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Research article

STATIS multivariate three-way method for evaluating quality of life after corneal surgery: Methodology and case study in Costa Rica

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Abstract: Vision-related quality of life (QoL) analyzes the visual function concerning individual well-being based on activity and social participation. Because QoL is a multivariate construct, a multivariate statistical method must be used to analyze this construct. In this paper, we present a methodology based on STATIS multivariate three-way methods to assess the real change in vision-related QoL for myopic patients by comparing their conditions before and after corneal surgery. We conduct a case study in Costa Rica to detect the outcomes of patients referred for myopia that underwent refractive surgery. We consider a descriptive, observational and prospective study. We utilize the NEI VFQ-25 instrument to measure the vision-related QoL in five different stages over three months. After applying this instrument/questionnaire, a statistically significant difference was detected between the perceived QoL levels. In addition, strong correlations were identified with highly similar structures ranging from 0.857 to 0.940. The application of the dual STATIS method found the non-existence of reconceptualization in myopic patients, but a statistically significant recalibration was identified. Furthermore, a real change was observed in all patients after surgery. This finding has not been stated previously due to the limitations of the existing statistical tools. We demonstrated that dual STATIS is a multivariate method capable of evaluating vision-related QoL data and detecting changes in recalibration and reconceptualization.

Keywords: multivariate statistics; myopia; NEI VFQ-25 questionnaire; principal components; recalibration; reconceptualization; response shift; three-way data analysis

1. Introduction

1.1. Vision-related quality of life

Myopia is a visual focus defect, considered a multifactorial disease that complicates the vision at a certain distance [1]. The uncoordinated development of eyeball parts causes this disease during emmetropization, which is affected by the environment and genes. It is a mismatch between the optical power and length of the eye, causing the incoming light to be focused in front of the retina [2].

Most visual disabilities are caused by refractive errors, as astigmatism, hypermetropia and myopia [3]. The high prevalence of myopia emphasizes the importance of understanding the eye's growth mechanisms and finding effective treatments that reduce the progression of ocular pathologies [4]. Excimer laser [5] is used in refractive surgery [6] for vision correction, based on argon fluoride at 193 nm, and produces effective results with low thermal damage on the adjacent cornea tissue. The laser can remove part of the tissue and reshape the corneal curvature, modifying its convergence through refractive surgery [7]. Corneal curvature changes allow the patients to improve their visual acuity with significant human perception, influencing their quality of life (QoL).

QoL is a multidisciplinary and subjective concept that includes positive and negative dimensions. As defined by the World Health Organization (WHO), "the QoL concept is the individual perception of the position in life within the context of the culture and value systems in which somebody lives and about the goals, expectations, standards and concerns" [8]. When this concept of individual well-being is associated with general ocular health and focused on a medical condition that influences vision, this is called vision-related QoL, such as indicated in [9]. The evaluation of the vision-related QoL enables us to assess how visual function affects the individual well-being for their regular performance and participation in society.

Some studies have used different questionnaires to measure the evolution of patients undergoing surgical procedures that improve their vision, reflecting positive changes in their QoL [10–12]. If a person experiences changes in the QoL through time, such change is known as a "response shift", which is a variation in the meaning of an individual's self-evaluation. This variation is the result of changes in the patient's internal self-evaluation (recalibration) and the construct's redefinition through time (reconceptualization) linked to QoL [13].

Several investigations have evaluated the response shift in different diseases under the presence of adverse events, for example, in cancer [14–18]. However, few studies have evaluated it in the presence of positive events. Our study is the first investigation conducted in Latin America to detect response shift in myopic patients. Therefore, in this study, we assess the change in vision-related QoL for myopic patients by comparing their conditions before and after corneal refractive surgery with an Excimer LASIK laser.

Recent works have indicated that the use of multivariate techniques applied in different areas of research demonstrates their methodological strengths and statistical rigor [25–31, 37, 38]. However, this methodology has never before been used to investigate a true/real change in vision-related QoL. For the evaluation of change, we propose the use of a multivariate method for three-way tables.

1.2. Response shift in QoL research

The concept of response refers to the fact that patients make an assessment, judgment, report, or rating of a health state. The notion of shift implies a change in the patients' response [19]. In medicine, response shift refers to a change resulting from an event, such as a therapy, in the meaning of patient's self-evaluation about QoL [20]. The phenomenon of response shift can occur in any field where self-report data are collected, focusing on changes in health status that are improving or deteriorating.

Scale recalibration was introduced in educational research [22]. An initial typology in this area establishes three types of changes: (i) alpha change, referring to true behavioral change; (ii) beta change, related to scale recalibration; and (iii) gamma change, associated with concept redefinition [21–23]. The first type is defined in terms of changes in the internal measurement standards of respondents. The second type of response shift is a change in the respondent's values over time. A specific construct may consist of various parts. The importance of these parts may change due to an event. For example, the health-related QoL construct has physical, psychological and social health parts. In some cases, the patient's concept about her/his QoL has not changed, and then the construct remains. The third type of response shift is a redefinition of the construct (reconceptualization). This happens when the parts that constitute the target construct change over time.

Response shift is defined [21] in terms of change in internal standards of measurement, but they introduced the component of reconceptualization in addition to the scale recalibration. While changes in values are inherent in the definition of reconceptualization [21], the description given in [13] includes this as a separate part that is relevant to the meaning of an individual's self-evaluation. The interconnections and the hierarchy among the dimensions are not well known. If internal standards, values, or reconceptualization of QoL change over time, then the answers to the same items by the same individuals may not be comparable. Response shift produces a better understanding of how QoL is affected by changes in health status. In clinical medicine, response shift is more than an aspect to be identified, measured and controlled for. Response shift is sometimes one of the primary goals of clinical care [24]. A theoretical model was proposed in [13] to explain how response shift may affect health-related QoL because of changes in health status. The model has the following components:

- Catalyst: change in the respondent's health status that may or not result from a treatment.
- Antecedents: individual's dispositional characteristics (personality, self-control, expectations).
- Mechanisms: behavioral, cognitive and affective processes to accommodate the catalyst (coping strategies, reframing expectations, engaging in spiritual practice).
- Perceived QoL: QoL as self-perceived by an individual.

The advantage of including response shift in a model to explain self-perceived QoL is that it conceptualizes three important change aspects. Considering these aspects explicitly in the model allows a better understanding of how QoL is affected by changes in health status.

1.3. The then-test method

The most common method to detect recalibration is the "then-test" proposed in [22]. This method detects changes in internal standards, conceptualization and patient's values. The then-test method assesses the effects of the QoL of a patient after a treatment carrying out three tests: (i) "pre-test", which records the current condition of the patient before starting the treatment; (ii) "then-test", which questions the patient's condition before starting the treatment; and (iii) "post-test", which is applied to evaluate the condition after the treatment. If significant differences are identified between the then-test and pre-test, a real change is manifested in the individual's standards, which means a recalibration. If there is a significant difference between the then-test and post-test, then a real change occurs; see Figure 1. The then-test method is based on a retrospective design that allows us to evaluate a change by comparing the scores with two other moments (occasions or instants of time). However, this method is sensitive to bias, challenging to employ in longitudinal secondary data analysis, and cannot be used to redefine the construct through time; this means reconceptualization [32]. In the present study, two groups are compared to detect statistical significance from the pre-test, then-test, and post-test results utilizing a t-test and standard deviations [13].

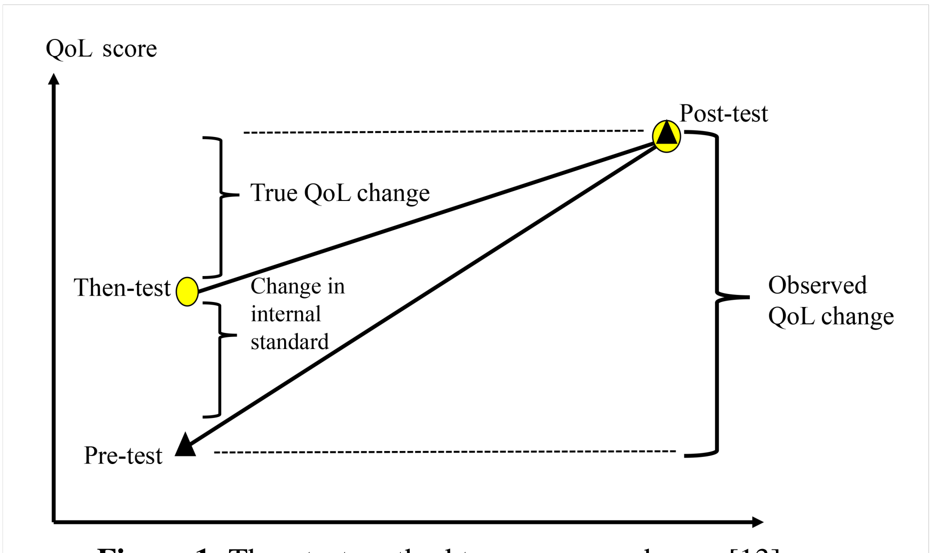


Figure 1. Then-test method to measure a change [13].

1.4. The STATIS methods

The "structuration de tableaux trois indices de la statistique" (STATIS) method can detect recalibration and reconceptualization. L'Hermier des Plantes [33] and Christine Lavit [**?**, 34] developed STATIS based on [36], which used linear algebra to reduce the dimensionality of the data, employing a similarity measure as the Euclidian distance between point configurations.

The STATIS method is a generalization of principal component analysis [78], whose objective is to study a set of variables collected from the same observations. STATIS obtains a joint called compromise to be stated through principal components analysis. This permits us to get a standard structure between the observations and project each of the original datasets onto the compromise system to explore commonalities and discrepancies [41]. Figure 2 presents tools for three-way table analysis that open new ways to detect changes in health-related QoL studies. Recent research about generalizations of principal component analysis for three-way tables was presented in [39, 40].

Dual STATIS is a multivariate three-way method that enables us to evaluate any phenomenon that includes individuals and variables at different times. This method provides a relevant contribution to the study of QoL, permitting the achievement of a comprehensive interpretation supported by a graphical representation. Its versatility, capable of simultaneously analyzing three components, has also been used in other contexts, such as engineering [63], mathematics [64,65] and plant sciences [66].

STATIS and dual STATIS are both multivariate three-ways methods. The first one identifies individuals' similarities through different times, while dual STATIS examines the structures of covariation between variables. There are several modifications to the STATIS methods described above that adapt these tools to more complex situations and that could be explored in this context shortly; see Figure 2.

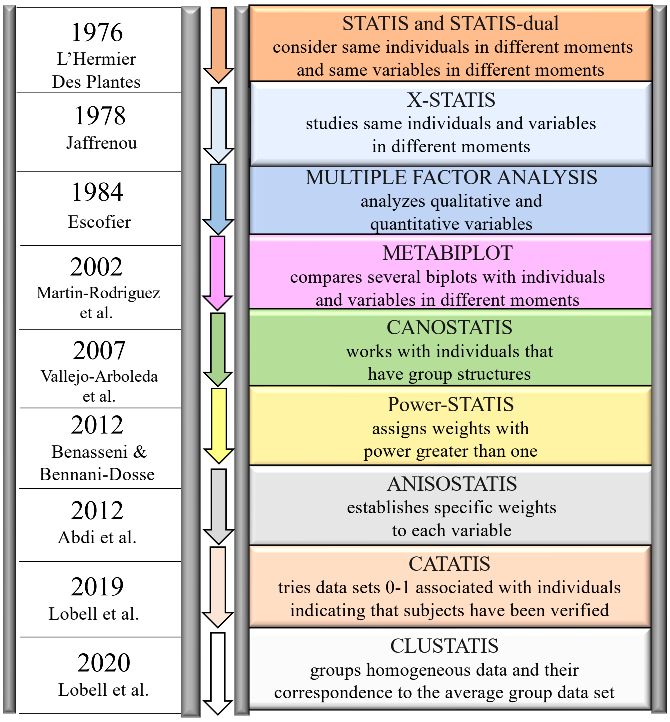


Figure 2. Chronology of STATIS and other alternative multivariate methods for QoL analysis [68–76].

1.5. Objectives and organization of the article

The literature on the study of QoL, considered a multivariate construct, is extensive with international coverage, including a large array of target groups, and uses different research designs and measurements [67]. Within the framework of health-related QoL, many medical conditions have been widely studied, allowing the comparison of different types of changes [77]. However, vision-related QoL has limited updated literature. To the best of our knowledge, no other studies focused on detecting the change in QoL for myopic patients or those with other ophthalmological pathologies have been proposed until now. Therefore, it is necessary to continue studying the topic of health-related QoL and statistical methodologies for its analysis.

After the literature review, we have detected that the dual STATIS method has not been used in any previous QoL or vision-related QoL study. Dual STATIS is proposed as a viable alternative method that analyzes three-way data and is mathematically robust for response shift detection. The main objective of the present investigation is to present a methodology based on STATIS methods to assess the change in vision-related QoL for myopic patients by comparing their conditions before and after corneal surgery. We conduct a case study in Costa Rica to detect the outcomes of patients referred for myopia that underwent refractive surgery.

Vision-related QoL is also studied to understand individuals' self-evaluations after undergoing different types of treatments and to improve visual conditions such as neuromyelitis optical spectrum disorder [49], dystonia [50], autologous internal membrane transplantation [51] and dry eyes [52], among others.

Our article is organized into sections. We detail the methodology in Section 2. Section 3 presents the case study. The results of our research and their discussion are stated in Section 4. We give some conclusions, limitations and ideas for further investigation in Section 5.

2. Methodology

2.1. Type of variables under analysis

STATIS is a non-supervised method since there is no response variable. STATIS studies the covariation structures among items in different times. The theory is developed with quantitative variables. However, in practice, STATIS is used with Likert scales, such as in factor analysis or principal component analysis, understanding that there is a continuous construct under a Likert scale.

2.2. The STATIS method and its use for identifying response shift

The STATIS method allows the evaluation of multiple tables simultaneously, where the individuals (I), measured in different occasions/situations or times (T), and the set of variables (J) may be the same under different conditions/tables or not. Therefore, STATIS performs simultaneous analysis of T matrices X_t. STATIS also analyzes the global congruence of the structure by evaluating the distance between points and the origin of coordinates in the plane of maximum inertia. Since the tables have the same individuals, the STATIS analysis privileges their position. Commonly, the STATIS method is explicitly used in the following situations: (i) when measurements are collected from the same individuals on different occasions, forming a matrix for each occasion and (ii) when the same variables are measured in different groups of individuals, forming a matrix for each group.

The STATIS method identifies recalibration and reconceptualization as follows:

- If the norms present different values, at least in one, there is recalibration.
- If angles formed by vectors joining the origin with points representing the matrices are close to zero degrees, factorial structures of the matrices are equal, and then there is no conceptualization.
- If all the matrices have the same factorial structure, a compromise matrix is calculated to build a global view of the latent dimensions of QoL throughout the treatment.

In the compromise matrix, the items of the original matrices project their path over time and detect elements that present reconceptualization.

Let X_t be the data matrix at the occasion t, with X_t having I rows, one for each patient, and J columns, one for each item (variable), stated as

$$X_t = \begin{bmatrix} \ddots & & \ddots \\ & X_{ij} & \\ \ddots & & \ddots \end{bmatrix}, \quad t \in \{1, \ldots, T\}.$$

Note that X_1 can be the data matrix at baseline time, X_2 the data matrix during treatment, X_3 the data matrix after treatment, and so on.

Two different configurations can capture the information for every time (X_t). The inner product among individuals is defined as $W_t = X_t X_t^\top$. The covariation/correlation matrix among variables (items) is stated as $C_t = X_t^\top X_t$. Every dataset is represented by its cross-product matrices, with W_t or C_t. Depending on our focus, we select a specific configuration. If our target set corresponds to the individuals (patients), we select W_t. If we want to compare covariation structures among variables (items), then we start with C_t. A configuration is used to represent it in a low-dimensional subspace. Observe that C_t is closely related to factor analysis when data are standardized. The factor loading is obtained from the eigenvalues and eigenvectors decomposition of C_t. Comparing configurations can be understood as comparing covariance or correlation structures on several occasions.

To evaluate response shift, we compare configurations. To do this, W_t (or C_t) can be defined as a correlation coefficient among matrices and/or a distance between them. Hence, we need to define an inner product between pairs of matrices. This product induces a norm and then a distance. We use the Hilbert-Schmidt (HS) inner product between two matrices W_t and $W_{t'}$, defined as $\langle W_t \setminus W_{t'} \rangle_{\mathrm{HS}} = \mathrm{trace}(W_t W_{t'})$ and $\langle C_t \setminus C_{t'} \rangle_{\mathrm{HS}} = \mathrm{trace}(C_t C_{t'})$. From this product, we can define an HS norm as $\|W_t\|^2_{\mathrm{HS}} = \langle W_t \setminus W_t \rangle_{\mathrm{HS}}$ and $\|C_t\|^2_{\mathrm{HS}} = \langle C_t \setminus C_t \rangle_{\mathrm{HS}}$ and an HS distance between two configurations by $d_{\mathrm{HS}}(W_t, W_{t'}) = \|W_t - W_{t'}\|$ and $d_{\mathrm{HS}}(C_t, C_{t'}) = \|C_t - C_{t'}\|$. STATIS and dual STATIS use the HS inner product to induce a distance between W_t (C_t). The vectorial correlation coefficient (RV) [36] is given by

$$\mathrm{RV}(W_t, W_t') = \frac{\langle W_t \setminus W_{t'} \rangle_{\mathrm{HS}}}{\sqrt{\langle W_t \setminus W_{t'} \rangle_{\mathrm{HS}} \, \langle W_{t'} \setminus W_{t'} \rangle_{\mathrm{HS}}}}. \tag{1}$$

If the vectorial correlation between two matrices stated in (1) is equal to unity, both matrices are equivalent because both structures are compatible. Then, there are no discrepancies between both studies. Thus, no reconceptualization is detected if the vectorial correlation between two matrices equals unity. In this context, equal to unity means that the difference is not statistically significant.

Note that $W = U \Delta U^\top$ and $C = V \Lambda V^\top$, where $\Delta = \mathrm{diag}(\delta_1, \ldots, \delta_I)$ and $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_J)$ are the eigenvalues of W, C respectively, and U, V are their eigenvectors. Coordinates can be calculated in the usual way for STATIS as $A = U \Delta^{-1/2} = W U \Delta^{-1/2}$ and for dual STATIS as $B = V \Delta^{-1/2} = C V \Lambda^{-1/2}$.

2.3. The dual STATIS method

Suppose the interest is in analyzing the covariation structure's stability or evolution between the variables. In this case, the dual STATIS method can be used based on $C_t = X_t^\top X_t$.

Dual STATIS is similar to STATIS, since it is applied when the same variables are measured in T groups of individuals that can be different. Unlike STATIS, dual STATIS privileges the relative position of the variables. The representative objects of the studies are, in this case, the matrices V_t of size $p \times p$ defined by $V_t = X_t^\top D_t X_t$, where V_t are the variance-covariance matrices of the tables X_t. The STATIS and dual STATIS methods consist of four stages: (i) inter-structure, (ii) compromise matrix, (iii) intra-structure, and (iv) path of individuals, defined as follows:

- *Stage 1 (inter-structure):* It consists of the global study of the data matrices' structure. It is necessary to define a representative matrix for each study, a metric, and then a Euclidean image of each object associated with a scalar product. To do this, one can use the distance calculated from the HS scalar product between two matrices W_t and $W_{t'}$, from which the table Z of size $T \times T$ is defined as $Z = [Z_{tt'}] = [\langle W_t \setminus W_{t'} \rangle_{\mathrm{HS}}]$. The diagonalization of Z provides a Euclidean image of the inter-structure after a possible weight assignment to the different studies.

- *Stage 2 (compromise matrix):* The analysis of the inter-structure permits us to identify the similarities between the different tables studied. If there are similarities between the tables, they contain similar covariance structures between their variables or individuals. It is possible to construct a compromise matrix or consensus matrix of the same nature as the objects to represent them. The compromise matrix is defined as a weighted average of objects W_t that, in the case of normed objects $V_t = W_t/\|W_t\|_{\mathrm{HS}}$, is formulated as $V = \sum_{t=1}^{T} \beta_t V_t$, where β_t are model coefficients. The compromise matrix V can be interpreted as an array of variances-covariances between variables averaged over time. The eigenvectors and eigenvalues of the matrix V provide a Euclidean compromise image that approximates the variables (intra-structure). Through the variables of each table, their trajectories are obtained in this image.
- *Stage 3 (intra-structure):* The study of the intra-structure is based on an analysis of all the different tables, enabling us to identify the similarities or differences between individuals or variables corresponding to these tables. It represents the cloud of individuals characterized by the set of T tables to obtain the Euclidean compromise image of the individuals. In addition, it states the correlations of the variables of the different tables with the axes of compromise, which permits us to interpret these axes and then to analyze the positions of the individuals on the plane.
- *Stage 4 (path of individuals):* The trajectory representation is carried out in the Euclidean compromise image, which aims to individually represent the deviations of individuals or variables from one study to another. These deviations are obtained with the inter-structure.

3. Case study

3.1. Participants

Our study is descriptive, observational and prospective. We conduct it to detect clinical outcomes of $n = 218$ patients selected by a sequential sampling referred for myopia who met the criteria for a refractive surgical intervention using an Excimer laser in Costa Rica. Applying STATIS methods and capturing recalibration or reconceptualization do not need a large sample size. These methods are standard techniques and are used to order patients. For this reason, it is not necessary to have many patients in the sequential sample. We employ the sample to test hypotheses and consider a confidence level of 95% based as usual in health-related studies.

The sample size was calculated for a finite population, where the prevalence of myopia in Costa Rica is around 16.8% of the total population [42]. In addition, a significance level of 5% was assumed. The dataset to be studied was collected from August 2012 to September 2013 in a single center named Eye and Laser Surgery Clinic in San José, Costa Rica. A selection of patients, who requested an evaluation of Excimer laser surgery to correct their myopia condition, was conducted as a sampling plan to state the participants under study who met the following inclusion criteria:

(i) Ages between 15 and 75 years old.
(ii) Spanish-speaking Latin Americans.
(iii) Myopia with or without astigmatism/presbyopia from -0.75 to -25.00 diopters.
(iv) Pre-operative visual acuity not corrected worse than 20/40.
(v) Corrected pre-operative visual ability better than 20/40.

3.2. Instrument

The instrument used is the reduced Spanish version which is officially translated for Latin American and Spain populations. All patients followed an initial pre-operative evaluation protocol to verify that: (a) they were candidates for LASIK surgery, (b) they were informed by a consent process and (c) they must accept to participate in the study. The NEI VFQ-25 questionnaire measures the impact of ocular pathology in vision-related QoL [43,44] through 25 items within 12 subscales related to the following:

(i) General health.
(ii) General vision.
(iii) Ocular pain.
(iv) Near vision activities.
(v) Distance vision activities.
(vi) Social functioning.
(vii) Mental health.
(viii) Role difficulties.
(ix) Dependency.
(x) Driving challenges.
(xi) Color vision.
(xii) Peripheral vision.

Under a Likert scale, scores range from 0 to 100, where 100 indicates existence of vision-related disabilities, and zero is their non-existence. The questionnaire is valid, reliable, widely used, and internally consistent [45]. To validate the sample, at least 5 to 10 individuals per item were considered [46]. Between 125 and 250 subjects is acceptable for applying the NEI-VFQ-25 questionnaire, indicating that $n = 218$ people is statistically representative.

3.3. Procedure

The study was initiated by applying the then-test method to identify recalibration. Then, five stages were required to determine changes. Three post-test methods were considered to detect changes in patients' conditions since they evolve in different ways and moments. The instrument was applied in five stages using the then-test method to identify recalibration as follows:

(i) Pre-test: pre-operative assessment, initial condition of QoL, myopia level, visual ability.
(ii) Post-test #1: during the next day after LASIK surgery.
(iii) Post-test #2: one month after surgery.
(iv) Post-test #3: after three months.
(v) Then-test: applied retrospectively to the patient to record condition before surgery.

The questionnaire lasted 5 to 10 minutes for each patient and was administered from August 2012 to September 2013, as mentioned. For the analysis of continuous variables, mean values were compared using a t-test from pre-and-post-surgery. The pre-test and post-test results were compared to detect a real change by means of differences utilizing a significance level of 5%. The then-test was employed to detect recalibration. The dual STATIS method was applied to assess reconceptualization and compare recalibration results [33]. We use `R` software [47] and an `R` code programmed in [48] for sparse dual STATIS, available at github.com/CCRM07/SparseSTATISdual.

4. Results and discussion

4.1. Results

The patients subject to Excimer laser refractive surgery were a heterogeneous sample, mainly formed by adults; see Table 1. Of 218 patients referred for myopia (432 operated eyes), 134 (61.5%) were women, with an average age of 31.8 $\pm$ 9.8 years old, all Latin Americans, with a majority from Costa Rica (97.3%). A large percentage (61.5%) were single, 52.3% were with university studies, and 62.8% had a profession. Patients' profiles accessing this type of surgery are people with high social-cultural and economic levels, adults under 60 years old, people economically active with a high educational level, and people with more skilled and better-paid jobs generating high incomes.

Table 1. Demographic characteristics of patients undergoing Excimer laser surgery, from August 2012 to September 2013.

Variables	Categories	Cases	
		Frequency	%
Gender	Female	134	61.5
	Male	84	38.5
Group of age	Young adult (18-19)	8	3.7
	Adult (20-59)	208	95.4
	Elder ($\geq$ 60)	2	0.9
Education	Elementary	4	1.8
	Middle	21	9.6
	High school	79	36.3
	Technical	13	6.0
	Bachelor	80	36.7
	Doctorate	21	9.6
Location	Alajuela	43	19.7
	Cartago	17	7.8
	Heredia	35	16.1
	Limón	6	2.7
	Managua, Nicaragua	1	0.5
	Puntarenas	5	2.3
	San José	111	50.9
Marital status	Single	131	60.1
	Married	74	33.9
	Separated/Divorced	11	5.1
	Widow	2	0.9
Occupation or profession	Housekeeper	5	2.4
	Student	31	14.2
	Technician	45	20.6
	Professional	137	62.8
Nationality	Costa Rican	212	97.2
	Others	6	2.8

The results obtained in the QoL scores before and after surgery demonstrated a real change and recalibration in vision-related QoL; see Table 2. The existence of recalibration in the patients' perception after myopia surgery is evidenced by a difference between the then-test and pre-test that is statistically significant at a level of 5% (p-value = 0.018). Also, a real change in patient's vision-related QoL was identified by comparing results between the post-test and then-test since QoL scores are above 83.1% at the third month of surgery (p-value < 0.001).

Table 2. QoL score before and after surgery, changes, and recalibration.

Moment of applying the NEI VFQ-25 questionnaire	Frequency	QoL score (X)	CI $_{95\%}$	Standard deviation
Pre-test (pre-operative)	209	46.75	[44.27; 49.23]	18.29
Post-test (3 months, post-operative)	215	89.26	[90.11; 90.11]	6.36
Then-test (retrospective from pre-test at three months from post-operative)	213	48.74	[51.28; 51.28]	18.97
Type of change		**How is it calculated?**	**Change in QoL score (%)**	**t-test p-value**
Observed change		post-test mean − pre-test mean	42.56 (90.93%)	< 0.001
Recalibration		then-test mean − pre-test mean	2.46 (4.26%)	0.018
Real change		post-test mean − then-test mean	40.49 (83.14%)	< 0.001

The dual STATIS method was applied to identify reconceptualization. A matrix structure was analyzed for four moments (pre-operative, the next day, the following month and three months later), comparing the correlation matrices. This comparison was based on the configuration of the NEI VFQ-25 items using the HS correlation coefficient. If the correlation between two matrices tends to be equal to one, they are equivalent, congruent, and comparable. From Table 3, note that strong correlations between 0.857 (minimum) and 0.940 (maximum) are obtained on the matrix, suggesting highly comparable structures. The correlations are strong at all four moments, demonstrating no reconceptualization in the vision-related QoL. The norms corresponding to the last three moments are practically identical. From pre-operative, we report that the variation in the perception of patients' vision-related QoL is similar after surgery. This indicates a recalibration in patients' standards after medical intervention, which is consistent with results obtained in the then-test.

Table 3. HS correlation matrix.

Moment	Pre-operative	The next day	The following month	Three months later	Norm
Pre-operative	1	-	-	-	0.170
The next day	0.940	1	-	-	0.135
The following month	0.857	0.864	1	-	0.134
Three months later	0.899	0.916	0.906	1	0.131

Table 4 shows that the first two axes absorb 96.53% of the total variability. The projections of each data matrix on the factorial plane of maximum inertia indicate an evident similarity between the four structures. The Euclidean space, where the four structures characterize similar matrices, shows a longer arrow corresponding to the pre-operative; see Figure 3. This explains a higher variability in the perception of the vision-related QoL before surgery compared to the following day, the next month, and three months after being operated on.

The behavior of each item of the NEI-VFQ-25 questionnaire at the four moments is presented in Figures 4, 5 and 6. The representation of Item 1 shows that, at the pre-operative moment, no significant change is detected until the next day after surgery. At the same time, in the following month and three months later, there are evidences of a broad trajectory identifying a change in vision-related QoL at similar times. This is a constant pattern with the rest of the items, with similar responses at pre-operative and the next day but different in the following month and the third month. Items 3, 7 and 13 show a less clear behavior, complicating their interpretation since their trajectories are more erratic. The then-test method allowed us to identify recalibration as a control test. After results were obtained with dual STATIS and compared to the then-test, there is evidence that this alternative method is also effective in identifying recalibration. Note that the then-test is not a suitable method to detect reconceptualization. However, dual STATIS is sufficiently robust to provide results identifying a change due to reconceptualization.

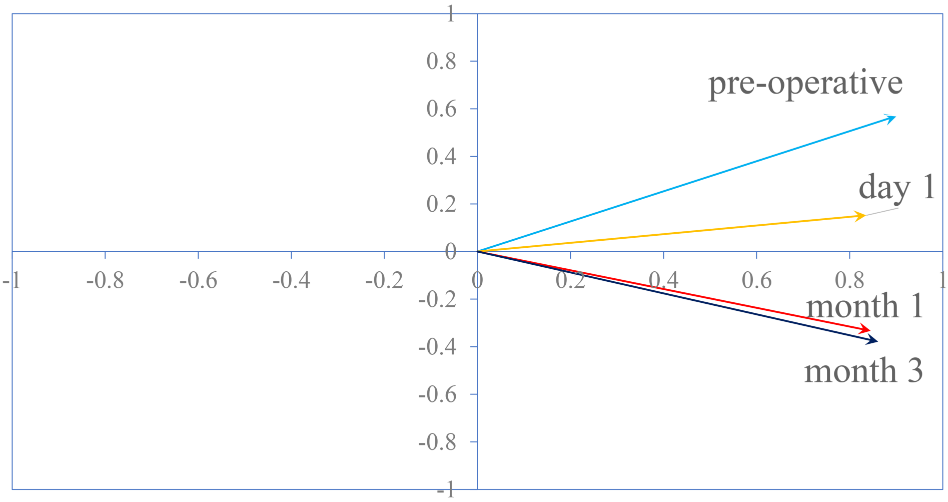


Figure 3. Euclidean space of axes 1-2 from the four structures related to moments: pre-operative, the next day (day 1), the following month (month 1) and three months later (month 3).

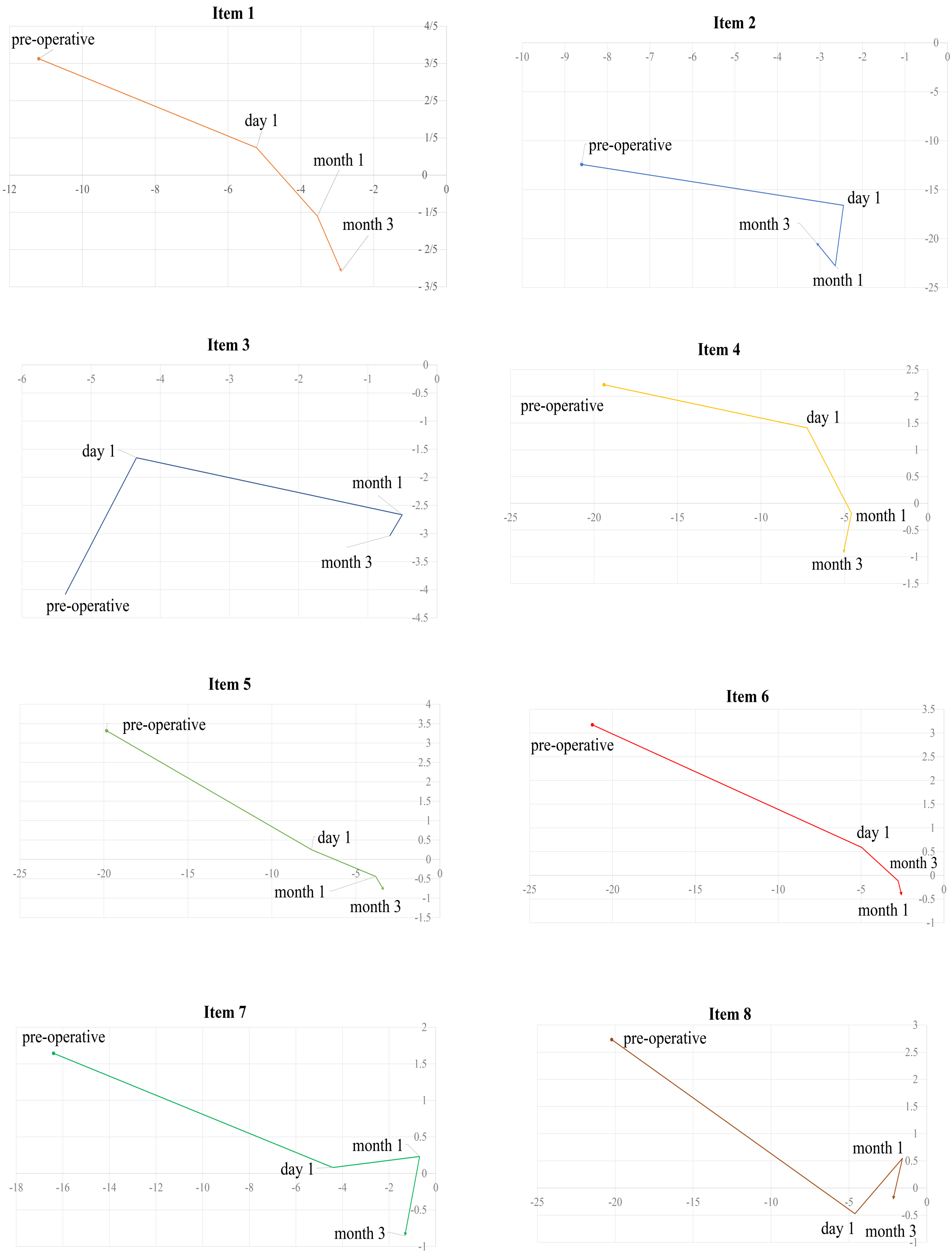


Figure 4. Changes in the indicated item at the four moments evaluated: pre-operative, the next day (day 1), the following month (month 1) and three months later (month 3).

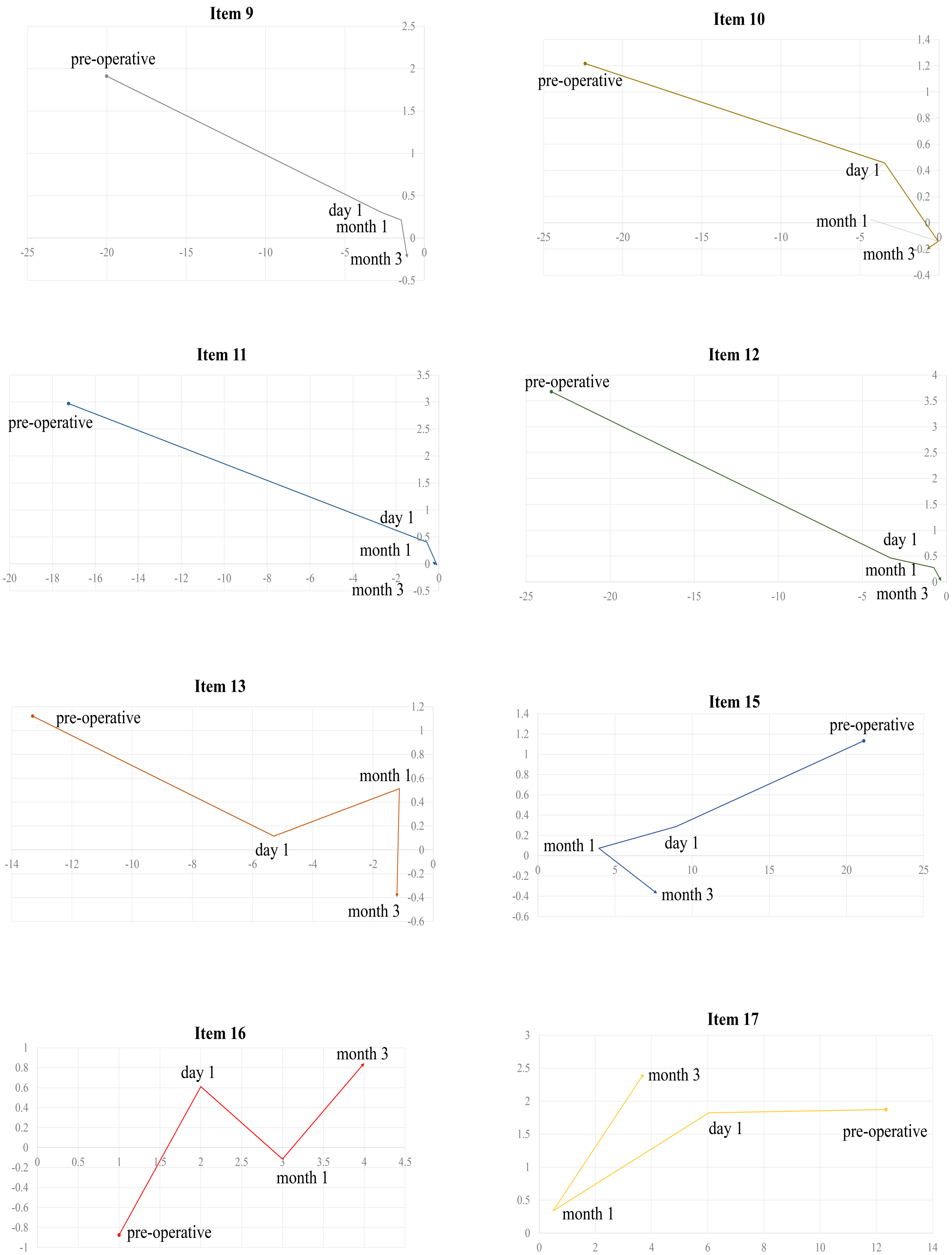
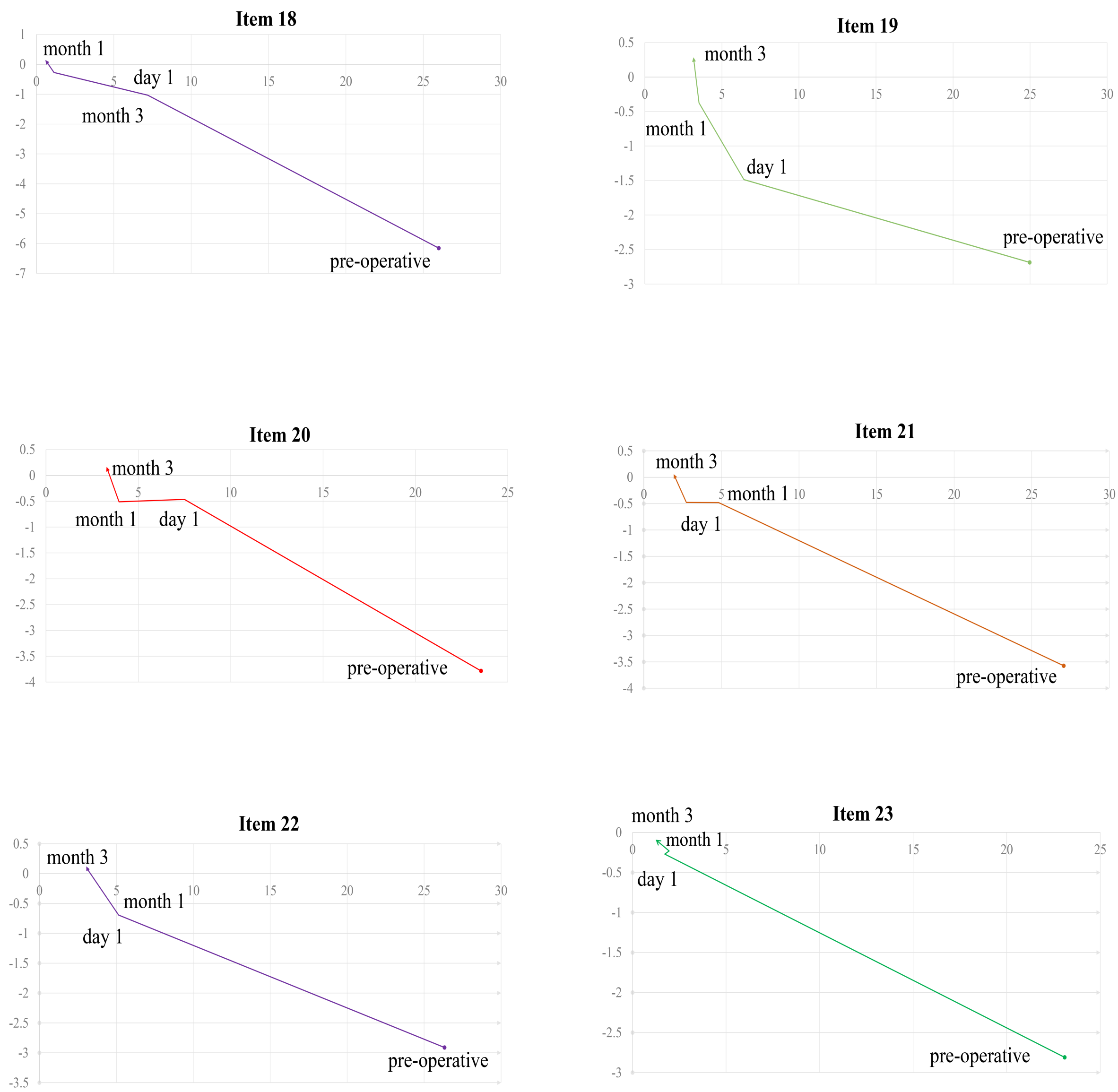


Figure 5. continued – Changes in the indicated item at the four moments evaluated: pre-operative, the next day (day 1), the following month (month 1) and three months later (month 3).



]

Figure 6. continued – Changes in the indicated item at the four moments evaluated: pre-operative, the next day (day 1), the following month (month 1) and three months later (month 3).

Table 4. Inertia decomposition.

Axis	Own values	Inertia %	Accumulated %
1	3.692	92.291	92.291
2	0.17	4.241	96.531

4.2. Discussion

This investigation evaluated the change in vision-related QoL for myopic patients using a non-traditional multivariate statistical method: dual STATIS. This method allowed us to analyze through a graphical representation (Euclidean space) the global congruence of the matrix structure and the maximum inertia (variability) of vectors (moments). The analysis began with the then-test method to obtain results about recalibration as a baseline to compare with the dual STATIS results and determine consistency in both methods. This comparison permitted us to evaluate whether the proposed method provides an alternative to assess vision-related QoL. There is no evidence of other studies focused on detecting the change in QoL in myopic patients or those with other ophthalmological pathologies.

Vision-related QoL was also studied to understand an individual's self-evaluation after undergoing different types of treatments to improve visual conditions such as neuromyelitis optical spectrum disorder [49], dystonia [50], autologous internal membrane transplantation [51] and dry eyes [52], among others.

The then-test method [13] has been widely used for the QoL analysis [53], allowing the identification of change (response shift) in our investigation. The results obtained in this study evidenced the existence of recalibration, which is consistent with other health-related investigations: oncology [54–56], orthopedics [57–59], oral health [60], gastroenterology [61] and neurology [62].

The employment of the dual STATIS method to detect response shift in this study evidenced the absence of reconceptualization in the perception of vision-related QoL and the existence of recalibration. This was statistically demonstrated with higher values after all patients underwent surgical treatment.

The Euclidean space, where the four structures represent the matrices of each of the four times (pre-operative, day 1, month 1, month 3), shows a longer arrow corresponding to the pre-operative; see Figure 3. This explains a higher variability in the perception of the vision-related QoL before surgery compared to the next day, the following month and three months after being operated on. Furthermore, the four structures are relatively closed and oriented in a similar direction, indicating a high correlation between the four moments.

The behavior of each item of the NEI-VFQ-25 questionnaire at the four times is presented in Figures 4, 5 and 6, which is called the item's path. If the trajectory of an item describes a low variability pattern, it indicates stability through time without changes in the item. In addition, patterns of high variability in the trajectory of an item may be influenced by response shift (change).

The representation of Item 1 shows that, at the pre-operative, no significant change is detected until the next day after surgery, whereas in the following month and three months later, there are patterns of high variability in the trajectory of an item identifying a change in vision-related QoL at similar times. This reading shows similar constant patterns in relation to the rest of the items, with similar responses at pre-operative and the next day, but different in the following month and the third month. Other items display erratic trajectories, indicating less clear behaviors.

5. Conclusions, limitations and further research

Vision-related QoL analyzes visual function concerning individual well-being based on activity and social participation. In this paper, we presented a methodology based on STATIS multivariate three-way methods to assess the change in vision-related QoL for myopic patients by comparing their conditions before and after corneal surgery. We conducted a case study in Costa Rica to detect the outcomes of patients referred for myopia that underwent refractive surgery. The NEI VFQ-25 instrument was employed to measure the vision-related QoL and applied in five different stages over three months. After applying the questionnaire, the results indicated a statistically significant difference at 5% (p-value $= 0.018$) between the then-test and pre-test and QoL scores above 83% (p-value ≤ 0.001). Strong correlations were recorded between 0.857 and 0.940, with highly comparable structures. The use of the dual STATIS method demonstrated the non-existence of reconceptualization in myopic patients. This finding has not been reported in this type of population due to the limitation of the existing statistical tools because QoL is a multivariate construct.

A disadvantage of the then-test is the possibility that data appear contaminated by memory problems and difficulties of patients in remembering their previous situation, especially in chronic cases of memory loss, affecting the validity of the test [79, 80]. While the then-test method can only identify recalibration-type response shift and cannot be used with secondary data studies, dual STATIS may detect reconceptualization and examine primary and secondary longitudinal data.

As mentioned, in this article, we have presented a multivariate statistical methodology that was used with QoL data by showing its ability to identify different response shifts. Due to its methodological nature, this contribution is suitable for strengthening research in different areas, not only in health-related studies. The medical community studies QoL, and the methodology commonly used is incorrect since researchers are limited in comparing the results before and after medical treatments (using t-test or ANOVA) without considering that the patient recalibrates, reprioritizes and even reconceptualizes the construct. The traditional process mixes all changes, and so it is impossible to isolate the real alpha change, for example when an individual is diagnosed with cancer or becomes almost blind to be capable of seeing the entire world's changes. In these examples, a psychological component needs to be considered. Note that QoL is perceived as a psychological condition, not a biological one. The psychological equilibrium may improve an individual's physical state. Comparing just two moments (before and after) will be like comparing height and weight in two moments, which is a biological condition.

The limitations of our investigation are related to the region where our case study was carried out and the epoch of the application of the NEI VFQ-25 instrument. However, the statistical framework is considered suitable and representative. Further analyses can be conducted to improve these limitations by applying methodologies that may be developed to obtain more accurate information from analyses derived in the present case study from Costa Rica.

The use of disjoint components has gained much strength in recent years [81]. This multivariate technique allows for the analysis of two-way and three-way datasets. We are considering incorporating disjoint components into our STATIS proposal. We can find possible applications of our work and related methods in areas such as chemometrics, economics, education, health, marketing, political sciences psychometrics and social sciences, where the study of three-way tables is critical, as we can see in [37, 39, 40].

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Conflict of interest

The authors declare there is no conflict of interest.

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