selling price) and related costs (ordering cost, purchasing cost, shortage cost, service investment, and customer satisfaction cost) are determined below.

4.2.1. Selling price

As retailers are in various regions, the selling price of products for each retailer is different. Though the purchasing cost of each retailer is equal, the revenue they earn from their business varies. The unit selling price of the product is y_r. Therefore, the revenue of all retailers per cycle is $\sum_{r=1}^{n} y_r d_r$.

4.2.2. Ordering cost

The manufacturer orders products whenever the inventory reaches the reorder point. While retailer r orders l_r quantity of products from the manufacturer, retailer r bears some ordering cost A_r. Subsequently, the total cost of ordering each retailer per cycle is taken as $\sum_{r=1}^{n} \frac{A_r d_r}{l_r}$.

4.2.3. Purchasing cost

Each retailer buys products from a single manufacturer. Every retailer pays the same purchasing amount to the manufacturer. In addition, retailers purchasing cost is similar to the manufacturer's wholesale price. While the unit purchasing cost is C_p and the ordering product is l_r, the entire purchasing cost per cycle is $C_p \sum_{r=1}^{n} d_r$.

4.2.4. Shortage cost

As retailers have stochastic safety stock, stockout occurs when the market demand exceeds safety stock. If $X_r > R_r$, then the shortage quantity for stockout is $E[X_r - R_r]^+$. As safety stock is stochastic, shortage with partial backorder is considered where the backorder rate of retailer r is $\phi_r, 0 < \phi_r < 1$. The partial backordered quantity for holding is $(1 - \phi_r)E[X_r - R_r]^+$. C_s is unit stockout cost and $C^0_{s_r}$ is the marginal profit from partial backorder quantity.

$$\sum_{r=1}^{n} \frac{\left[C_{s_r} + C^0_{s_r}(1-\phi_r)\right] d_r}{l_r} E(X_r - R_r)^+ \leq \sum_{r=1}^{n} \frac{\left[C_{s_r} + C^0_{s_r}(1-\phi_r)\right] d_r \sigma_r \sqrt{L_r}}{2 l_r} \left(\sqrt{1+q_r^2} - q_r\right).$$

See Appendix A.

4.2.5. Holding cost

As per the VMI contract, the manufacturer incurs the holding cost of each retailer. Retailer r holds products for safety stock $q_r \sigma_r \sqrt{L_r}$ along with the average inventory $\frac{l_r}{2}$. Partial backordered quantity is $(1 - \phi_r)E[X_r - R_r]^+$. If C_{h_r} is the unit cost for retailer r, then the entire holding cost is

$$\begin{aligned} &\sum_{r=1}^{n} C_{h_r} \left[\frac{l_r}{2} + R_r - M_r L_r + (1-\phi_r) E(X_r - R_r)^+\right] \\ = \quad &\sum_{r=1}^{n} C_{h_r} \left[\frac{l_r}{2} + q_r \sigma_r \sqrt{L_r} + (1-\phi_r) \frac{\sigma_r \sqrt{L_r}}{2} \left(\sqrt{1+q_r^2} - q_r\right)\right]. \end{aligned}$$

See Appendix A.

4.2.6. Investment due to service and cost for customer satisfaction

Retailer r provides some service s_r to their customers. For this customer service, the retailer invests some money with investment parameter I_r. Following that, investment due to service is observed as $\sum_{r=1}^{n} \frac{I_r {s_r}^2}{2}$. Customer satisfaction level depends on that provided service. If customers are still unsatisfied, retailer r provides extra facilities for their satisfaction. The cost for customer satisfaction is $\sum_{r=1}^{n}(1-s_r)^2 l_{c_r}$, where l_{c_r} is cost regarding customer satisfaction. Therefore, the total cost based on the customer service is given as $\sum_{r=1}^{n}\left[\frac{I_r {s_r}^2}{2} + (1-s_r)^2 l_{c_r}\right]$.

4.2.7. Total profit of retailers

From Eq (4.2), the expected total profit of the retailer per cycle is given as

$$\begin{aligned} EP_r(l_r, s_r, q_r) &= \sum_{r=1}^{n}\Bigg[(y_r - C_p)d_r - \frac{A_r d_r}{l_r} - \frac{\left[C_{s_r} + C^0_{s_r}(1-\phi_r)\right] d_r \sigma_r \sqrt{L_r}}{2 l_r}\left(\sqrt{1+q_r^2} - q_r\right) - \frac{I_r {s_r}^2}{2} \\ &\quad -(1-s_r)^2 l_{c_r}\Bigg]. \end{aligned} \tag{4.2}$$

4.2.8. Expected total profit of the supply chain

Consequently, the expected total profit (EP_c) of SCM per cycle is

$$\begin{aligned} EP_c(l_r, s_r, q_r, N_r) &= (C_p - C_m)\sum_{r=1}^{n} d_r - C_{h_m}\sum_{r=1}^{n}\frac{l_r}{2}\left[\frac{2d_r}{P} + N_r\left(1 - \frac{d_r}{P}\right) - 1\right] \\ &\quad - f\left[g_1\sum_{r=1}^{n} d_r + g_2\sum_{r=1}^{n}(J_{mr} + J_{rm})\frac{d_r}{N_r l_r}\right] - 2K_f\sum_{r=1}^{n}\frac{d_r}{N_r l_r} - K_v\sum_{r=1}^{n} d_r\left(J_{mr} + \beta J_{rm}\right) \\ &\quad - \sum_{r=1}^{n} C_{h_r}\left[\frac{l_r}{2} + q_r\sigma_r\sqrt{L_r} + (1-\phi_r)\frac{\sigma_r\sqrt{L_r}}{2}\left(\sqrt{1+q_r^2} - q_r\right)\right] \\ &\quad + T_c\left[M_c - \left(g_1\sum_{r=1}^{n} d_r + g_2 W\sum_{r=1}^{n}\frac{d_r}{N_r l_r}\right)\right] + \frac{T_c - P_c}{2} \\ &\quad \left(\sqrt{\left(g_1\sum_{r=1}^{n} d_r + g_2 W\sum_{r=1}^{n}\frac{d_r}{N_r l_r} - M_c\right)^2} + g_1\sum_{r=1}^{n} d_r + g_2 W\sum_{r=1}^{n}\frac{d_r}{N_r l_r} - M_c\right) \\ &\quad + \sum_{r=1}^{n}\Bigg[(y_r - C_p)d_r - \frac{A_r d_r}{l_r} - \frac{\left[C_{s_r} + C^0_{s_r}(1-\phi_r)\right] d_r\sigma_r\sqrt{L_r}}{2 l_r}\left(\sqrt{1+q_r^2} - q_r\right) - \frac{I_r {s_r}^2}{2} \\ &\quad -(1-s_r)^2 l_{c_r}\Bigg]. \end{aligned} \tag{4.3}$$

As expected, the total profit $EP_c(l_r, s_r, q_r, N_r)$ of SCM given by Eq (4.3) is a non-linear function of l_r, s_r, l_r, and N_r. Thus, the numerical method is portrayed to analyze the solution.

Optimum solutions of the expected total profit of the supply chain in Eq (4.3) are found by classical optimization. Equation (4.3) is a mixed-integer linear problem. Unique solutions of continuous variables l_r, s_r, q_r, and N_r are obtained using necessary conditions of classical optimization. The values of continuous decision variables are as follows:

$$\begin{aligned}
\frac{\partial EP_c}{\partial l_r} &= 0, \text{ i.e., } l_r^* = \sqrt{\frac{\left(\frac{\xi_1}{N_r} + A_r + \xi_2(\sqrt{(1+{q_r}^2)} - q_r\right) d_r}{(\xi_3 + \xi_4 N_r)}} \\
\frac{\partial EP_c}{\partial s_r} &= 0, \text{ i.e., } s_r^* = \frac{2l_{c_r}}{I_r + 2l_{c_r}} \\
\frac{\partial EP_c}{\partial q_r} &= 0, \text{ i.e., } q_r^* = \sqrt{\frac{\left(l_r C_{h_r} + d_r C_{s_r}^0\right)(1-\phi_r) + C_{s_r}}{l_r}}.
\end{aligned}$$

See Appendix B for ξ_1, ξ_2, ξ_3, and ξ_4.

The following proposition finds the optimum value of the integer variables.

Proposition. For given l_r^*, s_r^*, and q_r^*, the optimum value of Eq (4.3) is found for optimum N_r^* if

$$EP_c(l_r^*, s_r^*, q_r^*, N_r - 1) \leq EP_c(l_r^*, s_r^*, q_r^*, N_r^*) \geq EP_c(l_r^*, s_r^*, q_r^*, N_r + 1).$$

Table 3 depicts the formation of determining to mean $M_r L_r$, variance $\sigma_r\sqrt{L_r}$, and shortage quantity $E[X_r - R_r]^+$ for uniform and normal distributions.

Table 3. Procedure for obtaining $M_r L_r$, $\sigma_r\sqrt{L_r}$, and $E[X_r - R_r]^+$ of various distributions.

Distribution	Mean $(M_r L_r)$	Variance $(\sigma_r\sqrt{L_r})$	Expected shortage quantity $E[X_r - R_r]^+$
Uniform (a_r,b_r)	$\frac{(a_r+b_r)L_r}{2}$	$\sqrt{\frac{(b_r-a_r)^2}{12}L_r}$	$\frac{1}{2}\sqrt{\frac{(b_r-a_r)^2}{12}L_r}\left(\sqrt{1+q_r^2} - q_r\right)$
Normal (m_r,σ_r)	$m_r L_r$	$\sigma_r\sqrt{L_r}$	$\frac{\sigma_r\sqrt{L_r}}{2}\left(\sqrt{1+q_r^2} - q_r\right)$

5. Numerical discussion

A numerical example is provided to validate this model. All the input parameters are extracted from Guchhait and Sarkar [2]. There are four retailers in the SCM, that is, $r = 4$. Other parameters are $(C_m, C_p) = \$(80, 160)$/unit, $C_{h_m} = \$0.55$/unit, $(I_1, I_2, I_3, I_4) = \$(2, 3, 5, 4)$, $K_f = \$0.5$/shipment, $K_v = \$0.13$/km, $M_c = 6000$ gallons, $(y_1, y_2, y_3, y_4) = \$(200, 205, 208, 206)$/unit, $(C_{s_1}, C_{s_2}, C_{s_3}, C_{s_4}) = \$(8, 7, 9, 7)$/unit, $(C_{h_1}, C_{h_2}, C_{h_3}, C_{h_4}) = \$(44, 44, 41.6, 40.8)$/unit/year, $(T_c, P_c) = \$(30, 60)$/unit emission, $(J_{m1}, J_{m2}, J_{m3}, J_{m4}) = (45, 46, 45, 45)$km, $(L_1, L_2, L_3, L_4) = (3, 5, 5, 5)$week, $(d_1, d_2, d_3, d_4) = (300, 300, 300, 300)$unit/year, $(J_{1m}, J_{2m}, J_{3m}, J_{4m}) = (43, 42, 45, 43)$km, $(m_1, m_2, m_3, m_4) = (50, 50, 50, 50)$, $\beta = 0.5$, $f = \$3.1$/unit, $(l_{c_1}, l_{c_2}, l_{c_3}, l_{c_4}) = (0.3, 0.8, 0.4, 0.5)$, $(A_1, A_2, A_3, A_4) = \$(10, 50, 50, 50)$/order, $(a_1, a_2, a_3, a_4) = (0.9, 0.9, 0.9, 0.9)$, and $(b_1, b_2, b_3, b_4) = (4, 4, 4, 4)$, $(\phi_1, \phi_2, \phi_3, \phi_4) = (0.9, 0.9, 0.9, 0.9)$, $(C_{s_1}^0, C_{s_2}^0, C_{s_3}^0, C_{s_4}^0) = \$(60, 70, 60, 65)$/unit, $P = 1000$ unit/year.

Table 4 shows that the optimum results of this study are adequately mentioned. Figures 2–5 shows that the total profit of SCM $EP_c(l_r, s_r, q_r, N_r)$ is at a global maximum.

Table 4. Optimum findings of this research.

Mean value of demand follows	rth retailer's lot size (l_r)	Service given by the retailer r to customers (s_r)	Safety stock of retailer (q_r)	Number of production lots (N_r)	Total profit of SCM $EP_c(l_r, s_r, q_r, N_r)$
Uniform distribution	$(542, 667, 686, 563)$	$(0.41, 0.34, 0.15, 0.24)$	$(0.30, 0.87, 0.83, 0.87)$	$(3, 2, 2, 3)$	$\$253, 616.50$
Normal distribution	$(542, 667, 686, 563)$	$(0.41, 0.34, 0.15, 0.24)$	$(0.30, 0.87, 0.83, 0.87)$	$(3, 2, 2, 3)$	$\$252, 483.18$

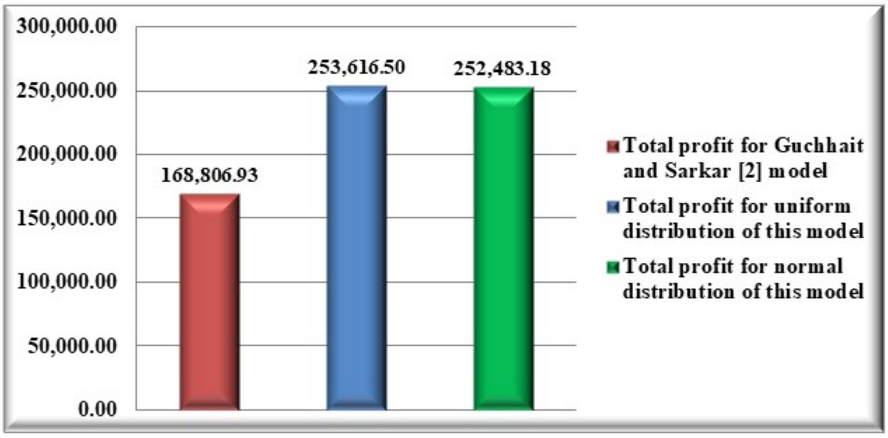

Figure 1. Comparison of total profit for this research and Guchhait and Sarkar [2] model.

In Guchhait and Sarkar's [2] model, the SCM's total profit was $\$168, 806.93$. Figure 1 shows that while the mean value of demand considers a uniform distribution, the proposed research achieved maximum profit compared to the Guchhait and Sarkar [2] model.

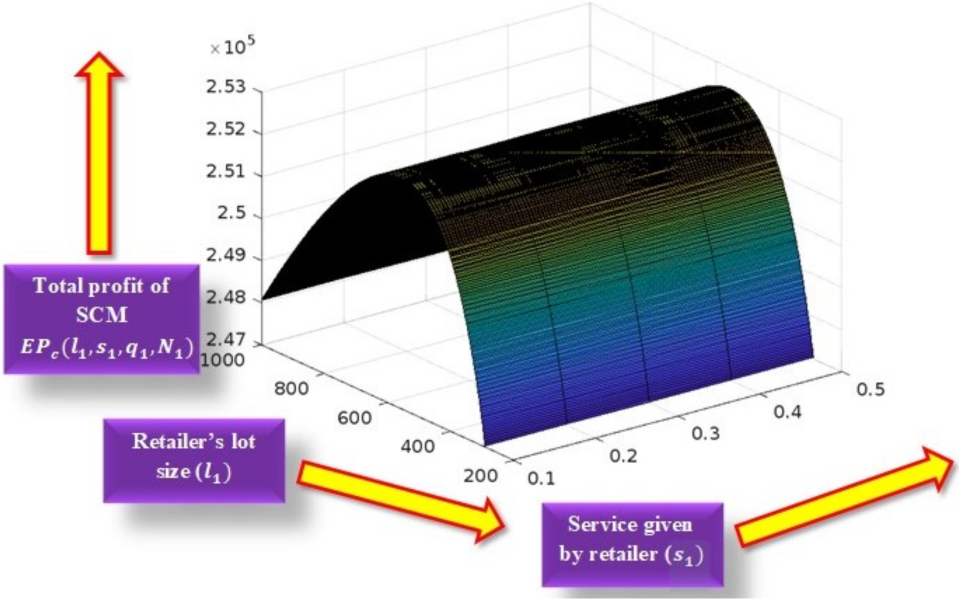

Figure 2. Total profit of supply chain $EP_c(l_1, s_1, q_1, N_1)$ versus retailer's lot size (l_1) and service given by retailer to customer (s_1).

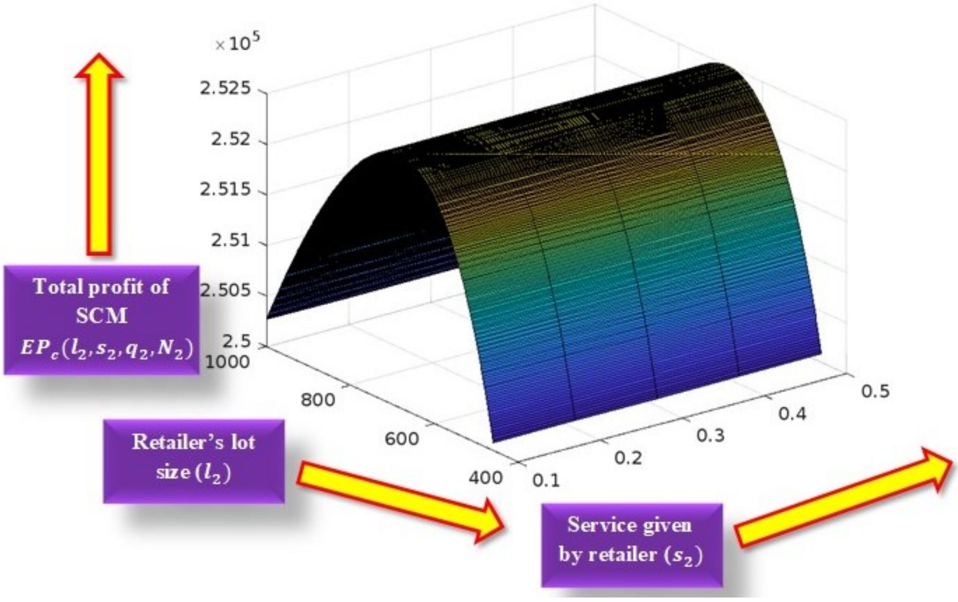

Figure 3. Total profit of supply chain $EP_c(l_2, s_2, q_2, N_2)$ versus retailer's lot size (l_2) and service given by retailer to customer (s_2).

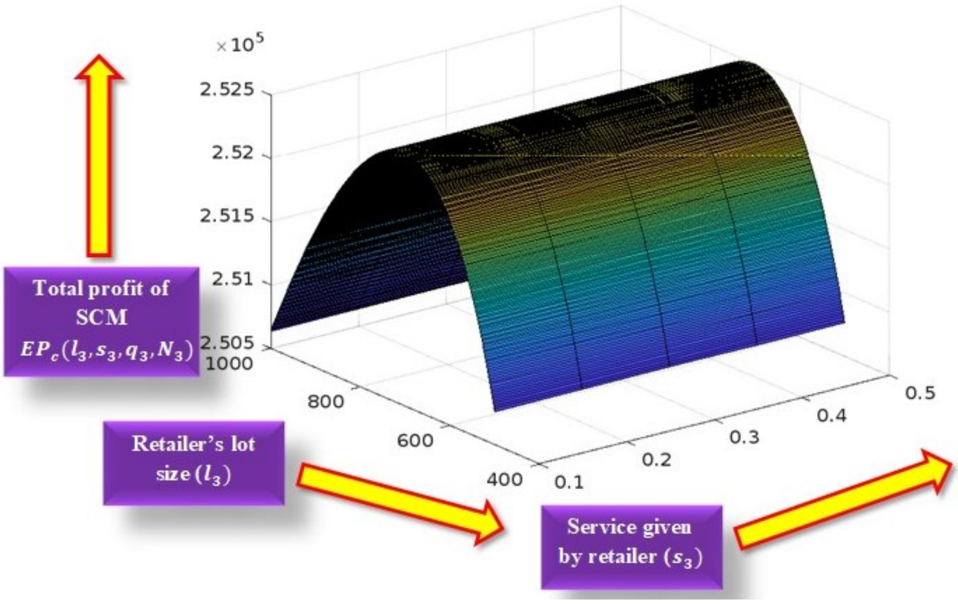

Figure 4. Total profit of supply chain $EP_c(l_3, s_3, q_3, N_3)$ versus retailer's lot size (l_3) and service given by retailer to customer (s_3).

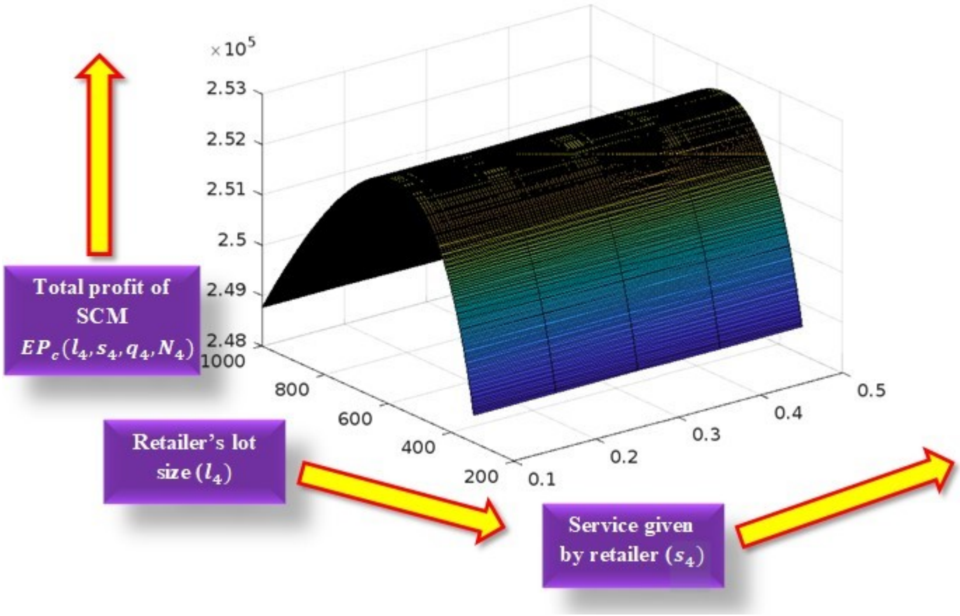

Figure 5. Total profit of supply chain $EP_c(l_4, s_4, q_4, N_4)$ versus retailer's lot size (l_4) and service given by retailer to customer (s_4).

5.1. Usefulness of numerical results in practice with the environmental perspective

Total carbon emissions from the entire SCM (T_e) are 2378.64 and 2378.63 gallons, respectively, whereas the mean value of demand follows uniform and normal distribution functions, respectively. From Table 5, for a uniform distribution, the production sector emits 90.81%, and the transportation sector emits 9.12% of the total emitted carbon. For the normal distribution function, the production sector emits 90.81%, and the transportation sector emits 9.12% of the total emitted carbon. Numerically, the carbon cap limit of the manufacturer (M_c) was 6000 gallons. Therefore, the total carbon emissions T_e are much lower than M_c for both distribution functions. Hence, an extra unused carbon cap is used for trading, contributing to manufacturer revenue.

Table 5. Interpretation of emitted carbon from production and transportation sector under two distribution functions.

Mean value of demand follows	Carbon emissions by production sector	Carbon emissions by transportation sector
Uniform distribution	2160.00 gallons	218.64 gallons
Normal distribution	2160.00 gallons	218.63 gallons

6. Sensitivity analysis

The effect of various parameters from −50% to +50% on the total profit of SCM is discussed in this section. Table 6 lists the numerical results of the sensitivity analysis. Pictorial illustrations of the sensitivity are presented in Figures 6–9.

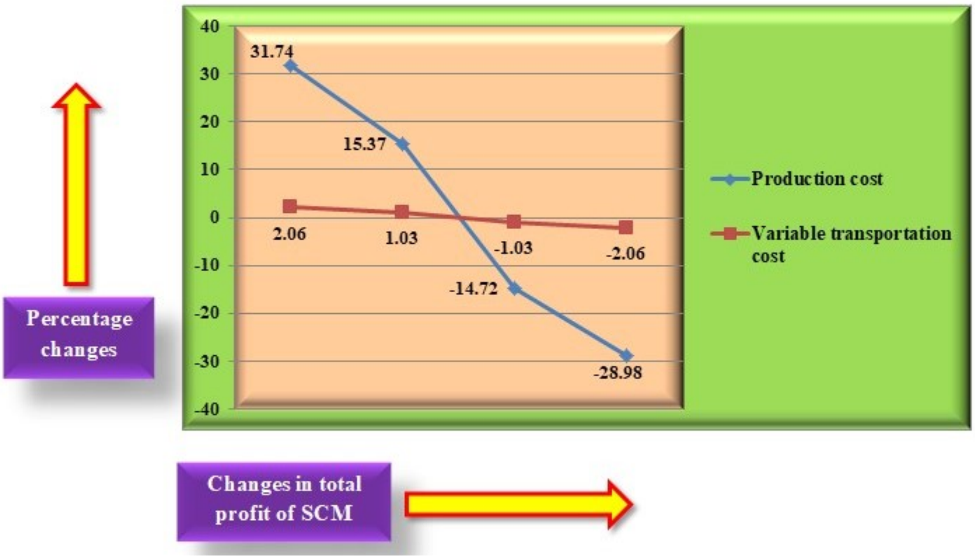

Figure 6. Impact of production and variable transportation costs on total profit of SCM.

(i) As soon as the manufacturer's production cost increases towards +50%, the total profit of SCM gradually decreases. The negative percentage change is more than the positive percentage change in production cost.

(ii) The negative and positive percentage changes of the carbon tax from −50% to +50% remain the same. Therefore, an increased ordering cost value implies a decreased value of SCM's total profit.

(iii) The scaling parameter regarding retailers' service investment is less sensitive than other parameters. Maximizing the retailers' service investment specifies that the total profit of SCM gradually diminishes.

(iv) The most sensitive parameter among all parameters is the retailers' selling price of products. This specific parameter is the main factor in raising the total profit of SCM. While the selling price is maximized, the SCM's total profit is increased. In this case, both percentage changes are similar.

(v) The total profit of SCM diminishes while the manufacturer's variable transportation cost rises. This parameter is a little bit sensitive. For this parameter, the positive percentage change is similar to the negative.

(vi) If one increases the cost related to customer satisfaction up to 50%, then the total profit of SCM decreases. Therefore, for this parameter, the negative percentage change equals the positive percentage change.

(vii) The percentage changes in the unit holding cost of retailers show that if one maximizes the unit holding cost, then the total profit of SCM decreases and vice-versa. Both the positive and negative percentage changes of this parameter are the same.

(viii) Within the range from −50% to +50%, changes in the percentage of ordering cost of retailers highlight that the total profit of SCM is changed. If the ordering cost increases within that range, then the total profit of SCM is minimized.

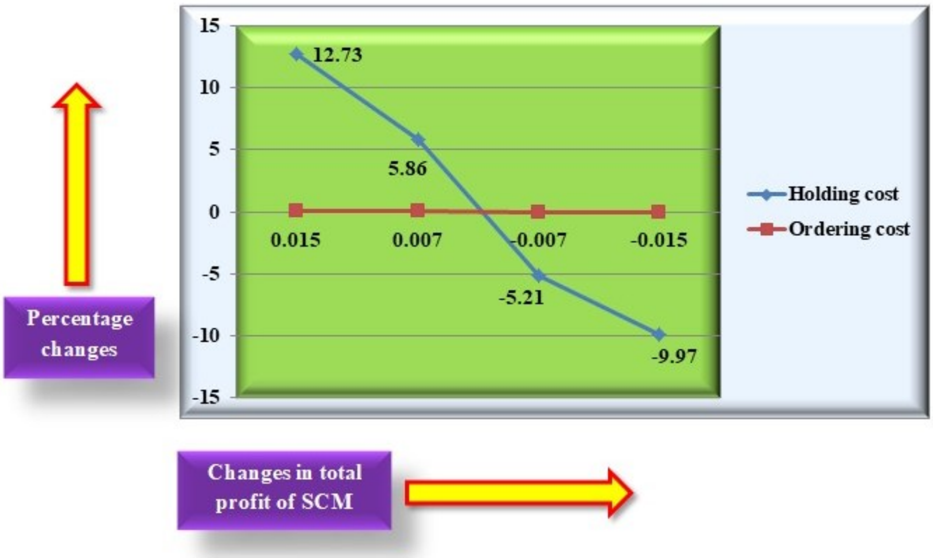

Figure 7. Impact of holding and ordering costs on total profit of SCM.

Table 6. Table for sensitivity analysis.

Parameters	Changes (in%)	Total profit of SCM
C_m	−50	+31.74
	−25	+15.37
	+25	−14.72
	+50	−28.98
f	−50	+0.0001
	−25	+0.00008
	+25	−0.00008
	+50	−0.0001
I_r	−50	+0.0001
	−25	+0.00004
	+25	−0.00003
	+50	−0.00005
y_r	−50	−48.66
	−25	−24.33
	+25	+24.33
	+50	+48.66
K_v	−50	+2.06
	−25	+1.03
	+25	−1.03
	+50	−2.06
l_{c_r}	−50	+0.0002
	−25	+0.0001
	+25	−0.0001
	+50	−0.0002
C_h	−50	+12.73
	−25	+5.86
	+25	−5.21
	+50	−9.97
A_r	−50	+0.015
	−25	+0.007
	+25	−0.007
	+50	−0.015

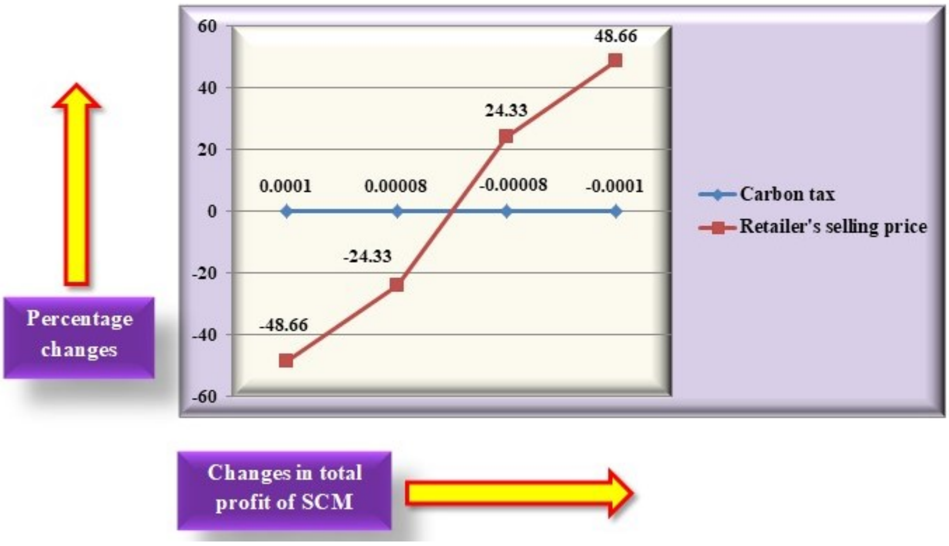

Figure 8. Impact of carbon tax and selling price on total profit of SCM.

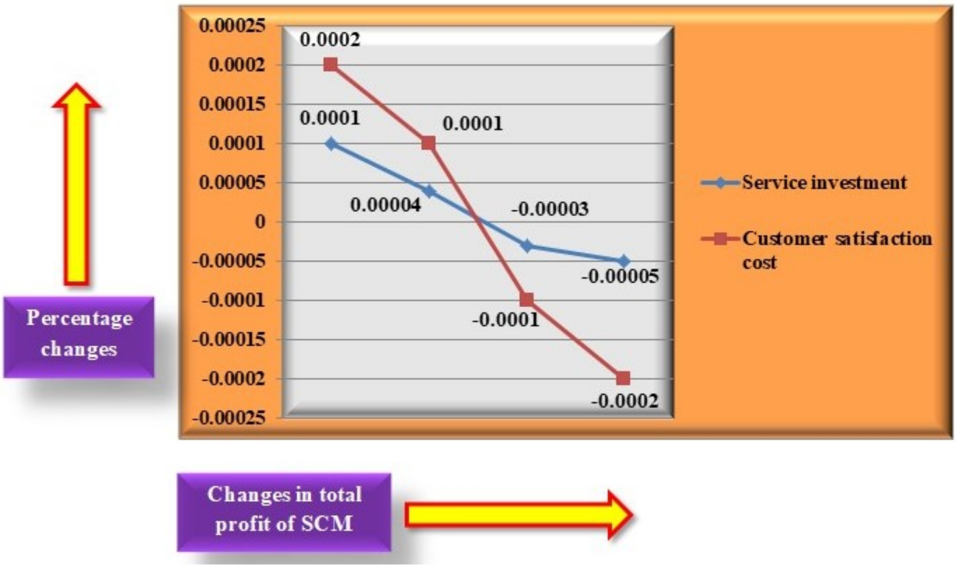

Figure 9. Impact of service investment and customer satisfaction cost on total profit of SCM.

7. Managerial insights

In this model, the CAPT policy is adapted by considering the environment. The amount of carbon emitted into the air is higher because of heavy transportation during forward and backward logistics. The CAPT policy can control this carbon emission level. In addition, the manufacturer pays an amount for each ton of carbon emitted (carbon tax). The CAPT policy sets the authorized maximum emission limit (i.e., the cap on industries). Using this CAPT policy, the actual carbon emissions can be detected.

In addition to the CAPT policy, manufacturers collect utilized products from retailers for recycling. Because of the environmental aspects, the CAPT policy and recycling would be the best options to protect the environment.

Here, the manufacturer implemented the VMI policy. In agreement with this policy, the manufacturer is responsible for meeting the requirements of each retailer, such as maintaining the quantity and quality of the required products from time to time. Under this VMI policy, the manufacturer shares the holding cost with each retailer. Under this contract, retailers do not need to bear any costs for holding finished goods. This would help retailers diminish their costs, as they do not manage the holding costs. Consequently, the industry will benefit if it applies a VMI policy during the business.

8. Conclusions, limitations, and future works

This research discussed untrustable issues of retailers and their viewpoints within SCM. Because multiple retailers involved in SCM are untrustable, information asymmetry was observed in the system. This study included VMI policy to eliminate the unreliability of the entire system. We analyzed the optimal retailer's lot size, customer service by the retailer, and retailer's reorder point under random demand. The retailer's lead time demand follows two random distribution functions, uniform and normal. Numerical findings show that the proposed study was economically 50.24% more beneficial for SCM than previous studies in the literature. The numerical results proved that this study maximized the supply chain's total profit compared to previous research. This study was beneficial from an environmental perspective. The manufacturer traded an extra carbon cap and earns profits under the CAPT policy. Thus, the CAPT policy was both economically and environmentally beneficial.

This research considered a single type of product. This approach can be extended by considering multiple products [24] and deteriorating products [25,26]. This study is limited to a two-echelon supply chain model. This model can be extended to a multi-echelon supply chain model. This study used a single period. Therefore, the multi-period multi-echelon model is a better extension of this study. In addition, it was considered a player of SCM with equal power. This is another major limitation.

For future works, this research recommended solving this model by applying the Stackelberg technique and expanding by utilizing robust optimization to deal with the unequal power of SCM players or with other methodologies [27]. Advertisement is one of the major tactics used to attract consumer attention to any enterprise. In the future, this research can be broadened by determining promotional effort [28], and price-dependent stochastic demands [29]. In addition, the application of a cross-docking scheme [30], water supply and wastewater collection system (WSWCS) under uncertainty [31], price discount policy [32], Internet of Things [33], and green products [34–36] will help achieve development goals.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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Appendix

Appendix A

$$
\begin{aligned}
E|X_r - R_r| &\leq \sqrt{E(X_r - R_r)^2} = \sqrt{E(X_r^2) + R_r^2 - 2R_rE(X_r)} \\
&= \sqrt{\left(\sigma_r\sqrt{L_r}\right)^2 + (M_rL_r)^2 + \left(M_rL_r + q_r\sigma_r\sqrt{L_r}\right)^2 - 2\left(M_rL_r + q_r\sigma_r\sqrt{L_r}\right)M_rL_r} \\
&\quad \text{[using Cauchy–Schwarz inequality]} \\
&= \sqrt{\sigma_r^2L_r + \left(M_rL_r - \left(M_rL_r + q_r\sigma_r\sqrt{L_r}\right)\right)^2} \\
&= \sqrt{\sigma_r^2L_r + q_r^2\sigma_r^2L_r} = \sigma_r\sqrt{L_r}\sqrt{1 + q_r^2} \\
E(X_r - R_r)^+ &= \frac{E|X_r - R_r| + E(X_r - R_r)}{2} \\
&\leq \frac{1}{2}\left[\sigma_r\sqrt{L_r}\sqrt{1 + q_r^2} + M_rL_r - \left(M_rL_r + q_r\sigma_r\sqrt{L_r}\right)\right] \\
&= \frac{1}{2}\left[\sigma_r\sqrt{L_r}\sqrt{1 + q_r^2} - q_r\sigma_r\sqrt{L_r}\right] \\
&= \frac{\sigma_r\sqrt{L_r}}{2}\left[\sqrt{1 + q_r^2} - q_r\right]
\end{aligned}
$$

Appendix B

$$
\begin{aligned}
\xi_1 &= f(J_{mr} + J_{rm}) + 2K_f + P_cg_2W \\
\xi_2 &= \frac{(C_{s_r} + C_{s_r}^0(1 - \phi_r))\sigma_r\sqrt{L_r}}{2} \\
\xi_3 &= \frac{C_{h_m}}{2}\left(\frac{2d_r}{P} - 1\right) + \frac{C_{h_r}}{2} \\
\xi_4 &= \frac{C_{h_m}}{2}\left(1 - \frac{d_r}{P}\right)
\end{aligned}
$$

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