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Research article

Spatiotemporal dynamics for impulsive eco-epidemiological model with Crowley-Martin type functional response

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Abstract: Spatiotemporal dynamics of an impulsive eco-epidemiological model with Crowley-Martin type functional responses in a heterogeneous space is studied. The ultimate boundedness of solutions is obtained. The conditions of persistence and extinction under impulsive controls are derived. Furthermore, the existence and globally asymptotic stability of a unique positive periodic solutions are proved. Numerical simulations are also shown to illustrate our theoretical results. Our results show that impulsive harvesting can accelerate the extinction of ecological epidemics.

Keywords: eco-epidemiological model; reaction-diffusion; Crowley-Martin type; impulsive effects; positive periodic solution

1. Introduction

Recently, it has been found that the occurrence of management and optimal control of some life phenomena is not a continuous process, which can not be described by ordinary differential equation or difference equation, but by impulsive differential system or the impulsive diffusion systems [1–3]. For example, when populations are locally stimulated with sufficient intensity (e.g., climate, drought, hunting, harvesting, reproduction, etc.), species numbers can change rapidly in a very short period of time [4–6]. Since the impulsive diffusion system can fully consider the influence of impulses and understand the roles of structural or spatial heterogeneity simultaneously, the impulsive diffusion system has been widely used in the modeling of population, infectious disease and pharmacokinetics. In addition, impulse differential equations have also been applied to the development of renewable resources, pest control, environmental culture and urban management [7–9]. Fazly et al. [10] studied a high-dimensional impulsive reaction-diffusion equation to describe the population dynamics of species with distinct reproductive and dispersal stages. Liang et al. [11] extended the classical Fisher reaction-diffusion equations by involving instant birth and control perturbations to study how multiple pulse perturbations affect the dynamics of the pest population. Meng et al. [12] considered a diffusive logistic population model with impulsive harvesting on a periodically evolving domain to understand how the combination of the evolution of a domain and impulsive harvesting affects the dynamics of a population. For other research, please see [13–19] and references cited therein.

Different from other functional responses, Crowley-Martin [20] supposed that predation would decrease because of high predator density even when prey density was high. Crowley-Martin type functional responses simultaneously describe the effects of handling time and disturbance between predators on population and community dynamics [21, 22]. Little is known about how the intensity of interference between predators is affected. Therefore, the Crowley-Martin type functional response is used to explain this phenomenon. Crowley-Martin also gives good descriptions of predator feeding over their prey. The per capita feeding rate in Crowley-Martin type functional response is as follows:

$$\mathscr{G}(U,V) = \frac{iU}{1 + aU + bV + abUV},$$

where i, a and b are positive constants, which describe the effects of capture rate, processing time and interference degree among predators respectively. If $a = 0$, $b = 0$, Crowley-Martin type functional response is reduced to Holling I type functional response, and if $a > 0, b = 0$, it is reduced to Holling II type functional response. A series of predator-prey models with Crowley-Martin type functional response have been studied in [23–25].

Many eco-epidemiological models have been studied in recent years since they can reflect both ecological and epidemiological cases simultaneously. Xie and Wang [26] proposed a new SIS (susceptible-infected-susceptible) eco-epidemiological model on complex networks with an infective medium. Cai et al. [27] studied positive periodic solutions of an eco-epidemic predator-prey model with Crowley-Martin type functional response and disease in prey population. Chang et al. [28] investigated spatiotemporal dynamics of an impulse eco-epidemiological systems driven by canine distemper viruses. Based on the above motivations, in order to understand how the combination of impulse and Crowley-Martin type functional response affect the dynamics of the population, we consider the following impulsive diffusion eco-epidemiological model with Crowley-Martin type functional response:

$$\begin{aligned}
\frac{\partial Q(t,x)}{\partial t} &= d_Q \Delta Q(t,x) + Q(t,x)\left[a_0(t,x) - b_0(t,\ x)Q(t,x)\right] - \frac{q_1(t,x)Q(t,x)R(t,x)}{Q(t,x)+c_1(t,x)R(t,x)} \\
&\quad - \frac{p_1(t,x)Q(t,x)X(t,x)}{1+\gamma_1(t,x)Q(t,x)+\gamma_2(t,x)X(t,x)+\gamma_1(t,x)\gamma_2(t,x)Q(t,x)X(t,x)}, \qquad (1.1)\\
\frac{\partial R(t,x)}{\partial t} &= d_R \Delta R(t,x) + R(t,x)\left[-a_1(t,x)\right] + \frac{q_1(t,x)Q(t,x)R(t,x)}{Q(t,x)+c_1(t,x)R(t,x)} \\
&\quad - \frac{p_2(t,x)R(t,x)X(t,x)}{1+\gamma_3(t,x)R(t,x)+\gamma_4(t,x)X(t,x)+\gamma_3(t,x)\gamma_4(t,x)R(t,x)X(t,x)} \\
&\quad - \frac{p_3(t,x)R(t,x)Y(t,x)}{1+\gamma_3(t,x)R(t,x)+\gamma_5(t,x)Y(t,x)+\gamma_3(t,x)\gamma_5(t,x)R(t,x)Y(t,x)}, \qquad (1.2)\\
\frac{\partial X(t,x)}{\partial t} &= d_X \Delta X(t,x) + X(t,x)\left[a_2(t,x) - b_1(t,\ x)X(t,x)\right] - \frac{q_2(t,x)X(t,x)Y(t,x)}{X(t,x)+c_2(t,x)Y(t,x)} \\
&\quad + \frac{p_4(t,x)Q(t,x)X(t,x)}{1+\gamma_1(t,x)Q(t,x)+\gamma_2(t,x)X(t,x)+\gamma_1(t,x)\gamma_2(t,x)Q(t,x)X(t,x)} \\
&\quad + \frac{p_5(t,x)R(t,x)X(t,x)}{1+\gamma_3(t,x)R(t,x)+\gamma_4(t,x)X(t,x)+\gamma_3(t,x)\gamma_4(t,x)R(t,x)X(t,x)}, \qquad (1.3)
\end{aligned}$$

$$\frac{\partial Y(t,x)}{\partial t} = d_Y \Delta Y(t,x) + Y(t,x)\left[-a_3(t,x)\right] + \frac{q_2(t,x)X(t,x)Y(t,x)}{X(t,x)+c_2(t,x)Y(t,x)} + \frac{p_6(t,x)R(t,x)Y(t,x)}{1+\gamma_3(t,x)R(t,x)+\gamma_5(t,x)Y(t,x)+\gamma_3((t,x)\gamma_5(t,x)R(t,x)Y(t,x)}, \tag{1.4}$$

$$Q(t_k^+,x) = Q(t_k,x) f_k(x, Q(t_k,x), R(t_k,x), X(t_k,x), Y(t_k,x)), \tag{1.5}$$

$$R(t_k^+,x) = R(t_k,x) g_k(x, Q(t_k,x), R(t_k,x), X(t_k,x), Y(t_k,x)), \tag{1.6}$$

$$X(t_k^+,x) = X(t_k,x) h_k(x, Q(t_k,x), R(t_k,x), X(t_k,x), Y(t_k,x)), \tag{1.7}$$

$$Y(t_k^+,x) = Y(t_k,x) \omega_k(x, Q(t_k,x), R(t_k,x), X(t_k,x), Y(t_k,x)), \tag{1.8}$$

$$\frac{\partial Q(t,x)}{\partial n} = \frac{\partial R(t,x)}{\partial n} = \frac{\partial X(t,x)}{\partial n} = \frac{\partial Y(t,x)}{\partial n} = 0, \tag{1.9}$$

$$Q(0,x) = Q_0(x) \geq (\not\equiv) 0, R(0,x) = R_0(x) \geq (\not\equiv) 0, \tag{1.10}$$

$$X(0,x) = X_0(x) \geq (\not\equiv) 0, Y(0,x) = Y_0(x) \geq (\not\equiv) 0. \tag{1.11}$$

Here, Q, R, X, and Y are the abbreviations of $Q(t,x)$, $R(t,x)$, $X(t,x)$, and $Y(t,x)$, respectively, representing the density of susceptible prey, infected prey, susceptible predator and infected predator at time t and location x in the bounded and smooth region $\Omega \in \mathbb{R}^n$; That is, it should indicate that Q, R, X and Y are abbreviations for $Q(t,x)$, $R(t,x)$, $X(t,x)$ and $Y(t,x)$, respectively. $a_0(t,x)$ and $a_2(t,x)$ represent the birth rates of Q and X; $a_1(t,x)$ and $a_3(t,x)$ represent the mortality of R and Y; $b_0(t,x)$ and $b_1(t,x)$ represent the self-limiting coefficient of Q and X, respectively; $q_1(t,x)$ represents the effective contact rate of Q and R; $q_2(t,x)$ represents the effective contact rate of X and Y; $c_1(t,x)$ and $c_2(t,x)$ indicate psychological inhibition effect; $p_1(t,x)$, $p_2(t,x)$ and $p_3(t,x)$ represent the maximum capture rates of X to Q, X to R and Y to R. $p_4(t,x)$, $p_5(t,x)$ and $p_6(t,x)$ represent the corresponding conversion coefficients; Obviously, we can get $p_1 > p_4$, $p_3 > p_5$ and $p_4 > p_6$. d_Q, d_R, d_X and d_Y represent the diffusion rate of the susceptible prey Q, infected prey R, susceptible predator X and infected predator Y; Denote by $\frac{\partial}{\partial t}$ the outward derivative; $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + ... + \partial^2/\partial x_n^2$ denotes Laplace operator. Both prey and predator populations obey transient impulse control f_k, g_k, h_k and ω_k at a fixed time t_k. Real number sequence $\{t_k\}$ satisfies $0 = t_0 < t_1 < ... < t_k < ...$ with $\lim_{k\to\infty} t_k = +\infty$. Suppose that the infected predator Y can only prey on R but not on Q. Neumann boundary conditions represent no boundary flow and describe the characteristics of no migration.

The rest of this paper is organized as follows. Some necessary lemmas for basic premises will be shown in Section 2. In Section 3, we find a sufficient condition for the ultimate boundedness of systems (1.1)–(1.11). The persistence of systems (1.1)–(1.11) is presented in Section 4. In Section 5, we mainly consider extinction conditions of epidemic. The existence and uniqueness of periodic solution are proved in Section 6. In Section 7, some numerical simulations are shown to verify the theoretical results. In addition, we explain the impact of some key parameters on the epidemic. We have a brief discussion in Section 8.

2. Notation and lemmas

In this section, we will present some premises and basic lemmas.

Let $\mathbb{N}_+$ and $\mathbb{R}$ represent the set of all positive integers and real numbers, respectively, and $\mathbb{R}_+ = [0,\infty)$. For convenience, we first give the following hypothesis:

(H1) Functions $a_i(t,x)(i=0,1,2,3)$, $b_i(t,x)(i=0,1)$, $p_i(t,x)(i=1,2...6)$, $\gamma_i(t,x)(i=0,1,2...5)$, $c_i(t,x)$ $(i=1,2)$ and $q_i(t,x)(i=1,2)\in\mathbb{C}^2(\mathbb{R}_+\times\overline{\Omega})$ are bounded and positive on $\mathbb{R}\times\overline{\Omega}$;

(H2) Functions $f_k(x,Q,R,X,Y), g_k(x,Q,R,X,Y), h_k(x,Q,R,X,Y)$ and $\omega_k(x,Q,R,X,Y), k\in\mathbb{N}_+$ are positive and continuously differentiable for all parameters;

(H3) Functions $a_i(t,x)(i=0,1,2,3), b_i(t,x)(i=0,1), p_i(t,x)(i=1,2...6), \gamma_i(t,x)(i=0,1,2...5), c_i(t,x)$ $(i=1,2)$ and $q_i(t,x)(i=1,2)\in\mathbb{C}^2(\mathbb{R}_+\times\overline{\Omega})$ are periodic function of period $\tau>0$;

(H4) There exists a number $p\in\mathbb{N}_+$ such that $t_{k+p}=t_k+\tau$ for all $k\geq 1$;

(H5) Sequences f_k, g_k, h_k and ω_k satisfy the following conditions:
$f_{k+p}(x,Q,R,X,Y)=f_k(x,Q,R,X,Y)$; $\quad g_{k+p}(x,Q,R,X,Y)=g_k(x,Q,R,X,Y)$;
$h_{k+p}(x,Q,R,X,Y)=h_k(x,Q,R,X,Y)$; $\quad \omega_{k+p}(x,Q,R,X,Y)=\omega_k(x,Q,R,X,Y)$
for all $k\geq 1$;

For convenience, we introduce the following notations: $G=\mathbb{R}_+\times\Omega, \overline{G}=\mathbb{R}_+\times\overline{\Omega}$,

$$\sum_k=\{(t,x)\mid t\in(t_{k-1},t_k), x\in\Omega\},\qquad k\in\mathbb{N}_+,\qquad \sum=\bigcup_{k\in\mathbb{N}_+}\sum_k.$$
$$\overline{\sum}_k=\left\{(t,x)\mid t\in(t_{k-1},t_k), x\in\overline{\Omega}\right\},\qquad k\in\mathbb{N}_+,\qquad \overline{\sum}=\bigcup_{k\in\mathbb{N}_+}\overline{\sum}_k,$$

and denote:

$$\psi_k=\left\{\phi:\overline{G}\to\mathbb{R}\ \middle|\ \begin{array}{l}(i)\phi(t,x)\in C^{1,2}_{t,x}(\sum_k),\phi(t,x)\in C^{1,2}_{t,x}(\overline{\sum}_k),\\ (ii)\lim_{t\to t_k^-}\phi(t,x)=\phi(t_k,x)\ exists,\\ (iii)\lim_{t\to t_k^+}\phi(t,x)=\phi(t_k^+,x)\ exists,\end{array}\right\},$$

where $C^{1,2}_{t,x}$ denotes function $\phi(t,x)$ is continuously differentiable with respect to parameter t and is twice partial exists with respect to x. A vector function $\{Q(t,x),R(t,x),X(t,x),Y(t,x)\}\in\psi_k\times\psi_k\times\psi_k\times\psi_k$ is called a solution of systems (1.1)–(1.8) if it satisfies Neumann boundary condition (1.9) and initial value conditions (1.10)–(1.11).

For continuous functions, we define $\phi^L=\inf_{(t,x)}\phi(t,x), \phi^M=\sup_{(t,x)}\phi(t,x)$.

Lemma 2.1. *(Lemma 1. [29]) Let T and d be positive constants, the function $U(t,x)$ is continuous in $[0,T]\times\overline{\Omega}$ which is continuously differentiable on $x\in\overline{\Omega}$, ΔU and $\frac{\partial U}{\partial t}$ are continuous on $(0,T]\times\Omega$. $U(t,x)$ satisfies the following inequality:*

$$\frac{\partial U}{\partial t}-d\Delta U+C(t,x)U\geq 0,\qquad (t,x)\in(0,T]\times\Omega,$$
$$\frac{\partial U}{\partial n}\geq 0,\qquad (t,x)\in(0,T]\times\partial\Omega,$$

where $C(t,x)$ is bounded on $(0,T]\times\Omega$ and n is the unit vector of $U(t,x)$. Then $U(t,x)\geq 0$ if $U(0,x)\geq 0$ on $(0,T]\times\partial\Omega$. Moreover, $U(t,x)$ is strictly positive if $U(t,x)\geq(\not\equiv)0$ on $(0,T]\times\partial\Omega$.

Lemma 2.2. *(Lemma 2.2. [30]) Suppose the vector functions $\nu(t,x)=(\nu_1(t,x),...\nu_m(t,x)), \varrho(t,x)=(\varrho_1(t,x),...\varrho_m(t,x))$, $m\geq 1$, satisfies the following conditions:*

(i) *$v(t,x)$ and $\varrho(t,x)$ are quadratic continuously differentiable with respect to x ($x \in \Omega$) and once continuously differentiable in $(t,x) \in [t_a,t_b] \times \overline{\Omega}$;*

(ii) *$v_t - \mu\Delta v - H(t,x,v) \le \varrho_t - \mu\Delta\varrho - H(t,x,\varrho)$; where $(t,x) \in [t_a,t_b] \times \Omega, \mu = (\mu_1,\mu_2,...,\mu_m) > 0$, the vector function $H(t,x,U) = (H_1(t,x,U)...,H_m(t,x,U))$ is continuously differentiable and quasi monotone increasing of U is equal to $U = (U_1,U_2,...,U_m)$:*

$$\frac{\partial H_i(t,x,U_1,U_2,...U_m)}{\partial U_j} \ge 0, i,j = 1,2,...,m, i \ne j;$$

(iii) $\frac{\partial v}{\partial \mu} = \frac{\partial \varrho}{\partial \mu} = 0, (t,x) \in [t_a,t_b] \times \partial\Omega$;

Then $v(t,x) \le \varrho(t,x)$ for $(t,x) \in [t_a,t_b] \times \partial\Omega$.

We now consider the following differential system with impulses on $t = t_k, k \in \mathbb{N}_+$,

$$\begin{cases} \dfrac{dX}{dt} = \alpha X(\beta - X), & t \ne t_k, \\ X(t_k^+) = X(y_k)\lambda_k(X(t_k)), & t \in \mathbb{N}_+, \end{cases} \tag{2.1}$$

where $X(t)$ is a positive function, $\alpha,\beta \in \mathbb{R}_+$, and the strictly increasing sequence $\{t_k, k \in \mathbb{N}_+\}$ satisfies the condition (H4). For all $X \in \mathbb{R}_+, k \in \mathbb{N}_+, \lambda_k$ is continuous positive functions satisfying $\lambda_{k+p}(X) = \lambda_k(X)$ for all $X \in \mathbb{R}_+$.

Lemma 2.3. *(Lemma 2.1. [31]) Every solution $X(t) = X(t,0,X_0), X_0 = X(0) = X(0^+) > 0$ of system (2.1) is positive and bounded for all $t \in [0,+\infty)$.*

Next, we prove the compactness of solutions for systems (1.1)–(1.11). Let $C^{\iota+\alpha}$ be the space of ι-times continuous differentiable functions $f : \Omega \to \mathbb{R}$ with ι-order derivatives which satisfies the Holder condition with exponent $0 < \alpha < 1$, and $\varphi = (Q,R,X,Y) \in L_m(\Omega) \times L_m(\Omega) \times L_m(\Omega) \times L_m(\Omega)$, where ι and m are two positive integers and $L_m(\Omega)$ is the Banach space in Ω. For sufficiently small $\xi > 0$, let

$$\xi = \begin{pmatrix} d_Q\Delta - \delta & 0 & 0 & 0 \\ 0 & d_R\Delta - \delta & 0 & 0 \\ 0 & 0 & d_X\Delta - \delta & 0 \\ 0 & 0 & 0 & d_Y\Delta - \delta \end{pmatrix},$$

$$\mathscr{P}(t,\varphi) = \begin{pmatrix} Q\left[a_0 - b_0 Q - \frac{q_1 R}{Q+c_1 R} - \frac{p_1 X}{1+\gamma_1 Q+\gamma_2 X+\gamma_1\gamma_2 QX} + \delta\right] \\ R\left[-a_1 + \frac{q_1 Q}{Q+c_1 R} - \frac{p_2 X}{1+\gamma_3 R+\gamma_4 X+\gamma_3\gamma_4 RX} - \frac{p_3 Y}{1+\gamma_3 R+\gamma_5 Y+\gamma_3\gamma_5 RY} + \delta\right] \\ X\left[a_2 - b_1 X - \frac{q_2 Y}{X+c_2 Y} + \frac{p_4 Q}{1+\gamma_1 Q+\gamma_2 X+\gamma_1\gamma_2 QX} + \frac{p_5 R}{1+\gamma_3 R+\gamma_4 X+\gamma_3\gamma_4 RX} + \delta\right] \\ Y\left[-a_3 + \frac{q_2 X}{X+c_2 Y} + \frac{p_6 R}{1+\gamma_3 R+\gamma_5 Y+\gamma_3\gamma_5 RY} + \delta\right] \end{pmatrix},$$

$$\mathscr{F}(\varphi(t_i)) = \begin{pmatrix} Q(t_i,x)f_k(x,Q(t_i,x),R(t_i,x),X(t_i,x),Y(t_i,x)) - Q(t_i,x) \\ R(t_i,x)g_k(x,Q(t_i,x),R(t_i,x),X(t_i,x),Y(t_i,x)) - R(t_i,x) \\ X(t_i,x)h_k(x,Q(t_i,x),R(t_i,x),X(t_i,x),Y(t_i,x)) - X(t_i,x) \\ Y(t_i,x)\omega_k(x,Q(t_i,x),R(t_i,x),X(t_i,x),Y(t_i,x)) - Y(t_i,x) \end{pmatrix}.$$

So systems (1.1)–(1.11) can be rewritten as

$$\begin{cases} \dfrac{d\varphi^T}{dt} = \xi\varphi^T + \mathscr{P}(t,\varphi), & t \neq t_k, x \in \Omega; \\ \varphi(t_i^+) = \varphi(t_i) + \mathscr{F}(\varphi(t_i)), & i \in \mathbb{N}_+; \\ \dfrac{\partial\varphi}{\partial v} = 0, \quad x \in \partial\Omega; \\ \varphi(0,x) = \varphi_0(x) \geq (\not\equiv) 0, & x \in \overline{\Omega}; \end{cases} \tag{2.2}$$

In system (2.2), $\mathscr{D}(\xi) = \left\{\varphi : \varphi \in \mathbb{N}^{2,m}(\Omega), \frac{\partial\varphi}{\partial v} |_{\partial\Omega}= 0\right\}$ is the domain of operator ξ, where $\varphi^{2,m}$ stands for Sobolev space of functions from $L_m(\Omega)$ which have two generalized derivatives. Functions $\mathscr{P}(t,\varphi(t))$ satisfy $\sup_t \parallel \mathscr{P}(t,\varphi(t)) \parallel < \infty, \mathscr{F}_i(\varphi)$ is periodic in i. Let Σ_ξ be the spectrum of sectorial operator ξ, therefore, $\mathrm{Re}\Sigma_\xi \leq \varsigma$ with reference to [32], for any $\alpha > 0$ is given as the power $\xi^{-\alpha}$ of the bounded bijective fraction

$$\xi^{-\alpha} = \frac{1}{\Gamma(\alpha)} \int e^{-s\xi} s^{\alpha-1} ds,$$

where Γ is the gamma function. It is obvious that $\xi^\alpha = (\xi^{-\alpha})^{-1}$ and $\mathscr{D}(\xi^\alpha) = \mathscr{R}(\xi^{-\alpha})$, where $\mathscr{R}(\cdot)$ denotes the range of operator $\xi^{-\alpha}$, and ξ^0 represents the identity operator in $L_m \times L_m \times L_m \times L_m$ which has a norm $\parallel \cdot \parallel$. Then a new space $X^\alpha = \mathscr{D}(\xi^\alpha)$ can be defined such that $\parallel x \parallel_\alpha = \parallel \xi^\alpha x \parallel_\alpha$ for $\alpha \in [0,1]$.

Lemma 2.4. *(Lemma 2.4. [33]) Assume that the function $\mathscr{F}_i$ is continuously differentiable, and there is a positive function $\eta(M)$, such that*

$$\sup_{\|\varphi\|_\alpha \leq M} \parallel \mathcal{F}_k(\varphi) \parallel \leq \eta(M), \qquad k \in \mathbb{N}_+, \tag{2.3}$$

for some $\alpha \in (\frac{1}{2} + \frac{n}{2m}, 1)$. Let $\varphi(t,\varphi_0), \varphi_0 = (\varphi_{00}, \varphi_{10}, ...\varphi_{n0}) \in X^\alpha$, be a bounded solution of (2.2)*, i.e.:*

$$\parallel \varphi(t,\varphi_0) \parallel_c \leq M', \qquad t > 0. \tag{2.4}$$

Then the set $\{\varphi(t,\varphi_0) : t > 0\}$ is relatively compact in $C^{1+v}(\overline{\Omega}, \mathbb{R}^{n+1})$ for $0 < v < 2\alpha - 1 - \frac{n}{m}$.

Similar to [28], using the upper and lower solutions of partial differential equations, there exists a classical solution for (1.1) under the conditions of systems (1.9) and (1.10). The analysis of R, X and Y are similar to that of Q. If the twice partial derivatives of Q, R, X and Y with respect to x exist in systems (1.1)–(1.4), and are continuously differentiable with respect to t, then the systems (1.1)–(1.4) and (1.9)–(1.11) classical solution are exist. In particular, the solution of systems (1.1)–(1.11) is defined as the classical solution of systems (1.1)–(1.4) for $t \in (0, t_1]$. According to the impulse conditions (1.5)–(1.8), the function $(Q(t_1^+, x), R(t_1^+, x), X(t_1^+, x), Y(t_1^+, x))$ is also continuously differentiable in x which satisfies the boundary condition (1.9). Therefore, we can make $(Q(t_1^+, x), R(t_1^+, x), X(t_1^+, x), Y(t_1^+, x))$ as a new initial function to compute for $t \in (t_1, t_2]$. Finally, using the same construction process, we can obtain the solution of $t \in \mathbb{R}_+$.

Since $Q(t,x)$, $R(t,x)$, $X(t,x)$ and $Y(t,x)$ describe the population density at x at time t, the solution of systems (1.1)–(1.11) should be non-negative. Therefore, we give the following lemma.

Lemma 2.5. *Assume that (H1)–(H5) are true, then the non-negative and positive quadrants of $\mathbb{R}^4$ are positive invariant for systems* (1.1)–(1.11).

Proof. Let $\widehat{Q}$ and $\overline{Q}$ be the solutions of the following two equations

$$\frac{\partial\widehat{Q}}{\partial t}-d_Q\Delta\widehat{Q}-\widehat{Q}\left[a_0^L-b_0^M\widehat{Q}-\frac{q_1^M}{c_1^L}-\frac{p_1^M}{\gamma_2^L}\right]=0,\qquad \widehat{Q}(0,x)=Q_0(X),$$

and

$$\frac{\partial\overline{Q}}{\partial t}-d_Q\Delta\overline{Q}-\overline{Q}\left[a_0^M-b_0^L\overline{Q}\right]=0,\qquad \overline{Q}(0,x)=Q_0(X).$$

Then $\widehat{Q}$ and $\overline{Q}$ are the upper and lower solutions of system (1.1). Since $Q_0(x)\geq 0$ and $Q_0(x)\not\equiv 0$. By Lemma 2.1, for $t\in(0,t_1]$, we get $\widehat{Q}(t,x)>0$ and $\overline{Q}(t,x)>0$. Because $Q(t,x)$ is bounded from below by positive function $\widehat{Q}(t,x)$, we have $Q(t,x)>0$ for $t\in[0,t_1]$. Considering the function f_k is positive, and we can repeat the same argument to show that it is positive $Q(t,x)$ for $t\in[t_1,t_2]$. By induction, we get $Q(t,x)>0$ for $t\in\mathbb{R}_+$.

For R,X and Y, using the same analysis, we finally have $R(t,x)>0, X(t,x)>0$ and $Y(t,x)>0$ for $t\in\mathbb{R}_+$.

Lemma 2.6. *(The Brouwers Fixed Point Theorem [34]) Let $\mathbb{S}$ be a bounded, closed and convex subset of $\mathbb{R}^n$ for $n\in\mathbb{N}_+$. $\partial\mathbb{S}$ represents the relative boundary of $\mathbb{S}$. If $\Upsilon\in\mathbb{C}(\mathbb{S},\mathbb{R}^n)$ and satisfies $\Upsilon(\partial\mathbb{S})\in\mathbb{S}$, then Υ must have a fixed point on $\mathbb{S}$.*

3. Ultimate boundedness

In order to prove the ultimate boundedness of solutions, we first give the definition of ultimate boundedness.

Definition 1. *(Definition 1. [31]) Solutions of systems* (1.1)–(1.11) *are said to be ultimate boundeness if there are positive constant M_0, M_1, M_2 and M_3 such that for every solution $\{Q(t,x,Q_0,R_0,X_0,Y_0),R(t,x,Q_0,R_0,X_0,Y_0),X(t,x,Q_0,R_0,X_0,Y_0),Y(t,x,Q_0,R_0,X_0,Y_0)\}$ of systems* (1.1)–(1.11)*, there is a time at which $t^*=t^*(Q_0,R_0,X_0,Y_0)>0$, such that*

$$\begin{aligned}&Q(t,x,Q_0,R_0,X_0,Y_0)\leq M_0, &&R(t,x,Q_0,R_0,X_0,Y_0)\leq M_1,\\ &X(t,x,Q_0,R_0,X_0,Y_0)\leq M_2, &&Y(t,x,Q_0,R_0,X_0,Y_0)\leq M_3,\end{aligned}$$

for all $x\in\Omega$ and $t\geq t^*$.

Next, we will verify that the ultimate boundedness of solutions for systems (1.1)–(1.11).

Theorem 3.1. *If (H_1)–(H_5) hold, furtheremore,*

(i) *there is a positive function $\xi(M)$, such that $f_k(x,Q,R,X,Y)\leq\xi(M)$ for $k\in\mathbb{N}_+$, $Q\leq M, R\geq 0, X\geq 0$ and $Y\geq 0$ for $x\in\overline{\Omega}$;*
(ii) *The inequality*

$$-\tau a_1^L+\sum_{0\leq t_k<t}\ln\sup_{(x,Q,R,X,Y)}g_k(x,Q,R,X,Y)<0$$

holds;

(iii) *There is a positive function* $\rho(K)$*, such that* $h_k(x, Q, R, X, Y) \le \rho(K)$ *for* $k \in \mathbb{N}_+$*,* $X \le K, Q \ge 0, R \ge 0$ *and* $Y \ge 0$ *for* $x \in \overline{\Omega}$*, in addition, the inequality* $a_2^M - \frac{p_4^M}{\gamma_1^L} - \frac{p_5^M}{\gamma_3^L} > 0$ *holds;*

(iv) *The inequality*

$$-\tau a_3^L + \sum_{0 \le t_k < t} \ln \sup_{(x,Q,R,X,Y)} \omega_k(x, Q, R, X, Y) < 0$$

holds.

Then all the solutions of systems (1.1)–(1.11) *are ultimately bounded.*

Proof. First, let $\overline{Q} = (t, x, Q_0)$ is the solution of the following equation

$$\frac{\partial \overline{Q}}{\partial t} - d_Q \Delta \overline{Q} - \overline{Q}\left[a_0^M - b_0^L \overline{Q}\right] = 0. \tag{3.1}$$

According to system (1.1), it is easy to get

$$\begin{aligned} 0 &= \frac{\partial Q}{\partial t} - d_Q \Delta Q - Q\left[a_0 - b_0 Q\right] + \frac{q_1 QR}{Q + c_1 R} + \frac{p_1 QX}{1 + \gamma_1 Q + \gamma_2 X + \gamma_1 \gamma_2 QX} \\ &\ge \frac{\partial Q}{\partial t} - d_Q \Delta Q - Q\left[a_0^M - b_0^L Q\right], \end{aligned} \tag{3.2}$$

by Lemma 2.5, we have

$$0 = \frac{\partial \overline{Q}}{\partial t} - d_Q \Delta \overline{Q} - \overline{Q}\left[a_0^M - b_0^L \overline{Q}\right] \ge \frac{\partial Q}{\partial t} - d_Q \Delta Q - Q\left[a_0^M - b_0^L Q\right].$$

Therefore, by Lemma 2.2, $Q(t, x, Q_0, R_0, X_0, Y_0) \le \overline{Q}(t, M_Q)$, where $M_Q \ge \max_{x \in \overline{\Omega}} \mid Q_0(x) \mid = \parallel Q_0(x) \parallel_C$, according to the uniqueness theorem, the solution of (3.2) is independent of x for $t > 0$, that is, $\overline{Q}(t, M_Q)$ satisfies

$$\frac{d\overline{Q}}{dt} = \overline{Q}\left[a_0^M - b_0^L \overline{Q}\right], \qquad \overline{Q}(0, M_Q) = M_Q.$$

Hence

$$\begin{aligned} &\parallel Q(t_k^+, x, Q_0, R_0, X_0, Y_0) \parallel_C \\ &= \parallel Q(t_k, x, Q_0, R_0, X_0, Y_0) f_k(x, Q(t_k, x), R(t_k, x), X(t_k, x), Y(t_k, x)) \parallel_C \\ &\le \overline{Q}(t_k, M_Q)\xi(\overline{Q}(t_k, M_Q)). \end{aligned}$$

According to Lemma 2.3, the solutions of the following impulsive ordinary differential equations are ultimately bounded:

$$\begin{cases} \dfrac{d\overline{Q}}{dt} = \overline{Q}\left[a_0^M - b_0^L \overline{Q}\right], \\ Q(t_k^+) = \overline{Q}(t_k)\xi(\overline{Q}(t_k, M_Q)). \end{cases}$$

Then Q is uniformly bounded, that is, there is a positive constant $M_Q^{'}$ such that $Q(t, x) \le M_Q^{'}$ from t_1^*.

For R, by the equation of (1.2), for $t \geq t_1^*$, we have

$$\begin{aligned} 0 =& \frac{\partial R}{\partial t} - d_R \Delta R - R\left[-a_1\right] - \frac{q_1 QR}{Q + c_1 R} \\ &+ \frac{p_2 RX}{1 + \gamma_3 R + \gamma_4 X + \gamma_3\gamma_4 RX} + \frac{p_3 RY}{1 + \gamma_3 R + \gamma_5 Y + \gamma_3\gamma_5 RY} \\ \geq& \frac{\partial R}{\partial t} - d_R \Delta R - R(-a_1^L) - \frac{q_1^M M_Q^{'}}{c_1^L}. \end{aligned} \tag{3.3}$$

So, $R(t, x, Q_0, R_0, X_0, Y_0) \leq \overline{R}(t, M_R)$ where $\overline{R}(t, M_R)$ satisfies the following Cauchy problem

$$\frac{d\overline{R}}{dt} = -a_1^L \overline{R} + \frac{q_1^M M_Q^{'}}{c_1^L}, \qquad \overline{R}(0, M_R) = M_R.$$

Now, we solve the following linear periodic impulsive ODE:

$$\begin{cases} \dfrac{d\overline{R}}{dt} = -a_1^L \overline{R} + \dfrac{q_1^M M_Q^{'}}{c_1^L}, \\ \overline{R}(t_k^+) = \sup\limits_{(x,Q,R,X,Y)} g_k(x, Q, R, X, Y)\overline{R}(t_k). \end{cases} \tag{3.4}$$

In view of [1], the form of the solution of impulsive equation (3.4) is $\overline{R}(t) = A_0(t) + C^* A(t)$, where C^* is a constant and $A_0(t)$ is a continuous function with period τ.

$$A(t) = \exp\left\{-a_1^L t + \sum_{0 \leq t_k < t} \ln \sup_{(x,Q,R,X,Y)} g_k(x, Q, R, X, Y)\right\}.$$

It is easy to know $\lim\limits_{t\to+\infty} A(t) = 0$ by according to condition (ii). Then all solutions of (3.4) are ultimately bounded. Thus, we get the ultimate boundedness of $R(t, x)$, that is to say, there exists a positive constant $M_R^{'}$, such that $R(t, x) \leq M_R^{'}$ from t_2^*.

For X, by the equation of (1.3), let $\overline{X} = (t, x, X_0)$ is the solution of the following equation

$$\frac{\partial \overline{X}}{\partial t} - d_X \Delta \overline{X} - \overline{X}\left[a_2^M - b_1^L \overline{X} - \frac{p_4^M}{\gamma_1^L} - \frac{p_5^M}{\gamma_3^L}\right] = 0. \tag{3.5}$$

According to system (1.3), it is easy to get

$$\begin{aligned} 0 =& \frac{\partial X}{\partial t} - d_X \Delta X - X\left[a_2 - b_1 X\right] + \frac{q_2 XY}{X + c_2 Y} \\ &- \frac{p_4 QX}{1 + \gamma_1 Q + \gamma_2 X + \gamma_1\gamma_2 RX} - \frac{p_5 RX}{1 + \gamma_3 R + \gamma_4 X + \gamma_3\gamma_4 RX} \\ \geq& \frac{\partial X}{\partial t} - d_X \Delta X - X\left[a_2^M - b_1^L X - \frac{p_4^M}{\gamma_1^L} - \frac{p_5^M}{\gamma_3^L}\right]. \end{aligned} \tag{3.6}$$

By Lemma 2.5, we have

$$0 = \frac{\partial \overline{X}}{\partial t} - d_X\Delta\overline{X} - \overline{X}\left[a_2^M - b_1^L\overline{X} - \frac{p_4^M}{\gamma_1^L} - \frac{p_5^M}{\gamma_3^L}\right]$$
$$\geq \frac{\partial X}{\partial t} - d_X\Delta X - X\left[a_2^M - b_1^L X - \frac{p_4^M}{\gamma_1^L} - \frac{p_5^M}{\gamma_3^L}\right].$$

Therefore, by Lemma 2.2, $X(t, x, Q_0, R_0, X_0, Y_0) \leq \overline{X}(t, M_X)$, where $M_X \geq \max_{x\in\overline{\Omega}} \mid X_0(x) \mid = \parallel X_0(x) \parallel_C$, according to the uniqueness theorem, the solution $\overline{X}(t, M_X)$ of (3.5) is independent of x for $t > 0$, that is, $\overline{X}(t, M_X)$ satisfies

$$\frac{d\overline{X}}{\partial t} = \overline{X}\left[a_2^M - b_1^L\overline{X} - \frac{p_4^M}{\gamma_1^L} - \frac{p_5^M}{\gamma_3^L}\right].$$

Hence

$$\begin{aligned}&\parallel X(t_k^+, x, Q_0, R_0, X_0, Y_0) \parallel_C\\ =& \parallel X(t_k, x, Q_0, R_0, X_0, Y_0)h_k(x, Q(t_k, x), R(t_k, x), X(t_k, x), Y(t_k, x)) \parallel_C\\ \leq& \overline{X}(t_k, M_X)\rho(\overline{X}(t_k, M_X)).\end{aligned}$$

According to Lemma 2.3, the solutions of the following impulsive ordinary differential equations are ultimately bounded:

$$\begin{cases}\dfrac{d\overline{X}}{dt} = \overline{X}\left[a_2^M - b_1^L\overline{X} - \dfrac{p_4^M}{\gamma_1^L} - \dfrac{p_5^M}{\gamma_3^L}\right],\\ X(t_k^+) = \overline{X}(t_k)\rho(\overline{X}(t_k, M_X)).\end{cases}$$

Then X is uniformly bounded if $a_2^M - \frac{p_4^M}{\gamma_1^L} - \frac{p_5^M}{\gamma_3^L} > 0$, that is, there is a positive constant $M_X^{'}$ such that $X(t, x) \leq M_X^{'}$ from t_3^*.

For Y, by the equation of (1.4), for $t \geq t_3^*$, we have

$$0 = \frac{\partial Y}{\partial t} - d_Y\Delta Y + a_3 Y - \frac{q_2 XY}{X + c_2 Y} - \frac{p_6 RY}{1 + \gamma_3 R + \gamma_5 Y + \gamma_3\gamma_5 RY}$$
$$\geq \frac{\partial Y}{\partial t} - d_Y\Delta Y + a_3^L Y - \frac{q_2^M M_X^{'}}{c_2^L} - \frac{p_6^M M_R^{'}}{\gamma_5^L}.$$

Thus, $Y(t, x, Q_0, R_0, X_0, Y_0) \leq \overline{Y}(t, M_Y)$ where $\overline{Y}(t, M_Y)$ satisfies the following Cauchy problem

$$\begin{cases}\dfrac{d\overline{Y}}{\partial t} = -a_3^L\overline{Y} + \dfrac{q_2^M M_X^{'}}{c_2^L} + \dfrac{p_6^M M_R^{'}}{\gamma_5^L},\\ \overline{Y}(0, M_Y) = M_Y.\end{cases}$$

According to [1], the solution of the following impulsive ordinary differential equation is: $\overline{Y}(t) = Y_0(t) + C^*Y(t)$,

$$\begin{cases}\dfrac{d\overline{Y}}{\partial t} = -a_3^L\overline{Y} + \dfrac{q_2^M M_X^{'}}{c_2^L} + \dfrac{p_6^M M_R^{'}}{\gamma_5^L},\\ \overline{Y}(t_k^+) = \sup\limits_{(x,Q,R,X,Y)} \omega_k(x, Q, R, X, Y)\overline{Y}(t_k),\end{cases}$$

where C^* is a constant and $Y_0(t)$ is a continuous function with period τ.

$$Y(t) = \exp\left\{-\tau a_3^L + \sum_{0\le t_k<t} \ln \sup_{(x,Q,R,X,Y)} \omega_k(x,Q,R,X,Y)\right\}.$$

According to (iv) condition of Theorem 3.1, there exists a normal number $M_Y^{'}$ such that $Y(t,x) \le M_Y^{'}$ from t_4^*.

The proof is completed.

4. Prevailing of epidemic

In this section, we will derive sufficient conditions for prevailing of epidemic in (1.1)–(1.11). Before this, a definition is given at first.

Definition 2. *(Definition 2. [31]) The epidemic is called prevailing if there exist positive constans m_0, m_1,m_2,m_3,M_0, M_1,M_2, M_3 and a moment of time $\tilde{t} = \tilde{t}(Q_0, R_0, X_0, Y_0)$ such that*

$$m_0 \le Q(t,x,Q_0,R_0,X_0,Y_0) \le M_0, \qquad m_1 \le R(t,x,Q_0,R_0,X_0,Y_0) \le M_1,$$
$$m_2 \le X(t,x,Q_0,R_0,X_0,Y_0) \le M_2, \qquad m_3 \le Y(t,x,Q_0,R_0,X_0,Y_0) \le M_3,$$

for all $x \in \Omega$ and $t \ge \tilde{t}$.

Then, we will give the conditions for prevailing of epidemic in systems (1.1)–(1.11).

Theorem 4.1. *If (H_1)–(H_5) hold, furthermore,*

(i) *Systems* (1.1)–(1.11) *is uniformly bounded, that is, there are positive constants M_0, M_1, M_2, M_3 and a time $\bar{t} = \bar{t}(Q_0, R_0, X_0, Y_0)$ such that $Q(t,x,Q_0,R_0,X_0,Y_0) \le M_0$, $R(t,x,Q_0,R_0,X_0,Y_0) \le M_1$, $X(t,x,Q_0,R_0,X_0,Y_0) \le M_2$ and $Y(t,x,Q_0,R_0,X_0,Y_0) \le M_3$ for $t > \bar{t}$.*

(ii) *The inequality*

$$\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} f_k(x,Q,R,X,Y) + \tau\left(a_0^L - \frac{p_1^M}{\gamma_2^L} - \frac{q_1^M}{c_1^L}\right) > 0, \tag{4.1}$$

$$-\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x,Q,R,X,Y) + \tau\left(a_1^M + \frac{p_2^M}{\gamma_4^L} + \frac{p_3^M}{\gamma_5^L}\right) > 0, \tag{4.2}$$

$$\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} h_k(x,Q,R,X,Y) + \tau\left(a_2^L - \frac{q_2^M}{c_2^L}\right) > 0, \tag{4.3}$$

and

$$-\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} \omega_k(x,Q,R,X,Y) + a_3^M\tau > 0 \tag{4.4}$$

hold, where $S = \{(Q,R,X,Y) \mid 0 < Q < M_0, 0 < R < M_1, 0 < X < M_2, 0 < Y < M_3\}$.

Then, there are positive constants $\sigma_0^*, \sigma_1^*, \sigma_2^*, \sigma_3^*$ *such that any solution of systems* (1.1)–(1.11) *satisfies*

$$\sigma_0^* \le Q(t,x) \le M_0, \qquad \sigma_1^* \le R(t,x) \le M_1,$$
$$\sigma_2^* \le X(t,x) \le M_2, \qquad \sigma_3^* \le Y(t,x) \le M_3,$$

for $t > \bar{t}^*$, *that is, the epidemic in systems* (1.1)–(1.11) *is prevailing.*

Proof. Lemma 2.1 means that if $Q_0(x) \ge 0, R_0(x) \ge 0, X_0(x) \ge 0, Y_0(x) \ge 0$, and $Q_0(x), R_0(x), X_0(x), Y_0(x) \not\equiv 0$, then

$$Q(t,x,Q_0,R_0,X_0,Y_0) > 0, R(t,x,Q_0,R_0,X_0,Y_0) > 0,$$

and

$$X(t,x,Q_0,R_0,X_0,Y_0) > 0, Y(t,x,Q_0,R_0,X_0,Y_0) > 0,$$

for all $x \in \overline{\Omega}$ and $t > 0$. For a small $\varepsilon > 0$ on interval $t \ge \varepsilon$, the initial value condition $\{Q(\varepsilon,x,Q_0,R_0,X_0,Y_0), R(\varepsilon,x,Q_0,R_0,X_0,Y_0), X(\varepsilon,x,Q_0,R_0,X_0,Y_0), Y(\varepsilon,x,Q_0,R_0,X_0,Y_0)\} \ne 0$. Without loss of generality, we assume that $\min_{x\in\overline{\Omega}} Q_0(x) = m_Q > 0, \min_{x\in\overline{\Omega}} R_0(x) = m_R > 0, \min_{x\in\overline{\Omega}} X_0(x) = m_X > 0, \min_{x\in\overline{\Omega}} Y_0(x) = m_Y > 0$, by (1.1), we have

$$\begin{aligned} 0 &= \frac{\partial Q}{\partial t} - d_Q\Delta Q - Q\,[a_0 - b_0 Q] + \frac{q_1 QR}{Q + c_1 R} + \frac{p_1 QX}{1 + \gamma_1 Q + \gamma_2 X + \gamma_1\gamma_2 QX} \\ &\le \frac{\partial Q}{\partial t} - d_Q\Delta Q - Q\left[a_0^L - b_0^M Q - \frac{p_1^M}{\gamma_2^L} - \frac{q_1^M}{c_1^L}\right], \end{aligned}$$

in view of Lemma 2.5, we obtain

$$\begin{aligned} 0 &= \frac{\partial \widehat{Q}}{\partial t} - d_Q\Delta \widehat{Q} - \widehat{Q}\left[a_0^L - b_0^M \widehat{Q} - \frac{q_1^M}{c_1^L} - \frac{p_1^M}{\gamma_2^L}\right] \\ &\le \frac{\partial Q}{\partial t} - d_Q\Delta Q - Q\left[a_0^L - b_0^M Q - \frac{p_1^M}{\gamma_2^L} - \frac{q_1^M}{c_1^L}\right]. \end{aligned}$$

Now, using Lemma 2.2 for $m = 1$, $Q(t,x,Q_0,R_0,X_0,Y_0) \ge \widehat{Q}(t, M_Q)$ for $t \in [0, t_1]$. Applying the last inequality for $t = t_1$, together with (1.1), we obtain that

$$Q(t_1^+, x, Q_0, R_0, X_0, Y_0) \ge \widehat{Q}(t, m_Q) \inf_{x\in\Omega,(Q,R,X,Y)\in S} f_k(x,Q,R,X,Y).$$

It then follows that the solution $Q(t,x,Q_0,R_0,X_0,Y_0)$ is bounded from below by a solution of the following equation with impulses:

$$\begin{cases} \dfrac{d\widehat{Q}}{dt} = \widehat{Q}\left[a_0^L - b_0^M \widehat{Q} - \dfrac{q_1^M}{c_1^L} - \dfrac{p_1^M}{\gamma_2^L}\right], \\ Q(t_k^+) = \widehat{Q}(t_k) \inf\limits_{x\in\Omega,(Q,R,X,Y)\in S} f_k(x,Q,R,X,Y). \end{cases} \tag{4.5}$$

According to (4.1) and Theorem 2.1 in [3], (4.5) has a unique piecewise continuous strictly positive periodic solution $\widehat{Q}(t)^*$ such that for every solutions $\widehat{Q}(t, Q_m)$ of (4.5) that satisfy $\widehat{Q}(t, Q_m) \to \widehat{Q}(t)^*$ as $t \to \infty$. Therefore, there is a positive constant σ_0^* such that $\widehat{Q}(t, Q_m) \geq \sigma_0^*$.

Because the solution $Q(t, x, Q_0, R_0, X_0, Y_0)$ of Q is defined by the solution $\widehat{Q}(t, Q_m)$ from below (4.5), we draw a conclusion from the following: $Q(t, x, Q_0, R_0, X_0, Y_0) \geq \sigma_0^*$ for $t > \widehat{t_0}$.

For R, we have

$$\begin{aligned}0 =&\frac{\partial R}{\partial t} - d_R \Delta R - R\left[-a_1\right] - \frac{q_1 QR}{Q + c_1 R}\\ &+ \frac{p_2 RX}{1 + \gamma_3 R + \gamma_4 X + \gamma_3\gamma_4 RX} + \frac{p_3 RY}{1 + \gamma_3 R + \gamma_5 Y + \gamma_3\gamma_5 RY}\\ \leq&\frac{\partial R}{\partial t} - d_R \Delta R - R\left[-a_1^M - \frac{p_2^M}{\gamma_4^L} - \frac{p_3^M}{\gamma_5^L}\right].\end{aligned}$$

In view of Lemma 2.5, we obtain

$$\begin{aligned}0 =& \frac{\partial \widehat{R}}{\partial t} - d_R \Delta \widehat{R} - \widehat{R}\left[-a_1^M - \frac{p_2^M}{\gamma_4^L} - \frac{p_3^M}{\gamma_5^L}\right]\\ \leq& \frac{\partial R}{\partial t} - d_R \Delta R - R\left[-a_1^M - \frac{p_2^M}{\gamma_4^L} - \frac{p_3^M}{\gamma_5^L}\right].\end{aligned}$$

Now, using Lemma 2.2 for $m = 1$, $R(t, x, Q_0, R_0, X_0, Y_0) \geq \widehat{R}(t, M_R)$ for $t \in [0, t_1]$. Applying the last inequality for $t = t_1$, together with (1.2), we obtain that

$$R(t_1^+, x, Q_0, R_0, X_0, Y_0) \geq \widehat{R}(t, m_R) \inf_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x, Q, R, X, Y).$$

It then follows that the solution $R(t, x, Q_0, R_0, X_0, Y_0)$ is bounded from below by a solution of the following equation with impulses:

$$\begin{cases} \dfrac{d\widehat{R}}{dt} = \widehat{R}\left[-a_1^M - \dfrac{p_2^M}{\gamma_4^L} - \dfrac{p_3^M}{\gamma_5^L}\right],\\ R(t_k^+) = \widehat{R}(t_k) \inf\limits_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x, Q, R, X, Y).\end{cases} \tag{4.6}$$

Then the solution of (4.6) is

$$\begin{aligned}\widehat{R}(\tau) &= \widehat{R}(0) \prod \inf_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x, Q, R, X, Y) \exp\left\{(-a_1^M - \frac{p_2^M}{\gamma_4^L} - \frac{p_3^M}{\gamma_5^L})\tau\right\}\\ &= \widehat{R}(0) \exp\left\{(-a_1^M - \frac{p_2^M}{\gamma_4^L} - \frac{p_3^M}{\gamma_5^L})\tau + \sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x, Q, R, X, Y)\right\}\\ &= \widehat{R}(0) Z(\tau).\end{aligned}$$

where $Z(\tau) = \exp\left\{(-a_1^M - \frac{p_2^M}{\gamma_4^L} - \frac{p_3^M}{\gamma_5^L})\tau + \ln \sum_{k=1}^{p} \inf_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x, Q, R, X, Y)\right\}$. Let's assume that $R^*(t)$ is an piecewise continuous strictly positive periodic solution of (4.6), and $R(t)$ is any solution of (4.6). Iet's prove that $\lim\limits_{t\to+\infty} \mid R(t) - R^*(t) \mid= 0$ when (4.2) holds.

$$\lim_{t\to+\infty} \mid \widehat{R}(t) - \widehat{R}^*(t) \mid= \lim_{t\to+\infty} \mid \widehat{R}(0)Z(\tau) + \widehat{R}^*(0)Z(\tau) \mid= \lim_{t\to+\infty} \mid Z(\tau)\left(\widehat{R_0} - \widehat{R_0^*}\right) \mid.$$

Since Eq (4.6) is ultimately bounded, $\widehat{R_0} - \widehat{R_0^*}$ is bounded. When condition (4.2) holds, we have

$$\lim_{t\to+\infty} Z(\tau) = \lim_{t\to+\infty} \exp\left\{(-a_1^M - \frac{p_2^M}{\gamma_4^L} - \frac{p_3^M}{\gamma_5^L})\tau + \sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x,Q,R,X,Y)\right\} = 0.$$

Therefore, the positive periodic solution $R^*(t)$ is globally asymptotically stable, i.e., $\lim_{t\to+\infty} R(t) = R^*(t)$. Thus, there is a positive constant σ_1^* such that $\widehat{R}(t, R_m) \geq \sigma_1^*$.

We can obtain that there is a normal number σ_1^*, such that $R(t, x, Q_0, R_0, X_0, Y_0) \geq \sigma_1^*$ for $t > \widehat{t_1}$.

For X, we have

$$\begin{aligned}
0 =& \frac{\partial X}{\partial t} - d_X \Delta X - X\left[a_2 - b_1 X\right] + \frac{q_2 XY}{X + c_2 Y} \\
&- \frac{p_4 QX}{1 + \gamma_1 Q + \gamma_2 X + \gamma_1\gamma_2 RX} - \frac{p_5 RX}{1 + \gamma_3 R + \gamma_4 X + \gamma_3\gamma_4 RX} \\
\leq& \frac{\partial X}{\partial t} - d_X \Delta X - X\left[a_2^L - b_1^M X - \frac{q_2^M}{c_2^L}\right].
\end{aligned}$$

According to condition (4.3) the same analysis as population Q, we obtain that there is a normal number σ_2^*, such that $X(t, x, Q_0, R_0, X_0, Y_0) \geq \sigma_2^*$ for $t > \widehat{t_2}$.

For population Y, we have

$$\begin{aligned}
0 =& \frac{\partial Y}{\partial t} - d_Y \Delta Y + Y a_3 - \frac{q_2 XY}{X + c_2 Y} - \frac{p_6 RY}{1 + \gamma_3 R + \gamma_5 Y + \gamma_3\gamma_5 RY} \\
\leq& \frac{\partial Y}{\partial t} - d_Y \Delta Y + a_3^M Y.
\end{aligned}$$

According to condition (4.4) the same analysis as population R, we obtain that there is a normal number σ_3^*, such that $Y(t, x, Q_0, R_0, X_0, Y_0) \geq \sigma_3^*$ for $t > \widehat{t_3}$.

The proof is completed.

5. The extinction of epidemic

Definition 3. *(Definition 3. [28]) It is called epidemics extinction if* $\lim_{t\to+\infty} R(t, x, Q_0(x), R_0(x), X_0(x), Y_0(x)) = 0$ *and* $\lim_{t\to+\infty} Y(t, x, Q_0(x), R_0(x), X_0(x), Y_0(x)) = 0$.

Now, we study the extinction of epidemic in (1.1)–(1.11).

Theorem 5.1. *If* (H_1)–(H_5) *hold, furthermore, inequality*

$$(-a_1^L + q_1^M)\tau + \sum_{k=1}^{p} \ln \sup_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x,Q,R,X,Y) < 0, \tag{5.1}$$

and

$$(-a_3^L + q_2^M + \frac{p_6^M}{\gamma_3^L})\tau + \sum_{k=1}^{p} \ln \sup_{x\in\Omega,(Q,R,X,Y)\in S} \omega_k(x,Q,R,X,Y) < 0, \tag{5.2}$$

hold.

Then $\lim_{t\to+\infty} R(t,x) = 0$, $\lim_{t\to+\infty} Y(t,x) = 0$.

Proof. Taking a positive constant M_R such that $M_R \geq R_0(x)$. Let $\overline{R}(t, M_R)$ be the solution to the following initial value problem

$$\begin{cases} \dfrac{d\overline{R}}{dt} = \overline{R}\left(-a_1^L + q_1^M\right), \\ \overline{R}(0, M_R) = M_R. \end{cases}$$

From the following inequality,

$$\begin{aligned} 0 =& \frac{\partial R}{\partial t} - d_R \Delta R - R\left[-a_1\right] - \frac{q_1 Q R}{Q + c_1 R} \\ &+ \frac{p_2 R X}{1 + \gamma_3 R + \gamma_4 X + \gamma_3 \gamma_4 R X} + \frac{p_3 R Y}{1 + \gamma_3 R + \gamma_5 Y + \gamma_3 \gamma_5 R Y} \\ \geq& \frac{\partial R}{\partial t} - d_R \Delta R - R\left[-a_1^L + q_1^M\right]. \end{aligned}$$

Using comparison theory, it is easy to know $R(t, x, Q, R, X, Y) \leq \overline{R}(t, M_R)$ for $t \leq t_1$.

By the impulse condition we get

$$R(t_1^+, x, Q, R, X, Y) \leq \overline{R}(t, M_R) \sup_{(x,Q,R,X,Y)} g_k(x, Q, R, X, Y).$$

Continuing in this manner, we conclude that every solution to R with impulse condition $R(t_k^+, x) = R(t_k, x) g_k(x, Q(t_k, x), R(t_k, x), X(t_k, x), Y(t_k, x))$ is bounded by the corresponding solution to the following impulse equation

$$\begin{cases} \dfrac{d\overline{R}}{dt} = \overline{R}\left(-a_1^L + q_1^M\right), \\ \overline{R}(t_k^+) = \overline{R}(t_k) \sup\limits_{(x,Q,R,X,Y)} g_k(x, Q, R, X, Y). \end{cases} \tag{5.3}$$

Then the solution of (5.3) is

$$\begin{aligned} \overline{R}(\tau) &= \overline{R}(0) \prod \sup_{x \in \Omega, (Q,R,X,Y) \in S} g_k(x, Q, R, X, Y) \exp\left\{(-a_1^L + q_1^M)\tau\right\} \\ &= \overline{R}(0) \exp\left\{(-a_1^L + q_1^M)\tau + \sum_{k=1}^{p} \ln \sup_{x \in \Omega, (Q,R,X,Y) \in S} g_k(x, Q, R, X, Y)\right\}. \end{aligned}$$

By (5.1), we can see that all the solutions of the (5.3) tend to 0 as $t \to \infty$.

For infected predator Y,

$$\begin{aligned} 0 &= \frac{\partial Y}{\partial t} - d_Y \Delta Y + Y a_3 - \frac{q_2 X Y}{X + c_2 Y} - \frac{p_6 R Y}{1 + \gamma_3 R + \gamma_5 Y + \gamma_3 \gamma_5 R Y} \\ &\geq \frac{\partial Y}{\partial t} - d_Y \Delta Y + Y\left(a_3^L - q_2^M - \frac{p_6^M}{\gamma_3^L}\right). \end{aligned} \tag{5.4}$$

We can obtain in the same way. Any solution of (5.4) goes to 0 when (5.2) is true.

The proof is completed.

6. Periodic solution

In this Section, we prove that systems (1.1)–(1.11) has a unique globally asymptotic stable periodic solution by constructing Lyapunov function.

Theorem 6.1. *If conditions* (H_1)*–*(H_5) *hold and Lemma 2.4 is true, and epidemic is prevailing, i.e., there exist two positive constants* σ *and* N*, so that any nonnegative and* $\not\equiv 0$ *solution satisfying the initial value and impulsive condition of the system satisfies:*

$$\{Q(t,x),R(t,x),X(t,x),Y(t,x)\}\in\prod=\{(Q,R,X,Y)\mid\sigma\le Q(t,x),R(t,x),X(t,x),Y(t,x)\le N\},$$

from a certain moment. In addition, let

$$\sum_{j=1}^{p}\ln K_j+\tau\lambda_M<0,\tag{6.1}$$

where

$$\begin{aligned}K_j=\max_{Q,R,X,Y\in\Pi,x\in\Omega}2\Bigg\{&f_j^2+\left(N\frac{\partial f_j}{\partial Q}\right)^2+\left(N\frac{\partial f_j}{\partial R}\right)^2+\left(N\frac{\partial f_j}{\partial X}\right)^2+\left(N\frac{\partial f_j}{\partial Y}\right)^2+g_j^2+\left(N\frac{\partial g_j}{\partial Q}\right)^2\\&+\left(N\frac{\partial g_j}{\partial R}\right)^2+\left(N\frac{\partial g_j}{\partial X}\right)^2+\left(N\frac{\partial g_j}{\partial Y}\right)^2+h_j^2+\left(N\frac{\partial h_j}{\partial Q}\right)^2+\left(N\frac{\partial h_j}{\partial R}\right)^2+\left(N\frac{\partial h_j}{\partial X}\right)^2\\&+\left(N\frac{\partial h_j}{\partial Y}\right)^2+\omega_j^2+\left(N\frac{\partial \omega_j}{\partial Q}\right)^2+\left(N\frac{\partial \omega_j}{\partial R}\right)^2+\left(N\frac{\partial \omega_j}{\partial X}\right)^2+\left(N\frac{\partial \omega_j}{\partial Y}\right)^2\Bigg\},\end{aligned}$$

and λ_M *is the principal eigenvalue of the following matrix*

$$\begin{pmatrix}M_{11}&M_{12}&M_{13}&M_{14}\\M_{21}&M_{22}&M_{23}&M_{24}\\M_{31}&M_{32}&M_{33}&M_{34}\\M_{41}&M_{42}&M_{43}&M_{44}\end{pmatrix},$$

where

$$\begin{aligned}M_{11}&=2\left[a_0^M-b_0^L\sigma-\frac{c_1^Lq_1^L}{(N/\sigma+c_1^M)^2}-\frac{\gamma_2^Lp_1^L\sigma+p_1^L}{(1+\gamma_1^M+\gamma_2^M+\gamma_1^M\gamma_2^MN)^2}\right],\\M_{22}&=2\left[-a_1^L+\frac{q_1^M}{(1+c_1^L\sigma/N)^2}-\frac{\gamma_4^Lp_2^L\sigma+p_2^L}{(1+\gamma_3^M+\gamma_4^M+\gamma_3^M\gamma_4^MN)^2}-\frac{\gamma_5^Lp_3^L\sigma+p_3^L}{(1+\gamma_3^M+\gamma_5^M+\gamma_3^M\gamma_5^MN)^2}\right],\\M_{33}&=2\left[a_2^M-b_1^L\sigma-\frac{c_2^Lq_2^L}{(N/\sigma+c_2^M)^2}+\frac{\gamma_1^Mp_4^MN+p_4^M}{(1+\gamma_1^L+\gamma_2^L+\gamma_1^L\gamma_2^L\sigma)^2}+\frac{\gamma_3^Mp_5^MN+p_5^M}{(1+\gamma_3^L+\gamma_4^L+\gamma_3^L\gamma_4^L\sigma)^2}\right],\\M_{44}&=2\left[-a_3^L+\frac{q_2^M}{(N/\sigma+c_2^M)^2}+\frac{\gamma_3^Mp_6^MN+p_6^M}{(1+\gamma_3^L+\gamma_5^L+\gamma_3^L\gamma_5^L\sigma)^2}\right],\\M_{13}&=M_{31}=\left[\frac{p_1^M}{\gamma_1^L}+\frac{p_1^M}{(\gamma_1^L)^2\sigma}+\frac{p_4^M}{\gamma_2^L}+\frac{p_4^M}{(\gamma_2^L)^2\sigma}\right],\end{aligned}$$

$$M_{23} = M_{32} = \left[\frac{p_2^M}{\gamma_3^L} + \frac{p_2^M}{(\gamma_3^L)^2\sigma} + \frac{p_5^M}{\gamma_4^L} + \frac{p_5^M}{(\gamma_4^L)^2\sigma}\right],$$
$$M_{24} = M_{42} = \left[\frac{p_3^M}{\gamma_3^L} + \frac{p_3^M}{(\gamma_3^L)^2\sigma} + \frac{p_6^M}{\gamma_5^L} + \frac{p_6^M}{(\gamma_5^L)^2\sigma}\right],$$
$$M_{12} = M_{21} = q_1^M + \frac{q_1^M}{c_1^L},$$
$$M_{34} = M_{43} = q_2^M + \frac{q_2^M}{c_2^L},$$
$$M_{14} = M_{41} = 0.$$

Systems (1.1)–(1.11) *have a strictly positive piecewise continuous τ periodic solution, and the periodic solution is globally asymptotic stable.*

Proof. First, we consider the operator $\Upsilon : \mathbb{R}^4 \to \mathbb{R}^4$,

$$\Upsilon(Q(t_0^+), R(t_0^+), X(t_0^+), Y(t_0^+)) = (Q(t_k^+), R(t_k^+), X(t_k^+), Y(t_k^+)), k \in \mathbb{N}_+.$$

The continuous operator Υ maps the closed bounded connected convex set $\mathbb{S}$ into itself because the disease is prevailing, Υ satisfies the conditions of the Lemma 2.6. In this case, $\mathbb{S} = \mathbb{R}^4$. Therefore, the operator Υ has at least one fixed point (Q^*, R^*, X^*, Y^*) on $\mathbb{R}^4$. Since the parameters are positive and periodic, we know that this fixed point is the positive periodic solution of system (1.1)-(1.11). This proves the existence of periodic solutions.

Second, we prove the global asymptotic stability of the periodic solution.

Suppose $\{Q(t,x), R(t,x), X(t,x), Y(t,x)\}$ and $\left\{\overline{Q}(t,x), \overline{R}(t,x), \overline{X}(t,x), \overline{Y}(t,x)\right\}$ are two solutions of systems (1.1)–(1.11), which satisfy $\sigma \le Q(t,x), R(t,x), X(t,x), Y(t,x) \le N$ and $\sigma \le \overline{Q}(t,x), \overline{R}(t,x), \overline{X}(t,x), \overline{Y}(t,x) \le N$. Consider the function

$$J(t) = \int_\Omega \left[(Q(t,x) - \overline{Q}(t,x))^2 + (R(t,x) - \overline{R}(t,x))^2 + (X(t,x) - \overline{X}(t,x))^2 + (Y(t,x) - \overline{Y}(t,x))^2\right] dx.$$

With derivative

$$\begin{aligned}
\frac{dJ(t)}{dt} =& 2\int_\Omega (Q - \overline{Q})(\frac{\partial Q}{\partial t} - \frac{\partial \overline{Q}}{\partial t})dx + 2\int_\Omega (R - \overline{R})(\frac{\partial R}{\partial t} - \frac{\partial \overline{R}}{\partial t})dx \\
&+ 2\int_\Omega (X - \overline{X})(\frac{\partial X}{\partial t} - \frac{\partial \overline{X}}{\partial t})dx + 2\int_\Omega (Y - \overline{Y})(\frac{\partial Y}{\partial t} - \frac{\partial \overline{Y}}{\partial t})dx \\
=& 2d_Q \int_\Omega (Q - \overline{Q})\Delta(Q - \overline{Q})dx + 2d_R \int_\Omega (R - \overline{R})\Delta(R - \overline{R})dx \\
&+ 2d_X \int_\Omega (X - \overline{X})\Delta(X - \overline{X})dx + 2d_Y \int_\Omega (Y - \overline{Y})\Delta(Y - \overline{Y})dx \\
&+ 2\int_\Omega (Q - \overline{Q})\left[Q(a_0 - b_0 Q - \frac{q_1 R}{Q + c_1 R} - \frac{p_1 X}{1 + \gamma_1 Q + \gamma_2 X + \gamma_1\gamma_2 QX})\right. \\
&\left. -\overline{Q}(a_0 - b_0\overline{Q} - \frac{q_1\overline{R}}{\overline{Q} + c_1\overline{R}} - \frac{p_1\overline{X}}{1 + \gamma_1\overline{Q} + \gamma_2\overline{X} + \gamma_1\gamma_2\overline{QX}})\right] dx
\end{aligned}$$

$$
\begin{aligned}
&+2\int_\Omega (R-\overline{R})\Big[-a_1R+\frac{q_1QR}{Q+c_1R}-\frac{p_2RX}{1+\gamma_3R+\gamma_4X+\gamma_3\gamma_4RX}\\
&-\frac{p_3RY}{1+\gamma_3R+\gamma_5Y+\gamma_3\gamma_5RY}+a_1\overline{R}-\frac{q_1\overline{QR}}{\overline{Q}+c_1\overline{R}}+\frac{p_2\overline{RX}}{1+\gamma_3\overline{R}+\gamma_4\overline{X}+\gamma_3\gamma_4\overline{RX}}\\
&+\frac{p_3\overline{RY}}{1+\gamma_3\overline{R}+\gamma_5\overline{Y}+\gamma_3\gamma_5\overline{RY}}\Big]dx+2\int_\Omega (X-\overline{X})\Big[a_2X-b_1X^2-\frac{q_2XY}{X+c_2Y}\\
&+\frac{p_4QX}{1+\gamma_1Q+\gamma_2X+\gamma_1\gamma_2QX}+\frac{p_5RX}{1+\gamma_3R+\gamma_4X+\gamma_3\gamma_4RX}-a_2\overline{X}+b_1\overline{X}^2+\frac{q_2\overline{YX}}{\overline{X}+c_2\overline{Y}}\\
&-\frac{p_4\overline{XQ}}{1+\gamma_1\overline{Q}++\gamma_2\overline{X}+\gamma_1\gamma_2\overline{XQ}}-\frac{p_5\overline{XR}}{1+\gamma_3\overline{R}+\gamma_4\overline{X}+\gamma_3\gamma_4\overline{XR}}\Big]dx\\
&+2\int_\Omega (Y-\overline{Y})\Big[-a_3Y+\frac{q_2XY}{X+c_2Y}+\frac{p_6RY}{1+\gamma_3R+\gamma_5Y+\gamma_3\gamma_5RY}+a_3\overline{Y}-\frac{q_2\overline{XY}}{\overline{X}+c_2\overline{Y}}\\
&-\frac{p_6\overline{YR}}{1+\gamma_3\overline{R}+\gamma_5\overline{Y}+\gamma_3\gamma_5\overline{YR}}\Big]dx\\
\le&2\int_\Omega (Q-\overline{Q})^2\Big[a_0-b_0(Q+\overline{Q})-\frac{c_1q_1R\overline{R}}{(Q+c_1R)(\overline{Q}+c_1\overline{R})}\\
&-\frac{\gamma_2p_1X\overline{X}+p_1\overline{X}}{(1+\gamma_1Q+\gamma_2X+\gamma_1\gamma_2QX)(1+\gamma_1\overline{Q}+\gamma_2\overline{X}+\gamma_1\gamma_2\overline{QX})}\Big]dx.\\
&+2\int_\Omega (R-\overline{R})^2\Big[-a_1+\frac{q_1Q\overline{Q}}{(Q+c_1R)(\overline{Q}+c_1\overline{R})}\\
&-\frac{\gamma_4p_2X\overline{X}+p_2\overline{X}}{(1+\gamma_3R+\gamma_4X+\gamma_3\gamma_4RX)(1+\gamma_3\overline{R}+\gamma_4\overline{X}+\gamma_3\gamma_4\overline{RX})}\\
&-\frac{\gamma_5p_3Y\overline{Y}+p_3\overline{Y}}{(1+\gamma_3R+\gamma_5Y+\gamma_3\gamma_5RY)(1+\gamma_3\overline{R}+\gamma_5\overline{Y}+\gamma_3\gamma_5\overline{RY})}\Big]dx\\
&+2\int_\Omega (X-\overline{X})^2\Big[a_2-b_1(X+\overline{X})-\frac{c_2q_2Y\overline{Y}}{(X+c_2Y)(\overline{X}+c_2\overline{Y})}\\
&+\frac{\gamma_1p_4Q\overline{Q}+p_4\overline{Q}}{(1+\gamma_1Q+\gamma_2X+\gamma_1\gamma_2QX)(1+\gamma_1\overline{Q}+\gamma_2\overline{X}+\gamma_1\gamma_2\overline{QX})}\\
&+\frac{\gamma_3p_5R\overline{R}+p_5\overline{R}}{(1+\gamma_3R+\gamma_4X+\gamma_3\gamma_4RX)(1+\gamma_3\overline{R}+\gamma_4\overline{X}+\gamma_3\gamma_4\overline{RX})}\Big]dx\\
&+2\int_\Omega (Y-\overline{Y})^2\Big[-a_3+\frac{q_2X\overline{X}}{(X+c_2Y)(\overline{X}+c_2\overline{Y})}\\
&+\frac{\gamma_3p_6R\overline{R}+p_6\overline{R}}{(1+\gamma_3R+\gamma_5Y+\gamma_3\gamma_5RY)(1+\gamma_3\overline{R}+\gamma_5\overline{Y}+\gamma_3\gamma_5\overline{RY})}\Big]dx\\
&-2\int_\Omega (Q-\overline{Q})(R-\overline{R})\left[\frac{q_1Q\overline{Q}-c_1q_1R\overline{R}}{(Q+c_1R)(\overline{Q}+c_1\overline{R})}\right]dx
\end{aligned}
$$

$$
\begin{aligned}
&-2\int_{\Omega}(Q-\overline{Q})(X-\overline{X})\left[\frac{\gamma_1 p_1 Q\overline{Q}+p_1 Q}{(1+\gamma_1 Q+\gamma_2 X+\gamma_1\gamma_2 QX)(1+\gamma_1\overline{Q}+\gamma_2\overline{X}+\gamma_1\gamma_2\overline{QX})}\right]dx\\
&-2\int_{\Omega}(R-\overline{R})(X-\overline{X})\left[\frac{\gamma_3 p_2 R\overline{R}+p_2 R}{(1+\gamma_3 R+\gamma_4 X+\gamma_3\gamma_4 RX)(1+\gamma_3\overline{R}+\gamma_4\overline{X}+\gamma_3\gamma_4\overline{RX})}\right]dx\\
&-2\int_{\Omega}(R-\overline{R})(Y-\overline{Y})\left[\frac{\gamma_3 p_3 R\overline{R}+p_3 R}{(1+\gamma_3 R+\gamma_5 Y+\gamma_3\gamma_5 RY)(1+\gamma_3\overline{R}+\gamma_5\overline{Y}+\gamma_3\gamma_5\overline{RY})}\right]dx\\
&-2\int_{\Omega}(X-\overline{X})(Q-\overline{Q})\left[\frac{\gamma_2 p_4 X\overline{X}+p_4 X}{(1+\gamma_1 Q+\gamma_2 X+\gamma_1\gamma_2 QX)(1+\gamma_1\overline{Q}+\gamma_2\overline{X}+\gamma_1\gamma_2\overline{QX})}\right]dx\\
&-2\int_{\Omega}(X-\overline{X})(R-\overline{R})\left[\frac{\gamma_4 p_5 X\overline{X}+p_5 X}{(1+\gamma_3 R+\gamma_4 X+\gamma_3\gamma_4 RX)(1+\gamma_3\overline{R}+\gamma_4\overline{X}+\gamma_3\gamma_4\overline{RX})}\right]dx\\
&-2\int_{\Omega}(Y-\overline{Y})(R-\overline{R})\left[\frac{\gamma_5 p_6 Y\overline{Y}+p_6 Y}{(1+\gamma_3 R+\gamma_5 Y+\gamma_3\gamma_5 RY)(1+\gamma_3\overline{R}+\gamma_5\overline{Y}+\gamma_3\gamma_5\overline{RY})}\right]dx\\
&-2\int_{\Omega}(X-\overline{X})(Y-\overline{Y})\left[\frac{q_2 X\overline{X}-c_2 q_2 Y\overline{Y}}{(X+c_2 Y)(\overline{X}+c_2\overline{Y})}\right]dx\\
\leq &2\int_{\Omega}(Q-\overline{Q})^2\left[a_0^M-b_0^L\sigma-\frac{c_1^L q_1^L}{(N/\sigma+c_1^M)^2}-\frac{\gamma_2^L p_1^L\sigma+p_1^L}{(1+\gamma_1^M+\gamma_2^M+\gamma_1^M\gamma_2^M N)^2}\right]dx\\
&+2\int_{\Omega}(R-\overline{R})^2\left[-a_1^L+\frac{q_1^M}{(1+c_1^L\sigma/N)^2}-\frac{\gamma_4^L p_2^L\sigma+p_2^L}{(1+\gamma_3^M+\gamma_4^M+\gamma_3^M\gamma_4^M N)^2}\right.\\
&\left.-\frac{\gamma_5^L p_3^L\sigma+p_3^L}{(1+\gamma_3^M+\gamma_5^M+\gamma_3^M\gamma_5^M N)^2}\right]dx\\
&+2\int_{\Omega}(X-\overline{X})^2\left[a_2^M-b_1^L\sigma-\frac{c_2^L q_2^L}{(N/\sigma+c_2^M)^2}+\frac{\gamma_1^M p_4^M N+p_4^M}{(1+\gamma_1^L+\gamma_2^L+\gamma_1^L\gamma_2^L\sigma)^2}\right.\\
&\left.+\frac{\gamma_3^M p_5^M N+p_5^M}{(1+\gamma_3^L+\gamma_4^L+\gamma_3^L\gamma_4^L\sigma)^2}\right]dx\\
&+2\int_{\Omega}(Y-\overline{Y})^2\left[-a_3^L+\frac{q_2^M}{(N/\sigma+c_2^M)^2}+\frac{\gamma_3^M p_6^M N+p_6^M}{(1+\gamma_3^L+\gamma_5^L+\gamma_3^L\gamma_5^L\sigma)^2}\right]dx\\
&+2\int_{\Omega}|Q-\overline{Q}||R-\overline{R}|\left(q_1^M+\frac{q_1^M}{c_1^L}\right)dx\\
&+2\int_{\Omega}|Q-\overline{Q}||X-\overline{X}|\left(\frac{p_1^M}{\gamma_1^L}+\frac{p_1^M}{(\gamma_1^L)^2\sigma}+\frac{p_4^M}{\gamma_2^L}+\frac{p_4^M}{(\gamma_2^L)^2\sigma}\right)dx\\
&+2\int_{\Omega}|R-\overline{R}||X-\overline{X}|\left(\frac{p_2^M}{\gamma_3^L}+\frac{p_2^M}{(\gamma_3^L)^2\sigma}+\frac{p_5^M}{\gamma_4^L}+\frac{p_5^M}{(\gamma_4^L)^2\sigma}\right)dx\\
&+2\int_{\Omega}|R-\overline{R}||Y-\overline{Y}|\left(\frac{p_3^M}{\gamma_3^L}+\frac{p_3^M}{(\gamma_3^L)^2\sigma}+\frac{p_6^M}{\gamma_5^L}+\frac{p_6^M}{(\gamma_5^L)^2\sigma}\right)dx\\
&+2\int_{\Omega}|X-\overline{X}||Y-\overline{Y}|\left(q_2^M+\frac{q_2^M}{c_2^L}\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&\leq \lambda_m \int_\Omega \left[(Q-\overline{Q})^2 + (R-\overline{R})^2 + (X-\overline{X})^2 + (Y-\overline{Y})^2\right] \\
&= \lambda_M J(t).
\end{aligned}
$$

So we obtain $J(t_{j+1}) \leq J(t_j^+) \exp\left\{\lambda_M (t_{j+1} - t_j)\right\}$, and

$$
\begin{aligned}
J(t_{j+1}^+) = &\int_\Omega \left[Q f_{j+1}(Q,R,X,Y) - \overline{Q} f_{j+1}(\overline{Q},\overline{R},\overline{X},\overline{Y})\right]^2 dx \\
&+ \int_\Omega \left[R g_{j+1}(Q,R,X,Y) - \overline{R} g_{j+1}(\overline{Q},\overline{R},\overline{X},\overline{Y})\right]^2 dx \\
&+ \int_\Omega \left[X h_{j+1}(Q,R,X,Y) - \overline{X} h_{j+1}(\overline{Q},\overline{R},\overline{X},\overline{Y})\right]^2 dx \\
&+ \int_\Omega \left[Y \omega_{j+1}(Q,R,X,Y) - \overline{Y} \omega_{j+1}(\overline{Q},\overline{R},\overline{X},\overline{Y})\right]^2 dx \\
\leq & K_{j+1} J(t_{j+1}) \leq K_{j+1} \exp\left\{\lambda_M (t_{j+1} - t_j)\right\} J(t_j^+).
\end{aligned}
$$

Therefore, we can easily get the change of the function in this period of time. We have

$$
J(t+\tau) \leq K_* J(t) = \prod_{i=1}^{p} K_i \exp(\lambda_M \tau) J(t).
$$

According to condition (6.1), we obtain $K_* < 1$. Therefore,

$$
J(m\tau + s) \leq K_*^m J(s) \to 0, m \to \infty.
$$

That is,

$$
\begin{aligned}
\lim_{t\to+\infty} \| Q(t,x) - \overline{Q}(t,x) \|_{L_2} = 0, \\
\lim_{t\to+\infty} \| R(t,x) - \overline{R}(t,x) \|_{L_2} = 0, \\
\lim_{t\to+\infty} \| X(t,x) - \overline{X}(t,x) \|_{L_2} = 0,
\end{aligned}
$$

and

$$
\lim_{t\to+\infty} \| Y(t,x) - \overline{Y}(t,x) \|_{L_2} = 0.
$$

Because solutions of systems (1.1)–(1.11) are bounded in the space $C^{1+\nu}$ by Lemma 2.4. Therefore,

$$
\lim_{t\to+\infty} \sup_{x\in\Omega} | Q(t,x) - \overline{Q}(t,x) | = 0, \tag{6.2}
$$

$$
\lim_{t\to+\infty} \sup_{x\in\Omega} | R(t,x) - \overline{R}(t,x) | = 0, \tag{6.3}
$$

$$
\lim_{t\to+\infty} \sup_{x\in\Omega} | X(t,x) - \overline{X}(t,x) | = 0, \tag{6.4}
$$

and

$$\lim_{t\to+\infty}\sup_{x\in\Omega} \mid Y(t,x)-\overline{Y}(t,x)\mid = 0. \tag{6.5}$$

Therefore, the solution of systems (1.1)–(1.11) is globally stable.

At last, we prove the uniqueness of periodic solutions.

For the sequence $\{Q(k\tau, x, Q_0, R_0, X_0, Y_0), R(k\tau, x, Q_0, R_0, X_0, Y_0), X(k\tau, x, Q_0, R_0, X_0, Y_0), Y(k\tau, x, Y_0, R_0, X_0, Y_0)\} = \upsilon(k\tau, \upsilon_0), k \in \mathbb{N}_+$. It is compact in the space $C(\overline{\Omega}) \times C(\overline{\Omega}) \times C(\overline{\Omega}) \times C(\overline{\Omega})$ from Lemma 2.4. Let $\overline{\upsilon}$ be the limit point of the sequence, $\overline{\upsilon} = \lim_{n\to+\infty} \upsilon(k_n\tau, \upsilon_0)$. Then $\upsilon(\tau, \overline{\upsilon}) = \overline{\upsilon}$. Actually, since $\upsilon\{\tau, \upsilon(k_n\tau, \upsilon_0)\} = \upsilon\{k_n\tau, \upsilon(\tau, \upsilon_0)\}$ and $\lim_{k_n\to+\infty}[\upsilon(k_n\tau, \upsilon(\tau, \upsilon_0)) - \upsilon(k_n\tau, \upsilon_0)] = 0$. We obtain

$$\begin{aligned}\lim_{n\to+\infty} \parallel \upsilon(\tau, \overline{\upsilon}) - \overline{\upsilon} \parallel_C &\leq \lim_{n\to+\infty} \parallel \upsilon(\tau, \overline{\upsilon}) - \upsilon(\tau, \upsilon(k_n\tau, \upsilon_0) \parallel_C \\ &+ \lim_{n\to+\infty} \parallel \upsilon(\tau, \upsilon(k_n\tau, \upsilon_0) - \upsilon(k_n\tau, \upsilon_0) \parallel_C + \lim_{n\to+\infty} \parallel \upsilon(k_n\tau, \upsilon_0) - \overline{\upsilon} \parallel_C = 0.\end{aligned}$$

The sequence $\{\upsilon(k\tau, \upsilon_0), K \in \mathbb{N}_+\}$ has a unique limit point. If not, assume that the sequence has two limit points $\overline{\upsilon} = \lim_{n\to+\infty} \upsilon(k_n\tau, \upsilon_0)$, and $\tilde{\upsilon} = \lim_{n\to+\infty} \upsilon(k_n\tau, \upsilon_0)$. Then, taking into account (6.3) to (6.5), we obtain

$$\lim_{n\to+\infty} \parallel \overline{\upsilon} - \tilde{\upsilon} \parallel_C \leq \lim_{n\to+\infty} \parallel \overline{\upsilon} - \upsilon(k_n\tau, \upsilon_0) \parallel_C + \lim_{n\to+\infty} \parallel \upsilon(k_n\tau, \upsilon_0) - \tilde{\upsilon} \parallel_C = 0.$$

Hence $\overline{\upsilon} = \tilde{\upsilon}$. The periodic solution $\left\{Q(t, x, \overline{Q}, \overline{R}, \overline{X}, \overline{Y}), R(t, x, \overline{Q}, \overline{R}, \overline{X}, \overline{Y}), X(t, x, \overline{Q}, \overline{R}, \overline{X}, \overline{Y}), Y(t, x, \overline{Q}, \overline{R}, \overline{X}, \overline{Y})\right\}$ of systems (1.1)–(1.11) is unique.

7. Numerical simulations

In this Section, similar to that of [28], we perform some numerical examples to illustrate our main results in $\Omega = [-2, 2]$ [28], the diffusion coefficients d_Q, d_R, d_X, d_Y are taken as $d_Q = d_R = d_X = d_Y = 0.5$ [28]. We will present our theoretical results through the following three examples.

7.1. Example 1

For the sake of convenience, we choose: $c_1 = c_2 = 1.5$ [28]; $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 6$. The impulsive functions $f_k = 0.8$ [28]; $g_k = 0.6$; $h_k = 0.7$ [28]; $\omega_k = 0.8$. For convenience, the initial values are taken as $Q_0 = 3$; $R_0 = 9$; $X_0 = 1$ and $Y_0 = 1$. Setting other parameters as follows:

Table 1. Parameter description in the model.

Parameters	Value	Reference	Parameters	Value	Reference
$a_0(t,x)$	$1.5\sin t + 1.5\cos x + 15$	[28]	$b_0(t,x)$	$1.5\cos t + 1.3\cos x + 5$	[28]
$a_3(t,x)$	$0.6\sin t + 0.6\cos x + 4$	[28]	$p_5(t,x)$	$0.5\sin t + 0.5\cos x + 5$	[28]
$q_2(t,x)$	$0.5\sin t + 0.5\cos x + 3$	[28]	$p_3(t,x)$	$\sin t + 0.8\cos x + 5$	[28]
$p_2(t,x)$	$1.5\sin t + \cos x + 7$	[28]	$p_1(t,x)$	$1.5\sin t + \cos x + 7$	[28]
$a_1(t,x)$	$0.4\sin t + 0.3\cos x + 0.2$	-	$b_1(t,x)$	$1.5\cos t + 1.5\cos x + 6$	-
$a_2(t,x)$	$0.4\sin t + 0.3\cos x + 3$	-	$p_6(t,x)$	$0.1\sin t + 0.1\cos x + 0.1$	-
$p_4(t,x)$	$0.5\sin t + 0.5\cos x + 7$	-	$q_1(t,x)$	$0.7\sin t + 0.7\cos x + 5$	-

At this time, we can easily get the following inequality:

$$-a_1^L\tau + \sum_{0\le t_k<t} \ln \sup_{(x,Q,R,X,Y)} g_k(x,Q,R,X,Y) = -6.207 < 0,$$

and

$$-\tau a_3^L + \sum_{0\le t_k<t} \ln \sup_{(x,Q,R,X,Y)} \omega_k(x,Q,R,X,Y) = -18.932 < 0.$$

In addition

$$\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} f_k(x,Q,R,X,Y) + \tau(a_0^L - \frac{p_1^M}{\gamma_2^L} - \frac{q_1^M}{c_1^L}) = 37.303 > 0,$$

$$-\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x,Q,R,X,Y) + \tau(a_1^M + \frac{p_2^M}{\gamma_4^L} + \frac{p_3^M}{\gamma_5^L}) = 25.684 > 0,$$

$$\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} h_k(x,Q,R,X,Y) + \tau(a_2^L - \frac{q_2^M}{c_2^L}) = 4.350 > 0,$$

and

$$-\sum_{k=1}^{p} \ln \inf_{x\in\Omega,(Q,R,X,Y)\in S} \omega_k(x,Q,R,X,Y) + a_3^M\tau = 34.011 > 0.$$

According to Theorems 3.1 and 4.1, the population is persistent, as shown in Figures 1–3.

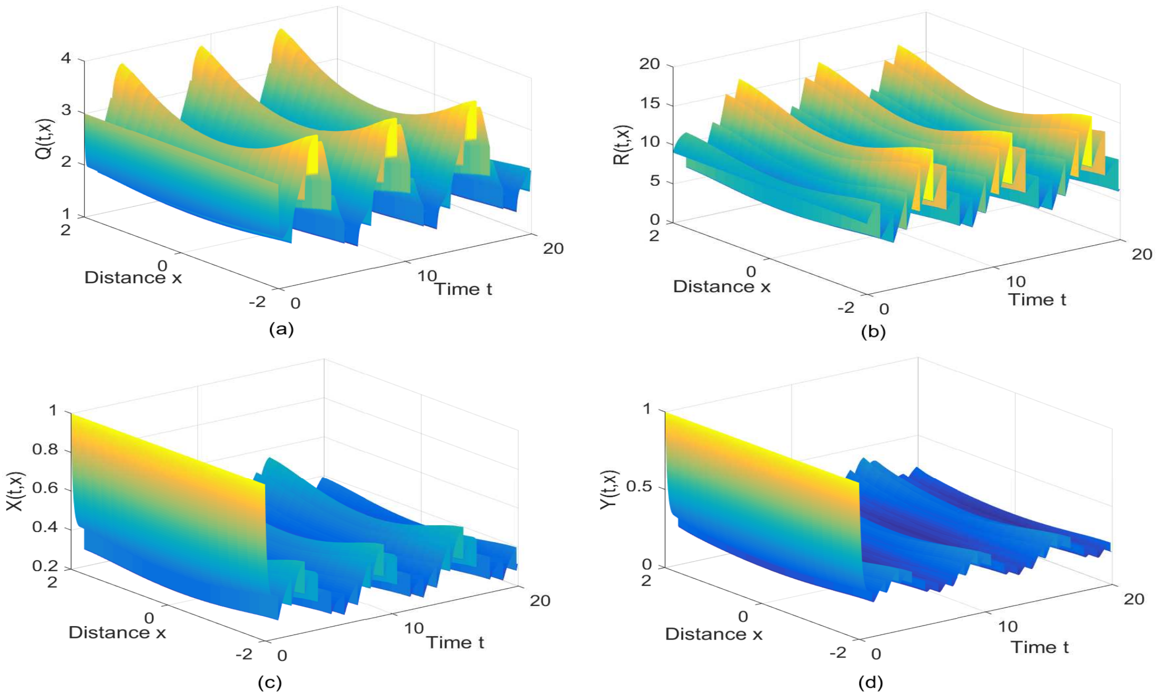

Figure 1. Permenence of populations $Q(t,x)$, $R(t,x)$, $X(t,x)$ and $Y(t,x)$.

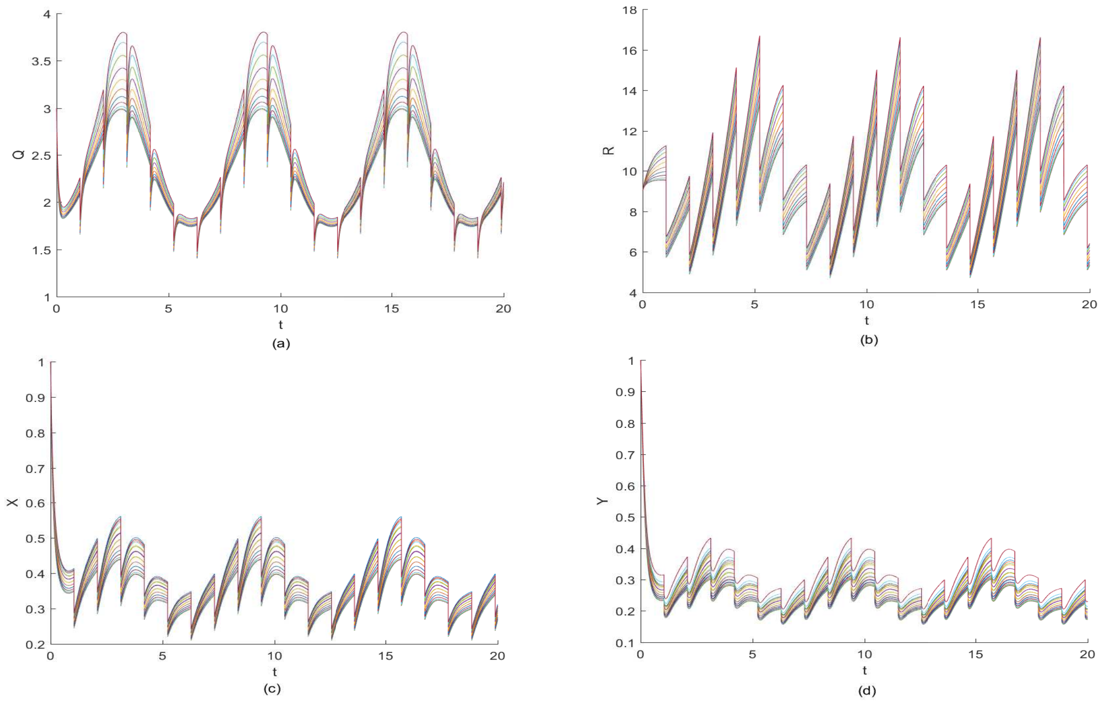

Figure 2. The projection of Figure 1 in $t - Q(t, x), t - R(t, x), t - X(t, x)$ and $t - Y(t, x)$.

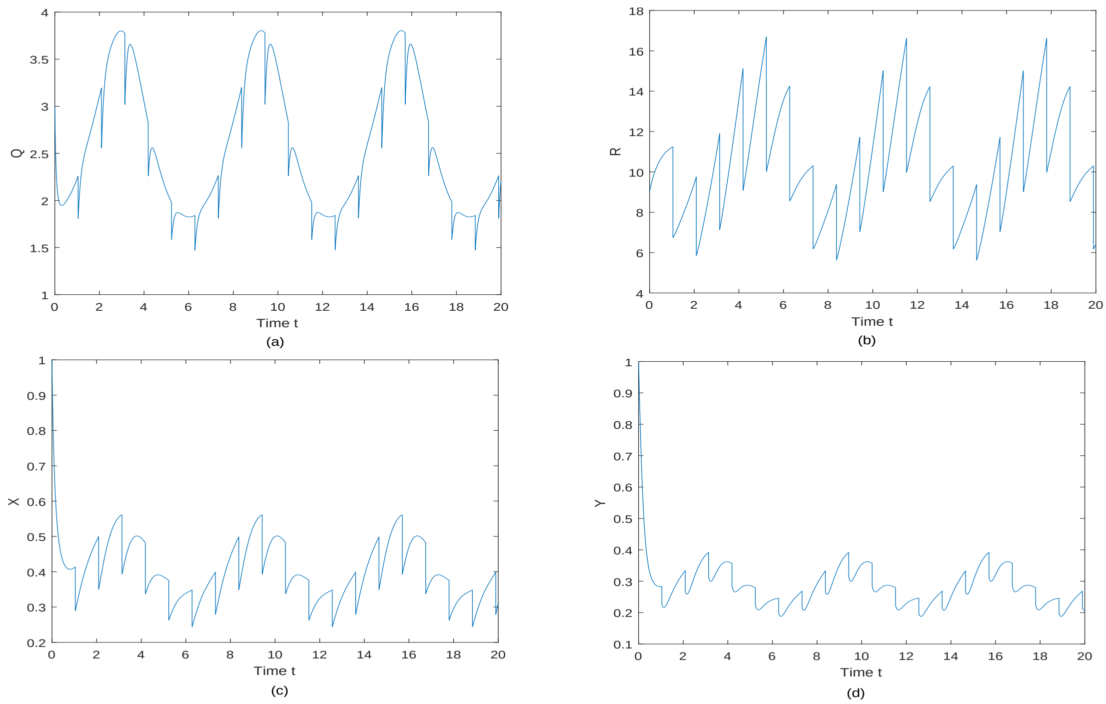

Figure 3. The section of Figure 1 with $x = 0$.

(a)–(d) in Figure 1 describe the persistence of four populations Q, R, X and Y respectively. Figure 2 is the projection of Figure 1 on the t-Q(R,X,Y) plane, and Figure 3 is the cross section of Figure 1 when $x = 0$. It is easy to verify that the parameters given satisfy the conditions of Theorems 3.1 and 4.1. As a result, Theorems 3.1 and 4.1 is thus verified.

7.2. Example 2

In this Section, we examine sufficient conditions for an epidemic extinction. Figures 4 and 5 are the numerical simulation results obtained by taking $d_Y = 0.3, a_1(t, x) = 0.4 \sin t + 0.3 \cos x + 2.8; a_3(t, x) = 1.6 \cos t + 1.6 \cos x + 7; q_1(t, x) = 0.3 \sin t + 0.3 \cos x + 3$; and $q_2(t, x) = 0.2 \sin t + 0.2 \cos x + 2$ on the basis of Example 1. At this point, these parameters satisfy the following inequality:

$$(-a_1^L + q_1^M)\tau + \ln \sum_{k=1}^{p} \sup_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x, Q, R, X, Y) - 16.260 < 0;$$

and

$$(-a_3^L + q_2^M + \frac{p_6^M}{\gamma_3^L})\tau + \ln \sum_{k=1}^{p} \sup_{x\in\Omega,(Q,R,X,Y)\in S} \omega_k(x, Q, R, X, Y) = -9.821 < 0;$$

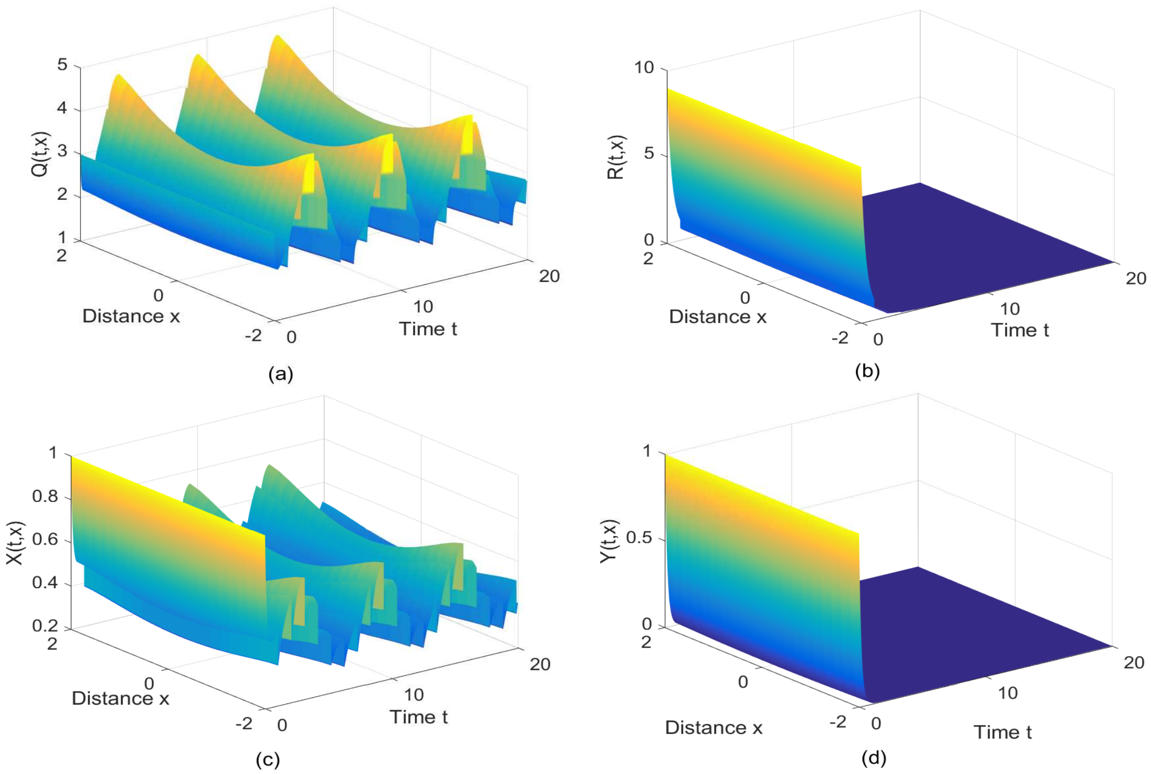

Figure 4. The persistence of population Q, X and the extinction of population R, Y.

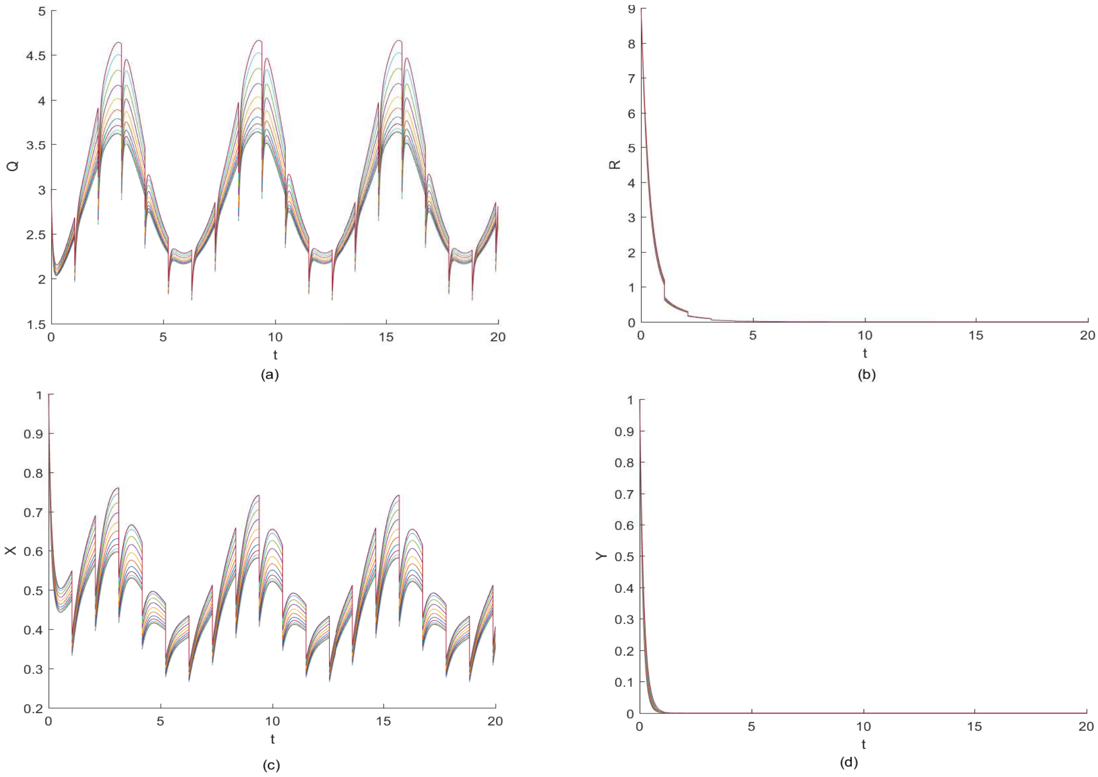

Figure 5. The projection of Figure 4 in $t - Q(t,x), t - R(t,x), t - X(t,x)$ and $t - Y(t,x)$.

Figures 4 and 5 show the extinction behavior of epidemics under certain conditions. As can be seen in Figures 4 and 5, epidemics can become extinct when the mortality rate of infected populations increase and the diffusion rate and effective contact rate decreases. This suggests that the extinction of epidemics can be accelerated by limiting the flow of population Y, which is consistent with our general understanding.

7.3. Example 3

In this Section, we are more concerned about how some other conditions affect the dynamic behavior of the population. Thus, this section mainly considers the effects of impulsive harvest, Crowley-Martin type response function and contact rate on population dynamics. For convenience, we consider only the dynamic behavior changes of Q and R based on Example 2.

To illustrate the impact of impulse function on epidemic dynamics, the following numerical simulation is performed. Figure 6 simulates the dynamic behavior without impulsive of population Q and R. Figure 7 is the result of numerical simulation when the impulse function is $g_k = 1.4$. At this time, these parameters meet the following inequality:

$$(-a_1^L + q_1^M)\tau + \ln \sum_{k=1}^{p} \sup_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x,Q,R,X,Y) = -3.636 < 0;$$

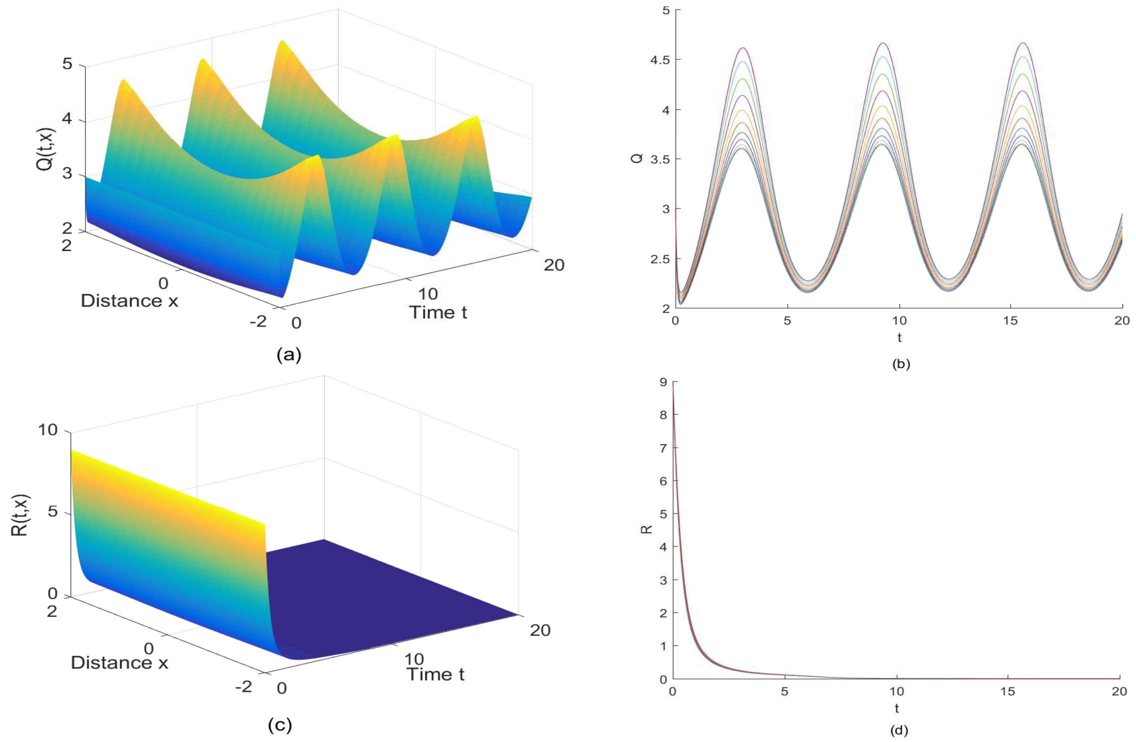

Figure 6. Extinction diagram of population Q and R without impulsive.

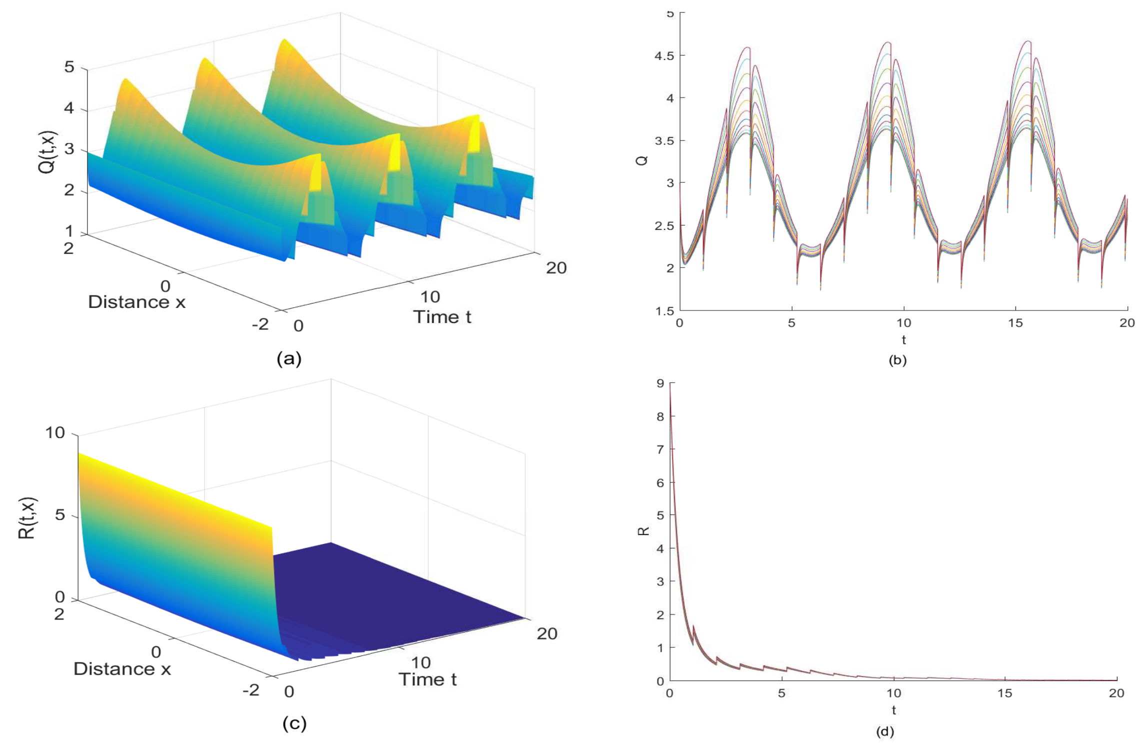

Figure 7. Extinction diagram of population Q and R with impulsive.

By comparing Figures 5(b) and 6(d), it can be concluded that the impulse effect can accelerate the extinction of epidemics. By comparing Figures 5(b), 6(d) and 7(d), it can be easily concluded that applying impulsive to the population can reduce the extinction of the population when impulse function $g_k > 1$, and applying an impulse to the population can accelerate the extinction of the population when the impulse function $g_k < 1$. This corresponds to the impulsive birth and impulsive death of species.

To illustrate the effect of the functional response function on epidemic dynamics, the following numerical simulation is considered. Figure 8 is the result of $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 0$ numerical simulation. At this time, the Crowley-Martin type response function is reflected into Holling I. Figure 9 shows the result of $\gamma_1 = \gamma_3 = 6, \gamma_2 = \gamma_4 = \gamma_5 = 0$. At this time, the Crowley-Martin type response function is reflected into Holling II.

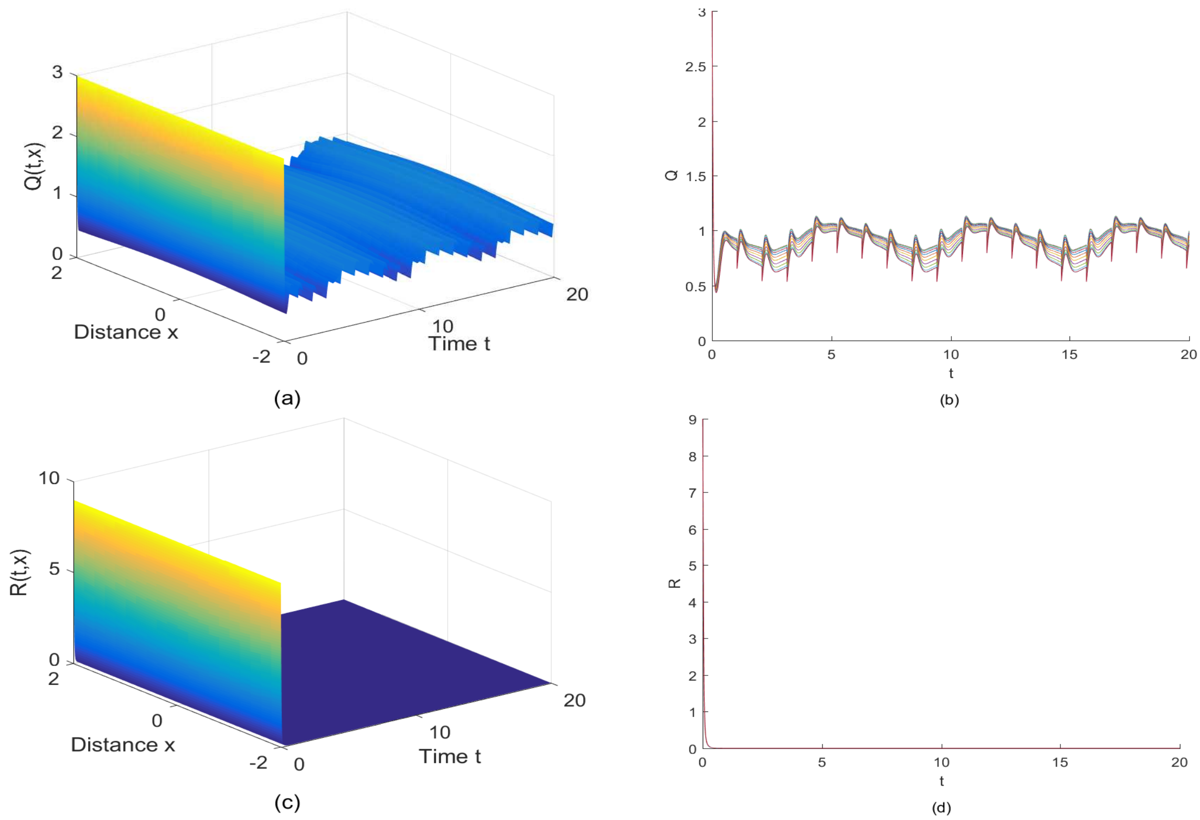

Figure 8. Epidemiological extinction diagram with Holling I type.

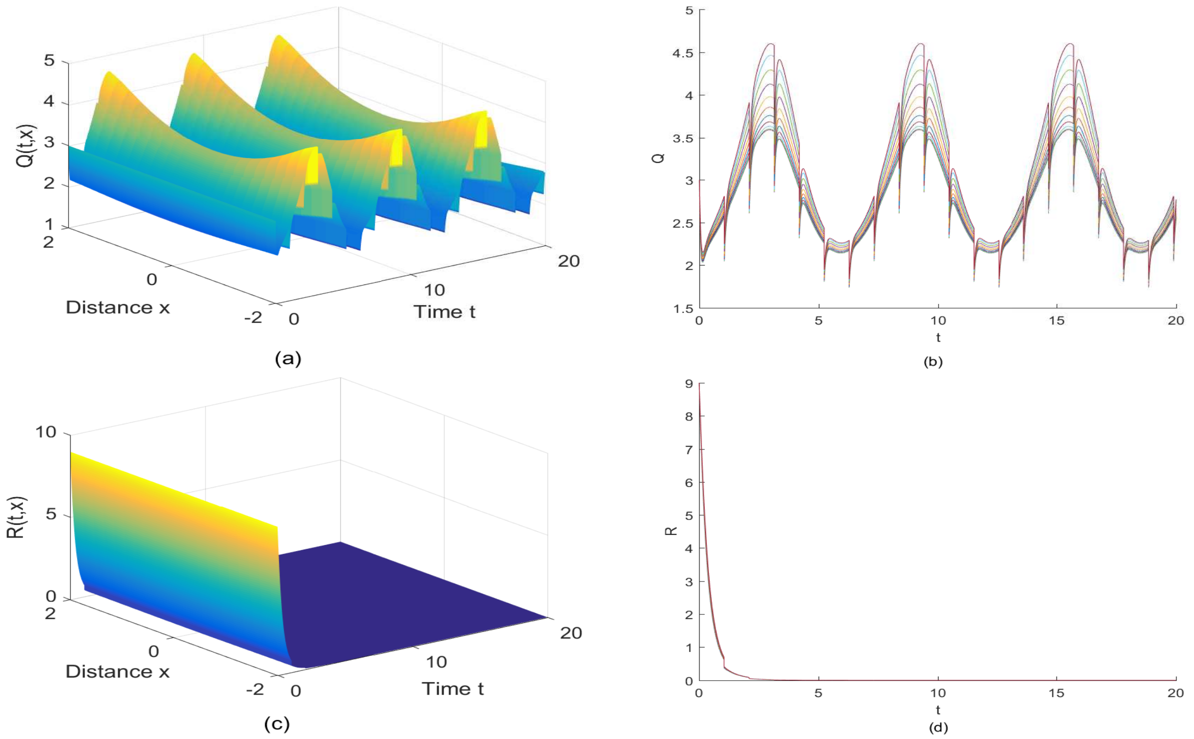

Figure 9. Epidemiological extinction diagram with Holling II type.

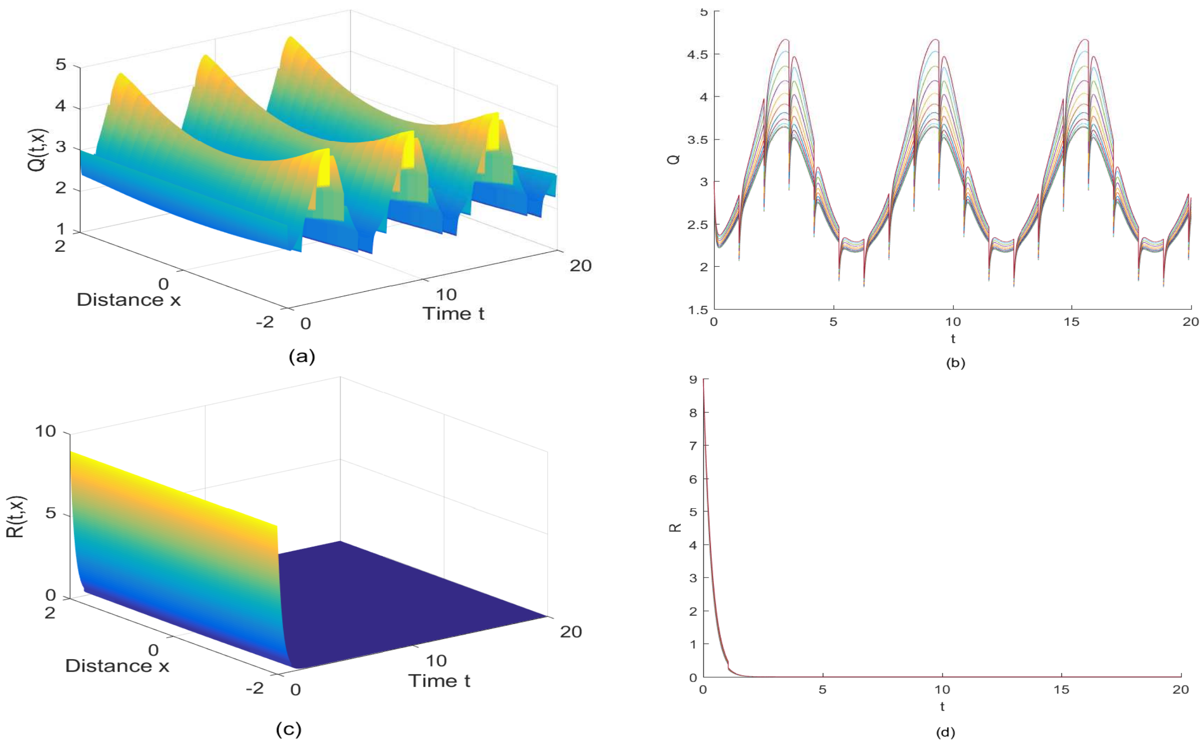

Figure 10. Effects of exposure rates on populations.

By comparing Figures 4(a), 8(a) and 5(a) and 8(b), it can be seen that Holling I type functional response reduced the population size of susceptible prey Q. By comparing Figures 4(b), 8(c) and 5(b) and 8(d), it can be concluded that Holling I type functional response accelerates the extinction of infected prey R. It can be seen from Figures 8 and 9 that Holling I type functional response reduced the population size of susceptible prey Q more than Holling II. The extinction rate of infected prey R accelerated by Holling I type functional response was higher than that of Holling II. This is because the predator's ability to hunt is higher than it actually is when processing time and predator interference are not taken into account. At this time, the number of susceptible prey Q decreased, while the extinction time of infected prey R was delayed.

To illustrate the effect of effective exposure rates on epidemic dynamics, the following numerical simulations were performed. Figure 10 shows the result of $q_1 = 0.1 \sin t + 0.1 \cos x + 0.5$. At this time, these parameters meet the following inequality:

$$(-a_1^L + q_1^M)\tau + \ln \sum_{k=1}^{p} \sup_{x\in\Omega,(Q,R,X,Y)\in S} g_k(x, Q, R, X, Y) = -11.861 < 0;$$

As can be seen from Figures 4(b), 10(c) and 5(b), 10(d), the epidemic will fade faster when the exposure rate of the population decreases. This illustrates the importance of isolation during an epidemic.

8. Concluding remarks

In recent years, the idea of epidemic modeling has always existed. When does the disease appear? When will the disease become extinct? This is an issue that people have always been concerned about. This paper presents an epidemic model (1.1)–(1.11). We studied the dynamics of prey populations by subdividing them into susceptible and infected populations. We consider a class of ecological epidemic models with impulses, we obtain sufficient conditions for the ultimate boundedness and persistence of the system by constructing upper and lower solutions, and we have proved the system periodic solutions. In order to prove the existence, uniqueness and global asymptotic stability of positive periodic solutions, compactness theory and methods based on the construction of appropriate auxiliary functions are applied. Furthermore, our theoretical results are verified by numerical simulation. The numerical simulation results also show that: (i) Under appropriate conditions, populations can coexist (periodic solutions exist). The existence of periodic solutions depends on the persistence of the system; (ii) Large (> 1) impulses prolong the extinction time of epidemics; Small (< 1) impulses accelerate the extinction of epidemics; (iii) The number of susceptible prey is highest and the extinction time of the epidemic is delayed when processing time and predator disturbance are taken into account; (iv) Reducing population movements and effective contact rates can better control the spread of epidemics.

These results indicated that dispersal rate, mortality rate, effective contact rate, maximum predation rate, impulse control and functional response function had significant effects on disease prevalence and population dynamics. Pulsed reaction-diffusion systems can also serve as a useful tool to study the dynamics of populations. In this paper, we study an ecological epidemic model with Crowley-Martin type functional response by model construction and analysis. We accounted for processing time and interference between predators. In other words, it takes some time in order to digest the prey and when the predator has captured the prey, the predator's capture behavior is interrupted during this time. We take them into account when we study the dynamics of the population. And studied the persistence of populations, from an ecological point of view, consistent persistence means that prey and predators can coexist at any time and in any location in an inhabited area. Thus, we can use some key arguments to control population persistence and extinction through some hybrid system of reaction-diffusion pulses, which is expected to be useful in the study of dynamic complexes of ecosystems. An important question is: what interesting things would happen if population Y could prey on both population R and population Q? This will be our main consideration in the later stage.

Acknowledgments

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Conflict of interest

The authors declare there is no conflict of interest.

References

1. A. M. Samoilenko, N. Perestyuk, Impulsive differential equations, *World Scientific*, 1995. https://doi.org/10.1142/2413
2. D. W. Rook, R. J. Fisher, Normative influences on impulsive buying behavior, *J. Consum. Res.*, **22** (1995), 305–313. https://doi.org/10.1086/209452
3. X. Liu, L. Chen, Global dynamics of the periodic logistic system with periodic impulsive perturbations, *J. Math. Anal. Appl.*, **289** (2004), 279–291. https://doi.org/10.1016/j.jmaa.2003.09.058
4. J. J. Nieto, D. ORegan, Variational approach to impulsive differential equations, *Nonlinear Anal.: Real World Appl.*, **10** (2009), 680–690. https://doi.org/10.1016/j.nonrwa.2007.10.022
5. S. Zavalishchin, A. N. Sesekin, Dynamic impulse systems: Theory and applications, *Math. Appl.*, **394** (1997). https://doi.org/10.1007/978-94-015-8893-5
6. Y. Xie, Z. Wang, A ratio-dependent impulsive control of an SIQS epidemic model with non-linear incidence, *Appl. Math. Comput.*, **423** (2022). https://doi.org/10.1016/j.amc.2022.127018
7. K. S. Jatav, J. Dhar, Hybrid approach for pest control with impulsive releasing of natural enemies and chemical pesticides: A plant-pest-natural enemy model, *Nonlinear Anal.: Hybrid Syst.*, **12** (2014), 79–92. https://doi.org/10.1016/j.nahs.2013.11.011
8. K. Chakraborty, K. Das, H. Yu, Modeling and analysis of a modified leslie-gower type three species food chain model with an impulsive control strategy, *Nonlinear Anal.: Hybrid Syst.*, **15** (2015), 171–184. https://doi.org/10.1016/j.nahs.2014.09.003
9. M. Zhao, X. Wang, H. Yu, J. Zhu, Dynamics of an ecological model with impulsive control strategy and distributed time delay, *Math. Comput. Simul.*, **82** (2012), 1432–1444. https://doi.org/10.1016/j.matcom.2011.08.009
10. M. Fazly, M. Lewis, H. Wang, On impulsive reaction-diffusion models in higher dimensions, *SIAM J. Appl. Math.*, **77** (2017). https://doi.org/10.1137/15M1046666

11. J. Liang, Q. Yan, C. Xiang, S. Tang, A reaction-diffusion population growth equation with multiple pulse perturbations, *Commun. Nonlinear Sci. Numer. Simul.*, **74** (2019), 122–137. https://doi.org/10.1016/j.cnsns.2019.02.015
12. Y. Meng, Z. Lin, M. Pedersen, Effects of impulsive harvesting and an evolving domain in a diffusive logistic model, *Nonlinearity*, **34** (2021). https://doi.org/10.1088/1361-6544/ac1f78
13. G. Liu, X. Meng, S. Liu, Dynamics for a tritrophic impulsive periodic plankton-fish system with diffusion in lakes, *Math. Methods Appl. Sci.*, **44** (2021), 3260–3279. https://doi.org/10.1002/mma.6938
14. Z. Liu, L. Zhang, P. Bi, J. Pang, B. Li, C. Fang, On the dynamics of one-prey-n-predator impulsive reaction-diffusion predator-prey system with ratio-dependent functional response, *J. Biol. Dyn.*, **12** (2018), 551–576. https://doi.org/10.1080/17513758.2018.1485974
15. Z. Liu, S. Zhong, C. Yin, W. Chen, On the dynamics of an impulsive reaction-diffusion predator-prey system with ratio-dependent functional response, *Acta Appl. Math.*, **115** (2011), 329–349. https://doi.org/10.1007/s10440-011-9624-8
16. C. Dai, H. Liu, Z. Jin, Q. Guo, Y. Wang, H. Yu, et al., Dynamic analysis of a heterogeneous diffusive prey-predator system in time-periodic environment, *Complexity*, **2020** (2020). https://doi.org/10.1155/2020/7134869
17. H. Liu, H. Yu, C. Dai, Q. Wang, J. Li, R. P. Agarwal, et al., Dynamic analysis of a reaction-diffusion impulsive hybrid system, *Nonlinear Anal.: Hybrid Syst.*, **33** (2019), 353–370. https://doi.org/10.1016/j.nahs.2019.03.001
18. X. Li, Q. Wang, R. Han, An impulsive predator-prey system with modified leslie-gower functional response and diffusion, *Qual. Theory Dyn. Syst.*, **20** (2021). https://doi.org/10.1007/s12346-021-00517-2
19. Z. Liu, S. Zhong, X. Liu, Permanence and periodic solutions for an impulsive reaction-diffusion food-chain system with ratio-dependent functional response, *Commun. Nonlinear Sci. Numer. Simul.*, **19** (2014), 173–188. https://doi.org/10.1016/j.cnsns.2013.05.030
20. P. H. Crowley, E. K. Martin, Functional responses and interference within and between year classes of a dragonfly population, *J. North Am. Benthol. Soc.*, **8** (1989). https://doi.org/10.2307/1467324
21. Q. Dong, W. Ma, M. Sun, The asymptotic behavior of a chemostat model with crowley-Martin type functional response and time delays, *J. Math. Chem.*, **51** (2013), 1231–1248. https://doi.org/10.1007/s10910-012-0138-z
22. X. Liu, S. Zhong, B. Tian, F. Zheng, Asymptotic properties of a stochastic predator-prey model with Crowley-Martin functional response, *J. Appl. Math. Comput.*, **43** (2013), 479–490. https://doi.org/10.1007/s12190-013-0674-0
23. X. Li, X. Lin, J. Liu, Existence and global attractivity of positive periodic solutions for a predator-prey model with Crowley-Martin functional response, *Electron. J. Differ. Equations*, **2018** (2018), 1–17. Available from: https://ejde.math.txstate.edu/Volumes/2018/191/li.pdf.
24. R. K. Upadhyay, R. K. Naji, Dynamics of a three species food chain model with Crowley-Martin type functional response, *Chaos, Solitons Fractals*, **42** (2009), 1337–1346. https://doi.org/10.1016/j.chaos.2009.03.020

25. X. Shi, X. Zhou, X. Song, Analysis of a stage-structured predator-prey model with Crowley-Martin function, *J. Appl. Math. Comput.*, **36** (2011), 459–472. https://doi.org/10.1007/s12190-010-0413-8
26. Y. Xie, Z. Wang, Transmission dynamics, global stability and control strategies of a modified SIS epidemic model on complex networks with an infective medium, *Math. Comput. Simul.*, **188** (2021), 23–34. https://doi.org/10.1016/j.matcom.2021.03.029
27. M. Cai, S. Yan, Z. Du, Positive periodic solutions of an eco-epidemic model with Crowley-Martin type functional response and disease in the prey, *Qual. Theory Dyn. Syst.*, **19** (2020). https://doi.org/10.1007/s12346-020-00392-3
28. Z. Chang, X. Xing, S. Liu, X. Meng, Spatiotemporal dynamics for an impulsive eco-epidemiological system driven by canine distemper virus, *Appl. Math. Comput.*, **402** (2021). https://doi.org/10.1016/j.amc.2021.126135
29. H. Smith, Dynamics of competition, *Math. Inspired Biol.*, **1714** (1999), 191–240. https://doi.org/10.1007/BFb0092378
30. W. Walter, Differential inequalities and maximum principles: Theory, new methods and applications, *Nonlinear Anal.: Theory Methods Appl.*, **30** (1997), 4695–4711. https://doi.org/10.1016/S0362-546X(96)00259-3
31. M. Akhmet, M. Beklioglu, T. Ergenc, V. Tkachenko, An impulsive ratio-dependent predator-prey system with diffusion, *Nonlinear Anal.: Real World Appl.*, **7** (2006), 1255–1267. https://doi.org/10.1016/j.nonrwa.2005.11.007
32. D. Henry, Geometric theory of semilinear parabolic equations, *Springer*, **840** (2006). https://doi.org/10.1007/bfb0089647
33. O. Struk, V. Tkachenko, On impulsive lotka-volterra systems with diffusion, *Ukr. Math. J.*, **54** (2002), 629–646. https://doi.org/10.1023/A:1021039528818
34. M. A. Krasnoselskij, P. P. Zabrejko, Geometrical methods of nonlinear analysis, *Springer*, **263** (1984).

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