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Research article

Single acceptance sampling plans based on truncated lifetime tests for two-parameter Xgamma distribution with real data application

Ghadah Alomani[1,*] and Amer I. Al-Omari[2,*]

[1] Department of Mathematical Sciences, College of Science, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia
[2] Department of Mathematics, Faculty of Science, Al al-Bayt University, Mafraq, Jordan

* **Correspondence:** Email: gaalomani@pnu.edu.sa, alomari_amer@yahoo.com.

Abstract: Recently, the two-parameter Xgamma distribution (TPXGD) is suggested as a new lifetime distribution for modeling some real data. The TPXGD is investigated in different areas and generalized to other forms by many of the researchers. The acceptance sampling plans are one of the main important statistical tools in production and engineering fields. In this paper, modified acceptance sampling plans for the TPXGD are proposed with the assumption that the lifetime is truncated at a predetermined level. The mean of the TPXGD model is utilized as a quality parameter. The variables of the acceptance sampling plans including the acceptance numbers, the minimum sample sizes, operating characteristic function and the producer's risk are investigated for various values of the model parameters. Numerical examples are offered to illustrate the process of the proposed plans. Also, a real data is fitted to the TPXGD and an application based on the suggested acceptance sampling plans is considered for explanation.

Keywords: two-parameter Xgamma distribution; acceptance sampling plans; truncated lifetime test; producer's risk; operating characteristic function; minimum sample size; consumer's risk

1. Introduction

The single acceptance sampling plans (SASP) are very important in the production sector to assert the acceptability of a lot based on its lifetime. The manufacturers are interested in producing a good product with less cost, while the customers are welling to get a suitable product with high quality which may survive for a long time. Indeed, compared to low-quality products, high-quality products have a greater acceptance rate. By inspecting a small sample of the entire batch, quality can be checked in accordance with the ASP rule. The plan known as the acceptance sampling plan specifies sampling policies as well as the standards for accepting or rejecting a lot. This ASP strategy can be used to test initial materials used in processes, or completed goods. There are two types of acceptance sampling plans: attribute acceptance sampling plans and variable acceptance sampling plans.

The SASP in terms of truncated lifetime tests is investigated by several researchers, presuming that a product's life span follows a particular probability distribution. For example, see reference [1] for SASP under the Ishita distribution and reference [2] for SASP for the Rama model. Al-Omari and Al-Nasser [3] considered the two parameters quasi Lindley model in SASP. Al-Omari et al. [4] considered SASP based on truncated life tests for Akash model. In [5] attribute chain sampling plans for Darna distribution are discussed. Al-Omari et al. [6] introduced SASP for two-parameter Quasi Shanker distribution. Reference [7] offered SASP for Tsallis q-exponential distribution. Reference [8] for SASP when the lifetime follows the exponential distribution. Gui and Aslam [9] considered ASP for weighted exponential distribution, and Al-Omaari [10] for the transmuted generalized inverse Weibull distribution.

Also, for other types and extensions of the ASP see Srinivasa [11] for double ASP (DASP) under the Marshall-Olkin extended exponential model. Ramaswamy and Sutharani [12] considered DASP based on truncated lifetime follows Rayleigh model. Reference [13] studied the odd generalized exponential log-logistic distribution in group ASP (GASP). In [14], two-sided group chain ASP for Pareto distribution of the 2nd kind is investigated. ASP based on percentiles for odds exponential log logistic distribution is proposed by [15]. Reference [16] investigated group ASP for generalized exponential distribution. Reference [17] studied single ASP to the new Weibull–Pareto distribution. Teh et al. [18] offered group chain ASP for log logistic distribution. Reference [19] evolved SASP for two-parameter Quasi Shanker distribution. When data are coming from a complex process or from an unpredictable environment, neutrosophic statistics, an extension of classical statistics, is used. Aslam et al. [20] offered variable ASP plan for Pareto distribution based on neutrosophic statistical interval method. Reference [21] studied ASP process loss index for multiple dependent state sampling under neutrosophic statistics. Aslam [22] proposed a modified attribute ASP with implementation of neutrosophic statistical interval method. In [23] fuzzy ASP for the Birnbaum-Saunders distribution is studied.

To our knowledge, this is the first study to suggest SASP for the two-parameter Xgamma distribution. The TPXGD is a well-known in the literature of lifetime distributions and it is generalized to other distribution with numerous applications.

The structure of this paper is as follows. In Section 2, the TPXGD with some of its statistical properties are introduced. The designation and the parameters of the suggested SASP are explained in Section 3. Section 4 is devoted to the minimum sample sizes, operating characteristic function (OC) function values and the minimum ratio of true mean lifetime to the specified mean lifetime for acceptability of a lot tables as well as some illustrated examples are discussed. An application of a real data set is offered in Section 5 to assert the importance of the suggested ASP. Conclusions and future work suggestions are given in Section 6.

2. The TPXGD

This section describes the TPXGD which is proposed by [24] as a modification to the well-known Xgamma distribution by adding a new parameter to the base distribution. The distribution function of the TPXGD is given by

$$F(x)=1-\frac{\alpha+\theta+\alpha\theta x+\frac{\alpha\theta^2x^2}{2}}{\alpha+\theta}e^{-\theta x},\quad x>0,\theta>0,\alpha>0, \tag{1}$$

with corresponding probability density function

$$f(x)=\frac{\theta^2}{\alpha+\theta}\left(1+\frac{\alpha\,\theta}{2}x^2\right)e^{-\theta x},\quad x>0,\theta>0,\alpha>0. \tag{2}$$

Plots of the distribution pdf are presented in Figure 1 which shows that the model is positively skewed.

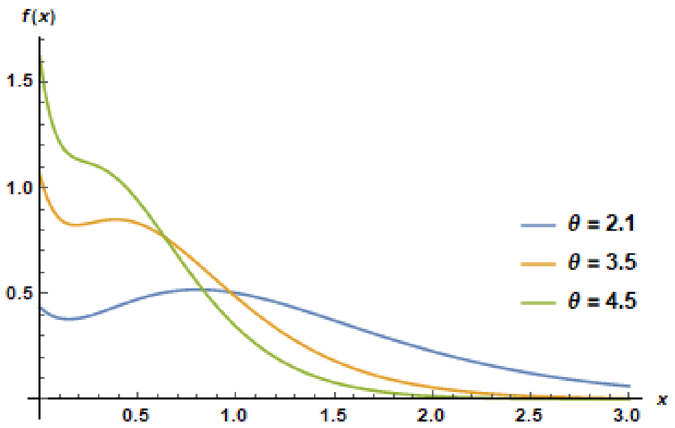

Figure 1. The plots of $f(x)$ when $\alpha=8$ and $\theta=2.1,3.5,4.5$.

The hazard rate and survival functions of the TPXG distributed random variable, respectively, are

$$H(x)=\frac{g(x)}{1-G(x)}=\frac{\theta^2\left(1+\frac{1}{2}\alpha\theta^2x^2\right)}{\alpha+\theta+\alpha\theta x+\frac{1}{2}\alpha\theta^2x^2}, \tag{3}$$

and

$$S(x)=\frac{\alpha\left(\alpha+\theta+\alpha\theta x+\frac{1}{2}\alpha\theta^2x^2\right)}{\alpha+\theta}e^{-\theta x}. \tag{4}$$

Figure 2 consists of the hazard and survival functions of the TPXGD for some parameters.

The rth moment the distribution is given by

$$E\left(X^r\right)=\frac{r!\left(2\theta+\alpha(r+1)(r+2)\right)}{2\theta^r(\alpha+\theta)},\ r=1,2,3,..., \tag{5}$$

with mean and variance, respectively, are given by

$$E(X)=\frac{\theta+3\alpha}{\theta(\theta+\alpha)},\text{ and } Var(X)=\frac{2\left(\theta^2+8\alpha\theta+3\alpha^2\right)}{\theta^2(\alpha+\theta)^2}. \tag{6}$$

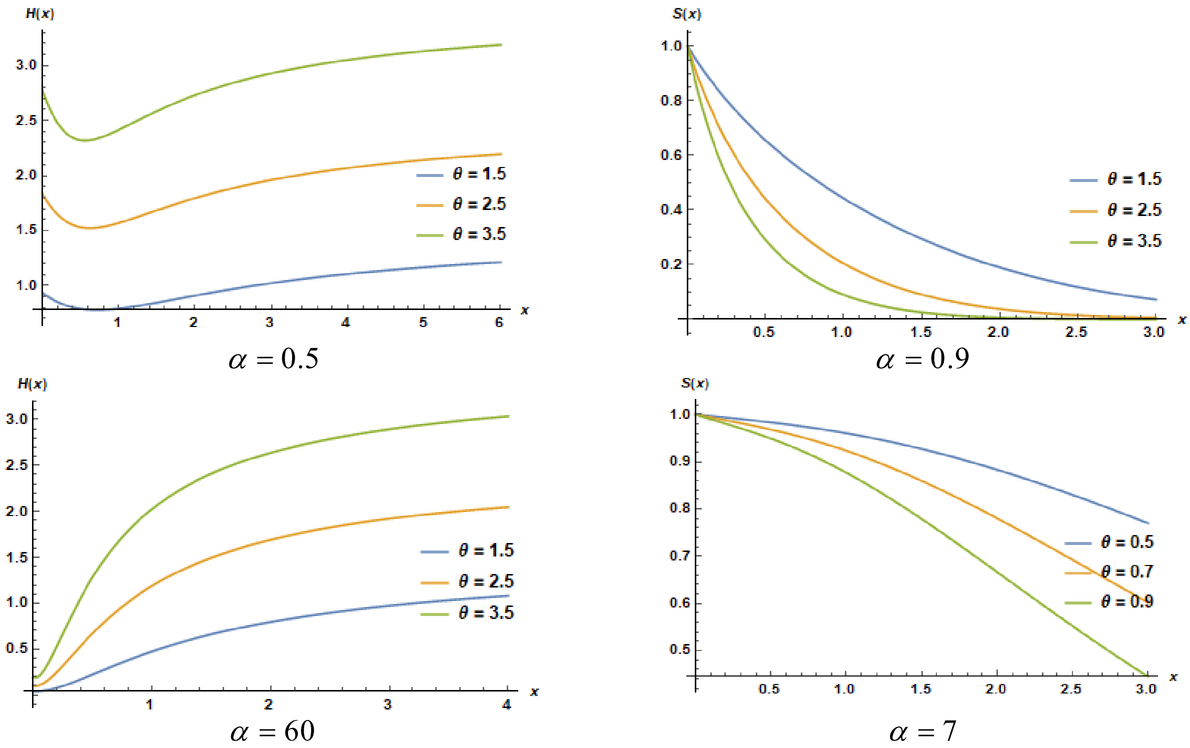

Figure 2. The plots of $H(x)$ and $S(x)$ for some parameters.

3. The suggested ASP

This section describes the single acceptance sampling plans suggested in the current study proposing that the lifetime tests follows the TPXGD. Here, the minimum sample size (MSS), operating characteristic function (OC), and the producer's risk (PR) are introduced. The method for implementing the SASP to get at a decision about the product can be explained as follows:

1) Draw a sample of size m randomly from the lot collected from the supplier or the final production.
2) The sample size m that is drawn from the lot to be tested and distinguish it to good or defective.
3) The test duration time, t.
4) An acceptance number of defective items, c such that if c or less failures occurred within the test time t, the lot is not rejected.
5) The minimum ratio t/μ_0, where μ_0 is the identified average lifetime.

To explain the process, let the consumer's risk is preassigned to be at most $1-P^*$. That is, the probability of the actual average lifetime of the quality parameter μ is not larger than $1-P^*$. For a given c, ratio of t/μ_0 and $0<P^*<1$, we are concerned in finding the optimum minimum sample size m provided that $\mu\geq\mu_0$, and $\mu_0=\dfrac{\theta_0+3\alpha_0}{\theta_0\left(\theta_0+\alpha_0\right)}$. Therefore, the desired sample size is the smallest positive integer satisfying the inequality

$$\sum_{i=0}^{c}\binom{m}{i}p^{i}(1-p)^{m-i}\le 1-P^{*},\tag{7}$$

where $\binom{m}{i}=\frac{m!}{i!(m-1)!}$ and $p=F(t;\mu_0)$ defined in (2) is the probability that the lifetime does not more than t for the true mean μ_0. The above equation is based on the assumption that the size of the lot is large as possible to use the binomial distribution theory.

When the number of failures up to the time t is found to be c or less, then based on (7) we have $F(t;\mu)\le F(t;\mu_0)$ with probability P^{*}, that emphasize $\mu\ge\mu_0$. For the proposed ASP, the minimum sample sizes that satisfying (7) are summarized in Table 1 for $t/\mu_0=0.628,\ 0.942$, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 with $P^{*}=0.75$, 0.90, 0.95, 0.99 and $c=0,1,2,\ldots,10$.

In actuality, the OC is a crucial indicator of how well the sampling plan performed. The OC function contributes in finding the probability of acceptance the lot and it is defined as

$$OC(p)=\sum_{i=0}^{c}\binom{m}{i}p^{i}(1-p)^{m-i}=1-I_{p}(c+1,m-c),\tag{8}$$

where $I_{p}(c+1,m-c)=\frac{m!}{c!(m-c-1)!}\int_{0}^{p}z^{c}(1-z)^{m-c-1}dz$ is known as the incomplete beta function. Table 2 is devoted to the OC values for the suggested SASP.

The producer's risk is defined as the probability of rejecting the lot if $\mu>\mu_0$, and it given as

$$P(p)=\sum_{i=c+1}^{m}\binom{m}{i}p^{i}(1-p)^{m-i}=I_{p}(c+1,m-c).\tag{9}$$

For the offered ASP with a given value of the PR, say β, it is essential to find the value of the ratio μ/μ_0, that keeps the PR at most β. Since $p=F(t;\mu)$ can be gained as a function of μ/μ_0, hence μ/μ_0 is the minimum positive number under which

$$F\left(\frac{x}{\mu_0};\frac{\mu}{\mu_0},\alpha,\theta\right)=1-\frac{\alpha+\theta+\alpha\frac{x}{\mu_0}\frac{\mu_0}{\mu}\left(\frac{\theta+3\alpha}{\theta+\alpha}\right)+\frac{\alpha}{2}\left(\frac{x}{\mu_0}\frac{\mu_0}{\mu}\left(\frac{\theta+3\alpha}{\theta+\alpha}\right)\right)^{2}}{\alpha+\theta}e^{-\frac{x}{\mu_0}\frac{\mu_0}{\mu}\left(\frac{\theta+3\alpha}{\theta+\alpha}\right)},\tag{10}$$

satisfies the

$$PR(p)=\sum_{i=c+1}^{m}\binom{m}{i}p^{i}(1-p)^{m-i}\le\beta.\tag{11}$$

For the suggested plan $(m,\,c,\,t/\mu_0)$ with a confidence level of P^{*}, the lowest values of μ/μ_0 satisfying (10) are obtainable in Table 3.

4. Discussion and examples

In this section, it is assumed that the lifetime model is the TPXG distribution with $\theta=3$ and $\alpha=2$. Table 1 involved the minimum sample size essential to assertion that the mean lifetime surpasses

μ_0 with a probability level at least P^*. Table 3 summarizes the minimum ratio of the true average lifetime to the identified mean lifetime.

For illustration, let $P^* = 0.90$, $c = 2$, and $t/\mu_0 = 0.942$, the tabulated minimum sample size values in Table 1 is 8. Which means that the mean lifetime should be at least 1000 hours and the time test can be terminated at 942 hours. Hence, if out the 8 units there are more than two failures, the lot is accepted and ignored otherwise.

Table 1. Minimum sample sizes required to the TPXG distribution SASP with $\theta = 3$ and $\alpha = 2$.

P^*		t/μ_0							
	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	0	3	2	2	1	1	1	1	1
	1	6	4	3	3	2	2	2	2
	2	8	6	5	4	3	3	3	3
	3	11	8	7	6	5	4	4	4
	4	13	10	8	7	6	5	5	5
0.75	5	16	12	10	9	7	6	6	6
	6	18	14	11	10	8	7	7	7
	7	21	15	13	11	9	8	8	8
	8	23	17	14	13	10	10	9	9
	9	25	19	16	14	12	11	10	10
	10	28	21	17	15	13	12	11	11
	0	4	3	2	2	1	1	1	1
	1	7	5	4	4	3	2	2	2
	2	10	8	6	5	4	3	3	3
	3	13	10	8	7	5	5	4	4
	4	16	12	9	8	6	6	5	5
0.90	5	18	14	11	10	8	7	6	6
	6	21	16	13	11	9	8	7	7
	7	24	17	14	12	10	9	8	8
	8	26	19	16	14	11	10	9	9
	9	29	21	17	15	12	11	11	10
	10	31	23	19	17	13	12	12	11
	0	5	4	3	2	2	1	1	1
	1	9	6	5	4	3	3	2	2
	2	12	9	7	6	4	4	3	3
	3	15	11	9	7	6	5	4	4
0.95	4	18	13	10	9	7	6	6	5
	5	20	15	12	10	8	7	7	6
	6	23	17	14	12	9	8	8	7
	7	26	19	15	13	11	9	9	8
	8	28	21	17	15	12	10	10	9

Continued on next page

P^*	t/μ_0								
	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.95	9	31	23	19	16	13	11	11	10
	10	34	25	20	18	14	13	12	11
0.99	0	8	6	4	3	2	2	2	1
	1	12	8	7	5	4	3	3	2
	2	15	11	9	7	5	4	4	4
	3	19	13	10	9	7	5	5	5
	4	22	16	12	10	8	7	6	6
	5	25	18	14	12	9	8	7	7
	6	28	20	16	14	10	9	8	8
	7	30	22	18	15	12	10	9	9
	8	33	24	19	17	13	11	10	10
	9	36	26	21	18	14	12	11	11
	10	39	28	23	20	15	13	12	12

The OC values under the proposed SASP are given in Table 2 with the indicated values of P^* with values of t/μ_0 when $c=2$. From the table, it is clear that the OC increases as the average life ratio increases, implies that accepting the lot of items in case it has larger probability. This motivates the producer to increase the mean life of the product. For the SASP $(m=8,\ c=2,\ t/\mu_0=0.942)$ the corresponding OC and PR values are:

μ/μ_0	2	4	6	8	10	12
$OC(p)$	0.374378	0.759977	0.890120	0.941580	0.965472	0.977966
PR	0.625622	0.240023	0.109880	0.058420	0.034528	0.022033

Now, for the SASP given above, with the ratio $\mu/\mu_0=4$, the OC value is 0.759977 and the corresponding PR is 0.240023. The PR values decrease as the values of the ratio μ/μ_0 increase.

Table 3 gives the smallest values of ratio of the true mean lifetime to the specified mean lifetime, μ/μ_0 keeping that the PR is not more than 0.05. For explanation, to the SASP $(m=8,\ c=2,\ t/\mu_0=0.942)$ we have μ/μ_0 is 12.26 with $P^*=0.90$; that is the product must have a mean lifetime of 12.26 times the identified mean lifetime in order to accept the lot with probability of 0.90 .

5. Applications of real data

In this section, a real data set is analyzed to demonstrate the practicality of the suggested ASP in practical situations. The data is given by [25] in which 20 items are tested till failure are discussed. The data values are: 11.24, 1.92, 12.74, 22.48, 9.60, 11.50, 8.86, 7.75, 5.73, 9.37, 30.42, 9.17, 10.20, 5.52, 5.85, 38.14, 2.99, 16.58, 18.92, 13.36. Some descriptive statistics of the data are given in Table 4. It is clear that the data is nonsymmetrical distributed where its positively skewed.

The unknown distribution parameters are estimated using the maximum likelihood estimation method. For fitting the data, we consider the negative log-likelihood values (-LL), Anderson-Darling

(AD), Cramer von Mises (CR), Akaike information criteria (AIC), consistent Akaike information criteria (CAIC), Baysian information criteria (BIC), and Hanan Quinn information criteria (HQIC) which are defined as

$$\text{AIC} = -LL + 2w, \qquad \text{CAIC} = -LL + \frac{2wm}{n-w-1}, \qquad \text{BIC} = -LL + w\,log(m),$$

$\text{HQIC} = 2log\left[log(m)(w-LL)\right]$, where w is the number of parameters and m is the sample size.

Table 2. OC values for SASP $(m, c=2, t/\mu_0)$ under the TPXG distribution for $\theta = 3, \alpha = 2$.

P^*	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	8	0.628	0.635837	0.896581	0.958435	0.979440	0.988392	0.992823
	6	0.942	0.616989	0.887124	0.953547	0.976676	0.986698	0.991716
	5	1.257	0.595297	0.877578	0.948621	0.973858	0.984952	0.990565
	4	1.571	0.653745	0.901958	0.959528	0.979527	0.988245	0.992638
	3	2.356	0.698753	0.921130	0.967825	0.983694	0.990594	0.994082
	3	3.141	0.526805	0.857826	0.938762	0.967833	0.980956	0.987778
	3	3.927	0.371298	0.782106	0.901644	0.946804	0.967819	0.978997
	3	4.712	0.247912	0.698753	0.857796	0.921130	0.951406	0.967825
0.90	10	0.628	0.456480	0.812994	0.918907	0.958235	0.975823	0.984796
	8	0.942	0.374378	0.759977	0.890120	0.941580	0.965472	0.977966
	6	1.257	0.425782	0.791478	0.905933	0.950255	0.970669	0.981305
	5	1.571	0.441512	0.801533	0.910567	0.952600	0.971981	0.982101
	4	2.356	0.386176	0.774694	0.895740	0.943527	0.966050	0.978025
	3	3.141	0.515125	0.850139	0.934710	0.965526	0.979534	0.986844
	3	3.927	0.361353	0.772235	0.895728	0.943208	0.965511	0.977438
	3	4.712	0.240445	0.687488	0.850108	0.916147	0.948079	0.965517
0.95	12	0.628	0.321593	0.726308	0.873051	0.932185	0.959837	0.974344
	9	0.942	0.286085	0.694325	0.853205	0.919872	0.951849	0.968920
	7	1.257	0.301402	0.705925	0.858723	0.922662	0.953393	0.969843
	6	1.571	0.287089	0.695685	0.851883	0.918102	0.950274	0.967641
	4	2.356	0.386176	0.774694	0.895740	0.943527	0.966050	0.978025
	4	3.141	0.203275	0.639358	0.817711	0.895763	0.934930	0.956693
	3	3.927	0.361353	0.772235	0.895728	0.943208	0.965511	0.977438
	3	4.712	0.240445	0.687488	0.850108	0.916147	0.948079	0.965517
0.99	15	0.628	0.178534	0.592205	0.791945	0.882826	0.928246	0.953100
	11	0.942	0.159108	0.563370	0.770407	0.868125	0.918140	0.945964
	9	1.257	0.138739	0.534669	0.748625	0.852972	0.907563	0.938407
	7	1.571	0.178516	0.588600	0.784643	0.875954	0.922689	0.948770
	5	2.356	0.192501	0.615615	0.801609	0.885884	0.928722	0.952623
	4	3.141	0.203275	0.639358	0.817711	0.895763	0.934930	0.956693
	4	3.927	0.095355	0.506229	0.730147	0.838322	0.895722	0.928879
	4	4.712	0.040839	0.386176	0.639300	0.774694	0.850426	0.895740

Table 3. Minimum ratio of the true mean life to the specified mean lifetime for acceptability of a lot with producer's risk of 0.05 under the TPXG distribution for $\theta = 3, \alpha = 2$.

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	0	64.10	64.09	86.20	53.08	80.63	108.17	135.76	163.30
	1	15.34	14.39	13.39	17.28	14.47	20.06	25.66	31.24
	2	7.45	7.99	8.53	7.54	6.66	9.76	12.86	15.95
	3	5.60	5.63	6.54	6.53	7.60	6.07	8.29	10.52
	4	3.98	4.39	4.16	4.22	5.15	4.23	5.97	7.74
0.75	5	3.48	3.62	3.75	4.12	3.73	3.25	4.62	6.08
	6	2.74	3.09	2.68	3.02	2.89	2.70	3.80	5.01
	7	2.54	2.24	2.58	2.33	2.40	2.37	3.28	4.30
	8	2.10	2.04	2.02	2.44	2.09	3.26	2.93	3.82
	9	1.78	1.89	2.01	2.03	2.49	2.88	2.68	3.47
	10	1.72	1.77	1.70	1.77	2.22	2.61	2.50	3.20
	0	86.14	97.15	86.20	108.23	80.63	108.17	135.76	163.30
	1	18.55	19.24	19.92	25.42	27.00	20.06	25.66	31.24
	2	10.27	12.26	11.42	11.23	12.51	9.76	12.86	15.95
	3	7.31	8.23	8.29	8.77	7.60	10.99	8.29	10.52
	4	5.79	6.23	5.43	5.84	5.15	7.79	5.97	7.74
0.90	5	4.41	5.03	4.72	5.34	5.66	5.89	4.62	6.08
	6	3.87	4.23	4.22	3.99	4.35	4.65	3.80	5.01
	7	3.47	3.18	3.21	3.07	3.47	3.82	3.28	4.30
	8	2.90	2.82	3.02	3.07	2.89	3.26	2.93	3.82
	9	2.69	2.54	2.41	2.51	2.49	2.88	4.07	3.47
	10	2.32	2.32	2.34	2.56	2.22	2.61	3.65	3.20
	0	108.18	130.21	130.32	108.23	163.33	108.17	135.76	163.30
	1	24.94	24.05	26.37	25.42	27.00	36.72	25.66	31.24
	2	13.07	14.37	14.26	14.82	12.51	17.46	12.86	15.95
	3	9.00	9.51	10.02	8.77	11.03	10.99	8.29	10.52
	4	6.97	7.13	6.67	7.41	7.65	7.79	10.43	7.74
0.95	5	5.32	5.73	5.67	5.34	5.66	5.89	8.07	6.08
	6	4.60	4.79	4.98	4.97	4.35	4.65	6.51	5.01
	7	4.08	4.13	3.85	3.87	4.70	3.82	5.42	4.30
	8	3.43	3.63	3.56	3.75	3.87	3.26	4.64	3.82
	9	3.15	3.24	3.33	3.06	3.26	2.88	4.07	3.47
	10	2.93	2.93	2.73	3.04	2.83	3.50	3.65	3.20
	0	174.30	196.33	174.43	163.37	163.33	218.42	273.59	163.30
	1	34.51	33.65	39.19	33.48	39.18	36.72	46.45	31.24
0.99	2	17.25	18.56	19.88	18.36	17.98	17.46	22.42	27.37
	3	12.35	12.05	11.73	13.09	14.32	10.99	14.38	17.76
	4	9.32	9.80	9.09	8.93	10.02	11.06	10.43	13.05

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P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.99	5	7.55	7.76	7.52	7.70	7.51	8.45	8.07	10.25
	6	6.39	6.44	6.47	6.86	5.86	6.73	6.51	8.40
	7	5.28	5.50	5.72	5.46	5.94	5.52	5.42	7.08
	8	4.71	4.81	4.63	5.11	4.91	4.64	4.64	6.11
	9	4.27	4.27	4.26	4.24	4.14	3.98	4.07	5.37
	10	3.92	3.84	3.96	4.07	3.54	3.50	3.65	4.81

Table 4. Some descriptive statistics to the data.

Mean	**Sd**	**Median**	**Mad**	**Min**	**Max**	**Range**	**Skew**	**Kurtosis**	**SE**
12.62	9.03	9.9	5.57	1.92	38.14	36.22	1.37	1.25	2.02

Also, we calculate the Kolmogorov-Smirov statistic (K-S) with the corresponding P-value where K-S $= \underset{X}{Sup}\left|F_m(x)-F(x)\right|$, $F_m(x)$ is empirical distribution function and $F(x)$ is cumulative distribution function. The results are presented in Table 5. Figure 3 displays the P-P and Q-Q plots of the TPXGD to the real data, while Figure 4 represents the density and TTT plots based on the real data.

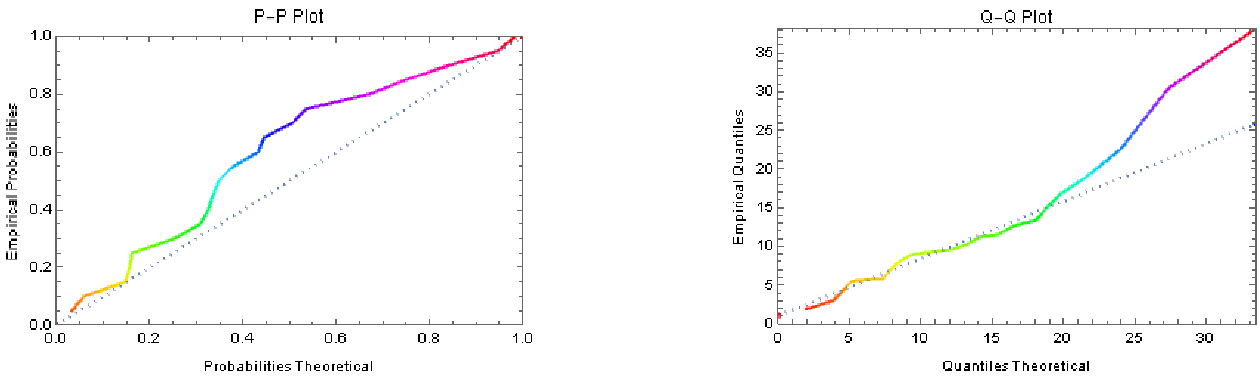

Figure 3. P-P and Q-Q plots for the TPXGD for the data.

Table 5. Model fitting summary for the data sets.

AIC	**CAIC**	**BIC**	**HQIC**	**MLE**	
139.4887	140.1946	141.4802	139.8775	$\hat{\theta}=0.237329$ $\hat{\alpha}=70.485903$	
-2MLL	**Error**	**W**	**A**	**KS**	**P. Value**
67.74436	0.028931	0.058147	0.329026	0.13506	0.8123

For this data set the maximum likelihood estimators of the model parameters are $\hat{\theta}=0.2373291$ and $\hat{\alpha}=70.4859029$. Consequently, the mean in the quality parameter in this study and the estimated mean of the data is $\hat{\mu}=\dfrac{\hat{\theta}+3\hat{\alpha}}{\hat{\theta}(\hat{\theta}+\hat{\alpha})}=12.6124$, and in Tables 6–8 the SASP parameters are obtained based on the estimated values $\hat{\theta}=0.2373291$ and $\hat{\alpha}=70.4859029.$ Assume that the identified mean lifetime is $\mu_0 = 12.6124$ with testing time $t_0 = 7.921$, then, whether the lot can be accepted? Hence, to the ratio $t_0 / \mu_0 = 0.628$, with $P^* = 0.90$ from Table 4 we found $m = 20$ when $c = 3$. Therefore, the deduced SASP is $(m = 20, c = 3, t / \mu_0 = 0.628)$. Now, if the number of failures earlier $t_0 = 7.921$ is less than or equal 3, then the lot can be accepted to the asserted mean lifetime 12.6124, with probability of 0.90. Now, since the number of failures earlier $t_0 = 7.921$ is 13, then the lot would be rejected.

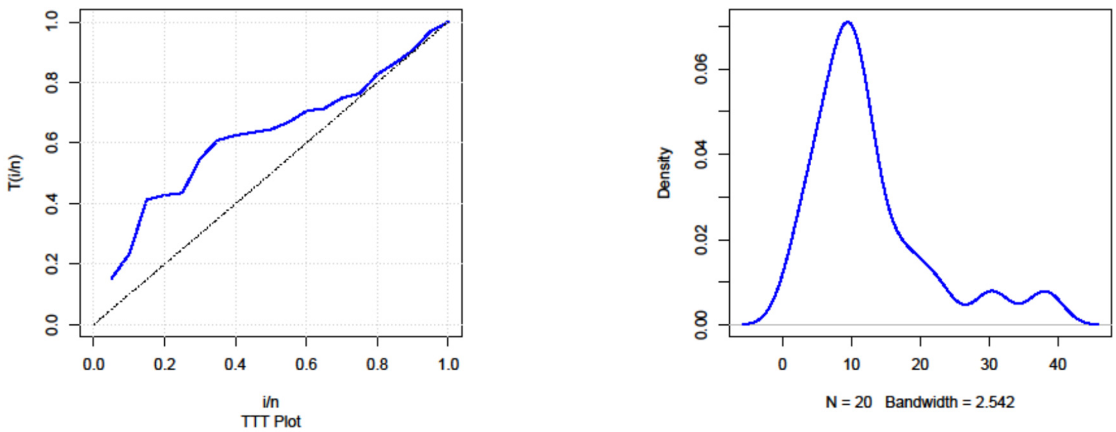

Figure 4. Density and TTT plots for the TPXGD for the data.

Table 6. Minimum sample sizes for a given μ_0 with p^* for c with $\hat{\alpha} = 70.4859029$, $\hat{\theta} = 0.2373291$ in the TPXGD.

p^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	5	2	2	1	1	1	1	1
	1	9	5	3	3	2	2	2	2
	2	13	7	5	4	3	3	3	3
	3	17	9	6	5	4	4	4	4
	4	21	11	8	6	5	5	5	5
	5	25	13	9	8	6	6	6	6
	6	28	15	11	9	7	7	7	7
	7	32	17	12	10	8	8	8	8
	8	36	19	14	11	9	9	9	9
	9	40	21	15	13	11	10	10	10
	10	43	23	17	14	12	11	11	11
0.90	0	7	3	2	2	1	1	1	1
	1	11	6	4	3	2	2	2	2
	2	15	8	6	5	3	3	3	3
	3	20	10	7	6	5	4	4	4

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$p*$	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.90	4	24	13	9	7	6	5	5	5
	5	27	15	10	8	7	6	6	6
	6	31	17	12	10	8	7	7	7
	7	35	19	13	11	9	8	8	8
	8	39	21	15	12	10	9	9	9
	9	42	23	16	14	11	10	10	10
	10	46	25	18	15	12	11	11	11
0.95	0	8	4	3	2	1	1	1	1
	1	13	7	5	4	3	2	2	2
	2	18	9	6	5	4	3	3	3
	3	22	12	8	6	5	4	4	4
	4	27	14	10	8	6	5	5	5
	5	31	16	11	9	7	6	6	6
	6	35	18	13	10	8	7	7	7
	7	39	21	14	12	9	8	8	8
	8	42	23	16	13	10	9	9	9
	9	46	25	18	14	11	10	10	10
	10	50	27	19	16	12	11	11	11
0.99	0	13	6	4	3	2	1	1	1
	1	18	9	6	5	3	2	2	2
	2	24	12	8	6	4	4	3	3
	3	28	15	10	8	5	5	4	4
	4	33	17	12	9	7	6	5	5
	5	37	19	13	10	8	7	6	6
	6	42	22	15	12	9	8	7	7
	7	46	24	17	13	10	9	8	8
	8	50	26	18	14	11	10	9	9
	9	54	29	20	16	12	11	10	10
	10	58	31	21	17	13	12	11	11

Table 7. OC values of the sampling plan $(n, c = 2, t/\mu_0)$ with $\hat{\alpha} = 70.4859029$, $\hat{\theta} = 0.2373291$ in the TPXGD.

$p*$	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	13		0.939372	0.999374	0.999967	0.999996	0.999999	1
	7		0.898246	0.998498	0.999919	0.999990	0.999998	0.999999
	5		0.845933	0.996722	0.999804	0.999976	0.999995	0.999999
	4		0.798061	0.994127	0.999605	0.999950	0.999990	0.999998
	3		0.678702	0.982136	0.998392	0.999766	0.999952	0.999987
	3		0.389604	0.926473	0.990725	0.998393	0.999635	0.999898

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p*	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	3		0.189019	0.820387	0.969014	0.993621	0.998391	0.999518
	3		0.082800	0.678702	0.926439	0.982136	0.995000	0.998392
0.90	15		0.831980	0.990785	0.998109	0.999313	0.999671	0.999815
	8		0.802684	0.991405	0.998674	0.999599	0.999828	0.999910
	6		0.709855	0.985877	0.998109	0.999503	0.999808	0.999907
	5		0.609339	0.976570	0.997009	0.999284	0.999746	0.999885
	3		0.660133	0.976331	0.997065	0.999378	0.999808	0.999924
	3		0.375541	0.914261	0.986781	0.997067	0.999120	0.999668
	3		0.181349	0.803300	0.961114	0.990478	0.997064	0.998902
	3		0.079262	0.660133	0.914224	0.976331	0.992295	0.997065
0.95	18		0.754915	0.984536	0.996735	0.998800	0.999421	0.999674
	9		0.745128	0.987655	0.998055	0.999407	0.999745	0.999866
	6		0.709855	0.985877	0.998109	0.999503	0.999808	0.999907
	5		0.609339	0.976570	0.997009	0.999284	0.999746	0.999885
	4		0.352073	0.925712	0.989522	0.997670	0.999267	0.999704
	3		0.375541	0.914261	0.986781	0.997067	0.999120	0.999668
	3		0.181349	0.803300	0.961114	0.990478	0.997064	0.998902
	3		0.079262	0.660133	0.914224	0.976331	0.992295	0.997065
0.99	24		0.592506	0.966383	0.992490	0.997179	0.998623	0.999218
	12		0.567685	0.971597	0.995237	0.998516	0.999354	0.999659
	8		0.505730	0.965841	0.995070	0.998669	0.999480	0.999745
	6		0.459494	0.958091	0.994327	0.998613	0.999503	0.999773
	4		0.352073	0.925712	0.989522	0.997670	0.999267	0.999704
	4		0.103416	0.770465	0.956500	0.989527	0.996733	0.998741
	3		0.181349	0.803300	0.961114	0.990478	0.997064	0.998902
	3		0.079262	0.660133	0.914224	0.976331	0.992295	0.997065

Table 8. Minimum ratio of the true mean life to the specified mean lifetime for acceptability of a lot with producer's risk of 0.05 $\hat{\alpha} = 70.4859029$, $\hat{\theta} = 0.2373291$ in the TPXGD.

p*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	112.1651	36.9226	74.4778	15.8193	63.282	109.8698	156.0667	202.0572
	1	3.1514	3.3659	3.2710	4.2056	4.7602	6.5806	8.5596	10.7273
	2	2.2992	2.4560	2.6775	2.8750	3.3963	4.5963	5.8360	7.1149
	3	1.9979	2.1073	2.1577	2.3431	2.8260	3.8034	4.8012	5.8176
	4	1.8362	1.9161	2.0743	2.0437	2.5004	3.3571	4.2271	5.1085
	5	1.7333	1.7935	1.8545	2.1078	2.2856	3.0644	3.8532	4.6500
	6	1.6289	1.7073	1.8375	1.9365	2.1311	2.8549	3.5865	4.3243
	7	1.5802	1.6430	1.7060	1.8075	2.0136	2.6958	3.3845	4.0782

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p^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	8	1.5421	1.5930	1.7073	1.7061	1.9204	2.5699	3.2251	3.8843
0.75	9	1.5113	1.5528	1.6158	1.7729	2.1858	2.4672	3.0951	3.7266
	10	1.4667	1.5197	1.6236	1.6944	2.0996	2.3814	2.9867	3.5951
	0	185.6924	93.3754	74.4778	111.3996	63.282	109.8698	156.0667	202.0572
	1	3.7900	3.8622	4.0304	4.2056	4.7602	6.5806	8.5596	10.7273
	2	2.5170	2.6738	3.0359	3.4020	3.3963	4.5963	5.8360	7.1149
	3	2.1986	2.2520	2.4141	2.7221	3.5724	3.8034	4.8012	5.8176
	4	1.9819	2.1294	2.2606	2.3492	3.1009	3.3571	4.2271	5.1085
0.90	5	1.8103	1.9654	2.0127	2.1078	2.7963	3.0644	3.8532	4.6500
	6	1.7247	1.8515	1.9651	2.1337	2.5803	2.8549	3.5865	4.3243
	7	1.6615	1.7673	1.8205	1.9844	2.4175	2.6958	3.3845	4.0782
	8	1.6127	1.7021	1.8045	1.8675	2.2896	2.5699	3.2251	3.8843
	9	1.5531	1.6501	1.7055	1.9062	2.1858	2.4672	3.0951	3.7266
	10	1.5233	1.6074	1.7021	1.8186	2.0996	2.3814	2.9867	3.5951
	0	222.3988	148.7354	148.6889	111.3996	63.282	109.8698	156.0667	202.0572
	1	4.6954	4.4117	4.8231	5.3021	6.8416	6.5806	8.5596	10.7273
	2	2.8504	2.8844	3.0359	3.4020	4.4343	4.5963	5.8360	7.1149
	3	2.3295	2.5229	2.6474	2.7221	3.5724	3.8034	4.8012	5.8176
	4	2.1223	2.2295	2.4327	2.6145	3.1009	3.3571	4.2271	5.1085
0.95	5	1.9579	2.0462	2.1581	2.3325	2.7963	3.0644	3.8532	4.6500
	6	1.8464	1.9195	2.0841	2.1337	2.5803	2.8549	3.5865	4.3243
	7	1.7651	1.8831	1.9272	2.1432	2.4175	2.6958	3.3845	4.0782
	8	1.6807	1.8043	1.8960	2.0121	2.2896	2.5699	3.2251	3.8843
	9	1.6339	1.7415	1.8698	1.9062	2.1858	2.4672	3.0951	3.7266
	10	1.5957	1.6902	1.7765	1.9324	2.0996	2.3814	2.9867	3.5951
	0	405.772	258.9609	222.3349	203.7371	203.2733	109.8698	156.0667	202.0572
	1	19.0427	5.9222	5.7521	6.5835	6.8416	6.5806	8.5596	10.7273
	2	3.6149	3.5084	3.7000	3.8830	4.4343	6.0916	5.8360	7.1149
	3	2.7226	2.9040	3.0737	3.3621	3.5724	4.8447	4.8012	5.8176
	4	2.3950	2.5122	2.7489	2.8553	3.5820	4.1837	4.2271	5.1085
0.99	5	2.1692	2.2740	2.4229	2.5352	3.2012	3.7626	3.8532	4.6500
	6	2.0480	2.1721	2.3031	2.4742	2.9340	3.4662	3.5865	4.3243
	7	1.9366	2.0450	2.2141	2.2892	2.7341	3.2440	3.3845	4.0782
	8	1.8524	1.9475	2.0656	2.1449	2.5778	3.0699	3.2251	3.8843
	9	1.7861	1.9111	2.0193	2.1426	2.4516	2.9290	3.0951	3.7266
	10	1.7325	1.8443	1.9155	2.0384	2.3470	2.8121	2.9867	3.5951

6. Conclusions

In this paper, new single ASP under truncated lifetime tests for the two parameters Xgamma distribution are offered. For different options of the distribution parameters, sample plans have been built. The essential tables of the minimum sample size required to affirm an assured mean lifetime of the test units are presented. The OC function values as well as the corresponding producer's risks are obtained for various plan parameters. A real data is fitted to the TPXGD distribution and an application to this data is discussed to illustrate the usefulness of the proposed ASP. It is indicated that the results encourage the researchers to use the advised ASP when the life duration of components follow the the TPXGD. The current study can be extended using neutrosophic statistics as future research.

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Conflicts of interest

The authors declare that they have no conflicts of interest.

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