Research article

Investigation of a nutrient-plankton model with stochastic fluctuation and impulsive control

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Abstract: In this paper, we investigate a stochastic nutrient-plankton model with impulsive control of the nutrient concentration and zooplankton population. Analytically, we find that the population size is nonnegative for a sufficiently long time. We derive some sufficient conditions for the existence of stable periodic oscillations, which indicate that the plankton populations will behave periodically. The numerical results show that the plankton system experiences a transition from extinction to the coexistence of species due to the emergence of impulsive control. Additionally, we observe that the nutrient pulse has a stronger relationship with phytoplankton growth than the zooplankton pulse. Although the frequency of impulsive control and appropriate environmental fluctuations can promote the coexistence of plankton populations, an excessive intensity of noise can result in the collapse of the entire ecosystem. Our findings may provide some insights into the relationships among nutrients, phytoplankton and zooplankton in a stochastic environment.

Keywords: nutrient-plankton model; pulse perturbations; stochastic fluctuation; positive T-periodic solutions

1. Introduction

Over the past several decades, researchers have shown an increased interest in plankton dynamics due to the numerous negative effects caused by algal blooms [1–3], which can threaten entire aquatic systems and result in economic losses, altering long-term species coexistence and influencing ecological stability. On the other hand, under conditions of nutrient shortage, the plankton community may suffer from nutrient deficiency and the aquatic system may evolve into species extinction. As important components of aquatic food webs, nutrients and the zooplankton population are widely recognized as the most important factors for phytoplankton growth [4]. Previous studies have established that a high nutrient load can be a major trigger for the increased frequency, intensity and duration of the plankton blooms [5, 6]. Moreover, the experiment by Vanni [7] has demonstrated that even small changes in the zooplankton size can enormously affect the phytoplankton community. Although numerous experimental and theoretical studies have been devoted to the study of phytoplankton growth response to the changes in nutrients and the zooplankton population [8, 9], the relationships between phytoplankton growth, nutrient concentration and the grazer zooplankton are still not well understood due to the fact that many events are pulsed and short-lived, such as nutrient delivery [10].

The aquatic ecosystems are inevitably subjected to instantaneous perturbations, leading to abrupt changes in nutrient concentration and zooplankton biomass. For example, heavy rainfall triggers increased nutrient input through river discharge, resulting in algal blooms in coastal areas [11]. These phenomena are commonly described as pulsed instantaneous behavior, also known as impulsive control, and they are characterized by the introduction or removal of some members from a population at a certain moment of time. Such pulsed instantaneous behavior cannot be considered continuously [12]. Therefore, mathematical models of ecological population dynamics have been developed in the study of plankton dynamics as they not only capture the ubiquitous stoichiometric constraints for the growth and interactions of species [13], but they also provide quantitative insights into population growth dynamics [14]. Actually, the theory of dynamical systems is generally recognized as two basic types: continuous and discontinuous [15]. In the natural world, many systems are characterized by the fact that they are subjected to pulse perturbations, causing discontinuous behaviors. Thus, impulsive differential equations have drawn increasing attention due to the advantage that the theory of impulsive differential equations can accurately capture the instantaneous state of a system. Based on the previous study of impulsive differential equations [16–18], in recent years, impulsive differential equations have been widely used to describe such instantaneous perturbations in the study of plankton dynamics [19–21]. Moreover, modelling approaches may have important implications for how plankton population growth responds to pulse perturbations of the nutrients and zooplankton population.

Impulsive control refers to the application of sudden and short-duration interventions or perturbations to a system. Impulsive control studies can help researchers to gain insight into how pulse perturbations influence the plankton dynamics. Additionally, studying impulsive control in plankton dynamics allows researchers to explore potential control strategies to achieve desired outcomes in terms of regulating plankton populations or managing ecological processes. In recent years, plankton dynamics induced by nutrient and zooplankton pulses have been reported in many aquatic food chain models [9, 22, 23]. The experimental study conducted by Spatharis et al. [24] emphasized that a sudden pulse of high nutrient concentration may promote the dominance of a single species and yield a negative effect on the aquatic ecological diversity. However, Cottingham et al. [25] demonstrated that the

phytoplankton show low sensitivity to nutrient pulses in systems with a high density of large zooplankton [26]. Furthermore, Zhao et al. [27] reported that both phytoplankton and zooplankton can coexist if the amount of the released zooplankton is within certain ranges. Their results illustrated that both a nutrient pulse and zooplankton pulse may alter the dynamical behavior of an ecosystem, and that some proper impulsive control strategies can maintain the balance of the ecosystem [12, 22, 28]. Despite all of our knowledge on the relationships between phytoplankton, nutrient and zooplankton, there is limited research on whether phytoplankton growth is more sensitive to a nutrient pulse or zooplankton pulse.

To the best of our knowledge, lots of studies on plankton dynamics are conducted in a stable environment [29–31]. However, numerous studies support that almost all natural aquatic ecosystems are often subject to environmental fluctuations, such as variation in temperature, light and water depth. Environmental fluctuations have the potential to disturb the steady state through direct or indirect effects on density or parameter values [32]. Specifically, Mao et al. [33] developed a simple stochastic population growth model, and reported that the stochastic noise has the capability to suppress the explosion of population. The findings of Deng et al. [34] demonstrated that noise can suppress/enhance the exponential growth of populations. These results revealed that environmental fluctuations can significantly affect the dynamical behavior of biological populations. Furthermore, algal blooms may exhibit stochastic scales and timing due to the fact that the growth responses of plankton are inevitably affected by random fluctuations [35, 36]. In recent years, several attempts have been made to investigate how environmental noise affects ecosystems [37, 38]. For instance, Yu et al. [39] demonstrated that the Markov chain is beneficial for the survival of plankton. Biologically, it is more realistic to consider environmental noise in the species dynamical model.

The interaction between nutrient pulses and phytoplankton growth under stochastic disturbance has been investigated in [40]. However, far too little attention has been paid to the synergistic effects of a nutrient pulse and zooplankton pulse on the plankton dynamics under environmental fluctuation. Understanding how a phytoplankton population responds to nutrient pulses and zooplankton pulses under stochastic fluctuations may provide insights into the complex interactions among phytoplankton, nutrients and zooplankton. In addition, nutrient pulse control and phytoplankton pulse control are common measures to control phytoplankton blooms. Thus, investigating the interacting dynamics of nutrient pulses, zooplankton pulses and phytoplankton population may allow us to develop better strategies to mitigate eutrophication and its associated bloom events. In order to provide a better understanding of how phytoplankton respond to a nutrient pulse and zooplankton pulse in a random environment, in the present paper, an aquatic food chain model comprised of nutrient, phytoplankton and zooplankton species is investigated under the conditions of a random environment. In the model, nutrient pulses and zooplankton pulses are adopted to respectively describe the instantaneous changes in nutrient concentration and zooplankton biomass. Here, we aim to explore whether phytoplankton growth is more affected by pulsed nutrients or zooplankton under environmental fluctuations.

The rest of this article is organized in the following way. A mathematical model for the nutrient-plankton model of the aquatic habitat with impulsive control and stochastic disturbance is introduced in next section. In Section 3, the stochastic-impulsive coupled model is analyzed analytically. Some numerical results are presented in Section 4 to deeply explore the effects of impulsive control and stochasticity on the interplay of nutrients and planktons in the aquatic habitat. Section 5 covers a discussion of this study and biologically implications of the obtained results. At the end, we close the

paper with the conclusion in Section 6.

2. The mathematical model

Numerous mathematical and ecological studies have been conducted to explore the growth dynamics of plankton populations in the aquatic ecosystems [41–44]. By considering the nutrients, phytoplankton and zooplankton as dynamical variables, we develop here a mathematical model to comprehend the responses of plankton populations to the pulsed nutrients and zooplankton in a fluctuating environment. In a real aquatic system, the predation of phytoplankton by zooplankton is very complex, e.g., copepods [45]. Thus, we adopt the Holling type II functional response to model the hunting behavior of zooplankton for phytoplankton. Moreover, some species of phytoplankton can produce toxins, which contribute to the death of the zooplankton population. Both the theoretical and experimental studies have proven that the toxin-producing phytoplankton can greatly influence the occurrence as well as termination of the plankton blooms [46]. Indeed, the liberation of toxins by phytoplankton can reduce the growth of zooplankton species and substantially contribute to their mortality. Actually, toxin-producing phytoplankton are not easily accessible by the zooplankton. Hence, the Holling type II functional form is a more common and intuitively obvious choice to describe the grazing phenomenon of phytoplankton by zooplankton [46]. Moreover, we consider the intraspecific competition among the phytoplankton communities and the zooplankton communities due to the limitation of the resources in the aquatic biome. At any time $t > 0$, let $N(t)$, $P(t)$ and $Z(t)$ respectively represent the concentration of nutrients, phytoplankton population and zooplankton population in a considered aquatic system. Based on the following ecological assumptions, we will formulate our mathematical model for the nutrient-plankton system.

(1) The nutrient concentration in the aquatic system is determined by its washout and uptake by phytoplankton population at the rates b and n, respectively.

(2) The growth of phytoplankton depends on the nutrient availability and the predation by zooplankton.

(3) We represent the biomass conversion rate and the death rate of the phytoplankton population by β and m, respectively. The zooplankton predates the phytoplankton at a rate followed by the Holling type II functional form [47], which is given by $\dfrac{cPZ}{h + P}$, where c denotes the phytoplankton capture rate for zooplankton and h is the half-saturation constant for such an uptake.

(4) We denote the biomass conversion rate and the natural mortality of zooplankton by d and k, respectively. The effect of toxin-producing phytoplankton on the zooplankton population is modeled by the term $\dfrac{\rho PZ}{h + P}$, where ρ is the rate by which the phytoplankton population releases the toxic chemicals.

(5) Evidently, intraspecific competition exists in a real natural aquatic ecosystem due to the limitation of resources [48]. Here, we also model the intraspecific competition among the phytoplankton communities and the zooplankton communities and denote the strengths of competition by r and v, respectively.

(6) The natural ecosystems are affected by environmental fluctuations [49], so we use white noise to represent their impact on the nutrient-plankton model.

By considering the aforementioned assumptions, we come up with the following mathematical model that involves stochastic fluctuation:

$$\begin{cases} dN(t) = [-b(t)N(t) - n(t)P(t)N(t)]\,dt + \sigma_1(t)N(t)dB_1(t), \\ dP(t) = \left[\beta(t)P(t)N(t) - \frac{c(t)P(t)Z(t)}{h(t)+P(t)} - m(t)P(t) - r(t)P(t)^2\right]dt + \sigma_2(t)P(t)dB_2(t), \\ dZ(t) = \left[\frac{d(t)P(t)Z(t)}{h(t)+P(t)} - k(t)Z(t) - \frac{\rho(t)P(t)Z(t)}{h(t)+P(t)} - v(t)Z(t)^2\right]dt + \sigma_3(t)Z(t)dB_3(t). \end{cases} \quad (2.1)$$

In model (2.1), $B_i(t)$ $(i = 1, 2, 3)$ denotes the independent standard Brownian motions and σ_i^2 denotes the intensities of the white noises. We assume that all of the parameters involved in model (2.1) are positive, bounded and continuous θ-periodic functions in $[0, +\infty)$. In Table 1, we provide the biological meanings of variables and parameters describing model (2.1) in a constant environment.

The aquatic ecosystems are inevitably affected by instantaneous perturbations, which leads to abrupt changes in the nutrient concentration and zooplankton biomass, so we adopt nutrient and zooplankton pulses in model (2.1) and obtain the following implusive-stochastic differential equations:

$$\begin{cases} \left.\begin{array}{l} dN(t) = [-b(t)N(t) - n(t)P(t)N(t)]\,dt + \sigma_1(t)N(t)dB_1(t), \\ dP(t) = \left[\beta(t)P(t)N(t) - \frac{c(t)P(t)Z(t)}{h(t)+P(t)} - m(t)P(t) - r(t)P(t)^2\right]dt + \sigma_2(t)P(t)dB_2(t), \\ dZ(t) = \left[\frac{d(t)P(t)Z(t)}{h(t)+P(t)} - k(t)Z(t) - \frac{\rho(t)P(t)Z(t)}{h(t)+P(t)} - v(t)Z(t)^2\right]dt + \sigma_3(t)Z(t)dB_3(t), \end{array}\right\} \quad t \neq t_k, k \in \mathbb{N}, \\ \left.\begin{array}{l} N(t_k^+) = (1 + \alpha_{1k})N(t_k), \\ P(t_k^+) = P(t_k), \\ Z(t_k^+) = (1 + \alpha_{2k})Z(t_k), \end{array}\right\} \qquad\qquad\qquad\qquad t = t_k, k \in \mathbb{N}. \end{cases}$$

$$(2.2)$$

Note that model (2.2) shares common biological meanings with model (2.1). Further, there exists a sequence of real numbers with $0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{t\to\infty} t_k = +\infty$. For biological reasons, we impose the following restriction on α_{ik} $(i = 1, 2)$:

$$1 + \alpha_{ik} > 0, \; i = 1, 2, \; k = 1, 2, \cdots .$$

For $\alpha_{ik} > 0$ $(i = 1, 2)$, the impulsive effects represent the process of species introductions whereas $\alpha_{ik} < 0$ $(i = 1, 2)$ denotes harvesting. Here, we only consider $\alpha_{ik} > 0$ $(i = 1, 2)$. Following the approach of [19, 50, 51], we assume that there exists a positive integer p such that $t_{k+p} = t_k + T$ and $\alpha_{i(k+p)} = \alpha_{ik}, i = 1, 2, \; k \in Z^+$. We also consider that $[0, T) \cap \{t_k, k \in Z^+\} = \{t_1, t_2, \cdots, t_p\}$.

3. The main results

We assume that $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P})$ is a complete probability space with a filtration $\mathcal{F}_{t(t\geq 0)}$ satisfying the usual normal conditions i.e., it is right continuous and $\mathcal{F}_0$ contains all of the $\mathbf{P}$-null sets.

3.1. Preliminaries

Assume that a product equals unity if the number of factors is zero. For a bounded and continuous function $f : [0, +\infty) \to \mathbb{R}^+$, we define

$$f^u = \sup_{t\geq 0} f(t), \; f^l = \inf_{t\geq 0} f(t).$$

Moreover, if the function f is integrable on $[0, +\infty) \to \mathbb{R}^+$, then

$$\langle f \rangle = \frac{1}{t} \int_0^t f(s)ds, \ \langle f \rangle_T = \frac{1}{T} \int_0^T f(s)ds.$$

Now, we present the following definitions and lemmas in order to facilitate our further discussions. Consider the n-dimensional stochastic differential equation

$$dx(t) = f(x(t), t)dt + g(x(t), t)dB(t), \ t \geq t_0 \tag{3.1}$$

with the initial value $x(t_0) = x_0 \in \mathbb{R}^n$, where $B(t)$ is an n-dimensional standard Brownian motion. The differential operator $\mathcal{L}$ associated with Eq (3.1) is defined as

$$\mathcal{L} = \frac{\partial}{\partial t} + \sum_{k=1}^n f_k(x, t) \frac{\partial}{\partial x_k} + \frac{1}{2} \sum_{k,j=1}^n \left[g^T(x, t)g(x, t) \right]_{kj} \frac{\partial^2}{\partial x_k \partial x_j}. \tag{3.2}$$

To analyze model (2.2), we adopt the following definitions.

Definition 3.1. *[52]. Consider the following stochastic differential equation with impulse:*

$$\begin{cases} dX(t) = F(t, X(t))dt + G(t, X(t))dB(t), & t \neq t_k, \quad t > 0, \\ X(t_k^+) - X(t_k) = \beta_k X(t_k), & t = t_k, k = 0, 1, 2, \ldots, \end{cases} \tag{3.3}$$

where $X(0) \in \mathbb{R}^n$. A stochastic process $X(t) = (X_1(t), X_2(t), \cdots, X_n(t))^T, t \in [0, \infty)$ is said to be a solution of model (3.3) if the following conditions hold:

(i) $X(t)$ is $\mathcal{F}_t$ adapted and continuous on the intervals $(0, t_1)$ and $(t_k, t_{k+1}) \in R_+, k \in N$; $F(t, X(t)) \in \mathcal{L}^1(R_+; R^n)$ and $G(t, X(t)) \in \mathcal{L}^2(R_+; R^n)$, where $\mathcal{L}^k(R_+; R^n)$ is the set of all R^n valued measurable $\mathcal{F}_t$-adapted processes $f(t)$ satisfying $\int_0^T |f(t)|^k dt < \infty$ a.s. for every T.

(ii) For any $t_k, k \in \mathbb{N}, X(t_k^+) = \lim\limits_{t \to t_k^+} X(t)$ exists and $X(t_k^-) = \lim\limits_{t \to t_k^-} X(t)$ with a probability of one.

(iii) For almost every $t \in (0, t_1)$, $X(t)$ satisfies the following integral equation:

$$X(t) = X(0) + \int_0^t F(s, X(s))ds + \int_0^t G(s, X(s))dB(s).$$

Further, for all $t = [t_k, t_{k+1}], k \in \mathbb{N}, X(t)$ satisfies the following integral equation:

$$X(t) = X(t_k^+) + \int_{t_k}^t F(s, X(s))ds + \int_{t_k}^t G(s, X(s))dB(s).$$

Lemma 3.1. *[53]. For any $u > 0$, the following inequality holds:*

$$u \leq 2(u + 1 - \log u) - (4 - 2\log 2).$$

Definition 3.2. *[54,55]. A stochastic process $\xi(t) = \xi(t, \omega), (-\infty < t < +\infty)$ is said to be periodic with period T if for every finite sequence of numbers $t_1, t_2, \cdots, t_n$, the joint distribution of random variables $\xi(t_1 + h), \xi(t_2 + h), \cdots, \xi(t_n + h)$ is independent of h, where $h = kT, k = \pm 1, \pm 2, \cdots.$*

Remark 1. *[56] A Markov process $\xi(t)$ is T-periodic if and only if its transition probability function is T-periodic and the function $P_0(t, A) = P\{\xi(t) \in A\}$ satisfies the following equation:*

$$P_0(s, A) = \int_{R^l} P_0(s, dx)P(s, x, s + T, A) \equiv p_0(s + T, A)$$

for every $A \in \mathcal{B}$.

For the following Itô differential equation:

$$dx(t) = b(t, x(t))dt + \sigma(t, x(t))dB(t), \tag{3.4}$$

let all of the coefficients be T-periodic in t and satisfy the linear growth condition and the Lipschitz condition in every cylinder $U_l \times R_+$ for $l > 0$, where $U_l = \{x : \|x\| \leq l\}$. If there exists a function $v = v(t, x)$ which is twice continuously differentiable with respect to x and once continuously differentiable with respect to t in $R^n \times R_+$, and if T is periodic in t and satisfies the following conditions:

$$\inf_{\|x\|>R} v(t, x) \to +\infty \text{ as } l \to +\infty, \tag{3.5}$$

$$\mathcal{L}v(t, x) \leq -1 \text{ outside some compact set}, \tag{3.6}$$

then there exists a solution of model (3.4) which is a T-periodic Markovian process [56].

3.2. Existence and uniqueness of the global positive solution

From the biological perspective, the population size should be nonnegative. Following the approach of [40], we will show that model (2.2) has a unique global positive solution. First, we consider the model without impulses as

$$
\begin{aligned}
dy_1(t) &= y_1(t)\left(-b(t) + \frac{1}{T}\sum_{j=1}^{p}\log(1 + \alpha_{1j}) - n(t)y_2(t)\right)dt + \sigma_1(t)y_1(t)dB_1(t), \\
dy_2(t) &= y_2(t)\left(\beta(t)A_1(t)y_1(t) - m(t) - \frac{c(t)A_2(t)y_3(t)}{h(t) + y_2(t)} - r(t)y_2(t)\right)dt + \sigma_2(t)y_2(t)dB_2(t), \\
dy_3(t) &= y_3(t)\left(\frac{d(t)y_2(t)}{h(t) + y_2(t)} + \frac{1}{T}\sum_{j=1}^{p}\log(1 + \alpha_{2j}) - k(t) - v(t)A_2(t)y_3(t) - \frac{\rho(t)y_2(t)}{h(t) + y_2(t)}\right)dt + \sigma_3(t)y_3(t)dB_3(t)
\end{aligned}
\tag{3.7}
$$

with

$$A_1(t) = \left[\prod_{j=1}^{p}(1 + \alpha_{1j})\right]^{-\left(\frac{t}{T}\right)}\prod_{0 \leq t_k < t}(1 + \alpha_{1k}),$$

$$A_2(t) = \left[\prod_{j=1}^{p}(1 + \alpha_{2j})\right]^{-\left(\frac{t}{T}\right)}\prod_{0 \leq t_k < t}(1 + \alpha_{2k}).$$

According to [54], $A_1(t)$ and $A_2(t)$ are periodic functions with periodic T. Thus, we obtain the following lemma.

Lemma 3.2. *Let* $N(t) = A_1(t)y_1(t)$, $P(t) = y_2(t)$ *and* $Z(t) = A_2(t)y_3(t)$.

(1) If $(N(t), P(t), Z(t))$ *is the solution of model* (2.2)*, then* $(y_1(t), y_2(t), y_3(t))$ *is the solution of model* *(3.7).*

(2) If $(y_1(t), y_2(t), y_3(t))$ *is the solution of model* (3.7)*, then* $(N(t), P(t), Z(t))$ *is the solution of model* *(2.2).*

The proof easily follows from [52], hence, it is omitted. Now, we state the following theorem.

Theorem 3.1. *For any initial value* $(N(0), P(0), Z(0)) \in R_+^3$*, model* (2.2) *exhibits a unique positive solution* $(N(t), P(t), Z(t))$*, which will remain in* R_+^3 *with a probability of one.*

Proof. Obviously, for any fixed $t \geq 0$, there exists a nonnegative integer $n \in \{0, 1, 2, \cdots\}$ such that $nT \leq t \leq (n+1)T$. In view of the equalities $t_{k+p} = t_k + T$ and $\alpha_{k+p} = \alpha_k$, one can get

$$t_{k+np} = t_{t+(n-1)p} + T = \cdots = t_k + nT, \quad \alpha_{k+np} = \alpha_{k+(n-1)p} = \cdots = \alpha_k.$$

Owing to $[0, \infty) \cap \{t_k, k \in \mathbb{N}\} = \{t_1, t_2, \cdots, t_p\}$, there exists a positive integer $r \in \{1, 2, \cdots, p\}$ such that

$$t_{r+np}, t_{r+1+np}, \cdots, t_{p+np} \in [t, (n+1)T),$$

$$t_{1+(n+1)p}, t_{2+(n+1)p}, \cdots, t_{r-1+(n+1)p} \in [(n+1)T, t+T).$$

Now, we will prove that model (3.7) has a unique globally positive solution $(y_1(t), y_2(t), y_3(t))$. Obviously, the coefficients of model (3.7) satisfy the local Lipschitz condition. Thus, there exists a unique local solution on $[0, \tau_e)$, where τ_e is the explosion time. This solution will be global if $\tau_e = \infty$ almost surely. We choose $\gamma_0 > 1$ sufficiently large so that $y_1(0)$, $y_2(0)$ and $y_3(0)$ all lie in the interval $\left[\frac{1}{\gamma_0}, \gamma_0\right]$. For any positive integer γ ($\gamma \geq \gamma_0$), we define the stopping time as follows:

$$\tau_\gamma = \inf\left\{t \in [0, \tau_e] : y_1(t) \notin \left(\frac{1}{\gamma}, \gamma\right) \text{ or } y_2(t) \notin \left(\frac{1}{\gamma}, \gamma\right) \text{ or } y_3(t) \notin \left(\frac{1}{\gamma}, \gamma\right)\right\}.$$

We set $\inf \emptyset = \infty$. Clearly, τ_γ increases as $\gamma \to \infty$. Let $\tau_\infty = \lim\limits_{\gamma \to +\infty} \tau_\gamma$. Obviously, if $\tau_\infty = \infty$, then $\tau_\infty \leq \tau_e$ almost surely, $\tau_e = \infty$ and $(y_1(t), y_2(2), y_3(t)) \in R_+^3$ almost surely.

Now, we define a C^3-function $V : R_+^3 \to R_+$ as

$$V(y_1(t), y_2(t), y_3(t)) = \beta(t)A_1(t)(y_1(t) + 1 - \log y_1(t)) + n(t)(y_2(t) + 1 - \log y_2(t)) + y_3(t) + 1 - \log y_3(t).$$

For any $T > 0$, we apply Itô's formula on $t \in [0, \tau_\gamma \wedge T]$ to get the following:

$$dV(y_1(t), y_2(t), y_3(t)) = \mathcal{L}V dt + \beta(t)A_1(t)\left(1 - \frac{1}{y_1(t)}\right)\sigma_1(t)y_1(t)dB_1(t)$$

$$+ n(t)\left(1 - \frac{1}{y_2(t)}\right)\sigma_2(t)y_2(t)dB_2(t) + \left(1 - \frac{1}{y_3(t)}\right)\sigma_3(t)y_3(t)dB_3(t),$$

where

$$
\begin{aligned}
\mathcal{L}V &= \beta(t)A_1(t)\left[(y_1(t)-1)\left(-b(t)+\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{1j})-n(t)y_2(t)\right)+\frac{1}{2}\sigma_1^2(t)\right] \\
&\quad +n(t)\left[(y_2(t)-1)\left(\beta(t)A_1(t)y_1(t)-\frac{A_2(t)c(t)y_3(t)}{h(t)+y_2(t)}-m(t)-r(t)y_2(t)\right)+\frac{1}{2}\sigma_2^2(t)\right] \\
&\quad +(y_3(t)-1)\left(\frac{d(t)y_2(t)}{h(t)+y_2(t)}+\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{2j})-k(t)-\frac{\rho(t)y_2(t)}{h(t)+y_2(t)}-v(t)A_2(t)y_3(t)\right) \\
&\quad +\frac{1}{2}\sigma_3^2(t) \\
&= \beta(t)A_1(t)\left(-b(t)y_1(t)+b(t)+y_1(t)\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{1j})-n(t)y_1(t)y_2(t)\right. \\
&\quad \left.-\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{1j})+n(t)y_2(t)\right)+n(t)\left(-\frac{c(t)A_2(t)y_2(t)y_3(t)}{h(t)+y_2(t)}-\beta(t)A_1(t)y_1(t)\right. \\
&\quad \left.+A_1(t)\beta(t)y_1(t)y_2(t)+\frac{c(t)A_2(t)y_3(t)}{h(t)+y_2(t)}-m(t)y_2(t)+m(t)-r(t)y_2^2(t)+r(t)y_2(t)\right) \\
&\quad +y_3(t)\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{2j})-\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{2j})+\frac{d(t)y_2(t)y_3(t)}{h(t)+y_2(t)}-\frac{d(t)y_2(t)}{h(t)+y_2(t)} \\
&\quad -k(t)y_3(t)+k(t)-\frac{\rho(t)y_2(t)y_3(t)}{h(t)+y_2(t)}+\frac{\rho(t)y_2(t)}{h(t)+y_2(t)}-v(t)A_2(t)y_3^2(t) \\
&\quad +v(t)A_2(t)y_3(t)+\frac{1}{2}\beta(t)A_1(t)\sigma_1^2(t)+\frac{1}{2}n(t)\sigma_2^2(t)+\frac{1}{2}\sigma_3^2(t) \\
&\leq \beta^u A_1^u b^u+n^u m^u+k^u+\rho^u+\frac{1}{2}\left(\beta^u A_1^u(\sigma_1^u)^2+n^u(\sigma_2^u)^2+(\sigma_3^u)^2\right) \\
&\quad +\beta(t)A_1(t)y_1(t)\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{1j})+\left(\beta^u A_1^u+r^u\right)n(t)y_2(t) \\
&\quad +\left(\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{2j})+\frac{n^u c^u A_2^u}{h^l}+d^u+v^u A_2^u\right)y_3(t) \\
&\leq F_1+2\beta(t)A_1(t)\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{1j})(y_1(t)+1-\log y_1(t)) \\
&\quad +2n(t)\left(\beta^u A_1^u+r^u\right)(y_2(t)+1-\log y_2(t)) \\
&\quad +2\left(\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{2j})+\frac{n^u c^u A_2^u}{h^l}+d^u+v^u A_2^u\right)(y_3(t)+1-\log y_3(t)) \\
&\leq F_1+F_2 V(y_1(t),y_2(t),y_3(t)),
\end{aligned}
$$

where

$$F_1 = \beta^u A_1^u b^u + n^u m^u + k^u + \rho^u + \frac{1}{2}\left(\beta^u A_1^u (\sigma_1^u)^2 + n^u (\sigma_2^u)^2 + (\sigma_3^u)^2\right),$$

$$F_2 = 2\left(\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{1j})\right) \vee 2\left(\beta^u A_1^u + r^u\right) \vee 2\left(\frac{1}{T}\sum_{j=1}^{p}\log(1+\alpha_{2j}) + \frac{n^u c^u A_2^u}{h^l} + d^u + v^u A_2^u\right).$$

Thus, we have

$$
\begin{aligned}
dV(y_1(t), y_2(t), y_3(t)) &\leq F_1 dt + F_2 V(y_1(t), y_2(t), y_3(t)) + \beta^u A_1^u \sigma_1(t)(y_1(t)-1)dB_1(t) \\
&\quad + n^u \sigma_2(t)(y_2(t)-1)dB_2(t) + \sigma_3(t)(y_3(t)-1)dB_3(t).
\end{aligned}
$$

On integrating both sides of the above inequality from 0 to $\tau_\gamma \wedge T$ and taking the expectation, we get

$$
\begin{aligned}
&\mathbf{E}V\left(y_1(\tau_\gamma \wedge T), y_2(\tau_\gamma \wedge T), y_3(\tau_\gamma \wedge T)\right) \\
&\leq V(y_1(0), y_2(0), y_3(0)) + F_1 T + F_2\mathbf{E}\int_0^{\tau_\gamma \wedge T} V(y_1(t), y_2(t), y_3(t))\, dt.
\end{aligned}
$$

By using Gronwall's inequality, we get

$$\mathbf{E}V\left(y_1(\tau_\gamma \wedge T), y_2(\tau_\gamma \wedge T), y_3(\tau_\gamma \wedge T)\right) \leq \left(V(y_1(0), y_2(0), y_3(0)) + F_1 T\right) e^{F_2 T}.$$

For $\gamma \geq \gamma_0$, let $\Omega_\gamma(\omega) = \{\omega \in \Omega : \tau_\gamma = \tau_\gamma(\omega) \leq T\}$; then, either $y_1(\tau_\gamma, \omega)$, $y_2(\tau_\gamma, \omega)$ or $y_3(\tau_\gamma, \omega)$ is equal to γ or $\frac{1}{\gamma}$ and

$$
\begin{aligned}
V\left(y_1\left(\tau_\gamma, \omega\right), y_2\left(\tau_\gamma, \omega\right), y_3\left(\tau_\gamma, \omega\right)\right) &\geq (\gamma + 1 - \log\gamma) \vee \left(\frac{1}{\gamma} + 1 + \log\gamma\right) \\
&\triangleq H(\gamma);
\end{aligned}
$$

$$
\begin{aligned}
\left(V(y_1(0), y_2(0), y_3(0)) + F_1 T\right) e^{F_2 T} &\geq \mathbf{E}I_{\Omega_\gamma}(\omega)V\left(y_1\left(\tau_\gamma, \omega\right), y_2\left(\tau_\gamma, \omega\right), y_3\left(\tau_\gamma, \omega\right)\right) \\
&\geq \mathbf{P}\{\tau_\gamma \leq T\}H(\gamma),
\end{aligned}
$$

where I_{Ω_γ} is the indicator function of Ω_γ. Let $\gamma \to \infty$; then, $\lim_{\gamma \to \infty} \mathbf{P}\{\tau_{\gamma \leq T}\} = 0$ implying that $\mathbf{P}\{\tau_{\gamma \leq T}\} = 0$. $T > 0$ is arbitrary, so $\mathbf{P}\{\tau_\infty < \infty\} = 0$. This implies that $\mathbf{P}\{\tau_\infty = \infty\} = 1$, as required. Thus, one can conclude that model (3.7) has a unique positive solution $(y_1(t), y_2(t), y_3(t))$ for all $t \geq 0$ almost surely.

Let $N(t) = A_1(t)y_1(t)$, $P(t) = y_2(t)$ and $Z(t) = A_2(t)y_3(t)$. From Lemma 3.2, it follows that $(N(t), P(t), Z(t))$ is the solution of model (2.2). Apparently, $N(t)$, $P(t)$ and $Z(t)$ are continuous on the intervals $(0, t_1)$ and (t_k, t_{k+1}), $k \in \mathbb{N}$. For $t \neq t_k$, we have

$$dN(t) = A_1'(t)y_1(t)dt + A_1(t)dy_1(t)$$

$$
\begin{aligned}
&= A_1(t)y_1(t)\left(-b(t) - n(t)y_2(t)\right)dt + \sigma_1(t)A_1(t)y_1(t)dB_1(t)\\
&= N(t)\left(-b(t) - e(t)P(t)\right) + \sigma_1(t)N(t)dB_1(t),\\
dZ(t) &= A_2'(t)y_3(t)dt + A_2(t)dy_3(t)\\
&= A_2(t)y_3(t)\left(\frac{d(t)y_2(t)}{h(t) + y_2(t)} - k(t) - \frac{\rho(t)y_2(t)}{h(t) + y_2(t)} - v(t)A_2(t)y_3(t)\right)dt\\
&\quad + \sigma_3(t)A_2(t)y_3(t)dB_3(t)\\
&= Z(t)\left(\frac{d(t)P(t)}{h(t) + P(t)} - k(t) - \frac{\rho(t)P(t)}{h(t) + P(t)} - v(t)Z_3(t)\right) + \sigma_3(t)Z(t)dB_3(t).
\end{aligned}
$$

For $t = t_k$, $k \in \mathbb{N}$, we have

$$
\begin{aligned}
N(t_k^+) &= \lim_{t \to t_k^+} A_1(t)y_1(t) = \left[\prod_{j=1}^{p}(1 + \alpha_{1j})\right]^{-\left(\frac{t_k}{T}\right)} \prod_{0 \le t_j \le t_k}(1 + \alpha_{1j})y_1(t_k^+)\\
&= (1 + \alpha_{1k})A_1(t_k^+)y_1(t_k^+) = (1 + \alpha_{1k})N(t_k),\\
N(t_k^-) &= \lim_{t \to t_k^-} A_1(t)y_1(t) = \left[\prod_{j=1}^{p}(1 + \alpha_{1j})\right]^{-\left(\frac{t_k}{T}\right)} \prod_{0 \le t_j < t_k}(1 + \alpha_{1j})y_1(t_k^-) = N(t_k).
\end{aligned}
$$

Similarly, we get

$$
Z(t_k^+) = (1 + \alpha_{2k})Z(t_k), \quad Z(t_k^-) = Z(t_k).
$$

Thus, one can claim that for any initial value $(N(0), P(0), Z(0)) \in R_+^2$, model (2.2) has a solution $(N(t), P(t), Z(t))$ for $t \ge 0$.

Now, we must approve that the solution of model (2.2) is unique. For $t \in [0, t_1]$, model (2.2) becomes

$$
\begin{cases}
dN(t) = \left[-b(t)N(t) - nP(t)N(t)\right]dt + \sigma_1(t)N(t)dB_1(t),\\
dP(t) = \left[\beta(t)P(t)N(t) - \frac{c(t)P(t)Z(t)}{h(t)+P(t)} - m(t)P(t) - r(t)P(t)^2\right]dt + \sigma_2(t)P(t)dB_2(t),\\
dZ(t) = \left[\frac{d(t)P(t)Z(t)}{h(t)+P(t)} - k(t)Z(t) - \frac{\rho(t)P(t)Z(t)}{h(t)+P(t)} - v(t)Z(t)^2\right]dt + \sigma_3(t)Z(t)dB_3(t)
\end{cases} \tag{3.8}
$$

with the initial value $(N(0), P(0), Z(0)) = (N_0, P_0, Z_0)$. The above model is equivalent to the following one:

$$
\begin{cases}
dN(t) = \left[-b(t) - ne^{v(t)} - \frac{\sigma_1^2(t)}{2}\right]dt + \sigma_1(t)dB_1(t),\\
dP(t) = \left[\beta(t)e^{u(t)} - \frac{c(t)e^{w(t)}}{h(t) + e^{v(t)}} - m(t) - r(t)e^{v(t)} - \frac{\sigma_2^2(t)}{2}\right]dt + \sigma_2(t)dB_2(t),\\
dZ(t) = \left[\frac{d(t)e^{v(t)}}{h(t) + e^{v(t)}} - k(t) - \frac{\rho(t)e^{v(t)}}{h(t) + e^{v(t)}} - v(t)e^{w(t)} - \frac{\sigma_3^2(t)}{2}\right]dt + \sigma_3(t)dB_3(t)
\end{cases} \tag{3.9}
$$

with the initial value $(u(0), v(0), w(0)) = (\log N_0, \log P_0, \log Z_0)$. Obviously, the coefficients of model (3.9) satisfy the local Lipschitz condition. Thus, by the theory of stochastic differential equation, model (3.9) has a unique solution $(u(0), v(0), w(0))$. In view of the Itô formula, $(N(t), P(t), Z(t)) = (e^u(t), e^v(t), e^w(t))$ is the unique solution of model (3.8).

Now, for $t \in (t_1, t_2]$, model (2.2) can be rewritten as follows:

$$
\begin{cases}
dN(t) = \left[-b(t)N(t) - nP(t)N(t)\right] dt + \sigma_1(t)N(t)dB_1(t), \\
dP(t) = \left[\beta(t)P(t)N(t) - \frac{c(t)P(t)Z(t)}{h(t)+P(t)} - m(t)P(t) - r(t)P(t)^2\right] dt + \sigma_2(t)P(t)dB_2(t), \\
dZ(t) = \left[\frac{d(t)P(t)Z(t)}{h(t)+P(t)} - k(t)Z(t) - \frac{\rho(t)P(t)Z(t)}{h(t)+P(t)} - v(t)Z(t)^2\right] dt + \sigma_3(t)Z(t)dB_3(t), \\
\left(N(t_1^+), P(t_1^+), Z(t_1^+)\right) = \left((1 + \alpha_{11})N(t_1^+), P(t_1^+), (1 + \alpha_{21})Z(t_1^+)\right).
\end{cases}
\tag{3.10}
$$

Using the same argument as above, one can show that model (3.10) admits a unique solution for $t \in (t_1, t_2]$. By the analogy, we can say that model (2.2) has a unique solution with the initial value $(N(0), P(0), Z(0))$ for each interval $(t_k, t_{k+1}]$. $\qquad \square$

One can conclude from the above theorem that for model (2.2), the population size will remain nonnegative for sufficiently large values of t.

3.3. Existence of positive T-periodic solution

Here, we derive conditions for the existence and uniqueness of a positive periodic solution of model (2.2).

Lemma 3.3. *[39]. For any initial value $(y_1(0), y_2(0), y_3(0)) \in R_+^3$, the solution $(y_1(t), y_2(t), y_3(t))$ of model (3.7) has the following properties:*

$$
\limsup_{t \to \infty} y_1(t) < \infty \text{ a.s.}, \ \limsup_{t \to \infty} y_2(t) < \infty \text{ a.s.}, \ \limsup_{t \to \infty} y_3(t) < \infty \text{ a.s.}
$$

Further, there exist three positive constants H_1, H_2 and H_3 such that

$$
y_1(t) \le H_1, \ y_2(t) \le H_2 \text{ and } y_3(t) \le H_3 \text{ for all } t \ge 0 \text{ a.s.}
$$

Define

$$
\phi_1 \triangleq \frac{1}{T} \sum_{j=1}^{p} \log\left(1 + \alpha_{1j}\right) - \left\langle b(t) + \sigma_1^2(t)\right\rangle_T - n^u H_2,
$$

$$
\phi_2 \triangleq \left\langle m(t) + \sigma_2^2(t)\right\rangle_T - \frac{c^u A_2^u H_3}{h^l},
$$

$$
\phi_3 \triangleq \frac{1}{T} \sum_{j=1}^{p} \log\left(1 + \alpha_{2j}\right) - \left\langle k(t) + \sigma_3^2(t)\right\rangle_T - \rho^u.
$$

Now, we present the following theorem.

Theorem 3.2. *If ϕ_1, ϕ_2, $\phi_3 > 0$, then model (2.2) has a positive T-periodic solution.*

Proof. In view of Theorem 3.1, it suffices to show that a periodic solution exists for the equivalent model (3.7) without impulses. Thus, we only need to verify conditions (3.5) and (3.6).

We define a C^3-function $V : [0, +\infty) \times R_+^3 \to R_+$ as

$$V(t, y_1, y_2, y_3) = \left(\frac{e^{w_1(t)}}{y_1} + \log y_1\right) + \left(\frac{e^{w_2(t)}}{y_2} + \log y_2\right) + \left(\frac{e^{w_3(t)}}{y_3} + \log y_3\right)$$
$$= V_1(t, y_1) + V_2(t, y_2) + V_3(t, y_3),$$

where

$$\begin{align}
w_1'(t) &= \left\langle b(t) + \sigma_1^2(t)\right\rangle_T - b(t) - \sigma_1^2(t),\\
w_2'(t) &= -\left\langle m(t) + \sigma_2^2(t)\right\rangle_T - m(t) - \sigma_2^2(t), \tag{3.11}\\
w_3'(t) &= \left\langle k(t) + \sigma_3^2(t)\right\rangle_T - k(t) - \sigma_3^2(t).
\end{align}$$

Following [40], $w_1(t)$, $w_2(t)$ and $w_3(t)$ are T-periodic functions. According to condition (3.5), we need to verify that

$$\inf_{(t,y_1,y_2,y_3)\in[0,+\infty)\times(R_+^3\setminus U_\epsilon)} V(t, y) \to \infty, \quad \epsilon \to \infty,$$

where $U_\epsilon = \left(\frac{1}{\epsilon}, \epsilon\right) \times \left(\frac{1}{\epsilon}, \epsilon\right) \times \left(\frac{1}{\epsilon}, \epsilon\right)$. This trivially holds as

$$\frac{1}{z} + \log z \to +\infty \ a.s. \ z \to 0^+, \quad \frac{1}{z} + \log z \to +\infty \ a.s. \ z \to +\infty.$$

Now, we prove condition (3.6). Using Itô's formula, we have

$$\begin{align}
\mathcal{L}V_1 &= -\frac{e^{w_1(t)}}{y_1(t)}\left(-w_1'(t) - b(t) + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{1j}\right) - n(t)y_2(t) - \sigma_1^2(t)\right)\\
&\quad + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{1j}\right) - b(t) - n(t)y_2(t) - \frac{\sigma_1^2(t)}{2}\\
&\leq -\frac{e^{w_1(t)}}{y_1(t)}\left(-w_1'(t) - b(t) - \sigma_1^2(t) + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{1j}\right) - n^u H_2\right)\\
&\quad + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{1j}\right) - n^l y_2(t) - b^l - \frac{(\sigma_1^l)^2}{2};
\end{align}$$

$$\begin{align}
\mathcal{L}V_2 &= -\frac{e^{w_2(t)}}{y_2(t)}\left(-w_2'(t) - m(t) - \sigma_2^2(t) + \beta(t)A_1(t)y_1(t) - r(t)y_2(t)\right.\\
&\quad \left. -\frac{c(t)A_2(t)y_3(t)}{h(t) + y_2(t)}\right) + \beta(t)A_1(t)y_1(t) - \frac{c(t)A_2(t)y_3(t)}{h(t) + y_2(t)} - m(t)\\
&\quad - r(t)y_2(t) - \frac{\sigma_2^2(t)}{2}
\end{align}$$

$$\leq -\frac{e^{w_2(t)}}{y_2(t)}\left(-w_2'(t) - m(t) - \sigma_2^2(t) - \frac{c^u A_2^u H_3}{h^l}\right) - \frac{e^{w_2^l}\beta^l A_1^l y_1(t)}{H_2}$$

$$+ r^u e^{w_2^u} + \beta^u A_1^u H_1 - \frac{c^l A_2^l y_3(t)}{h^u + H_2} - m^l - r^l y_2(t) - \frac{(\sigma_2^l)^2}{2};$$

$$\mathcal{L}V_3 = -\frac{e^{w_3(t)}}{y_3(t)}\left(-w_3'(t) - k(t) - \sigma_3^2(t) + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{2j}\right) + \frac{d(t)y_2(t)}{h(t) + y_2(t)}\right.$$

$$\left. -\frac{\rho(t)y_2(t)}{h(t) + y_2(t)} - v(t)A_2(t)y_3(t)\right) + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{2j}\right) + \frac{d(t)y_2(t)}{h(t) + y_2(t)}$$

$$-\frac{\rho(t)y_2(t)}{h(t) + y_2(t)} - v(t)A_2(t)y_3(t) - k(t) - \frac{\sigma_3^2(t)}{2}$$

$$\leq -\frac{e^{w_3(t)}}{y_3(t)}\left(-w_3'(t) - k(t) - \sigma_3^2(t) + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{2j}\right) - \rho^u\right)$$

$$+ e^{w_3^u}v^u A_2^u + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{2j}\right) + d^u - k^l - \frac{\rho^l y_2(t)}{h^u + H_2} - v^l A_2^l y_3(t) - \frac{(\sigma_3^l)^2}{2}.$$

Now, from Eq (3.11), we get

$$\mathcal{L}V = \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3$$

$$\leq -\frac{e^{w_1(t)}}{y_1(t)}\left(\frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{1j}\right) - \left\langle b(t) + \sigma_1^2(t)\right\rangle_T - n^u H_2\right)$$

$$-\frac{e^{w_2(t)}}{y_2(t)}\left(\left\langle m(t) + \sigma_2^2(t)\right\rangle_T - \frac{c^u A_2^u H_3}{h^l}\right)$$

$$-\frac{e^{w_3(t)}}{y_3(t)}\left(\frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{2j}\right) - \left\langle k(t) + \sigma_3^2(t)\right\rangle_T - \rho^u\right)$$

$$+ \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{1j}\right) + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{2j}\right) + \beta^u A_1^u H_1 + e^{w_2^u}r^u$$

$$+ e^{w_3^u}v^u A_2^u + d^u - b^l - m^l - k^l - \frac{(\sigma_1^l)^2}{2} - \frac{(\sigma_2^l)^2}{2} - \frac{(\sigma_3^l)^2}{2}.$$

Let

$$\Theta^* = \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{1j}\right) + \frac{1}{T}\sum_{j=1}^{p}\log\left(1 + \alpha_{2j}\right) + \beta^u A_1^u H_1 + e^{w_2^u}r^u$$

$$+ e^{w_3^u}v^u A_2^u + d^u - b^l - m^l - k^l - \frac{(\sigma_1^l)^2}{2} - \frac{(\sigma_2^l)^2}{2} - \frac{(\sigma_3^l)^2}{2}.$$

Then, we have

$$\begin{aligned}
\mathcal{L}V \leq & \Theta^* - \frac{e^{w_1^l}}{y_1(t)}\phi_1 - \frac{e^{w_2^l}}{y_2(t)}\phi_2 - \frac{e^{w_3^l}}{y_3(t)}\phi_3 - \frac{e^{w_2^l}\beta^l A_1^l}{H_2}y_1(t) \\
& - \left(n^l + r^l + \frac{\rho^l}{h^u + H_2}\right)y_2(t) - \left(\frac{c^l A_2^l}{h^u + H_2} + v^l A_2^l\right)y_3(t) \\
\triangleq & \Gamma(y_1, y_2, y_3).
\end{aligned} \tag{3.12}$$

Obviously,

$$\begin{aligned}
\Gamma(y_1, y_2, y_3) &\to -\infty \text{ a.s. } y_1 \to 0^+ \text{ or } y_2 \to 0^+ \text{ or } y_3 \to 0^+, \\
\Gamma(y_1, y_2, y_3) &\to -\infty \text{ a.s. } y_1 \to +\infty \text{ or } y_2 \to +\infty \text{ or } y_3 \to +\infty.
\end{aligned} \tag{3.13}$$

From Eqs (3.12) and (3.13), we can take ϵ sufficiently small such that

$$\mathcal{L}V \leq -1, \text{ for } (y_1, y_2, y_3) \in R_+^3 \setminus U,$$

where $U = \left[\epsilon, \frac{1}{\epsilon}\right] \times \left[\epsilon, \frac{1}{\epsilon}\right] \times \left[\epsilon, \frac{1}{\epsilon}\right]$. Therefore, for the initial values $y_1(0) > 0$, $y_2(0) > 0$ and $y_3(0) > 0$, the solution $y(t) = (y_1(t), y_2(t), y_3(t))$ of model (3.7) is a positive T-periodic Markov process. Based on the previous analysis, one can get that

$$\begin{aligned}
N(t) &= A_1(t)y_1(t) = \left[\prod_{j=1}^{p}(1 + \alpha_{1j})\right]^{-\left(\frac{t}{T}\right)} \prod_{0 \leq t_k < t}(1 + \alpha_{1k})y_1(t), \\
P(t) &= y_2(t), \\
Z(t) &= A_2(t)y_3(t) = \left[\prod_{j=1}^{p}(1 + \alpha_{2j})\right]^{-\left(\frac{t}{T}\right)} \prod_{0 \leq t_k < t}(1 + \alpha_{2k})y_3(t),
\end{aligned}$$

where $A_1(t)$ and $A_2(t)$ are T-periodic functions. Thus, one can say that $(N(t), P(t), Z(t))$ is a positive T-periodic solution of model (2.2). □

From a biological point of view, the above theorem tells that model (2.2) exhibits a positive T-periodic solution for an appropriate noise intensity, and that the phytoplankton population will oscillate periodically.

4. Numerical simulations

Table 1. Biological explanations of variables/parameters in model (2.1) and the numerical values of parameters used for simulation results.

Variables/ Parameters	Descriptions	Units	Values	References
N	Concentration of nutrients	μg/L		
P	Biomass of phytoplankton	μg/L		
Z	Biomass of zooplankton	μg/L		
b	Washout rate of nutrients	day^{-1}	0.05	0.05/0.07[66]
n	Maximal uptake rate of nutrients by phytoplankton	day^{-1}	0.2	[0.2, 1][13]
β	Conversion rate of nutrients into the biomass of phytoplankton	day^{-1}	0.5	0.4[45]
d	Rate of conversion of biomass of phytoplankton into that of zooplankton	day^{-1}	0.8	0.92[45]
m	Natural death rate of phytoplankton	day^{-1}	0.3	0.2[65]
k	Natural death rate of zooplankton	day^{-1}	0.1	0.1[65]
c	Grazing rate of phytoplankton by zooplankton	day^{-1}	0.2	0.1/0.3[66]
h	Half saturation constant for the phytoplankton population	μg/L	5	—
ρ	Rate of toxin liberation by toxin-producing phytoplankton	$L/\mu g/day$	0.01	0.0186[64]
r	Strength of intraspecific competition among the phytoplankton population	$L/\mu g$/day	0.1	0.1[64]
v	Strength of intraspecific competition among the zooplankton population	$L/\mu g$/day	0.1	—

In order to have better insight into how impulsive control and environmental fluctuation affects the distributions of plankton populations in aquatic ecosystems, we present here some numerical results. To solve the stochastic system, we have used the Euler-Maruyama method whereas the Runge-Kutta method is employed to solve the deterministic system. If not specified in the text, we adopt the values of the parameters as in Table 1. To keep the model tractable, we set a common period of 40 d^{-1} for all parameters in model (2.2). Further, to capture the impact of periodic environment, we set the following forms of a model parameters:

$$
\begin{aligned}
b &= 0.05 + 0.01 \sin(\pi t/20), \ n = 0.2 + 0.01 \sin(\pi t/20), \ \beta = 0.5 + 0.01 \sin(\pi t/20), \\
c &= 0.2 + 0.01 \sin(\pi t/20), \ h = 5 + 0.01 \sin(\pi t/20), \ m = 0.3 + 0.01 \sin(\pi t/20), \\
d &= 0.8 + 0.01 \sin(\pi t/20), \ k = 0.1 + 0.01 \sin(\pi t/20), \ \rho = 0.01 + 0.01 \sin(\pi t/20), \\
v &= 0.1 + 0.01 \sin(\pi t/20), \ r = 0.1 + 0.01 \sin(\pi t/20).
\end{aligned}
$$

Here, we mainly focus on the impulsive control and the noise intensity as the key factors in controlling the plankton blooms.

4.1. Effects of impulsive control
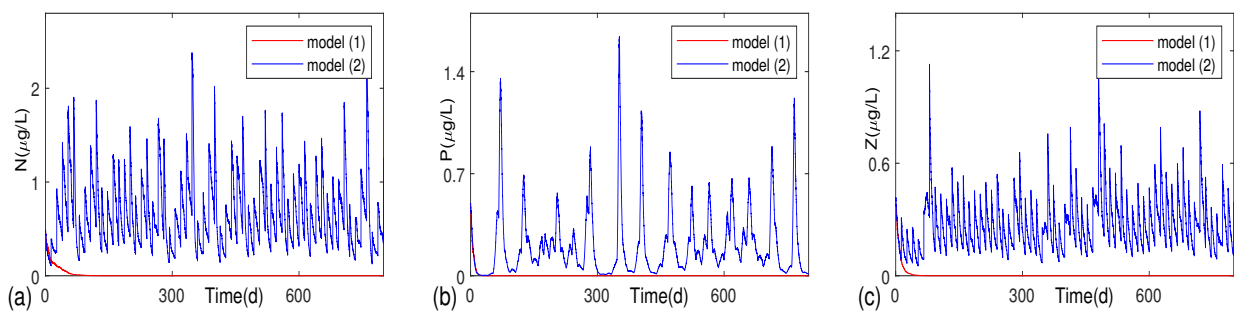
Figure 1. Time evolutions of (a) nutrients, (b) phytoplankton and (c) zooplankton in models (2.1) and (2.2).

In real aquatic ecosystems, the oscillatory behavior of population density is observed due to the existence of ubiquitous pulse perturbations and stochastic fluctuation. For instance, industrial wastewater discharge can lead to rapid increment in the nutrient concentrations in water bodies, which produces transient effects on the state of aquatic ecosystems and may trigger the rapid growth of algae. In recent years, the importance of impulsive control has been widely recognized by researchers worldwide. Here, we present the effects of impulsive control on the plankton populations. The paths of nutrients, phytoplankton and zooplankton are depicted in Figure 1 for a fixed noise intensity, pulse intensity and number of pulses per period ($\sigma_1 = \sigma_2 = \sigma_3 = 0.02 + 0.01 \sin(\pi t/20)$, $\alpha_1 = \exp(2)$, $\alpha_2 = \exp(2)$, $p = 3$). One can see in the figure that without impulsive fluctuation, both nutrient and plankton populations tend to extinct. However, when the impulsive control is applied on the stochastic model, nutrient and plankton populations can coexist. Obviously, the results displayed by Figure 1 are induced by impulsive control, indicating that impulsive control has the potential to alter population dynamics and may promote the coexistence of plankton populations.

In order to investigate how plankton populations respond to the combined effects of nutrient and zooplankton pulses, in Figure 2, we present the mean for plankton biomasses in the (α_1, α_2) plane. Apparently, for relatively small nutrient pulses, the phytoplankton population disappears from the aquatic system even if a high pulse of zooplankton is applied (see Figure 2(c)). One possible explanation for this behavior of phytoplankton might be the deficiency of nutrients in the aquatic system. Figure 2 depicts that an increment in the nutrient pulse may cause a significant increase in the mean of plankton biomasses indicating that as the nutrient pulse increases, so does the oscillatory behavior of plankton
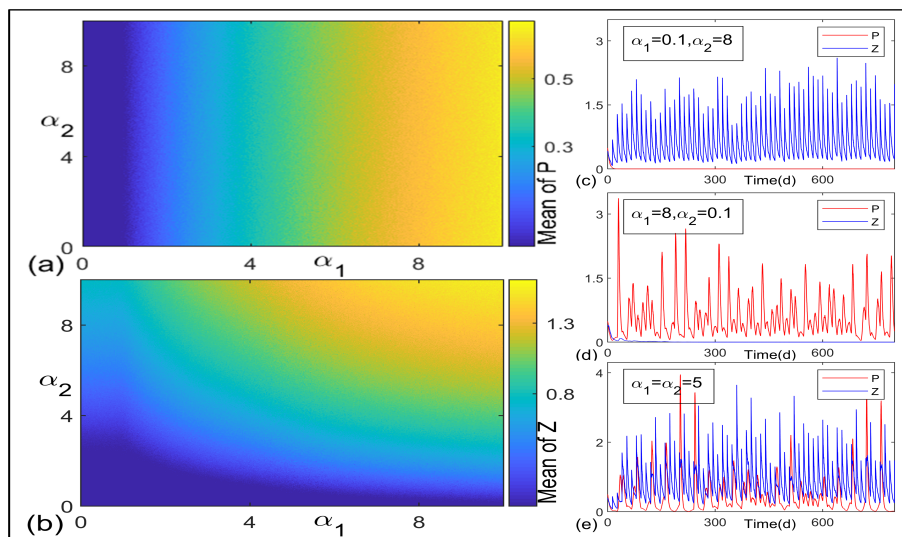
Figure 2. Results for model (2.2) with $\sigma_1 = \sigma_2 = \sigma_3 = 0.05 + 0.01 \sin(\pi t/20)$. (a) The mean for phytoplankton biomass in the (α_1, α_2) plane; (b) the mean for zooplankton biomass in the (α_1, α_2) plane; (c) the evolution of a single path of the phytoplankton population and zooplankton population in model (2.2) with $\alpha_1 = 0.1, \alpha_2 = 8$; (d) the evolution of a single path of the phytoplankton population and zooplankton population in model (2.2) with $\alpha_1 = 8, \alpha_2 = 0.1$; (e) the evolution of a single path of the phytoplankton population and zooplankton population in model (2.2) with $\alpha_1 = 5, \alpha_2 = 5$.

populations. The variance and the third order central moment of plankton populations are presented in Figure 3 by taking three different sets of pulse intensity. Apparently, the plankton populations disappear from the aquatic system if there is no nutrient pulse. One can also see that the variance of plankton biomasses show overall an increasing trend as the nutrient pulse increases, but there is a sharp increase in the skewness of the phytoplankton population. These results indicate that the nutrient pulse has the potential to alter the survival probability of plankton populations in the aquatic system. Next, we set the nutrient pulse as $\alpha_1 = 3$ and show the variance and the third order central moment of plankton populations by differing the zooplankton impulse; see Figure 3(c)–(d). Notably, the variance and the skewness of the phytoplankton population is unlikely to change significantly with a rise in the zooplankton impulse. Thus, we can say that the nutrient concentration in the aquatic reservoir is an important driver in schemes to control phytoplankton growth. Here, our findings concur with the results reported by the authors of [57]. Specifically, in comparison to the impulsive control of the zooplankton population, the phytoplankton growth has a stronger relationship with the nutrient pulse.

Further, the frequency of pulses of plankton populations is also expected to rise due to the number of pulses per period, which may play an important role in stabilizing the plankton systems [19]. Figure 4 shows the mean and the variance of plankton biomasses with respect to the frequency of pulses. We observed that plankton populations tend to extinction when the number of pulses per period is small. This might happen because a small number of pulses per period can lessen the concentration of nutrients in the aquatic ecosystem, which may not support the survival of plankton populations. However, an increment in the number of pulses may induce a significant increase in the mean and the variance of plankton biomasses indicating that the densities of plankton populations show stronger
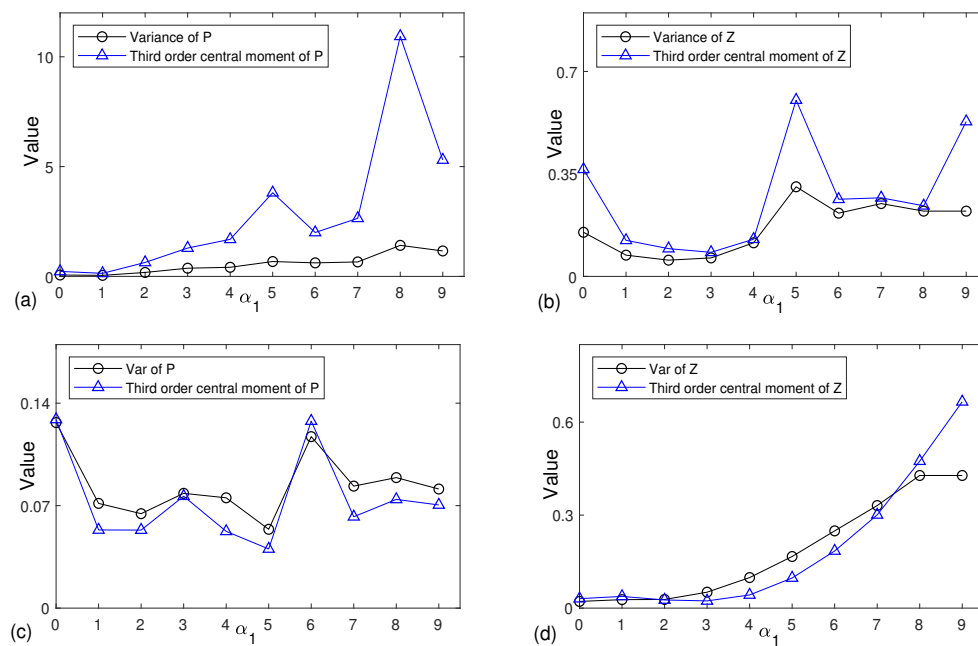
Figure 3. Results for $\sigma_1 = \sigma_2 = \sigma_3 = 0.05 + 0.01 \sin(\pi t/20)$. (a) The variance and the third-order central moment of phytoplankton population and (b) zooplankton population versus different nutrient pulses with $\alpha_2 = 4$; (c) the variance and the third order central moment of the phytoplankton population and (d) zooplankton population versus different zooplankton pulses with $\alpha_1 = 3$.

oscillatory behavior. These results reveal that an appropriate frequency of pulses can contribute to the survival of plankton populations in the aquatic ecosystem.

4.2. Effects of stochastic perturbations

It is well known that all populations show substantial stochasticity in their ecological dynamics [37]. Importantly, the random fluctuation plays a key role in determining the survivorship of plankton populations in aquatic systems [39]. But, comparatively little attention has been given to the effect of environmental fluctuation on an aquatic system with pulse perturbations. As one can see in Figure 5, the means of nutrient concentration and plankton biomass are less affected by stochastic perturbations, but environmental noise can significantly affect the variance of nutrient concentration and phytoplankton biomass. It means that the presence of environmental noise can promote the oscillatory behavior of nutrients and phytoplankton population.

Studies have shown that an excessive noise intensity can potentially affect the species richness and completely destroy the plankton system of aquatic reservoirs [8]. For example, a dramatic decrease in the temperature can cause the complete disappearance of the *Cylindrospermopsis* filaments [58]. Figure 6, depicts that for a smaller range of noise intensity, the amplitude of oscillations in the plankton densities increases as the noise intensity increases. But, when the noise intensity is high, the nutrient concentration becomes zero while the plankton populations disappear from the aquatic system. Similar results were reported by the authors of [44]. These findings indicate that the control of environmental
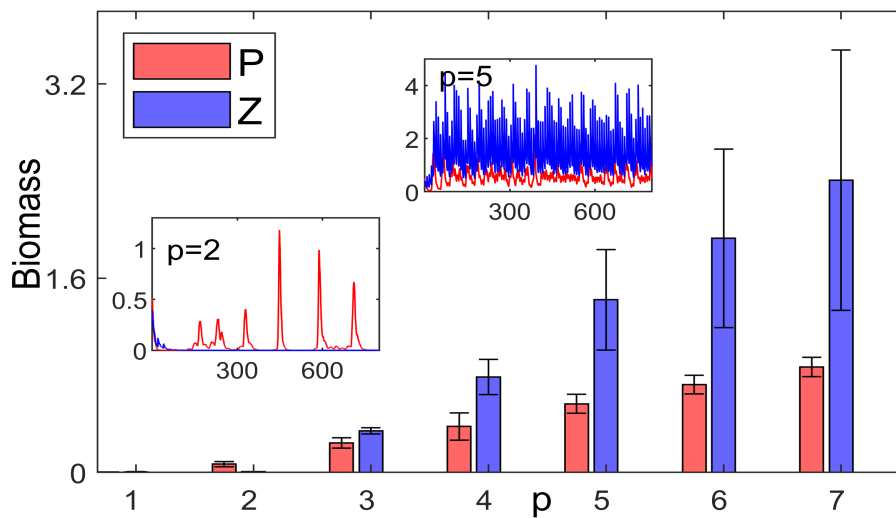
Figure 4. Results for model (2.2) with $\sigma_1 = \sigma_2 = \sigma_3 = 0.05 + 0.01 \sin(\pi t/20)$ and $\alpha_1 = \alpha_2 = \exp(1)$, showing the mean and the variance of phytoplankton biomass and zooplankton biomass with respect to number of pulses per period. The path of plankton populations indicates the changes of plankton biomasses for $p = 2$ and $p = 5$, respectively.

noise can have a dramatic role in the survival of both phytoplankton and zooplankton populations in aquatic ecosystems.

5. Discussion

Over the past few decades, the control of algal blooms has received considerable attention among research scientists. There has been an increasing focus on the roles of nutrients and zooplankton in the growth of phytoplankton [59–61]. Due to the advantage of impulsive differential equations, modeling approaches have become a widely used tool to enhance our understanding of pulsed instantaneous behaviors that exist in aquatic ecosystems. Here, we have investigated a nutrient-plankton model with impulsive control and stochastic disturbance. Our theoretical results showed the existence of a global positive solution for the considered model. Moreover, we found that the amplitudes of oscillations in the plankton density exhibit a more or less periodic nature under certain conditions.

Numerically, we saw the combined effects of nutrient and zooplankton pulses on the plankton population growth; see Figure 2. We observed that nutrient pulses have a more significant and enduring impact on the phytoplankton biomass compared to the zooplankton pulse, and that plankton populations will tend to extinction when the nutrient pulse and zooplankton pulse are low. Importantly, when the nutrient pulse is low, the phytoplankton population has become extinct in the aquatic system due to a lack of nutrients, as shown in Figure 3(a). This situation may arise in the aquatic systems because an inadequate nutrient pulse leads to insufficient nutrient concentration for the uptake of phytoplankton. However, as the nutrient pulse increases, the variance of phytoplankton and zooplankton populations exhibit an overall increasing trend, indicating stronger oscillatory behavior. One possible explanation is that an increment in the phytoplankton biomass due to a higher nutrient pulse may provide sufficient food for the grazer zooplankton [5]. We also noted that the density of the phytoplankton population
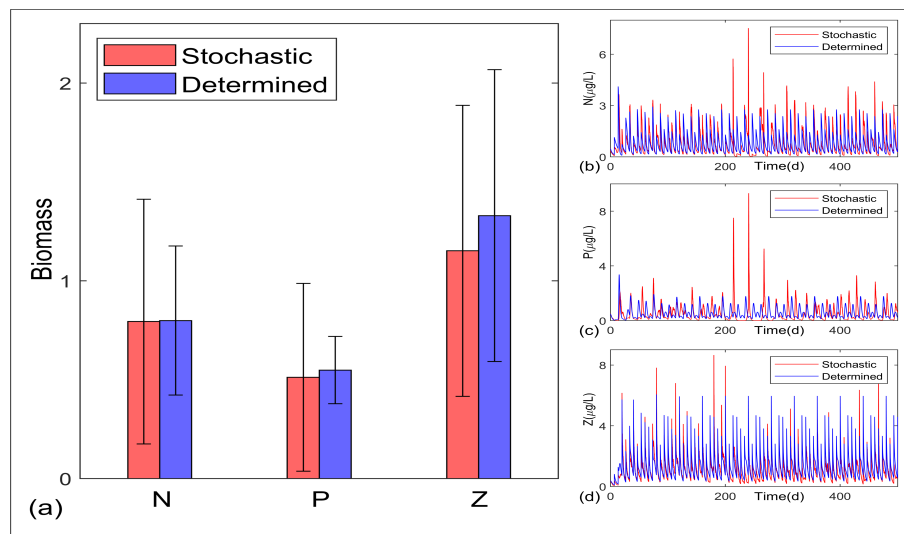
Figure 5. Results for model (2.2) with $\sigma_1 = \sigma_2 = \sigma_3 = 0.1 + 0.01\sin(\pi t/20)$ and $\alpha_1 = \alpha_2 = \exp(2)$. (a) The mean and the variance of phytoplankton and zooplankton biomasses; evolution of a single path of (b) nutrient concentration, (c) phytoplankton population, (e) zooplankton population.

does not change much in response to changing the pulse intensity of zooplankton even if a high pulse of zooplankton is applied, as shown in Figure 3(c). These results further emphasize that the nutrient pulse is strongly related to phytoplankton growth [62]. Overall, Figure 3 shows a positive association between the nutrient pulse and the amplitudes of oscillations in the densities of plankton populations. Notably, the zooplankton pulse does not significantly affect the phytoplankton population. These results suggest that the growth of phytoplankton is directly correlated with the concentration of nutrients in the aquatic system.

Apart from the impulsive intensity, the frequency of impulsive control also has a direct role in regulating plankton dynamics [14]. Our simulation results showed a significant positive correlation between the number of pulses per period and the intensities of oscillations in the densities of plankton populations, as shown in Figure 4. Biologically, Figure 4 tells that a low number of pulses per period leads to stagnation in the growth of phytoplankton as the nutrient concentration declines in the aquatic system [63]. However, a high frequency of impulsive control leads to an increased input of nutrients into aquatic ecosystems, which can support the survivorship of the plankton populations. Thus, one can say that an increasing number of pulses per period may contribute to the survival of plankton populations in an aquatic habitat.

Environmental disturbances could not be neglected in the process of phytoplankton growth as environmental noise has the potential to alter population dynamics [37]. For instance, in [58], it was experimentally demonstrated that a dramatic drop in the temperature can promote the disappearance of the *Cylindrospermopsis* filaments. Therefore, we present Figures 5 and 6 to show the effects of stochastic fluctuations on the impulsive control. It follows from Figure 5 that the noise intensity has a significant effect on the variance of nutrients and phytoplankton, which implies that oscillations in the densities of nutrient concentration and phytoplankton population have a positive relationship with the environmental fluctuations. However, the results in Figure 6 suggest that the entire plankton pop-
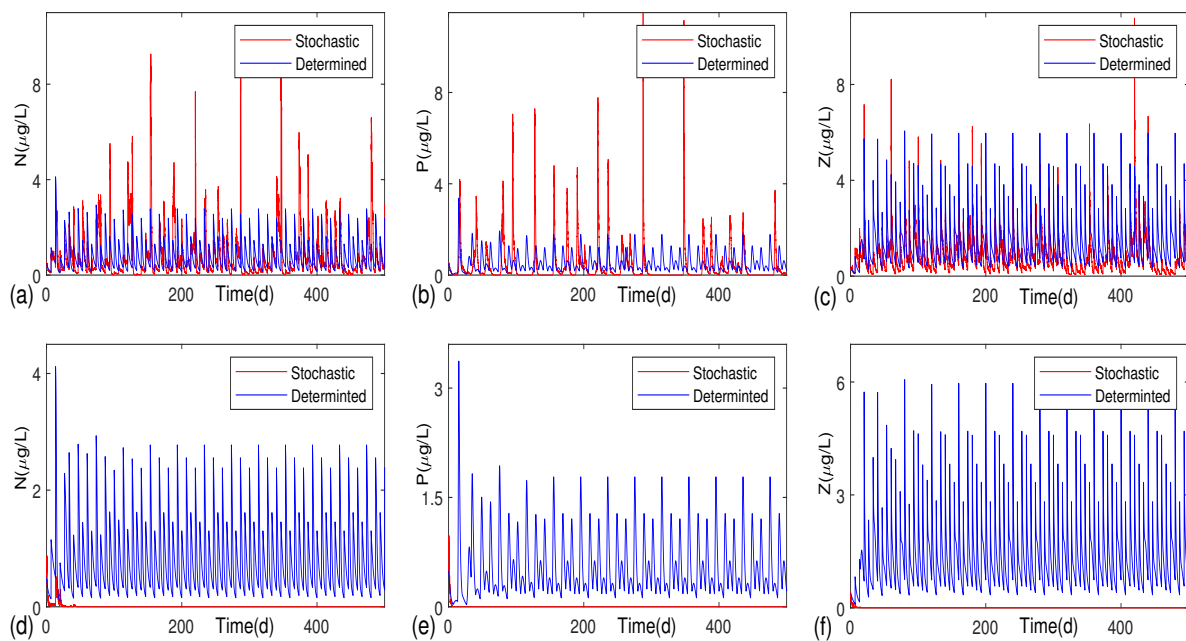
Figure 6. Time evolutions of nutrients (first column), phytoplankton (second column) and zooplankton (third column) for model (2.2) and the corresponding deterministic model. Here, we have chosen $\alpha_1 = \alpha_2 = \exp(2)$ and $p = 3$; $\sigma_1 = \sigma_2 = \sigma_3 = 0.2 + 0.01 \sin(\pi t/20)$ in $(a - c)$ and $\sigma_1 = \sigma_2 = \sigma_3 = 0.5 + 0.01 \sin(\pi t/20)$ in $(d - f)$.

ulation may collapse in response to increased noise intensity. This finding is in line with the results reported in [58]. It suggests that the environmental fluctuations are likely to influence the plankton dynamics whereas the noise control could be of practical significance for the mitigation of planktonic growth. From the results obtained here, one can conclude that nutrient pulses may be more likely to affect phytoplankton dynamics, and that the environmental fluctuations could have larger impacts on aquatic ecosystems.

6. Conclusions

In this paper, a stochastic-impulsive coupled nutrient-plankton model was investigated. Our theoretical analysis showed that model (2.2) has a unique globally positive solution. Some sufficient conditions have been derived for the existence of a positive T-periodic solution. Numerical results revealed that due to the existence of impulsive control, the plankton dynamics experienced a transition from extinction to species coexistence. The results indicated that the nutrient pulse is more effective than the zooplankton pulse in controlling the phytoplankton growth. We observed that an increased impulse frequency and an appropriate noise intensity can increase the amplitudes of oscillations in the plankton density whereas an excessive noise intensity may cause a collapse of the entire aquatic ecosystem. It is worth noting that our results may not predict the specific events within a particular lake/ocean, but it contributes to the studies on how the coupling between stochasticity and impulse influences the growth of phytoplankton species. Overall, the findings of this study may provide some insights into the research on the relationships between the phytoplankton population, nutrients and

zooplanktonic predators in a stochastic environment.

Use of AI tools declaration

The authors declare that they have not used artificial intelligence tools in the creation of this article.

Acknowledgments

Conflict of interest

The author declares that there are no conflicts of interest.

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