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Research article

Fixed-deviation stabilization and synchronization for delayed fractional-order complex-valued neural networks

Bingrui Zhang and Jin-E Zhang*

School of Mathematics and Statistics, Hubei Normal University, Huangshi 435002, China

* **Correspondence:** Email: zhang86021205@163.com; Tel: 8671465712019; Fax: 8671465712019.

Abstract: In this paper, we study fixed-deviation stabilization and synchronization for fractional-order complex-valued neural networks with delays. By applying fractional calculus and fixed-deviation stability theory, sufficient conditions are given to ensure the fixed-deviation stabilization and synchronization for fractional-order complex-valued neural networks under the linear discontinuous controller. Finally, two simulation examples are presented to show the validity of theoretical results.

Keywords: fractional-order complex-valued neural networks; time delays; fixed-deviation dynamics; discontinuous control

1. Introduction

Fractional calculus is a theory of differentiation and integration of arbitrary order, which is an extension of integer order calculus. Initially, the study of fractional calculus theory was mainly conducted in the field of pure number theory, but as it developed further, fractional calculus was widely used in fluid mechanics [1], mechanical systems [2], signal processing [3, 4], system identification [5], and many other fields. Fractional calculus has become an essential theory in many fields. Many scholars have applied fractional-order derivatives to neural networks and have built fractional-order neural networks (FONNs). So far, the study of FONNs has yielded some interesting results [6–17]. Zhang and Zeng [18] showed asymptotic stability of nonlinear FONNs with unbounded time-varying delays and asymptotic synchronization of FONNs under a linear controller. Ding et al. [19] investigated the robust finite-time stability of FONNs.

Complex-valued neural networks (CVNNs), whose input/output signals, connection weights, and activation functions are derived from the complex domain. Unlike real-valued neural networks, functions that are both bounded and analytic in the complex domain must be constant according to Liouville's theorem [20]. Therefore, the study of the dynamics of CVNNs is essential. In recent years, the dynamic behavior of fractional-order CVNNs (FOCVNNs) has been reported in many kinds of literatures, including finite-time stability [21, 22], impulse stability and synchronization [23], and Mittag-Leffler stability and synchronization [24, 25].

In neural networks, time delays are prevalent. Failure to take into account time delays will cause stable systems to be unstable and lead to a reduction in the capabilities of the neural network [26, 27]. Therefore it is relevant to study FOCVNNs with time delays in practical applications. Bao et al. [28] obtained sufficient conditions to guarantee the synchronization of FOCVNNs with time delays using linear delay feedback control and fractional-order inequalities. Liu and Yu [29] derived several conditions for quasi-projective synchronization and complete synchronization of FOCVNNs with time delays based on generalized discrete fractional Halanay inequality and Lyapunov generalized function methods without dividing the complex-valued neural network into two real-valued systems.

Deviation dynamics is particularly important for the evolutionary characterization of control systems. Fixed-deviation stabilization and synchronization are very important dynamical behaviors of discontinuous neural network systems. There have been some important findings about fixed-deviation dynamics [30, 31]. Chen et al. [30] initially proposed the concept of fixed-deviation stability to describe the stability properties of discontinuous systems, and sufficient conditions to ensure globally uniform asymptotic fixed-deviation stability of delayed fractional-order memristive neural networks were given. Based on the theory of fixed-deviations in [30], Zhang [31] used linear-type discontinuous control and fractional-order calculus methods to address fixed-deviation stability and synchronization problems of FONNs. Clearly, the investigation of fixed-deviation dynamics for FONNs is an important topic. But so far, there are few results on the fixed-deviation dynamics of FOCVNNs.

In the above view, we present the problems of fixed-deviation stability and synchronization of FOCVNNs. Continuous FONNs are difficult to achieve fixed-deviation stability and synchronization, and a special control method needs to be imposed to make the continuous system generate fixed-deviation dynamics behavior. A natural idea is to add a discontinuous controller so that continuous FOCVNNs turn into the discontinuous system under the discontinuous controller, and then impose complex-valued conditions to make the FOCVNNs achieve fixed-deviation stability and synchronization. Also based on the theory of fixed-deviations in [30], fractional-order calculus and Lyapunov method, sufficient conditions for the formation of fixed-deviation stability and synchronization of FOCVNNs under linear discontinuous controllers are obtained.

2. Model description and preliminaries

In this section, necessary definitions and lemmas will be provided for the proof of the theorem in Section 3.

The Caputo's fractional derivative of a function $\mathscr{H}(t) \in C^{\lambda+1}([t_0, +\infty), \mathbb{R})$ with order $\alpha > 0$ is defined by

$$ {}^{C}D_{t_0}^{\alpha}\mathscr{H}(t) = \frac{1}{\Gamma(\lambda - \alpha)} \int_{t_0}^{t} \frac{\mathscr{H}^{(\lambda)}(s)}{(t-s)^{\alpha-\lambda+1}} \mathrm{d}s, $$

where $t \geq t_0$, $\lambda - 1 < \alpha < \lambda$, λ is positive integer, α is a positive constant and $\Gamma(\cdot)$ is Gamma function, that is

$$ \Gamma(\alpha) = \int_{0}^{\infty} t^{\alpha-1} e^{-t} \mathrm{d}t. $$

The Riemann-Liouville fractional derivative of order $\alpha > 0$ for a function $\mathscr{H}(t) \in C^{\lambda+1}([t_0, +\infty), \mathbb{R})$ is defined by

$$^{RL}D_{t_0}^{\alpha}\mathscr{H}(t) = \frac{1}{\Gamma(\lambda-\alpha)}\frac{\mathrm{d}^{\lambda}}{\mathrm{d}t^{\lambda}}\int_{t_0}^{t}\frac{\mathscr{H}(s)}{(t-s)^{\alpha-\lambda+1}}\mathrm{d}s,$$

where $\lambda - 1 < \alpha < \lambda$, $\lambda > 0$.

By the above definition, the following relation holds:

$$^{C}D_{t_0}^{\alpha}\mathscr{H}(t) = {}^{RL}D_{t_0}^{\alpha}\mathscr{H}(t) - \frac{\mathscr{H}(t_0)}{\Gamma(1-\alpha)}(t-t_0)^{-\alpha}.$$

Now, we introduce delayed FOCVNNs as follows:

$$^{C}D_{t_0}^{\alpha}z_k(t) = -a_k z_k(t) + \sum_{\ell=1}^{n} b_{k\ell} f_\ell(z_\ell(t)) + \sum_{\ell=1}^{n} d_{kl} g_\ell(z_\ell(t-\varpi_{kl}(t))) + U_k(t), \tag{2.1}$$

where $0 < \alpha < 1$, $z_k(t) \in C$ denotes the state variable; $a_k > 0$ is the self-feedback connective weight of the kth neuron; $b_{k\ell}$ and $d_{k\ell}$ are the connective weights matrix without and with time delay respectively; $f_\ell(z_\ell(t))$, $g_\ell(z_\ell(t-\varpi_{k\ell}(t)))$ represent the complex-valued state activation functions at time t and $t-\varpi_{k\ell}(t)$; $\varpi_{k\ell}(t)$ is the time-varying delay satisfying $0 \le \varpi_{k\ell}(t) \le \varpi$; $U_k(t)$ stands for the external input.

Let $C_\varpi = C([-\varpi, 0], \mathbb{R}^n)$ be the Banach space of continuous functions mapping $[-\varpi, 0]$ into $\mathbb{R}^n$. For $\psi \in C_\varpi$, $\|\psi\|_c = \sup\limits_{-\varpi \le s \le 0} \|\psi(s)\|$.

Note the initial conditions of delayed FOCVNNs (2.1) as

$$z_k(t_0+s) = \psi_k^R(s) + \psi_k^I(s), \ -\varpi \le s \le 0, \ k = 1, \cdots, n. \tag{2.2}$$

Let $z = z^R + iz^I \in C$. For any ℓ, $f_\ell(z)$ and $g_\ell(z(t-\varpi))$ can be shown by dividing into its real and imaginary parts as

$$\begin{aligned} f_\ell(z) =& f_\ell^R(z^R, z^I) + i f_\ell^I(z^R, z^I), \\ g_\ell(z(t-\varpi)) =& g_\ell^R(z^R(t-\varpi), z^I(t-\varpi)) + i g_\ell^I(z^R(t-\varpi), z^I(t-\varpi)). \end{aligned} \tag{2.3}$$

Let $z_k(t) = z_k^R(t) + iz_k^I(t)$. Delayed FOCVNNs (2.1) can be described as the following equation:

$$\begin{aligned} ^{C}D_{t_0}^{\alpha}z_k^R(t) =& -a_k z_k^R(t) + \sum_{\ell=1}^{n} b_{k\ell}^R f_\ell^R(z_\ell(t)) - \sum_{\ell=1}^{n} b_{k\ell}^I f_\ell^I(z_\ell(t)) \\ &+ \sum_{\ell=1}^{n} d_{k\ell}^R g_\ell^R(z_\ell(t-\varpi_{k\ell}(t))) - \sum_{\ell=1}^{n} d_{k\ell}^I g_\ell^I(z_\ell(t-\varpi_{k\ell}(t))) + U_k^R(t), \end{aligned}$$

$$\begin{aligned} ^{C}D_{t_0}^{\alpha}z_k^I(t) =& -a_k z_k^I(t) + \sum_{\ell=1}^{n} b_{k\ell}^R f_\ell^I(z_\ell(t)) + \sum_{\ell=1}^{n} b_{k\ell}^I f_\ell^R(z_\ell(t)) \\ &+ \sum_{\ell=1}^{n} d_{k\ell}^R g_\ell^I(z_\ell(t-\varpi_{k\ell}(t))) + \sum_{\ell=1}^{n} d_{k\ell}^I g_\ell^R(z_\ell(t-\varpi_{k\ell}(t))) + U_k^I(t). \end{aligned} \tag{2.4}$$

Definition 1 ([30]): FOCVNNs (2.1) is called globally uniformly β-stable if for any $\xi > 0$ and any initial values $\phi, \varphi \in C_{\varpi}$, $||\phi - \varphi||_C \leq \xi$, there is a constant $T(\xi) \geq 0$, such that

$$||z(t, t_0, \phi) - z(t, t_0, \varphi)|| \leq \beta$$

for all $t \geq t_0 + T(\xi)$, where $\beta > 0$.

Remark 1: β-stability, also known as fixed-deviation stability, specifically, when the difference between two different initial values of the described neural network are kept in a certain range, the difference among final values of the system trajectories starting from these two initial values will be maintained in a fixed-deviation degree.

Definition 2: The zero solution of delayed FOCVNNs (2.1) is called globally uniformly β-stable if for any $\psi \in C_{\varpi}$, $\xi > 0$, $||\psi||_C \leq \xi$, there is a constant $T(\xi) \geq 0$, such that

$$||z(t, t_0, \psi)|| \leq \beta$$

for all $t \geq t_0 + T(\xi)$, where $\beta > 0$ is a constant.

In this paper, we propose the below assumptions:

(i) The activation functions $f_\ell(\cdot)$ and $g_\ell(\cdot)$ satisfy $f_\ell(0) = g_\ell(0) = 0$.

(ii) For functions $f_\ell^R(\cdot,\cdot)$, $f_\ell^I(\cdot,\cdot)$, $g_\ell^R(\cdot,\cdot)$, $g_\ell^I(\cdot,\cdot)$, there exist positive constants F_ℓ^{RR}, F_ℓ^{RI}, F_ℓ^{IR}, F_ℓ^{II}, G_ℓ^{RR}, G_ℓ^{RI}, G_ℓ^{IR},G_ℓ^{II}, such that

$$\begin{cases} |f_\ell^R(\tilde{z}^R, \tilde{z}^I) - f_\ell^R(z^R, z^I)| \leq F_\ell^{RR}|\tilde{z}^R - z^R| + F_\ell^{RI}|\tilde{z}^I - z^I| \\ |f_\ell^I(\tilde{z}^R, \tilde{z}^I) - f_\ell^I(z^R, z^I)| \leq F_\ell^{IR}|\tilde{z}^R - z^R| + F_\ell^{II}|\tilde{z}^I - z^I| \\ |g_\ell^R(\tilde{z}^R, \tilde{z}^I) - g_\ell^R(z^R, z^I)| \leq G_\ell^{RR}|\tilde{z}^R - z^R| + G_\ell^{RI}|\tilde{z}^I - z^I| \\ |g_\ell^I(\tilde{z}^R, \tilde{z}^I) - g_\ell^I(z^R, z^I)| \leq G_\ell^{IR}|\tilde{z}^R - z^R| + G_\ell^{II}|\tilde{z}^I - z^I|. \end{cases} \tag{2.5}$$

Remark 2: Condition (i) holds if and only if both its real and imaginary parts are 0, i.e., $f_\ell^R(0,0) = f_\ell^I(0,0) = 0$ and $g_\ell^R(0,0) = g_\ell^I(0,0) = 0$ for any $\ell \in R$.

Next, we present two necessary lemmas.

Lemma 1 ([30]): If functions $f(t)$ and $g(t)$ together with their derivatives are continuous in $[t_0, t]$, then fractional differentiation of the Leibniz rule is in the form

$${}^{RL}D_{t_0}^{\alpha}(p(t)q(t)) = \sum_{m=0}^{n} \binom{\alpha}{m} \frac{d^m p(t)}{dt^m} \, {}^{RL}D_{t_0}^{\alpha-m} q(t) - I_n^{\alpha}(t),$$

where $n \geq \alpha + 1$,

$$\binom{\alpha}{m} = \frac{\Gamma(\alpha + 1)}{m! \, \Gamma(\alpha - m + 1)},$$

and

$$\begin{aligned}I_n^\alpha(t) =&\frac{(-1)^n(t-\alpha)^{n-\alpha+1}}{n!\,\Gamma(-\alpha)}\int_0^1\int_0^1 F_\alpha(t,\zeta,\hbar)\mathrm{d}\zeta\mathrm{d}\hbar,\\ F_\alpha(t,\zeta,\hbar) =&q(t_0+\hbar(t-t_0))p^{(n+1)}(t_0+(t-t_0)(\zeta+\hbar-\zeta\hbar)).\end{aligned}$$

Lemma 2: For a continuous differentiable function $\mathcal{P}(t):[\,t_0,+\infty)\to[\,0,+\infty)$ and $Q(t)=(t-t_0+\sigma)^\alpha\mathcal{P}(t)$, then

$$^{C}D_{t_0}^{\alpha}Q(t)\le(t-t_0+\sigma)^\alpha\,{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{1-\alpha+\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)}\overline{Q}(t),$$

where $t\ge t_0$, $\sigma>0$ and $\overline{Q}(t)=\sup\limits_{t_0\le s\le t}Q(s)$.

Proof: From Lemma 1, we know

$$\begin{aligned}^{C}D_{t_0}^{\alpha}Q(t)&={}^{RL}D_{t_0}^{\alpha}Q(t)-\frac{Q(t_0)}{\Gamma(1-\alpha)}(t-t_0)^{-\alpha}\\ &=(t-t_0+\sigma)^\alpha\,{}^{RL}D_{t_0}^{\alpha}\mathcal{P}(t)+\alpha^2(t-t_0+\sigma)^{\alpha-1}\,{}^{RL}D_{t_0}^{\alpha-1}\mathcal{P}(t)-R_2^\alpha(t)-\frac{\sigma^\alpha\mathcal{P}(t_0)}{\Gamma(1-\alpha)}(t-t_0)^{-\alpha}\\ &\le(t-t_0+\sigma)^\alpha({}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{\mathcal{P}(t_0)}{\Gamma(1-\alpha)}(t-t_0)^{-\alpha})-\frac{\sigma^\alpha\mathcal{P}(t_0)}{\Gamma(1-\alpha)}(t-t_0)^{-\alpha}+\alpha^2(t-t_0+\sigma)^{\alpha-1}\,{}^{RL}D_{t_0}^{\alpha-1}\mathcal{P}(t)\\ &\le(t-t_0+\sigma)^\alpha\,{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{\mathcal{P}(t_0)}{\Gamma(1-\alpha)}+\alpha^2(t-t_0+\sigma)^{\alpha-1}\,{}^{RL}D_{t_0}^{\alpha-1}\mathcal{P}(t)\\ &\le(t-t_0+\sigma)^\alpha\,{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{\overline{Q}(t)}{\sigma^\alpha\,\Gamma(1-\alpha)}+\alpha^2(t-t_0+\sigma)^{\alpha-1}\,{}^{RL}D_{t_0}^{\alpha-1}\mathcal{P}(t).\end{aligned}$$

Also by the definition of Riemann-Liouville fractional derivative,

$$\begin{aligned}&\alpha^2(t-t_0+\sigma)^{\alpha-1}\,{}^{RL}D_{t_0}^{\alpha-1}\mathcal{P}(t)\\ &=\frac{\alpha^2}{\Gamma(1-\alpha)}(t-t_0+\sigma)^{\alpha-1}\int_{t_0}^{t}(t-s)^{-\alpha}\mathcal{P}(s)\mathrm{d}s\le\frac{\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)}\overline{Q}(t).\end{aligned}$$

Therefore,

$$\begin{aligned}^{C}D_{t_0}^{\alpha}Q(t)&\le(t-t_0+\sigma)^\alpha\,{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{\overline{Q}(t)}{\sigma^\alpha\,\Gamma(1-\alpha)}+\frac{\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)}\overline{Q}(t)\\ &=(t-t_0+\sigma)^\alpha\,{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{1-\alpha+\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)}\overline{Q}(t)\end{aligned}$$

for $t\ge t_0$. Proof of Lemma 2 is finished.

3. Main results

In this section, we will provide some sufficient conditions to guarantee fixed-deviation stability and synchronization of delayed FOCVNNs (2.1).

3.1. Fixed-deviation stability

We design linear discontinuous control for system (2.1):

$$U_k(t) = \mathcal{M}_k z_k(t) + \mathcal{N}_k[\mathrm{sgn}(z_k^R(t)) + i\,\mathrm{sgn}(z_k^I(t))], \tag{3.1}$$

where $k = 1, \cdots, n$.

Thus by controller (3.1), system (2.4) is converted as

$$\begin{aligned}{}^C D_{t_0}^{\alpha} z_k^R(t) =& - a_k z_k^R(t) + \sum_{\ell=1}^{n} b_{k\ell}^R f_\ell^R(z_\ell(t)) - \sum_{\ell=1}^{n} b_{k\ell}^I f_\ell^I(z_\ell(t)) \\ &+ \sum_{\ell=1}^{n} d_{k\ell}^R g_\ell^R(z_\ell(t-\varpi_{k\ell}(t))) \sum_{\ell=1}^{n} d_{k\ell}^I g_\ell^I(z_\ell(t-\varpi_{k\ell}(t))) + \mathcal{M}_k z_k^R(t) + \mathcal{N}_k \mathrm{sgn}(z_k^R(t)),\end{aligned}$$

$$\begin{aligned}{}^C D_{t_0}^{\alpha} z_k^I(t) =& - a_k z_k^I(t) + \sum_{\ell=1}^{n} b_{k\ell}^R f_\ell^I(z_\ell(t)) + \sum_{\ell=1}^{n} b_{k\ell}^I f_\ell^R(z_\ell(t)) \\ &+ \sum_{\ell=1}^{n} d_{k\ell}^R g_\ell^I(z_\ell(t-\varpi_{k\ell}(t))) + \sum_{\ell=1}^{n} d_{k\ell}^I g_\ell^R(z_\ell(t-\varpi_{k\ell}(t))) + \mathcal{M}_k z_k^I(t) + \mathcal{N}_k \mathrm{sgn}(z_k^I(t)).\end{aligned} \tag{3.2}$$

Theorem 1: If there are positive constants $\sigma > \varpi \geq 0$ and $\mu_r > 0, v_r > 0$ $(r = 1, \cdots, n)$ such that the following conditions

$$\begin{aligned} a_r - |\mathcal{M}_r| - \frac{1-\alpha+\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)} - \frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\, & |b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI}) + |b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II}) \\ & + \Big(|d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI}) + |d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^{\alpha}\Big]\mu_\ell > 0 \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} a_r - |\mathcal{M}_r| - \frac{1-\alpha+\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)} - \frac{1}{v_r}\sum_{\ell=1}^{n}\Big[\, & |b_{r\ell}^R|(F_\ell^{IR} + F_\ell^{II}) + |b_{r\ell}^I|(F_\ell^{RR} + F_\ell^{RI}) \\ & + \Big(|d_{r\ell}^R|(G_\ell^{IR} + G_\ell^{II}) + |d_{r\ell}^I|(G_\ell^{RR} + G_\ell^{RI})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^{\alpha}\Big]v_\ell > 0 \end{aligned} \tag{3.4}$$

hold, then delayed FOCVNNs (3.2) is globally uniformly β-stable, that is, delayed FOCVNNs (2.1) is globally uniformly β-stable via control rule (3.1).

Proof: Construct an auxiliary function as follows

$$\mathcal{P}(t) = \max_{1\le k\le n} \max\left\{\frac{|z_k^R(t)|}{\mu_k}, \frac{|z_k^I(t)|}{\nu_k}\right\}.$$

Let

$$Q(t) = (t - t_0 + \sigma)^\alpha \mathcal{P}(t), \quad \overline{Q}(t) = \sup_{t_0-\sigma\le s\le t} Q(s).$$

There exists $r \in \{1, \cdots, n\}$ for given $t \ge t_0$ having

$$\mathcal{P}(t) = \max\left\{\frac{|z_r^R(t)|}{\mu_r}, \frac{|z_r^I(t)|}{\nu_r}\right\}.$$

Then we get $\mathcal{P}(t) = \frac{|z_r^R(t)|}{\mu_r}$, $\mathcal{P}(t) = \frac{|z_r^I(t)|}{\nu_r}$. Now, we let $\mathcal{P}(t) = \frac{|z_r^R(t)|}{\mu_r}$, and another case is similar. By (2.5) and (2.7) it follows that

$$\begin{aligned}
{}^C D_{t_0}^\alpha \mathcal{P}(t) =& \frac{1}{\mu_r} {}^C D_{t_0}^\alpha |z_r^R(t)| \le \frac{\mathrm{sgn}(z_r^R(t))}{\mu_r} {}^C D_{t_0}^\alpha z_r^R(t) \\
\le& \frac{-(a_r - |\mathcal{M}_r|)}{\mu_r}|z_r^R(t)| + \frac{|\mathcal{N}_r|}{\mu_r} + \frac{1}{\mu_r}\sum_{\ell=1}^n |b_{r\ell}^R|(F_\ell^{RR}|z_\ell^R(t)| + F_\ell^{RI}|z_\ell^I(t)|) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^n |b_{r\ell}^I|(F_\ell^{IR}|z_\ell^R(t)| + F_\ell^{II}|z_\ell^I(t)|) + \frac{1}{\mu_r}\sum_{\ell=1}^n |d_{r\ell}^R|(G_\ell^{RR}|z_\ell^R(t - \varpi_{r\ell}(t))| + G_\ell^{RI}|z_\ell^I(t - \varpi_{r\ell}(t))|) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^n |d_{r\ell}^I|(G_\ell^{IR}|z_\ell^R(t - \varpi_{r\ell}(t))| + G_\ell^{II}|z_\ell^I(t - \varpi_{r\ell}(t))|) \\
\le& -(a_r - |\mathcal{M}_r|)\mathcal{P}(t) + \frac{|\mathcal{N}_r|}{\mu_r} \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^n |b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI})\mu_\ell\mathcal{P}(t) + \frac{1}{\mu_r}\sum_{\ell=1}^n |b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II})\mu_\ell\mathcal{P}(t) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^n |d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI})\mu_\ell\mathcal{P}(t - \varpi_{r\ell}(t)) + \frac{1}{\mu_r}\sum_{\ell=1}^n |d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\mu_\ell\mathcal{P}(t - \varpi_{r\ell}(t)) \\
=& \left\{-(a_r - |\mathcal{M}_r|) + \frac{1}{\mu_r}\sum_{\ell=1}^n \left[|b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI}) + |b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II})\right]\mu_\ell\right\}\mathcal{P}(t) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^n \left[|d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI}) + |d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\right]\mu_\ell\mathcal{P}(t - \varpi_{r\ell}(t)) + \frac{|\mathcal{N}_r|}{\mu_r}.
\end{aligned} \tag{3.5}$$

By using Lemma 2 and (3.5), then

$$
\begin{aligned}
{}^{C}D_{t_0}^{\alpha}Q(t) \leq&(t-t_0+\sigma)^{\alpha}\,{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}\overline{Q}(t)\\
\leq&\Big\{-(a_r-|\mathcal{M}_r|)+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\Big]\mu_{\ell}\Big\}Q(t)\\
&+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big]\mu_{\ell}\Big(\frac{t-t_0+\sigma}{t-\varpi_{r\ell}(t)-t_0+\sigma}\Big)^{\alpha}Q(t-\varpi_{r\ell}(t))\\
&+(t-t_0+\sigma)^{\alpha}\frac{|\mathcal{N}_r|}{\mu_r}+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}\overline{Q}(t)\\
\leq&\Big\{-(a_r-|\mathcal{M}_r|)+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\Big]\mu_{\ell}\Big\}Q(t)\\
&+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big]\mu_{\ell}\Big(\frac{t-t_0+\sigma}{t-\varpi_{r\ell}(t)-t_0+\sigma}\Big)^{\alpha}\overline{Q}(t)\\
&+(t-t_0+\sigma)^{\alpha}\frac{|\mathcal{N}_r|}{\mu_r}+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}\overline{Q}(t).
\end{aligned}
$$

It is known that $\dfrac{\sigma+\mathscr{E}}{\mathscr{E}-\varpi_{r\ell}(t)+\sigma}$ is monotone non-increasing for $\mathscr{E}\geq 0$, and thus

$$
\frac{t-t_0+\sigma}{t-\varpi_{r\ell}(t)-t_0+\sigma}\leq\frac{\sigma}{-\varpi_{rl}(t)+\sigma}\leq\frac{\sigma}{-\varpi+\sigma},
$$

therefore,

$$
\begin{aligned}
&{}^{C}D_{t_0}^{\alpha}Q(t)\\
&\leq\Big\{-(a_r-|\mathcal{M}_r|)+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\\
&\quad+\Big(|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^{\alpha}\Big]\mu_{\ell}\Big\}Q(t)+(t-t_0+\sigma)^{\alpha}\frac{|\mathcal{N}_r|}{\mu_r}\\
&\leq-\mathscr{A}Q(t)+(t-t_0+\sigma)^{\alpha}\mathscr{B}
\end{aligned}
\tag{3.6}
$$

when $Q(t)=\overline{Q}(t)$, for $t\geq t_0$, where

$$
\begin{aligned}
\mathscr{A}\triangleq&\min_{1\leq r\leq n}\Big\{a_r-|\mathcal{M}_r|-\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}-\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\\
&+\Big(|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^{\alpha}\Big]\mu_{\ell}\Big\},\\
\mathscr{B}\triangleq&\max_{1\leq r\leq n}\Big(\frac{|\mathcal{N}_r|}{\mu_r}\Big).
\end{aligned}
$$

Next, from the definition $\overline{Q}(t)=\sup\limits_{t_0-\sigma\leq s\leq t}Q(s)$, we will divide into three cases to prove fixed-deviation stable.

Case 1: $\overline{Q}(s) > Q(s)$ for any $t_0 < s \le t$. Now, we consider $\overline{Q}(t)$ is the maximum value of $Q(s)$ at moment t_0, that is

$$\overline{Q}(t) = \overline{Q}(t_0), \quad \forall t \ge t_0.$$

Hence,

$$\begin{aligned}\|z(t)\| \le& \|\mu\|\mathcal{P}(t) = \frac{\|\mu\|}{(t-t_0+\sigma)^\alpha}Q(t) \le \frac{\|\mu\|}{(t-t_0+\sigma)^\alpha}\overline{Q}(t) = \frac{\|\mu\|}{(t-t_0+\sigma)^\alpha}\overline{Q}(t_0)\\ \le& \frac{\|\mu\|\sigma^\alpha}{(t-t_0+\sigma)\mu_{\min}}\|\psi\|_C \le \frac{\|\mu\|\sigma^\alpha\xi}{(t-t_0+\sigma)\mu_{\min}},\end{aligned}$$

when $\|\psi\|_C \le \xi$, where $\mu_{\min} = \min\limits_{1\le r\le n}\{\mu_r\}$.

Case 2: $\overline{Q}(t) = Q(t)$. We obtain

$$^C D_{t_0}^\alpha \overline{Q}(t) \le {}^C D_{t_0}^\alpha Q(t), \quad t \ge t_0. \tag{3.7}$$

From divisional integration method, we have

$$\begin{aligned}\int_{t_0}^t \frac{\overline{Q}'(s) - Q'(s)}{(t-s)^\alpha}\mathrm{d}s =& \lim_{s\to t^-}\frac{\overline{Q}(s)-Q(s)}{(t-s)^\alpha} - \frac{\overline{Q}(t_0)-Q(t_0)}{(t-t_0)^\alpha} - \alpha\int_{t_0}^t \frac{\overline{Q}(s)-Q(s)}{(t-s)^{\alpha+1}}\mathrm{d}s\\ =& \lim_{s\to t^-}\frac{1}{-\alpha}[\overline{Q}'(s) - Q'(s)](t-s)^{1-\alpha} - \frac{\overline{Q}(t_0)-Q(t_0)}{(t-t_0)^\alpha} - \alpha\int_{t_0}^t \frac{\overline{Q}(s)-Q(s)}{(t-s)^{\alpha+1}}\mathrm{d}s\\ =& -\frac{\overline{Q}(t_0)-Q(t_0)}{(t-t_0)^\alpha} - \alpha\int_{t_0}^t \frac{\overline{Q}(s)-Q(s)}{(t-s)^{\alpha+1}}\mathrm{d}s \le 0,\end{aligned}$$

thus, (3.7) holds.

Next, we demand

$$\mathcal{P}(t) \le \frac{\mathscr{B}}{\mathscr{A}}, \quad t \ge t_0. \tag{3.8}$$

Otherwise, from (3.6) and (3.7) we have

$$\begin{aligned}{}^C D_{t_0}^\alpha \overline{Q}(t) \le& {}^C D_{t_0}^\alpha Q(t) \le -\mathscr{A}Q(t) + (t-t_0+\sigma)^\alpha\mathscr{B}\\ \le& -\mathscr{A}(t-t_0+\sigma)^\alpha\mathcal{P}(t) + (t-t_0+\sigma)^\alpha\mathscr{B} < 0.\end{aligned}$$

It is known that $\overline{Q}(t)$ is monotonically increasing, so $\overline{Q}'(t) \ge 0$, then

$$^C D_{t_0}^\alpha \overline{Q}(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^t \frac{\overline{Q}'(s)}{(t-s)^\alpha}\mathrm{d}s \ge 0,$$

which is a contradiction. Hence, (3.8) is true.

Therefore,

$$\|z(t)\| \le \|\mu\|\mathcal{P}(t) \le \frac{\|\mu\|\mathscr{B}}{\mathscr{A}}$$

for $t \geq t_0$.

Case 3: $\overline{Q}(\hat{t}) = Q(\hat{t})$, $t_0 \leq \hat{t} < t$, and $\overline{Q}(s) > Q(s)$, for $\forall s \in (\hat{t}, t]$.

Combining Cases 1 and 2, we get

$$\mathcal{P}(\hat{t}) \leq \frac{\mathscr{B}}{\mathscr{A}}$$

and

$$Q(t) < \overline{Q}(t) = \overline{Q}(\hat{t}) = Q(\hat{t}) = (\hat{t} - t_0 + \sigma)^{\alpha}\mathcal{P}(\hat{t}) \leq (\hat{t} - t_0 + \sigma)^{\alpha}\frac{\mathscr{B}}{\mathscr{A}}.$$

Therefore, for $t \geq t_0$

$$\|z(t)\| \leq \|\mu\|\mathcal{P}(t) = \frac{\|\mu\|Q(t)}{(t - t_0 + \sigma)^{\alpha}} \leq \frac{\|\mu\|\mathscr{B}}{\mathscr{A}}.$$

In conclusion, let

$$T(\xi) = \max\left\{\left[\left(\frac{\mathscr{A}\xi}{\mathscr{B}\mu_{\min}}\right)^{\frac{1}{\alpha}} - 1\right], 0\right\},$$

then

$$\|z(t)\| \leq \frac{\|\mu\|\mathscr{B}}{\mathscr{A}} \triangleq \beta$$

for all $t \geq t_0 + T(\xi)$, when $\|\psi\|_C \leq \xi$. So, it can be inferred that then delayed FOCVNNs (2.1) is globally uniformly β-stable via control rule (3.1).

Corollary 1: If there are n positive constants μ_r, ν_r such that

$$a_r - |\mathcal{M}_r| - \frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[|b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI}) + |b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II}) + \Big(|d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI}) + |d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\Big)\Big]\mu_\ell > 0 \tag{3.9}$$

and

$$a_r - |\mathcal{M}_r| - \frac{1}{\nu_r}\sum_{\ell=1}^{n}\Big[|b_{r\ell}^R|(F_\ell^{IR} + F_\ell^{II}) + |b_{r\ell}^I|(F_\ell^{RR} + F_\ell^{RI}) + \Big(|d_{r\ell}^R|(G_\ell^{IR} + G_\ell^{II}) + |d_{r\ell}^I|(G_\ell^{RR} + G_\ell^{RI})\Big)\Big]\nu_\ell > 0 \tag{3.10}$$

hold, then delayed FOCVNNs (2.1) is globally uniformly fixed-deviation stable via control rule (3.1).

Proof: Let

$$\mathscr{L}(\vartheta) = a_r - |\mathcal{M}_r| - \frac{1 - \alpha + \alpha^2}{\vartheta^{\alpha}\,\Gamma(2 - \alpha)} - \frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[|b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI}) + |b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II}) + \Big(|d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI}) + |d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\Big)\Big(\frac{\vartheta}{-\varpi + \vartheta}\Big)^{\alpha}\Big]\mu_\ell,$$

$$\mathscr{X}(\vartheta) = a_r - |\mathcal{M}_r| - \frac{1-\alpha+\alpha^2}{\vartheta^\alpha\,\Gamma(2-\alpha)} - \frac{1}{v_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^R|(F_\ell^{IR}+F_\ell^{II}) + |b_{r\ell}^I|(F_\ell^{RR}+F_\ell^{RI}) + \Big(|d_{r\ell}^R|(G_\ell^{IR}+G_\ell^{II}) + |d_{r\ell}^I|(G_\ell^{RR}+G_\ell^{RI})\Big)\Big(\frac{\vartheta}{-\varpi+\vartheta}\Big)^\alpha\Big]v_\ell,$$

where $\vartheta > \varpi$, then from conditions (3.9), (3.10),

$$\lim_{\vartheta\to+\infty}\mathscr{L}(\vartheta) = a_r - |\mathcal{M}_r| - \frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^R|(F_\ell^{RR}+F_\ell^{RI}) + |b_{r\ell}^I|(F_\ell^{IR}+F_\ell^{II}) + \Big(|d_{r\ell}^R|(G_\ell^{RR}+G_\ell^{RI}) + |d_{r\ell}^I|(G_\ell^{IR}+G_\ell^{II})\Big)\Big]\mu_\ell > 0,$$

$$\lim_{\vartheta\to+\infty}\mathscr{X}(\vartheta) = a_r - |\mathcal{M}_r| - \frac{1}{v_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^R|(F_\ell^{IR}+F_\ell^{II}) + |b_{r\ell}^I|(F_\ell^{RR}+F_\ell^{RI}) + \Big(|d_{r\ell}^R|(G_\ell^{IR}+G_\ell^{II}) + |d_{r\ell}^I|(G_\ell^{RR}+G_\ell^{RI})\Big)\Big]v_\ell > 0.$$

By the property of the limit, there is a constant $\sigma > \varpi$ such that $\mathscr{L}(\sigma) > 0$ and $\mathscr{X}(\sigma) > 0$. So (3.3) and (3.4) hold. The proof is completed.

3.2. Fixed-deviation synchronization

Regard the following system (3.11) as the drive system,

$$^{C}D_{t_0}^{\alpha} z_k(t) = -a_k z_k(t) + \sum_{\ell=1}^{n} b_{k\ell} f_\ell(z_\ell(t)) + \sum_{\ell=1}^{n} d_{k\ell} g_\ell(z_\ell(t-\varpi_{k\ell}(t))), \tag{3.11}$$

and the response system is defined by the following:

$$^{C}D_{t_0}^{\alpha} \tilde{z}_k(t) = -a_k \tilde{z}_k(t) + \sum_{\ell=1}^{n} b_{k\ell} f_\ell(\tilde{z}_\ell(t)) + \sum_{\ell=1}^{n} d_{k\ell} g_\ell(\tilde{z}_\ell(t-\varpi_{k\ell}(t))) + U_k(t). \tag{3.12}$$

where $\tilde{z}_k(t) = \tilde{z}_k^R(t) + i\tilde{z}_k^I(t)$.

The initial values of system (3.12) is given by

$$\tilde{z}_k(t_0+s) = \tilde{\psi}_k^R(s) + \tilde{\psi}_k^I(s), \ -\varpi \le s \le 0.$$

Define $\Lambda_k(t) = \tilde{z}_k(t) - z_k(t)$,

$$\Lambda_k^R(t) = \tilde{z}_k^R(t) - z_k^R(t),\ \Lambda_k^I(t) = \tilde{z}_k^I(t) - z_k^I(t),$$

then we consider the following error system

$$^{C}D_{t_0}^{\alpha} \Lambda_k(t) = -a_k \Lambda_k(t) + \sum_{\ell=1}^{n} b_{k\ell} f_\ell(\Lambda_\ell(t)) + \sum_{\ell=1}^{n} d_{k\ell} g_\ell(\Lambda_\ell(t-\varpi_{k\ell}(t))) + U_k(t), \tag{3.13}$$

where

$$f_\ell(\Lambda_\ell(t)) = f_\ell(\tilde{z}_\ell(t)) - f_\ell(z_\ell(t)),$$
$$g_\ell(\Lambda_\ell(t-\varpi_{k\ell}(t))) = g_\ell(\tilde{z}_\ell(t-\varpi_{k\ell}(t))) - g_\ell(z_\ell(t-\varpi_{kl}(t))).$$

The initial value of the error system (3.13) is noted in the following form:

$$\Lambda_k(t_0+s) = \tilde{\psi}_k(s) - \psi_k(s) = \Psi_k(s), \ -\varpi \le s \le 0.$$

For error system (3.13), we construct the following controller:

$$U_k(t) = \mathcal{M}_k \Lambda_k(t) + \mathcal{N}_k[\mathrm{sgn}(\Lambda_k^R(t)) + i\mathrm{sgn}(\Lambda_k^I(t))]. \tag{3.14}$$

Thus by controller (3.14), system (3.13) is converted as

$$\begin{aligned}
{}^C D_{t_0}^\alpha \Lambda_k^R(t) =& -a_k\Lambda_k^R(t) + \sum_{\ell=1}^n b_{k\ell}^R f_\ell^R(\Lambda_\ell(t)) - \sum_{\ell=1}^n b_{k\ell}^I f_\ell^I(\Lambda_\ell(t)) \\
&+ \sum_{\ell=1}^n d_{k\ell}^R g_\ell^R(\Lambda_\ell(t-\varpi_{k\ell}(t))) - \sum_{\ell=1}^n d_{k\ell}^I g_\ell^I(\Lambda_\ell(t-\varpi_{k\ell}(t))) + \mathcal{M}_k\Lambda_k^R(t) + \mathcal{N}_k\mathrm{sgn}(\Lambda_k^R(t)), \\
{}^C D_{t_0}^\alpha \Lambda_k^I(t) =& -a_k\Lambda_k^I(t) + \sum_{\ell=1}^n b_{k\ell}^R f_\ell^I(\Lambda_\ell(t)) + \sum_{\ell=1}^n b_{k\ell}^I f_\ell^R(\Lambda_\ell(t)) \\
&+ \sum_{\ell=1}^n d_{k\ell}^R g_\ell^I(\Lambda_\ell(t-\varpi_{k\ell}(t))) + \sum_{\ell=1}^n d_{k\ell}^I g_\ell^R(\Lambda_\ell(t-\varpi_{k\ell}(t))) + \mathcal{M}_k\Lambda_k^I(t) + \mathcal{N}_k\mathrm{sgn}(\Lambda_k^I(t)).
\end{aligned} \tag{3.15}$$

Under assumption (ii), the following inequality holds:

$$\begin{cases}
\left|f_\ell^R(\Lambda_\ell(t))\right| \le F_\ell^{RR}\left|\Lambda_\ell^R(t)\right| + F_\ell^{RI}\left|\Lambda_\ell^I(t)\right| \\
\left|f_\ell^I(\Lambda_\ell(t))\right| \le F_\ell^{IR}\left|\Lambda_\ell^R(t)\right| + F_\ell^{II}\left|\Lambda_\ell^I(t)\right| \\
\left|g_\ell^R(\Lambda_\ell(t-\varpi_{k\ell}(t)))\right| \le G_\ell^{RR}\left|\Lambda_\ell^R(t-\varpi_{k\ell}(t))\right| + G_\ell^{RI}\left|\Lambda_\ell^I(t-\varpi_{k\ell}(t))\right| \\
\left|g_\ell^I(\Lambda_\ell(t-\varpi_{k\ell}(t)))\right| \le G_\ell^{IR}\left|\Lambda_\ell^R(t-\varpi_{k\ell}(t))\right| + G_\ell^{II}\left|\Lambda_\ell^I(t-\varpi_{k\ell}(t))\right|.
\end{cases} \tag{3.16}$$

Theorem 2: If there are positive constants $\sigma > \varpi \ge 0$ and $\mu_r > 0, \nu_r > 0$ $(r = 1, \cdots, n)$ such that the following conditions

$$\begin{aligned}
a_r - |\mathcal{M}_r| - \frac{1-\alpha+\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)} - \frac{1}{\mu_r}\sum_{\ell=1}^n \Big[& |b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI}) + |b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II}) \\
&+ \Big(|d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI}) + |d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^\alpha\Big]\mu_\ell > 0
\end{aligned} \tag{3.17}$$

and

$$\begin{aligned}
a_r - |\mathcal{M}_r| - \frac{1-\alpha+\alpha^2}{\sigma^\alpha\,\Gamma(2-\alpha)} - \frac{1}{\nu_r}\sum_{\ell=1}^n \Big[& |b_{r\ell}^R|(F_\ell^{IR} + F_\ell^{II}) + |b_{r\ell}^I|(F_\ell^{RR} + F_\ell^{RI}) \\
&+ \Big(|d_{r\ell}^R|(G_\ell^{IR} + G_\ell^{II}) + |d_{r\ell}^I|(G_\ell^{RR} + G_\ell^{RI})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^\alpha\Big]\nu_\ell > 0
\end{aligned} \tag{3.18}$$

hold, then delayed FOCVNNs (3.15) is globally uniformly β-stable. In other words, fixed-deviation synchronization between the drive system (3.11) and the response system (3.12) can be achieved.

Proof: Construct an auxiliary function as follows

$$\mathcal{P}(t) = \max_{1\le k\le n} \max\left\{\frac{|\Lambda_k^R(t)|}{\mu_k}, \frac{|\Lambda_k^I(t)|}{\nu_k}\right\}.$$

Let

$$Q(t) = (t - t_0 + \sigma)^{\alpha}\mathcal{P}(t), \quad \overline{Q}(t) = \sup_{t_0-\sigma\le s\le t} Q(s).$$

There exists $r \in \{1, \cdots, n\}$ for given $t \ge t_0$ having

$$\mathcal{P}(t) = \max\left\{\frac{|\Lambda_r^R(t)|}{\mu_r}, \frac{|\Lambda_r^I(t)|}{\nu_r}\right\}.$$

Then we get $\mathcal{P}(t) = \frac{|\Lambda_r^R(t)|}{\mu_r}$, $\mathcal{P}(t) = \frac{|\Lambda_r^I(t)|}{\nu_r}$. Now, we let $\mathcal{P}(t) = \frac{|\Lambda_r^R(t)|}{\mu_r}$, another case is similar. By (3.15) and (3.16) it follows that

$$\begin{aligned}
{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t) =& \frac{1}{\mu_r}\, {}^{C}D_{t_0}^{\alpha}|\Lambda_r^R(t)| \le \frac{\mathrm{sgn}(\Lambda_r^R(t))}{\mu_r}\, {}^{C}D_{t_0}^{\alpha}\Lambda_r^R(t) \\
\le& \frac{-(a_r - |\mathcal{M}_r|)}{\mu_r}|\Lambda_r^R(t)| + \frac{|\mathcal{N}_r|}{\mu_r} + \frac{1}{\mu_r}\sum_{\ell=1}^{n}|b_{r\ell}^R|(F_\ell^{RR}|\Lambda_\ell^R(t)| + F_\ell^{RI}|\Lambda_\ell^I(t)|) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^{n}|b_{r\ell}^I|(F_\ell^{IR}|\Lambda_\ell^R(t)| + F_\ell^{II}|\Lambda_\ell^I(t)|) + \frac{1}{\mu_r}\sum_{\ell=1}^{n}|d_{r\ell}^R|(G_\ell^{RR}|\Lambda_\ell^R(t - \varpi_{r\ell}(t))| + G_\ell^{RI}|\Lambda_\ell^I(t - \varpi_{r\ell}(t))|) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^{n}|d_{r\ell}^I|(G_\ell^{IR}|\Lambda_\ell^R(t - \varpi_{r\ell}(t))| + G_\ell^{II}|\Lambda_\ell^I(t - \varpi_{r\ell}(t))|) \\
\le& -(a_r - |\mathcal{M}_r|)\mathcal{P}(t) + \frac{|\mathcal{N}_r|}{\mu_r} \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^{n}|b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI})\mu_\ell\mathcal{P}(t) + \frac{1}{\mu_r}\sum_{\ell=1}^{n}|b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II})\mu_\ell\mathcal{P}(t) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^{n}|d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI})\mu_\ell\mathcal{P}(t - \varpi_{r\ell}(t)) + \frac{1}{\mu_r}\sum_{\ell=1}^{n}|d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\mu_\ell\mathcal{P}(t - \varpi_{r\ell}(t)) \\
=& \left\{-(a_r - |\mathcal{M}_r|) + \frac{1}{\mu_r}\sum_{\ell=1}^{n}\left[|b_{r\ell}^R|(F_\ell^{RR} + F_\ell^{RI}) + |b_{r\ell}^I|(F_\ell^{IR} + F_\ell^{II})\right]\mu_\ell\right\}\mathcal{P}(t) \\
&+ \frac{1}{\mu_r}\sum_{\ell=1}^{n}\left[|d_{r\ell}^R|(G_\ell^{RR} + G_\ell^{RI}) + |d_{r\ell}^I|(G_\ell^{IR} + G_\ell^{II})\right]\mu_\ell\mathcal{P}(t - \varpi_{r\ell}(t)) + \frac{|\mathcal{N}_r|}{\mu_r}.
\end{aligned} \tag{3.19}$$

By applying Lemma 2 and (3.19), we have

$$\begin{aligned}
{}^{C}D_{t_0}^{\alpha}\mathcal{Q}(t) \leq &(t-t_0+\sigma)^{\alpha}\,{}^{C}D_{t_0}^{\alpha}\mathcal{P}(t)+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}\overline{\mathcal{Q}}(t)\\
\leq &\Big\{-(a_r-|\mathcal{M}_r|)+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\Big]\mu_{\ell}\Big\}\mathcal{Q}(t)\\
&+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big]\mu_{\ell}\Big(\frac{t-t_0+\sigma}{t-\varpi_{r\ell}(t)-t_0+\sigma}\Big)^{\alpha}\mathcal{Q}(t-\varpi_{r\ell}(t))\\
&+(t-t_0+\sigma)^{\alpha}\frac{|\mathcal{N}_r|}{\mu_r}+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}\overline{\mathcal{Q}}(t)\\
\leq &\Big\{-(a_r-|\mathcal{M}_r|)+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\Big]\mu_{\ell}\Big\}\mathcal{Q}(t)\\
&+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big]\mu_{\ell}\Big(\frac{t-t_0+\sigma}{t-\varpi_{r\ell}(t)-t_0+\sigma}\Big)^{\alpha}\overline{\mathcal{Q}}(t)\\
&+(t-t_0+\sigma)^{\alpha}\frac{|\mathcal{N}_r|}{\mu_r}+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}\overline{\mathcal{Q}}(t).
\end{aligned}$$

It is known that $\dfrac{\sigma+\mathscr{E}}{\mathscr{E}-\varpi_{r\ell}(t)+\sigma}$ is monotone non-increasing for $\mathscr{E}\geq 0$, and thus

$$\frac{t-t_0+\sigma}{t-\varpi_{r\ell}(t)-t_0+\sigma}\leq\frac{\sigma}{-\varpi_{r\ell}(t)+\sigma}\leq\frac{\sigma}{-\varpi+\sigma},$$

therefore,

$$\begin{aligned}
{}^{C}D_{t_0}^{\alpha}\mathcal{Q}(t) \leq &\Big\{-(a_r-|\mathcal{M}_r|)+\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}+\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\\
&+\Big(|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^{\alpha}\Big]\mu_{\ell}\Big\}\mathcal{Q}(t)+(t-t_0+\sigma)^{\alpha}\frac{|\mathcal{N}_r|}{\mu_r}\\
\leq &-\mathscr{A}\mathcal{Q}(t)+(t-t_0+\sigma)^{\alpha}\mathscr{B},
\end{aligned}\tag{3.20}$$

when $\mathcal{Q}(t)=\overline{\mathcal{Q}}(t)$, for $t\geq t_0$, where

$$\begin{aligned}
\mathscr{A}\triangleq &\min_{1\leq r\leq n}\Big\{a_r-|\mathcal{M}_r|-\frac{1-\alpha+\alpha^2}{\sigma^{\alpha}\,\Gamma(2-\alpha)}-\frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[\,|b_{r\ell}^{R}|(F_{\ell}^{RR}+F_{\ell}^{RI})+|b_{r\ell}^{I}|(F_{\ell}^{IR}+F_{\ell}^{II})\\
&+\Big(|d_{r\ell}^{R}|(G_{\ell}^{RR}+G_{\ell}^{RI})+|d_{r\ell}^{I}|(G_{\ell}^{IR}+G_{\ell}^{II})\Big)\Big(\frac{\sigma}{-\varpi+\sigma}\Big)^{\alpha}\Big]\mu_{\ell}\Big\},\\
\mathscr{B}\triangleq &\max_{1\leq r\leq n}\Big(\frac{|\mathcal{N}_r|}{\mu_r}\Big).
\end{aligned}$$

Similar to cases 1–3 in Theorem 1, we finally obtain

$$\|z(t)\|\leq\frac{\|\mu\|\mathscr{B}}{\mathscr{A}}\triangleq\beta$$

for all $t \geq t_0 + T(\xi)$, when $\|\Psi\|_C \leq \xi$, where

$$T(\xi) = \max\left\{\left[\left(\frac{\mathscr{A}\xi}{\mathscr{B}\mu_{\min}}\right)^{\frac{1}{\alpha}} - 1\right], 0\right\}.$$

So, fixed-deviation synchronization between the drive system (3.11) and the response system (3.12) can be achieved.

Corollary 2: If there are n positive constants μ_r, v_r such that

$$\begin{aligned} a_r - |\mathcal{M}_r| - \frac{1}{\mu_r}\sum_{\ell=1}^{n}\Big[& |b^R_{r\ell}|(F^{RR}_\ell + F^{RI}_\ell) + |b^I_{r\ell}|(F^{IR}_\ell + F^{II}_\ell) \\ & + \Big(|d^R_{r\ell}|(G^{RR}_\ell + G^{RI}_\ell) + |d^I_{r\ell}|(G^{IR}_\ell + G^{II}_\ell)\Big)\Big]\mu_\ell > 0 \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} a_r - |\mathcal{M}_r| - \frac{1}{v_r}\sum_{\ell=1}^{n}\Big[& |b^R_{r\ell}|(F^{IR}_\ell + F^{II}_\ell) + |b^I_{r\ell}|(F^{RR}_\ell + F^{RI}_\ell) \\ & + \Big(|d^R_{r\ell}|(G^{IR}_\ell + G^{II}_\ell) + |d^I_{r\ell}|(G^{RR}_\ell + G^{RI}_\ell)\Big)\Big]v_\ell > 0 \end{aligned} \tag{3.22}$$

hold, then delayed FOCVNNs (3.15) is globally uniformly β-stable. In other words, fixed-deviation synchronization between the drive system (3.11) and the response system (3.12) can be achieved.

Proof: The proof of Corollary 2 is similar to the proof of Corollary 1.

4. Numerical examples

Example 1: We consider the following delayed FOCVNNs:

$$\begin{aligned} {}^{C}D^{0.9}_{t_0} z_k(t) &= -2z_1(t) + b_{11}f_1(z_1(t)) + b_{12}f_2(z_2(t)) + d_{11}g_1(z_1(t-1)) + d_{12}g_2(z_2(t-1)) + U_1(t), \\ {}^{C}D^{0.9}_{t_0} z_k(t) &= -4z_2(t) + b_{21}f_1(z_1(t)) + b_{22}f_2(z_2(t)) + d_{21}g_1(z_1(t-1)) + d_{22}g_2(z_2(t-1)) + U_2(t), \end{aligned} \tag{4.1}$$

where $t_0 = 0$, $z_k(t) = z^R_k(t) + iz^I_k(t)$, $f_\ell(z_\ell) = g_\ell(z_\ell) = tanh(z^R_\ell) + tanh(z^I_\ell)i$, $\ell = 1, 2$,

$$B = (b_{k\ell})_{2\times 2} = \begin{pmatrix} 0.03 + 0.05i & -0.05 + 0.04i \\ 0.02 - 0.01i & -0.03 + 0.02i \end{pmatrix}, \quad D = (d_{k\ell})_{2\times 2} = \begin{pmatrix} -0.07 - 0.02i & 0.03 + 0.01i \\ -0.05 + 0.03i & 0.01 + 0.05i \end{pmatrix}.$$

It's not hard to choose that $F^{RR}_\ell + F^{RI}_\ell = F^{IR}_\ell + F^{II}_\ell = 2$, $G^{RR}_\ell + G^{RI}_\ell = G^{IR}_\ell + G^{II}_\ell = 2$.

Let $\mathcal{M}_1 = 0.06$, $\mathcal{M}_2 = 0.04$, $\sigma = 10$, $\mu_1 = \mu_2 = 1$, $v_1 = v_2 = 1$, then (3.3) and (3.4) hold. Hence, system (4.1) is globally uniformly β-stable from Theorem 1. Figure 1 shows the numerical simulation of FOCVNNs (4.1) under discontinuous control rules $U_1(t) = 0.06z_1(t) + 1.19[\mathrm{sgn}(z^R_1(t)) + i\mathrm{sgn}(z^I_1(t))]$ and $U_2(t) = 0.04z_2(t) + 1.19[\mathrm{sgn}(z^R_2(t)) + i\mathrm{sgn}(z^I_2(t))]$.

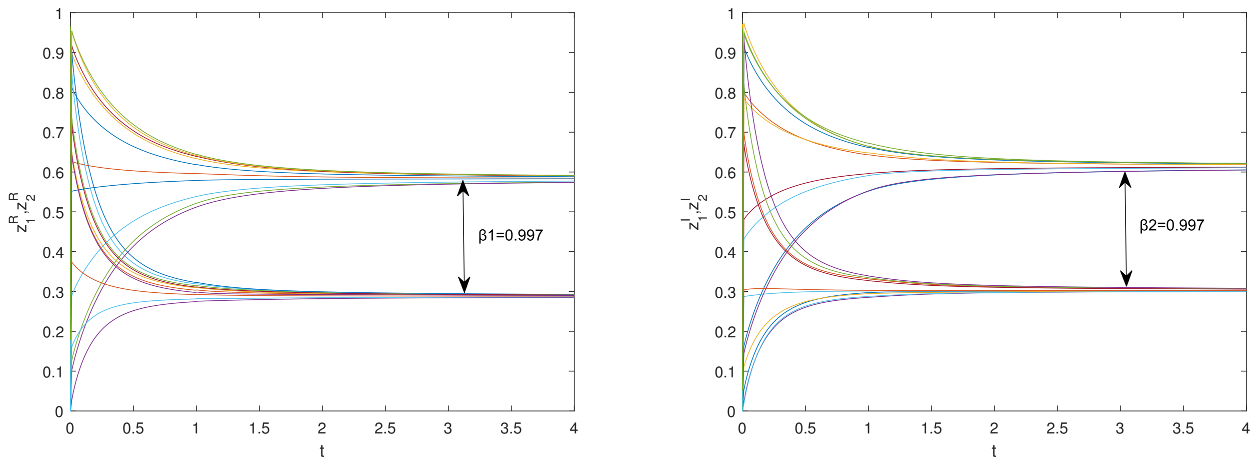

Figure 1. The fixed-deviation stabilization of system (4.1), where $\beta = N(\beta_1 + \beta_2), N = 10$.

Example 2: Regard the following FOCVNNs (4.2) as the drive system:

$$
\begin{aligned}
{}^{C}D_{t_0}^{0.95} z_k(t) &= -5z_1(t) + b_{11}f_1(z_1(t)) + b_{12}f_2(z_2(t)) + d_{11}g_1(z_1(t-1)) + d_{12}g_2(z_2(t-1)),\\
{}^{C}D_{t_0}^{0.95} z_k(t) &= -3z_2(t) + b_{21}f_1(z_1(t)) + b_{22}f_2(z_2(t)) + d_{21}g_1(z_1(t-1)) + d_{22}g_2(z_2(t-1)),
\end{aligned}
\tag{4.2}
$$

where $t_0 = 0$, $f_\ell(z_\ell) = g_\ell(z_\ell) = cos(z_\ell^R) + cos(z_\ell^I)i$, $\ell = 1, 2$,

$$
B = (b_{k\ell})_{2\times 2} = \begin{pmatrix} -0.02 + 0.05i & 0.04 + 0.06i \\ 0.02 - 0.01i & -0.07 + 0.09i \end{pmatrix}, \quad D = (d_{k\ell})_{2\times 2} = \begin{pmatrix} 0.03 - 0.02i & -0.08 + 0.09i \\ -0.05 + 0.03i & 0.01 + 0.05i \end{pmatrix}.
$$

The response system is given by

$$
\begin{aligned}
{}^{C}D_{t_0}^{0.95} \tilde{z}_k(t) &= -5\tilde{z}_1(t) + b_{11}f_1(\tilde{z}_1(t)) + b_{12}f_2(\tilde{z}_2(t)) + d_{11}g_1(\tilde{z}_1(t-1)) + d_{12}g_2(\tilde{z}_2(t-1)) + U_1(t),\\
{}^{C}D_{t_0}^{0.95} \tilde{z}_k(t) &= -3z_2(t) + b_{21}f_1(\tilde{z}_1(t)) + b_{22}f_2(\tilde{z}_2(t)) + d_{21}g_1(\tilde{z}_1(t-1)) + d_{22}g_2(\tilde{z}_2(t-1)) + U_2(t),
\end{aligned}
\tag{4.3}
$$

where the parameters $b_{k\ell}$, $d_{k\ell}$, $f_\ell(\cdot)$, $g_\ell(\cdot)$ are all the same as in FOCVNNs (4.2).

As above choose that $F_\ell^{RR} + F_\ell^{RI} = F_\ell^{IR} + F_\ell^{II} = 2$, $G_\ell^{RR} + G_\ell^{RI} = G_\ell^{IR} + G_\ell^{II} = 2$.

Let $\mathcal{M}_1 = 0.02$, $\mathcal{M}_2 = 0.04$, $\sigma = 20$, $\mu_1 = \mu_2 = 1$, $\nu_1 = \nu_2 = 1$, then (3.17) and (3.18) hold. Hence, it can be seen that drive system (4.2) and response system (4.3) are fixed-deviation synchronization from Theorem 2. Figure 2 shows the numerical simulation of error system under discontinuous control rules $U_1(t) = 0.02\Lambda_1(t) + 2.22[\text{sgn}(\Lambda_1^R(t)) + i\text{sgn}(\Lambda_1^I(t))]$ and $U_2(t) = 0.04\Lambda_2(t) + 2.22[\text{sgn}(\Lambda_2^R(t)) + i\text{sgn}(\Lambda_2^I(t))]$.

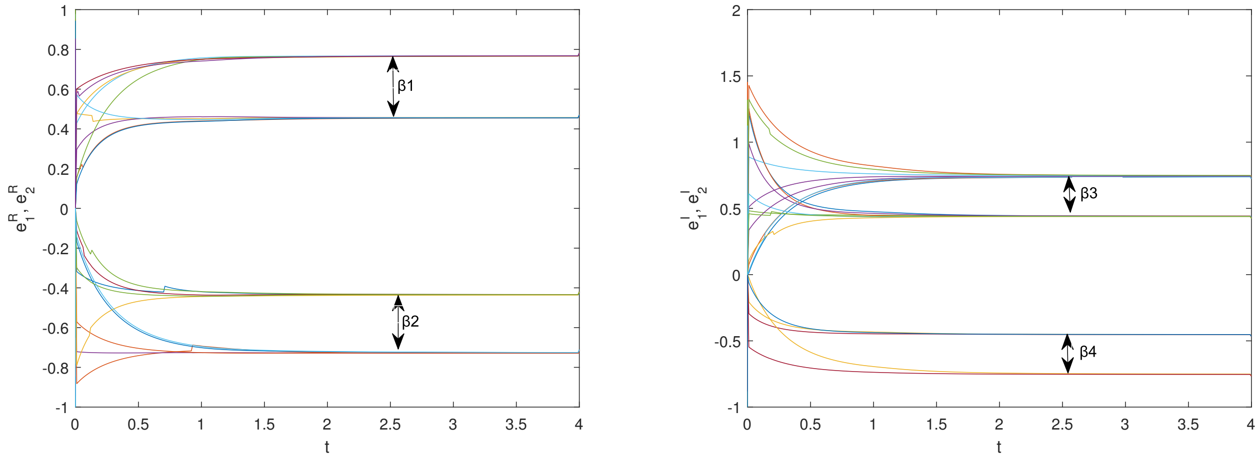

Figure 2. The fixed-deviation synchronization of drive system (4.2) and response system (4.3), where $\beta = N(\beta_1 + \beta_2 + \beta_3 + \beta_4), \beta_1 + \beta_2 = \beta_3 + \beta_4 = 0.996, N = 10$.

5. Conclusions

This paper discusses fixed-deviation stability and synchronization of FOCVNNs. The system investigated in this paper is a continuous neural network, and a discontinuous controller is introduced to address this problem. Under the discontinuous controller, fixed-deviation stability theory and fractional calculus method are used to observe the fixed-deviation dynamical behavior of delayed FOCVNNs. In this paper, a continuous system is transformed into a discontinuous system by imposing a discontinuous controller to achieve fixed-deviation dynamics, this technique can be extended to other more complex systems, which would be a future direction of research.

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Conflict of interest

The authors declare there is no conflict of interest.

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