Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/mbe

MBE, 20(8): 15431–15455.
DOI: 10.3934/mbe.2023689
Received: 16 May 2023
Revised: 05 July 2023
Accepted: 18 July 2023
Published: 24 July 2023

Research article

Lyapunov approach to manifolds stability for impulsive Cohen–Grossberg-type conformable neural network models

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Abstract: In this paper, motivated by the advantages of the generalized conformable derivatives, an impulsive conformable Cohen–Grossberg-type neural network model is introduced. The impulses, which can be also considered as a control strategy, are at fixed instants of time. We define the notion of practical stability with respect to manifolds. A Lyapunov-based analysis is conducted, and new criteria are proposed. The case of bidirectional associative memory (BAM) network model is also investigated. Examples are given to demonstrate the effectiveness of the established results.

Keywords: Cohen–Grossberg neural networks; conformable derivative; impulses; manifolds; practical stability

1. Introduction

The neural network modeling approach in the form of differential equations has been intensively applied by numerous researchers in different areas of science, engineering and medicine. The construction of a neuronal network model and its analysis are key goals in many practical applications. Indeed, the applications of such models include a variety of problems, such as optimization problems, associative memory, parallel processing, linear and nonlinear programming, including algorithms, computer vision, pattern recognition and many others [1, 2].

The Cohen–Grossberg-type of neural networks [3] are an important class of neural network models that are widely used as a framework in the study of partial memory storage and global pattern formation phenomena [4–6]. Due to its important applications, the properties of different Cohen–Grossberg neural network models have been intensively studied, including some very recent results [7, 8]. This class of neural networks also generalizes some of the essential neural network models, such as Hopfield neural networks [9] and cellular neural networks [10, 11].

In order to create more adequate neural network models, researchers have generalized and extended the classical Cohen–Grossberg-type of neural networks in different directions. One line of generalization relates to the consideration of the affects of impulsive disturbances on their qualitative behavior. In fact, short-term perturbations are natural for most applications of such neural networks, and their presence may affect the dynamic properties of the models. The apparatus of impulsive differential equations [12–14] has been widely applied to develop and investigate impulsive neural network models. Using the impulsive modeling approach, different classes of impulsive Cohen–Grossberg neural networks have been proposed [15–17]. The closely related impulsive control approach [18, 19] has also been examined effectively [20, 21].

The fractional calculus approach has also been applied in the neural network modeling. Indeed, models with fractional order dynamics generalize the integer-order models and have numerous advantages, including universality, more flexibility and hereditary properties [22–25]. Numerous researchers contributed to the development of fractional-order Cohen–Grossberg neural network models and the study of their properties [26–30].

In the development of the fractional calculus fruitful direction of generalization, the most used fractional order derivatives are these of the Grunwald–Letnikov, Caputo and Riemann–Liouville types. Note that there are some restrictions when the above notions are applied to real-world models. Some of the restrictions are related to their properties and mainly to the nonexistence of a straightforward rule for the derivatives of compositions of functions. To overcome such difficulties, several researchers proposed new approaches, such as Caputo–Fabrizio [31], Caputo–Hadamard [32], including variable fractional order derivatives [33, 34] without singular kernels, and generated new fractional models.

One of the recently applied extended fractional calculus approaches is based on the use of the conformable or fractional-like derivatives [35, 36] which offer some computational simplifications related to derivatives of compositions of functions. Some interesting and important results for systems with conformable derivatives are proposed in [37–40]. Physical interpretations of the conformable derivatives have been given in [41, 42]. Due to the advantages of such derivatives that are limit based and follow a very simple chain rule, the conformable calculus begins to be used in the modeling of several phenomena studied in economics [43], medicine [44] and forecasting [45].

The impulsive generalization is also applied to the conformable calculus approach. However, the results in this direction are very few and the theory of impulsive conformable systems is still not completed [46–48]. Also, to our best knowledge, the combined impulsive conformable modeling approach has not been applied to Cohen–Grossberg neural network models, and this is the main goal of this article. In fact, the proposed conformable technique seems to be very suitable for neural networks because of the simplifications in calculations offered.

In practical implementations, the global exponential stability for integer-order models [49, 50] and Mittag–Leffler stability for models with fractional dynamics [51, 52] are ones of the most important neural network's behaviors. Hence, there are numerous results that provided efficient asymptotic stability, exponential stability and Mittag–Leffler stability criteria for Cohen–Grossberg neural networks [3, 5–7, 15, 21, 26, 27, 30]. However, there are also cases when the classical asymptotic and Mittag–Leffler stability strategies cannot be applied [53]. In some of these cases, when the trajectories of a neural network model are not mathematically stable, but the system behavior is admissible from the practical point of view, the concept of practical stability is more relevant [54, 55].

In the past decade, the practical stability concept has been rapidly developed [56, 57], and has been extended to the practical stability with respect to manifolds notion [58]. Indeed, the practical stability is useful in many applied systems when the dynamic of the model is contained within particular bounds during a fixed time interval. In such cases, the global asymptotic notion is not applicable. The practical stability strategy is also very efficient when a neuronal state is unstable in the classical sense and yet, state trajectories may oscillate sufficiently near the desired state such that its performance is admissible, which does not imply stability or convergence of trajectories. In addition, there are many applied systems that are stable or asymptotically stable in the classical sense, but actually useless in practice because of the small or inappropriate stability domain or attraction domain. The stability with respect to manifolds concepts are generalizations of the stability notions for single trajectories and are related to the study of stability properties of a manifold of states. Such concepts are of considerable interest to networks capable of approaching not only one steady state. The extended practical stability with respect to manifolds concept has been applied to integer-order Cohen–Grossberg models in [59] and to some conformable models [60]. However, there is no one existing result on practical stability with respect to manifolds for impulsive conformable Cohen–Grossberg neural networks. It is clear that this task is attractive and important in the analysis of such neural network models.

The main goal of this research is to propose an impulsive conformable Cohen–Grossberg-type neural network model. Also, the adoption of the practical stability with respect to manifolds notion is an essential motivation for our study.

The innovations of this article can be described as follows:

(1) We propose a new impulsive conformable Cohen–Grossberg neural network model. The advantages of the conformable modelling approach can be used in the analysis of the properties of the introduced model. The impulsive perturbations can also be used to stabilize and control the proposed model.

(2) We introduce the practical stability with respect to manifolds notion. Using modifications of the Lyapunov technique, new criteria are proposed, which guarantee the practical stability behavior of manifolds related to the newly suggested model.

(3) The BAM generalization of the introduced model is analysed, and the corresponding sufficient conditions for practical stability with respect to manifolds are established.

(4) We verify the efficiency of the established criteria by examples.

The structuring of the rest of the paper pursue the following scheme. In Section 2 some definitions and lemmas from conformable calculus are given. The conformable impulsive Cohen–Grossberg neural network model is introduced and the practical stability with respect to manifolds concept is adopted to it. Some preliminaries related to the Lyapunov functions method are also given. Section 3 is devoted to the main practical stability results established on the basis of a Lyapunov-type analysis. The case of BAM impulsive conformable Cohen–Grossberg-type neural networks is studied, and sufficient conditions for the practical exponential stability of the trajectories are proposed in Section 4. Illustrative examples are elaborated in Section 5. Some concluding comments are stated in Section 6.

Notations: $\mathbb{R}$ is the set of all real numbers, the set of all nonnegative real numbers will be denoted by $\mathbb{R}_+ = [0, \infty)$, $\mathbb{R}^n$ denotes the n-dimensional real space with the norm $||y|| = \sum_{\iota=1}^{n} |y_\iota|$ of an $y \in \mathbb{R}^n$, $y = (y_1, y_2, \ldots, y_n)^T$.

2. Basic definitions, problem formulation and preliminary results

2.1. Conformable calculus definitions and lemmas

Here, we will present some basic definitions and lemmas related to the conformable calculus. Let $\tau_0 \in \mathbb{R}_+$.

Definition 1. [47,48] The generalized conformable derivative of order α, $0 < \alpha \leq 1$ for a function $y(t) : [\tilde{t}, \infty) \to \mathbb{R}^n$, $\tilde{t} \geq \tau_0$ is defined by

$$\mathcal{D}^{\alpha}_{\tilde{t}} y(t) = \lim \left\{ \frac{y(t + \phi(t - \tilde{t})^{1-\alpha}) - y(t)}{\phi},\ \phi \to 0 \right\}.$$

Let the time instants $\tau_1, \tau_2, \ldots,$ are such that

$$\tau_0 < \tau_1 < \tau_2 \cdots < \tau_k < \tau_{k+1} < \ldots,\ \lim_{k\to\infty} \tau_k = \infty$$

and $y(\tau_k^+) = \lim \{y(\tau_k + \phi),\ \phi \to 0^+\}$, $y(\tau_k^-) = \lim \{y(\tau_k - \phi),\ \phi \to 0^+\}$.

For $k = 1, 2, \ldots,$ we have [47,48]

$$\mathcal{D}^{\alpha}_{\tau_k} y(\tau_k^+) = \lim_{t\to\tau_k^+} \mathcal{D}^{\alpha}_{\tau_k} y(t),$$

$$\mathcal{D}^{\alpha}_{\tau_{k-1}} y(\tau_k^-) = \lim \left\{ \frac{y(\tau_k^- + \phi(\tau_k^- - \tau_{k-1})^{1-\alpha}) - y(\tau_k^-)}{\phi},\ \phi \to 0 \right\}.$$

Any function that has an α-generalized conformable derivatives for any $t \in (\tilde{t}, \infty)$ is called α-generalized conformable differentiable on $(\tilde{t}, \infty)$, and we will denote the set of all such functions by $C^{\alpha}[(\tilde{t}, \infty), \mathbb{R}^n]$ [48].

Definition 2. [47] Let $y : [\tilde{t}, \infty) \to \mathbb{R}^n$. The generalized conformable integral of order $0 < \alpha \leq 1$ of y is given by

$$I^{\alpha}_{\tilde{t}} y(t) = \int_{\tilde{t}}^{t} (o - \tilde{t})^{\alpha-1} y(o)do.$$

The following properties of the generalized conformable derivatives and integrals will be applied in our future analysis [46–48].

Lemma 3. *[48] If the function $y \in C^{\alpha}[(\tilde{t}, \infty), \mathbb{R}]$ for $0 < \alpha \leq 1$, then*

$$I^{\alpha}_{\tilde{t}}(\mathcal{D}^{\alpha}_{\tilde{t}} y(t)) = y(t) - y(\tilde{t}),\ t > \tilde{t}.$$

Lemma 4. *[48] Assume that $X(y(t)) : (\tilde{t}, \infty) \to \mathbb{R}$ and $0 < \alpha \leq 1$. If $X(\cdot)$ is differentiable with respect to y and $y \in C^{\alpha}[(\tilde{t}, \infty), \mathbb{R}]$, then for $t \in [\tilde{t}, \infty)$ and $y(t) \neq 0$, we have*

$$\mathcal{D}^{\alpha}_{\tilde{t}} X(y(t)) = X'(y(t))\mathcal{D}^{\alpha}_{\tilde{t}}(y(t)),$$

where X' is the derivative of $X(\cdot)$

Remark 5. The result represented in Lemma 5 is a simple chain rule which is not applicable for the classical fractional-order derivatives. More results on the conformable derivatives can be found in [35–40, 46–48].

Remark 6. To emphasize more advantages of the application of fractional derivatives, we note the following properties for $\alpha \in (0,1]$ and $x, y \in C^{\alpha}[(\tilde{t},\infty),\mathbb{R}]$ which follow directly by Definition 1 [35, 36, 41, 48]:

(a) $\mathcal{D}^{\alpha}_{\tilde{t}}(ax(t)+by(t)) = a\mathcal{D}^{\alpha}_{\tilde{t}}(x(t)) + b\mathcal{D}^{\alpha}_{\tilde{t}}(y(t))$ for all $a, b \in \mathbb{R}$;
(b) $\mathcal{D}^{\alpha}_{\tilde{t}}(t^p) = pt^{p-1}(t-\tilde{t})^{1-\alpha}$ for any $p \in \mathbb{R}$;
(c) $\mathcal{D}^{\alpha}_{\tilde{t}}(x(t)y(t)) = x(t)\mathcal{D}^{\alpha}_{\tilde{t}}(y(t)) + y(t)\mathcal{D}^{\alpha}_{\tilde{t}}(x(t))$;
(d) $\mathcal{D}^{\alpha}_{\tilde{t}}\left(\dfrac{x(t)}{y(t)}\right) = \dfrac{y(t)\mathcal{D}^{\alpha}_{\tilde{t}}(x(t)) - x(t)\mathcal{D}^{\alpha}_{\tilde{t}}(y(t))}{y^2(t)}$;
(e) $\mathcal{D}^{\alpha}_{\tilde{t}}(x(t)) = 0$ for any $x(t) = \lambda$, where λ is an arbitrary constant.

For the classical fractional derivatives [22–25], including the Riemann-Liouville fractional derivative

$$D^{RL}_{\tilde{t}}x(t) = \frac{d}{dt}\left[\frac{1}{\Gamma(1-\alpha)}\int_{\tilde{t}}^{t}\frac{x(s)}{(t-s)^{\alpha}}ds\right],$$

where $0 < \alpha < 1$ and $\Gamma(z) = \int_0^{\infty} e^{-t}t^{z-1}dt$ is the Gamma function, and Caputo fractional derivative

$$D^{C}_{\tilde{t}}x(t) = \frac{1}{\Gamma(1-\alpha)}\int_{\tilde{t}}^{t}\frac{x'(s)}{(t-s)^{\alpha}}ds$$

the properties (b)–(e) do not hold, except for the statement (e) for a fractional derivative of Caputo. The reason for this is the application of the integral in the known definitions of fractional derivatives.

2.2. The Gohen–Grossberg impulsive conformable neural network model

In this section we propose an impulsive Cohen–Grossberg-type neural network model with generalized conformable derivatives as follows

$$\begin{cases} \mathcal{D}^{\alpha}_{\tau_k}y_\iota(t) = -a_\iota(y_\iota(t))\left[b_\iota(y_\iota(t)) - \sum_{j=1}^{n}c_{\iota j}f_j(y_j(t)) - U_\iota(t)\right], \quad t \neq \tau_k,\ k = 0,1,2\ldots, \\ \\ y_\iota(\tau_k^+) = y_\iota(\tau_k) + P_{\iota k}(y_\iota(\tau_k)),\ k = 1,2,\ldots, \end{cases} \tag{1}$$

where $\iota = 1,2,\ldots,n,\ n \geq 2$ represents the number of the units, $y_\iota(t)$ is the state of the ι-th node at time t, the functions $a_\iota, b_\iota, f_j, U_\iota \in C[\mathbb{R},\mathbb{R}_+]$ and $c_{\iota j} \in \mathbb{R}_+$, $\iota, j = 1,2,\ldots,n$ are the system's parameters in the continuous part, a_ι denote the amplification functions, b_ι are appropriate behaved functions, f_j represent the activation functions, U_ι denotes the external bias on the ιth node at time t, $c_{\iota j}$ represent the connection weights, $y_\iota(\tau_k) = y_\iota(\tau_k^-)$ and $y_\iota(\tau_k^+)$ in the impulsive condition are, respectively, the neuronal states of the ι-th node before and after an impulsive perturbation at τ_k and the impulsive functions $P_{\iota k} \in C[\mathbb{R}, \mathbb{R}]$ represent the effects of the impulse controls on the node y_ι at τ_k, $\iota = 1, 2, \ldots, n$, $k = 1, 2, \ldots$.

For $\tau_0 \in \mathbb{R}_+$ we define the initial condition corresponding to the model (1) as

$$y(\tau_0^+) = y(\tau_0) = y_0, \tag{2}$$

where $y_0 \in \mathbb{R}^n$.

A function $\varphi \in \mathbb{R}^n$ that satisfies the system (1) and the initial condition (2) will be called a solution of the initial value problem (1), (2), and will be denoted by $\varphi(t) = \varphi(t; \tau_0, y_0)$, where $\varphi(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T$.

According to the theory of impulsive conformable systems [46–48], the solution $\varphi(t)$ of the initial value problem (1), (2) is an α-generalized conformable differentiable function, which is piecewise continuous with points of discontinuity of the first kind τ_k, $k = 1, 2, \ldots$, at which it is left continuous. At the impulsive moments, the functions $\varphi(t)$ is such that

$$\varphi_\iota(\tau_k^-) = \varphi_\iota(\tau_k),\ \varphi_\iota(\tau_k^+) = \varphi_\iota(\tau_k) + P_{\iota k}(\varphi_\iota(\tau_k)),\ \iota = 1, 2, \ldots, n,\ k = 1, 2, \ldots. \tag{3}$$

The set of all such functions will be denoted by $PC^\alpha[\mathbb{R}_+, \mathbb{R}^n]$.

For example, for the solution of an impulsive scalar generalized conformable neural network of the type

$$\begin{cases} \mathcal{D}^\alpha_{\tau_k} u(t) = a(t)u(t) - a(t)b(t),\ t \neq t_k,\ k = 0, 1, \ldots, \\ u(\tau_k^+) = u(\tau_k) + p_k u(\tau_k),\ k = 1, 2, \ldots, \end{cases} \tag{4}$$

where $u \in \mathbb{R}$, $a, b \in C[\mathbb{R}_+, \mathbb{R}_+]$, $p_k \in \mathbb{R}$, $k = 1, 2, \ldots$, we obtain

$$\begin{aligned} u(t) &= u(\tau_0^+) \prod_{j=1}^{k} (1 + p_j) \exp\left\{ \int_{\tau_k}^{t} \frac{a(o)}{(o - \tau_k)^{1-\alpha}} do + \sum_{l=1}^{k} \int_{\tau_{l-1}}^{\tau_l} \frac{a(o)}{(o - \tau_{l-1})^{1-\alpha}} do \right\} \\ &- \int_{\tau_k}^{t} \exp\left\{ \int_{\tau_k}^{t} \frac{a(o)}{(o - \tau_k)^{1-\alpha}} do - \int_{\tau_k}^{s} \frac{a(s)}{(s - \tau_k)^{1-\alpha}} ds \right\} \frac{a(o)b(o)}{(o - \tau_k)^{1-\alpha}} d0 \\ &- \sum_{l=1}^{k} \prod_{j=l}^{k} (1 + p_j) \int_{\tau_{l-1}}^{\tau_l} \exp\left\{ \int_{\tau_k}^{t} \frac{a(o)}{(o - \tau_k)^{1-\alpha}} do - \int_{\tau_{l-1}}^{o} \frac{a(s)}{(s - \tau_{l-1})^{1-\alpha}} ds \right. \\ &\left. + \sum_{m=l}^{k} \int_{\tau_{m-1}}^{\tau_m} \frac{a(o)}{(o - \tau_{m-1})^{1-\alpha}} do \right\} \frac{a(o)b(o)}{(o - \tau_{l-1})^{1-\alpha}} do. \end{aligned}$$

We will study some stability properties of the introduced model (1) under the following hypotheses:

H1. The continuous functions $a_\iota \in \mathbb{R}_+$, $\iota = 1, 2, \ldots, n$ are such that there exist positive constants $\underline{a}_\iota$ and $\overline{a}_\iota$ for which $1 < \underline{a}_\iota \leq a_\iota(\chi) \leq \overline{a}_\iota$ for $\chi \in \mathbb{R}$, and $\underline{a} = \min_{1 \leq \iota \leq n} a_\iota$, $\overline{a} = \max_{1 \leq \iota \leq n} a_\iota$.

H2. The continuous functions $b_\iota \in \mathbb{R}_+$ are such that there exist constants $\beta_\iota > 0$ with

$$b_\iota(\chi) \geq \beta_\iota |\chi|,$$

for any $\chi \in \mathbb{R}$ and $\iota = 1, 2, \ldots, n$.

H3. There exist Lipschitz constants $L_j^f > 0$ such that

$$|f_j(\chi_1) - f_j(\chi_2)| \leq L_j^f |\chi_1 - \chi_2|, \ f_j(0) = 0$$

for all $\chi_1, \chi_2 \in \mathbb{R}$, $j = 1, 2, \ldots, n$.

Remark 7. The developed Cohen–Grossberg type model (1) with generalized conformable derivatives is an extension of the existing integer-order neural network models studied in [3, 5]. It also extends some impulsive models investigated in [15–17, 20, 21] to the conformable setting. Since the concept of the conformable derivative overcomes some limitations related to the classical fractional-order derivatives, the introduced model is more suitable for applications than the models considered in [26, 27, 29, 30]. In addition, considering impulsive perturbations will allow for the application of efficient impulsive control strategies [18–21, 58, 59].

2.3. Practical stability with respect to manifolds formalism

In order to apply the practical stability with respect to manifolds method [55, 58, 59] to the model (1), we consider a continuous function $H : [\tau_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^m$, $H = (H_1, H_2, \ldots, H_m)^T$, $m \leq n$ which defines the following $(n - m)$-dimensional manifold:

$$\mathcal{M}_t^H = \{y \in \mathbb{R}^n : \ H_1(t, y) = H_2(t, y) = \cdots = H_m(t, y) = 0, \ t \in [\tau_0, \infty)\}. \tag{5}$$

The defined above manifold, will be called a H-manifold.

Consider, also

$$\mathcal{M}_t^H(\varepsilon) = \{y \in \mathbb{R}^n : \ ||H(t, y)|| < \varepsilon, \ t \in [\tau_0, \infty)\}, \ \varepsilon > 0,$$

and adopt the following notions [55–59].

Definition 8. If for given $(\lambda, \ A)$ with $0 < \lambda < A$, $y_0 \in \mathcal{M}_{\tau_0}^H(\lambda)$ implies $y(t; \tau_0, y_0) \in \mathcal{M}_t^H(A)$, $t \geq \tau_0$ for some $\tau_0 \in \mathbb{R}_+$, then the manifold $\mathcal{M}_t^H$ is said to be practically stable for the model (1).

Definition 9. If for given $(\lambda, \ A)$ with $0 < \lambda < A$, $y_0 \in \mathcal{M}_{\tau_0}^H(\lambda)$ implies

$$y(t; \tau_0, y_0) \in \mathcal{M}_t^H(A + \mu||H(\tau_0, y_0)||E_\alpha(-\delta, t - \tau_0)), \ t \geq \tau_0, \ \text{for some } \tau_0 \in \mathbb{R}_+,$$

where $0 < \alpha < 1$, $\mu, \delta > 0$, and $E_\alpha(\nu, o)$ is the conformable exponential function given as [47]

$$E_\alpha(\nu, o) = \exp\left(\nu \frac{o^\alpha}{\alpha}\right), \ \nu \in \mathbb{R}, \ o \in \mathbb{R}_+.$$

then the manifold $\mathcal{M}_t^H$ is said to be practically exponentially stable for the model (1).

Remark 10. Definitions 8 and 9 generalize and extend the classical stability notions to the combined practical stability with respect to manifolds case. The combined setting increases the benefits of both approaches. In some particular cases, the defined concept can be reduced to simple practical stability notions that are used widely in the applied problems [60–63]. For example, if the function that defines the manifold $H(t, y) = 0$ only for $y = 0$, then definitions 8 and 9 are the classical definitions of practical stability and exponential practical stability of the zero solution of the model (1), respectively. Similarly, if the manifold $\mathcal{M}_t^H$ includes only an equilibrium state y^*, i.e., $H(t, y^*) = 0$, then the above definitions are reduced to the practical stability and exponential practical stability definitions of y^*, respectively. Other particular cases, such as practical stability and exponential practical stability of periodic and almost periodic solutions can be considered analogously. Thus, the proposed practical stability with respect to manifolds technique offers a high-powered mechanism which can be applied not only for single solutions, but also in the cases when manifolds of solutions are attractors for the models. This technique is also appropriate to investigate such manifolds that are not stable in the classical Lyapunov sense, but their dynamic is acceptable from the practical point of view [55, 58].

We will also need the following additional hypothesis related to the specifics of the stability with respect to manifolds concepts.

H4. Each solution $y(t; \tau_0, y_0)$ of the initial value problem (1), (2) such that

$$\|H(t, y(t; \tau_0, y_0))\| \leq \Omega < \infty,$$

where $\Omega > 0$ is a constant, is defined on $[\tau_0, \infty)$.

2.4. Conformable Lyapunov functions method

Consider the sets $\mathcal{G}_k = (\tau_{k-1}, \tau_k) \times \mathbb{R}^n,\ k = 1, 2, \ldots, \mathcal{G} = \bigcup_{k=1}^{\infty} \mathcal{G}_k$.

In the further study, we apply a modified conformable Lyapunov function approach. A class of Lyapunov-like functions $\mathcal{L}^{\alpha}_{\tau_k}$ is defined as [47] a set of functions $L(t, y) : \mathcal{G} \to \mathbb{R}_+$ that are continuous on $\mathcal{G}$, α-generalized conformable differentiable in t, locally Lipschitz continuous with respect to y on each of the sets $\mathcal{G}_k$, $L(t, 0) = 0$ for $t \geq \tau_0$, and for each $k = 1, 2, \ldots$ and $y \in \mathbb{R}^n$, there exist the finite limits

$$L(\tau_k^-, y) = \lim_{\substack{t \to \tau_k \\ t < \tau_k}} L(t, y), \quad L(\tau_k^+, y) = \lim_{\substack{t \to \tau_k \\ t > \tau_k}} L(t, y),$$

with $L(\tau_k^-, y) = L(\tau_k, y)$.

For a function $L \in \mathcal{L}^{\alpha}_{\tau_k}$, the upper right conformable derivative is defined by [47]

$$^{+}\mathcal{D}^{\alpha}_{\tau_k} L(t, y) = \limsup \left\{ \frac{L(t + \phi(t - \tau_k)^{1-\alpha},\ y(t + \phi(t - \tau_k)^{1-\alpha})) - L(t, y)}{\phi},\ \phi \to 0^+ \right\}. \tag{6}$$

Denote by $F(t, y) = (F_1(t, y), F_2(t, y), \ldots, F_n(t, y))^T$, where $y = (y_1, y_2, \ldots, y_n)^T$ and

$$F_\iota(t, y) = -a_\iota(y_\iota) \left[b_\iota(y_\iota) - \sum_{j=1}^{n} c_{\iota j} f_j(y_j) - U_\iota(t) \right]$$

for $\iota = 1, 2, \ldots, n$.

Then, the generalized conformable derivative of the function $L(t, y)$ with respect to system (1), (2) is [47]

$$^{+}\mathcal{D}^{\alpha}_{\tau_k} L(t, y)|_{(1)} = \limsup \left\{ \frac{L(t + \phi(t - \tau_k)^{1-\alpha},\ y + \phi(t - \tau_k)^{1-\alpha} F(t, y)) - L(t, y)}{\phi},\ \phi \to 0^+ \right\}. \tag{7}$$

If $L(t, y(t)) = L(y(t))$, $0 < \alpha \leq 1$, the function L is differentiable on y, and the function $y(t)$ is α-generalized conformable differentiable on $[\tau_0, \infty)$, then Lemma 4 can applied componentwise.

It follows from (6) and (7) that

$$^{+}\mathcal{D}^{\alpha}_{\tau_k}\ L(t, y(t; \tau_k, y_k)) =^{+}\mathcal{D}^{\alpha}_{\tau_k}\ L(t, y)|_{(1)},$$

where $y_k = y(\tau_k^+)$, $k = 0, 1, 2, \ldots$.

For more information about the conformable modifications of the Lyapunov functions approach for impulsive control models, we refer to [46–48].

The next Lemma from [47] will be also necessary.

Lemma 11. *If for the function $L \in \mathcal{L}^{\alpha}_{\tau_k}$ and for $t \in [\tau_0, \infty)$, $y \in \mathbb{R}^n$, we have:*

(i)

$$L(\tau_k^+, y) \le L(\tau_k, y),\ \ k = 1, 2, \ldots,$$

(ii)

$$^{+}\mathcal{D}^{\alpha}_{\tau_k}\ L(t, y) \le -\delta L(t, y) + \varrho(t),\ t \ne \tau_k,\ k = 0, 1, \ldots$$

for $\delta = const > 0$, $\varrho \in\ C^{\alpha}[\mathbb{R}, \mathbb{R}_+]$, then

$$L(t, y(t)) \le L(\tau_0^+, y_0)E_{\alpha}(-\delta, t - \tau_0) + \int_{\tau_k}^{t} \frac{W^{\alpha}(t - \tau_k, o - \tau_k)\varrho(o)}{(o - \tau_k)^{1-\alpha}} do$$

$$+ \sum_{j=1}^{k} \prod_{l=k-j+1}^{k} E_{\alpha}(-\delta, \tau_l - \tau_{l-1}) \int_{\tau_{k-j}}^{\tau_{k-j+1}} \frac{W^{\alpha}(t - \tau_k, o - \tau_{k-j})\varrho(o)}{(o - \tau_{k_j})^{1-\alpha}} do,\ t \ge \tau_0,$$

where $W^{\alpha}(t - \tau_k, o - \tau_k) = E_{\alpha}(-\delta, t - \tau_k)E_{\alpha}(\delta, o - \tau_k)$, $0 < \alpha < 1$, $k = 0, 1, 2, \ldots$.

3. Main practical stability with respect to manifolds criteria

In this Section, we will apply the modified conformable Lyapunov-function approach to establish our main practical stability with respect to the manifold $\mathcal{M}^{H}_{t}$ criteria for the conformable impulsive Cohen–Grossberg neural network model (1).

Theorem 12. *Assume that $0 < \lambda < A$, hypotheses H1–H4 hold and:*

(i) there exists a function $H : [\tau_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^m$, $m \le n$, such that

$$||H(t, y)|| \le ||y|| \le \Gamma(\Omega)||H(t, y)||,\ (t, y) \in [\tau_0, \infty) \times \mathbb{R}^n, \tag{8}$$

where $\Gamma = \Gamma(\Omega)$ is a constant, $\Gamma(\Omega) \ge 1$, $\Gamma(\Omega)\lambda < A$ for any $0 < \Omega < \infty$;

(ii) the parameters and the impulsive control functions of the impulsive conformable Cohen–Grossberg neural network model (1) satisfy:

$$\min_{1\le\iota\le n} \Big(\underline{a}_{\iota}\beta_{\iota} - \overline{a}_{\iota}L^{f}_{\iota} \sum_{j=1}^{n} c_{j\iota}\Big) > 0, \tag{9}$$

$$U_{\iota}(t) = 0,\ t \in [\tau_0, \infty),\ \iota = 1, 2, \ldots, n, \tag{10}$$

$$P_{\iota k}(y_{\iota}(\tau_k)) = -\gamma_{\iota k} y_{\iota}(\tau_k),\ 0 \le \gamma_{\iota k} \le 2,\ \iota = 1, 2, \ldots, n,\ k = 1, 2, \ldots. \tag{11}$$

Then, the manifold $\mathcal{M}^{H}_{t}$ is practically stable for the model (1).

Proof. For the given $0 < \lambda < A$ and $y_0 \in \mathbb{R}^n$ let suppose that $y_0 \in \mathcal{M}^H_{\tau_0}(\lambda)$ and consider the solution $y(t; \tau_0, y_0)$ of the impulsive conformable Cohen–Grossberg model (1) corresponding to the initial condition (2).

We construct a Lyapunov-type function as

$$L_1(t, y) = \|y(t)\| = \sum_{\iota=1}^{n} |y_\iota(t)|. \tag{12}$$

From condition (11) of Theorem 12, at the impulsive control instants $t = \tau_k$, $k = 1, 2, \ldots$, we have:

$$L_1(\tau_k^+, y(\tau_k^+)) = \sum_{\iota=1}^{n} |y_\iota(\tau_k) + P_{\iota k}(y_\iota(\tau_k))| = \sum_{\iota=1}^{n} |1 - \gamma_{\iota k}||y_\iota(\tau_k)|$$

$$\leq \sum_{\iota=1}^{n} |y_\iota(\tau_k)| = L_1(\tau_k, y(\tau_k)), \tag{13}$$

for $k = 1, 2, \ldots$.

From H1-H3 and conditions (9) and (10) of Theorem 12, for $t \neq \tau_k$, $k = 0, 1, 2, \ldots$, we obtain:

$${}^{+}\mathcal{D}^{\alpha}_{\tau_k} L_1(t, y(t)) \leq \sum_{\iota=1}^{n} \Big[-\underline{a}_\iota \beta_\iota |y_\iota(t)| + \overline{a}_\iota \sum_{j=1}^{n} c_{ij} L^f_j |y_\iota(t)| \Big]$$

$$\leq -\min_{1 \leq \iota \leq n} \Big(\underline{a}_\iota \beta_\iota - \overline{a}_\iota L^f_\iota \sum_{j=1}^{n} c_{j\iota} \Big) L_1(t, y(t)) \leq 0. \tag{14}$$

Now, from (13) and (14), according to Lemma 11 for $\delta = \varrho(t) = 0$, $t \in [\tau_0, \infty)$, we have

$$L_1(t, y(t; \tau_0, y_0)) \leq L_1(\tau_0^+, y_0), \; t \geq \tau_0.$$

Hence, from condition (8) of Theorem 12, we obtain

$$\|H(t, y(t; \tau_0, y_0))\| \leq \|y(t; \tau_0, y_0)\| \leq \|y_0\| \leq \Gamma(\Omega)\|H(t, y_0)\| < \Gamma(\Omega)\lambda < A, \tag{15}$$

and, therefore, the manifold $\mathcal{M}^H_t$ is practically stable for the model (1). □

Next, criteria for the practical exponential stability of the manifold $\mathcal{M}^H_t$ for the model (1) will be presented.

Theorem 13. *Assume that* $0 < \lambda < A$*, hypotheses H1–H4 hold and:*

(i) there exists a function $H : [\tau_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^m$*,* $m \leq n$*, which satisfies (8) for* $\Gamma(\Omega) \geq 1$ *any* $0 < \Omega < \infty$*;*

(ii) condition (11) of Theorem 12 holds and there exists a positive δ_1 *such that*

$$\min_{1 \leq \iota \leq n} \Big(\underline{a}_\iota \beta_\iota - \overline{a}_\iota L^f_\iota \sum_{j=1}^{n} c_{j\iota} \Big) \geq \delta_1 > 0, \tag{16}$$

and

$$V_1(t) = \int_{\tau_0}^{\infty} \frac{W^\alpha(t-\tau_k, o-\tau_k)}{(o-\tau_0)^{1-\alpha}} \sum_{\iota=1}^{n} \overline{a}_\iota U_\iota(o) do$$

$$+ \sum_{j=1}^{k} \prod_{l=k-j+1}^{k} E_\alpha(-\delta, \tau_l - \tau_{l-1}) \int_{\tau_{k-j}}^{\tau_{k-j+1}} \frac{W^\alpha(t-\tau_k, o-\tau_{k-j})}{(o-\tau_{k_j})^{1-\alpha}} \sum_{\iota=1}^{n} \overline{a}_\iota U_\iota(o) do < \infty. \tag{17}$$

Then the manifold $\mathcal{M}_t^H$ is practically exponentially stable for the model (1).

Proof. For the given $0 < \lambda < A$ and $y_0 \in \mathbb{R}^n$ let again suppose that $y_0 \in \mathcal{M}_{\tau_0}^H(\lambda)$ and consider the solution $y(t;\tau_0,y_0)$ of the impulsive conformable Cohen–Grossberg model (1) corresponding to the initial condition (2). Without loss generality, we can consider $1 < \lambda < V_1(t) < A$, $t \in [\tau_0, \infty)$.

Consider again the Lyapunov-type function (12).

Following the same reasoning as in the proof of Theorem 12, at the impulsive control instants $t = \tau_k$, $k = 1, 2, \ldots$, we obtain (13).

Using H1–H3 and conditions (16) of Theorem 13, for $t \neq \tau_k$, $k = 0, 1, 2, \ldots$, we have

$$^{+}\mathcal{D}_{\tau_k}^{\alpha} L_1(t, y(t)) \leq \sum_{\iota=1}^{n} \Big[-\underline{a}_\iota \beta_\iota |y_\iota(t)| + \overline{a}_\iota \sum_{j=1}^{n} c_{\iota j} L_j^f |y_\iota(t)| + \overline{a}_\iota U_\iota(t) \Big]$$

$$\leq -\min_{1\leq\iota\leq n} \Big(\underline{a}_\iota \beta_\iota - \overline{a}_\iota L_\iota^f \sum_{j=1}^{n} c_{j\iota}\Big) L_1(t, y(t)) + \sum_{\iota=1}^{n} \overline{a}_\iota U_\iota(t) \leq -\delta_1 L_1(t, y(t)) + \varrho_1(t), \tag{18}$$

where $\varrho_1(t) = \sum_{\iota=1}^{n} \overline{a}_\iota U_\iota(t)$.

Now, from (13) and (18), using again Lemma 11, we have

$$L_1(t, y(t;\tau_0,y_0)) \leq L_1(\tau_0^+, y_0) E_\alpha(-\delta_1, t-\tau_0) + V_1(t), \ t \geq \tau_0.$$

Hence, from condition (i) of Theorem 13 and (17), and from the choice of the function $L_1 \in \mathcal{L}_k^\alpha$, we have

$$\begin{aligned} ||H(t, y(t;\tau_0,y_0))|| &\leq L_1(t, y(t;\tau_0,y_0)) \\ &< A + \Gamma(\Omega)||H(\tau_0^+, y_0)||E_\alpha(-\delta_1, t-\tau_0), \quad t \geq \tau_0. \end{aligned}$$

Therefore,

$$y(t;\tau_0,y_0) \in \mathcal{M}_t^H(A + \Gamma(\Omega)||H(\tau_0,y_0)||E_\alpha(-\delta_1, t-\tau_0), \ t \geq \tau_0, \ \text{for some } \tau_0 \in \mathbb{R}_+,$$

which implies that the manifold $\mathcal{M}_t^H$ is practically exponentially stable for the model (1). □

In the next result, we will use a different Lyapunov function in order to establish more relaxed sufficient conditions for the parameters in the continuous part of model (1). The price is, more restrictive conditions for the jump functions.

Theorem 14. *Assume that* $0 < \lambda < A$*, hypotheses H1–H4 hold and:*

(i) there exists a function $H : [\tau_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^m$*,* $m \leq n$*, such that*

$$||H(t, y)|| \leq \sum_{\iota=1}^{n} \int_0^{y_\iota(t)} \frac{sgn(o)}{a_\iota(o)} do \leq \Gamma(\Omega)||H(t, y)||, \ (t, y) \in [\tau_0, \infty) \times \mathbb{R}^n, \tag{19}$$

where $\Gamma(\Omega) \geq 1$ *for any* $0 < \Omega < \infty$*;*

(ii) there exists a positive δ *such that the parameters and the impulsive control functions of the impulsive conformable Cohen–Grossberg neural network model (1) satisfy:*

$$\underline{a} \min_{1 \leq \iota \leq n} \Big(\beta_\iota - L_\iota^f \sum_{j=1}^{n} c_{j\iota} \Big) \geq \delta > 0, \tag{20}$$

$$P_{\iota k}(y_\iota(\tau_k)) = -\eta_{\iota k} y_\iota(\tau_k), \ |1 - \eta_{\iota k}| \leq \frac{\overline{a}}{\underline{a}}, \ \iota = 1, 2, \ldots, n, \ k = 1, 2, \ldots, \tag{21}$$

and

$$V(t) = \int_{\tau_0}^{\infty} \frac{W^\alpha(t - \tau_k, o - \tau_k)}{(o - \tau_0)^{1-\alpha}} \sum_{\iota=1}^{n} U_\iota(o) do$$

$$+ \sum_{j=1}^{k} \prod_{l=k-j+1}^{k} E_\alpha(-\delta, \tau_l - \tau_{l-1}) \int_{\tau_{k-j}}^{\tau_{k-j+1}} \frac{W^\alpha(t - \tau_k, o - \tau_{k-j})}{(o - \tau_{k_j})^{1-\alpha}} \sum_{\iota=1}^{n} U_\iota(o) do < \infty. \tag{22}$$

Then the manifold $\mathcal{M}_t^H$ *is practically exponentially stable for the model (1).*

Proof. Let $0 < \lambda < A$. No generality is lost by making the assumption $1 < \lambda < U(t) < A$, $t \in [\tau_0, \infty)$.

We construct a new Lyapunov-type function as

$$L(t, y) = \sum_{\iota=1}^{n} \int_{0}^{y_\iota(t)} \frac{sgn(o)}{a_\iota(o)} do, \tag{23}$$

for which, we have

$$\frac{1}{\overline{a}} \|y(t)\| \leq L(t, y(t)) \leq \frac{1}{\underline{a}} \|y(t)\|, \ t \in [\tau_0, \infty) \tag{24}$$

for any solution $y(t) = y(t; \tau_0, y_0)$ of the impulsive conformable Cohen–Grossberg model (1) corresponding to the initial condition (2) with $y_0 \in \mathcal{M}_{\tau_0}^H(\lambda)$.

From (21) and (24), at the impulsive control instants $t = \tau_k$, $k = 1, 2, \ldots$, we have:

$$L(\tau_k^+, y(\tau_k^+)) \leq \frac{1}{\underline{a}} \|y(\tau_k^+)\| = \frac{1}{\underline{a}} \sum_{\iota=1}^{n} |y_\iota(\tau_k) + P_{\iota k}(y_\iota(\tau_k))|$$

$$= \frac{1}{\underline{a}} \sum_{\iota=1}^{n} |1 - \eta_{\iota k}| |y_\iota(\tau_k)| \leq \frac{1}{\overline{a}} \sum_{\iota=1}^{n} |y_\iota(\tau_k)| = \frac{1}{\overline{a}} \|y(\tau_k)\| \leq L(\tau_k, y(\tau_k)), \tag{25}$$

for $k = 1, 2, \ldots$.

Using H1–H3, (20) and (24) from the choice of the function $L \in \mathcal{L}_k^\alpha$ for $t \neq \tau_k$, $k = 0, 1, 2, \ldots$, we have

$${}^{+}\mathcal{D}_{\tau_k}^{\alpha} L(t, y(t)) \leq \sum_{\iota=1}^{n} \Big[-\beta_\iota |y_\iota(t)| + \sum_{j=1}^{n} c_{\iota j} L_j^f |y_\iota(t)| + U_\iota(t) \Big]$$

$$\leq -\min_{1 \leq \iota \leq n} \Big(\beta_\iota - L_\iota^f \sum_{j=1}^{n} c_{j\iota} \Big) \|y(t)\| + \sum_{\iota=1}^{n} U_\iota(t)$$

$$\leq -\underline{a}\min_{1\leq\iota\leq n}\Big(\beta_\iota - L_\iota^f\sum_{j=1}^n c_{j\iota}\Big)L(t,y(t)) + \sum_{\iota=1}^n U_\iota(t) \leq -\delta L(t,y(t)) + \varrho(t), \tag{26}$$

where $\varrho(t) = \sum_{\iota=1}^n U_\iota(t)$.

Now, from (25) and (26), using again Lemma 11, we have

$$L(t,y(t;\tau_0,y_0)) \leq L(\tau_0^+,y_0)E_\alpha(-\delta,t-\tau_0) + V(t),\ t\geq\tau_0.$$

Hence, from condition (i) of Theorem 14 and (22), we have

$$\|H(t,y(t;\tau_0,y_0))\| \leq L(t,y(t;\tau_0,y_0))$$

$$< A + \Gamma(\Omega)\|H(\tau_0^+,y_0)\|E_\alpha(-\delta,t-\tau_0),\quad t\geq\tau_0.$$

Therefore,

$$y(t;\tau_0,y_0)\in\mathcal{M}_t^H(A+\Gamma(\Omega)\|H(\tau_0,y_0)\|E_\alpha(-\delta,t-\tau_0),\ t\geq\tau_0,\ \text{for some } \tau_0\in\mathbb{R}_+,$$

which implies that the manifold $\mathcal{M}_t^H$ is practically exponentially stable for the model (1). □

Following the same steps as in the proof of Theorem 14, we can establish the following practical stability criteria.

Corollary 15. *If in Theorem 14 condition (20) is replaced by*

$$\underline{a}\min_{1\leq\iota\leq n}\Big(\beta_\iota - L_\iota^f\sum_{j=1}^n c_{j\iota}\Big) > 0 \tag{27}$$

and condition (22) is replaced by condition (10) of Theorem 12, then the manifold $\mathcal{M}_t^H$ is practically stable for the model (1).

Remark 16. The results in this section generalize and extend many existing stability results for integer-order Cohen–Grossberg neural network models [3–8], impulsive Cohen–Grossberg neural network models [15–17], as well as, Cohen–Grossberg neural network models with classical fractional-order derivatives [26,27,29,30] to the generalized impulsive conformable setting. Also, since the concept of practical stability with respect to manifolds is more general, results on practical stability, stability with respect to manifolds, stability and practical stability of a single solution can be obtained as corollaries. Thus, the proposed results are very general and include many particular stability notions. For example, if in Definition 9, $A = 0$ and the manifold $\mathcal{M}_t^H$ consists only of a single solution y^*, Theorems 13 and 14 offered criteria for exponential stability of the solution y^* of the impulsive conformable Cohen–Grossberg neural network model (1).

Remark 17. The proposed stability of manifold results are also consistent with the results in the pioneering work of Cohen and Grossberg [3], where models capable of approaching infinitely many equilibrium points in response to arbitrary initial data, have been considered.

Remark 18. The impulsive functions $P_{\iota k}$, $\iota = 1, 2, \dots, n$, $k = 1, 2, \dots$ are the impulsive control functions in the introduced model (1). The suitable choice of these functions is crucial for the elaboration of effective impulsive control strategies that can be applied to the trajectories of the model to control their stability properties. Hence, our results contribute to the development of this direction of research. Indeed, the impulsive conformable Cohen–Grossberg neural network model (1) can be considered as a closed-loop control system represented as

$$\mathcal{D}^{\alpha}_{\tau_k} y_\iota(t) = -a_\iota(y_\iota(t))\left[b_\iota(y_\iota(t)) - \sum_{j=1}^{n} c_{\iota j} f_j(y_j(t)) - U_\iota(t)\right] + u_\iota(t),\ t > \tau_0,$$

where

$$u_\iota(t) = \sum_{k=1}^{\infty} P_{\iota k}(y_\iota(t))\delta(t - \tau_k),\ \iota = 1, 2, ..., n$$

is the control input, $\delta(t)$ is the Dirac impulsive function. The controller $u(t) = (u_1(t), ..., u_n(t))^T$ has an effect on sudden change of the states of (1) at the time instants τ_k where each state $y_\iota(t)$ changes from the position $y_\iota(\tau_k)$ into the position $y_\iota(\tau_k^+)$, $P_{\iota k}$ are the functions that characterize the magnitudes of the impulse effects on the units y_ι at the moments τ_k, i.e., $u(t)$ is an impulsive control of the conformable model without impulses

$$\mathcal{D}^{\alpha}_{\tau_k} y_\iota(t) = -a_\iota(y_\iota(t))\left[b_\iota(y_\iota(t)) - \sum_{j=1}^{n} c_{\iota j} f_j(y_j(t)) - U_\iota(t)\right],\ t > \tau_0.$$

Therefore, our stability results also presents a general design method of impulsive control law $u(t)$ for the impulse free conformable Cohen–Grossberg neural network model. The constants $\gamma_{\iota k}$ in condition (ii) of Theorem 12 characterize the control gains of synchronizing impulses. Hence, our results can be used to design impulsive control law under which the controlled neural networks of type (1) are practically (practically exponentially) synchronized onto systems without impulses.

4. Impulsive conformable BAM Cohen–Grossberg neural networks

BAM neural networks have attracted an increased interest of numerous researchers since their introduction by Kosko [61, 62], due mainly to their important applications in many areas. Some typical applications are related to associative phenomena in two-layer hetero-associative circuits existing in biology, medicine, engineering and computer sciences.

Significant progress has been made in the theory and applications of different classes of integer-order BAM Cohen–Grossberg neural networks [63, 64], including impulsive models [21, 65, 66]. A practical stability analysis has also been proposed for some integer-order BAM Cohen–Grossberg neural network models [67].

Correspondingly, there are not reported results in the existing literature on BAM Cohen–Grossberg neural networks with conformable derivatives. In this Section, we will propose results on practical stability of such models with respect to manifolds as corollaries of the results offered in Section 3.

We consider the following BAM Cohen–Grossberg neural network model with conformable derivatives

$$
\begin{cases}
\mathcal{D}^{\alpha}_{\tau_k} y_\iota(t) = -a_\iota(y_\iota(t))\Big[b_\iota(y_\iota(t)) - \sum_{j=1}^{N} c^*_{j\iota} f^*_j(z_j(t)) - U_\iota(t)\Big], \quad t \neq \tau_k,\ k = 0,1,2\ldots, \\
\mathcal{D}^{\alpha}_{\tau_k} z_j(t) = -a^*_j(z_j(t))\Big[b^*_j(z_j(t)) - \sum_{\iota=1}^{n} c_{\iota j} f_\iota(y_\iota(t)) - U^*_j(t)\Big], \quad t \neq \tau_k,\ k = 0,1,2\ldots, \\
y_\iota(\tau_k^+) = y_\iota(\tau_k) + P_{\iota k}(y_\iota(\tau_k)),\ k = 1,2,\ldots, \\
z_j(\tau_k^+) = z_j(\tau_k) + P^*_{jk}(z_j(\tau_k)),\ k = 1,2,\ldots,
\end{cases}
\tag{28}
$$

where $t \geq \tau_0$, $y_\iota(t)$ and $z_j(t)$ represent the states of the ιth neuron in the first Y layer and jth neuron in the second Z-layer, respectively, at time t, f_j, f^*_ι are the activation functions, a_ι, a^*_j represent the amplification functions, b_ι, b^*_j denote well behaved functions, $c_{j\iota}$, $c^*_{\iota j}$ are the connection weights, the continuous functions U_ι, U^*_j are external inputs, $\iota = 1,2,\ldots,n$, $j = 1,2,\ldots,N$. The impulsive functions $P_{\iota k}, P^*_{jk} \in C[\mathbb{R},\mathbb{R}]$ represent the effects of the impulse controls on the nodes y_ι and z_j, respectively, at τ_k, $\iota = 1,2,\ldots,n$, $j = 1,2,\ldots,N$, $k = 1,2,\ldots$.

Following the definitions of solutions of the one layer Cohen–Grossber model (1) will denote by $u(t)$, $u(t) = (y(t), z(t))$, where $y(t) = y(t;\tau_0, y_0)$, $z(t) = z(t;\tau_0, z_0)$ the solution of the BAM neural network model (28) that satisfies the initial conditions:

$$y(\tau_0^+) = y_(\tau_0) = y_0, \quad z(\tau_0^+) = z(\tau_0) = z_0, \tag{29}$$

where $y_0 \in \mathbb{R}^n$, $z_0 \in \mathbb{R}^N$ and $u_0 = (y_0, z_0) \in \mathbb{R}^{n+N}$.

We will apply the practical stability notion with respect to a H^*-manifold

$$\mathcal{M}^{H^*}_t = \{u \in \mathbb{R}^{n+N} :\ H^*_1(t,u) = H^*_2(t,u) = \cdots = H^*_m(t,u) = 0,\ t \in [\tau_0, \infty)\} \tag{30}$$

defined by a function $H^* = H^*(t,u)$, $H^* : [\tau_0, \infty) \times \mathbb{R}^{n+N} \to \mathbb{R}^m$, using the following definition.

Definition 19. The manifold $\mathcal{M}^{H^*}_t$ is called practically exponentially stable for the model (28) if given $(\lambda,\ A)$ with $0 < \lambda < A$, $u_0 \in \mathcal{M}^{H^*}_{\tau_0}(\lambda)$ implies

$$u(t;\tau_0, u_0) \in \mathcal{M}^{H^*}_t(A + \mu\|H^*(\tau_0, u_0)\|E_\alpha(-\delta, t - \tau_0)),\ t \geq \tau_0,\ \text{for some } \tau_0 \in \mathbb{R}_+,$$

where $0 < \alpha < 1, \mu, \delta > 0$.

For the BAM neural network model (28) we can use the demonstrated Lyapunov function approach in Section 3, and obtain similar results. For example, as a corollary of Theorem 14, we can establish the following practical exponential stability criteria for the manifold $\mathcal{M}^{H^*}_t$ of the system (28).

Theorem 20. *Assume that* $0 < \lambda < A$, *hypotheses H1–H4 and conditions (20), (21) of Theorem 14 hold and:*

(i) the continuous functions $a^*_j \in \mathbb{R}_+$, $j = 1,2,\ldots,N$ *are such that there exist positive constants* $\underline{a}^*_j$ *and* $\overline{a}^*_j$ *for which* $1 < \underline{a}^*_j \leq a^*_j(\chi) \leq \overline{a}^*_j$ *for* $\chi \in \mathbb{R}$, *and* $\underline{a}^* = \min_{1\leq j\leq N} a^*_j$, $\overline{a}^* = \max_{1\leq j\leq N} a^*_j$;

(ii) the continuous functions $b_j^* \in \mathbb{R}_+$ *are such that there exist constants* $\beta_j^* > 0$ *with*

$$b_j^*(\chi) \geq \beta_j^* |\chi|,$$

for any $\chi \in \mathbb{R}$ *and* $j = 1, 2, \ldots, N$*;*

(iii) there exist Lipschitz constants $L_j^{f^*} > 0$ *such that*

$$|f_j^*(\chi_1) - f_j^*(\chi_2)| \leq L_j^{f^*} |\chi_1 - \chi_2|, \; f_j^*(0) = 0$$

for all $\chi_1, \chi_2 \in \mathbb{R},\; j = 1, 2, \ldots, N$*;*

(iv) there exists a function $H^* : [\tau_0, \infty) \times \mathbb{R}^{n+N} \to \mathbb{R}^m$*,* $m \leq n$*, such that*

$$||H^*(t,u)|| \leq \sum_{\iota=1}^{n} \int_0^{y_\iota(t)} \frac{sgn(o)}{a_\iota(o)} do + \sum_{j=1}^{N} \int_0^{z_j(t)} \frac{sgn(o)}{a_j^*(o)} do$$

$$\leq \Gamma(\Omega) ||H^*(t,u)||, \; (t,u) \in [\tau_0, \infty) \times \mathbb{R}^{n+N},$$

where $\Gamma(\Omega) \geq 1$ *for any* $0 < \Omega < \infty$*;*

(v) each solution $u(t) = (y(t), z(t)),\; y(t) = y(t; \tau_0, y_0),\; z(t) = z(t; \tau_0, z_0)$*, of the initial value problem (28), (29) such that*

$$||H^*(t, u(t))|| \leq \Omega < \infty$$

is defined on $[\tau_0, \infty)$*;*

(vi) there exists a positive δ^* *such that the parameters and the impulsive control functions of the impulsive conformable Cohen–Grossberg neural network model (28) satisfy:*

$$\underline{a}^* \min_{1 \leq j \leq N} \Big(\beta_j^* - L_j^{f^*} \sum_{\iota=1}^{n} c_{\iota j}^* \Big) \geq \delta^* > 0, \tag{31}$$

$$P_{jk}^*(z_j(\tau_k)) = -\eta_{jk}^* z_j(\tau_k), \; |1 - \eta_{jk}^*| \leq \frac{\underline{a}^*}{\overline{a}^*}, \; j = 1, 2, \ldots, N, \; k = 1, 2, \ldots, \tag{32}$$

and

$$S(t) = \int_{\tau_0}^{\infty} \frac{W^\alpha(t - \tau_k, o - \tau_k)}{(o - \tau_0)^{1-\alpha}} \left(\sum_{\iota=1}^{n} U_\iota(o) + \sum_{j=1}^{N} U_j^*(o) \right) do$$

$$+ \sum_{j=1}^{k} \prod_{l=k-j+1}^{k} E_\alpha(-\delta, \tau_l - \tau_{l-1}) \int_{\tau_{k-j}}^{\tau_{k-j+1}} \frac{W^\alpha(t - \tau_k, o - \tau_{k-j})}{(o - \tau_{k_j})^{1-\alpha}} \left(\sum_{\iota=1}^{n} U_\iota(o) + \sum_{j=1}^{N} U_j^*(o) \right) do < \infty. \tag{33}$$

Then the manifold $\mathcal{M}_t^{H^*}$ *is practically exponentially stable for the model (28).*

Remark 21. As a Corollary of Theorem 14, Theorem 20 proposes criteria for practical exponential stability of a manifold for the BAM impulsive conformable Cohen–Grossberg model (28). Similarly, criteria for practical stability of the manifold $\mathcal{M}_t^{H^*}$ can be established as a Corollary of Theorem 12.

5. Illustrative examples

In this section, examples are constructed to illustrate the usefulness of the proposed results.

Example 22. We consider the 2-dimensional impulsive Cohen–Grossberg-type neural network model with generalized conformable derivatives

$$\begin{cases} \mathcal{D}^{\alpha}_{t_k} y_1(t) = -a_1(y_1(t))\Big[b_1(y_1(t)) - \sum_{j=1}^{2} c_{1j} f_j(y_j(t)) - U_1(t)\Big], \quad t \neq \tau_k,\ k = 0, 1, 2 \ldots, \\ \mathcal{D}^{\alpha}_{t_k} y_2(t) = -a_2(y_2(t))\Big[b_2(y_2(t)) - \sum_{j=1}^{2} c_{2j} f_j(y_j(t)) - U_2(t)\Big], \quad t \neq \tau_k,\ k = 0, 1, 2 \ldots, \\ y_1(\tau_k^+) = \dfrac{y_1(t_k)}{8},\ k = 1, 2, \ldots, \quad y_2(\tau_k^+) = \dfrac{y_2(t_k)}{6},\ k = 1, 2, \ldots, \end{cases} \tag{34}$$

where $\alpha = 0.98$, $t > 0$, $0 = \tau_0 < \tau_1 < \tau_2 < \ldots$, $k = 1, 2, \ldots$, $\lim_{k\to\infty} \tau_k = \infty$, $f_j(y_j) = \frac{1}{2}\Big(|y_j + 1| - |y_j - 1|\Big)$, $U_1(t) = \sin t$, $U_2(t) = 1$, $b_1(y_1) = 3y_1$, $b_2(y_2) = 2y_2$,

$$(c_{ij}) = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 1.5 & 1.1 \\ 1.3 & 0.1 \end{pmatrix}.$$

(A) If in the model (34), we have $a_1(y_1) = 2 - 0.5\sin(y_1)$, $a_2(y_2) = 2 - 0.6\cos(y_2)$, then hypotheses H1–H3 are satisfied for

$$\underline{a}_1 = 1.5, \quad \overline{a}_1 = 2.5, \quad \underline{a}_2 = 1.4, \quad \overline{a}_2 = 2.6, \quad \underline{a} = 1.4, \quad \overline{a} = 2.6,$$
$$\beta_1 = 3, \quad \beta_2 = 2, \quad L_1^f = L_2^f = 1.$$

Also, conditions (20), (21) and (22) of Theorem 14 are satisfied for

$$\underline{a} \min_{1\le\iota\le2} \Big(\beta_\iota - L_\iota^f \sum_{j=1}^{2} c_{j\iota}\Big) = 0.56 \geq \delta > 0,$$

$$|1 - \eta_{1k}| = \frac{1}{8} = 0.125 \leq \frac{\underline{a}}{\overline{a}} = 0.538,\ |1 - \eta_{12k}| = \frac{1}{6} = 0.167 \leq \frac{\underline{a}}{\overline{a}} = 0.538,\ k = 1, 2, \ldots,$$

and $\varrho(t) = 1 + \sin t$.

If we consider a manifold

$$\mathcal{M}_t^H = \{y \in \mathbb{R}^2 : H_1 = H_2 = 0\}, \tag{35}$$

containing the zero solution $y^* = (0, 0)^T$ of the model (34), defined by a function $H : \mathbb{R}^2 \to \mathbb{R}^2$, $H = (H_1, H_2)^T$, then according to Theorem 14 the manifold (35) is practically exponentially stable for the model (34). The graphs of the trajectories of the model (34) with corresponding fixed τ_k are shown in Figure 1.

Note that for the particular choice of the parameters, condition (16) of Theorem 13 is not satisfied, since

$$\min_{1\le\iota\le 2}\left(\underline{a}_\iota\beta_\iota - \overline{a}_\iota L^f_\iota \sum_{j=1}^{2} c_{j\iota}\right) = -2 < 0.$$

Hence, the criteria provided by Theorem 13 for the parameters in the continuous part are more restrictive.

(B) If in the model (34), we have $a_1(y_1) = a_2(y_2) = 1.4$, then, hypotheses H1–H3 are satisfied for

$$\underline{a}_1 = \overline{a}_1 = \underline{a}_2 = \overline{a}_2 = \underline{a} = \overline{a} = 1.4,$$
$$\beta_1 = 3, \quad \beta_2 = 2, \quad L^f_1 = L^f_2 = 1.$$

In this case, conditions of Theorem 13 and Theorem 14 are satisfied for $\delta = \delta_1 = 0.56 > 0$. Therefore, the practical exponential stability of the manifold (35) can be provided by either one of the results.

(C) If in the model (34), we have

$$\eta_{1k} = 0.3,$$

then condition (21) of Theorem 14 is not satisfied, however, condition (11) of Theorem 12 holds. Hence, in this case we can make some conclusions about the practical exponential behavior of the manifold (35) only by means of Theorem 12.

In addition, if

$$\eta_{\iota k} = -1, \ \iota = 1, 2, \ k = 1, 2, \ldots,$$

then no one of the conditions about the impulsive functions in theorems 13 and 14 are not true, and we cannot make any conclusion about the practical exponential behavior of the manifold (35).

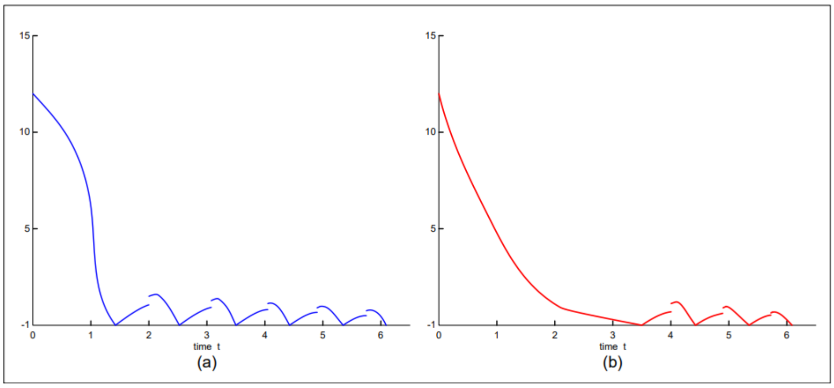

Figure 1. The graphs of the practically exponentially stable trajectories of the model (34) for $y_0 = (y_{01}, y_{02})^T = (12, 12)^T$ and corresponding fixed τ_k: (a) the trajectory of $y_1(t)$; (b) the trajectory of $y_2(t)$.

Remark 23. The demonstrated Example 22 illustrates the validity of theorems 13 and 14 for practical exponential stability of a manifold with respect to an impulsive conformable Cohen–Grossberg neural network model of the type (1). Similar demonstration for the practical stability behavior can be provided for a particular choice of the model's parameters. We show that the conditions for the parameters in the continuous part in Theorem 14 are more relaxed than these in Theorem 13. Also, we demonstrate that the choice of the impulsive functions is very important to control the practical stability behavior of manifolds related to models of type (1).

Example 24. Consider the BAM Cohen–Grossberg impulsive conformable neural network model (28) with $n = N = 2$, $\alpha = 0.93$, where $t > 0$,

$$y(t) = \begin{pmatrix} y_1(t) \\ \\ y_2(t) \end{pmatrix}, \quad z(t) = \begin{pmatrix} z_1(t) \\ \\ z_2(t) \end{pmatrix}, \quad U_1 = U_2 = U_1^* = U_2^* = 1,$$

$$a_1(y_1) = a_2(y_2) = a_1^*(z_1) = a_2^*(z_2) = 1.1, \quad b_\iota(y_\iota) = 4y_\iota, \quad b_j^*(z_j) = 4z_j,$$

$$f_\iota(y_\iota) = \frac{|y_\iota + 1| - |y_\iota - 1|}{2}, \quad f_j^*(z_j) = \frac{|z_j + 1| - |z_j - 1|}{2}, \quad \iota = 1,2, \ j = 1,2,$$

$$(c_{\iota j})_{2\times 2} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 1.3 & 1.5 \\ 1.1 & 1.4 \end{pmatrix}, \ (c^*_{j\iota})_{2\times 2} = \begin{pmatrix} c^*_{11} & c^*_{12} \\ c^*_{21} & c^*_{22} \end{pmatrix} = \begin{pmatrix} 1.4 & 1.3 \\ 1.2 & 1.1 \end{pmatrix},$$

with impulsive perturbations of the type

$$\begin{aligned} y(\tau_k^+) - y(\tau_k) &= \begin{pmatrix} -1 + \frac{1}{3k} & 0 \\ 0 & -1 + \frac{1}{3k} \end{pmatrix} y(\tau_k), \quad k = 1,2,\ldots, \\ z(\tau_k^+) - z(\tau_k) &= \begin{pmatrix} -1 + \frac{1}{4k} & 0 \\ 0 & -1 + \frac{1}{4k} \end{pmatrix} z(\tau_k), \quad k = 1,2,\ldots, \end{aligned} \tag{36}$$

$0 = \tau_0 < \tau_1 < \tau_2 < \ldots$, $k = 1,2,\ldots$, $\lim\limits_{k\to\infty} \tau_k = \infty$.

We can verify that all assumptions of Theorem 20 are satisfied for

$$L_1^f = L_2^f = 1, \quad L_1^{f^*} = L_2^{f^*} = 1, \quad \beta_1 = \beta_2 = \beta_1^* = \beta_2^* = 4,$$
$$\underline{a}_1 = \overline{a}_1 = \underline{a}_2 = \overline{a}_2 = \underline{a} = \overline{a} = 1.1$$
$$\underline{a}_1^* = \overline{a}_1^* = \underline{a}_2^* = \overline{a}_2^* = \underline{a}^* = \overline{a}^* = 1.1,$$
$$\delta = 1.32, \quad \delta^* = 1.43, \quad \eta_{\iota k} = 1 - \frac{1}{3k}, \quad \eta^*_{jk} = 1 - \frac{1}{4k}, \quad \iota, j = 1,2, \ k = 1,2,\ldots.$$

Therefore, according to Theorem 20, we conclude that the manifold

$$\mathcal{M}_t^{H^*} = \{u \in \mathbb{R}^4 : H_1 = H_2 = H_3 = H_4 = 0\}, \tag{37}$$

defined by a function $H^* : \mathbb{R}^4 \to \mathbb{R}^4$, $H^* = (H_1^*, H_2^*, H_3^*, H_4^*)^T = (y_1 - y_1^*, y_2 - y_2^*, z_1 - z_1^*, z_2 - z_2^*)^T$, where $u^* = (y_1^*, y_2^*, z_1^*, z_2^*)^T$ is a solution of the model (28), is practically exponentially stable for the model (28).

6. Concluding comments

In this paper, using the impulsive generalization of the conformable calculus approach, an impulsive conformable Cohen–Grossberg neural network model is proposed. The introduced model extends and complements numerous existing integer-order Cohen–Grossberg neural network models [3–8], impulsive Cohen–Grossberg neural network models [15–17] as well as Cohen–Grossberg neural network models with classical fractional-order derivatives [26, 27, 29, 30] to the generalized impulsive conformable setting. The proposed model possesses the flexibility of the fractional-order models. Moreover, since the conformable derivatives simplify the computational procedures, it overcomes some limitations existing in the applications of the classical fractional-order problems. We adopt the combined concept of practical stability with respect to manifolds to the introduced model. Using a Lyapunov-based analysis, sufficient conditions are derived to ensure the practical stability and practical exponential stability with respect to manifolds. The case of BAM impulsive conformable Cohen–Grossberg neural network models is also studied to contribute to the development of the stability and control theories. Two illustrative examples are given to demonstrate the effectiveness of the contributed results. The application of the proposed conformable calculus approach to more neural network models is an interesting and challenging topic for future research. Also, it is possible to extend the proposed results to the delayed case and study the delay effects on the qualitative behavior of the neuronal states. In fact, delay effects are unavoidable in most of the neuronal network models. In addition, time delays, also known as transmission delays, may affect the dynamical properties of neural networks.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare there is no conflict of interest.

References

1. M. A. Arbib, *Brains, Machines, and Mathematics*, 2nd edition, Springer-Verlag, New York, 1987.
2. S. Haykin, *Neural Networks: A Comprehensive Foundation*, 2nd edition, Prentice-Hall, Englewood Cliffs, 1998.
3. M. A. Cohen, S. M. A. Grossberg, Absolute stability of global pattern formation and parallel memory storage by competitive neural networks, *IEEE Trans. Syst. Man Cyber.*, **13** (1983), 815–826. https://doi.org/10.1109/TSMC.1983.6313075
4. C. Aouiti, E. A. Assali, Nonlinear Lipschitz measure and adaptive control for stability and synchronization in delayed inertial Cohen–Grossberg-type neural networks, *Int. J. Adapt. Control Signal Process.*, **33** (2019), 1457–1477. https://doi.org/10.1002/acs.3042

5. W. Lu, T. Chen, Dynamical behaviors of Cohen–Grossberg neural networks with discontinuous activation functions, *Neural Networks*, **18** (2015), 231–242. https://doi.org/10.1016/j.neunet.2004.09.004
6. N. Ozcan, Stability analysis of Cohen–Grossberg neural networks of neutral-type: Multiple delays case, *Neural Networks*, **113** (2019), 20–27. https://doi.org/10.1016/j.neunet.2019.01.017
7. S. Han, C. Hu, J. Yu, H. Jiang, S. Wen, Stabilization of inertial Cohen-Grossberg neural networks with generalized delays: A direct analysis approach, *Chaos Solitons Fractals*, **142** (2021), 110432. https://doi.org/10.1016/j.chaos.2020.110432
8. D. Peng, J. Li, W. Xu, X. Li, Finite-time synchronization of coupled Cohen-Grossberg neural networks with mixed time delays, *J. Franklin Inst.*, **357** (2020), 11349–11367. https://doi.org/10.1016/j.jfranklin.2019.06.025
9. J. J. Hopfield, Neural networks and physical systems with emergent collective computational abilities, *Proc. Nat. Acad. Sci.*, **79** (1982), 2554–2558. https://www.pnas.org/doi/10.1073/pnas.79.8.2554
10. L. O. Chua, L. Yang, Cellular neural networks: Theory, *IEEE Trans. Circuits Syst.*, **35** (1988), 1257–1272. https://doi.org/10.1109/31.7600
11. L. O. Chua, L. Yang, Cellular neural networks: Applications, *IIEEE Trans. Circuits Syst.*, **35** (1988), 1273–1290. https://doi.org/10.1109/31.7601
12. W. M. Haddad, V. S. Chellaboina, S. G. Nersesov, *Impulsive and Hybrid Dynamical Systems, Stability, Dissipativity, and Control*, 1st edition, Princeton University Press, Princeton, 2006.
13. X. Li, S. Song, *Impulsive Systems with Delays: Stability and Control*, 1st edition, Science Press & Springer, Singapore, 2022.
14. I. Stamova, G. Stamov, *Applied Impulsive Mathematical Models*, 1st edition, Springer, Cham, 2016.
15. C. Aouiti, F. Dridi, New results on impulsive Cohen–Grossberg neural networks, *Neural Process. Lett.*, **49** (2019), 1459–1483. https://doi.org/10.1007/s11063-018-9880-y
16. B. Lisena, Dynamical behavior of impulsive and periodic Cohen–Grossberg neural networks, *Nonlinear Anal.*, **74** (2011), 4511–4519. https://doi.org/10.1016/j.na.2011.04.015
17. C. Xu, Q. Zhang, On anti–periodic solutions for Cohen–Grossberg shunting inhibitory neural networks with time–varying delays and impulses, *Neural Comput.*, **26** (2014), 2328–2349. https://dl.acm.org/doi/10.1162/NECO_a_00642
18. T. Yang, *Impulsive Control Theory*, 1st edition, Springer, Berlin, 2001.
19. X. Yang, D. Peng, X. Lv, X. Li, Recent progress in impulsive control systems, *Math. Comput. Simul.*, **155** (2019), 244–268. https://doi.org/10.1016/j.matcom.2018.05.003
20. J. Cao, T. Stamov, S. Sotirov, E. Sotirova, I. Stamova, Impulsive control via variable impulsive perturbations on a generalized robust stability for Cohen–Grossberg neural networks with mixed delays, *IEEE Access*, **8** (2020), 222890–222899. https://doi.org/10.1109/ACCESS.2020.3044191

21. X. Li, Exponential stability of Cohen–Grossberg-type BAM neural networks with time-varying delays via impulsive control, *Neurocomputing*, **73** (2009), 525–530. https://doi.org/10.1016/j.neucom.2009.04.022
22. D. Baleanu, K. Diethelm, E. Scalas, J. J. Trujillo, *Fractional Calculus: Models and Numerical Methods*, 1st edition, World Scientific, Singapore, 2012. https://doi.org/10.1100/2012/738423
23. R. Magin, *Fractional Calculus in Bioengineering*, 1st edition, Begell House, Redding, 2006.
24. I. Podlubny, *Fractional Differential Equations*, 1st edition, Academic Press, San Diego, 1999.
25. I. M. Stamova, G. T. Stamov, *Functional and Impulsive Differential Equations of Fractional Order: Qualitative Analysis and Applications*, 1st edition, CRC Press, 2017.
26. P. Anbalagan, E. Hincal, R. Ramachandran, D. Baleanu, J. Cao, C. Huang, et al., Delay-coupled fractional order complex Cohen–Grossberg neural networks under parameter uncertainty: Synchronization stability criteria, *AIMS Math.*, **6** (2021), 2844–2873. https://doi.org/10.3934/math.2021172
27. A. Pratap, R. Raja, J. Cao, C. P. Lim, O. Bagdasar, Stability and pinning synchronization analysis of fractional order delayed Cohen–Grossberg neural networks with discontinuous activations, *Appl. Math. Comput.*, **359** (2019), 241–260. https://doi.org/10.1016/j.amc.2019.04.062
28. C. Rajivganthi, F. A. Rihan, S. Lakshmanan, P. Muthukumar, Finite-time stability analysis for fractional-order Cohen–Grossberg BAM neural networks with time delays, *Neural Comput. Appl.*, **29** (2018), 1309–1320. https://doi.org/10.1007/s00521-016-2641-9
29. I. Stamova, S. Sotirov, E. Sotirova, G. Stamov, Impulsive fractional Cohen–Grossberg neural networks: Almost periodicity analysis, *Fractal Fractional*, **5** (2021), 78. https://doi.org/10.3390/fractalfract5030078
30. F. Zhang, Z. Zeng, Multiple Mittag-Leffler stability of delayed fractional-order Cohen–Grossberg neural networks via mixed monotone operator pair, *IEEE Trans. Cyber.*, **51** (2021), 6333–6344. https://doi.org/10.1109/TCYB.2019.2963034
31. M. Caputo, M. Fabrizio, A new definition of fractional derivative without singular kernel, *Prog. Fractional Differ. Appl.*, **1** (2015), 1–13. https://dx.doi.org/10.12785/pfda/010201
32. C. Derbazi, H. Hammouche, Caputo–Hadamard fractional differential equations with nonlocal fractional integro-differential boundary conditions via topological degree theory, *AIMS Math.*, **5** (2020), 2694–2709. https://doi.org/10.3934/math.2020174
33. R. Almeida, Caputo–Hadamard fractional derivatives of variable order, *Numer. Functional Anal. Optim.*, **38** (2017), 1–19. https://doi.org/10.1080/01630563.2016.1217880
34. A. Benkerrouche, M. S. Souid, G. Stamov, I. Stamova, Multiterm impulsive Caputo–Hadamard type differential equations of fractional variable order, *Axioms*, **11** (2022), 634. https://doi.org/10.3390/axioms11110634
35. T. Abdeljawad, On conformable fractional calculus, *J. Comput. Appl. Math.*, **279** (2015), 57–66. https://doi.org/10.1016/j.cam.2014.10.016

36. R. Khalil, M. Al-Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative, *J. Comput. Appl. Math.*, **264** (2014), 65–70. https://doi.org/10.1016/j.cam.2014.01.002

37. H. Kiskinov, M. Petkova, A. Zahariev, M. Veselinova, Some results about conformable derivatives in Banach spaces and an application to the partial differential equations, *AIP Conf. Proc.*, **2333** (2021), 120002. https://doi.org/10.1063/5.0041758

38. A. A. Martynyuk, I. M. Stamova, Fractional-like derivative of Lyapunov-type functions and applications to the stability analysis of motion, *Electron. J. Differ. Equations*, **2018** (2018), 1–12.

39. M. Pospíšil, L. Pospíšilova Škripkova, Sturm's theorems for conformable fractional differential equation, *Math. Commun.*, **21** (2016), 273–281.

40. A. Souahi, A. B. Makhlouf, M. A. Hammami, Stability analysis of conformable fractional-order nonlinear systems, *Indagationes Math.*, **28** (2017), 1265–1274. https://doi.org/10.1016/j.indag.2017.09.009

41. A. A. Martynyuk, G. Stamov, I. Stamova, Integral estimates of the solutions of fractional-like equations of perturbed motion, *Nonlinear Anal. Modell. Control*, **24** (2019), 138–149. https://doi.org/10.15388/NA.2019.1.8

42. D. Zhao, M. Luo, General conformable fractional derivative and its physical interpretation, *Calcolo*, **54** (2017), 903–917. https://doi.org/110.1007/s10092-017-0213-8

43. M. Bohner, V. F. Hatipoğlu, Cobweb model with conformable fractional derivatives, *Math. Methods Appl. Sci.*, **41** (2018), 9010–9017. https://doi.org/10.1002/mma.4846

44. A. Harir, S. Malliani, L. S. Chandli, Solutions of conformable fractional-order SIR epidemic model, *Int. J. Differ. Equations*, **2021** (2021), 6636686. https://doi.org/10.1155/2021/6636686

45. W. Xie, C. Liu, W. Z. Wu, W. Li, C. Liu, Continuous grey model with conformable fractional derivative, *Chaos Solitons Fractals*, **139** (2020), 110285. https://doi.org/10.1016/j.chaos.2020.110285

46. S. Sitho, S. K. Ntouyas, P. Agarwal, J. Tariboon, Noninstantaneous impulsive inequalities via conformable fractional calculus, *J. Inequalities Appl.*, **2018** (2018), 261. https://doi.org/10.1186/s13660-018-1855-z

47. G. Stamov, A. Martynyuk, I. Stamova, Impulsive fractional-like differential equations: Practical stability and boundedness with respect to $h-$manifolds, *Fractal Fractional*, **3** (2019), 50. https://doi.org/10.3390/fractalfract3040050

48. J. Tariboon, S. K. Ntouyas, Oscillation of impulsive conformable fractional differential equations, *Open Math.*, **14** (2016), 497–508. https://doi.org/10.1515/math-2016-0044

49. X. Li, D. Peng, J. Cao, Lyapunov stability for impulsive systems via event-triggered impulsive control, *IEEE Trans. Autom. Control*, **65** (2020), 4908–4913. https://doi.org/10.1109/TAC.2020.2964558

50. X. Li, S. Song, J. Wu, Exponential stability of nonlinear systems with delayed impulses and applications, *IEEE Trans. Autom. Control*, **64** (2019), 4024–4034. https://doi.org/10.1109/TAC.2019.2905271

51. T. Stamov, I. Stamova, Design of impulsive controllers and impulsive control strategy for the Mittag–Leffler stability behavior of fractional gene regulatory networks, *Neurocomputing*, **424** (2021), 54–62. https://doi.org/10.1016/j.neucom.2020.10.112

52. I. Stamova, G. Stamov, Impulsive control strategy for the Mittag–Leffler synchronization of fractional-order neural networks with mixed bounded and unbounded delays, *AIMS Math.*, **6** (2021), 2287–2303. https://doi.org/10.3934/math.2021138

53. G. Stamov, I. Stamova, Extended stability and control strategies for impulsive and fractional neural networks: A review of the recent results, *Fractal Fractional*, **7** (2023), 289. https://doi.org/10.3390/fractalfract7040289

54. G. Ballinger, X Liu, Practical stability of impulsive delay differential equations and applications to control problems, in *Optimization Methods and Applications. Applied Optimization* (eds. X. Yang, K. L. Teo and L. Caccetta), Springer, (2001), 3–21.

55. V. Lakshmikantham, S. Leela, A. A. Martynyuk, *Pract. Stab. Nonlinear Syst.*, 1st edition, World Scientific, Teaneck, 1990. https://doi.org/10.1142/1192

56. T. Stamov, Neural networks in engineering design: Robust practical stability analysis, *Cybern. Inf. Technol.*, **21** (2021), 3–14. https://doi.org/10.2478/cait-2021-0039

57. Y. Tian, Y. Sun, Practical stability and stabilisation of switched delay systems with non-vanishing perturbations, *IET Control Theory Appl.*, **13** (2019), 1329–1335. https://doi.org/10.1049/iet-cta.2018.5332

58. G. Stamov, I. M. Stamova, X. Li, E. Gospodinova, Practical stability with respect to h-manifolds for impulsive control functional differential equations with variable impulsive perturbations, *Mathematics*, **7** (2019), 656. https://doi.org/10.3390/math7070656

59. G. Stamov, E. Gospodinova, I. Stamova, Practical exponential stability with respect to $h-$manifolds of discontinuous delayed Cohen–Grossberg neural networks with variable impulsive perturbations, *Math. Modell. Control*, **1** (2021), 26–34. https://doi.org/10.3934/mmc.2021003

60. A. A. Martynyuk, G. Stamov, I. Stamova, Practical stability analysis with respect to manifolds and boundedness of differential equations with fractional-like derivatives, *Rocky Mt. J. Math.*, **49** (2019), 211–233. https://doi.org/10.1216/RMJ-2019-49-1-211

61. B. Kosko, Bidirectional associative memories, *IEEE Trans. Syst. Cybern.*, **18** (1988), 49–60. https://doi.org/10.1109/21.87054

62. B. Kosko, *Neural Networks and Fuzzy Systems: A Dynamical System Approach to Machine Intelligence*, 1st edition, Prentice-Hall, Englewood Cliffs, 1992.

63. M. Syed Ali, S. Saravanan, M. E. Rani, S. Elakkia, J. Cao, A. Alsaedi, et al., Asymptotic stability of Cohen–Grossberg BAM neutral type neural networks with distributed time varying delays, *Neural Process. Lett.*, **46** (2017), 991–1007. https://doi.org/10.1007/s11063-017-9622-6

64. H. Jiang, J. Cao, BAM-type Cohen–Grossberg neural networks with time delays, *Math. Comput. Modell.*, **47** (2008), 92–103. https://doi.org/10.1016/j.mcm.2007.02.020

65. X. Li, Existence and global exponential stability of periodic solution for impulsive Cohen–Grossberg-type BAM neural networks with continuously distributed delays, *Appl. Math. Comput.*, **215** (2009), 292–307. https://doi.org/10.1016/j.amc.2009.05.005

66. C. Maharajan, R. Raja, J. Cao, G. Rajchakit, A. Alsaedi, Impulsive Cohen–Grossberg BAM neural networks with mixed time-delays: An exponential stability analysis issue, *Neurocomputing*, **275** (2018), 2588–2602. https://doi.org/10.1016/j.neucom.2017.11.028

67. T. Stamov, Discrete bidirectional associative memory neural networks of the Cohen–Grossberg type for engineering design symmetry related problems: Practical stability of sets analysis, *Symmetry*, **14** (2022), 216. https://doi.org/10.3390/sym14020216

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