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Research article

Event-triggered tracking control for switched nonlinear systems

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Abstract: In this paper, we study the output tracking control problem based on the event-triggered mechanism for cascade switched nonlinear systems. Firstly, an integral controller based on event-triggered conditions is designed, and the output tracking error of the closed-loop system can converge to a bounded region under the switching signal satisfying the average dwell time. Secondly, it is proved that the proposed minimum inter-event interval always has a positive lower bound and the Zeno behavior is successfully avoided during the sampling process. Finally, the numerical simulation is given to verify the feasibility of the proposed method.

Keywords: switched nonlinear systems; event-triggered control; tracking control; Zeno behavior

1. Introduction

A switched system is a dynamic system consisting of a series of continuous or discrete subsystems and a switching signal that coordinates the switching between these subsystems. This switching signal, also known as the switching law, is a piecewise constant function that depends on the time or the state of the system [1]. A switched system provides a uniform framework for mathematical model of many physical or man-made systems displaying switching features, such as temperature control systems, chemical procedure systems and mechanical manufacturing procedure systems [2–4]. In general, the switched control has become one of the hottest topics in the control field.

The majority of control systems are nowadays implemented on digital platforms. The advantage is that the digital controller is more intelligent and easier to implement complex algorithms. However, there are some intractable problems to be solved. To this end, periodic sampling mechanisms are proposed to solve related problems [5–7].

Time-triggered control systems are often implemented by periodic sampling of the sensors and zero-order holder of the actuators [8–10]. The advantage of periodic sampling mechanism is that the system analysis process can be simplified by using a fixed sampling period [11]. Nevertheless, the control input can only be updated at a fixed sampling instant, and the controller can only apply data at discrete sampling moments. The real-time state of the system was not considered. Thus, when the amount of data transmitted by the system is relatively larger, the sampling period will be relatively smaller, and the sampling scheme will produce a large amount of redundant sampling information. This method generally results in a waste of resources [12].

Therefore, an event-triggered mechanism (ETM) is considered as an effective way to reduce communication burden [13–17]. The key of event-triggered mechanism is to reduce data transmission load in the network, then the performance of the system will be improved, ensuring stability of the system. A simple event-based PID controller was first presented in the late 1990s [18]. It explained the idea of event-triggered mechanism from the perspective of simulation and experimentation, and confirmed that the method can effectively replace periodic sampling control. Wang et al. [19] investigated the consensus tracking problem for a class of uncertain high-order nonlinear systems with event-triggered communication mechanism. It is shown that the output consensus tracking errors will converge to a compact set with the presented distributed adaptive consensus control scheme and the event-triggered communication mechanism. Based on the event-triggered mechanism, a sampled-data-based controller was developed to achieve stabilization for the switched linear system [20].

The importance of the study of tracking control for switched systems arises from the extensive applications in robot tracking control, and guided missile tracking control, etc. [21–23]. Output tracking control causes the output of the system to be as close as possible to track an external reference signal by designing the controller. Yang et al. [24] proposed a state-dependent switching rule and the switching regions to solve an output tracking problem for a class of delayed switched linear systems via the state-dependent switching law and the dynamic output feedback control. Pezeshki et al. [25] studied the problems of stability and H_∞ model reference tracking performance for a class of switched nonlinear systems with uncertain input delay. Tallapragada and Chopra [26] assumed that the desired trajectory and the exogenous input to the reference system are uniformly bounded. An event-based controller that not only guarantees uniform ultimate boundedness of the tracking error, but also ensures non-accumulation of inter-execution times. Lu et al. [27] studied the event-triggered optimal tracking control method for discrete-time nonlinear systems. For the time-invariant desired trajectory, the tracking error is asymptotically stable. For the time-varying desired trajectory, it is shown that the tracking error is uniformly ultimately bounded. The triggering condition reduces communication costs by relaxing the restriction of the asymptotic stability of the system.

At present, more and more results of switched systems based on event-triggered mechanism have been obtained. However, there are few results about the tracking control. Tracking control as one of the basic problems of control theory is necessary and meaningful to study for the switched nonlinear system. Motivated by the above discussion, this paper mainly studied the event-triggered tracking control for switched nonlinear systems by average dwell time method. The main contributions of this paper are summarized as follows:

1) An integral controller combining the state of the system and tracking error integration is designed, and the original system is converted into an augmented system. For the cascaded switched nonlinear system, an event-triggered control scheme is presented, under which the communication resources

are effectively saved. Based on the event-triggered mechanism, the output tracking error of a closed-loop system can converge to a bounded region under the switching signal that satisfies the average dwell time.

2) Using the event-triggered mechanism to study the tracking problem of a switched nonlinear system, it is clear that infinite events may happen in a limited time interval. Therefore, the minimum interval of inter-event lower bound is calculated, and the Zeno behavior is successfully excluded during the sampling process.

2. Problem statement and preliminaries

Consider a continuous-time cascade switched nonlinear system

$$\begin{aligned} \dot{x}_1(t) &= A_{1\sigma(t)}x_1(t) + A_{2\sigma(t)}x_2(t) + B_{\sigma(t)}u_{\sigma(t)}(t) \\ \dot{x}_2(t) &= f_{2\sigma(t)}(x_2(t)) \\ y(t) &= Cx_1(t) \end{aligned} \tag{2.1}$$

where $x_1(t) \in R^{n-d}$, $x_2(t) \in R^d$ are the system states, $y(t) \in R$ is the output, which tracks a given reference signal $y_d(t) \in R$. $\sigma : [0, \infty) \to M = \{1, ..., N\}$ denotes the switching signal, where M is a finite index set. When $\sigma(t) = i$, the $i-$th subsystem is active; $u_i(t) \in R^m$ is the control input, $\{(A_{1i}, A_{2i}, B_i, C) : i \in M\}$ are constant matrices with appropriate dimensions, $f_{2i}(x_2(t))$ are known smooth vector fields with appropriate dimensions. The switching signal $\sigma(t)$ can be represented by the following switching sequence

$$\sum = \{x_{t_0}; (s_0, t_0), (s_1, t_1), ..., (s_i, t_i), ... s_i \in M, i \in N\} \tag{2.2}$$

which means that the s_i-th subsystem is active when $t \in [t_i, t_{i+1})$, where t_i is the switching instant, x_{t_0} is the initial state of the system.

The following assumption is necessary for the output of the system (2.1) to track the reference signal $y_d(t)$.

Assumption 1. *The desired output $y_d(t)$ is known, bounded and continuous, and* $\max \|y_d(t)\| = \rho$, *where ρ is a positive constant.*

Now, consider the following integral controller

$$\begin{aligned} \dot{z}(t) &= y(t) - y_d(t) \\ u(t) &= K_\sigma x_1(t) + L_\sigma z(t) \end{aligned} \tag{2.3}$$

where K_i, L_i, $i \in M$ are constant matrices with appropriate dimensions.

Set $\bar{x}_1(t) = [x_1(t), z(t)]^T$. The controller (2.3) can be rewritten as

$$\begin{aligned} \dot{z}(t) &= \bar{C}\bar{x}_1(t) - y_d(t) \\ u(t) &= \bar{K}_\sigma \bar{x}_1(t) \end{aligned} \tag{2.4}$$

where $\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$, $\bar{K}_\sigma = \begin{bmatrix} K_\sigma & L_\sigma \end{bmatrix}$.

Next, the event-triggered condition can be described as follows

$$e^T(t)\, e(t) \geq \eta \bar{x}_1^T(t)\, \bar{x}_1(t) + \varepsilon \tag{2.5}$$

where $e(t) = \bar{x}_1(t) - \bar{x}_1(\tilde{t}_k)$ means the event-triggered error, $\bar{x}_1(t)$ is real-time state, $\bar{x}_1(\tilde{t}_k)$ is last state of event-triggered, η and ε are positive parameters.

Once the event-triggered condition (2.5) is satisfied, sampling occurs immediately. The sampling mechanism obtains the latest sampled state information at sampling instants and transmits it to the controller.

Denoting instant with an event happens by $\{\tilde{t}_k\}_{k=0}^{\infty}$. Without loss of generality, assume that the first event occurs at the time $\tilde{t}_0$. With the state $\bar{x}_1(\tilde{t}_k)$ sampled at the time $\tilde{t}_k$, we can describe the next sampling instant $\tilde{t}_{k+1}$ by

$$\tilde{t}_{k+1} = \inf\left\{t > \tilde{t}_k | e^T(t)\, e(t) = \eta \bar{x}_1^T(t)\, \bar{x}_1(t) + \varepsilon\right\} \tag{2.6}$$

In the above event-triggered mechanism, $[\tilde{t}_k, \tilde{t}_{k+1})$ is the event-triggered interval, and the controller only transmits at the sampling time $\tilde{t}_k$, in which the form of the controller is as follows

$$\begin{aligned} \dot{z}(t) &= \bar{C}\bar{x}_1(\tilde{t}_k) - y_d(t) \\ u(t) &= \bar{K}_\sigma \bar{x}_1(\tilde{t}_k) \end{aligned} \tag{2.7}$$

In the following, we consider the augmented system in the form of

$$\begin{aligned} \dot{\bar{x}}_1(t) &= \bar{A}_{1\sigma}\bar{x}_1(t) + \bar{A}_{2\sigma}x_2(t) + \bar{B}_\sigma e(t) + r(t) \\ \dot{x}_2(t) &= f_{2\sigma}(x_2(t)) \end{aligned} \tag{2.8}$$

Denote

$$\bar{A}_{1\sigma} = \begin{bmatrix} A_{1\sigma} + B_\sigma K_\sigma & B_\sigma L_\sigma \\ C & 0 \end{bmatrix}, \bar{A}_{2\sigma} = \begin{bmatrix} A_{2\sigma} \\ 0 \end{bmatrix}, \bar{B}_\sigma = \begin{bmatrix} -B_\sigma K_\sigma & -B_\sigma L_\sigma \\ -C & 0 \end{bmatrix}, r(t) = \begin{bmatrix} 0 \\ -y_d(t) \end{bmatrix}.$$

To obtain the main results, we give a definition and a lemma firstly.

Definition 1. *[28] If there exists a constant $\tau_d > 0$ such that any two switches are separated by at least τ_d, then τ_d is called the dwell time. If there exists a positive constant $\tau_a > \tau_d$ and $N_0 \geq 0$ such that $N_\sigma(s,t) \leq N_0 + \frac{t-s}{\tau_a}$, $\forall t \geq s \geq 0$, then $\tau_a > 0$ is called the average dwell time.*

Lemma 1. *[29] For any vectors $a, b \in R^n$, and positive definite matrix $H \in R^{n\times n}$, the following inequality holds*

$$2a^T b \leq a^T H a + b^T H^{-1} b \tag{2.9}$$

3. Main results

In this section, we consider to design the event-triggered controller and the switching rule for system (2.8), under which the tracking error can converge to a bounded region. Then, we give a strictly positive lower bound between any event-triggered interval.

3.1. Event-triggered tracking control

Now, we give the following main result in this section.

Theorem 1. *Consider the closed-loop system* (2.8)*, for the given scalars* $\varepsilon > 0$*,* $\eta > 0$*,* $\delta > 0$*,* $\xi > 0$*,* $k_1 > 0$*,* $k_2 > 0$*,* $\beta > 0$*,* $\lambda_i > 0$*, if there exist matrices* $P_i > 0$*,* K_i*,* L_i*, the function* $W_i(x_2(t))$*,* $\forall i \in M$*, satisfying following inequalities*

$$\begin{bmatrix} \bar{A}_{1i}^T P_i + P_i \bar{A}_{1i} + P_i \bar{B}_i \bar{B}_i^T P_i + \eta I + P_i P_i + \lambda_i P_i & \bar{A}_{2i}^T P_i \\ * & (\lambda_i \xi k_2 - \xi\beta) I \end{bmatrix} < 0 \tag{3.1}$$

$$P_i \le \delta P_j, \forall i, j \in M \tag{3.2}$$

$$k_1 \|x_2(t)\|^2 \le W_i(x_2(t)) \le k_2 \|x_2(t)\|^2 \tag{3.3}$$

$$\frac{\partial W(x_2(t))}{\partial (x_2(t))} f_{2i}(x_2(t)) \le -\beta \|x_2(t)\|^2 \tag{3.4}$$

then for any switching signal σ *satisfying*

$$\tau_a > \frac{\ln \hat{\delta}}{\lambda}, \hat{\delta} = \max\left\{\delta, \frac{k_2}{k_1}\right\}, \lambda = \min_{i \in M} \lambda_i > 0 \tag{3.5}$$

the tracking error of the system (2.8) *will converge to a bounded region.*

Proof. For the system (2.8), we construct a Lyapunov function as follows

$$V(t) = \bar{x}_1^T(t) P_{\sigma(t)} \bar{x}_1(t) + \xi W_{\sigma(t)}(x_2(t)) \tag{3.6}$$

When the $i-$th subsystem is active, the derivative of V_i is

$$\begin{aligned} \dot{V}_i(t) &= \dot{\bar{x}}_1^T(t) P_i \bar{x}_1(t) + \bar{x}_1^T(t) P_i \dot{\bar{x}}_1(t) + \xi \tfrac{\partial W_i(x_2(t))}{\partial (x_2(t))} \dot{x}_2(t) \\ &= \bar{x}_1^T(t)\left(\bar{A}_{1i}^T P_i + P_i \bar{A}_{1i}\right)\bar{x}_1(t) + x_2^T(t) \bar{A}_{2i}^T P_i \bar{x}_1(t) + \bar{x}_1^T(t) P_i \bar{A}_{2i} x_2(t) \\ &+ e^T(t) \bar{B}_i^T P_i \bar{x}_1(t) + \bar{x}_1^T(t) P_i \bar{B}_i e(t) + r^T(t) P_i \bar{x}_1(t) \\ &+ \bar{x}_1^T(t) P_i r(t) + \xi \tfrac{\partial W_i(x_2(t))}{\partial (x_2(t))} f_{2i}(x_2(t)) \end{aligned} \tag{3.7}$$

According to Lemma 1 and event-triggered condition (2.5), we know that

$$\begin{aligned} \dot{V}_i(t) &\le \bar{x}_1^T(t)\left(\bar{A}_{1i}^T P_i + P_i \bar{A}_{1i}\right)\bar{x}_1(t) + 2\bar{x}_1^T(t) P_i \bar{A}_{2i} x_2(t) + e^T(t) e(t) \\ &+ \bar{x}_1^T(t) P_i \bar{B}_i \bar{B}_i^T P_i \bar{x}_1(t) + r^T(t) r(t) + \bar{x}_1^T(t) P_i P_i \bar{x}_1(t) \\ &- \xi\beta x_2^T(t) x_2(t) \\ &\le \bar{x}_1^T(t)\left(\bar{A}_{1i}^T P_i + P_i \bar{A}_{1i}\right)\bar{x}_1(t) + 2\bar{x}_1^T(t) P_i \bar{A}_{2i} x_2(t) + \eta \bar{x}_1^T(t) \bar{x}_1(t) \\ &+ \bar{x}_1^T(t) P_i \bar{B}_i \bar{B}_i^T P_i \bar{x}_1(t) + r^T(t) r(t) + \bar{x}_1^T(t) P_i P_i \bar{x}_1(t) \\ &- \xi\beta x_2^T(t) x_2(t) + \varepsilon \end{aligned} \tag{3.8}$$

Therefore,

$$\begin{aligned} \dot{V}_i(t) + \lambda_i V_i(t) &\le \bar{x}_1^T(t)\left(\bar{A}_{1i}^T P_i + P_i \bar{A}_{1i} + P_i \bar{B}_i \bar{B}_i^T P_i + \eta I + P_i P_i + \lambda_i P_i\right)\bar{x}_1(t) \\ &+ \bar{x}_1^T(t) P_i \bar{A}_{2i} x_2(t) + x_2^T(t) \bar{A}_{2i}^T P_i \bar{x}_1(t) \\ &+ (\lambda_i \xi k_2 - \xi\beta) x_2^T(t) x_2(t) + \|r(t)\|^2 + \varepsilon \end{aligned} \tag{3.9}$$

According to inequality (3.9), we get

$$\dot{V}_i(t) + \lambda_i V_i(t) - \|r(t)\|^2 - \varepsilon \le \varphi^T(t) \psi_i \varphi(t) \tag{3.10}$$

where

$$\varphi(t) = \begin{bmatrix} \bar{x}_1^T(t) & x_2^T(t) \end{bmatrix}^T$$

$$\psi_i = \begin{bmatrix} \bar{A}_{1i}^T P_i + P_i \bar{A}_{1i} + P_i \bar{B}_i \bar{B}_i^T P_i + \eta I + P_i P_i + \lambda_i P_i & \bar{A}_{2i}^T P_i \\ * & (\lambda_i \xi k_2 - \xi\beta) I \end{bmatrix} \tag{3.11}$$

Then, we have

$$\dot{V}_i(t) \leq -\lambda_i V_i(t) + \varepsilon + \|r(t)\|^2 \tag{3.12}$$

Integrating (3.12) from t_i to t , we can get

$$V_i(t) \leq V_i(t_i) e^{-\lambda_i(t-t_i)} + \left(\varepsilon + \|r(t)\|^2\right) \int_{t_i}^{t} e^{-\lambda_i(t-s)} ds \tag{3.13}$$

Let $\lambda = \min_{i \in M} \lambda_i > 0$. Combining inequalities (3.2) with (3.6), we conclude

$$V_i \leq \hat{\delta} V_j, \forall i, j \in M, \hat{\delta} = \max\left\{\delta, \frac{k_2}{k_1}\right\} \tag{3.14}$$

According to inequality (3.13), we have

$$\begin{aligned}
& V(t) = V_i(t) \\
& \leq \hat{\delta} V_i\left(t_i^-\right) e^{-\lambda(t-t_i)} + \left(\varepsilon + \|r(t)\|^2\right) \int_{t_i}^{t} e^{-\lambda(t-s)} ds \\
& \leq \hat{\delta} V_i\left(t_i^-\right) e^{-\lambda(t-t_i)} + \tfrac{\varepsilon + \|r(t)\|^2}{\lambda}\left(1 - e^{-\lambda(t-t_i)}\right) \\
& \leq \hat{\delta} e^{-\lambda(t-t_i)}\left(e^{-\lambda(t-t_i)} V_{i-1}(t_{i-1}) + \tfrac{\varepsilon + \|r(t)\|^2}{\lambda}\left(1 - e^{-\lambda(t-t_i)}\right)\right) + \tfrac{\varepsilon + \|r(t)\|^2}{\lambda}\left(1 - e^{-\lambda(t-t_i)}\right) \\
& \leq \hat{\delta}^2 e^{-\lambda(t-t_{i-1})} V_{i-2}\left(t_{i-1}^-\right) + \tfrac{\left(\varepsilon + \|r(t)\|^2\right)\hat{\delta}}{\lambda}\left(e^{-\lambda(t-t_i)} - e^{-\lambda(t-t_{i-1})}\right) + \tfrac{\varepsilon + \|r(t)\|^2}{\lambda}\left(1 - e^{-\lambda(t-t_i)}\right) \\
& \vdots \\
& \leq e^{-\lambda(t-t_0)} \hat{\delta}^{N_\sigma(t_0,t)} V(t_0) + \tfrac{\left(\varepsilon + \|r(t)\|^2\right)\hat{\delta}^{N_\sigma(t_1,t)}}{\lambda}\left(e^{-\lambda(t-t_2)} - e^{-\lambda(t-t_1)}\right) \\
& + \tfrac{\left(\varepsilon + \|r(t)\|^2\right)\hat{\delta}^{N_\sigma(t_2,t)}}{\lambda}\left(e^{-\lambda(t-t_3)} - e^{-\lambda(t-t_2)}\right) \\
& + \ldots + \tfrac{\left(\varepsilon + \|r(t)\|^2\right)\hat{\delta}}{\lambda}\left(e^{-\lambda(t-t_i)} - e^{-\lambda(t-t_{i-1})}\right) + \tfrac{\left(\varepsilon + \|r(t)\|^2\right)}{\lambda}\left(1 - e^{-\lambda(t-t_i)}\right) \\
& \leq e^{-\lambda(t-t_0)} \hat{\delta}^{N_\sigma(t_0,t)}\left(V(t_0) - \tfrac{\varepsilon + \|r(t)\|^2}{\lambda\hat{\delta}}\right) + \tfrac{\left(\varepsilon + \|r(t)\|^2\right)\left(\hat{\delta} - 1\right)}{\lambda} \sum_{k=0}^{N_\sigma(t_2,t)} \hat{\delta}^k e^{-\lambda(t-t_{i-k})} \\
& + \tfrac{\varepsilon + \|r(t)\|^2}{\lambda} \\
& \leq e^{-\left(\lambda - \frac{\ln\hat{\delta}}{\tau_a}\right)(t-t_0)}\left(V(t_0) - \tfrac{\left(\varepsilon + \|r(t)\|^2\right)\hat{\delta}}{\lambda}\right) + \tfrac{\left(\varepsilon + \|r(t)\|^2\right)\left(\hat{\delta} - 1\right)}{\lambda} \sum_{k=0}^{N_\sigma(t_2,t)} e^{k\left(\ln\hat{\delta} - \lambda\tau_a\right)} \\
& + \tfrac{\varepsilon + \|r(t)\|^2}{\lambda}
\end{aligned} \tag{3.15}$$

From (3.3) and (3.6), we know that

$$V(t) \geq \min_{\forall i \in M}(\lambda(P_i))\|\bar{x}_1(t)\|^2 + \xi k_1 \|x_2(t)\|^2 \geq a\|\tilde{x}(t)\|^2 \tag{3.16}$$

$$V(t_0) \leq \max_{\forall i \in M}(\lambda(P_i))\|\bar{x}_1(t_0)\|^2 + \xi k_2 \|x_2(t_0)\|^2 \leq b\|\tilde{x}(t_0)\|^2 \tag{3.17}$$

where $a = \min\left\{\min_{\forall i \in M}(\lambda(P_i)), \xi k_1\right\}$, $b = \max\left\{\max_{\forall i \in M}(\lambda(P_i)), \xi k_2\right\}$.

Thus, Combining the inequalities (3.15)–(3.17), we have

$$\|\tilde{x}(t)\|^2 \le \frac{b}{a} e^{-\left(\lambda - \frac{\ln\hat{\delta}}{\tau_a}\right)(t-t_0)} \left(\|\tilde{x}(t_0)\|^2 - \frac{\varepsilon+\rho}{\lambda\hat{\delta}b} \right) + \frac{(\varepsilon+\rho)\left(\hat{\delta}-1\right)}{a\lambda} \sum_{k=0}^{N_\sigma(t_2,t)} e^{k\left(\ln\hat{\delta}-\lambda\tau_a\right)} + \frac{\varepsilon+\rho}{a\lambda} \tag{3.18}$$

In addition, the condition $\tau_a > \frac{\ln\hat{\delta}}{\lambda}$ means that $\ln\hat{\delta} - \lambda\tau_a < 0$. Thus, inequality (3.18) can guarantee the uniform bounded of the error.

The tracking error can converge to a bounded region

$$\Omega = \left\{ y(t) - y_d(t) \le \|y(t)\| + \|y_d(t)\| = c\sqrt{\Theta} + \rho \right\} \tag{3.19}$$

where $\Theta = \frac{(\varepsilon+\rho)\left(\hat{\delta}-1\right)}{a\lambda} \sum\limits_{k=0}^{N_\sigma(t_2,t)} e^{k\left(\ln\hat{\delta}-\lambda\tau_a\right)} + \frac{\varepsilon+\rho}{a\lambda}$, $\| C \| = c$.

3.1.1. Minimum inter-event interval

From another perspective, we know that event-triggered control easily causes infinite triggered behavior (i.e., Zeno behavior) within a finite time. Therefore, we need to show that there always exists a positive lower bound of the minimum inter-event interval for the event-triggered sampling condition (2.5).

Theorem 2. *Consider the switched nonlinear system* (2.1) *and the controller* (2.7)*. With the event-triggered condition* (2.5)*, the Zeno behavior can be avoided during the control process.*

Proof. To exclude the Zeno behavior, namely, we need to find a lower bound on the triggered interval, and show that infinite trigged event does not occur in a finite time. Suppose that n samplings happen during an interval $[t_i, t_{i+1})$ and $\tilde{t}_{k+1}, \ldots, \tilde{t}_{k+n}$ are n sampling instants, respectively. For $\forall t \in [t_i, \tilde{t}_{k+1}), [\tilde{t}_{k+1}, \tilde{t}_{k+2}), \ldots, [\tilde{t}_{k+n}, t_{i+1})$, the state $\bar{x}_1(\tilde{t}_{k+l})$ are constants and $e(t) = \bar{x}_1(t) - \bar{x}_1(\tilde{t}_{k+l})$ holds for all $l = 1, 2, \ldots, n$. Hence, for $\forall t \in [t_i, t_{i+1})$, we can obtain that

$$\begin{aligned} \dot{e}(t) &= \bar{A}_{1i}\bar{x}_1(t) + \bar{A}_{2i}x_2(t) + \bar{B}_i e(t) + r(t) \\ &= \bar{A}_{1i}\left(e(t) + \bar{x}_1(\tilde{t}_{k+l})\right) + \bar{A}_{2i}x_2(t) + \bar{B}_i e(t) + r(t) \\ &= \left(\bar{A}_{1i} + \bar{B}_i\right) e(t) + \bar{A}_{1i}\bar{x}_1(\tilde{t}_{k+l}) + \bar{A}_{2i}x_2(t) + r(t) \end{aligned} \tag{3.20}$$

Let $D_i = \bar{A}_{1i} + \bar{B}_i$. Therefore

$$\dot{e}(t) = D_i e(t) + \bar{A}_{1i}\bar{x}_1(\tilde{t}_{k+l}) + \bar{A}_{2i}x_2(t) + r(t) \tag{3.21}$$

Integral to both sides of the Eq (3.21)

$$e(t) = e^{D_i\left(t-\tilde{t}_{k+l}\right)} e(\tilde{t}_{k+l}) + \int_{\tilde{t}_{k+l}}^{t} e^{D_i(t-s)} \left(\bar{A}_{1i}\bar{x}_1(\tilde{t}_{k+l}) + A_{2i}x_2(s) + r(s) \right) ds \tag{3.22}$$

Due to $e(\tilde{t}_{k+l}) = \bar{x}_1(\tilde{t}_{k+l}) - \bar{x}_1(\tilde{t}_{k+l})$, we have

$$e(t) = \int_{\tilde{t}_{k+l}}^{t} e^{D_i(t-s)} \left(\bar{A}_{1i}\bar{x}_1(\tilde{t}_{k+l}) + A_{2i}x_2(s) + r(s) \right) ds \tag{3.23}$$

Therefore,

$$\|e(t)\| \leq \int_{\tilde{t}_{k+l}}^{t} e^{\|D_i\|(t-s)}\|\bar{A}_{1i}\|\|\bar{x}_1(\tilde{t}_{k+l})\|ds + \int_{\tilde{t}_{k+l}}^{t} e^{\|D_i\|(t-s)}(\|A_{2i}\|\|x_2(s)\| + \|r(s)\|)ds \tag{3.24}$$

According to (3.18), we can find a positive constant ℓ such that

$$\begin{aligned}\|e(t)\| &\leq \int_{\tilde{t}_{k+l}}^{t} e^{\|D_i\|(t-s)}\|\bar{A}_{1i}\|\|\bar{x}_1(\tilde{t}_{k+l})\|ds + \int_{\tilde{t}_{k+l}}^{t} e^{\|D_i\|(t-s)}\left(\|A_{2i}\|\sqrt{\ell} + \|r(s)\|\right)ds \\ &\leq \phi(\tilde{t}_{k+l})\int_{\tilde{t}_{k+l}}^{t} e^{\|D_i\|(t-s)}ds\end{aligned} \tag{3.25}$$

where $\phi(\tilde{t}_{k+l}) = \|\bar{A}_{1i}\|\|\bar{x}_1(\tilde{t}_{k+l})\| + \|A_{2i}\|\sqrt{\ell} + \rho$.

If $\|D_i\| \neq 0$, then, we have

$$\|e(t)\| \leq \frac{\phi(\tilde{t}_{k+l})}{\|D_i\|}\left(e^{\|D_i\|(t-\tilde{t}_{k+l})} - 1\right) \tag{3.26}$$

We know that the next event will happen when the event-triggered mechanism (2.6) is satisfied. Thus, let $T = t - \tilde{t}_{k+l}$ denote the lower bound of inter-event interval, we have

$$\frac{\phi(\tilde{t}_{k+l})}{\|D_i\|}\left(e^{\|D_i\|(t-\tilde{t}_{k+l})} - 1\right) \geq \sqrt{\eta\|\bar{x}_1(t)\|^2+\varepsilon} \tag{3.27}$$

$$e^{\|D_i\|T} \geq \frac{\|D_i\|\sqrt{\eta\|\bar{x}_1(t)\|^2+\varepsilon}}{\phi(\tilde{t}_{k+l})} + 1 \tag{3.28}$$

$$T \geq \frac{1}{\|D_i\|}\ln\left(\frac{\|D_i\|\sqrt{\eta\|\bar{x}_1(t)\|^2+\varepsilon}}{\phi(\tilde{t}_{k+l})} + 1\right) \tag{3.29}$$

If $\|D_i\|=0$, then $\phi(\tilde{t}_{k+l})(t - \tilde{t}_{k+l}) \geq \sqrt{\eta\|\bar{x}_1(t)\|^2+\varepsilon}$

$$T \geq \frac{\sqrt{\eta\|\bar{x}_1(t)\|^2+\varepsilon}}{\phi(\tilde{t}_{k+l})} \tag{3.30}$$

It is known that Zeno behavior does not occur in the event-triggered control of the nonlinear switched system.

4. Simulation example

In this section, we will show the feasibility of the proposed methods by applying it to a numerical example.

Consider a cascade switched nonlinear system

$$\begin{aligned}\dot{x}_1(t) &= A_{1\sigma}x_1(t) + A_{2\sigma}x_2(t) + B_\sigma u_\sigma(t) \\ \dot{x}_2(t) &= f_{2\sigma}(x_2(t)) \\ \dot{z}(t) &= Cx_1(t) - y_d(t)\end{aligned}$$

with

$A_{11} = \begin{bmatrix} -4 & 0 \\ -1 & -3 \end{bmatrix}$, $A_{21} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0.3 \\ 2 \end{bmatrix}$, $A_{12} = \begin{bmatrix} -3 & -1 \\ 0 & -5 \end{bmatrix}$, $A_{22} = \begin{bmatrix} 1 \\ 0.6 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 0.3 & 0.25 \end{bmatrix}$, $f_{21}(x_2) = -x_2 - x_2\sin^2 x_2$, $f_{22}(x_2) = -2x_2 - x_2\cos^2 x_2$, $y_d(t) = 0.4\sin(2.5t)$, let $\eta = 0.5$, $\delta = 1.2$, $\xi = 1$, $\lambda_0 = 1.8$, $\varepsilon = 0.01$, $\lambda = 0.4$, $\beta = 1.3$, $k_1 = 0.3$, $k_2 = 0.5$.

Solving the inequality (3.1) and (3.2) yields

$$P_1 = \begin{bmatrix} 0.9650 & -0.1439 & 0.0185 \\ -0.1439 & 0.7264 & 0.1105 \\ 0.0185 & 0.1105 & 0.5591 \end{bmatrix}, P_2 = \begin{bmatrix} 0.6125 & 0.1615 & -0.2086 \\ 0.1615 & 0.4756 & 0.3293 \\ -0.2086 & 0.3293 & 0.2890 \end{bmatrix}$$

Consequently, the controller gains are obtained as

$$\bar{K}_1 = \begin{bmatrix} 0.5311 & 1.3013 & 0.2715 \end{bmatrix}, \bar{K}_2 = \begin{bmatrix} 0.7921 & 1.2141 & 0.3755 \end{bmatrix}$$

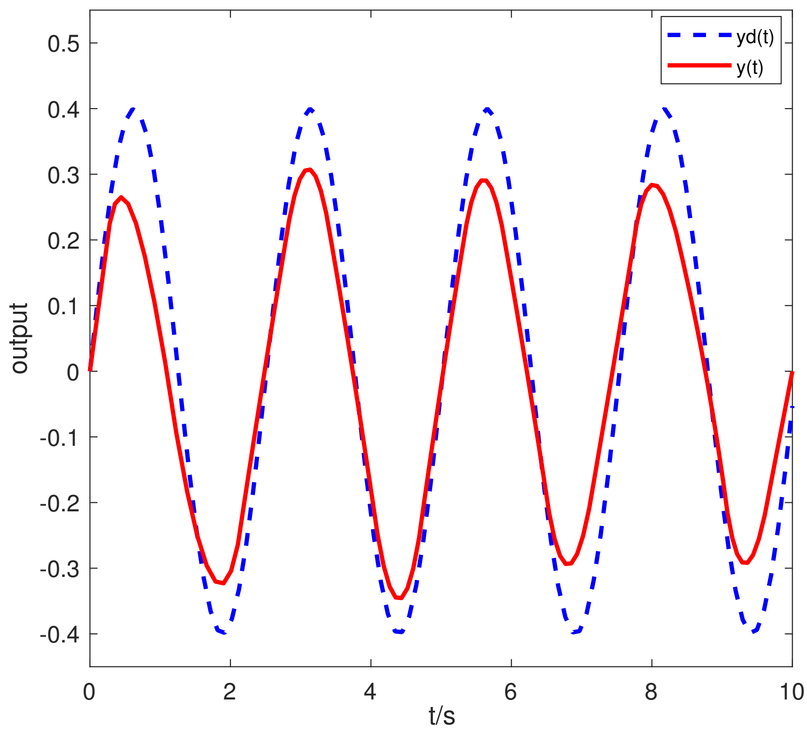

Figure 1. Output of the switched system and the reference signal.

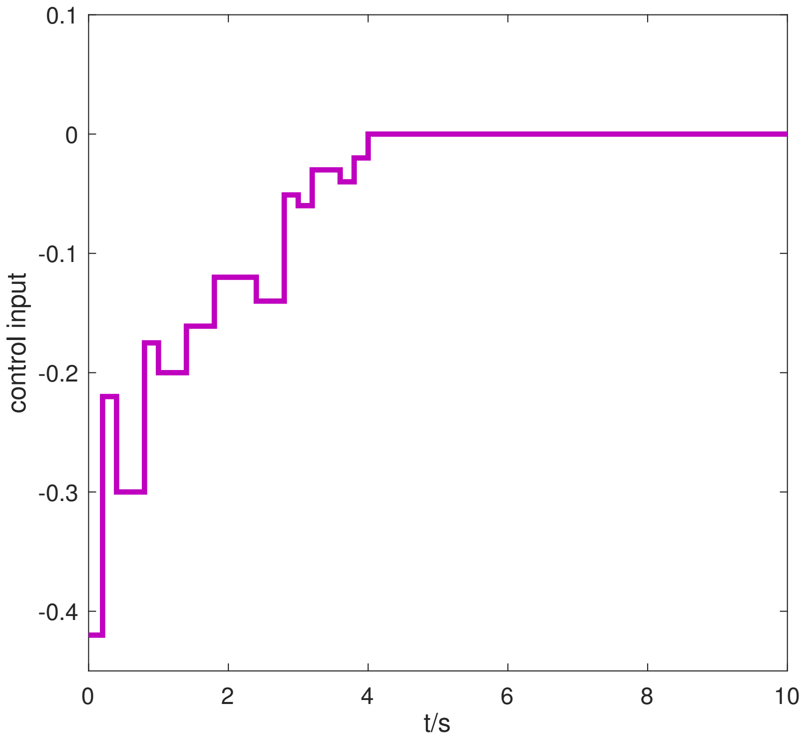

Figure 2. Control input of the switched system.

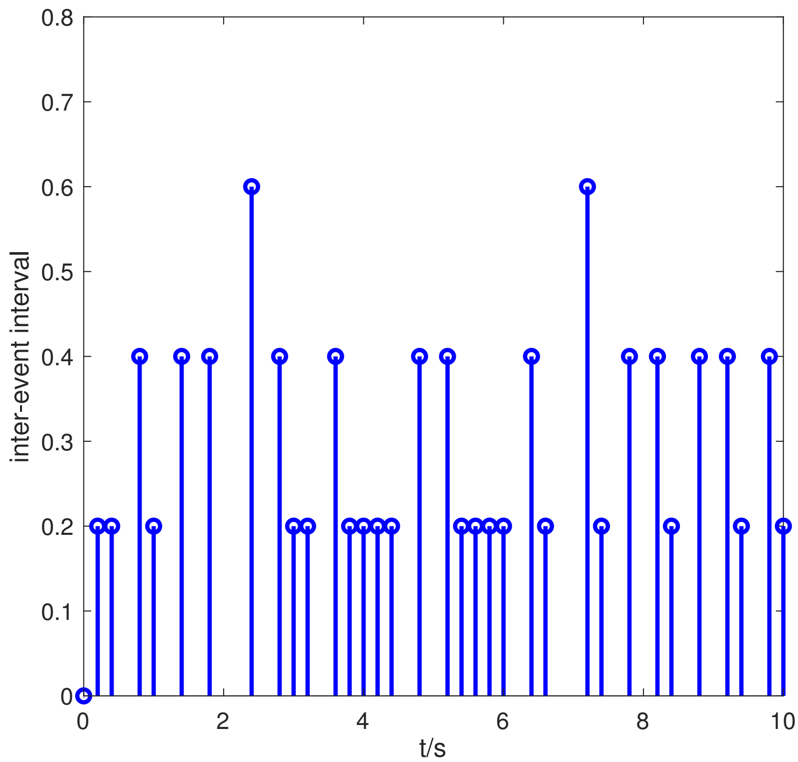

Figure 3. Inter-event interval of ETM.

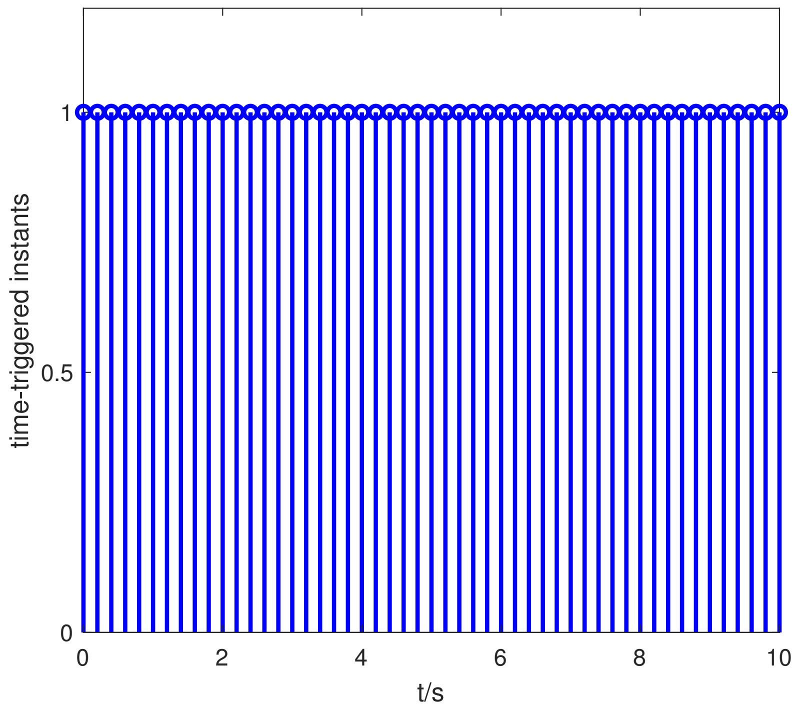

Figure 4. Time-triggered instants.

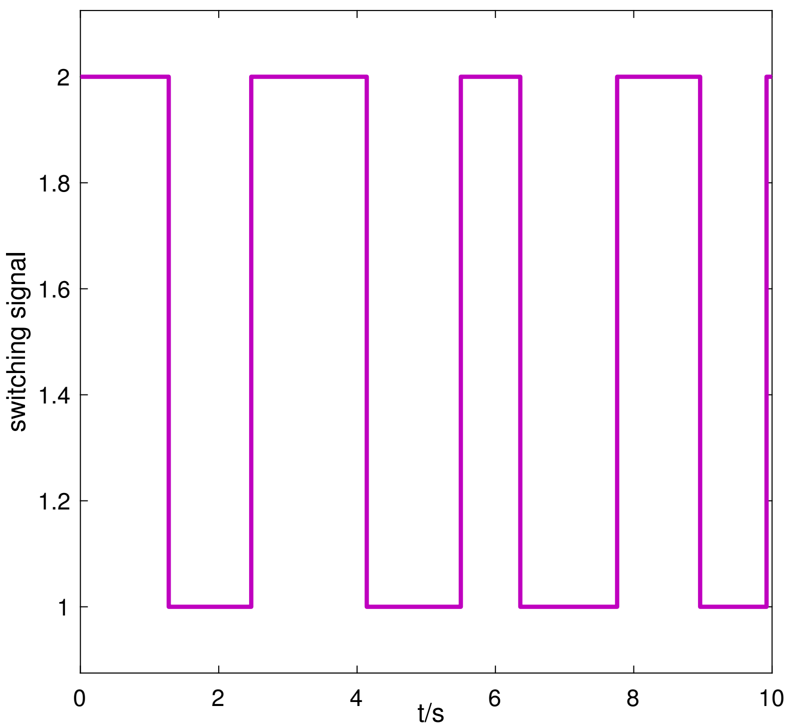

Figure 5. Switching signal.

We obtain the average dwell time $\tau_a^* = \frac{\ln\hat{\delta}}{\lambda} = 1.2771$. Meanwhile, by solving inequalities (3.19) in Theorem 1, we get $\Omega = 2.4117$. Figure 1 shows that the tracking error can converge to this region. Figure 2 displays the control input of the system. The event-triggered controller can ensure the dynamic performance of the system while reducing the number of system information transmission and the calculation amount of the controller. Figures 3 and 4 depict the event-triggered interval and time-triggered instants, respectively. Compared to the time-triggered scheme, 67% data information is used to track the reference signal, which proves that the designed event-triggered scheme in (2.5) can effectively reduce the number of sampling and save the communication resource effectively. Figure 5 demonstrates the switching signal. This simulation example demonstrates the effectiveness of the proposed method in this paper.

5. Conclusions

In this paper, the tracking control of cascaded switched nonlinear system is studied by an event-triggered mechanism. By using the average dwell time method, the sufficient conditions for the output tracking error of the system can converge to a bounded region are given. Moreover, this paper proves that the proposed minimum event interval is strictly positive, excluding the Zeno behaviour.

Although the event-triggered mechanism has many advantages, there are still many problems to be solved, such as the event-triggered tracking control for stochastic switched system, the event-triggered tracking control of switching system under the state dependent switching signal.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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