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Research article

Propagation of lump-type waves in nonlinear shallow water wave

Hong-Yang Guan[1,*] **and Jian-Guo Liu**[2,*]

[1] Department of Information Engineering, Liaoning University of Traditional Chinese Medicine, Shenyang 110847, China

[2] College of Computer, Jiangxi University of Chinese Medicine, Jiangxi 330004, China

* **Correspondence:** Email: ghyang1982yang@126.com, 20101059@jxutcm.edu.cn.

Abstract: In this work, a new extended shallow water wave equation in (3+1) dimensions was studied, which represents abundant physical meaning in a nonlinear shallow water wave. We discussed the interaction between a lump wave and a single solitary wave, which is an inelastic collision. Further, the interaction between a lump wave and two solitary waves and the interaction between a lump wave and a periodic wave was also studied using the Hirota bilinear method. Finally, the interaction among lump, periodic and one solitary wave was investigated. The dynamic properties of the obtained results are shown and analyzed by some three-dimensional images.

Keywords: shallow water wave; lump wave; periodic wave; bilinear method

1. Introduction

A lump wave is a rational soliton with a large amplitude, which is localized only in space and will not disappear due to time change. In 2015, an effective algebraic method to obtain lump solutions of integrable systems was proposed by Ma [1]. Subsequently, He [2] also provided theoretical support for this method and proved it, which made great progress in the solution of the lump wave and attracted the attention of a large number of researchers, such as Tian [3–5], Lü [6–8], He [9–11], Wen [12–14], Su [15–17], Lan [18–20], Chen [21–23], Li [24–26] and Ma [27,28] et al.

The shallow water wave equation has become a hot topic in recent years. The wave equation for shallow water is a model in which the depth of water is less than the wavelength of the free surface disturbance. Examples of shallow water wave equations are widely used in the field of oceanography and the atmosphere to simulate the dynamic behavior of water wave propagation. For example, the shallow water wave equation in a (2+1) dimension [29–31] is given below:

$$\Phi_{yt} - 3\Phi_x\Phi_{xy} - 3\Phi_y\Phi_{xx} + \Phi_{xxxy} = 0, \tag{1}$$

and

$$\Phi_{yt} - 4\Phi_x\Phi_{xy} - 2\Phi_y\Phi_{xx} + \Phi_{xxxy} = 0, \tag{2}$$

and the shallow water wave equation in (3+1) dimension:

$$\Phi_{yt} - \Phi_{xz} - 3\Phi_x\,\Phi_{xy} - 3\,\Phi_y\,\Phi_{xx} + \Phi_{xxxy} = 0. \tag{3}$$

Recently, a new extended shallow water wave equation in a (3+1) dimension was proposed by Wazwaz as follows [31]:

$$\alpha\Phi_{xx} + \beta\Phi_{yy} + \gamma\Phi_{xy} + \delta\Phi_{yz} + \Phi_{yt} - 3\Phi_x\Phi_{xy} - 3\Phi_y\Phi_{xx} + \Phi_{xxxy} = 0, \tag{4}$$

where $\Phi = \Phi(x, y, z, t)$, α, β, γ and δ are arbitrary constants. Wazwaz [31] obtained the multiple soliton and lump solutions. In addition, this equation has not been studied in other literatures.

Under the transformation

$$\Phi = -2\,(\ln\xi)_x, \tag{5}$$

where $\xi = \xi(x, y, z, t)$, Eq (4) has the following Hirota bilinear form

$$\begin{aligned}
&[D_yD_t + D_x^3D_y + \beta D_y^2 + \alpha D_x^2 + \gamma D_yD_x + \delta D_yD_z]\xi\cdot\xi = -\alpha\xi_x^2 - \beta\xi_y^2\\
&+3\xi_{xy}\xi_{xx} - \xi_y\xi_{xxx} + \xi\left(\alpha\xi_{xx} + \gamma\xi_{xy} + \delta\xi_{yz} + \beta\xi_{yy} + \xi_{xxxy} + \xi_{yt}\right)\\
&-\xi_t\xi_y - \gamma\xi_x\xi_y - \delta\xi_z\xi_y - 3\xi_x\xi_{xxy} = 0.
\end{aligned} \tag{6}$$

The organization of this paper is as follows: Section two investigates the interaction between a lump wave and one soliton of Eq (4). Section three obtains the interaction solutions between a lump and two solitary waves for Eq (4). Section four studies the interaction between a lump and a periodic wave of Eq (4); Section five discusses the interaction among a lump, periodic and one solitary wave. Section six makes the conclusions.

2. Lump-1-soliton solution

The lump solutions of Eq (4) have been obtained by Wazwaz. On this basis, we will further investigate the interaction between the lump wave and one soliton. For this reason, we assume

$$\begin{aligned}
\xi &= (\mathcal{G}_4t + \mathcal{G}_1x + \mathcal{G}_2y + \mathcal{G}_3z)^2 + (\mathcal{G}_8t + \mathcal{G}_5x + \mathcal{G}_6y + \mathcal{G}_7z)^2\\
&+ \mathcal{G}_9 + k_1e^{\mathcal{G}_{13}t+\mathcal{G}_{10}x+\mathcal{G}_{11}y+\mathcal{G}_{12}z},
\end{aligned} \tag{7}$$

where $\mathcal{G}_i(i = 1, 2, \cdots, 12)$ and k_1 are undetermined constants. Substituting Eq (7) into (6), we obtain

$$(I)\qquad \mathcal{G}_3 = \frac{\alpha\left(-\mathcal{G}_2\mathcal{G}_1^2 - 2\mathcal{G}_5\mathcal{G}_6\mathcal{G}_1 + \mathcal{G}_2\mathcal{G}_5^2\right)}{\delta\left(\mathcal{G}_2^2 + \mathcal{G}_6^2\right)} - \frac{\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4}{\delta},$$

$$\mathcal{G}_8 = -[\mathcal{G}_6\left(\alpha\mathcal{G}_5^2 + \beta\mathcal{G}_6^2 + \gamma\mathcal{G}_6\mathcal{G}_5 + \delta\mathcal{G}_6\mathcal{G}_7\right) - \alpha\mathcal{G}_6\mathcal{G}_1^2 + 2\alpha\mathcal{G}_2\mathcal{G}_5\mathcal{G}_1$$

$$+\mathcal{G}_2^2\left(\beta\mathcal{G}_6+\gamma\mathcal{G}_5+\delta\mathcal{G}_7\right)]/(\mathcal{G}_2^2+\mathcal{G}_6^2),$$

$$\mathcal{G}_9=-\frac{3\left(\mathcal{G}_1^2+\mathcal{G}_5^2\right)\left(\mathcal{G}_1\mathcal{G}_2+\mathcal{G}_5\mathcal{G}_6\right)\left(\mathcal{G}_2^2+\mathcal{G}_6^2\right)}{\alpha\left(\mathcal{G}_2\mathcal{G}_5-\mathcal{G}_1\mathcal{G}_6\right)^2},$$

$$\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12},\mathcal{G}_{11}=\epsilon_1\frac{\sqrt{\mathcal{G}_2^2+\mathcal{G}_6^2}\mathcal{G}_{10}}{\sqrt{\mathcal{G}_1^2+\mathcal{G}_5^2}},$$

$$\mathcal{G}_{10}=\epsilon_2\frac{\sqrt{\frac{2}{3}}\sqrt{\alpha\mathcal{G}_1\mathcal{G}_2+\alpha\mathcal{G}_5\mathcal{G}_6+\epsilon_1\alpha\sqrt{\mathcal{G}_1^2+\mathcal{G}_5^2}\sqrt{\mathcal{G}_2^2+\mathcal{G}_6^2}}}{\sqrt{\mathcal{G}_2^2+\mathcal{G}_6^2}}. \tag{8}$$

$$(II)\qquad \mathcal{G}_3=\frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\mathcal{G}_5\left(\alpha\mathcal{G}_5^2-\beta\mathcal{G}_2^2-\gamma\mathcal{G}_1\mathcal{G}_2-\mathcal{G}_4\mathcal{G}_2\right)}{\delta\mathcal{G}_1\mathcal{G}_2},$$

$$\mathcal{G}_8=\frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2-\alpha\mathcal{G}_5^2\right)-\alpha\mathcal{G}_5^4+\mathcal{G}_2\mathcal{G}_5^2\left(\beta\mathcal{G}_2+\mathcal{G}_4\right)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5},\mathcal{G}_9=0,\mathcal{G}_5=\mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},$$

$$\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12},\mathcal{G}_{11}=\pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}. \tag{9}$$

$$(III)\qquad \mathcal{G}_3=-\frac{\mathcal{G}_5\mathcal{G}_7}{\mathcal{G}_1},\mathcal{G}_6=-\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5},\mathcal{G}_7=\frac{\mathcal{G}_1\left(\mathcal{G}_2\left(\beta\mathcal{G}_2+\gamma\mathcal{G}_1+\mathcal{G}_4\right)-\alpha\mathcal{G}_5^2\right)}{\delta\mathcal{G}_2\mathcal{G}_5},$$

$$\mathcal{G}_8=-\frac{\mathcal{G}_1\left(\gamma\mathcal{G}_1+\mathcal{G}_4\right)}{\mathcal{G}_5}-\gamma\mathcal{G}_5,\mathcal{G}_9=0,\mathcal{G}_5=\mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},$$

$$\mathcal{G}_{13}=-\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}}-\beta\mathcal{G}_{11}-\mathcal{G}_{10}\left(\gamma+\mathcal{G}_{10}^2\right)-\delta\mathcal{G}_{12},\mathcal{G}_{11}=\pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}. \tag{10}$$

All other parameters are free and unrestricted. By substituting the results of Eqs (8)–(10) into Eqs (5) and (7), we can get the corresponding interaction solution for Eq (4). In order to understand the dynamic properties of the interaction solutions between the lump wave and one soliton, we take Eq (8) as an example and select special values of parameters (see Figure 1) to obtain a special solution of the equation as follows:

$$\Phi=-576[73e^{\frac{19t}{6}+x}(7t-16x+9z)+192e^{\frac{2y}{3}+3z}]/[73e^{\frac{19t}{6}+x}[2041t^2+t(2560y$$
$$-2016x+1134z)+9(16x-9z)^2+1024y^2]+55296e^{\frac{2y}{3}+3z}]. \tag{11}$$

The dynamic properties of Eq (11) are shown in Figure 1. In Figure 1(a), we can observe that a soliton and a lump wave propagate forward respectively. In Figure 1(b), the soliton and the lump wave slowly close together, and the amplitude of the lump wave begins to decrease. Until Figure 1(c), the soliton and the lump wave gradually converge, and the amplitude of the lump wave becomes smaller. It can be seen that the interaction between the soliton and the lump wave is an inelastic collision where the energy is consumed.

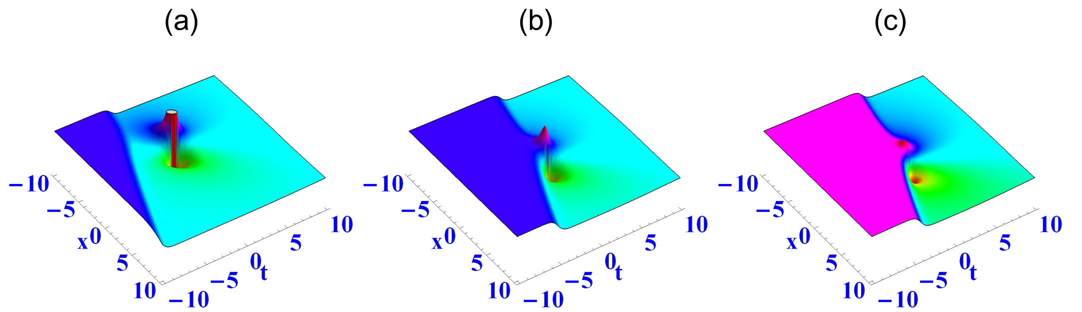

Figure 1. Solution (11). $\mathcal{G}_2 = \alpha = \beta = \delta = \gamma = 1$, $\mathcal{G}_1 = -4$, $\mathcal{G}_4 = \mathcal{G}_{12} = 3$, $k_1 = 6$, $y = 0$, $\mathcal{G}_{10} = -1$, (a) $z = -5$, (b) $z = 0$, (c) $z = 1$.

3. Lump-2-soliton solution

Next, we want to further consider the interaction between the lump wave and double solitons. Therefore, we assume

$$\begin{aligned}\xi &= (\mathcal{G}_4 t + \mathcal{G}_1 x + \mathcal{G}_2 y + \mathcal{G}_3 z)^2 + (\mathcal{G}_8 t + \mathcal{G}_5 x + \mathcal{G}_6 y + \mathcal{G}_7 z)^2 \\ &+ \mathcal{G}_9 + k_1 \cosh(\mathcal{G}_{13} t + \mathcal{G}_{10} x + \mathcal{G}_{11} y + \mathcal{G}_{12} z). \end{aligned} \tag{12}$$

Substituting Eq (12) into (6), we derive

$$\begin{aligned}(I) \quad & \mathcal{G}_3 = \frac{\alpha\left(-\mathcal{G}_2\mathcal{G}_1^2 - 2\mathcal{G}_5\mathcal{G}_6\mathcal{G}_1 + \mathcal{G}_2\mathcal{G}_5^2\right)}{\delta\left(\mathcal{G}_2^2 + \mathcal{G}_6^2\right)} - \frac{\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4}{\delta}, \\ & \mathcal{G}_8 = -[\mathcal{G}_6\left(\alpha\mathcal{G}_5^2 + \beta\mathcal{G}_6^2 + \gamma\mathcal{G}_6\mathcal{G}_5 + \delta\mathcal{G}_6\mathcal{G}_7\right) - \alpha\mathcal{G}_6\mathcal{G}_1^2 + 2\alpha\mathcal{G}_2\mathcal{G}_5\mathcal{G}_1 \\ & + \mathcal{G}_2^2(\beta\mathcal{G}_6 + \gamma\mathcal{G}_5 + \delta\mathcal{G}_7)]/(\mathcal{G}_2^2 + \mathcal{G}_6^2), \\ & \mathcal{G}_9 = -\frac{3\left(\mathcal{G}_2^2 + \mathcal{G}_6^2\right)\left(4\mathcal{G}_2\mathcal{G}_1^3 + 4\mathcal{G}_5\mathcal{G}_6\mathcal{G}_1^2 + 4\mathcal{G}_2\mathcal{G}_5^2\mathcal{G}_1 + 4\mathcal{G}_5^3\mathcal{G}_6 + \mathcal{G}_{10}^3\mathcal{G}_{11}k_1^2\right)}{4\alpha\left(\mathcal{G}_2\mathcal{G}_5 - \mathcal{G}_1\mathcal{G}_6\right)^2}, \\ & \mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{11} = \epsilon_1\frac{\sqrt{\mathcal{G}_2^2 + \mathcal{G}_6^2}\mathcal{G}_{10}}{\sqrt{\mathcal{G}_1^2 + \mathcal{G}_5^2}}, \\ & \mathcal{G}_{10} = \epsilon_2\frac{\sqrt{\frac{2}{3}}\sqrt{\alpha\mathcal{G}_1\mathcal{G}_2 + \alpha\mathcal{G}_5\mathcal{G}_6 + \epsilon_1\alpha\sqrt{\mathcal{G}_1^2 + \mathcal{G}_5^2}\sqrt{\mathcal{G}_2^2 + \mathcal{G}_6^2}}}{\sqrt{\mathcal{G}_2^2 + \mathcal{G}_6^2}}. \end{aligned} \tag{13}$$

$$\begin{aligned}(II) \quad & \mathcal{G}_3 = \frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5}, \mathcal{G}_6 = -\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\alpha\mathcal{G}_5^3 - \mathcal{G}_2\mathcal{G}_5(\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4)}{\delta\mathcal{G}_1\mathcal{G}_2}, \\ & \mathcal{G}_{11} = \pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}, \mathcal{G}_8 = \frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2 - \alpha\mathcal{G}_5^2\right) - \alpha\mathcal{G}_5^4 + \mathcal{G}_2\mathcal{G}_5^2(\beta\mathcal{G}_2 + \mathcal{G}_4)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5}, \end{aligned}$$

$$\mathcal{G}_9 = \frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2 + 9\mathcal{G}_2^2\mathcal{G}_{10}^4}, \mathcal{G}_5 = \mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},$$
$$\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}. \tag{14}$$

$$(III) \qquad \mathcal{G}_3 = -\frac{\mathcal{G}_5\mathcal{G}_7}{\mathcal{G}_1}, \mathcal{G}_6 = -\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\mathcal{G}_1\left(\mathcal{G}_2\left(\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4\right) - \alpha\mathcal{G}_5^2\right)}{\delta\mathcal{G}_2\mathcal{G}_5},$$
$$\mathcal{G}_8 = -\frac{\mathcal{G}_1\left(\gamma\mathcal{G}_1 + \mathcal{G}_4\right)}{\mathcal{G}_5} - \gamma\mathcal{G}_5, \mathcal{G}_9 = \frac{2\alpha^2\mathcal{G}_{10}^2k_1^2}{4\alpha^2\mathcal{G}_1^2 + 9\mathcal{G}_2^2\mathcal{G}_{10}^4}, \mathcal{G}_5 = \mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha},$$
$$\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{11} = \pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}. \tag{15}$$

All other parameters are free and unrestricted. By substituting the results of Eqs (13)–(15) into Eqs (5) and (12), the corresponding interaction solution of Eq (4) can be obtained. In order to understand the dynamic properties of the interaction solutions between the lump wave and two solitons, we take Eq (14) as an example and select special values of parameters (see Figure 2) to obtain a special solution of the equation as follows:

$$\Phi = [96\sinh(\frac{19t}{6} + x - \frac{2y}{3} - 3z) - 25(7t + 8x - 15z)]/[8[\frac{72}{25} + [3t + 2x + y - \frac{15z}{4}]^2 + \frac{(17t - 72x + 64y + 135z)^2}{2304} - 6\cosh(\frac{19t}{6} + x - \frac{2y}{3} - 3z)]]. \tag{16}$$

The dynamic properties of Eq (16) are described in Figure 2. In Figure 2(a), we can see that the lump wave interacts with one of the solitons. In Figure 2(b), the lump wave begins to move to the middle and interact with another soliton. At this time, the amplitude of the lump wave becomes larger. Until Figure 2(c), the lump wave shifts to another soliton and its amplitude decreases.

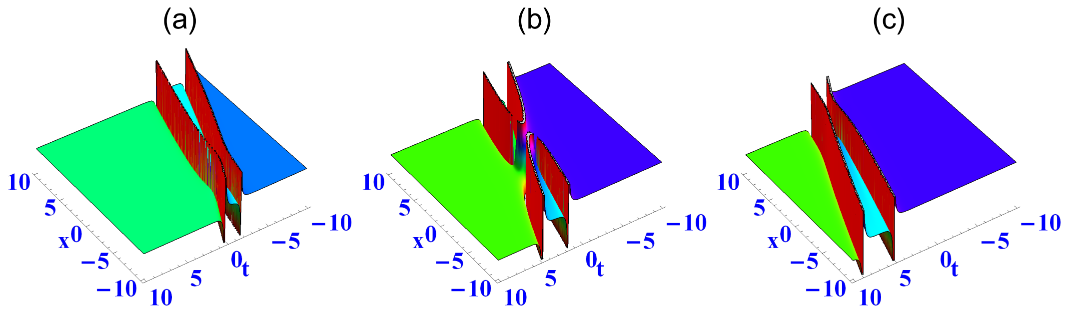

Figure 2. Solution (16). $\mathcal{G}_2 = \alpha = \beta = \delta = \gamma = 1$, $\mathcal{G}_1 = 2$, $\mathcal{G}_4 = \mathcal{G}_{12} = 3$, $k_1 = -6$, $y = 0$, $\mathcal{G}_{10} = -1$, (a) $z = -4$, (b) $z = 0$, (c) $z = 4$.

4. Lump-periodic solution

In this section, we intend to investigate the interaction between the lump and periodic waves, so we suppose

$$\begin{aligned}\xi &= (\mathcal{G}_4 t + \mathcal{G}_1 x + \mathcal{G}_2 y + \mathcal{G}_3 z)^2 + (\mathcal{G}_8 t + \mathcal{G}_5 x + \mathcal{G}_6 y + \mathcal{G}_7 z)^2 \\ &+ \mathcal{G}_9 + k_1 \cos(\mathcal{G}_{13} t + \mathcal{G}_{10} x + \mathcal{G}_{11} y + \mathcal{G}_{12} z). \end{aligned} \tag{17}$$

Substituting Eq (17) into (6), we give

$$(I) \quad \begin{aligned} &\mathcal{G}_3 = \frac{\mathcal{G}_1\mathcal{G}_7}{\mathcal{G}_5}, \mathcal{G}_6 = -\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\alpha\mathcal{G}_5^3 - \mathcal{G}_2\mathcal{G}_5(\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4)}{\delta\mathcal{G}_1\mathcal{G}_2}, \\ &\mathcal{G}_{11} = \pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}, \mathcal{G}_8 = \frac{\mathcal{G}_1^2\left(\beta\mathcal{G}_2^2 - \alpha\mathcal{G}_5^2\right) - \alpha\mathcal{G}_5^4 + \mathcal{G}_2\mathcal{G}_5^2(\beta\mathcal{G}_2 + \mathcal{G}_4)}{\mathcal{G}_1\mathcal{G}_2\mathcal{G}_5}, \\ &\mathcal{G}_9 = -\frac{2\alpha^2\mathcal{G}_{10}^2 k_1^2}{4\alpha^2\mathcal{G}_1^2 + 9\mathcal{G}_2^2\mathcal{G}_{10}^4}, \mathcal{G}_5 = \pm\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha}, \\ &\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}. \end{aligned} \tag{18}$$

$$(II) \quad \begin{aligned} &\mathcal{G}_3 = -\frac{\mathcal{G}_5\mathcal{G}_7}{\mathcal{G}_1}, \mathcal{G}_6 = -\frac{\mathcal{G}_1\mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\mathcal{G}_1\left(\mathcal{G}_2(\beta\mathcal{G}_2 + \gamma\mathcal{G}_1 + \mathcal{G}_4) - \alpha\mathcal{G}_5^2\right)}{\delta\mathcal{G}_2\mathcal{G}_5}, \\ &\mathcal{G}_8 = -\frac{\mathcal{G}_1(\gamma\mathcal{G}_1 + \mathcal{G}_4)}{\mathcal{G}_5} - \gamma\mathcal{G}_5, \mathcal{G}_9 = -\frac{2\alpha^2\mathcal{G}_{10}^2 k_1^2}{4\alpha^2\mathcal{G}_1^2 + 9\mathcal{G}_2^2\mathcal{G}_{10}^4}, \mathcal{G}_5 = \mp\frac{3\mathcal{G}_2\mathcal{G}_{10}^2}{2\alpha}, \\ &\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{11} = \pm\frac{\mathcal{G}_2\mathcal{G}_{10}}{\mathcal{G}_5}. \end{aligned} \tag{19}$$

All other parameters are free and unrestricted. By substituting the results of Eqs (18) and (19) into Eqs (17) and (5), we can get the corresponding interaction solution between the lump and periodic waves of Eq (4). In order to understand the dynamic properties of the interaction solutions between lump and periodic waves, we take Eq (18) as an example and select special values of parameters (see Figure 3) to obtain a special solution of the equation as follows:

$$\begin{aligned}&\Phi = [16\sin(\frac{31t}{6} - x - \frac{2y}{3} - 3z) - 25(7t + 8x - 15z)]/[8[(3t + 2x + y - \\ &\frac{15z}{4})^2 + \frac{(17t - 72x + 64y + 135z)^2}{2304} - \cos(\frac{31t}{6} - x - \frac{2y}{3} - 3z) - \frac{2}{25}]]. \end{aligned} \tag{20}$$

The dynamic properties of Eq (20) are described in Figure 3. Different from the previous two sections, the lump wave and periodic wave have been entangled and propagated forward.

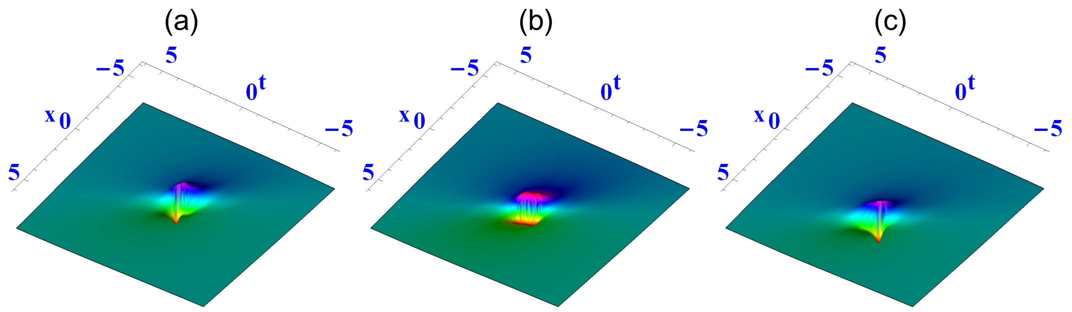

Figure 3. Solution (20). $\mathcal{G}_2 = \alpha = \beta = \delta = \gamma = \mathcal{G}_{10} = 1$, $\mathcal{G}_1 = -3$, $k_1 = -1$, $\mathcal{G}_4 = \mathcal{G}_{12} = 3$, $y = 0$, (a) $z = -2$, (b) $z = 0$, (c) $z = 2$.

5. Lump-periodic-1-soliton solutions

In order to investigate the interaction among the lump, periodic and one solitary wave, we suppose

$$\begin{aligned} \xi &= (\mathcal{G}_4 t + \mathcal{G}_1 x + \mathcal{G}_2 y + \mathcal{G}_3 z)^2 + (\mathcal{G}_8 t + \mathcal{G}_5 x + \mathcal{G}_6 y + \mathcal{G}_7 z)^2 + \mathcal{G}_9 \\ &+ k_2 e^{\mathcal{G}_{17} t + \mathcal{G}_{14} x + \mathcal{G}_{15} y + \mathcal{G}_{16} z} + k_1 \cos(\mathcal{G}_{13} t + \mathcal{G}_{10} x + \mathcal{G}_{11} y + \mathcal{G}_{12} z), \end{aligned} \tag{21}$$

where $\mathcal{G}_i (i = 14, 15, 16, 17)$ and k_2 are undetermined constants. Substituting Eq (21) into (6), we obtain

$$(I) \quad \begin{aligned} &\mathcal{G}_3 = \frac{\mathcal{G}_1 \mathcal{G}_7}{\mathcal{G}_5}, \mathcal{G}_6 = -\frac{\mathcal{G}_1 \mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\alpha \mathcal{G}_5^3 - \mathcal{G}_2 \mathcal{G}_5 (\beta \mathcal{G}_2 + \gamma \mathcal{G}_1 + \mathcal{G}_4)}{\delta \mathcal{G}_1 \mathcal{G}_2}, \\ &\mathcal{G}_{11} = \frac{\mathcal{G}_2 \mathcal{G}_{10}}{\mathcal{G}_5}, \mathcal{G}_8 = \frac{\mathcal{G}_1^2 \left(\beta \mathcal{G}_2^2 - \alpha \mathcal{G}_5^2\right) - \alpha \mathcal{G}_5^4 + \mathcal{G}_2 \mathcal{G}_5^2 (\beta \mathcal{G}_2 + \mathcal{G}_4)}{\mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_5}, \\ &\mathcal{G}_9 = -\frac{2\alpha^2 \mathcal{G}_{10}^2 k_1^2}{4\alpha^2 \mathcal{G}_1^2 + 9 \mathcal{G}_2^2 \mathcal{G}_{10}^4}, \mathcal{G}_5 = \frac{3 \mathcal{G}_2 \mathcal{G}_{10}^2}{2\alpha}, \\ &\mathcal{G}_{13} = -\frac{\alpha \mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta \mathcal{G}_{11} - \mathcal{G}_{10} \left(\gamma + \mathcal{G}_{10}^2\right) - \delta \mathcal{G}_{12}, \mathcal{G}_{15} = -\frac{2\alpha \mathcal{G}_{14}}{3 \mathcal{G}_{10}^2}, \\ &\mathcal{G}_{17} = -\frac{\alpha \mathcal{G}_{14}^2}{\mathcal{G}_{15}} - \beta \mathcal{G}_{15} - \mathcal{G}_{14} \left(\gamma + \mathcal{G}_{14}^2\right) - \delta \mathcal{G}_{16}, \mathcal{G}_{14} = \pm \mathcal{G}_{10}. \end{aligned} \tag{22}$$

$$(II) \quad \begin{aligned} &\mathcal{G}_3 = \frac{\mathcal{G}_1 \mathcal{G}_7}{\mathcal{G}_5}, \mathcal{G}_6 = -\frac{\mathcal{G}_1 \mathcal{G}_2}{\mathcal{G}_5}, \mathcal{G}_7 = \frac{\alpha \mathcal{G}_5^3 - \mathcal{G}_2 \mathcal{G}_5 (\beta \mathcal{G}_2 + \gamma \mathcal{G}_1 + \mathcal{G}_4)}{\delta \mathcal{G}_1 \mathcal{G}_2}, \\ &\mathcal{G}_{11} = -\frac{\mathcal{G}_2 \mathcal{G}_{10}}{\mathcal{G}_5}, \mathcal{G}_8 = \frac{\mathcal{G}_1^2 \left(\beta \mathcal{G}_2^2 - \alpha \mathcal{G}_5^2\right) - \alpha \mathcal{G}_5^4 + \mathcal{G}_2 \mathcal{G}_5^2 (\beta \mathcal{G}_2 + \mathcal{G}_4)}{\mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_5}, \\ &\mathcal{G}_9 = -\frac{2\alpha^2 \mathcal{G}_{10}^2 k_1^2}{4\alpha^2 \mathcal{G}_1^2 + 9 \mathcal{G}_2^2 \mathcal{G}_{10}^4}, \mathcal{G}_5 = -\frac{3 \mathcal{G}_2 \mathcal{G}_{10}^2}{2\alpha}, \end{aligned}$$

$$\mathcal{G}_{13} = -\frac{\alpha\mathcal{G}_{10}^2}{\mathcal{G}_{11}} - \beta\mathcal{G}_{11} - \mathcal{G}_{10}\left(\gamma + \mathcal{G}_{10}^2\right) - \delta\mathcal{G}_{12}, \mathcal{G}_{15} = -\frac{2\alpha\mathcal{G}_{14}}{3\mathcal{G}_{10}^2},$$
$$\mathcal{G}_{17} = -\frac{\alpha\mathcal{G}_{14}^2}{\mathcal{G}_{15}} - \beta\mathcal{G}_{15} - \mathcal{G}_{14}\left(\gamma + \mathcal{G}_{14}^2\right) - \delta\mathcal{G}_{16}, \mathcal{G}_{14} = \pm\mathcal{G}_{10}. \tag{23}$$

All other parameters are free and unrestricted. By substituting the results of Eqs (22) and (23) into Eqs (21) and (5), we can derive the corresponding interaction solution among the lump, periodic and one solitary wave. In order to understand the dynamic properties of the interaction solutions among lump, periodic and one solitary wave, we take Eq (22) as an example and select special values of parameters (see Figure 4) to obtain a special solution of the equation as follows:

$$\Phi = -[2[3\left(\frac{17t}{48} + \frac{3x}{2} + \frac{4y}{3} - \frac{3z}{16}\right) + 4e^{\frac{13t}{6}+x-\frac{2y}{3}-2z} - 4\left(3t - 2x + y + \frac{z}{4}\right)$$
$$- \sin\left(\frac{31t}{6} - x - \frac{2y}{3} - 3z\right)]]/[\left(\frac{17t}{48} + \frac{3x}{2} + \frac{4y}{3} - \frac{3z}{16}\right)^2 + 4e^{\frac{13t}{6}+x-\frac{2y}{3}-2z}$$
$$+\left(3t - 2x + y + \frac{z}{4}\right)^2 - \cos\left(\frac{31t}{6} - x - \frac{2y}{3} - 3z\right) - \frac{2}{25}]. \tag{24}$$

The dynamic properties of Eq (24) are described in Figure 4.

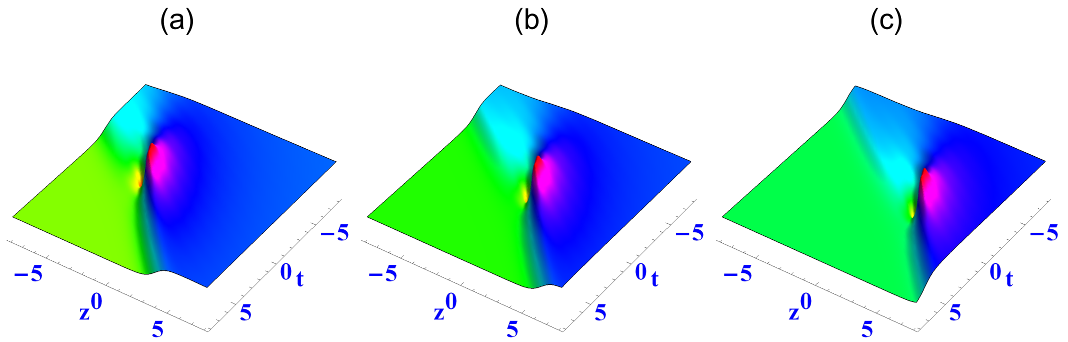

Figure 4. Solution (24). $\mathcal{G}_2 = \alpha = \beta = \delta = \gamma = \mathcal{G}_{10} = 1$, $\mathcal{G}_1 = \mathcal{G}_{16} = -2$, $k_1 = -1$, $y = 0$, $\mathcal{G}_4 = \mathcal{G}_{12} = 3$, $k_2 = 4$, (a) $x = -5$, (b) $x = 0$, (c) $x = 5$.

6. Conclusions

In this article, we investigated a new extended shallow water wave equation in (3+1) dimensions based on the Hirota bilinear form and symbolic computation [32–44]. The interaction between the lump wave and single solitary wave is studied. The interaction between the lump wave and two solitary waves as well as the interaction between lump wave and periodic wave was also discussed. Finally, we obtained the interaction solutions among the lump, periodic and one solitary wave and discribed the dynamic properties of the obtained results in Figures 1–4. Since its discovery, the Hirota bilinear method has been widely used in solving lump wave solutions of nonlinear integrable systems, and this method is simple, direct, and effective. In addition to the lump wave solution, this method can also be used to obtain hybrid rogue wave and breather solutions and has been promoted by many famous scholars in China [45–48].

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

References

1. W. X. Ma, Lump solutions to the Kadomtsev-Petviashvili equation, *Phys. Lett. A*, **379** (2015), 1975–1978. https://doi.org/10.1016/j.physleta.2015.06.061
2. W. X. Ma, Y. Zhou, Lump solutions to nonlinear partial differential equations via Hirota bilinear forms, *J. Differ. Equations*, **264** (2018), 2633–2659. https://doi.org/10.1016/j.jde.2017.10.033
3. Y. Sun, B. Tian, L. Liu, H. P. Chai, Y. Q. Yuan, Rogue waves and lump solitons of the (3+1)-dimensional generalized B-type Kadomtsev–Petviashvili equation for water waves, *Commun. Theor. Phys.*, **68** (2017), 693–700. https://doi.org/10.1142/S0217984917501226
4. M. Wang, B, Tian, S. H. Liu, W. R. Shan, Y. Jiang, Soliton, multiple-lump and hybrid solutions of a (2+1)-dimensional generalized Hirota-Satsuma-Ito equation for the water waves, *Eur. Phys. J. Plus*, **136** (2021), 635. https://doi.org/10.1140/epjp/s13360-021-01588-5
5. H. Yin, B. Tian, X. Zhao, C. R. Zhang, C. C. Hu, Breather-like solitons, rogue waves, quasi-periodic/chaotic states for the surface elevation of water waves, *Nonlinear Dyn.*, **97** (2019), 21–31. https://doi.org/10.1007/s11071-019-04904-y
6. X. Lü, W. X. Ma, S. T. Chen, M. K. Chaudry, A note on rational solutions to a Hirota-Satsuma-like equation, *Appl. Math. Lett.*, **58** (2016), 13–18. https://doi.org/10.1016/j.aml.2015.12.019
7. S. J. Chen, X. Lü, M. G. Li, F. Wang, Derivation and simulation of the M-lump solutions to two (2+1)-dimensional nonlinear equations. *Phys. Scr.*, **96** (2021), 095201. https://doi.org/10.1088/1402-4896/abf307
8. X. Lü, S. J. Chen, Interaction solutions to nonlinear partial differential equations via Hirota bilinear forms: one-lump-multi-stripe and one-lump-multi-soliton types, *Nonlinear Dyn.*, **103** (2021), 947–977. https://doi.org/10.1007/s11071-020-06068-6

9. J. Rao, K. W. Chow, D. Mihalache, J. He, Completely resonant collision of lumps and line solitons in the Kadomtsev-Petviashvili I equation, *Stud. Appl. Math.*, **147** (2021), 1007–1035. https://doi.org/10.1111/sapm.12417

10. Y. Jiang, J. Rao, D. Mihalache, J. He, Y. Cheng, Rogue breathers and rogue lumps on a background of dark line solitons for the Maccari system, *Commun. Nonlin. Sci. Numer. Simul.*, **102** (2021), 105943. https://doi.org/10.1016/j.cnsns.2021.105943

11. Y. L. Cao, Y. Cheng, J. S. He, Y. R. Chen, High-order breather, M-kink lump and semi-rational solutions of potential Kadomtsev-Petviashvili equation, *Commun. Theor. Phys.*, **73** (2021), 035004. https://doi.org/10.1088/1572-9494/abdaa6

12. H. T. Wang, X. Y. Wen, Modulational instability and mixed breather-lump interaction solutions in the (2+1)-dimensional KMN equation, *Mod. Phys. Lett. B*, **34** (2020), 2050092. https://doi.org/10.1142/S021798492050092X

13. Y. Q. Liu, X. Y. Wen, Soliton, breather, lump and their interaction solutions of the (2 +1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation, *Adv. Differ. Equations*, **1** (2019), 332. https://doi.org/10.1186/s13662-019-2271-5

14. Y. Q. Liu, X. Y. Wen, D. S. Wang, Novel interaction phenomena of localized waves in the generalized (3+1)-dimensional KP equation, *Comput. Math. Appl.*, **78** (2019), 1–19. https://doi.org/10.1016/j.camwa.2019.03.005

15. J. J. Su, S. Zhang, Nth-order rogue waves for the AB system via the determinants, *Appl. Math. Lett.*, **112** (2021), 06714. https://doi.org/10.1016/j.aml.2020.106714

16. J. J. Su, G. F. Deng, Quasi-periodic waves and irregular solitary waves of the AB system, *Waves Random Complex Medium*, **32** (2022), 856–866. https://doi.org/10.1080/17455030.2020.1804091

17. J. J. Su, Y. T. Gao, G. F. Deng, T. T. Jia, Solitary waves, breathers, and rogue waves modulated by long waves for a model of a baroclinic shear flow, *Phys. Rev. E*, **100** (2019), 042210. https://doi.org/10.1103/PhysRevE.100.042210

18. R. F. Zhang, M. C. Li, M. Albishari, F. C. Zheng, Z. Z. Lan, Generalized lump solutions, classical lump solutions and rogue waves of the (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada-like equation, *Appl. Math. Comput.*, **403** (2021), 126201. https://doi.org/10.1016/j.amc.2021.126201

19. Z. Z. Lan, J. J. Su, Solitary and rogue waves with controllable backgrounds for the non-autonomous generalized AB system, *Nonlinear Dyn.*, **96** (2019), 2535–2546. https://doi.org/10.1007/s11071-019-04939-1

20. Z. Z. Lan, Periodic, breather and rogue wave solutions for a generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation in fluid dynamics, *Appl. Math. Lett.*, **94** (2019), 126–132. https://doi.org/10.1016/j.aml.2018.12.005

21. Z. L. Zhao, Y. Chen, B. Han, Lump soliton, mixed lump stripe and periodic lump solutions of a (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation, *Mod. Phys. Lett. B*, **31** (2017), 1750157. https://doi.org/10.1142/S0217984917501573

22. Y. F. Yue, Y. Chen, Dynamics of localized waves in a (3+1)-dimensional nonlinear evolution equation, *Mod. Phys. Lett. B*, **33** (2019), 1950101. https://doi.org/10.1142/S021798491950101X

23. X. L. Tang, Y. Chen, Lumps, breathers, rogue waves and interaction solutions to a (3+1)-dimensional Kudryashov-Sinelshchikov equation, *Mod. Phys. Lett. B*, **34** (2020), 2050117. https://doi.org/10.1142/S0217984920501171

24. B. Q. Li, Y. L. Ma, Hybrid soliton and breather waves, solution molecules and breather molecules of a (3+1)-dimensional Geng equation, *Phys. Lett. A*, **463** (2023), 128672. https://doi.org/10.1016/j.physleta.2023.128672

25. B. Q. Li, Y. L. Ma, Breather, soliton molecules, soliton fusions and fissions, and lump wave of the Caudrey-Dodd-Gibbon equation, *Phys. Scr.*, **98** (2023), 095214. https://doi.org/10.1088/1402-4896/aceb25

26. B. Q. Li, Loop-like kink breather and its transition phenomena for the Vakhnenko equation arising from high-frequency wave propagation in electromagnetic physics, *Appl. Math. Lett.*, **112** (2021), 106822. https://doi.org/10.1016/j.aml.2020.106822

27. Y. L. Ma, B. Q. Li, Interaction behaviors between solitons, breathers and their hybrid forms for a short pulse equation, *Qual. Theory Dyn. Syst.*, **22** (2023), 146. https://doi.org/10.1007/s12346-023-00844-6

28. Y. L. Ma, A. M. Wazwaz, B. Q. Li, Soliton resonances, soliton molecules, soliton oscillations and heterotypic solitons for the nonlinear Maccari system, *Nonlinear Dyn.*, **111** (2023), 18331–18344. https://doi.org/10.1007/s11071-023-08798-9

29. C. R. Gilson, J. J. C. Nimmo, R. Willox, A (2+1)-dimensional generalization of the AKNS shallow water wave equation, *Phys. Lett. A*, **180** (1993), 337–345. https://doi.org/10.1016/0375-9601(93)91187-A

30. A. C. Petter, L. M. Elizabeth, Symmetry reductions and exact solutions of shallow water wave equations, *Acta Appl. Math.*, **39** (1995), 245–276. https://doi.org/10.1007/BF00994636

31. A. M. Wazwaz, New integrable (2+1)-and (3+1)-dimensional shallow water wave equations: multiple soliton solutions and lump solutions, *Int. J. Numer. Methods Heat Fluid Flow*, **32** (2022), 138–149. https://doi.org/10.1108/HFF-01-2021-0019

32. G. Q. Xu, Y. P. Liu, W. Y. Cui, Painlevé analysis, integrability property and multiwave interaction solutions for a new (4+1)-dimensional KdV-Calogero-Bogoyavlenkskii-Schiff equation, *Appl. Math. Lett.*, **132** (2022), 108184. https://doi.org/10.1016/j.aml.2022.108184

33. A. M. Wazwaz, Two new Painlevé integrable KdV-Calogero-Bogoyavlenskii-Schiff (KdV-CBS) equation and new negative-order KdV-CBS equation, *Nonlinear Dyn.*, **104** (2021), 4311–4315. https://doi.org/10.1007/s11071-021-06537-6

34. K. J. Wang, J. H. Liu, Diverse optical solitons to the nonlinear Schrödinger equation via two novel techniques, *Eur. Phys. J. Plus*, **138** (2023), 74. https://doi.org/10.1140/epjp/s13360-023-03710-1

35. Y. L. Ma, A. M. Wazwaz, B. Q. Li, A new (3+1)-dimensional Kadomtsev-Petviashvili equation and its integrability, multiple-solitons, breathers and lump waves, *Math. Comput. Simulat.*, **187** (2021), 505–519. https://doi.org/10.1016/j.matcom.2021.03.012

36. F. Baronio, S. Wabnitz, Y. Kodama, Optical Kerr spatiotemporal dark-lump dynamics of hydrodynamic origin, *Phys. Rev. Lett.*, **116** (2016), 173901. https://doi.org/10.1103/PhysRevLett.116.173901

37. Y. L. Ma, B. Q. Li, Bifurcation solitons and breathers for the nonlocal Boussinesq equations, *Appl. Math. Lett.*, **124** (2023), 107677. https://doi.org/10.1016/j.aml.2021.107677
38. R. F. Zhang, M. C. Li, H. M. Yin, Rogue wave solutions and the bright and dark solitons of the (3+1)-dimensional Jimbo-Miwa equation, *Nonlinear Dyn.*, **103** (2021), 1071–1079. https://doi.org/10.1007/s11071-020-06112-5
39. Y. L. Ma, B. Q. Li, Mixed lump and soliton solutions for a generalized (3+1)-dimensional Kadomtsev-Petviashvili equation, *AIMS Math.*, **5** (2020), 1162–1176. https://doi.org/10.3934/math.2020080
40. Y. Y. Gu, L. W. Liao, Closed form solutions of Gerdjikov-Ivanov equation in nonlinear fiber optics involving the beta derivatives, *Int. J. Mod. Phys. B*, **36** (2022), 2250116. https://doi.org/10.1142/S0217979222501168
41. B. Q. Li, Y. L. Ma, Multiple-lump waves for a (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation arising from incompressible fluid, *Comput. Math. Appl.*, **76** (2023), 204–214. https://doi.org/10.1016/j.camwa.2018.04.015
42. Y. Y. Gu, C. F. Wu, X. Yao, W. J. Yuan, Characterizations of all real solutions for the KdV equation and W_R, *Appl. Math. Lett.*, **107** (2020), 106446. https://doi.org/10.1016/j.aml.2020.106446
43. Y. L. Ma, A. M. Wazwaz, B. Q. Li, New extended Kadomtsev-Petviashvili equation: multiple soliton solutions, breather, lump and interaction solutions, *Nonlinear Dyn.*, **104** (2021), 1581–1594. https://doi.org/10.1007/s11071-021-06357-8
44. Y. Y. Gu, W. J. Yuan, N. Aminakbari, J. M. Lin, Meromorphic solutions of some algebraic differential equations related Painleve equation IV and its applications, *Math. Methods Appl. Sci.* , **41** (2018), 3832–3840. https://doi.org/10.1002/mma.4869
45. B. Q. Li, Y. L. Ma, Extended generalized Darboux transformation to hybrid rogue wave and breather solutions for a nonlinear Schrödinger equation, *Appl. Math. Comput.*, **386** (2020), 125469. https://doi.org/10.1016/j.amc.2020.125469
46. B. Q. Li, Y. L. Ma, Interaction dynamics of hybrid solitons and breathers for extended generalization of Vakhnenko equation, *Nonlinear Dyn.*, **102** (2020), 1787–1799. https://doi.org/10.1007/s11071-020-06024-4
47. B. Q. Li, Hybrid breather and rogue wave solution for a (2+1)-dimensional ferromagnetic spin chain system with variable coefficients, *Int. J. Comput. Math.*, **99** (2022), 506–519. https://doi.org/10.1080/00207160.2021.1922678
48. Y. L. Ma, B. Q. Li, Hybrid rogue wave and breather solutions for a complex mKdV equation in few-cycle ultra-short pulse optics, *Eur. Phys. J. Plus*, **137** (2022), 861. https://doi.org/10.1140/epjp/s13360-022-03080-0

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