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Research article

A fractional order model of the COVID-19 outbreak in Bangladesh

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Abstract: In this study, we propose a Caputo-based fractional compartmental model for the dynamics of the novel COVID-19. The dynamical attitude and numerical simulations of the proposed fractional model are observed. We find the basic reproduction number using the next-generation matrix. The existence and uniqueness of the solutions of the model are investigated. Furthermore, we analyze the stability of the model in the context of Ulam-Hyers stability criteria. The effective numerical scheme called the fractional Euler method has been employed to analyze the approximate solution and dynamical behavior of the model under consideration. Finally, numerical simulations show that we obtain an effective combination of theoretical and numerical results. The numerical results indicate that the infected curve predicted by this model is in good agreement with the real data of COVID-19 cases.

Keywords: COVID-19; fractional model; existence and uniqueness; stability; numerical simulations

1. Introduction

Coronavirus disease (COVID-19) is an infectious disease caused by the SARS-CoV-2 virus. Most people infected with the virus will experience mild to moderate respiratory illness and recover without requiring special treatment. However, some will become seriously ill and require medical attention. Coronaviruses belong to a large family. They are fatal and live in the throat cells. People carrying this virus may not be symptomatic for several days. There are SARS-CoV (the cause of an outbreak of severe acute respiratory syndrome in 2002), MERS-CoV (the cause of middle east respiratory syndrome in 2012), and SARS-CoV-2, which is a novel beta coronavirus that is the cause of coronavirus disease 2019 (COVID-19). These three coronaviruses cause the most severe and fatal respiratory infections in humans than other coronaviruses and are responsible for significant outbreaks of deadly pneumonia in the 21st century. It is well known that COVID-19 is transmitted by means of either direct or indirect contact, droplet spray such as sneezing in short-range transmission and airborne transmission such as aerosol in long-range transmission [1]. It is important to practice respiratory etiquette, for example by coughing into a flexed elbow, and to stay home and self-isolate until you recover if you feel unwell.

On October 2, 2022 (BSS-Bangladesh Songbad Sangstha, National News Agency Of Bangladesh) reported two COVID-19 deaths with 696 coronavirus-positive cases as 5801 samples were tested. According to WHO reports, Bangladesh reached 2,026,908 coronavirus cases, 29,371 deaths, 1,966,645 recovered, 30,892 infected/active cases, 696 daily cases, 29,371 total deaths, and 2 new deaths as of October 3, 2022 [2].

Infectious outbreaks have a critical effect on health and finance. Therefore, it is important to study the dynamics of transmission. According to the Institute of Epidemiology Disease Control and Research (IEDCR), the first three coronavirus cases were detected among approximately 111 tests on March 8, 2020, which included two men and one woman aged between 20 and 35 years. On March 18, Bangladesh recorded its first death due to COVID-19. Authorities tried to implement protective measures to reduce the spread of the COVID-19 outbreak in the country. The measures included wearing surgical masks, cleaning hands thoroughly, covering the nose and mouth when coughing and sneezing, increasing consciousness, lockdowns in several areas, home quarantine, social distancing, and local or international flight restrictions.

In Bangladesh, from January 3, 2020, to 6.04 pm CEST, September 1, 2022, there have been 2,011,946 confirmed cases of COVID-19 including 29,323 deaths, with 1198 new cases and 3 new deaths and globally 600,555,262 confirmed cases including 6,472,914 deaths, were reported to WHO [3]. To date, 176 countries, including Bangladesh, have reported 537,808 confirmed cases of COVID-19, leading to 24,127 deaths worldwide as of March 27 [4]. The first COVID-19 case was identified in Bangladesh on March 7, 2020. Since then, five deaths out of 48 confirmed cases have been reported in Bangladesh as of March 27, 2020 [5]. In this stage, it is crucial to have a perfect prediction of new cases due to COVID-19 for hospitals to be prepared and administration to ensure a proper strategy in advance. Furthermore, an acute course of action is necessary for the country to handle the situation. The government can control the outbreak if a movement control order (MCO) is issued.

The goal of the present paper is two folds, first, we want to establish both the mathematical and epidemiological well-posedness of the integer-order model and employ an approximate analytical technique to obtain long-term dynamics of the disease. Second, we modify and extend the existing epidemic model using a dimensionally consistent Caputo derivative operator which has been extensively demonstrated in the literature to be one of the most useful and powerful derivative operators to describe more efficiently memory effect dynamics that exist in real-world phenomena.

This study aims to investigate the fractional-order COVID-19 epidemic model using actual numerical data from a case study in Bangladesh and explore the role of time using the Caputo fractional derivative. All necessary graphical simulations were performed to determine the characteristics of the acquired solutions in the Caputo fractional-order derivative method. We analyzed the role of time in the coronavirus epidemic using graphical simulations for different fractional orders and actual values of time.

This paper is structured as follows: Section 1, the introduction; Section 2, the preliminary definitions and notations; Section 3, the model formulation, the qualitative properties of the solution, the positivity and existence of unique solutions, the equilibria and basic reproduction number; Section 4, the stability analysis; Sections 5, the sensitivity analysis; Section 6, the numerical simulations; Section 7, the discussion and conclusion.

2. Preliminaries and notations

Fractional calculus has different well-known definitions and results that are relevant to the current article. The most common are the Riemann-Liouville type and Caputo-type fractional derivatives, which are more practical and essential for real applications and theory. For details and appropriate studies, we refer to [6–8].

Definition 2.1. *([9]) Suppose* $\alpha > 0$ *and* $g \in L^1([0,b],\mathbb{R})$ *where* $[0,b] \subset \mathbb{R}_+$. *The fractional integral of order* α *of function g in the sense of Riemann-Liouville is defined as follows:*

$$I^{\alpha}_{0^+}g(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} g(\tau)d\tau, t > 0,$$

where $\Gamma(\cdot)$ *is the classical gamma function defined by*

$$\Gamma(\alpha) = \int_0^{\infty} \tau^{\alpha-1} e^{-\tau} d\tau.$$

Definition 2.2. *([9]) Let* $n-1 < \nu < n, n \in N$, *and* $g \in C^n[0,b]$. *The Caputo fractional derivative of order* α *for a function g is defined as*

$$^{C}D^{\alpha}_{0+}g(t) = I^{n-\alpha}D^n g(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} g^{(n)}(\tau)d\tau, t > 0.$$

where $n-1 < \alpha < n, n \in N$, *and* $[\alpha]$ *represent the smallest integer that is less or equal to* α.

Let η_1, η_2 *be two positive numbers, then the Mittag-Leffler function is given by*

$$E_{\eta_1\eta_2}(s) = \sum_{k=o}^{k=\infty} \frac{s^k}{\Gamma(\eta_1 k + \eta_2)}. \tag{2.1}$$

Lemma 2.1. *([9]) Let* $Re(\alpha) > 0, n = [Re(\alpha)] + 1$, *and* $g \in AC^n(0,b)$. *Then*

$$(J^{\alpha}_{0^+}\,{}^{C}D^{\alpha}_{0^+}g)(t) = g(t) - \sum_{k=1}^{m} \frac{g^k(0)}{k!} t^k.$$

In particular, if $0 < \alpha \leq 1$, *then* $(J^{\alpha}_{0^+}\,{}^{C}D^{\alpha}_{0^+}g)(t) = g(t) - g_0$.

3. Fractional order model

Fractional derivatives are generally believed to model disease epidemics more realistically because of their capability to capture the memory effect often associated with the human body's response to diseases.

3.1. The model formulation

Herein, we analyze a fractional *SAHIAqIqR* model consisting of seven compartments. The seven compartments of that population are S for susceptible, A for exposed but not hospitalized, H for hospitalized, I for infectious, Aq for isolated exposed, Iq for isolated infectious, and R for recovered. The compartment of people who are not yet infected but can contract the disease are the susceptible individuals (S). The compartment of people who get infected are the exposed but not hospitalized individuals (A). The compartment of people who are hospitalized after infection are the hospitalized individuals (H). The compartment of people who can transmit the disease to others are the infected individuals (I). The compartment of people who are quarantined from the exposed but not hospitalized are the isolated exposed individuals (Aq). The isolated infectious individuals (Iq) those who are tested positive for the virus and are quarantined from the rest of the population. Those infected and isolated individuals who are cured are recovered individuals (R). A flowchart of the spread of COVID-19 is shown in Figure 1.

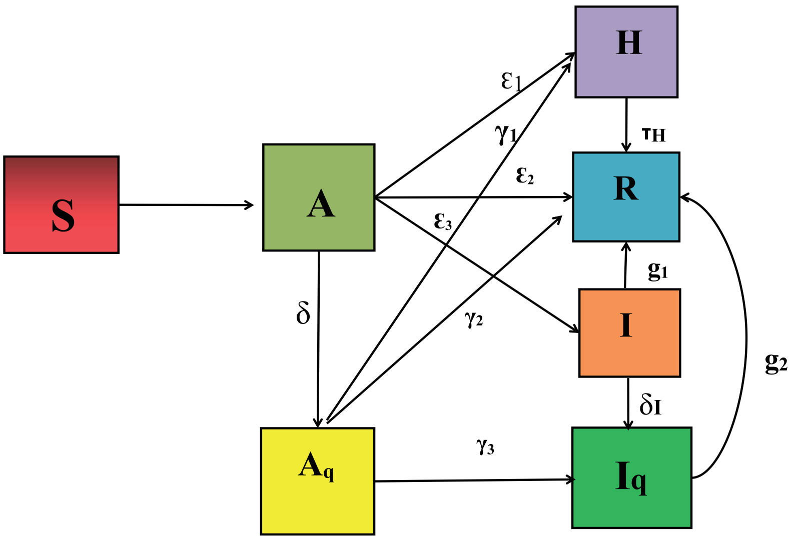


Figure 1. The SAHIAqIqR model diagram for COVID-19 dynamics.

In this section, we present the fractional model

$$\begin{cases} {}^{C}D^{\alpha}_{0^+}S(t) &=& \Lambda^{\alpha} - \mu^{\alpha}S - \beta^{\alpha}_1 AS - \beta^{\alpha}_2 IS, \\ {}^{C}D^{\alpha}_{0^+}A(t) &=& \beta^{\alpha}_1 AS + \beta^{\alpha}_2 IS - (\epsilon^{\alpha}_1 + \epsilon^{\alpha}_2 + \epsilon^{\alpha}_3)A - \delta^{\alpha}A - \mu^{\alpha}A, \\ {}^{C}D^{\alpha}_{0^+}H(t) &=& \epsilon^{\alpha}_1 A + \gamma^{\alpha}_1 A_q - \tau^{\alpha}H - \mu^{\alpha}H, \\ {}^{C}D^{\alpha}_{0^+}I(t) &=& \epsilon^{\alpha}_3 A - g^{\alpha}_1 I - \delta^{\alpha}I - \mu^{\alpha}I, \\ {}^{C}D^{\alpha}_{0^+}A_q(t) &=& \delta^{\alpha}A - (\gamma^{\alpha}_1 + \gamma^{\alpha}_2 + \gamma^{\alpha}_3)A_q - \mu^{\alpha}A_q, \\ {}^{C}D^{\alpha}_{0^+}I_q(t) &=& \gamma^{\alpha}_3 A_q + \delta^{\alpha}I - g^{\alpha}_2 I_q - \mu^{\alpha}I_q, \\ {}^{C}D^{\alpha}_{0^+}R(t) &=& \tau^{\alpha}H + \gamma^{\alpha}_2 A_q + g^{\alpha}_1 I + g^{\alpha}_2 I_q + \epsilon^{\alpha}_2 A - \mu^{\alpha}R. \end{cases} \tag{3.1}$$

With initial conditions as follows:

$$S(0) = S_0 = n_1, A(0) = A_0 = n_2, H(0) = H_0 = n_3, I(0) = I_0 = n_4,$$

$$Aq(0) = Aq_0 = n_5, Iq(0) = Iq_0 = n_6, R(0) = R^0 = n_7.$$

where ${}^{C}D_t^{\alpha}$ denotes Caputo fractional derivative of order $0 < \alpha \leq 1$ and the total human population *N(t)* are divided into seven groups as follows:

$$N(t) = S(t) + A(t) + H(t) + I(t) + Aq(t) + Iq(t) + R(t).$$

The different parameters used in this fractional model, along with their values and references, are listed in Table 1.

Table 1. Description of parametric values for Bangladesh.

Parameter	Interpretation	Values	Reference
β_1	transmission rate of A to S	3.13	Assumed
β_2	transmission rate of I to S	2.55	Assumed
g_1	recovered rate of I	0.23	[10]
g_2	recovered rate of Iq	6.6×10^{-6}	[Estimated]
ϵ_1	confirmed rate of A	0.037	[10]
ϵ_2	self-recovered rate of A	0.1	Assumed
ϵ_3	clinical rate	0.0974	Assumed
τ	rate of recovered hospitalized patients	0.1	Fitted
γ_1	confirmed rate of Aq	0.2599	[Estimated]
γ_2	self-recovered rate of Aq	0.1	Assumed
γ_3	clinical rate	0.0974	Assumed
Λ	birth rate	17.71	Fitted
μ	death rate	5.54	Fitted
δ	quarantine rate	0.6185	[Estimated]

3.2. Qualitative properties of solution

In this section, we examine the mathematical and biological well-posedness of the fractional order model. In essence, we prove that solution of the fractional model is bounded and remains positive as long as a positive initial condition is given. Furthermore, we prove the existence and uniqueness of the solution to the proposed model. The theory of existence and uniqueness of solutions is one of the most dominant fields in the theory of fractional-order differential equations. In this section, we discuss the existence and uniqueness of solutions of the proposed model using fixed point theorems. We simplify the proposed model (3.1) in the following setting:

$$\begin{cases} {}^{C}D_{0^+}^{\alpha}S(t) &= \Theta_1(t, S, A, H, I, A_q, I_q, R), \\ {}^{C}D_{0^+}^{\alpha}A(t) &= \Theta_2(t, S, A, H, I, A_q, I_q, R), \\ {}^{C}D_{0^+}^{\alpha}H(t) &= \Theta_3(t, S, A, H, I, A_q, I_q, R), \\ {}^{C}D_{0^+}^{\alpha}I(t) &= \Theta_4(t, S, A, H, I, A_q, I_q, R), \\ {}^{C}D_{0^+}^{\alpha}A_q(t) &= \Theta_5(t, S, A, H, I, A_q, I_q, R), \\ {}^{C}D_{0^+}^{\alpha}I_q(t) &= \Theta_6(t, S, A, H, I, A_q, I_q, R), \\ {}^{C}D_{0^+}^{\alpha}R(t) &= \Theta_7(t, S, A, H, I, A_q, I_q, R). \end{cases} \tag{3.2}$$

Let $\phi(t) = (S, A, H, I, A_q, I_q, R)^T$ and $\kappa(t, \phi(t)) = (\Theta_i)^T, i = 1......7$ where

$$\begin{cases} \Theta_1(t,S,A,H,I,A_q,I_q,R) &=& \Lambda^\alpha - \mu^\alpha S - \beta_1^\alpha AS - \beta_2^\alpha IS, \\ \Theta_2(t,S,A,H,I,A_q,I_q,R) &=& \beta_1^\alpha AS + \beta_2^\alpha IS - (\epsilon_1^\alpha + \epsilon_2^\alpha + \epsilon_3^\alpha)A - \delta^\alpha A - \mu^\alpha A, \\ \Theta_3(t,S,A,H,I,A_q,I_q,R) &=& \epsilon_1^\alpha A + \gamma_1^\alpha A_q - \tau^\alpha H - \mu^\alpha H, \\ \Theta_4(t,S,A,H,I,A_q,I_q,R) &=& \epsilon_3^\alpha A - g_1^\alpha I - \delta^\alpha I - \mu^\alpha I, \\ \Theta_5(t,S,A,H,I,A_q,I_q,R) &=& \delta^\alpha A - (\gamma_1^\alpha + \gamma_2^\alpha + \gamma_3^\alpha)A_q - \mu^\alpha A_q, \\ \Theta_6(t,S,A,H,I,A_q,I_q,R) &=& \gamma_3^\alpha A_q + \delta^\alpha I - g_2^\alpha I_q - \mu^\alpha I_q, \\ \Theta_7(t,S,A,H,I,A_q,I_q,R) &=& \tau^\alpha H + \gamma_2^\alpha A_q + g_1^\alpha I + g_2^\alpha I_q + \epsilon_2^\alpha A - \mu^\alpha R. \end{cases} \tag{3.3}$$

Thus, the proposed model (3.1) takes the form

$$\begin{cases} {}^C D_0^\alpha \phi(t) &=& \kappa(t,\phi(t)), t \in J = [0,b], 0 < \alpha \le 1, \\ \phi(0) &=& \phi_0 \ge 0. \end{cases} \tag{3.4}$$

on condition that

$$\begin{cases} \phi(t) &=& (S,A,H,I,A_q,I_q,R)^T, \\ \phi(0) &=& (S_0,A_0,H_0,I_0,A_{q0},I_{q0},R_0)^T, \\ \kappa(t,\phi(t)) &=& (\Theta_i(S,A,H,I,A_q,I_q,R))^T, i = 1,.....7, \end{cases} \tag{3.5}$$

where $(\cdot)^T$ represents the transpose operation.

Lemma 3.1. *the integral representation of problem (3.4) is given by*

$$\begin{aligned} \phi(t) &= \phi_0 + J_{0+}^\alpha \kappa(t,\phi(t)) \\ &= \phi_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \kappa(\tau,\phi(\tau)) d\tau. \end{aligned} \tag{3.6}$$

Next, we shall analyze model (3.1) through the integral representation above. For that purpose, let $\mathbb{E} = C([0,b];\mathbb{R})$ *denote the Banach space of all continuous functions from* $[0,b]$ *to* $\mathbb{R}$ *endowed with the norm defined by*

$$\|\phi\|_{\mathbb{E}} = \sup_{t\in J} |\phi(t)|,$$

where

$$|\phi(t)| = |S(t)| + |A(t)| + |H(t)| + |I(t)| + |A_q(t)| + |I_q(t)| + |R(t)|. \tag{3.7}$$

Note that $S,A,H,I,A_q,I_q,R \in C([0,b],\mathbb{R})$. *Furthermore, we define the operator* $P : \mathbb{E} \to \mathbb{E}$ *by*

$$(P\phi)(t) = \phi_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \kappa(\tau,\phi(\tau)) d\tau. \tag{3.8}$$

Note that operator P is well defined due to the obvious continuity of κ, 3.2.1. Positivity and existence of unique solution

Here, we establish existence, uniqueness and uniform stability of solutions. The following preliminary result is in order.

Theorem 3.1. *The closed set*

$$\Upsilon = \{(S(t)+A(t)+H(t)+I+A_q(t)+I_q(t)+R(t)) \in \mathbb{R}^7_+ : 0 \leq S(t)+A(t)+H(t)+I(t)+A_q(t)+I_q(t)+R(t) \leq P_1\}$$

is a positive invariant set for the proposed fractional order system (3.1) To prove that the system of Eq (3.1) has a non-negative solution, the system of Eq (3.1) implies

$$\begin{cases} {}_0^C D_t^\alpha S|_{S=0} &= \Lambda^\alpha > 0 \\ {}_0^C D_t^\alpha A|_{A=0} &= \beta_2^\alpha IS \geq 0, \\ {}_0^C D_t^\alpha H|_{H=0} &= \epsilon_1^\alpha A + \gamma_1^\alpha A_q \geq 0, \\ {}_0^C D_t^\alpha I|_{I=0} &= \epsilon_3^\alpha A \geq 0, \\ {}_0^C D_t^\alpha A_q|_{A_q=0} &= \delta^\alpha A \geq 0, \\ {}_0^C D_t^\alpha I_q|_{I_q=0} &= \gamma_3^\alpha A_q + \delta^\alpha I \geq 0. \\ {}_0^C D_t^\alpha I_q|_{I_q=0} &= \tau^\alpha H + \gamma_2^\alpha A_q + g_1^\alpha I + g_2^\alpha I_q + \epsilon_2^\alpha A \geq 0. \end{cases} \tag{3.9}$$

Thus, the fractional system (3.1) has non-negative solutions. In the end, from the seven equations of the fractional system (3.1), we obtain

$${}_0^C D_t^\alpha (S(t)+A(t)+H(t)+I(t)+A_q(t)+I_q(t)+R(t)) \leq \Lambda^\alpha - \Psi(S(t)+A(t)+H(t)+I(t)+A_q(t)+I_q(t)+R(t)) \tag{3.10}$$

where $\Psi = \min(\Lambda^\alpha, \mu^\alpha)$. *Solving the above inequality we obtain*

$$\begin{aligned} (S(t) + A(t) + H(t) + I(t) + A_q(t) + I_q(t) + R(t)) &\leq (S(0) + A(0) + H(0) + I(0) \\ +A_q(0) + I_q(0) + R(0) - \frac{\Lambda^\alpha}{\Psi}) E_\alpha(-\Psi t^\alpha) + \frac{\Lambda^\alpha}{\Psi}. \end{aligned} \tag{3.11}$$

So by the asymptotic behavior of Mittag-Leffler function [9], we obtain

$$S(t) + A(t) + H(t) + I(t) + A_q(t) + I_q(t) + R(t) \leq \frac{\Lambda^\alpha}{\Psi} \cong P_1$$

Hence, the closed set Υ *is a positive invariant region for the proposed fractional-order model (3.1).*

Lemma 3.2. *Let* $\bar{\phi} = (\bar{S}, \bar{A}, \bar{H}, \bar{I}, \bar{A}_q, \bar{I}_q, \bar{R})^T$. *The function* $\kappa = (\Theta_i)^T$ *defined above satisfies*

$$||\kappa(t, \phi(t)) - \kappa(t, \bar{\phi(t)})|| \leq L_\kappa ||\phi - \bar{\phi}||_\varepsilon,$$

for some $L_\kappa > 0$.

Proof. From the first component of κ, we observe that

$$\begin{aligned} |\Theta_1(t, \phi(t)) - \Theta_1(t, \bar{\phi}(t))| &= |\beta_1^\alpha (A(t)S(t) - \bar{A}(t)\bar{S}(t)) - \beta_2^\alpha (I(t)S(t) - \bar{I}(t)\bar{S}(t)) - \mu^\alpha (S(t) - \bar{S}(t))| \\ &\leq \beta_1^\alpha |(A(t)S(t) - \bar{A}(t)\bar{S}(t))| + \beta_2^\alpha |(I(t)S(t) - \bar{I}(t)\bar{S}(t))| + \mu^\alpha |(S(t) - \bar{S}(t))| \end{aligned}$$

However,

$$|(A(t)S(t) - \bar{A}(t)\bar{S}(t))| \leq f_1(t)|S(t) - \bar{S}(t)| + f_2(t)|A(t) - \bar{A}(t)|$$
$$|(I(t)S(t) - \bar{I}(t)\bar{S}(t))| \leq g_1(t)|S(t) - \bar{S}(t))| + g_2(t)|I(t) - \bar{I}(t)|,$$

where

$$f_1(t) = A + \bar{A} + A\bar{S} - \bar{A}S, f_2(t) = S + \bar{S} + S\bar{A} - \bar{S}A, g_1(t) = I + \bar{I} + I\bar{S} - \bar{I}S, g_2(t) = S + \bar{S} + S\bar{I} - \bar{S}I.$$

Altogether, we have

$$|\Theta_1(t, \phi(t)) - \Theta_1(t, \bar{\phi}(t)) \leq (\mu^\alpha + \beta_1^\alpha f_1(t) + \beta_2^\alpha g_1(t))|S(t) - \bar{S}(t)| \\ +\beta_1^\alpha f_2(t)|A(t) - \bar{A}(t)| + \beta_2^\alpha g_2(t)|I(t) - \bar{I}(t)|,$$

$$L_1(|S(t) - \bar{S}(t)| + |A(t) - \bar{A}(t)| + |I(t) - \bar{I}(t)|,$$

where

$$L_1 = \mu^\alpha + max_{t\in[0,b]}(\beta_1 f_1(t) + \beta_2 g_1(t) + \beta_1 f_2(t) + \beta_2 g_2(t)).$$

In a similar manner, one obtains

$$|\Theta_2(t, \phi(t)) - \Theta_2(t, \bar{\phi}(t))| \leq L_2(|S(t) - \bar{S}(t)| + |A(t) - \bar{A}(t)| + |I(t) - \bar{I}(t)|),$$

where

$$L_2 = \epsilon_1^\alpha + \epsilon_2^\alpha + \epsilon_3^\alpha + \delta^\alpha + \mu^\alpha + max_{t\in[0,b]}\beta_1^\alpha f_1(t) + \beta_2^\alpha g_1(t) + \beta_1^\alpha f_2(t) + \beta_2^\alpha g_2(t).$$

For the remaining components of κ, it holds

$$\begin{aligned}
|\Theta_3(t, \phi(t)) - \Theta_3(t, \bar{\phi}(t))| &\leq (L_3(|A(t) - \bar{A}(t)| + |H(t) - \bar{H}(t)| + |A_q(t) - \bar{A}_q(t)|), \\
|\Theta_4(t, \phi(t)) - \Theta_4(t, \bar{\phi}(t))| &\leq L_4(|A(t) - \bar{A}(t)| + |I(t) - \bar{I}(t)|), \\
|\Theta_5(t, \phi(t)) - \Theta_5(t, \bar{\phi}(t))| &\leq L_5(|A(t) - \bar{A}(t)| + |A_q(t) - \bar{A}_q(t)|), \\
|\Theta_6(t, \phi(t)) - \Theta_6(t, \bar{\phi}(t))| &\leq L_6(|I(t) - \bar{I}(t)| + |A_q(t) - \bar{A}_q(t)| + |I_q(t) - \bar{I}_q(t)|), \\
|\Theta_7(t, \phi(t)) - \Theta_7(t, \bar{\phi}(t))| &\leq L_7(|A(t) - \bar{A}(t)| + |H(t) - \bar{H}(t)| + |I(t) - \bar{I}(t)| \\
&\quad + |A_q(t) - \bar{A}_q(t)| + |I_q(t) - \bar{I}_q(t)| + |R(t) - \bar{R}(t)|).
\end{aligned}$$

with

$$\begin{aligned}
L_3 &= \epsilon_1^\alpha + \gamma_1^\alpha + \tau^\alpha + \mu^\alpha, \\
L_4 &= \epsilon_3^\alpha + g_1^\alpha + \delta^\alpha + \mu^\alpha, \\
L_5 &= \delta^\alpha + \gamma_1^\alpha + \gamma_2^\alpha + \gamma_3^\alpha + \mu^\alpha, \\
L_6 &= \gamma_3^\alpha + \delta^\alpha + g_2^\alpha + \mu^\alpha, \\
L_7 &= \tau^\alpha + \gamma_2^\alpha + g_1^\alpha + g_2^\alpha + \epsilon_2^\alpha + \mu^\alpha.
\end{aligned}$$

Consequently,

$$\begin{aligned}
&\|\kappa(t, \phi(t)) - \kappa(t, \bar{\phi(t)})\| \leq \|\phi(t) - \bar{\phi}(t)\|_\varepsilon, \\
&= sup_{t\in[0,b]} \sum_{i=1}^{7} |\Theta_i(t, \phi(t)) - \Theta_i(t, \bar{\phi}(t))|, \\
&\leq L_\kappa(|\phi(t) - \bar{\phi}(t)\|_\varepsilon,
\end{aligned}$$

where

$$L_\kappa = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7.\square$$

Theorem 3.2. *Suppose that the function $\kappa \in C([J,\mathbb{R}])$ and maps a bounded subset of $J \times \mathbb{R}^7$ into relatively compact subsets of $\mathbb{R}$. In addition, there exists constant $L_\kappa > 0$ such that*

(A_1) $|\kappa(t,\phi_1(t)) - \kappa(t,\phi_2(t))| \le L_\kappa|\phi_1(t) - \phi_2(t)|$ for all $t \in J$ and each $\phi_1, \phi_2 \in C([J,\mathbb{R}])$. Then problem (3.4) which is equivalent to the proposed model (3.1) has a unique solution provided that $\Omega L_\kappa < 1$, where

$$\Omega = \frac{\Lambda^\alpha}{\Gamma(\alpha+1)}.$$

Proof. Consider the operator $P : \mathbb{E} \to \mathbb{E}$ defined by

$$(P\phi)(t) = \phi_0 + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}\kappa(\tau,\phi(\tau))d\tau. \tag{3.12}$$

Obviously, the opereator P is well defined and the unique solution of model (3.1) is just the fixed point of P. Indeed, let us take $\sup_{t\in J}\|\kappa(t,0)\| = M_1$. Thus, it is enough to show that $P\mathbb{B}_\kappa \subset \mathbb{B}_\kappa$, where the set $\mathbb{B}_\kappa = \phi \in \mathbb{E} : \|\phi\| \le \kappa$ is closed and convex. Now, for any $\phi \in \mathbb{B}_\kappa$, it yields

$$\begin{aligned}(P\phi)(t) &\le |\phi_0| + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}|\kappa(\tau,\phi(\tau))|d\tau \\ &\le \phi_0 + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}[|\kappa(\tau,\phi(\tau)) - \kappa(\tau,0)| + |\kappa(\tau,0)|]d\tau \\ &\le \phi_0 + \frac{(L_{\kappa^\kappa} + M_1)}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}d\tau \\ &\le \phi_0 + \frac{(L_{\kappa^\kappa} + M_1)}{\Gamma(\alpha+1)}b^\alpha \\ &\le \phi_0 + \Omega(L_{\kappa^\kappa} + M_1).\end{aligned} \tag{3.13}$$

Hence, the results follow. Also, given any $\phi_1, \phi_2 \in \mathbb{E}$, we get

$$\begin{aligned}|(P\phi_1)(t) - (P\phi_2)(t)| &\le \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}|\kappa(\tau,\phi_1(\tau)) - \kappa(\tau,\phi_2(\tau))|d\tau \\ &\le \frac{L_\kappa}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}|\phi_1(\tau) - \phi_2(\tau)|d\tau \\ &\le \Omega L_\kappa|\phi_1(t) - \phi_2(t)|,\end{aligned} \tag{3.14}$$

which implies that $\|(P\phi_1) - (P\phi_2)\| \le \Omega L_\kappa\|\phi_1 - \phi_2\|$. Therefore, as a consequence of the Banach contraction principle, the proposed model (3.1) has a unique solution.

Next, we prove the existence of solutions of problem (3.4) which is equivalent to the proposed model (3.1) by employing the concept of Schauder fixed point theorem. Thus, the following assumption is needed.

(A_2) Suppose that there exist $\sigma_1, \sigma_2 \in \mathbb{E}$ such that

$$|\kappa(t, \phi(t))| \leq \sigma_1(t) + \sigma_2|\phi(t)|$$

for any $\phi \in \mathbb{E}, t \in J$,

such that $\sigma_1^* = \sup_{t\in J} |\sigma_1(t)|$, $\sigma_2^* = \sup_{t\in J} |\sigma_2(t)| < 1$,

Lemma 3.3. *The operator P defined in (3.12) is completely continuous.*

Proof. Obviously, the continuity of the function κ gives the continuity of the operator P. Thus, for any $\phi \in \mathbb{B}_\kappa$, where $\mathbb{B}_\kappa$ is defined above, we get

$$\begin{aligned} |(P\phi)(t)| &= |\phi_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} |\kappa(\tau, \phi(\tau)) d\tau| \\ &\leq \|\phi_0\| + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} |\kappa(\tau, \phi(\tau))| d\tau. \\ &\leq \|\phi_0\| + \frac{(\sigma_1^* + \sigma_2^*\|\phi\|)}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} d\tau. \\ &\leq \|\phi_0\| + \frac{(\sigma_1^* + \sigma_2^*\|\phi\|)}{\Gamma(\alpha+1)} b^\alpha \\ &\leq \|\phi_0\| + \Omega(\sigma_1^* + \sigma_2^*\|\phi\|) \leq +\infty. \end{aligned} \tag{3.15}$$

So, the operator P is uniformly bounded. Next, we prove the equicontinuity of P. To do so, we let $\sup_{(t,\phi)\in J\times\mathbb{B}_\kappa} |\kappa(t, \phi(t))| = \kappa^*$. Then, for any $t_1, t_2 \in J$ such that $t_2 \geq t_1$, it gives

$$\begin{aligned} |(P\phi)(t_2) - (P\phi)(t_1)| &= \frac{1}{\Gamma(\alpha)} | \int_0^{t_1} [(t_2-\tau)^{\alpha-1} - (t_1-\tau)^{\alpha-1}] \kappa(\tau, \phi(\tau)) d\tau \\ &\quad + \int_{t_1}^{t_2} (t_2-\tau)^{\alpha-1} \kappa(\tau, \phi(\tau)) d\tau| \\ &\leq \frac{\kappa^*}{\Gamma(\alpha)} [2(t_2-t_1)^\alpha + (t_2^\alpha - t_1^\alpha)] \to 0, \ as\ t_2 \to t_1. \end{aligned} \tag{3.16}$$

Hence, the operator P is equicontinuous and so is relatively compact on $\mathbb{B}_\kappa$. Therefore, as a consequence of Arzelá-Ascoli theorem, P is completely continuous.

Theorem 3.3. *Suppose that the function $\kappa : J \times \mathbb{R}^5 \to \mathbb{R}$ is continuous and satisfies assumption (A2). Then problem (3.4) which is equivalent with the proposed model (3.1) has at least one solution.*

Proof. We define a set $U = \phi \in \mathbb{E} : \phi = o(P\phi)(t), 0 < o < 1$. Clearly, in view of Lemma 2, the operator $P : U \rightarrow \mathbb{E}$ as defined in (3.12) is completely continuous. Now, for any $\phi \in U$ and assumption (A2), it yields

$$\begin{aligned} |(\phi)(t)| &= |o(P\phi)(t)| \\ &\leq |\phi_0| + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} |\kappa(\tau, \phi(\tau))| d\tau. \\ &\leq \|\phi_0\| + \frac{(\sigma_1^* + \sigma_2^* \|\phi\|)}{\Gamma(\alpha+1)} b^{\alpha} \\ &\leq \|\phi_0\| + \Omega(\sigma_1^* + \sigma_2^* \|\phi\|) \\ &\leq +\infty. \end{aligned} \tag{3.17}$$

Thus, the set U is bounded. So the operator P has at least one fixed point which is just the solution of the proposed model (3.1). Hence the desired result.

3.3. Equilibria and basic reproduction number

The coordinates of the equilibrium (S, A, H, I, Aq, Iq, R) of system (3.1) satisfy the following equations:

$$\begin{cases} \Lambda^{\alpha} - \mu^{\alpha} S - \beta_1^{\alpha} AS - \beta_2^{\alpha} IS = 0 \\ \beta_1^{\alpha} AS + \beta_2^{\alpha} IS - (\epsilon_1^{\alpha} + \epsilon_2^{\alpha} + \epsilon_3^{\alpha}) A - \delta^{\alpha} A - \mu^{\alpha} A = 0 \\ \epsilon_1^{\alpha} A + \gamma_1^{\alpha} A_q - \tau^{\alpha} H - \mu^{\alpha} H = 0 \\ \epsilon_3^{\alpha} A - g_1^{\alpha} I - \delta^{\alpha} I - \mu^{\alpha} I = 0 \\ \delta^{\alpha} A - (\gamma_1^{\alpha} + \gamma_2^{\alpha} + \gamma_3^{\alpha}) A_q - \mu^{\alpha} A_q = 0 \\ \gamma_3^{\alpha} A_q + \delta^{\alpha} I - g_2^{\alpha} I_q - \mu^{\alpha} I_q = 0 \\ \tau^{\alpha} H + \gamma_2^{\alpha} A_q + g_1^{\alpha} I + g_2^{\alpha} I_q + \epsilon_2^{\alpha} A - \mu^{\alpha} R = 0. \end{cases} \tag{3.18}$$

The disease-free equilibrium E_0 of Eq (14) are $S_0 = N$, $A_0 = 0$, $H_0 = 0$, $I_0 = 0$, $Aq_0 = 0$, $Iq_0 = 0$, $R^0 = 0$.

We calculate the reproduction number of the fractional model (3.1) using the next-generation matrix method and the basic reproduction number presented in [11, 12]. We define a vector $X = [A, H, I, Aq, Iq, R]^T$.

$$\mathrm{f} = \begin{bmatrix} (\beta_1^{\alpha} AS + \beta_2^{\alpha} IS) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathrm{v} = \begin{bmatrix} (\epsilon_1^{\alpha} + \epsilon_2^{\alpha} + \epsilon_3^{\alpha}) A + \delta^{\alpha} A + \mu^{\alpha} A \\ -\epsilon_1^{\alpha} A - \gamma_1^{\alpha} Aq + \tau^{\alpha} H + \mu^{\alpha} H \\ -\epsilon_3^{\alpha} A + g_1^{\alpha} I + \delta^{\alpha} I + \mu^{\alpha} I \\ -\delta^{\alpha} A + (\gamma_1^{\alpha} + \gamma_2^{\alpha} + \gamma_3^{\alpha}) A_q + \mu^{\alpha} A_q \\ -\gamma_3^{\alpha} A_q - \delta^{\alpha} I + g_2^{\alpha} I_q + \mu^{\alpha} I_q \\ (-\tau^{\alpha} H - \gamma_2^{\alpha} A_q - g_1^{\alpha} I - g_2^{\alpha} I_q - \epsilon_2^{\alpha} A + \mu^{\alpha} R) \end{bmatrix}, \tag{3.19}$$

The Jacobian matrix at the disease-free equilibrium point (DFE) is

$$\mathcal{F}=\begin{bmatrix} \beta_1^\alpha S_0 & 0 & \beta_2^\alpha S_0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \mathcal{V}=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26}\\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36}\\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46}\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56}\\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}, \tag{3.20}$$

where $a_{11}=(\epsilon_1^\alpha+\epsilon_2^\alpha+\epsilon_3^\alpha+\delta^\alpha+\mu^\alpha)$, $a_{12}=0$, $a_{13}=0$, $a_{14}=0$, $a_{15}=0$, $a_{16}=0$, $a_{21}=-\epsilon_1^\alpha$, $a_{22}=(\mu^\alpha+\tau^\alpha)$, $a_{23}=0$, $a_{24}=-\gamma_1^\alpha$, $a_{25}=0$, $a_{26}=0$, $a_{31}=-\epsilon_3^\alpha$, $a_{32}=0$, $a_{33}=(\mu^\alpha+\delta^\alpha+g_1^\alpha)$, $a_{34}=0$, $a_{35}=0$, $a_{36}=0$, $a_{41}=-\delta^\alpha$, $a_{42}=0$, $a_{43}=0$, $a_{44}=(\gamma_1^\alpha+\gamma_2^\alpha+\gamma_3^\alpha+\mu^\alpha)$, $a_{45}=0$, $a_{46}=0$, $a_{51}=0$, $a_{52}=0$, $a_{53}=-\delta^\alpha$, $a_{54}=-\gamma_3^\alpha$, $a_{55}=(\mu^\alpha+g_2^\alpha)$, $a_{56}=0$, $a_{61}=-\epsilon_2^\alpha$, $a_{62}=-\tau^\alpha$, $a_{63}=-g_1^\alpha$, $a_{64}=-\gamma_2^\alpha$, $a_{65}=-g_2^\alpha$, $a_{66}=\mu^\alpha$.

Thus, the basic reproduction number of model (3.1) is

$$R_0=\rho(FV^{-1})=\frac{\beta_1^\alpha(\mu^\alpha+\delta^\alpha+g_1^\alpha)+\beta_2^\alpha\epsilon_3^\alpha}{(\epsilon_1^\alpha+\epsilon_2^\alpha+\epsilon_3^\alpha+\delta^\alpha+\mu^\alpha)(\mu^\alpha+\delta^\alpha+g_1^\alpha)}\frac{\Lambda^\alpha}{\mu^\alpha}.$$

4. Stability analysis of equilibrium

4.1. Analysis of DFE

By simplifying the stability of the disease-free equilibrium, we assume that the DFE is $E_0=(S_0, A_0, H_0, I_0, Aq_0, Iq_0, R^0)=(\frac{\Lambda^\alpha}{\mu^\alpha}, 0, 0, 0, 0, 0, 0)$. The Jacobian matrix of system (3.1) can be written as

$$J=\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} & b_{17}\\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} & b_{27}\\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} & b_{37}\\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} & b_{47}\\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} & b_{57}\\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} & b_{67}\\ b_{71} & b_{72} & b_{73} & b_{74} & b_{75} & b_{76} & b_{77} \end{bmatrix}, \tag{4.1}$$

where $b_{11}=-(\mu^\alpha+\beta_1^\alpha A+\beta_2^\alpha I)$, $b_{12}=-\beta_1^\alpha S_0$, $b_{13}=0$, $b_{14}=-\beta_2^\alpha S_0$, $b_{15}=0$, $b_{16}=0$, $b_{17}=0$, $b_{21}=(\beta_1^\alpha A_0+\beta_2^\alpha I_0)$, $b_{22}=\beta_1^\alpha S_0-(\epsilon_1^\alpha+\epsilon_2^\alpha+\epsilon_3^\alpha+\delta^\alpha+\mu^\alpha)$, $b_{23}=0$, $b_{24}=\beta_2^\alpha S_0$, $b_{25}=0$, $b_{26}=0$, $b_{27}=0$, $b_{31}=0$, $b_{32}=\epsilon_1^\alpha$, $b_{33}=-(\mu^\alpha+\tau^\alpha)$, $b_{34}=0$, $b_{35}=\gamma_1^\alpha$, $b_{36}=0$, $b_{37}=0$, $b_{41}=0$, $b_{42}=\epsilon_3^\alpha$, $b_{43}=0$, $b_{44}=-(\mu^\alpha+\delta^\alpha+g_1^\alpha)$, $b_{45}=0$, $b_{46}=0$, $b_{47}=0$, $b_{51}=0$, $b_{52}=\delta^\alpha$, $b_{53}=0$, $b_{54}=0$, $b_{55}=-(\gamma_1^\alpha+\gamma_2^\alpha+\gamma_3^\alpha+\mu^\alpha)$, $b_{56}=0$, $b_{57}=0$, $b_{61}=0$,$b_{62}=0$, $b_{63}=0$, $b_{64}=\delta^\alpha$, $b_{65}=\gamma_3^\alpha$, $b_{66}=-(\mu^\alpha+g_2^\alpha)$, $b_{67}=0$, $b_{71}=0$, $b_{72}=\epsilon_2^\alpha$, $b_{73}=\tau^\alpha$, $b_{74}=g_1^\alpha$, $b_{75}=\gamma_2^\alpha$, $b_{76}=g_2^\alpha$,
$b_{77}=-\mu^\alpha$.

By calculating the Jacobian matrix J at E_0 and solving for $\det(J-\lambda I)$, we obtain

$$P_j(x)=(\lambda+\mu^\alpha)^2(\lambda+\mu^\alpha+\tau^\alpha)(\lambda+\mu^\alpha+g_2^\alpha)(\lambda+\gamma_1^\alpha+\gamma_2^\alpha+\gamma_3^\alpha+\mu^\alpha)(\lambda^2+A\lambda+B),$$

where $A=(\epsilon_1^\alpha+\epsilon_2^\alpha+\epsilon_3^\alpha+\delta^\alpha+\mu^\alpha)+(\mu^\alpha+\delta^\alpha+g_1^\alpha)-\beta_1^\alpha S_0$, and $B=(\epsilon_1^\alpha+\epsilon_2^\alpha+\epsilon_3^\alpha+\delta^\alpha+\mu^\alpha-\beta_1^\alpha S_0)(\mu^\alpha+\delta^\alpha+g_1^\alpha)-\beta_2^\alpha S_0\epsilon_3^\alpha$.

It is easy to prove that, if $\mathcal{R}_0 < 1$, then $A > 0$ and $B > 0$. This polynomial $\lambda^2 + A\lambda + B$ has two roots with negative real parts. Therefore, E_0 is locally stable because the real parts of the seven eigenvalues of the matrix $J(E_0)$ are all negative. Therefore, we can conclude that the DFE is stable when $B > 0$ and DFE is unstable when $B < 0$.

4.2. Analysis of the endemic equilibria

The endemic equilibria of the proposed fractional model (3.1) are denoted by

$$E^* = (S^*, A^*, H^*, I^*, A_q^*, I_q^*, R^*) = (\frac{\Lambda^\alpha}{(\mu^\alpha + \beta_1^\alpha A^* + \beta_2^\alpha I^*)},$$
$$\frac{(g_1^\alpha + \delta^\alpha + \mu^\alpha)I^*}{\epsilon_3^\alpha}, \frac{(\epsilon_1^\alpha A^* + \gamma_1^\alpha A_q^*)}{(\mu^\alpha + \tau^\alpha)},$$
$$\frac{\epsilon_3^\alpha A^*}{(\mu^\alpha + \delta^\alpha + g_1^\alpha)}, \frac{\delta^\alpha A^*}{(\gamma_1^\alpha + \gamma_2^\alpha + \gamma_3^\alpha + \mu^\alpha)}, \frac{(\gamma_3^\alpha A_q^* + \delta^\alpha I^*)}{(\mu^\alpha + g_2^\alpha)},$$
$$\frac{(\tau^\alpha H^* + \epsilon_2^\alpha A^* + \gamma_2^\alpha A_q^* + g_1^\alpha I^* + g_2^\alpha I_q^*)}{\mu^\alpha}).$$

Now, we consider the following algebraic system.

$$\begin{cases} \Lambda^\alpha - \mu^\alpha S^* - \beta_1^\alpha A^* S^* - \beta_2^\alpha I^* S^* = 0 \\ \beta_1^\alpha A^* S^* + \beta_2^\alpha I^* S^* - (\epsilon_1^\alpha + \epsilon_2^\alpha + \epsilon_3^\alpha)A^* - \delta^\alpha A^* - \mu^\alpha A^* = 0 \\ \epsilon_1^\alpha A^* + \gamma_1^\alpha A_q^* - \tau^\alpha H^* - \mu^\alpha H^* = 0 \\ \epsilon_3^\alpha A^* - g_1^\alpha I^* - \delta^\alpha I^* - \mu^\alpha I^* = 0 \\ \delta^\alpha A^* - (\gamma_1^\alpha + \gamma_2^\alpha + \gamma_3^\alpha)A_q^* - \mu^\alpha A_q^* = 0 \\ \gamma_3^\alpha A_q^* + \delta^\alpha I^* - g_2^\alpha I_q^* - \mu^\alpha I_q^* = 0 \\ \tau^\alpha H^* + \gamma_2^\alpha A_q^* + g_1^\alpha I^* + g_2^\alpha I_q^* + \epsilon_2^\alpha A^* - \mu^\alpha R^* = 0 \end{cases} \tag{4.2}$$

From the above equations, we can write

$$\begin{cases} \Lambda^\alpha - \mu^\alpha S^* - \beta_1^\alpha A^* S^* - \beta_2^\alpha I^* S^* = 0 \\ \beta_1^\alpha A^* S^* + \beta_2^\alpha I^* S^* - (\epsilon_1^\alpha + \epsilon_2^\alpha + \epsilon_3^\alpha + \delta^\alpha + \mu^\alpha)A^* = 0 \\ \epsilon_1^\alpha A^* + \gamma_1^\alpha A_q^* - (\tau^\alpha + \mu^\alpha)H^* = 0 \\ \epsilon_3^\alpha A^* - (g_1^\alpha + \delta^\alpha + \mu^\alpha)I^* = 0 \\ \delta^\alpha A^* - (\gamma_1^\alpha + \gamma_2^\alpha + \gamma_3^\alpha + \mu^\alpha)A_q^* = 0 \\ \gamma_3^\alpha A_q^* + \delta^\alpha I^* - (g_2^\alpha + \mu^\alpha)I_q^* = 0 \\ \tau^\alpha H^* + \gamma_2^\alpha A_q^* + g_1^\alpha I^* + g_2^\alpha I_q^* + \epsilon_2^\alpha A^* - \mu^\alpha R^* = 0. \end{cases} \tag{4.3}$$

Let, $(\epsilon_1^\alpha + \epsilon_2^\alpha + \epsilon_3^\alpha + \delta^\alpha + \mu^\alpha) = m_1$, $(\tau^\alpha + \mu^\alpha) = m_2$, $(g_1^\alpha + \delta^\alpha + \mu^\alpha) = m_3$, and $(\gamma_1^\alpha + \gamma_2^\alpha + \gamma_3^\alpha + \mu^\alpha) = m_4$, $(g_2^\alpha + \mu^\alpha) = m_5$.

We obtain the following solutions using some algebraic manipulations of system (4.3).

$$A^* = \frac{m_3 I^*}{\epsilon_3^\alpha}, A_q^* = \frac{\delta^\alpha m_3 I^*}{\epsilon_3^\alpha m_4}, H^* = \frac{(m_3 m_4 \epsilon_1^\alpha - \gamma_3^\alpha \delta^\alpha m_3)I^*}{m_2 m_4 \epsilon_3^\alpha},$$

$$I_q^* = \frac{(\gamma_3^\alpha \delta^\alpha m_3 + \delta^\alpha m_4 \epsilon_3^\alpha) I^*}{m_4 m_5 \epsilon_3^\alpha}, S^* = \frac{\epsilon_3^\alpha \Lambda^\alpha}{\beta_1^\alpha m_3 I^* + \beta_2^\alpha \epsilon_3^\alpha I^* + \mu^\alpha \epsilon_3^\alpha},$$

$$R^* = \frac{(\tau^\alpha m_5 m_3 (m_4 \epsilon_1^\alpha - \gamma_3^\alpha \delta^\alpha) + [\gamma_2^\alpha m_3 m_5 \delta^\alpha + g_1^\alpha m_4 m_5 \epsilon_3^\alpha + g_2^\alpha (\gamma_3^\alpha \delta^\alpha m_3 + \delta^\alpha m_4 \epsilon_3^\alpha) + \epsilon_2^\alpha m_3 m_4 m_5] m_2) I^*}{m_2 m_4 m_5 \mu^\alpha \epsilon_3^\alpha}.$$

Now, $\beta_1^\alpha A^* S^* + \beta_2^\alpha I^* S^* - (\epsilon_1^\alpha + \epsilon_2^\alpha + \epsilon_3^\alpha + \delta^\alpha + \mu^\alpha) A^* = 0$.

By substituting the values of S^*, A^* in the above equation, we obtain

$$I^*[\frac{\Lambda^\alpha \beta_1^\alpha \epsilon_3^\alpha m_3}{\epsilon_3^\alpha (\beta_1^\alpha m_3 I^* + \beta_2^\alpha \epsilon_3^\alpha I^* + \mu^\alpha \epsilon_3^\alpha)}] + I^*[\frac{\Lambda^\alpha \beta_2^\alpha \epsilon_3^\alpha}{\beta_1^\alpha m_3 I^* + \beta_2^\alpha \epsilon_3^\alpha I^* + \mu^\alpha \epsilon_3^\alpha}] - I^*[\frac{m_1 m_3}{\epsilon_3^\alpha}] = 0.$$

So, $I^* = \frac{\epsilon_3^\alpha (\beta_1^\alpha \Lambda^\alpha m_3 + \beta_2^\alpha \Lambda^\alpha \epsilon_3^\alpha - \mu^\alpha m_1 m_3)}{m_1 m_3 (\beta_1^\alpha m_3 + \beta_2^\alpha \epsilon_3^\alpha)}$, for $I^* > 0$ implies, $R_0 > 1$.

Therefore, there is a unique value for I^* and a unique endemic equilibrium $E^* = (S^*, A^*, H^*, I^*, A_q^*, I_q^*, R^*)$ when $R_0 > 1$.

4.3. Global stability analysis

We establish the global stability of the fractional model (3.1) in the sense of Ulam-Hyers [13]. Recently the authors in [14] established Ulam-Hyers stability of a nonlinear fractional model of COVID-19 pandemic.

For clarity of the discussion that follows, let us introduce the inequality given by

$$|{}^C D_t^\alpha \phi(t) - \kappa(t, \phi(t))| \leq \epsilon, t \in [0, b]. \tag{4.4}$$

We say a function $\bar{\phi} \in \mathbb{E}$ is a solution of (4.4) if and only if there exists $h \in \mathbb{E}$ satisfying

i. $|h(t)| \leq \epsilon$.

ii. ${}^C D_t^\alpha \bar{\phi}(t) = \kappa(t, \bar{\phi}(t)) + h(t), t \in [0, b]$.

It is important to observe that by invoking (3.6) and property ii. above, simple simplification yields the fact that any function $\bar{\phi} \in \mathbb{E}$ satisfying (4.4) also satisfies the integral inequality

$$|\bar{\phi}(t) - \bar{\phi}(0) - \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \kappa(\tau, \bar{\phi}(\tau))| \leq \Omega_\epsilon. \tag{4.5}$$

Definition 4.1. *The fractional order model (3.4) (and equivalently (3.1)) is Ulam-Hyers stable if there exists $C_\kappa > 0$ such that for every $\epsilon > 0$, and for each solution $\bar{\phi} \in \mathbb{E}$ satisfying (4.4), there exists a solution $\phi \in \mathbb{E}$ of (3.4) with $\|\bar{\phi}(t) - \phi(t)\|_\varepsilon \leq C_\kappa \epsilon, t \in [0, b]$*

Definition 4.2. *The fractional order model (3.4) (and equivalently (3.1)) is said to be generalized Ulam-Hyers stable if there exists a continuous function $\Theta_\kappa : \mathbb{R}^+ \to \mathbb{R}^+$ with $\Theta_\kappa(0) = 0$, such that, for each solution $\bar{\phi} \in \mathbb{E}$ of (4.4), there exists a solution $\phi \in \mathbb{E}$ of (3.4) such that*

$$\|\bar{\phi}(t) - \phi(t)\|_\varepsilon \leq \Theta_\kappa \epsilon, t \in [0, b].$$

We now present our result on the stability of the fractional order model.

Theorem 4.1. *Let the hypothesis and result of Lemma 3.2 hold,* $\Omega = \frac{\Lambda^\alpha}{\Gamma(\alpha+1)}$ *and* $1 - \Omega L_\kappa > 0$. *Then, the fractional order model (3.4) (and equivalently (3.1)) is Ulam-Hyers stable and consequently generalized Ulam-Hyers stable.*

Proof. Let ϕ be a unique solution of (3.4) guranted by theorem 3.2; $\bar{\phi}$ satisfies (4.4). Then recalling the expressions (3.6),(4.5), we have for $\epsilon > 0, t \in [0, b]$ that

$$\|\bar{\phi} - \phi\|_\varepsilon = sup_{t\in[0,b]}|\bar{\phi}(t) - \phi(t)|$$

$$= sup_{t\in[0,b]}|\bar{\phi}(t) - \phi_0 - \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}\kappa(\tau, \phi(\tau))d\tau|,$$

$$\leq sup_{t\in[0,b]}|\bar{\phi}(t) - \bar{\phi}_0 - \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}\kappa(\tau, \bar{\phi}(\tau))d\tau|$$

$$+ sup_{t\in[0,b]}\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}|\kappa(\tau, \bar{\phi}(\tau)) - \kappa(\tau, \phi(\tau))|d\tau,$$

$$\leq \Omega_\epsilon + \frac{L_\kappa}{\Gamma(\alpha)} sup_{t\in[0,b]}\int_0^t (t-\tau)^{\alpha-1}|\bar{\phi}(\tau) - \phi(\tau)|d\tau,$$

$$\leq \Omega_\epsilon + \Omega L_\kappa\|\bar{\phi} - \phi\|_\varepsilon,$$

from which we obtain $\|\bar{\phi} - \phi\|_\varepsilon \leq C_\kappa\epsilon$ where $C_\kappa = \frac{\Omega}{1-\Omega L_\kappa}$.

5. Sensitivity analysis

Sensitivity analysis is beneficial and can help identify parameters that require control strategies. It provides an effective technique for preventing and restraining the dis ease. The disease can be controlled and mitigated if the parameter values change. A systematic description of the sensitivity analysis of the different parameters in R_0 for the model is as follows:

$$\Upsilon_\phi^{R_0} = \frac{dR_0}{d\phi}\frac{\phi}{R_0}.$$

Therefore, the basic reproduction number is

$$R_0 = \rho(FV^{-1}) = \frac{\beta_1^\alpha(\mu^\alpha + \delta^\alpha + g_1^\alpha) + \beta_2^\alpha\epsilon_3^\alpha}{(\epsilon_1^\alpha + \epsilon_2^\alpha + \epsilon_3^\alpha + \delta^\alpha + \mu^\alpha)(\mu^\alpha + \delta^\alpha + g_1^\alpha)}\frac{\Lambda^\alpha}{\mu^\alpha}.$$

It is easy to verify that

$$A = \frac{dR_0}{d\beta_1}\frac{\beta_1}{R_0} = \frac{a\eta}{bh + a\eta} > 0,$$

$$B = \frac{dR_0}{d\beta_2}\frac{\beta_2}{R_0} = \frac{bh}{bh + a\eta} > 0,$$

$$C = \frac{dR_0}{d\mu}\frac{\mu}{R_0} = -\frac{c^2\eta(i(bh + a\eta))}{c\eta\eta_1^2} + \frac{i(bh + a\eta)}{c\eta^2\eta_1} + \frac{i(bh + a\eta)}{c^2\eta\eta_1} - \frac{ai}{c\eta i(bh + a\eta)} < 0,$$

$$D = \frac{dR_0}{d\delta}\frac{\delta}{R_0} = -\frac{bc\eta(i(bh + a\eta))}{c\eta\eta_1^2} + \frac{i(bh + a\eta)}{c\eta^2\eta_1} - \frac{a}{c\eta(bh + a\eta)} < 0,$$

$$E = \frac{dR_0}{dg_1}\frac{g_1}{R_0} = -\frac{e(i(bh + a\eta))}{\eta^2\eta_1} - \frac{a}{c(bh + a\eta)} < 0,$$

$$F = \frac{dR_0}{d\epsilon_1}\frac{\epsilon_1}{R_0} = -\frac{f}{\eta_1} < 0, G = \frac{dR_0}{d\epsilon_2}\frac{\epsilon_2}{R_0} = -\frac{g}{\eta_1} < 0,$$

$$H = \frac{dR_0}{d\epsilon_3}\frac{\epsilon_3}{R_0} = -\frac{ch(i(bh + a\eta))}{c\eta\eta_1^2} - \frac{b}{c(bh + a\eta)} < 0, I = \frac{dR_0}{d\Lambda}\frac{\Lambda}{R_0} = 1 > 0.$$

where $\beta_1^\alpha = a, \beta_2^\alpha = b, \mu^\alpha = c, \delta^\alpha = d, g_1^\alpha = e, \epsilon_1^\alpha = f, \epsilon_2^\alpha = g, \epsilon_3^\alpha = h, \Lambda^\alpha = i, (c + d + e) = \eta,$ $(c + d + f + g + h) = \eta_1.$

From the above simplification, we assumed that the sensitivity indices are sign-related. This means that R_0 is more sensitive to the parameters $(\beta_1, \beta_2, \Lambda)$ in increasing order and is positively impacted by them, and thus reducing the value of these parameters will reduce R_0. The following parameters (μ, δ, g_1, ϵ_1, ϵ_2, ϵ_3) have a negative impact on R_0, and an increase in these parameters reduces R_0. After obtaining the above analytical results, we now perform a sensitivity analysis to find perfect ways to choose the various parameters in R_0. The following can be inferred from the sensitivity analysis.

1) If we can reduce the value of the transmission rates β_1, β_2 could be an effective control measure to stop the spread of the coronavirus.

2) If we can increase the quarantine rate δ or put infected people in isolation, they will not affect other susceptible individuals.

6. Numerical simulations

This section provides some illustrative numerical simulations to explain the dynamical behavior of the Caputo fractional order of COVID-19 mathematical model. Herein Caputo fractional operator is numerically simulated via first-order convergent numerical techniques. These numerical techniques of a mathematical model are accurate, conditionaly stable, and convergent for solving fractional-order both linear and nonlinear systems of ordinary differential equations. Consider a general Cauchy problem of fractional order having autonomous nature

$$^*D_{0^+}^\alpha y(t)) = g(y(t)), \alpha \in (0, 1], t \in [0, T], y(0) = y_0, \tag{6.1}$$

where $y = (a, b, c, d, e, f, g) \in \mathbb{R}_+^7$ is a real-valued continuous vector function which satisfies the Lipschitz criterion given as

$$||g(y_1(t)) - g(y_2(t))|| \leq M||y_1(t) - y_2(t)||, \tag{6.2}$$

where M is a positive real Lipschitz constant. Using the fractional-order integral operators, one obtains

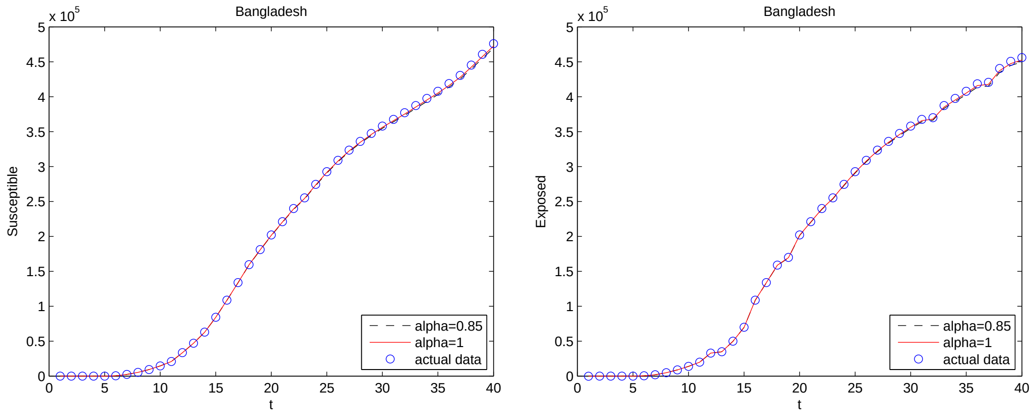
$$y(t) = y_0 + J_{0,t}^\alpha g(y(t)), t \in [0, T], \tag{6.3}$$

where $J_{0,t}^\alpha$ is the fractional-order integral operator . Consider an equi-spaced integration intervals over [0,T] with the fixed step size $h(= 10^{-2}$ for simulation $) = \frac{T}{n}, n \in N$. Suppose that y_q is the approximation of y(t) at $t = t_q$ for $q = 0, 1, ..., n$. The numerical technique for the governing model under Caputo fractional derivative operator takes the form

$$\begin{aligned}
{}^{c}S_{p+1} &= a_0 + \frac{h^\alpha}{\Gamma\alpha+1} \times \Sigma_{k=0}^{p}((p-k+1)^\alpha - (p-k)^\alpha) \\
&\qquad (\Lambda - \mu S - \beta_1 AS - \beta_2 IS), \\
{}^{c}A_{p+1} &= b_0 + \frac{h^\alpha}{\Gamma\alpha+1} \times \Sigma_{k=0}^{p}((p-k+1)^\alpha - (p-k)^\alpha) \\
&\qquad (\beta_1 AS + \beta_2 IS - (\epsilon_1 + \epsilon_2 + \epsilon_3)A - \delta A - \mu A), \\
{}^{c}H_{p+1} &= c_0 + \frac{h^\alpha}{\Gamma\alpha+1} \times \Sigma_{k=0}^{p}((p-k+1)^\alpha - (p-k)^\alpha) \\
&\qquad (\epsilon_1 A + \gamma_1 A_q - \tau H - \mu H), \\
{}^{c}I_{p+1} &= d_0 + \frac{h^\alpha}{\Gamma\alpha+1} \times \Sigma_{k=0}^{p}((p-k+1)^\alpha - (p-k)^\alpha) \\
&\qquad (\epsilon_3 A - g_1 I - \delta I - \mu I), \\
{}^{c}Aq_{p+1} &= e_0 + \frac{h^\alpha}{\Gamma\alpha+1} \times \Sigma_{k=0}^{p}((p-k+1)^\alpha - (p-k)^\alpha) \\
&\qquad (\delta A - (\gamma_1 + \gamma_2 + \gamma_3)A_q - \mu A_q), \\
{}^{c}Iq_{p+1} &= f_0 + \frac{h^\alpha}{\Gamma\alpha+1} \times \Sigma_{k=0}^{p}((p-k+1)^\alpha - (p-k)^\alpha) \\
&\qquad (\gamma_3 A_q + \delta I - g_2 I_q - \mu I_q), \\
{}^{c}R_{p+1} &= g_0 + \frac{h^\alpha}{\Gamma\alpha+1} \times \Sigma_{k=0}^{p}((p-k+1)^\alpha - (p-k)^\alpha) \\
&\qquad (\tau H + \gamma_2 A_q + g_1 I + g_2 I_q + \epsilon_2 A - \mu R).
\end{aligned} \tag{6.4}$$

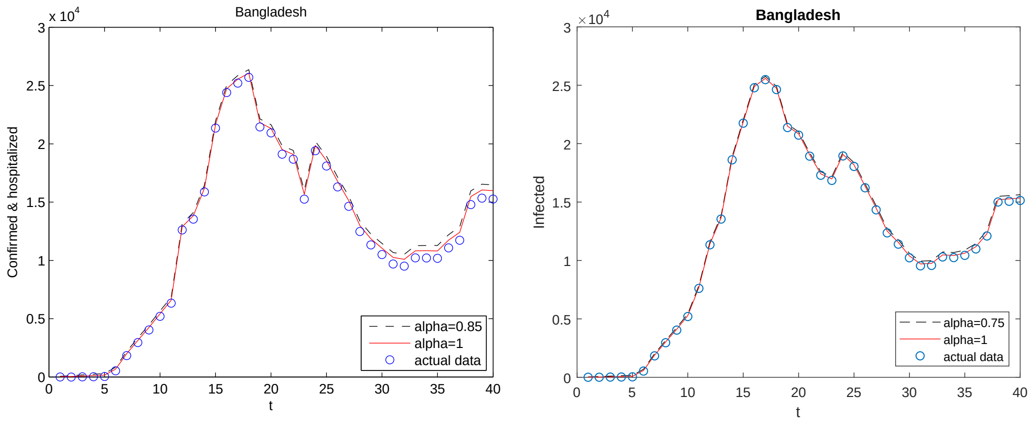
Now we discuss the obtained numerical outcomes of the governing model in respect of the approximate solutions. To this aim, we employed the effective Euler method under the Caputo fractional operator to do the job. Observing the numerical simulations of the proposed model (3.1) is vital. We use different parametric values for the numerical simulations based on a case study of Bangladesh cited from the literature; some are fitted, some are estimated, and some are referred. We use the total population of Bangladesh, N = 164,689,383 [15]. We have $N = S(0) + A(0) + H(0) + I(0) + A_q(0) + I_q(0) + R(0)$, The initial conditions are assumed as $S(0) = n_1 = 1000$, $A(0) = n_2 = 500$, $H(0) = n_3 = 300$, $I(0) = n_4 = 100$, $Aq(0) = n_5 = 0$, $Iq(0) = n_6 = 0$, $R(0) = n_7 = 0$ and the parameter values are taken as in Table 1. Considering the values in the table, we depicted the profiles of each variable under Caputo fractional derivative in Figure 2 with the fractional-order value α while Figures 2–7 are the illustration and dynamical outlook of each variable with different fractional-order values. From Figure 2(a), one can see that the susceptible class *S(t)* shows increasing behavior with the values of α and actual data, the rate of decreasing starts to disappear and the rate of increasing starts becoming higher. With the same values as can be seen in Figure 2(b), the exposed class *A(t)* has also increasing-decreasing behavior with the values of α and actual data. The decreasing rate also starts to disappear and the rate of increasing starts becoming higher. In Figure 3(a), the hospitalized class *H(t)* is virtually having the increasing-decreasing nature with fractional-order values, the class totally becomes stable. In Figure 3(b), the infected class *I(t)* is virtually retaining the increasing-decreasing nature, whereas the class is likely to be at stake. An interesting behavior can be noticed, one can see that there is a strongly increasing nature, in this case, and this could be due to the dangers associated with the class. In Figure

4(a), the isolated exposed class *Aq(t)* shows from decreasing to increasing nature with fractional-order values and actual data. In Figure 4(b), the isolated infectious class *Iq(t)* starts to disappear and the rate of increasing starts becoming higher. In Figure 5(a), the recovered class *R(t)* also starts to disappear and the rate of increasing starts becoming higher. In Figure 5(b), the daily recoveries class starts to higher and the rate of increasing starts becoming lower with different fractional order values. In Figure 6(a), the death class starts to disappear and the rate of increasing starts becoming higher. In Figure 6(b), the new case class starts to higher and the rate of increasing starts becoming lower with different fractional order values. In Figure 7(a), the new death class starts to higher and the rate of increasing starts becoming lower with different fractional order values. In Figure 7(b), the total tested class starts to higher and the rate of increasing starts becoming stable with different fractional order values.



(a) (b)

Figure 2. Numerical simulation of (a) Suspected individual *S(t)* (b) Exposed but not hospitalized individual *E(t)* for different values of α and actual values with time (weeks).



(a) (b)

Figure 3. Numerical simulation of (a) Hospitalized *H(t)* (b) Infectious *I(t)* for different values of α and actual values with time (weeks).

We used some reference values given in [10] and estimated the parameters. Furthermore, the basic reproduction number of the disease-free equilibrium point= $(\frac{\Lambda^\alpha}{\mu^\alpha}, 0, 0, 0, 0, 0, 0)$= (1.787946, 0, 0, 0, 0, 0, 0) for $\alpha = 0.5$ was computed as $R_0 = 0.7 < 1$, showing the fulfillment of the necessary and sufficient conditions for local asymptotic stability of the disease-free equilibrium. We also found out that for integer and fractional orders considered namely $\alpha \in (1, 0.9)$ and the corresponding computed $R_0 = (1.58458799, 0.90)$ showing that the COVID-19 pandemic is controllable and will effectively die out as long as there is compliance with social distancing/lockdown regulations, and if infectious and infected individuals are appropriately quarantined, thereby preventing contamination of the environment through virus shedding.

Susceptible (S), exposed but not hospitalized (A), hospitalized (H), infectious (I), isolated exposed (Aq), isolated infectious (Iq), and recovered (R) are shown in Figures 2–7.

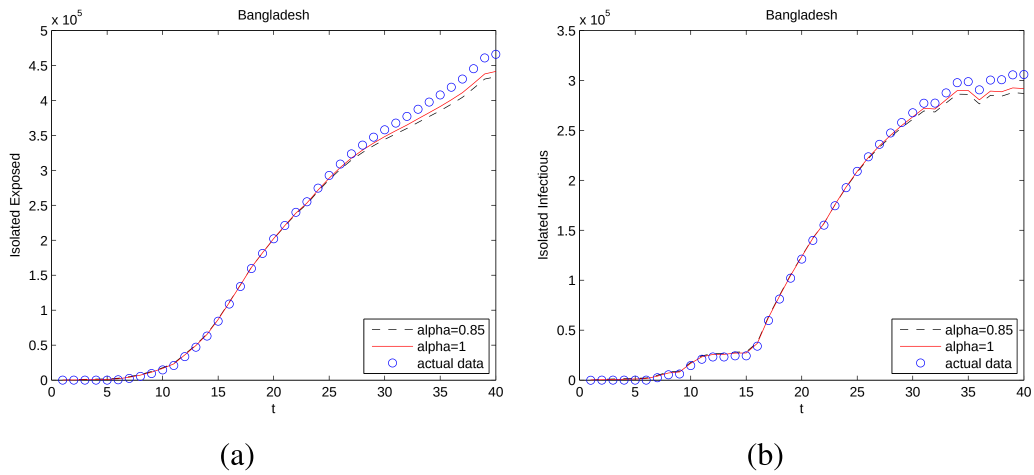


Figure 4. Numerical simulation of (a) Isolated Exposed *Aq(t)* (b) Isolated Infectious *Iq(t)* for different values of α and actual values with time (weeks).

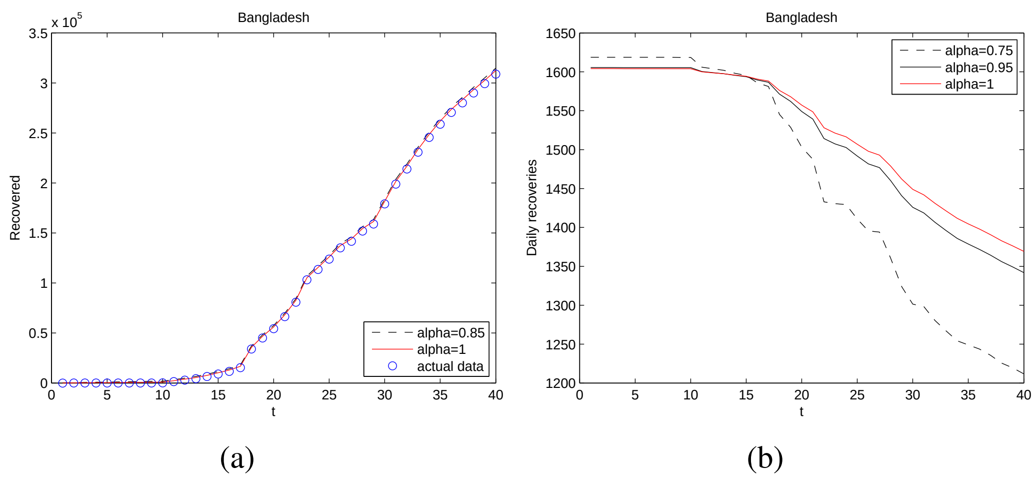
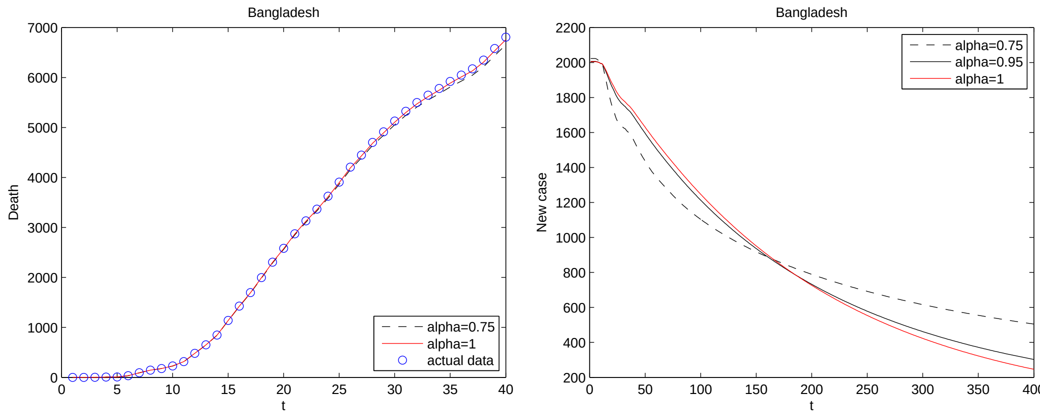
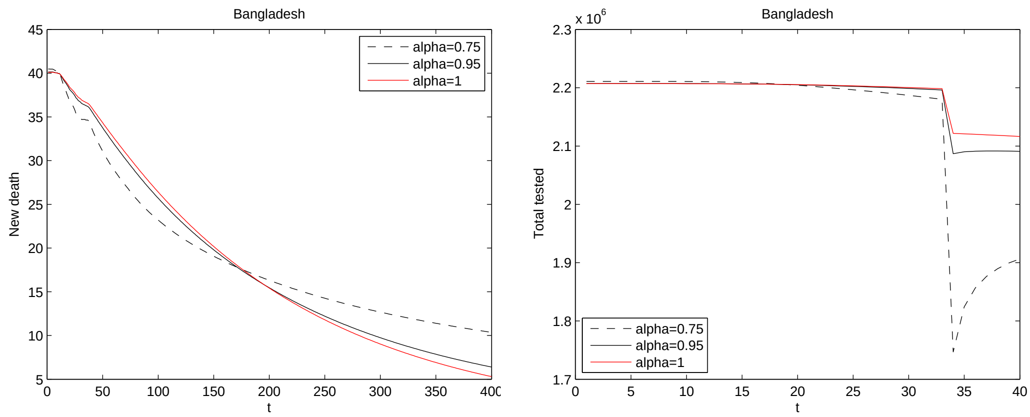


Figure 5. Numerical simulation of (a) Recovered *R(t)* (b) Daily Recoveries for different values of α and actual values with time (weeks).



(a) (b)

Figure 6. Numerical simulation of (a) Death (b) New Case for different values of α and actual values with time (weeks).



(a) (b)

Figure 7. Numerical simulation of (a) New Death (b) Total Tested for different values of α and actual values with time (weeks).

7. Discussion and conclusions

Fractional epidemic modeling is an effective process for trying to mitigate the global pandemic COVID-19 situation if different parameters can be estimated and fitted accurately. This paper analyzed a fractional COVID-19 compartmental model via Caputo FDs. The fixed point theorems of Schauder and Banach respectively are employed to prove the existence and uniqueness of solutions of the proposed model. We studied the existence and uniqueness of the solution and the local and global stability of the model. Stability analysis in the frame of Ulam-Hyers and generalized Ulam-Hyers was established. The fractional variant of the model under consideration via Caputo fractional operator has numerically been simulated via a first-order convergent numerical technique called the fractional Euler method. The illustration and dynamical outlook of each variable with different fractional-order values were examined. Thus, these results show the actual model was fitted using accurate COVID-19 data from Bangladesh. These fractional-order derivatives require new and perfect parameters to control the outbreak. All graphical simulations were performed as per the specified nature of the achieved solutions in the Caputo non-integer order derivative sense. Different compartments are plotted simultaneously using graphical simulations for other fractional orders of α and actual data from Bangladesh. The data of infected and death cases due to COVID-19 were collected from [3] to perform the numerical simulations. Table 2 represents the weekly data from March 2, 2020, to November 30, 2020.

Table 2. COVID-19 weekly data of Bangladesh.

week	1	2	3	4	5	6
cases	3	4	17	24	40	533
week	7	8	9	10	11	12
cases	1835	2960	4039	5202	7611	11,342
week	13	14	15	16	17	18
cases	13,543	18,616	21,751	24,786	25,481	24,630
week	19	20	21	22	23	24
cases	21,378	20,730	18,928	17,293	16,854	18,949
week	25	26	27	28	29	30
cases	18,049	16,224	14,335	12,363	11,396	10,232
week	31	32	33	34	35	36
cases	9542	9576	10,303	10,246	10,437	10,986
week	37	38	39	40		
cases	12,095	15,008	15,066	15,138		

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Conflict of interest

The authors declare that they have no competing interests.

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