Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/MBE

MBE, 19(7): 7201–7231.
DOI: 10.3934/mbe.2022340
Received: 01 March 2022
Revised: 04 April 2022
Accepted: 21 April 2022
Published: 16 May 2022

Research article

An enhanced VIKOR method for multi-criteria group decision-making with complex Fermatean fuzzy sets

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Abstract: This paper aims to propose a new decision-making approach retaining the fascinating traits of the conventional VIKOR method in the context of the enrich multidimensional complex Fermatean fuzzy N-soft set. The VIKOR technique is contemplated as the most reliable decision-making approach among others which employs a strategy to identify the compromise solution with advantageous distance from the positive ideal solution possesses maximum majority utility and minimum individual regret. At the same time, the paramount characteristic of the complex Fermatean fuzzy N-soft set considers the proclivity to capture two-dimensional uncertain and imprecise information along with the multi-valued parameters. This article expands the literature to handle the multi-attribute group decision-making strategy by introducing a technique, namely, the complex Fermatean fuzzy N-soft VIKOR method that amalgamates the unconventional traits of complex Fermatean fuzzy N-soft with the capability of the VIKOR method. The proposed technique permits the assignment of the N-soft grades to the decision-makers, alternatives, and attributes based on their performances. Firstly, we unify these individual opinions of all decision-makers about the alternatives by employing the complex Fermatean fuzzy N-soft weighted average operator. After that, all entities of the aggregated decision matrix are converted into crisp data by utilizing the score function. Furthermore, we calculate the ranking measures of the group utility and the individual regret by assigning the weight of strategy belongs to the interval $[0, 1]$. To find the compromise solution, we arrange the ranking measures in ascending order, and the alternative that possesses the conditions of compromise solution is selected. We demonstrate the presented multi-attribute group decision-making technique by selecting the best location for a nuclear power plant. We conduct the comparative analysis of the presented technique with Fermatean fuzzy TOPSIS to endorse the veracity and accuracy of our method. Finally, we explain the merits and limitations of our strategy and give some concluding remarks.

Keywords: Fermatean fuzzy N-soft sets; VIKOR method; decision matrix; ranking measure

1. Introduction

Decision-making has become a crucial aspect of our real life especially in business, household, job, education, marketing, medical, engineering, social sciences, economics, and so forth. Therefore, it is essential to take the best possible decisions for a comfortable and balancing lifestyle. Decision-making is the process of finding the solution to the challenges after investigating the multiple choices based on conflicting criteria. Owing to the growing concern of human beings to choose the best alternative, the mathematicians have broad their spadework by establishing various multi-attribute decision-making (MADM) techniques such as VIekriterijumsko KOmpromisno Rangiranje (VIKOR) [1], Technique for the Order of Preference by Similarity to an Ideal Solution (TOPSIS) [2], analytic hierarchy process (AHP) [3], and Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE) [4], et cetera. Yu [5], and Zeleny [6] developed the framework of compromise solution for the dilemma of MADM, which is helpful for decision-makers to reach a workable result of the problem. The underlying concept of a compromise solution is that it obsesses the maximum distance from the ideal solution. Over here, compromise means an agreement set up by mutual concession.

1.1. Related work

In this article, our main focus is to establish the VIKOR method in the environment of complex Fermatean fuzzy N-soft set (CFFNS$_f$S). VIKOR is a MADM technique for the complex structure, which was introduced by Opricovic [1] in 1998 intending to deal with crisp information, and later, it has been expanded to rationalize a broaden informational settings thenceforth. The basic objective of this technique is to determine the compromise solution of the MADM problem that has two crucial traits consists of the maximum group utility (or democracy) and the minimum individual regret of the opponents. The following L_p metric was deployed by Opricovic by means of aggregation function to identify the two paramount features of the feasible compromise solution:

$$L_{p,c} = \left\{ \sum_{d=1}^{k} \left[\beth_d \left(\frac{u_d^+ - u_{cd}}{u_d^+ - u_d^-} \right) \right]^p \right\}^{\frac{1}{p}}, \quad 1 \le p \le \infty, \quad d = 1, 2, 3, \dots, k.$$

In fact, the strength of the VIKOR technique is the L_p metric that effectively allows the approach to handle precise data in the process of decision-making. Particularly, $L_{1,c}$ and $L_{\infty,k}$ are used to formulate ranking measures as maximum group utility and minimum individual regret, respectively. Figure 1 is the pictorial representation of the compromise solution U^c and the ideal solution U^*. A comparative analysis of VIKOR methodology with different outranking approaches such as TOPSIS, ELECTRE, and PROMETHEE was presented by Opricovic, and Tzeng [7, 8]. Bazzazi et al. [9] introduced a revised adaption of the VIKOR technique that combined the entropy weights with the AHP method. These methodologies, designed for exact information, were inept to capture the ambiguity of the real-life decision-making problems.

In the traditional MADM techniques, the crisp information is used to assess the alternatives and attributes, even though in everyday unpredictable and dubious circumstances, it is a very unrealistic premise that a decision-maker possesses exact and rigorous illustration about the judgmental preferences situations. Because human nature, inclusive of priorities, is very imprecise and unclear. Hence, it is not appropriate for a decision-maker to evaluate the tendency of his problem with precise information. Consequently, to manipulate the uncertainty in the nature of mankind, fuzzy set (FS) theory is

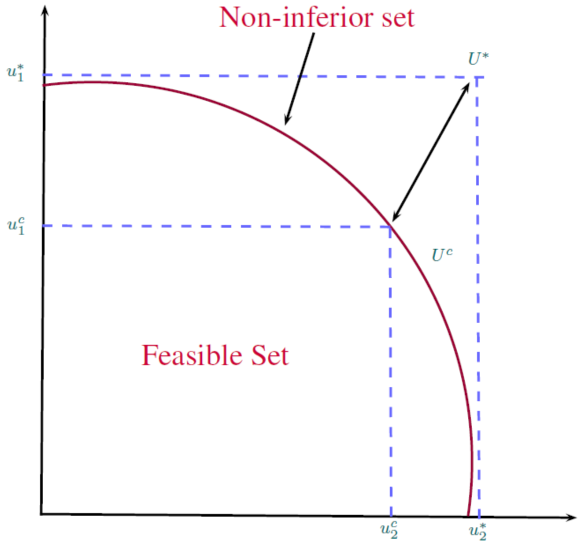

Figure 1. Ideal and compromise solution.

widely used, which was introduced by Zadeh [10] in 1965. While functioning the decision-making in the realm of FS, fuzzy membership values are taken from the interval $[0, 1]$. Bellmen and Zadeh [11] originated apply the fuzzy numbers (FNs) for the MADM process. Later, many authors have applied the FS theory to practical examples. Wang and Chang [12] presented the idea of the fuzzy VIKOR (F-VIKOR) approach to capture multi-attribute group decision-making (MAGDM). F-VIKOR approach was utilized by Chang [13] to categorize the hospitals of Taiwan according to their facilities and services. Some other applications were proposed by Mishra et al. [14], Sanayei et al. [15], and Shemshadi et al. [16] to handle the MAGDM problems related to the selection of suppliers by employing the F-VIKOR method. Opricovic [17] altered the traditional F-VIKOR approach for water resource management at the Mlava river. Ju et al. [18] presented an extension of the VIKOR method for trapezoidal FNs. Another application of the F-VIKOR method in the groundwork of triangular FNs was underlined by Rostamzadeh et al. [19] related to green supply chain management. Wang et al. [20] introduced the F-VIKOR technique with linguistic variables in the form of triangular FNs, to find out the optimal software company. An extended interpretation of F-VIKOR was initiated by Büyüközkan [21] for the ranking of web-based learning methodologies. In recent years, Taylan et al. [22] applied different decision-making methodologies, such as fuzzy AHP, F-VIKOR, and fuzzy TOPSIS to assess the power systems of Saudi Arabia for investment. All these variants of VIKOR method were able to process the data only in favor of an object according to the capacity of FS. So, these technique fails to perform in the presence of dissatisfaction degree.

Since the FS was incompetent to capture the dissatisfaction of human nature, ergo, Atanassov [23] put forward the notion of intuitionistic FS that is very fruitful to handle imprecise information via satisfaction $\mathfrak{m}$ and $\mathfrak{o}$ degrees with the limitation $\mathfrak{m} + \mathfrak{o} \leq 1$. Many authors such as Gupta et al. [24], Hu et al. [25], Mousavi et al. [26], and Wan et al. [27] have provided the approach of VIKOR for MAGDM problems relative to the different areas in the context of intuitionistic FS. The aptitude of these methodologies was utilizable until the sum of membership and non-membership grades does not exceed 1. In 2013, Yager [28, 29] established the concept of Pythagorean FS to overcome the flaws of intuitionistic FS with the easygoing condition of $\mathfrak{m}^2 + \mathfrak{o}^2 \leq 1$. Cui et al. [30] and Gul et al. [31] developed the decision-making method on account for Pythagorean FS based on the VIKOR method to select the best place for the electronic automobile station and to evaluate the safety measure in the industry of mine. Although Pythagorean FS was potent in handling the vague data, it fails in the circumstances when the square sum of satisfaction and dissatisfaction degrees exceed from 1. Consequently, Yager [32] set up the groundwork of q-rung orthopair FS with the endorsement satisfy the general constraint $\mathfrak{m}^q + \mathfrak{o}^q \leq 1$. The strict and limited space of intuitionistic and Pythagorean FSs was a major incentive to look for a broader model that can proficiently operate outside the space of these existing structures. Therefore, Senapati and Yager [33] proposed the marvelous model of Fermatean FS which allow the membership and nonmembership degrees subjected to more powerful condition than intuitionistic FS and Pythagorean FS that is $\mathfrak{m}^3 + \mathfrak{o}^3 \leq 1$. Ghorabaee et al. [34] utilized the Fermatean FSs and WASPAS for the evaluation process of green suppliers. Liu et al. [35] employed the Fermatean FS with a linguistic set on MADM problems. Garg et al. [36] utilized the Yager aggregation operators on the Fermatean fuzzy along with the application related to the COVID-19 testing facility. Several valuable decision-making strategies have been examined in [37–43]. Presently, the VIKOR technique with trapezoidal bipolar fuzzy numbers has been addressed by Shumaiza et al. [44]. Figure 2 is the graphical representation of the spaces of three fuzzy systems inclusive of intuitionistic FS, Pythagorean FS, and Fermatean FS. Although traditional models of FS theory are capable enough to

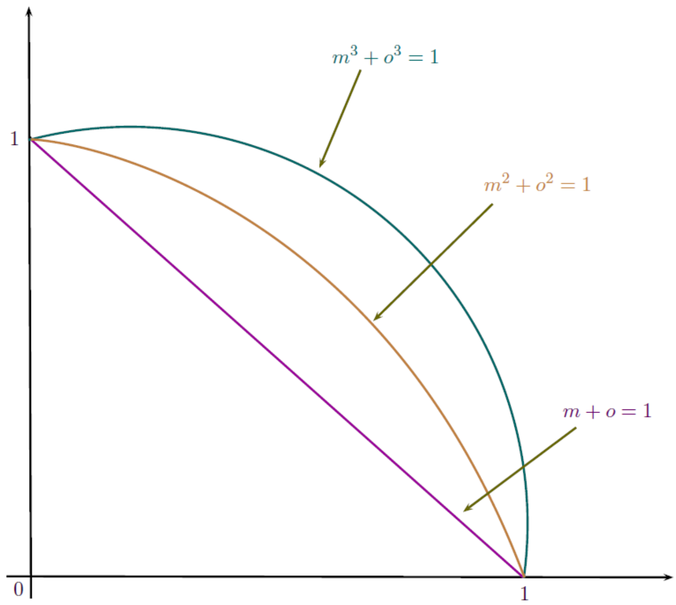

Figure 2. Spaces of fuzzy systems.

interpret the one-dimensional vague and imprecise data, they are inept to handle the two-dimensional periodic information. To overcome this shortcoming, Ramot et al. [45] proposed the notion of complex FS, in which the phase term of membership is involved along with amplitude term and defined as $\mathfrak{m} = me^{i\varphi}$ where m is amplitude term and φ is the phase term. The range of m and φ is $[0, 1]$ and $[0, 2\pi]$, respectively. Precisely, in complex FS, the membership grades lie on the unit disk. The idea of complex fuzzy class was introduced by Tamir et al. [46]. Since the complex fuzzy system cannot handle the dissatisfaction part, hence, to get over the deficiencies, Alkouri and Salleh [47] put forward the extended notion of complex intuitionistic FS in which the membership grades ($\mathfrak{m} = me^{i\varphi}$) and non-membership grades ($\mathfrak{o} = oe^{i\eta}$) both have the range unit circle as well as $m + n \leq 1$ and $\frac{\varphi}{2\pi} + \frac{\eta}{2\pi} \leq 1$. The space of complex intuitionistic FS was limited due to strict conditions on amplitude and phase terms. Therefore, Ullah et al. [48] generalized the model by presenting complex Pythagorean FS with the same range but flexible conditions that $m^2 + n^2 \leq 1$ and $(\frac{\varphi}{2\pi})^2 + (\frac{\eta}{2\pi})^2 \leq 1$. Ma et al. [49] has worked on the VIKOR technique in the environment of complex Pythagorean fuzzy data to address the MAGDM problems. Recently, Akram et al. [50] has also applied the VIKOR method on complex spherical FS for group decision-making.

All the discussed models in the literature suchlike FSs [10], complex FSs [45], complex intuitionistic FSs [47], complex Pythagorean FSs [48], and rough sets [51] et cetera are capable enough to handle vagueness and uncertainty encapsulated in the system efficiently. Still, all these methodologies have their own framework, influence, characteristics, and inherent flaws. One main built-in restriction is that none of them can work with the parameterized framework. To address this issue, Molodtsov [52, 53] introduced a new theory, namely, soft set, which is efficacious to cope with parameterized structure. After noticing the lavish potential of the soft sets, the scholars have expanded the literature to solve real problems in several ways, including forecasting, mathematical analysis, information systems, optimization theory, and many more.

Overtime, the researchers have proposed many hybrid models with the amalgamate of soft set theory to expand the vision of soft set. Maji et al. [54], and Peng et al. [55] put forward the groundwork of two hybrid models, namely, fuzzy soft set (FS_fS) and Pythagorean FS_fS, respectively. Shahzadi and Akram [56] established the hybrid model of Fermatean FS_fS with an application to choose the finest anti-virus mask. Salsabeela and John [57] exploited the Fermatean FS_fS on the TOPSIS method and presented the solution to MAGDM problems. Thirunavukarasu et al. [58] enhanced the publications by evoking the idea of complex FS_fS. The idea of complex intuitionistic FS_fS with some distance measures was proposed by Kumar and Bajaj [59].

Taking into account the above-explained theories, it is obvious that the hybrid models combined with soft sets allow only to do binary (0 or 1) evaluation or assessment by real numbers belong to $[0, 1]$. But in real phenomena, decision-making situations include multi-valued discrete data structure that is highly involved in ranking or rating-based systems, where the rating of alternatives (hotels, cities, movies, schools, and so on) can be delineated by using dots, whole numbers, starts, diamonds, etc. [60, 61].

Since the traditional soft set cannot tackle the multi-valued parameterized information. Thereby, there was a need to propose a new model that can handle these situations. Hence, Fatimah et al. [61] launched the notion of N-soft set (NS_fS) in conjunction with some decision-making algorithms that give prominence to ordered grades in realistic illustrations. Eventually, Akram et al. [62–64] initiated the hybrid models of hesitant NS_fS and fuzzy NS_fS by unifying the notion of NS_fS with hesitant set and fuzzy set, respectively, and with the fusion of both models proposed the hesitant fuzzy NS_fS. Akram et al. [65] presented another hybrid structure, that is, intuitionistic fuzzy NS_fS with the appropriate consolidation of NS_fS along with intuitionistic FSs. Zhang et al. [66] put forward the new hybrid model, namely, Pythagorean fuzzy NS_fSs, which incorporate the idea of NS_fS and Pythagorean FS. These models can not precisely present the parameterized data within two-dimensional context. This fact led Akram et al. [67, 68] to innovate the models of complex Pythagorean fuzzy NS_fSs and complex spherical fuzzy NS_fSs by merging the NS_fS alongside to complex Pythagorean FSs and complex spherical FSs, respectively. The strict inherent conditions of complex Pythagorean FSs on the structure of complex Pythagorean fuzzy NS_fSs restricted the frequent application of this elegant model. Akram et al. [69] is accredited to compile the broader structure of complex Fermatean FSs with the striking theory of NS_fSs in a single framework to develop a more practical hybrid model, namely, complex Fermatean fuzzy NS_fS and proposed the TOPSIS method for this prevalent model. Fatimah and Alcantud [70] have also introduced the new model, known as, multi-fuzzy NS_fS with decision-making applications. For exploration of useful group decision-making approaches readers are referred to [71–74].

1.2. Motivation along with contribution

The motivation of the proposed approach concentrates on the following points:

- The VIKOR technique, mainly, gives the compromise solution nearest to the ideal solution by adjusting the group utility and individual regret according to the preference of decision-maker. Further, it employs the L_p metric as an aggregation function to provide the ordered ranking of choices.

- In the practical world, many MADM problems involve the phenomena of periodicity. It is impossible to capture those problems with traditional models such as FSs, intuitionistic FSs, Pythagorean FSs, and Fermatean FSs. The proposed strategy has the edge to overcome all these shortcomings owing to the advantageous, broader and practical framework of complex Fermatean FSs that competently capture the periodic information. Beside these advantages, the complex Fermatean FSs were still inept to deal with parameterized information.

- The hybrid model of complex Pythagorean fuzzy NS_f has the ability to tackle the two-dimensional fuzzy information along with multi-valued parameters but it is limited to deal with data only when $m^2 + o^2 \leq 1$ and $(\frac{\varphi}{2\pi})^2 + (\frac{\eta}{2\pi})^2 \leq 1$.

- The complex Fermatean fuzzy NS_fS present a broader extension of complex intuitionistic fuzzy NS_fS and complex Pythagorean fuzzy NS_fS that integrate the dominant characteristics of complex Fermatean FSs with efficacy of N-soft sets. Hence, the complex Fermatean fuzzy NS_fS, competent to capture the parameterized two dimensional information, provides more trustworthy decision when fused with the accuracy of VIKOR technique to solve the practical MADM and MAGDM problems.

Get the lead out by these integral concerns; this research article put forward a new technique grounded on VIKOR methodology. In the proposed technique, the appointed jury and criteria initially are assessed through complex Fermatean fuzzy NS_f ordered grades to erect the normalized weights. After that, the individual opinions of decision-makers are taken, and then the complex Fermatean fuzzy N-soft weighted average operator is utilized to form the aggregated complex Fermatean fuzzy N-soft decision matrix. The entries of aggregated complex Fermatean fuzzy N-soft decision matrix are converted into crisp data with the help of the score function. Furthermore, after evaluating the best and worst values, ranking measures are determined. Decisively, the ascending order ranking is associated with the alternatives and the best opt is picked up by testing both conditions. The proposed method is justified with an application to select a suitable location for nuclear power plant. In the end, a comparative analysis is depicted with the Fermatean fuzzy TOPSIS approach to confirm the veracity of the proposed method.

In summation, the main contributions of this manuscript are as follows:

- This study mainly focuses to present the novel tremendous methodology, termed as, complex Fermatean fuzzy N-soft VIKOR methodology, in order to obtain the compromise solution nearest. The presented approach seeks help from the L_p metric for the aggregation purpose to provide the ordered ranking of choices.
- The presented approach is privileged to compile the parameterized and two-dimensional information simultaneously that empower it to run over the deficiencies of all other decision-making approaches designed for preceding and incompetent models.
- An example of a MAGDM problem regarding the selection of location for a nuclear power plant is demonstrated to emphasize the accuracy and veracity of the presented approach. Furthermore, a comparative study with Fermatean fuzzy TOPSIS method is provided as a solid proof to manifest the authenticity of the proposed methodology.
- Additionally, we narrate the merits, and advantages of our put forward MAGDM technique to explain its importance, necessity, superiority, and reliability in contract with existing MADM and MAGDM approaches.

1.3. Manuscript's structure

The arrangement and sections of this article are as follows: Section 2 put forward the preliminary concepts. Section 3 explains the step-by-step procedure and flowchart of the proposed complex Fermatean fuzzy NS_f VIKOR approach. Section 4 provides the numerical example related to the selection of the best location for nuclear power plant. Section 5 demonstrates the reliability and flexibility of the presented methodology by the indentation of comparative analysis. Further, Section 6 spotlights the limitations and merits of the proposed technique as compared to existing methods for decision-making. Section 7 illustrates the conclusion of the whole article and outlines some future directions of research.

2. Preliminaries

Definition 2.1 ([52])**.** Let W be a universal set and E be the set of all criteria. Let $P(W)$ be the collection of all subsets of W and $C \subseteq E$. A pair $\mathfrak{S} = (S, C)$ is called a *soft set* over W where S is a set-valued mapping defined as: $S : C \longrightarrow P(W)$. Mathematically, $\mathfrak{S}$ over W can be defined as follows:

$$\mathfrak{S} = \{(c_v, S(c_v)) \mid c_v \in C, S(c_v) \in P(W)\}.$$

Definition 2.2 ([61])**.** Let W be a non-empty set and E be the set of all criteria, $C \subseteq E$. Let $G = \{0, 1, \ldots, N-1\}$ be a set of ordered grades where $N \in \{2, 3, \ldots\}$. A triple $\mathfrak{R} = (S, C, N)$ is known as an *N-soft set* over W if $S : C \longrightarrow 2^{W \times G}$, with the condition that for each $c \in C$ and $w \in W$ there exists a unique $(w, g_c) \in W \times G$ such that $(w, g_c) \in S(c), w \in W, g \in G$. In set notation, the N-soft set $\mathfrak{R}$ over W can be defined as:

$$\mathfrak{R} = \{(c_v, S(c_v)) \mid c_v \in C, S(c_v) \in 2^{W \times G}\}.$$

Definition 2.3 ([33]). Let W be a non-empty universal set. A *Fermatean FS* $\mathfrak{Q}$ on U is characterized as an object of the form:

$$\mathfrak{Q} = \{(w, m_{\mathfrak{Q}}(w), o_{\mathfrak{Q}}(w)) \mid w \in W\},$$

where the $m_{\mathfrak{Q}}(w)$ and $o_{\mathfrak{Q}}(w) \in [0,1]$, and $\forall\ w \in W, 0 \leq (m_{\mathfrak{Q}}(w))^3 + (o_{\mathfrak{Q}}(w))^3 \leq 1$.
The value $\chi_{\mathfrak{Q}}(w) = \sqrt[3]{1 - (m_{\mathfrak{Q}}(w))^3 - (o_{\mathfrak{Q}}(w))^3}$ is known as indeterminacy degree of $w \in W$ to the Fermatean FS $\mathfrak{Q}$.
The pair of satisfaction and dissatisfaction degrees $(m_{\mathfrak{Q}}(w), o_{\mathfrak{Q}}(w))$ is called a Fermatean fuzzy number.

Definition 2.4 ([69]). A *complex Fermatean FS* $\mathfrak{CF}$ on the universe of discourse W is characterized as:

$$\mathfrak{CF} = \{\langle w, \mathfrak{m}_{\mathfrak{CF}}(w), \mathfrak{o}_{\mathfrak{CF}}(w)\rangle \mid w \in W\},$$

where $i = \sqrt{-1}$, $\mathfrak{m}_{\mathfrak{CF}}(w) : W \longrightarrow \{w \mid w \in \mathbb{C}, \mid w \mid \leq 1\}$, $\mathfrak{o}_{\mathfrak{CF}}(w) : W \longrightarrow \{w' \mid w' \in \mathbb{C}, \mid w \mid \leq 1\}$,

such that $\mathfrak{m}_{\mathfrak{CF}}(w) = m_{\mathfrak{CF}}(w)e^{i\varphi_{\mathfrak{CF}}(w)}$, $\mathfrak{o}_{\mathfrak{CF}}(w) = o_{\mathfrak{CF}}(w)e^{i\eta_{\mathfrak{CF}}(w)}$, and $m_{\mathfrak{CF}}(w)$ and $o_{\mathfrak{CF}}(w) \in [0,1]$,

$\varphi_{\mathfrak{CF}}(w), \eta_{\mathfrak{CF}}(w) \in [0, 2\pi]$.

Here, $m_{\mathfrak{CF}}(w), o_{\mathfrak{CF}}(w)$ are known as the amplitude terms and $\varphi_{\mathfrak{CF}}(w), \eta_{\mathfrak{CF}}(w)$ are known as the phase terms, with $0 \leq (m_{\mathfrak{CF}}(w))^3 + (o_{\mathfrak{CF}}(w))^3 \leq 1$, and $0 \leq (\frac{\varphi_{\mathfrak{CF}}(w)}{2\pi})^3 + (\frac{\eta_{\mathfrak{CF}}(w)}{2\pi})^3 \leq 1$.

The indeterminacy of the complex Fermatean fuzzy set $\mathfrak{CF}$ is calculated as $\pi_{\mathfrak{CF}}(w) = j_{\mathfrak{CF}}(w)e^{i\vartheta_{\mathfrak{CF}}(w)}$,

where $j_{\mathfrak{CF}}(w) = \sqrt[3]{1 - (m_{\mathfrak{CF}}(w))^3 - (o_{\mathfrak{CF}}(w))^3}$ and $\vartheta_{\mathfrak{CF}}(w) = \sqrt[3]{1 - (\frac{\varphi_{\mathfrak{CF}}(w)}{2\pi})^3 - (\frac{\eta_{\mathfrak{CF}}(w)}{2\pi})^3}$.

The pair $(m_{\mathfrak{CF}}(w)e^{i\varphi_{\mathfrak{CF}}(w)}, o_{\mathfrak{Q}}(w)e^{i\varphi_{\mathfrak{CF}}(w)})$ is called complex Fermatean fuzzy numbers.

Definition 2.5 ([69]). Let W be a universe of discourse and E be the set of all criteria, $C \subseteq E$. Let $\mathcal{P}(W)$ represents the collection of all complex Fermatean fuzzy subsets over U. A pair $\mathfrak{T} = (T, C)$ is called a *complex Fermatean* FS_fS over W, where T is a set-valued mapping: $T : C \longrightarrow \mathcal{P}(W)$, and defined as:

$$\mathfrak{T} = \{(c_v, T(c_v)) \mid c_v \in C, T(c_v) \in \mathcal{P}(W)\}.$$

Definition 2.6 ([69]). Let W be a universal set and E be the set of all criteria under consideration, $C \subseteq E$. Let $G = \{0, 1, 2, \ldots, N-1\}$ be the set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple $(\mathfrak{H}, \mathfrak{I}, N)$ is called a *complex Fermatean fuzzy N-soft set* (CFFNS_fS) over W, if $\mathfrak{I} = (T, C, N)$ is an N-soft set over W and $\mathfrak{H} : C \longrightarrow CFF^{(W\times G)}$, where $CFF^{(W\times G)}$ is the collection of all complex Fermatean FSs over $W \times G$. The CFFNS_fS $\mathfrak{D} = (\mathfrak{H}, \mathfrak{I}, N)$ over W can be represented as:

$$\mathfrak{D} = \{\langle c_v, \mathfrak{H}(c_v)\rangle \mid c_v \in C, \mathfrak{H}(c_v) \in CFF^{(W\times G)}\},$$

where

$$\mathfrak{H}(c_v) = \{\langle (w_n, g_{nv}), m_{nv}(w_n, g_{nv})e^{i\varphi_{nv}(w_n, g_{nv})}, o_{nv}(w_n, g_{nv})e^{i\eta_{nv}(w_n, g_{nv})}\rangle \mid c_v \in C, (w_n, g_{nv}) \in W \times G\}$$

is a representation of complex Fermatean FS over W. The amplitude terms $m_{nv}(w_n, g_{nv}), o_{nv}(w_n, g_{nv}) \in [0,1]$ and phase terms $\varphi_{nv}(w_n, g_{nv}), \eta_{nv}(w_n, g_{nv}) \in [0, 2\pi]$ satisfy the following conditions:

$$0 \leq (m_{nv}(w_n, g_{nv}))^3 + (o_{nv}(w_n, g_{nv}))^3 \leq 1,$$

$$0 \leq (\frac{\varphi_{nv}(w_n, g_{nv})}{2\pi})^3 + (\frac{\eta_{nv}(w_n, g_{nv})}{2\pi})^3 \leq 1.$$

The degree of indeterminacy for all $(w_n, g_{nv}) \in W \times G$ can be calculated as follows:

$$\pi_{nv}(w_n, g_{nv}) = j_{nv}(w_n, g_{nv})e^{i\vartheta_{nv}(w_n,g_{nv})},$$

where $j_{nv}(w_n, g_{nv}) = \sqrt[3]{1 - (m_{nv}(w_n, g_{nv}))^3 - (o_{nv}(w_n, g_{nv}))^3}$ and $\vartheta_{nv}(w_n, g_{nv}) = \sqrt[3]{1 - (\frac{\varphi_{nv}(w_n,g_{nv})}{2\pi})^3 - (\frac{\eta_{nv}(w_n,g_{nv})}{2\pi})^3}$.

Remark 2.1. For convenience, $\mathfrak{H}(c_v) = \langle (w_n, g_{nv}), m_{nv}(w_n, g_{nv})e^{i\varphi_{nv}(w_n,g_{nv})}, o_{nv}(w_n, g_{nv})e^{i\eta_{nv}(w_n,g_{nv})}\rangle$ is denoted by $\mathfrak{H}_{nv} = \langle g_{nv}, (m_{nv}e^{i\varphi_{nv}}, o_{nv}e^{i\eta_{nv}}\rangle$ which represents a CFFNS$_f$ number (CFFNS$_f$N).

Definition 2.7 ([69]). Let $\mathfrak{H}_{nv} = \langle g_{nv}, (m_{nv}e^{i\varphi_{nv}}, o_{nv}e^{i\eta_{nv}})\rangle$ be any CFFNS$_f$N over U. The score function of $\mathfrak{H}_{nv}$ is defined as follows:

$$f(\mathfrak{H}_{nv}) = (\frac{g_{nv}}{N-1})^3 + (m_{nv})^3 - (o_{nv})^3 + ((\frac{\varphi_{nv}}{2\pi})^3 - (\frac{\eta_{nv}}{2\pi})^3),$$

where $f(\mathfrak{H}_{nv}) \in [-2, 3]$.

Definition 2.8 ([69]). Let $\mathfrak{H}_{nv} = \langle g_{nv}, (m_{nv}e^{i\varphi_{nv}}, o_{nv}e^{i\eta_{nv}})\rangle$ be any CFFNS$_f$N over U. The accuracy function of $\mathfrak{H}_{nv}$ is defined as follows:

$$A(\mathfrak{H}_{nv}) = (\frac{g_{nv}}{N-1})^3 + (m_{nv})^3 + (o_{nv})^3 + ((\frac{\varphi_{nv}}{2\pi})^3 + (\frac{\eta_{nv}}{2\pi})^3),$$

where $A(\mathfrak{H}_{nv}) \in [0, 3]$.

Definition 2.9 ([69]). For any two distinct CFFNS$_f$Ns $\mathfrak{H}_{11}$ and $\mathfrak{H}_{21}$ we have:

1. if $f(\mathfrak{H}_{11}) < f(\mathfrak{H}_{21})$, then $\mathfrak{H}_{11} \prec \mathfrak{H}_{21}$ ($\mathfrak{H}_{11}$ precedes $\mathfrak{H}_{21}$),

2. if $f(\mathfrak{H}_{11}) > f(\mathfrak{H}_{21})$, then $\mathfrak{H}_{11} \succ \mathfrak{H}_{21}$ ($\mathfrak{H}_{11}$ succeeds $\mathfrak{H}_{21}$),

3. if $f(\mathfrak{H}_{11}) = f(\mathfrak{H}_{21})$, then

- **a.** if $A(\mathfrak{H}_{11}) > A(\mathfrak{H}_{21})$, then $\mathfrak{H}_{11} \succ \mathfrak{H}_{21}$ ($\mathfrak{H}_{11}$ succeeds $\mathfrak{H}_{21}$),
- **b.** if $A(\mathfrak{H}_{11}) < A(\mathfrak{H}_{21})$, then $\mathfrak{H}_{11} \prec \mathfrak{H}_{21}$ ($\mathfrak{H}_{11}$ precedes $\mathfrak{H}_{21}$),
- **c.** if $A(\mathfrak{H}_{11}) = A(\mathfrak{H}_{21})$, then $\mathfrak{H}_{11} \sim \mathfrak{H}_{21}$ ($\mathfrak{H}_{11}$ is equivalent to $\mathfrak{H}_{21}$).

Definition 2.10 ([69]). Let $\mathfrak{H}_{nv} = \langle g_{nv}, (m_{nv}e^{i\varphi_{nv}}, o_{nv}e^{i\eta_{nv}})\rangle$ $n = 1, 2$ and $\mathfrak{H} = \langle g, (me^{i\varphi}, oe^{i\eta})\rangle$ be three CFFNS$_f$Ns over U and $\varsigma > 0$. Then some operations for CFFNS$_f$Ns are:

1. $\mathfrak{H}_{11} \cup \mathfrak{H}_{21} = \langle \max(r_{11}, r_{21}), (\max(m_{11}, m_{21})e^{i\max(\varphi_{11},\varphi_{21})}, \min(o_{11}, o_{21})e^{i\min(\eta_{11},\eta_{21})})\rangle$,
2. $\mathfrak{H}_{11} \cap \mathfrak{H}_{21} = \langle \min(r_{11}, r_{21}), (\min(m_{11}, m_{21})e^{i\min(\varphi_{11},\varphi_{21})}, \max(o_{11}, o_{21})e^{i\max(\eta_{11},\eta_{21})})\rangle$,
3. $\mathfrak{H}^c = \langle g, (oe^{i\eta}, me^{i\varphi})\rangle$,
4. $\mathfrak{H}_{11} \bigoplus \mathfrak{H}_{21} = \Big\langle \max(r_{11}, r_{21}), \Big(\sqrt[3]{(m_{11})^3 + (m_{21})^3 - (m_{11})^3(m_{21})^3} e^{i2\pi \sqrt[3]{(\frac{\varphi_{11}}{2\pi})^3 + (\frac{\varphi_{21}}{2\pi})^3 - (\frac{\varphi_{11}}{2\pi})^3(\frac{\varphi_{21}}{2\pi})^3}},$
$o_{11}o_{21}e^{i2\pi(\frac{\eta_{11}}{2\pi})(\frac{\eta_{21}}{2\pi})}\Big)\Big\rangle$,

5. $\mathfrak{H}_{11} \bigotimes \mathfrak{H}_{21} = \Big\langle \min(r_{11}, r_{21}), \Big(m_{11}m_{21}e^{i2\pi(\frac{\varphi_{11}}{2\pi})(\frac{\varphi_{21}}{2\pi})}, \sqrt[3]{(o_{11})^3 + (o_{21})^3 - (o_{11})^3(o_{21})^3}$
$e^{i2\pi \sqrt[3]{(\frac{\eta_{11}}{2\pi})^3 + (\frac{\eta_{21}}{2\pi})^3 - (\frac{\eta_{11}}{2\pi})^3(\frac{\eta_{21}}{2\pi})^3}}\Big)\Big\rangle$,

6. $\varsigma\mathfrak{H} = \langle g, (\sqrt[3]{1-(1-m^3)^\varsigma}e^{i2\pi \sqrt[3]{1-(1-(\frac{\varphi}{2\pi})^3)^\varsigma}}, o^\varsigma e^{i2\pi(\frac{\eta}{2\pi})^\varsigma})\rangle$,

7. $\mathfrak{H}^\varsigma = \langle g, (m^\varsigma e^{i2\pi(\frac{\varphi}{2\pi})^\varsigma}, \sqrt[3]{1-(1-o^3)^\varsigma}e^{i2\pi \sqrt[3]{1-(1-(\frac{\eta}{2\pi})^3)^\varsigma}})\rangle$.

3. Complex Fermatean fuzzy N-soft VIKOR method

In this section, we will propose a new approach, namely, CFFNS$_f$-VIKOR method. This method will effectively handle the MAGDM problems in the environment of CFFNS$_f$S. Our primary goal is to identify a compromise solution that maximum group utility and minimum individual regret. This technique is extremely efficient in addressing the MAGDM problems with two-dimensional ambiguous information.

Let $\mathbb{E} = \{\mathbb{E}_1, \mathbb{E}_2, \mathbb{E}_3, \ldots, \mathbb{E}_x\}$ be the set of x decision-makers which are appointed to judge the feasibility, functionality and potential of choices from the set of s alternatives $\mathbb{A} = \{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3 \ldots, \mathbb{A}_s\}$ with respect to specific criteria. Let $C = \{\rfloor_1, \rfloor_2, \rfloor_3, \ldots, \rfloor_p\}$ be the set of p criteria on the basis of which feasibility of choices are judged. Mathematical description and detailed procedure of CFFNS$_f$S is described as follows:

Step 1: The decision-makers examine the abilities of alternatives according to the elected criteria and depict their importance by means of linguistic variable which is represented as CFFNS$_f$N $D_{nv}^{(t)} = \langle g_{nv}^{(t)}, (\mathfrak{m}_{nv}^{(t)}, \mathfrak{o}_{nv}^{(t)})\rangle = \langle g_{nv}^{(t)}, (m_{nv}^{(t)}e^{i\varphi_{nv}^{(t)}}, o_{nv}^{(t)}e^{i\eta_{nv}^{(t)}})\rangle$, $t = \{1, 2, 3, \ldots, x\}$, $n = \{1, 2, 3, \ldots, s\}$, and $v = \{1, 2, 3, \ldots, p\}$. The CFF$NS_f$ decision matrix (CFFNS$_f$DM) corresponding to the evaluation by expert (t) can be arranged as follows:

$$\mathfrak{D}^{(t)} = (D_{nv}^{(t)})_{s\times p} = \begin{pmatrix} \langle g_{11}^{(t)}, (\mathfrak{m}_{11}^{(t)}, \mathfrak{o}_{11}^{(t)})\rangle & \langle g_{12}^{(t)}, (\mathfrak{m}_{12}^{(t)}, \mathfrak{o}_{12}^{(t)})\rangle & \cdots & \langle g_{1p}^{(t)}, (\mathfrak{m}_{1p}^{(t)}, \mathfrak{o}_{1p}^{(t)})\rangle \\ \langle g_{21}^{(t)}, (\mathfrak{m}_{21}^{(t)}, \mathfrak{o}_{21}^{(t)})\rangle & \langle g_{22}^{(t)}, (\mathfrak{m}_{22}^{(t)}, \mathfrak{o}_{22}^{(t)})\rangle & \cdots & \langle g_{2p}^{(t)}, (\mathfrak{m}_{2p}^{(t)}, \mathfrak{o}_{2p}^{(t)})\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle g_{s1}^{(t)}, (\mathfrak{m}_{s1}^{(t)}, \mathfrak{o}_{s1}^{(t)})\rangle & \langle g_{s2}^{(t)}, (\mathfrak{m}_{s2}^{(t)}, \mathfrak{o}_{s2}^{(t)})\rangle & \cdots & \langle g_{sp}^{(t)}, (\mathfrak{m}_{sp}^{(t)}, \mathfrak{o}_{sp}^{(t)})\rangle \end{pmatrix}.$$

Step 2: All decision-makers have its own suggestions as well as opinions and they may not be evenly important. Hence, significance of decision-makers can be evaluated by using linguistic grades elaborated as CFFNS$_f$Ns. These linguistic grades are assigned to decision-makers as per their experience. Let $\Omega_t = \langle g_t, (\mathfrak{m}_t, \mathfrak{o}_t)\rangle = \langle g_t, (m_t e^{i\varphi_t}, o_t e^{i\eta_t})\rangle$, be the CFF$NS_f$N articulating the significance of decision maker $\mathbb{E}_t$. The normalized weight of each decision maker $\mathbb{E}_t$ can be erected as follows:

$$\kappa_t = \frac{\frac{g_t}{N-1} + m_t + (m_t^3 + o_t^3)\Big(\frac{m_t}{m_t + o_t}\Big) + \frac{\varphi_t}{2\pi} + ((\frac{\varphi_t}{2\pi})^3 + (\frac{\eta_t}{2\pi})^3)\Big(\frac{\frac{\varphi_t}{2\pi}}{\frac{\varphi_t}{2\pi} + \frac{\eta_t}{2\pi}}\Big)}{\sum_{t=1}^{x}\Big(\frac{g_t}{N-1} + m_t + (m_t^3 + o_t^3)\Big(\frac{m_t}{m_t + o_t}\Big) + \frac{\varphi_t}{2\pi} + ((\frac{\varphi_t}{2\pi})^3 + (\frac{\eta_t}{2\pi})^3)\Big(\frac{\frac{\varphi_t}{2\pi}}{\frac{\varphi_t}{2\pi} + \frac{\eta_t}{2\pi}}\Big)\Big)}, \tag{3.1}$$

where $\kappa_t \in [0, 1]$ $(t = 1, 2, 3, \ldots, x)$ and $\sum_{t=1}^{x} \kappa_t = 1$.

Step 3: The individual decisions of decision-makers are accumulated with the help of CFFNS_fWA operator to acquire a solution which is acceptable for all members in the decision panel. Hence, the aggregated CFFNS_fDM is constructed by using CFFNS_fWA operator as follows:

$$\begin{aligned} D_{nv} &= CFFNS_fWA_\kappa(D_{nv}^{(1)}, D_{nv}^{(2)}, \ldots, D_{nv}^{(x)}) \\ &= \kappa_1 D_{nv}^{(1)} \oplus \kappa_2 D_{nv}^{(2)} \oplus \ldots \oplus \kappa_x D_{nv}^{(x)} \\ &= \left\langle \max_{\mathrm{t}=1}^{x} g_{nv}^{(\mathrm{t})}, \left(\sqrt[3]{1 - \prod_{\mathrm{t}=1}^{x} (1-(m_{nv}^{(\mathrm{t})})^3)^{\kappa_\mathrm{t}}}\, e^{i2\pi \sqrt[3]{1-\prod_{\mathrm{t}=1}^{x}(1-(\frac{\varphi_{nv}^{(\mathrm{t})}}{2\pi})^3)^{\kappa_\mathrm{t}}}}, \prod_{\mathrm{t}=1}^{x} (o_{nv}^{(\mathrm{t})})^{\kappa_\mathrm{t}} e^{i2\pi \prod_{\mathrm{t}=1}^{x}(\frac{\eta_{nv}^{(\mathrm{t})}}{2\pi})^{\kappa_\mathrm{t}}} \right) \right\rangle \quad (3.2) \\ &= \langle g_{nv}, (m_{nv} e^{i\varphi_{nv}}, o_{nv} e^{i\eta_{nv}}) \rangle \\ &= \langle g_{nv}, (\mathfrak{m}_{nv}, \mathfrak{o}_{nv}) \rangle, \end{aligned}$$

where $D_{nv}^{(\mathrm{t})} = \langle g_{nv}^{(\mathrm{t})}, (\mathfrak{m}_{nv}^{(\mathrm{t})}, \mathfrak{o}_{nv}^{(\mathrm{t})}) \rangle = \langle g_{nv}^{(\mathrm{t})}, (m_{nv}^{(\mathrm{t})} e^{i\varphi_{nv}^{(\mathrm{t})}}, o_{nv}^{(\mathrm{t})} e^{i\eta_{nv}^{(\mathrm{t})}}) \rangle$ depicts the independent opinion of expert $\mathbb{E}^{(\mathrm{t})}$ about the crucial factor $\mathbb{A}_n$ relative to criterion $\rfloor_v$.

Step 4: Each decision maker evaluates all criteria according to the requirement of the MAGDM problem. All decision-makers assign the linguistic variable in form of CFFNS_fNs to each criterion to exhibit the importance of criteria in MAGDM problem and these independent opinions are assembled to construct the CFFNS_f weight matrix. Let the perspective of decision maker $\mathbb{E}_\mathrm{t}$ regarding criterion $\rfloor$ can be exhibited in the form of CFFNS_fN $\tau_v = \langle g_v, (\mathfrak{m}_v, \mathfrak{o}_v) \rangle$. The CFF$NS_f$ weight vector $\tau = (\tau_1\ \tau_2\ \ldots\ \tau_p)^T$, where $\tau_v = \langle g_\tau(\rfloor_v), (\mathfrak{m}_\tau(\rfloor_v), \mathfrak{o}_\tau(\rfloor_v)) \rangle = \langle g_\tau(\rfloor_v), (m_\tau(\rfloor_v) e^{i\varphi_\tau(\rfloor_v)}, o_\tau(\rfloor_v) e^{i\eta_\tau(\rfloor_v)}) \rangle$ is calculated as:

$$\begin{aligned} \tau_v &= CFFNS_fWA_\kappa(\tau_v^{(1)}, \tau_v^{(2)}, \ldots, \tau_v^{(x)}) \\ &= \kappa_1 \tau_v^{(1)} \oplus \kappa_2 \tau_v^{(2)} \oplus \ldots \oplus \kappa_x \tau_v^{(x)} \\ &= \left\langle \max_{\mathrm{t}=1}^{x} g_v^{(\mathrm{t})}, \left(\sqrt[3]{1 - \prod_{\mathrm{t}=1}^{x} (1-(m_v^{(\mathrm{t})})^3)^{\kappa_\mathrm{t}}}\, e^{i2\pi \sqrt[3]{1-\prod_{\mathrm{t}=1}^{x}(1-(\frac{\varphi_v^{(\mathrm{t})}}{2\pi})^3)^{\kappa_\mathrm{t}}}}, \prod_{\mathrm{t}=1}^{x} (o_v^{(\mathrm{t})})^{\kappa_\mathrm{t}} e^{i2\pi \prod_{\mathrm{t}=1}^{x}(\frac{\eta_v^{(\mathrm{t})}}{2\pi})^{\kappa_\mathrm{t}}} \right) \right\rangle. \quad (3.3) \end{aligned}$$

After computing the CFFNS_f weight τ_v of each criterion, the normalized weights w_v can be reckoned as follows:

$$w_v = \frac{\frac{g_\tau(\rfloor_v)}{N-1} + m_\tau(\rfloor_v) + ((m_\tau(\rfloor_v))^3 + (o_\tau(\rfloor_v))^3)\left(\frac{m_\tau(\rfloor_v)}{m_\tau(\rfloor_v)+o_\tau(\rfloor_v)}\right) + \frac{\varphi_\tau(\rfloor_v)}{2\pi} + \left((\frac{\varphi_\tau(\rfloor_v)}{2\pi})^3 + (\frac{\eta_\tau(\rfloor_v)}{2\pi})^3\right)\left(\frac{\frac{\varphi_\tau(\rfloor_v)}{2\pi}}{\frac{\varphi_\tau(\rfloor_v)}{2\pi}+\frac{\eta_\tau(\rfloor_v)}{2\pi}}\right)}{\sum_{\mathrm{t}=1}^{x}\left(\frac{g_\tau(\rfloor_v)}{N-1} + m_\tau(\rfloor_v) + ((m_\tau(\rfloor_v))^3 + (o_\tau(\rfloor_v))^3)\left(\frac{m_\tau(\rfloor_v)}{m_\tau(\rfloor_v)+o_\tau(\rfloor_v)}\right) + \frac{\varphi_\tau(\rfloor_v)}{2\pi} + \left((\frac{\varphi_\tau(\rfloor_v)}{2\pi})^3 + (\frac{\eta_\tau(\rfloor_v)}{2\pi})^3\right)\left(\frac{\frac{\varphi_\tau(\rfloor_v)}{2\pi}}{\frac{\varphi_\tau(\rfloor_v)}{2\pi}+\frac{\eta_\tau(\rfloor_v)}{2\pi}}\right)\right)} \quad (3.4)$$

Step 5: The entries of aggregated CFFNS_fDM are converted to crisp number by means of score function. The assemblage of score matrix $\mathcal{F}$ is given as follows:

$$\mathcal{F} = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1p} \\ f_{21} & f_{22} & \cdots & f_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ f_{s1} & f_{s2} & \cdots & f_{sp} \end{pmatrix},$$

where f_{nv} is computed with the help of the formula:

$$f_{nv} = (\frac{g_{nv}}{N-1})^3 + (m_{nv})^3 - (o_{nv})^3 + \Big((\frac{\varphi_{nv}}{2\pi})^3 - (\frac{\eta_{nv}}{2\pi})^3\Big). \tag{3.5}$$

Step 6: Based on score values, best and worst values of all criteria are selected. Let $\mathfrak{C}_B$ and $\mathfrak{C}_K$ are the benefit-type and cost-type criterion, then the best value f_v^+ and worst value f_v^- of criterion $\rfloor_v$ are determined as follows:

$$f_v^+ = \begin{cases} \max\limits_{1\le n\le s} f_{nv}, & \text{if } \rfloor_v \in \mathfrak{C}_B, \\ \min\limits_{1\le n\le s} f_{nv}, & \text{if } \rfloor_v \in \mathfrak{C}_K. \end{cases} \tag{3.6}$$

$$f_v^- = \begin{cases} \min\limits_{1\le n\le s} f_{nv}, & \text{if } \rfloor_v \in \mathfrak{C}_B, \\ \max\limits_{1\le n\le s} f_{nv}, & \text{if } \rfloor_v \in \mathfrak{C}_K. \end{cases} \tag{3.7}$$

Step 7: Our next goal is to find out the ranking measures for all alternatives. The group utility measure $\mathcal{S}_n$ and individual regret measure $\mathcal{R}_n$ corresponding to the alternative $\mathbb{A}_n$ are identified as follows:

$$\mathcal{S}_n = \sum_{v=1}^{p} w_v\Big(\frac{f_v^+ - f_{nv}}{f_v^+ - f_v^-}\Big), \tag{3.8}$$

$$\mathcal{R}_n = \max_{v=1}^{p} w_v\Big(\frac{f_v^+ - f_{nv}}{f_v^+ - f_v^-}\Big). \tag{3.9}$$

Now, we find out the minimum and maximum values of the group utility measure $\mathcal{S}$ and individual regret measure $\mathcal{R}$ as follows:

$$\mathcal{S}^* = \max_n \mathcal{S}_n \quad , \quad \mathcal{S}' = \min_n \mathcal{S}_n,$$
$$\mathcal{R}^* = \max_n \mathcal{R}_n \quad , \quad \mathcal{R}' = \min_n \mathcal{R}_n.$$

Finally, the ranking measure $\mathcal{Q}_n$ related to $\mathbb{A}_n$ can be calculated by using $\mathcal{S}_n$ and $\mathcal{R}_n$ as given:

$$\mathcal{Q}_n = q\Big(\frac{\mathcal{S}_n - \mathcal{S}'}{\mathcal{S}^* - \mathcal{S}'}\Big) + (1-q)\Big(\frac{\mathcal{R}_n - \mathcal{R}'}{\mathcal{R}^* - \mathcal{R}'}\Big), \tag{3.10}$$

where the parameter q is known as the weight of strategy of the maximum group utility of the attribute and its value can be chosen from $[0, 1]$ depend upon the MAGDM problem. Usually, q is considered as 0.5 in order to obtain the compromise solution that possesses maximum $\mathcal{S}$ and minimum $\mathcal{R}$. For $q = 1$, compromise solution only emphasizes on $\mathcal{R}^*$. Also, for $q = 0$, the compromise solution emphasizes to $\mathcal{R}'$.

Step 8: In this step, the alternatives are arranged in ascending order relative to $\mathcal{S}, \mathcal{R}$, and $\mathcal{Q}$. These three ranking indexes are useful to evaluate the compromise solution and the alternative having minimum $\mathcal{Q}$ value is superb than others.

Step 9: Let $\mathbb{A}^{(1)}$ and $\mathbb{A}^{(2)}$ ranked at the first and second place regarding to $\mathcal{Q}$, respectively. The compromise solution will be $\mathbb{A}^{(1)}$ if it satisfies the following conditions:

$\mathfrak{C}1$: "Acceptable advantage"

$$Q(\mathbb{A}^{(2)}) - Q(\mathbb{A}^{(1)}) \geq DQ,$$

where $DQ = \frac{1}{s-1}$ and s is the total number of alternatives.

$\mathfrak{C}2$: "Acceptable stability in decision-making"

In accordance to $\mathfrak{C}2$, the alternative $\mathbb{A}^{(1)}$, which is superior according to Q is also ranked at first number regarding to $\mathcal{S}$ or $\mathcal{R}$. The resultant compromise solution is consistent across a decision-making process: that perhaps "voting by majority rule" (for $q > 0.5$) or "by consensus" (for $q = 0.5$) or "with veto" (for $q < 0.5$).

In case of one or both of these requirements do not obey then the set of compromise solution can be identified as follows:

$\mathfrak{C}\mathfrak{S}1$: $\mathbb{A}^{(1)}$ and $\mathbb{A}^{(2)}$ both are in the set of compromise solution, if only one condition $\mathfrak{C}2$ does not satisfy.

If the first condition $\mathfrak{C}1$ does not satisfy, then the set of compromise solution comprises $\mathbb{A}^{(1)}, \mathbb{A}^{(2)}, \ldots, \mathbb{A}^{(\mathfrak{p})}$, where $\mathbb{A}^{(\mathfrak{p})}$ is originated by employing the inequality $Q(\mathbb{A}^{(\mathfrak{p})}) - Q(\mathbb{A}^{(1)}) < DQ$, for maximum $\mathfrak{p}$.

The structure of proposed CFFNS$_f$-VIKOR approach is summarized in Figure 3.

4. Selection of the best location for Nuclear Power Plant

A nuclear power plant is a thermal power station which converts heat energy into electrical energy and the resource of heat is nuclear reactor. It is basically a large electricity generating facility. It is very essential to select the best site for a nuclear power station, as it requires a massive potential of position and land to sustain the dynamic and static pressure during the whole process. Suppose that the government of Pakistan wants to start a new project of planting a thermal power station to meet the requirements of electricity. After the initial survey, six locations, such as $\mathbb{A}_1$ = Lahore, $\mathbb{A}_2$ = Karachi, $\mathbb{A}_3$ = Islamabad, $\mathbb{A}_4$ = Peshawar, $\mathbb{A}_5$ = Quetta, and $\mathbb{A}_6$ = Mianwali are chosen for further assessment. A panel of three decision-makers is designated to deeply examine all sites based on the following criteria:

$\rfloor_1$: Radioactive waste disposal facility (maximum disposal facility is preferable),
$\rfloor_2$: Economical cost of land (least cost is preferable),
$\rfloor_3$: Water availability (maximum water resources are preferable),
$\rfloor_4$: Shipping resources (maximum transportation with least cost is preferable),
$\rfloor_5$: Storage space of fuel (maximum space for fuel storage is preferable).

Here, $\rfloor_2, \rfloor_4$ are cost-type criteria, whereas $\rfloor_1, \rfloor_3$, and $\rfloor_5$ are benefit-type criteria.

Hierarchy structured of the problem with the required alternatives and criteria is demonstrated in Figure 4. The mathematical steps to find out the solution of this MAGDM by using CFFNS$_f$-VIKOR method are as follows:

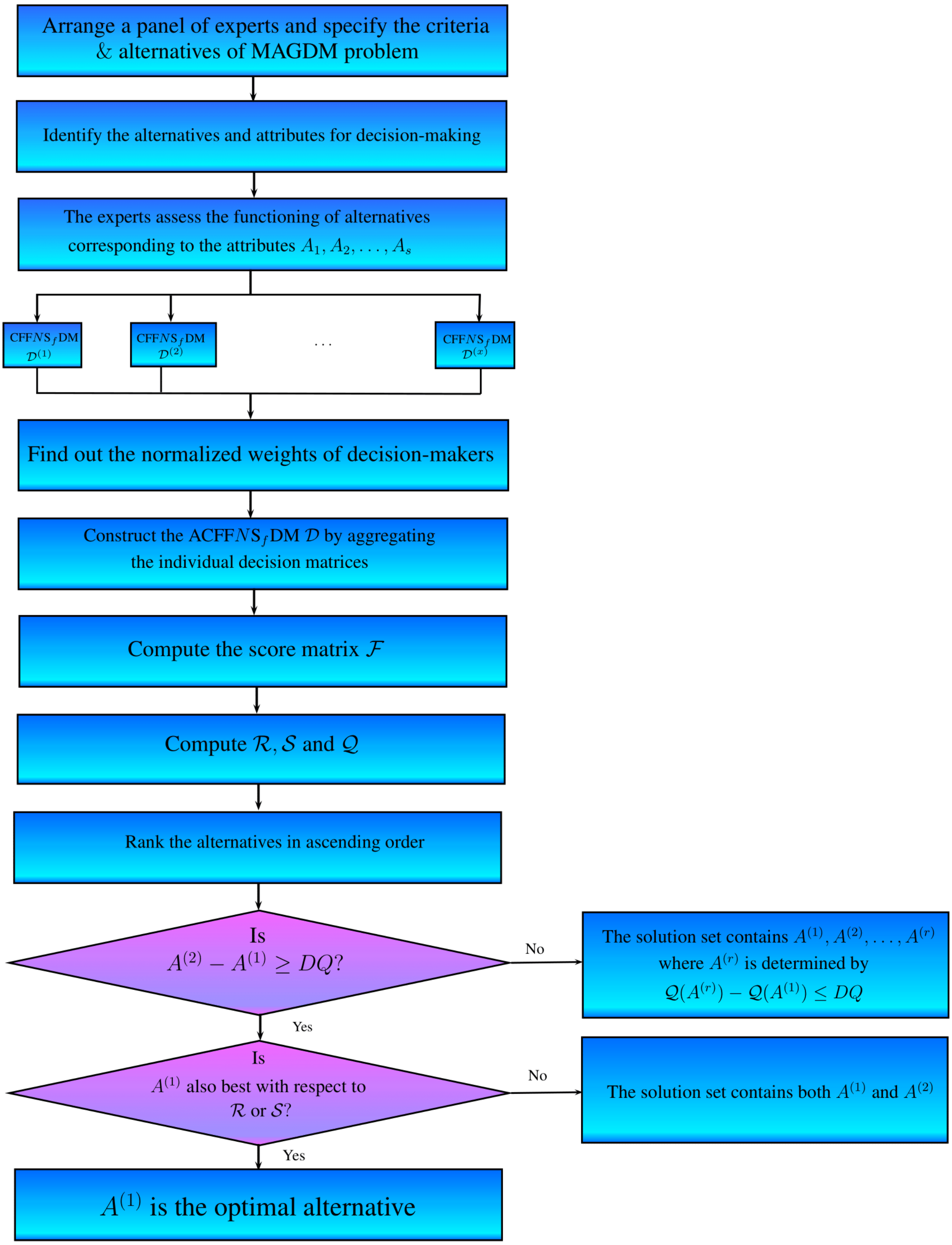

Figure 3. Flow chart of CFF$\mathcal{NS}_f$-VIKOR method.

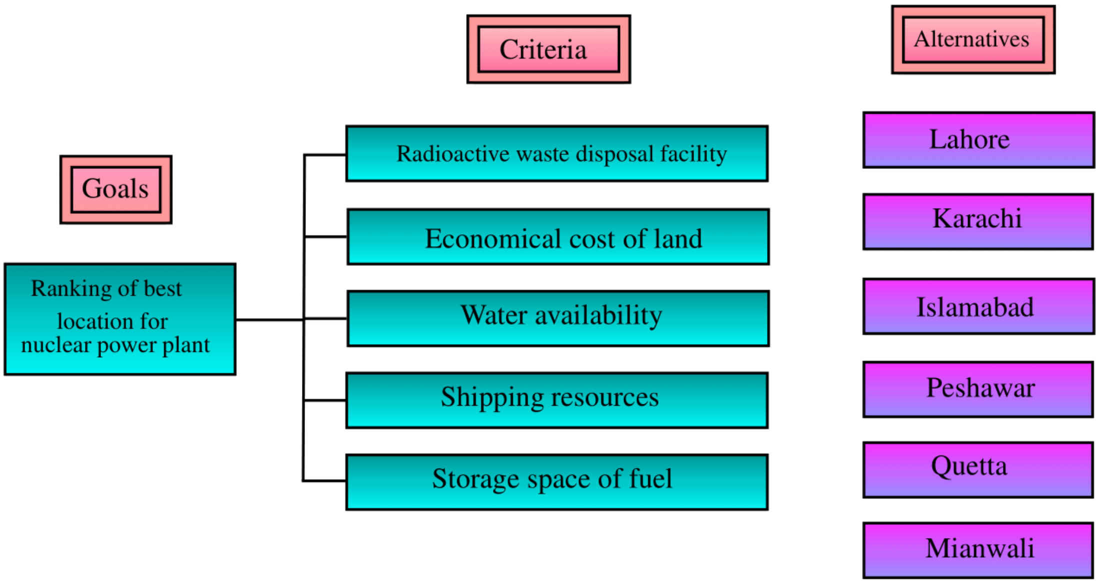

Figure 4. Hierarchical Structure of suitable Location for Nuclear Power Plant Evaluation.

Step 1. The priority 5-soft rating of alternatives, in the form of linguistic terms, to depict the importance of criteria is arranged in Table 1, where

✠✠✠✠ symbolize 'Remarkable',
✠✠✠ symbolize 'Excellent'
✠✠ symbolize 'Ordinary',
✠ symbolizes 'Acceptable',
△ symbolizes 'Substandard'.

The individual decision matrices of decision-makers $\mathbb{E}_1, \mathbb{E}_2$, and $\mathbb{E}_3$, which are formed by using Table 1 and corresponding CFFNS_fNs, are given in Tables 2, 3, and 4.

Step 2. The linguistic 5-soft grades and associated CFFNS_f weights as well as normalized weights, calculated by employing Equation 3.1, of decision-makers are given in Table 5.

Step 3. The individual CFFNS_fDM of decision-makers along with their normalized weights are accumulated by utilizing Equation 3.2 and the results are arranged in Table 6.

Step 4. Each criterion has its own contribution and worth in the MAGDM problem. Hence, the decision-makers judge all criteria and assigned them linguistic grades along with CFFNS_fDM according to their impact on the problem. These weights of criteria, assigned by decision-makers $\mathbb{E}_1, \mathbb{E}_2$, and $\mathbb{E}_3$ are arranged in Tables 7 and 8. The corresponding CFFNS_f weights and normalized weights of attributes, computed by Equations 3.3 and 3.4, are assembled in Table 9.

Step 6. The score of the entries of aggregated CFFNS_fDM, shown in Table 6, is calculated by employing Equation 3.5 and the values are arranged in Table 10.

Table 1. Performance rating of alternatives corresponding to each criterion.

Criterion	**Alternatives**	$\mathbb{E}_1$	$\mathbb{E}_2$	$\mathbb{E}_3$
$\rfloor_1$	$\mathbb{A}_1$	✠ = 1	✠ = 1	✠ = 1
	$\mathbb{A}_2$	✠✠ = 2	✠✠ = 2	✠✠ = 2
	$\mathbb{A}_3$	✠✠✠ = 3	✠✠✠ = 3	✠✠✠ = 3
	$\mathbb{A}_4$	✠✠ = 2	✠✠ = 2	✠✠ = 2
	$\mathbb{A}_5$	✠ = 1	✠ = 1	✠✠ = 1
	$\mathbb{A}_6$	△ = 0	✠ = 1	△ = 0
$\rfloor_2$	$\mathbb{A}_1$	✠ = 1	✠✠ = 2	✠✠ = 2
	$\mathbb{A}_2$	✠✠ = 2	✠ = 1	✠✠ = 2
	$\mathbb{A}_3$	✠✠✠ = 3	✠✠✠ = 3	✠✠✠ = 3
	$\mathbb{A}_4$	✠✠ = 2	✠✠ = 2	✠✠ = 2
	$\mathbb{A}_5$	✠ = 1	△ = 0	✠ = 1
	$\mathbb{A}_6$	△ = 0	△ = 0	△ = 0
$\rfloor_3$	$\mathbb{A}_1$	✠✠✠ = 3	✠✠✠ = 3	✠✠✠ = 3
	$\mathbb{A}_2$	✠✠ = 2	✠✠ = 2	✠✠✠ = 3
	$\mathbb{A}_3$	△ = 0	△ = 0	△ = 0
	$\mathbb{A}_4$	✠ = 1	✠ = 1	✠ = 1
	$\mathbb{A}_5$	△ = 0	△ = 0	△ = 0
	$\mathbb{A}_6$	✠✠ = 2	✠✠ = 2	✠✠✠ = 3
$\rfloor_4$	$\mathbb{A}_1$	✠✠ = 2	✠✠ = 2	✠✠ = 2
	$\mathbb{A}_2$	✠ = 1	△ = 0	✠ = 1
	$\mathbb{A}_3$	△ = 0	✠ = 1	△ = 0
	$\mathbb{A}_4$	✠✠ = 2	✠✠✠ = 3	✠✠✠ = 3
	$\mathbb{A}_5$	✠✠ = 2	✠ = 1	✠ = 1
	$\mathbb{A}_6$	✠ = 1	✠ = 1	✠ = 1
$\rfloor_5$	$\mathbb{A}_1$	✠ = 1	✠ = 1	△ = 0
	$\mathbb{A}_2$	✠✠✠ = 3	✠✠ = 2	✠✠✠ = 3
	$\mathbb{A}_3$	✠✠ = 2	✠✠ = 2	✠✠ = 2
	$\mathbb{A}_4$	△ = 0	✠ = 1	✠ = 1
	$\mathbb{A}_5$	△ = 0	△ = 0	△ = 0
	$\mathbb{A}_6$	✠ = 1	✠ = 1	✠ = 1

Step 7. In the proposed MAGDM problem, radioactive waste disposal facility, water availability and storage space of fuel are benefit type attributes; whereas economical cost of land and shipping resources are cost type criteria. According to nature of the attributes, best and worst values are

Table 2. Tabulated form of CFF$\mathcal{N}$S$_f$DM $\mathfrak{D}^{(1)}$ of expert $\mathbb{E}_1$.

$\mathfrak{D}^{(1)}$	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$	$\rfloor_4$	$\rfloor_5$
$\mathbb{A}_1$	$\langle 1, (0.39e^{i0.41\pi}, 0.66e^{i1.32\pi})\rangle$	$\langle 1, (0.39e^{i0.73\pi}, 0.75e^{i1.42\pi})\rangle$	$\langle 3, (0.82e^{i1.53\pi}, 0.27e^{i0.62\pi})\rangle$	$\langle 2, (0.63e^{i0.85\pi}, 0.61e^{i1.14\pi})\rangle$	$\langle 1, (0.31e^{i0.76\pi}, 0.67e^{i1.59\pi})\rangle$
$\mathbb{A}_2$	$\langle 2, (0.41e^{i0.92\pi}, 0.54e^{i1.15\pi})\rangle$	$\langle 2, (0.46e^{i0.81\pi}, 0.51e^{i1.21\pi})\rangle$	$\langle 2, (0.47e^{i1.13\pi}, 0.52e^{i0.83\pi})\rangle$	$\langle 1, (0.21e^{i0.47\pi}, 0.66e^{i1.58\pi})\rangle$	$\langle 3, (0.75e^{i1.64\pi}, 0.34e^{i0.74\pi})\rangle$
$\mathbb{A}_3$	$\langle 3, (0.65e^{i1.32\pi}, 0.21e^{i0.45\pi})\rangle$	$\langle 3, (0.69e^{i1.45\pi}, 0.26e^{i0.54\pi})\rangle$	$\langle 0, (0.05e^{i0.23\pi}, 0.93e^{i1.84\pi})\rangle$	$\langle 0, (0.09e^{i0.27\pi}, 0.91e^{i1.83\pi})\rangle$	$\langle 2, (0.62e^{i0.85\pi}, 0.62e^{i1.16\pi})\rangle$
$\mathbb{A}_4$	$\langle 2, (0.51e^{i0.94\pi}, 0.41e^{i1.18\pi})\rangle$	$\langle 2, (0.56e^{i0.92\pi}, 0.50e^{i1.11\pi})\rangle$	$\langle 1, (0.24e^{i0.55\pi}, 0.76e^{i1.43\pi})\rangle$	$\langle 2, (0.56e^{i0.87\pi}, 0.60e^{i1.22\pi})\rangle$	$\langle 0, (0.10e^{i0.36\pi}, 0.86e^{i1.77\pi})\rangle$
$\mathbb{A}_5$	$\langle 1, (0.21e^{i0.52\pi}, 0.67e^{i1.34\pi})\rangle$	$\langle 1, (0.38e^{i0.63\pi}, 0.74e^{i1.41\pi})\rangle$	$\langle 0, (0.12e^{i0.34\pi}, 0.95e^{i1.86\pi})\rangle$	$\langle 2, (0.64e^{i0.95\pi}, 0.59e^{i1.26\pi})\rangle$	$\langle 0, (0.19e^{i0.29\pi}, 0.88e^{i1.78\pi})\rangle$
$\mathbb{A}_6$	$\langle 0, (0.02e^{i0.01\pi}, 0.92e^{i1.94\pi})\rangle$	$\langle 0, (0.17e^{i0.03\pi}, 0.89e^{i1.93\pi})\rangle$	$\langle 2, (0.57e^{i1.16\pi}, 0.53e^{i1.10\pi})\rangle$	$\langle 1, (0.27e^{i0.77\pi}, 0.85e^{i1.57\pi})\rangle$	$\langle 1, (0.22e^{i0.75\pi}, 0.68e^{i1.61\pi})\rangle$

determined by using Equations 3.6 and 3.7. The results are arranged in Table 11.

Step 8. Table 12 represents the $\mathcal{S}, \mathcal{R}$, and $\mathcal{Q}$ values which are calculated by employing Equations 3.8, 3.9 and 3.10, respectively. While computing $\mathcal{Q}$, the weight of strategy q is taken 0.5.

Step 9. The ascending order ranking of alternatives corresponding to the values of $\mathcal{S}, \mathcal{R}$ and $\mathcal{Q}$ is given by Table 13.

Step 10. The alternative $\mathbb{A}_2$ has least value of $\mathcal{Q}$ and satisfies both conditions as follows:

- $\mathcal{Q}(\mathbb{A}_6) - \mathcal{Q}(\mathbb{A}_2) = 0.311415 - 0 = 0.311415 \geq DQ = \frac{1}{p-1} = \frac{1}{6-1} = 0.2$.
- $\mathbb{A}_2$ is the best choice according to both $\mathcal{S}$ and $\mathcal{R}$.

Step 11. Hence, it is concluded that site $\mathbb{A}_2$ is the most preferable choice that possesses the maximum group utility along with minimum individual regret of opponent. The optimal ranking of suitable locations for nuclear power plant $\mathbb{A}_2 \succ \mathbb{A}_6 \succ \mathbb{A}_1 \succ \mathbb{A}_3 \succ \mathbb{A}_4 \succ \mathbb{A}_5$. Thus, the best site to start the project of planting nuclear power station is $\mathbb{A}_2$, namely, Karachi.

Table 3. Tabulated form of CFFNS$_f$DM $\mathfrak{D}^{(2)}$ of expert $\mathbb{E}_2$.

$\mathfrak{D}^{(2)}$	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$
$\mathbb{A}_1$	$\langle 1,(0.38e^{i0.42\pi},0.68e^{i1.33\pi})\rangle$	$\langle 2,(0.55e^{i0.82\pi},0.49e^{i1.29\pi})\rangle$	$\langle 3,(0.82e^{i1.58\pi},0.28e^{i0.64\pi})\rangle$
$\mathbb{A}_2$	$\langle 2,(0.42e^{i0.89\pi},0.51e^{i1.14\pi})\rangle$	$\langle 1,(0.37e^{i0.72\pi},0.73e^{i1.39\pi})\rangle$	$\langle 2,(0.48e^{i0.85\pi},0.54e^{i1.26\pi})\rangle$
$\mathbb{A}_3$	$\langle 3,(0.76e^{i1.33\pi},0.22e^{i0.47\pi})\rangle$	$\langle 3,(0.68e^{i1.47\pi},0.25e^{i0.53\pi})\rangle$	$\langle 0,(0.06e^{i0.33\pi},0.96e^{i1.87\pi})\rangle$
$\mathbb{A}_4$	$\langle 2,(0.52e^{i0.98\pi},0.42e^{i0.89\pi})\rangle$	$\langle 2,(0.45e^{i0.91\pi},0.48e^{i1.12\pi})\rangle$	$\langle 1,(0.25e^{i0.45\pi},0.77e^{i1.44\pi})\rangle$
$\mathbb{A}_5$	$\langle 1,(0.22e^{i0.53\pi},0.69e^{i1.35\pi})\rangle$	$\langle 0,(0.15e^{i0.32\pi},0.87e^{i1.94\pi})\rangle$	$\langle 0,(0.14e^{i0.04\pi},0.97e^{i1.85\pi})\rangle$
$\mathbb{A}_6$	$\langle 0,(0.03e^{i0.21\pi},0.93e^{i1.96\pi})\rangle$	$\langle 0,(0.13e^{i0.22\pi},0.88e^{i1.92\pi})\rangle$	$\langle 2,(0.58e^{i0.94\pi},0.56e^{i1.13\pi})\rangle$
	$\rfloor_4$	$\rfloor_5$	
$\mathbb{A}_1$	$\langle 2,(0.59e^{i0.95\pi},0.58e^{i1.28\pi})\rangle$	$\langle 1,(0.32e^{i0.58\pi},0.69e^{i1.62\pi})\rangle$	
$\mathbb{A}_2$	$\langle 0,(0.18e^{i0.25\pi},0.92e^{i1.82\pi})\rangle$	$\langle 2,(0.54e^{i1.18\pi},0.63e^{i0.89\pi})\rangle$	
$\mathbb{A}_3$	$\langle 1,(0.33e^{i0.48\pi},0.84e^{i1.56\pi})\rangle$	$\langle 2,(0.43e^{i0.99\pi},0.64e^{i0.91\pi})\rangle$	
$\mathbb{A}_4$	$\langle 3,(0.83e^{i1.62\pi},0.33e^{i0.63\pi})\rangle$	$\langle 1,(0.23e^{i0.69\pi},0.70e^{i1.64\pi})\rangle$	
$\mathbb{A}_5$	$\langle 1,(0.28e^{i0.62\pi},0.83e^{i1.55\pi})\rangle$	$\langle 0,(0.01e^{i0.38\pi},0.89e^{i1.76\pi})\rangle$	
$\mathbb{A}_6$	$\langle 1,(0.33e^{i0.73\pi},0.82e^{i1.54\pi})\rangle$	$\langle 1,(0.33e^{i0.57\pi},0.71e^{i1.45\pi})\rangle$	

5. Comparative study

In this section, above MAGDM problem related to "selection of the best location for Nuclear Power Plant" is solved by the Fermatean fuzzy TOPSIS approach, introduced by Senapati and Yager [33], to show the authenticity of our proposed CFFNS$_f$-VIKOR method. The process for the selection of most preferable location for nuclear power plant compromises of the following steps:

Step 1. To start the procedure, the collective opinion of all judges is taken, given by Table 6, and convert it into the environment of Fermatean fuzzy set by omitting the grades and phase terms. Required Fermatean fuzzy decision matrix is arranged in Table 14. Moreover, the weights of criteria, assigned by decision-makers, is as follows:

$$\zeta = (0.130178 \;\; 0.166890 \;\; 0.279169 \;\; 0.228613 \;\; 0.19515)^T.$$

Step 2. To find out the positive ideal solution and negative ideal solution of criteria, score values of all Fermatean fuzzy numbers, given in Table 14, are computed and arranged in Table 15 by

Table 4. Tabulated form of CFFNS$_f$DM $\mathfrak{D}^{(3)}$ of expert $\mathbb{E}_3$.

$\mathfrak{D}^{(3)}$	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$	$\rfloor_4$	$\rfloor_5$
$\mathbb{A}_1$	$\langle 1,(0.37e^{i0.43\pi},0.70e^{i1.36\pi})\rangle$	$\langle 2,(0.44e^{i0.83\pi},0.47e^{i1.28\pi})\rangle$	$\langle 3,(0.70e^{i1.54\pi},0.29e^{i0.69\pi})\rangle$	$\langle 2,(0.49e^{i0.91\pi},0.57e^{i1.19\pi})\rangle$	$\langle 0,(0.08e^{i0.28\pi},0.87e^{i1.74\pi})\rangle$
$\mathbb{A}_2$	$\langle 2,(0.43e^{i1.17\pi},0.43e^{i0.92\pi})\rangle$	$\langle 2,(0.54e^{i0.93\pi},0.46e^{i1.14\pi})\rangle$	$\langle 3,(0.81e^{i1.59\pi},0.30e^{i0.68\pi})\rangle$	$\langle 1,(0.32e^{i0.59\pi},0.81e^{i1.52\pi})\rangle$	$\langle 3,(0.74e^{i1.66\pi},0.37e^{i0.72\pi})\rangle$
$\mathbb{A}_3$	$\langle 3,(0.66e^{i1.36\pi},0.23e^{i0.42\pi})\rangle$	$\langle 3,(0.77e^{i1.49\pi},0.24e^{i0.59\pi})\rangle$	$\langle 0,(0.07e^{i0.35\pi},0.98e^{i1.89\pi})\rangle$	$\langle 0,(0.08e^{i0.33\pi},0.94e^{i1.81\pi})\rangle$	$\langle 2,(0.40e^{i0.95\pi},0.65e^{i1.20\pi})\rangle$
$\mathbb{A}_4$	$\langle 2,(0.53e^{i1.26\pi},0.44e^{i0.88\pi})\rangle$	$\langle 2,(0.61e^{i0.84\pi},0.45e^{i1.27\pi})\rangle$	$\langle 1,(0.26e^{i0.74\pi},0.78e^{i1.45\pi})\rangle$	$\langle 3,(0.84e^{i1.61\pi},0.32e^{i0.65\pi})\rangle$	$\langle 1,(0.36e^{i0.68\pi},0.72e^{i1.66\pi})\rangle$
$\mathbb{A}_5$	$\langle 1,(0.23e^{i0.54\pi},0.71e^{i1.37\pi})\rangle$	$\langle 1,(0.36e^{i0.61\pi},0.72e^{i1.38\pi})\rangle$	$\langle 0,(0.16e^{i0.24\pi},0.99e^{i1.88\pi})\rangle$	$\langle 1,(0.27e^{i0.48\pi},0.80e^{i1.47\pi})\rangle$	$\langle 0,(0.16e^{i0.19\pi},0.90e^{i1.72\pi})\rangle$
$\mathbb{A}_6$	$\langle 0,(0.04e^{i0.31\pi},0.91e^{i1.97\pi})\rangle$	$\langle 0,(0.11e^{i0.02\pi},0.86e^{i1.91\pi})\rangle$	$\langle 3,(0.78e^{i1.53\pi},0.31e^{i0.67\pi})\rangle$	$\langle 1,(0.31e^{i0.78\pi},0.79e^{i1.46\pi})\rangle$	$\langle 1,(0.25e^{i0.56\pi},0.73e^{i1.68\pi})\rangle$

Table 5. Importance weight of decision-makers.

Decision-makers	**Linguistic grades**	**Weights**	**Normalized weights**
$\mathbb{E}_1$	✠✠✠ = 3	$\langle 3,(0.69e^{i1.45\pi},0.25e^{i0.59\pi})\rangle$	0.274353
$\mathbb{E}_2$	✠✠✠ = 3	$\langle 3,(0.73e^{i1.57\pi},0.33e^{i0.72\pi})\rangle$	0.296102
$\mathbb{E}_3$	✠✠✠✠ = 4	$\langle 4,(0.92e^{i1.87\pi},0.13e^{i0.33\pi})\rangle$	0.429545

employing the following formula:

$$f_{nv} = (m_{nv})^3 - (o_{nv})^3. \tag{5.1}$$

Step 3. In the proposed MAGDM problem, radioactive waste disposal facility, water availability and storage space of fuel are benefit type attributes. Whereas economical cost of land and shipping resources are cost type criteria. According to nature of the attributes, positive ideal solution and

Table 6. Aggregated CFFNS_fDM.

$\mathfrak{D}$	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$
$\mathbb{A}_1$	$\langle 1,(0.378644e^{i0.421715\pi}, 0.682904e^{i1.340027\pi})\rangle$	$\langle 2,(0.469031e^{i0.802080\pi}, 0.540929e^{i1.320013\pi})\rangle$	$\langle 3,(0.778317e^{i1.549737\pi}, 0.281430e^{i0.655285\pi})\rangle$
$\mathbb{A}_2$	$\langle 2,(0.421732e^{i1.039657\pi}, 0.481449e^{i1.042192\pi})\rangle$	$\langle 2,(0.479852e^{i0.845376\pi}, 0.542550e^{i1.228860\pi})\rangle$	$\langle 3,(0.686278e^{i1.360559\pi}, 0.415189e^{i0.862132\pi})\rangle$
$\mathbb{A}_3$	$\langle 3,(0.693069e^{i1.340526\pi}, 0.221397e^{i0.442521\pi})\rangle$	$\langle 3,(0.726251e^{i1.473471\pi}, 0.248312e^{i0.557840\pi})\rangle$	$\langle 0,(0.062623e^{i0.318817\pi}, 0.960141e^{i1.870248\pi})\rangle$
$\mathbb{A}_4$	$\langle 2,(0.521714e^{i1.115801\pi}, 0.425654e^{i0.956948\pi})\rangle$	$\langle 2,(0.558896e^{i0.884453\pi}, 0.472134e^{i1.179224\pi})\rangle$	$\langle 1,(0.251827e^{i0.628246\pi}, 0.771508e^{i1.441528\pi})\rangle$
$\mathbb{A}_5$	$\langle 1,(0.221862e^{i0.531683\pi}, 0.692907e^{i1.355787\pi})\rangle$	$\langle 1,(0.331202e^{i0.560784\pi}, 0.767243e^{i1.535471\pi})\rangle$	$\langle 0,(0.144966e^{i0.255883\pi}, 0.972964e^{i1.865584\pi})\rangle$
$\mathbb{A}_6$	$\langle 0,(0.033525e^{i0.249596\pi}, 0.918627e^{i1.95877\pi})\rangle$	$\langle 0,(0.136998e^{i0.146825\pi}, 0.874058e^{i1.918430\pi})\rangle$	$\langle 3,(0.689349e^{i1.328675\pi}, 0.427867e^{i0.896116\pi})\rangle$
	$\rfloor_4$	$\rfloor_5$	
$\mathbb{A}_1$	$\langle 2,(0.566608e^{i0.907198\pi}, 0.583704e^{i1.201733\pi})\rangle$	$\langle 1,(0.263112e^{i0.574401\pi}, 0.756113e^{i1.661952\pi})\rangle$	
$\mathbb{A}_2$	$\langle 1,(0.264115e^{i0.495612\pi}, 0.795168e^{i1.620391\pi})\rangle$	$\langle 3,(0.701858e^{i1.565021\pi}, 0.423221e^{i0.772424\pi})\rangle$	
$\mathbb{A}_3$	$\langle 1,(0.223697e^{i0.377210\pi}, 0.901154e^{i1.737294\pi})\rangle$	$\langle 2,(0.492998e^{i0.938034\pi}, 0.638689e^{i1.095389\pi})\rangle$	
$\mathbb{A}_4$	$\langle 3,(0.792850e^{i1.511352\pi}, 0.383712e^{i0.765447\pi})\rangle$	$\langle 1,(0.288957e^{i0.626657\pi}, 0.749688e^{i1.683427\pi})\rangle$	
$\mathbb{A}_5$	$\langle 2,(0.454392e^{i0.711337\pi}, 0.743951e^{i1.431413\pi})\rangle$	$\langle 0,(0.153897e^{i0.295910\pi}, 0.891514e^{i1.748116\pi})\rangle$	
$\mathbb{A}_6$	$\langle 1,(0.306727e^{i0.763100\pi}, 0.814971e^{i1.513101\pi})\rangle$	$\langle 1,(0.272922e^{i0.627241\pi}, 0.710063e^{i1.589663\pi})\rangle$	

Table 7. Performance rating of criteria according to decision-makers.

Decision-makers	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$	$\rfloor_4$	$\rfloor_5$
$\mathbb{E}_1$	✠✠ = 2	✠✠✠ = 3	✠✠✠✠ = 4	✠✠✠ = 3	✠✠✠ = 3
$\mathbb{E}_2$	✠✠ = 2	✠✠ = 2	✠✠✠ = 3	✠✠✠ = 3	✠✠✠ = 3
$\mathbb{E}_3$	✠✠ = 2	✠✠ = 2	✠✠✠ = 3	✠✠✠ = 3	✠✠✠ = 3

Table 8. CFFNS_fNs corresponding to the importance grade of criteria.

Decision-makers	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$
$\mathbb{E}_1$	$\langle 2,(0.41e^{i0.82\pi}, 0.63e^{i1.25\pi})\rangle$	$\langle 3,(0.67e^{i0.21\pi}, 1.65e^{i0.75\pi})\rangle$	$\langle 4,(0.95e^{i1.87\pi}, 0.17e^{i0.27\pi})\rangle$
$\mathbb{E}_2$	$\langle 2,(0.44e^{i0.93\pi}, 0.52e^{i1.12\pi})\rangle$	$\langle 2,(0.46e^{i0.88\pi}, 0.59e^{i1.14\pi})\rangle$	$\langle 3,(0.71e^{i0.28\pi}, 1.49e^{i0.72\pi})\rangle$
$\mathbb{E}_3$	$\langle 2,(0.43e^{i0.97\pi}, 0.55e^{i1.03\pi})\rangle$	$\langle 2,(0.49e^{i0.95\pi}, 0.61e^{i1.07\pi})\rangle$	$\langle 3,(0.81e^{i0.37\pi}, 1.55e^{i0.75\pi})\rangle$
	$\rfloor_4$	$\rfloor_5$	
$\mathbb{E}_1$	$\langle 3,(0.78e^{i0.27\pi}, 1.52e^{i0.61\pi})\rangle$	$\langle 3,(0.81e^{i0.23\pi}, 1.45e^{i0.54\pi})\rangle$	
$\mathbb{E}_2$	$\langle 3,(0.73e^{i0.35\pi}, 1.65e^{i0.56\pi})\rangle$	$\langle 3,(0.79e^{i0.34\pi}, 1.34e^{i0.43\pi})\rangle$	
$\mathbb{E}_3$	$\langle 3,(0.68e^{i0.25\pi}, 1.38e^{i0.47\pi})\rangle$	$\langle 2,(0.52e^{i0.97\pi}, 0.57e^{i1.28\pi})\rangle$	

negative ideal solution are determined and the results are arranged in Table 16.

Step 4. To find out the distances of each location $\mathbb{A}_n$ from Fermatean fuzzy positive ideal solution $\mathfrak{D}^+$ and Fermatean fuzzy negative ideal solution $\mathfrak{D}^-$ following equations are employed [33]:

$$d(\mathbb{A}_n, \mathfrak{D}^+) = \frac{1}{2}\sum_{v=1}^{p}\zeta_v\sqrt{\frac{1}{2}\Big\{(m_{nv}^3 - (m_v^+)^3)^2 + (o_{nv}^3 - (o_v^+)^3)^2 + (\pi_{nv}^3 - (\pi_v^+)^3)^2\Big\}},$$

Table 9. Normalized weights of criteria.

Decision-makers	**Weights**	**Normalized weights**
$\rfloor_1$	$\langle 2, (0.427821e^{i0.921691\pi}, 0.561457e^{i1.113462\pi})\rangle$	0.130178
$\rfloor_2$	$\langle 3, (0.550247e^{i0.834352\pi}, 0.793608e^{i0.988995\pi})\rangle$	0.166890
$\rfloor_3$	$\langle 4, (0.856737e^{i1.442302\pi}, 0.835426e^{i0.559863\pi})\rangle$	0.279169
$\rfloor_4$	$\langle 3, (0.726880e^{i0.291682\pi}, 1.494071e^{i0.531733\pi})\rangle$	0.228613
$\rfloor_5$	$\langle 3, (0.722808e^{i0.748796\pi}, 0.948517e^{i0.731311\pi})\rangle$	0.195150

Table 10. Score matrix $\mathcal{F}$.

$\mathcal{F}$	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$	$\rfloor_4$	$\rfloor_5$
$\mathbb{A}_1$	-0.53997	-0.51310	1.301148	-0.01558	-0.94855
$\mathbb{A}_2$	0.087382	-0.08066	0.908246	-0.98534	1.11335
$\mathbb{A}_3$	1.034220	1.167804	-1.69855	-1.35372	-0.07683
$\mathbb{A}_4$	0.253991	0.075844	-0.77107	1.239239	-0.94717
$\mathbb{A}_5$	-0.59886	-0.83016	-1.72755	-0.51455	-1.36945
$\mathbb{A}_6$	-1.71265	-1.54736	0.874376	-0.87428	-0.79335

Table 11. Tabulated arrangement of the best and worst values of each attribute.

Criteria	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$	$\rfloor_4$	$\rfloor_5$
f_v^+	1.034220	-1.54736	1.301148	-1.35372	1.11335
f_v^-	-1.71265	1.167804	-1.72755	1.239239	-1.36945

Table 12. Values of $\mathcal{S}, \mathcal{R}$, and Q for alternatives.

Sites	$\mathcal{S}$	$\mathcal{R}$	Q
$\mathbb{A}_1$	0.440350	0.162067	0.420148
$\mathbb{A}_2$	0.203718	0.090152	0
$\mathbb{A}_3$	0.536936	0.276497	0.816690
$\mathbb{A}_4$	0.718326	0.228613	0.866267
$\mathbb{A}_5$	0.669783	0.279169	0.952836
$\mathbb{A}_6$	0.361653	0.149868	0.311415

Table 13. Ranking of location.

Sites	$\mathcal{S}$	$\mathcal{R}$	$\mathcal{Q}$
$\mathbb{A}_1$	3	3	3
$\mathbb{A}_2$	1	1	1
$\mathbb{A}_3$	4	5	4
$\mathbb{A}_4$	6	4	5
$\mathbb{A}_5$	5	6	6
$\mathbb{A}_6$	2	2	2

Table 14. Fermatean fuzzy decision matrix.

	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$
$\mathbb{A}_1$	(0.378644, 0.682904)	(0.469031, 0.540929)	(0.778317, 0.281430)
$\mathbb{A}_2$	(0.421732, 0.481449)	(0.479852, 0.542550)	(0.686278, 0.415189)
$\mathbb{A}_3$	(0.693069, 0.221397)	(0.726251, 0.248312)	(0.062623, 0.960141)
$\mathbb{A}_4$	(0.521714, 0.425654)	(0.558896, 0.472134)	(0.251827, 0.771508)
$\mathbb{A}_5$	(0.221862, 0.692907)	(0.331202, 0.767243)	(0.144966, 0.972964)
$\mathbb{A}_6$	(0.033525, 0.918627)	(0.136998, 0.874058)	(0.689349, 0.427867)

	$\rfloor_4$	$\rfloor_5$
$\mathbb{A}_1$	(0.566608, 0.583704)	(0.263112, 0.756113)
$\mathbb{A}_2$	(0.264115, 0.795168)	(0.701858, 0.423221)
$\mathbb{A}_3$	(0.223697, 0.901154)	(0.492998, 0.638689)
$\mathbb{A}_4$	(0.792850, 0.383712)	(0.288957, 0.749688)
$\mathbb{A}_5$	(0.454392, 0.743951)	(0.153897, 0.891514)
$\mathbb{A}_6$	(0.306727, 0.814971)	(0.272922, 0.710063)

$$d(\mathbb{A}_n, \mathfrak{D}^-) = \frac{1}{2}\sum_{v=1}^{p} \zeta_v \sqrt{\frac{1}{2}\Big\{(m_{nv}^3 - (m_v^-)^3)^2 + (o_{nv}^3 - (o_v^-)^3)^2 + ((\pi_{nv}^3 - (\pi_v^-)^3)^2)\Big\}}.$$

The results are collected in Table 17.

Step 5. In this step, the closeness index of all alternatives and their descending order ranking is given in Table 18, where the closeness index is calculated by employing the following equation:

$$\Psi(\mathbb{A}_n) = \frac{d(\mathbb{A}_n, \mathfrak{D}^-)}{d_{\max}(\mathbb{A}_n, \mathfrak{D}^-)} - \frac{d(\mathbb{A}_n, \mathfrak{D}^+)}{d_{\min}(\mathbb{A}_n, \mathfrak{D}^-)}, \quad n = 1, 2, 3, \ldots, s, \tag{5.2}$$

Table 15. Score matrix of Fermatean fuzzy values.

$\mathcal{F}$	$\rfloor_1$	$\rfloor_2$	$\rfloor_3$	$\rfloor_4$	$\rfloor_5$
$\mathbb{A}_1$	-0.26419	-0.05510	0.449197	-0.016967	-0.41406
$\mathbb{A}_2$	-0.03659	-0.04922	0.251651	-0.484355	0.269933
$\mathbb{A}_3$	0.322059	0.367743	-0.88488	-0.720614	-0.14071
$\mathbb{A}_4$	0.064883	0.069335	-0.44325	0.441898	-0.39722
$\mathbb{A}_5$	-0.32176	-0.41531	-0.91802	-0.31793	-0.70493
$\mathbb{A}_6$	-0.77517	-0.66519	0.249250	-0.512428	-0.33768

Table 16. Fermatean Fuzzy positive and negative ideal solutions.

Attributes	**Positive ideal solution** ($\mathfrak{D}^+$)	**Negative ideal solution** ($\mathfrak{D}^-$)
$\rfloor_1$	$(0.693069, 0.221397)$	$(0.033525, 0.918627)$
$\rfloor_2$	$(0.136998, 0.874058)$	$(0.726251, 0.248312)$
$\rfloor_3$	$(0.778317, 0.281430)$	$(0.144966, 0.972964)$
$\rfloor_4$	$(0.223697, 0.901154)$	$(0.792850, 0.383712)$
$\rfloor_5$	$(0.701858, 0.423221)$	$(0.153897, 0.891514)$

Table 17. Distance of each alternative from ideal solution.

Alternatives	$d(\mathbb{A}_n, \mathfrak{D}^+)$	$d(\mathbb{A}_n, \mathfrak{D}^-)$
$\mathbb{A}_1$	0.145489	0.214696
$\mathbb{A}_2$	0.097371	0.270975
$\mathbb{A}_3$	0.172152	0.156725
$\mathbb{A}_4$	0.216528	0.147373
$\mathbb{A}_5$	0.232860	0.105463
$\mathbb{A}_6$	0.111452	0.238118

where

$$
\begin{aligned}
d_{\max}(\mathbb{A}_n, \mathfrak{D}^-) &= \max_{1\le n\le s} d(\mathbb{A}_n, \mathfrak{D}^-),\\
d_{\min}(\mathbb{A}_n, \mathfrak{D}^+) &= \min_{1\le n\le s} d(\mathbb{A}_n, \mathfrak{D}^+).
\end{aligned}
$$

Table 18. Revised closeness index and optimal ranking of each alternative.

Sites	$\Psi(\mathbb{A}_n)$	Ranking
$\mathbb{A}_1$	−0.70186	3
$\mathbb{A}_2$	0	1
$\mathbb{A}_3$	−1.18962	4
$\mathbb{A}_4$	−1.67987	5
$\mathbb{A}_5$	−2.00227	6
$\mathbb{A}_6$	−0.26587	2

Table 19. Comparison between CFFNS_f-VIKOR and Fermatean fuzzy TOPSIS method.

Methods	Ranking of the most suitable location for nuclear power plant	Best city
CFFNS_f-VIKOR method (proposed)	$\mathbb{A}_2 > \mathbb{A}_6 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_5$	$\mathbb{A}_2$
Fermatean fuzzy TOPSIS method [33]	$\mathbb{A}_2 > \mathbb{A}_6 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_5$	$\mathbb{A}_2$

Step 6. Table 18 demonstrates the optimal ranking of suitable locations for nuclear power plant $\mathbb{A}_2 > \mathbb{A}_6 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_5$. Thus, the best site to start the project of planting nuclear power station is $\mathbb{A}_2$, namely, Karachi.

5.1. Discussion

From the comparison with Fermatean fuzzy TOPSIS method, the following results are derived:

- The summarized results of CFFNS_f-VIKOR method and Fermatean fuzzy TOPSIS approach, are arranged in Table 19, which depicts that in both cases, $\mathbb{A}_2$ is the optimal solution. This implies the conformity of the proposed decision-making method.

- Figure 5 represents a graph between the alternatives and ranking measures $Q(\mathbb{A}_n)$ and $\Psi(\mathbb{A}_n)$, in case of CFFNS_f-VIKOR method and Fermatean fuzzy TOPSIS method, respectively. It is clear from the graph that not only the ranking of best alternative is same, but the optimal order of other alternatives is also same.

- Both decision-making approaches are used to choose the suitable alternatives depending upon the given data and environment of the MAGDM problem. In the CFFNS_f-VIKOR technique, to find out the closeness of alternatives to the ideal solution, L_p-metric is used. Whereas the Fermatean fuzzy TOPSIS approach ranks the alternatives on the basis of closeness index.

- The proposed CFFNS_f-VIKOR approach has potential to handle Fermatean fuzzy data by taking grades and phase terms equal to zero. But the existing Fermatean fuzzy TOPSIS method cannot deal with only two-dimensional data along with grading of parameters. Therefore, this trait proves that our presented technique is more powerful and superior over Fermatean fuzzy TOPSIS method.

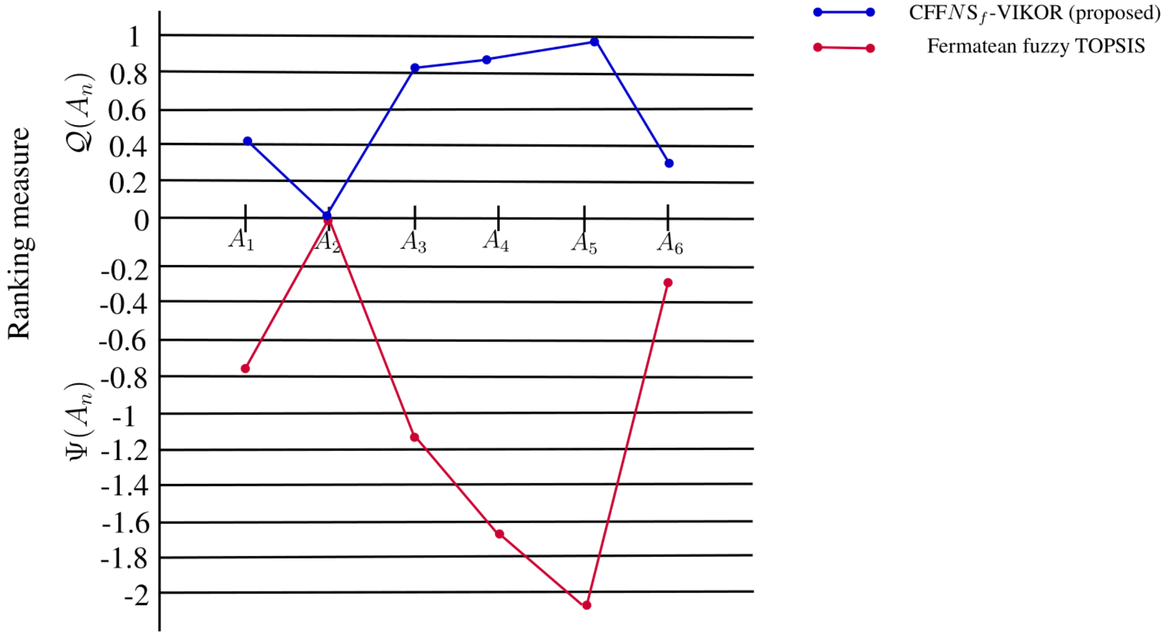

Figure 5. Comparative Study.

- Another advantage of proposed CFF$\mathcal{N}$S$_f$-VIKOR method is that it can efficiently deal with MAGDM problem, whereas Fermatean fuzzy TOPSIS approach does not discuss the decision-making problems having single decision-makers.

6. Merits and limitations of CFF$\mathcal{N}$S$_f$-VIKOR approach

- The proposed CFF$\mathcal{N}$S$_f$-VIKOR technique is competent to capture the two-dimensional data having periodicity and uncertainty along with the rating of parameters at the same time. It is a great tool to operate the obstacles of other proposed models.

- The main aim of the CFF$\mathcal{N}$S$_f$-VIKOR method is to produce a compromise solution that minimizes the individual regret and maximizes the group utility.

- The presented CFF$\mathcal{N}$S$_f$-VIKOR approach portray a general decision-making framework that can excellently deal with not only the CFF$\mathcal{N}$S$_f$ model, but this can be efficiently applied on the other fuzzy models having two-dimensional data which satisfy the condition that cubic sum of membership and nonmembership degrees of amplitude and phase term is less than or equal to 1 such as complex FS, complex intuitionistic FS, complex Pythagorean FS.

- Another edge of our CFF$\mathcal{N}$S$_f$-VIKOR method is that it can also capture one-dimensional models such as intuitionistic FS, Pythagorean FS, and Fermatean FS by taking grades and phase terms equal to 0.

- The adaptability and versatility of our put forward CFF$\mathcal{N}$S$_f$-VIKOR approach is not only restricted to address one and two dimensions, but this method demonstrates the same veracity when applied to parameterized information inclusive of Pythagorean fuzzy N soft set and complex Pythagorean fuzzy N soft set. Thereby, our presented technique is an authentic and flexible approach that effectively manipulates the three one-dimensional, two-dimensional, and parameterized vague information with precision.

- Although, the proposed CFFNS$_f$S along with the VIKOR approach has the potential to handle the uncertain and vague periodic data along with multi-valued parameter grades, it is failed if the cubic sum of membership and nonmembership degrees of amplitude term or phase term or both exceeds from 1.

7. Conclusion

In practical MADM or MAGDM problems, it is extremely rare and even in many cases not possible to opt for the feasible alternative regarding all crucial factors. To get rid of these difficulties, VIKOR is applied that is very technical and provides a compromise solution by employing the L_p metric on crisp data. It broadly applies in engineering, HR department, medical, educational recruitment process, mine industry, automobile manufacturing industry and logistics. Maximum group utility and minimum individual regret to the opponent are two primary features of VIKOR methods. The weight of strategy of the vast majority lies within the unit interval. This method provides an ordered ranking of alternatives that play the vital role to find out the feasible compromise solution under multiple attributes that differ.

This article has emerged the characteristics of the CFFNS$_f$ model with VIKOR technique for MAGDM, namely, CFFNS$_f$-VIKOR method for decision-making. Firstly, the preferences N-soft grades have been assigned to the alternatives, attributes, and decision-makers. After that, the individual CFFNS$_f$ decision matrices of alternatives corresponding to the N-soft grades have been arranged as well as the work done to find out the normalized weights of decision-makers. Then, the aggregated decision matrix has been computed by employing the CFFNS$_f$WA operator, and the resultant entries have been converted into crisp data with the help of the score function. Moreover, the worst and best values of each attribute have been determined. Eventually, the ranking measures have been calculated, and the alternatives have been arranged in ascending order. Lately, based on the satisfaction of one or both conditions, the optimal compromise solution nearest to the ideal solution has been selected. Furthermore, for practical illustration of the proposed technique by selecting the best location for a nuclear power plant has been calculated. Finally, for the sake of showing the veracity and authenticity, a comparative analysis has been performed with Fermatean fuzzy TOPSIS approach. Despite the conviction that the presented strategy excellently deals with uncertain periodic data and multivalued parameterized information N-soft grades information. However, the model drops its flexibility if the cubic sum of amplitude or phase terms or both exceed 1. Thereby, our target is to apply the VIKOR method to a complex q-rung N-soft environment.

In the future, we aim to broaden the literature on some more MAGDM approaches under the environment of CFFNS$_f$, inclusive CFFNS$_f$-AHP method, CFFNS$_f$-ELECTRE I method, and CFFNS$_f$-ELECTRE II method.

Conflict of interest

The authors declare that they have no conflict of interest.

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