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Research article

Multi-attribute decision-making method with triangular fuzzy numbers based on regret theory and the catastrophe progression method

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Abstract: The purpose of this paper was to develop a novel triangular fuzzy method for multi-attribute decision-making to eliminate the influence of indicator weights on scheme selection and account for the regret psychology of decision-makers. Therefore, considering the consequences of regret aversion and subjective weighting, we propose a multi-attribute decision-making method with triangular fuzzy number based on regret theory and catastrophe progression. First, to eliminate the influence of various dimensions on the decision-making results, the decision matrix is described by a triangular fuzzy number, and the regret value matrix and rejoicing value matrix are independently constructed by applying regret theory. Second, the importance ranking of attributes is improved to eliminate the influence of subjective weighting by employing the maximizing deviation method; and the comprehensive catastrophe progression attribute is calculated to rank the alternatives. Finally, an instance of investment project selection is provided to prove the availability and superiority. In conclusion, the proposed method not only considers decision-makers' bounded rationality for decision-making, but it also expands the application of catastrophe progression methods under the condition of a triangular fuzzy environment.

Keywords: regret theory; catastrophe progression; triangular fuzzy number; multi-attribute decision-making

1. Introduction

Multi-attribute decision-making (MADM) is a process in which a decision-maker (DM) comprehensively evaluates a limited set of possible alternatives based on multiple attributes. It has a wide range of applications in performance assessment [1], program planning [2], scheme selection [3] and other areas. In a realistic situation, the decision-making environment is complicated, as it is necessary to combine multiple formats of decision-making information and comprehensively consider various factors to achieve scientific and reasonable decision-making results. Due to the dynamic nature of the decision-making process and the limited knowledge of DMs, evaluation information has the characteristics of ambiguity and uncertainty, which is difficult to describe with precise numerical values [4–7]; such processes include group MADM [8] and risk MADM [9]. Thus, fuzzy sets constitute the solution in an uncertain case. Triangular fuzzy numbers (TFNs) have been widely applied to decision-making problems, making them one of the most common types of evaluation formats for uncertain information; they also limit the field of the possible evaluation. Biswas et al. proposed single-valued neutrosophic sets of TFNs and applied them to MADM [10]. Huang and Luo presented a similarity relationship algorithm for uncertain TFN-based MADM with unknown attribute weights [11]. Yin et al. considered the ambiguity and grayness in actual decision-making, and provided a novel method with interval gray TFNs [8]. Dong et al. proposed a new fuzzy best-worst (BW) method based on TFNs for MADM problems [12]. Wang et al. improved the traditional BW method based on TFNs and used the interval VIKOR method to obtain the final evaluation results [13]. Tan and Zhang applied a triangular fuzzy VIKOR method to rank the scheme in risky MADM [9]. Wang designed a preference degree-based algorithm to rank TFNs and proposed a new method to solve triangular fuzzy multi-attribute group decision-making (MAGDM) problems [14]. A TFN is a special kind of fuzzy set defined on the real number set, and it reflects the membership by the function, allowing for more accurate expression of DM information [13,14]. These methods are effective in decision-making under the conditions of a triangular fuzzy environment, as they are based on the presumption that the DM is completely rational. Besides, it is subjective when a DM will determine the weights in these methods. Therefore, considering the above two factors, it is necessary to understand how to solve MADM problems with TFNs.

The rationality of DM is usually between completely rational and irrational. The expected utility theory is used to describe the rationality of DMs [15,16]. Under the conditions of an uncertain environment, completely rational decision-making based on expected utility theory cannot explain the actual decision-making behavior. Therefore, behavioral decision theory is used to express the irrational behavior of DMs, because the cognitive limitations and subjective psychological factors of DMs are taken into consideration [17–19]; examples of such factors include reference dependence, loss aversion and diminishing sensitivity in decision-making processes [20–26]. Que et al. provided an improved TOPSIS method combined with cumulative prospect theory [27]. Behavioral decision-making methods based on prospect theory usually require DMs to give or determine a reference point. And, the corresponding equation involves many different parameters that would influence the result of the decision analysis. Then, many scholars began to study behavioral decision methods based on regret theory; with such methods, DMs do not need to give reference points and there are few parameters [28,29]. Liang and Wang proposed an interval neutrosophic MADM method based on regret theory with asymmetric evaluation information [30]. Qian et al. introduced regret theory to construct the gray regret-rejoicing function and gray perception utility function to solve the gray risky

MADM problem [31]. Liu et al. presented a stochastic decision-making method based on regret theory and group satisfaction [32]. In summary, the regret theory provides a better solution to the MADM problem in terms of accounting for the effect of irrational psychological behavior of a DM.

Traditional MADM methods generally take into account the influence of attribute weights [33,34]. Due to the complexity of the actual decision-making problem, uncertain weight information is determined via the weighting method, which introduces the prejudice of a DM and can even lead to the wrong decision result. The catastrophe progression method can be used to solve this problem; it was developed based on catastrophe theory and fuzzy mathematics theory [35,36]. Its biggest feature is its ability to reference the relative importance degree of each evaluation attribute, thereby eliminating the influences caused by subjective weighting of the attribute in the traditional evaluation model [37,38]. Guo et al. combined the catastrophe progression method with the balanced score to propose an improved decision model based on the catastrophe progression method for the selection of science and technology park projects [39]. Zhang et al. used the catastrophe progression method to design a risk assessment index system for cold chain logistics to account for the dynamic nature [35]. Lv et al. used the catastrophe progression method to decompose the evaluation dimensions at multiple levels, and they evaluated the national ecological security capabilities with the aid of a visual analysis method [40]. Zhang et al. developed a cold chain e-commerce logistics service quality evaluation model by using the catastrophe progression method [41]. In conclusion, the catastrophe progression method has been widely applied to MADM problems because of its objective decision-making results and simple calculations.

According to the preceding reviews, the motivation for conducting this study can be explained as follows. 1) As the decision environment becomes more complex, the evaluation value of each attribute and the subjective psychological behavior of the DM must now be considered in the decision-making process in order to make the results more realistic. If only the regret psychology of the DM is to be considered in the decision, then the influence of subjective weights on the attributes should be avoided in the calculation process. 2) In the real world, the definite value cannot be obtained under most conditions. If an indicator value cannot be accurately described by real numbers, then fuzzy sets need to be used.

To the best of our knowledge, the research on scheme selection that accounts for the DM's regret perception and eliminates the influence of subjective weights on the attributes under the conditions of a triangular fuzzy environment is still in uncharted territory. To solve the problem mentioned above, the proposed method has been extended to the triangular fuzzy environment. Besides, the attribute evaluation with regret-aversion psychology has been applied to the catastrophe progression method; thus, it not only considers the DM's psychological behaviors, but it also avoids the interference of subjective weighting factors. The main contributions of the work are discussed as follows. 1) Regret theory is introduced into the method proposed in this paper; and, the DM's regret psychology toward the solution is taken as an important reference for solution evaluation. 2) The catastrophe progression method has been applied to rank the solutions without determining the weights by comparing the relative importance of the indicators. 3) The applicability of the MADM method under the conditions of a triangular fuzzy environment has been enriched and extended.

The remainder of this paper is organized as follows. Section 2 reviews the basic knowledge. Section 3 introduces the MADM approach based on regret theory and the catastrophe progression method; the decision information is described as TFNs and the regret and rejoicing values are calculated. Section 4 discusses how the proposed model was applied to select suitable investment projects; the comparative analysis and sensitivity analysis are further discussed. Section 5 concludes the paper.

2. Preliminaries

This section introduces the concepts of TFNs, regret theory and the catastrophe progression method.

2.1. Triangular fuzzy number

Definition 1 [42]. A TFN can be defined by a triplet $\tilde{T}=\left(x^L,x^M,x^U\right)$, and the membership function $\mu_{\tilde{x}}\left(x\right)$ is defined as:

$$\mu_{\tilde{T}}\left(x\right)=\begin{cases} 0 & for\ x<x^L \\ \dfrac{x-x^L}{x^M-x^L} & for\ x^L\le x\le x^M \\ \dfrac{x-x^M}{x^U-x^M} & for\ x^M\le x\le x^U \\ 0 & for\ x^U<x \end{cases} \tag{1}$$

where x^L and x^U stand for the minimum and maximum values of x, respectively, and x^M represents the information preference value.

Definition 2 [42]. The defuzzified value of the TFN is

$$Crisp\left(\tilde{T}\right)=\frac{x^L+kx^M+x^U}{k+2},\ k\in N^+ \tag{2}$$

2.2. Regret theory

Regret theory describes that the DM would compare the consequences of the selected alternative with the possible consequences to those of other alternatives in the decision-making process [29]. If DMs find that choosing other alternatives can yield better results, DMs would feel regret. Otherwise, DMs would feel joy, and this psychological perception affects the final decision-making result of the DM. The regret theory indicates that the function of perceived utility is composed of a regret utility function and rejoicing utility function. Let $v\left(x\right)$ and $v\left(y\right)$ denote the utility obtained by the DM from Alternative a and Alternative b; then, the perceived utility of the DM for Alternative a is

$$u\left(x,y\right)=v\left(x\right)+R\left(v\left(x\right)-v\left(y\right)\right) \tag{3}$$

The function $R\left(v\left(x\right)-v\left(y\right)\right)$ is a concave monotonically increasing function that represents regret-rejoicing utility. If $R\left(v\left(x\right)-v\left(y\right)\right)<0$, there is regret utility for Alternative a when compared to Alternative b. If $R\left(v(x)-v(y)\right)>0$, there is rejoicing utility for Alternative a when compared to Alternative b [43].

The original regret theory only applies to the comparison between two alternatives; Quiggin extended it to the selection problem with multiple alternatives [29]. There are m possible alternatives $a_1, a_2, \cdots, a_m$, where a_i stands for the i th alternative, $i=\{1,2,\cdots m\}$. $x_1, x_2, \cdots, x_m$ respectively represents the outcome of choosing $a_1, a_2, \cdots, a_m$, where x_i denotes the result of Alternative a_i; then, the perceived utility of the DM for Alternative a_i is

$$u(x_i)=v(x_i)+R\left(v(x_i)-v(x^+)\right) \tag{4}$$

where $v(x_i)$ is the utility that the DM would acquire from Alternative a_i immediately and $x^+=\max\{x_i\}$. When $R\left(v(x_i)-v(x^+)\right)<0$, $R\left(v(x_i)-v(x^+)\right)$ denotes the regret value relative to the positive ideal result. When $R\left(v(x_i)-v(x^+)\right)>0$, $R\left(v(x_i)-v(x^+)\right)$ denotes the rejoicing value relative to the positive ideal result [44].

Table 1. Potential functions and equations for normalized catastrophe progression [35].

Catastrophe type	Potential function	Bifurcation set equation	Control dimension	Normalization equation
Fold catastrophe	$f(x)=x^3+ax$	$a=-3x^2$	1	$x_a=\sqrt{a}$
Cusp catastrophe	$f(x)=x^4+ax^2+bx$	$a=-6x^2, b=8x^3$	2	$x_a=\sqrt{a}$, $x_b=\sqrt[3]{b}$
Swallowtail catastrophe	$f(x)=x^5+ax^3+bx^2+cx$	$a=-6x^2, b=8x^3, c=-3x^4$	3	$x_a=\sqrt{a}$, $x_b=\sqrt[3]{b}$, $x_c=\sqrt[4]{c}$
Butterfly catastrophe	$f(x)=x^6+ax^4+bx^3+cx^2+dx$	$a=-10x^2, b=20x^3, c=-15x^4, d=4x^5$	4	$x_a=\sqrt{a}$, $x_b=\sqrt[3]{b}$, $x_c=\sqrt[4]{c}$, $x_d=\sqrt[5]{d}$
Shack catastrophe	$f(x)=x^7+ax^6+bx^4+cx^3+dx^2+ex$	$a=-x^2, b=2x^3, c=-2x^4, d=4x^5, e=-5x^6$	5	$x_a=\sqrt{a}$, $x_b=\sqrt[3]{b}$, $x_c=\sqrt[4]{c}$, $x_d=\sqrt[5]{d}$, $x_e=\sqrt[6]{e}$

Table 2. Normalization equations for the catastrophe progression.

Catastrophe type	Status dimension	Control dimension	Normalization equation	
			Regret normalization	Rejoicing normalization
Fold catastrophe	1	1	$Rx_1 = \sqrt{\lvert R_1 \rvert}$	$Gx_1 = \sqrt{G_1}$
Cusp catastrophe	1	2	$Rx_1 = \sqrt{\lvert R_1 \rvert}, Rx_2 = \sqrt[3]{\lvert R_2 \rvert}$	$Gx_1 = \sqrt{G_1}, Gx_2 = \sqrt[3]{G_2}$
Swallowtail catastrophe	1	3	$Rx_1 = \sqrt{\lvert R_1 \rvert}, Rx_2 = \sqrt[3]{\lvert R_2 \rvert}, Rx_3 = \sqrt[4]{\lvert R_3 \rvert}$	$Gx_1 = \sqrt{G_1}, Gx_2 = \sqrt[3]{G_2}, Gx_3 = \sqrt[4]{G_3}$
Butterfly catastrophe	1	4	$Rx_1 = \sqrt{\lvert R_1 \rvert}, Rx_2 = \sqrt[3]{\lvert R_2 \rvert}, \; Rx_3 = \sqrt[4]{\lvert R_3 \rvert}, Rx_4 = \sqrt[5]{\lvert R_4 \rvert}$	$Gx_1 = \sqrt{G_1}, Gx_2 = \sqrt[3]{G_2}, \; Gx_3 = \sqrt[4]{G_3}, Gx_4 = \sqrt[5]{G_4}$
Shack catastrophe	1	5	$Rx_1 = \sqrt{\lvert R_1 \rvert}, Rx_2 = \sqrt[3]{\lvert R_2 \rvert}, Rx_3 = \sqrt[4]{\lvert R_3 \rvert}, Rx_4 = \sqrt[5]{\lvert R_4 \rvert}, Rx_5 = \sqrt[6]{\lvert R_5 \rvert}$	$Gx_1 = \sqrt{G_1}, Gx_2 = \sqrt[3]{G_2}, \; Gx_3 = \sqrt[4]{G_3}, Gx_4 = \sqrt[5]{G_4}, \; Gx_5 = \sqrt[6]{G_5},$

2.3. Catastrophe progression method

The catastrophe progression method is based on the elementary catastrophe model and incorporates the idea of a fuzzy membership function [45]. The type of catastrophe is mainly determined by the numbers of control and state variables. The common types of elementary catastrophe systems include folding catastrophe, cusp catastrophe, swallowtail catastrophe and butterfly catastrophe systems. The control variables range from 1 to 4. The control variable of shack catastrophe is 5, and this is not an elementary catastrophe; nevertheless, it is often used in decision-making methods. Its potential function is shown in Table 1. Let the first derivative be $f'(x)=0$ and the second derivative be $f''(x)=0$ of the potential function. Then, $f'(x)=0$ and $f''(x)=0$ are combined to eliminate the state variables, and the bifurcation point set equations (bifurcation set) of the system can be obtained. The final bifurcation equation derivation results are shown in Table 1. By further deriving the bifurcation point set equations, five equations of normalized catastrophe models can be obtained, as shown in Table 2. Finally, the catastrophe theory is combined with the fuzzy membership function and the different qualitative states of the various attributes are transformed into a comparable qualitative state. By recursively calculating the various attributes in the system, the catastrophe value of each control variable can be obtained and the total mutation membership function value that represents the state characteristics of system can be calculated [35].

3. MADM approach based on regret theory and Catastrophe progression method

This section presents the MADM approach based on regret theory and Catastrophe progression method. The specific steps are shown in Figure 1.

3.1. Description of problem

Considering MADM problems with TFNs, let a_i indicate the i th alternative, where $i=1,2,\cdots,m$; also, c_j indicates the j th attribute, $j=1,2,\cdots,n$, where w_j is the weight of Attribute a_i , $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$. N_b and N_c represent the subscript set of the benefit attribute and cost attribute, respectively, where $N_b \cup N_c = N$ and $N_b \cap N_c = \varnothing$. The decision matrix can be represented as $\mathbf{D}=\left[\tilde{x}_{ij}\right]_{m\times n}$, where $\tilde{x}_{ij}=\left(x_{ij}^{L}, x_{ij}^{M}, x_{ij}^{U}\right)$ is the evaluation value for Alternative a_i in Attribute c_j , and $\tilde{x}_{ij}$ is a TFN, as shown in Table 3.

Table 3. Fuzzy initial decision matrix.

	c_1	c_2	$\cdots$	c_n
a_1	$\tilde{x}_{11}$	$\tilde{x}_{12}$	$\cdots$	$\tilde{x}_{1n}$
a_2	$\tilde{x}_{21}$	$\tilde{x}_{22}$	$\cdots$	$\tilde{x}_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_m	$\tilde{x}_{m1}$	$\tilde{x}_{m2}$	$\cdots$	$\tilde{x}_{mn}$

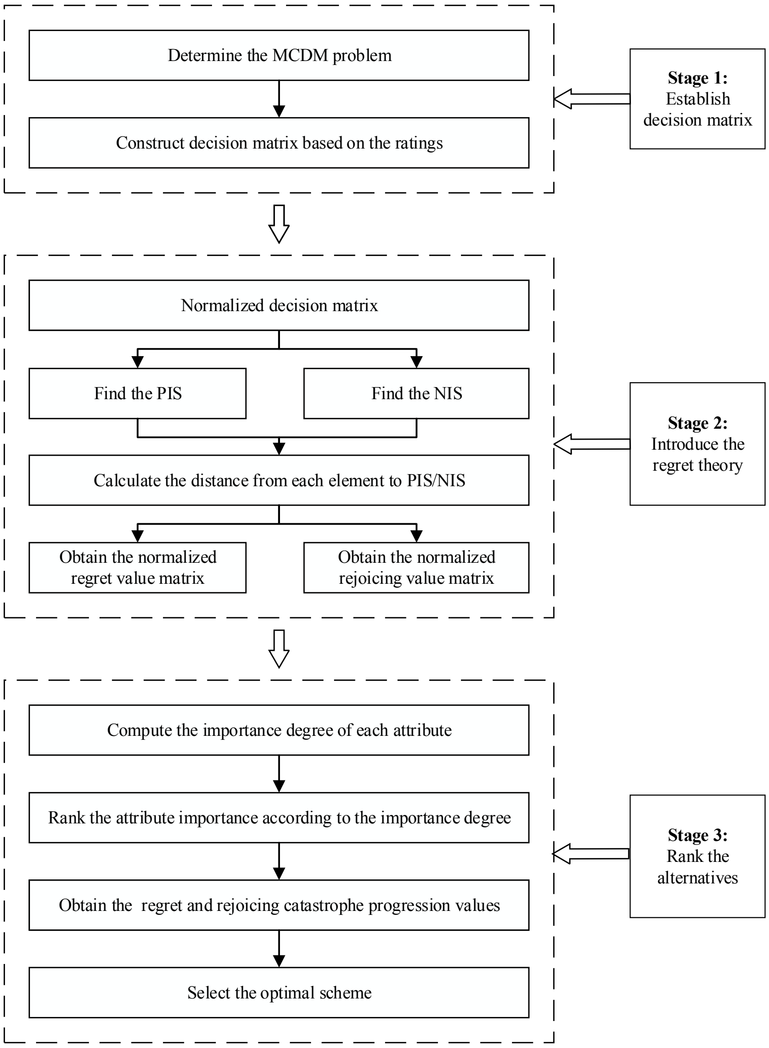

Figure 1. Decision-making framework.

3.2. Decision-making process

In summary, the steps of a MADM method with TFNs based on regret theory and catastrophe progression are presented below.

Step 1: Obtain the normalized decision matrix $\tilde{\mathbf{D}}' = \left[\tilde{x}_{ij}'\right]_{m\times n}$.

Benefit attribute normalization:
$$\tilde{x}_{ij}' = \left(\begin{array}{l} x_{ij}^{L} \Big/ \sqrt{\sum_{i=1}^{m}\left(\left(x_{ij}^{L}\right)^2 + \left(x_{ij}^{M}\right)^2 + \left(x_{ij}^{U}\right)^2\right)}, \\ x_{ij}^{M} \Big/ \sqrt{\sum_{i=1}^{m}\left(\left(x_{ij}^{L}\right)^2 + \left(x_{ij}^{M}\right)^2 + \left(x_{ij}^{U}\right)^2\right)}, \\ x_{ij}^{U} \Big/ \sqrt{\sum_{i=1}^{m}\left(\left(x_{ij}^{L}\right)^2 + \left(x_{ij}^{M}\right)^2 + \left(x_{ij}^{U}\right)^2\right)} \end{array} \right) \tag{5}$$

Cost attribute normalization: $$\tilde{x}_{ij}{}'=\begin{pmatrix}\left(1/x_{ij}{}^{L}\right)\Big/\sqrt{\sum_{i=1}^{m}\left(\left(1/x_{ij}{}^{L}\right)^{2}+\left(1/x_{ij}{}^{M}\right)^{2}+\left(1/x_{ij}{}^{U}\right)^{2}\right)},\\ \left(1/x_{ij}{}^{M}\right)\Big/\sqrt{\sum_{i=1}^{m}\left(\left(1/x_{ij}{}^{L}\right)^{2}+\left(1/x_{ij}{}^{M}\right)^{2}+\left(1/x_{ij}{}^{U}\right)^{2}\right)},\\ \left(1/x_{ij}{}^{U}\right)\Big/\sqrt{\sum_{i=1}^{m}\left(\left(1/x_{ij}{}^{L}\right)^{2}+\left(1/x_{ij}{}^{M}\right)^{2}+\left(1/x_{ij}{}^{U}\right)^{2}\right)}\end{pmatrix} \tag{6}$$

Step 2: Get the normalized regret value matrix $\tilde{\mathbf{R}}=\left[\tilde{r}_{ij}\right]_{m\times n}$ and the normalized rejoicing value matrix $\tilde{\mathbf{G}}=\left[\tilde{g}_{ij}\right]_{m\times n}$.

$$\tilde{r}_{ij}=r_{ij}\big/q_{j}^{\max} \tag{7}$$

$$\tilde{g}_{ij}=g_{ij}\big/q_{j}^{\max} \tag{8}$$

where $r_{ij}=1-\exp\left(\gamma\Delta v_{ij}^{+}\right)$, $g_{ij}=1-\exp\left(-\gamma\Delta v_{ij}^{-}\right)$, $q_{j}^{\max}=\max\left\{\max_{i\in m}\left\{\left|r_{ij}\right|\right\},\max_{i\in m}\left\{g_{ij}\right\}\right\}$, $\tilde{r}_{ij}\in[-1,0]$ and $\tilde{g}_{ij}\in[0,1]$. $\Delta v_{ij}^{+}=\sqrt{\frac{1}{3}\left[\left(\tilde{x}_{ij}{}'^{L}-\tilde{x}_{j}{}'^{+L}\right)^{2}+\left(\tilde{x}_{ij}{}'^{M}-\tilde{x}_{j}{}'^{+M}\right)^{2}+\left(\tilde{x}_{ij}{}'^{U}-\tilde{x}_{j}{}'^{+U}\right)^{2}\right]}$ and $\Delta v_{ij}^{-}=\sqrt{\frac{1}{3}\left[\left(\tilde{x}_{ij}{}'^{L}-\tilde{x}_{j}{}'^{-L}\right)^{2}+\left(\tilde{x}_{ij}{}'^{M}-\tilde{x}_{j}{}'^{-M}\right)^{2}+\left(\tilde{x}_{ij}{}'^{U}-\tilde{x}_{j}{}'^{-U}\right)^{2}\right]}$ denote the distances from the positive ideal point $\tilde{x}_{j}{}'^{+}=\left[\max_{1\le i\le m,1\le j\le n}\tilde{x}_{ij}{}'^{L},\max_{1\le i\le m,1\le j\le n}\tilde{x}_{ij}{}'^{M},\max_{1\le i\le m,1\le j\le n}\tilde{x}_{ij}{}'^{U}\right]$ and the negative ideal point $\tilde{x}_{j}{}'^{-}=\left[\min_{1\le i\le m,1\le j\le n}\tilde{x}_{ij}{}'^{L},\min_{1\le i\le m,1\le j\le n}\tilde{x}_{ij}{}'^{M},\min_{1\le i\le m,1\le j\le n}\tilde{x}_{ij}{}'^{U}\right]$, respectively. The parameter γ denotes the DM's regret-aversion coefficient, and $\gamma>0$.

Step 3: Calculate the importance degree w_{j}^{*} of each attribute to rank the attributes in the catastrophe progression method.

To eliminate the evaluation bias caused by the traditional subjective weighting method, we adopted the maximizing deviation method to improve the catastrophe progression; this ensures that the problem of ranking the importance degree of attributes can be objectively solved. The maximizing deviation method determines the importance degree of the attribute by calculating the ratio of the total deviation value of the k th attribute to the total deviation of all attributes. Through the above analysis, a deviation optimization model can be constructed:

$$\begin{cases}Max\ H\left(w\right)=\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{j=1}^{n}\left|z_{ij}-z_{kj}\right|w_{j}\\ s.t.\qquad \sum_{j=1}^{n}w_{j}^{2}=1,0\le w_{j}\le 1\end{cases} \tag{9}$$

where $H\left(w\right)$ is the sum of the deviation values and $z_{ij}=\dfrac{\tilde{x}_{ij}{}'^{L}+2\tilde{x}_{ij}{}'^{M}+\tilde{x}_{ij}{}'^{U}}{4}$.

The optimal parameter w_{j}^{*} can be obtain by solving the following model:

$$w_j^* = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\left|z_{ij} - z_{kj}\right|}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\left|z_{ij} - z_{kj}\right|\right)^2}} \tag{10}$$

Step 4: Determine the final comprehensive catastrophe progression values $\Phi(a_i)$.

According to the PROMETHEE method idea [46], the final comprehensive catastrophe progression value is calculated.

Complementary comprehensive catastrophe progression value is:

$$\Phi(a_i) = \frac{sum(Gx_{ij})}{n} - \frac{sum(Rx_{ij})}{n}, i \in m, j \in n \tag{11}$$

where the importance degree of each attribute is ranked according to w_j^*; the regret catastrophe progression value Rx_{ij} and rejoicing catastrophe progression value Gx_{ij} of each attribute can be obtained by solving the equations in Table 1.

Step 5: Rank the order of alternatives by $\Phi(a_i)$ to determine the DM's overall psychological perception of the Alternative a_i. A larger $\Phi(a_i)$ means a better Alternative a_i.

4. Illustrative example

To illustrate the proposed method, we considered a venture capital enterprise that is committed to seeking suitable investment projects. There are four alternatives, and the attributes are environmental impact c_1, expected returns c_2, growth c_3 and social benefits c_4. The first attribute is cost type; also, the others are all benefit types. The decision matrix is presented in Table 4.

Table 4. Fuzzy initial decision matrix.

	c_1	c_2	c_3	c_4
a_1	(4.3,4.6,4.7)	(88,90,92)	(0.6,0.7,0.8)	(4,5,7)
a_2	(2.8,2.9,3.1)	(91,92,94)	(0.4,0.5,0.6)	(3,4,5)
a_3	(4.0,4.1,4.2)	(90,92,94)	(0.2,0.3,0.4)	(5,6,7)
a_4	(3.5,3.7,3.8)	(80,82,83)	(0.4,0.5,0.7)	(3,4,6)

Table 5. Normalized decision matrix.

	c_1	c_2	c_3	c_4
a_1	(0.245,0.229,0.224)	(0.261,0.294,0.300)	(0.323,0.377,0.431)	(0.227,0.284,0.397)
a_2	(0.377,0.364,0.340)	(0.297,0.300,0.307)	(0.215,0.269,0.323)	(0.170,0.227,0.284)
a_3	(0.264,0.257,0.251)	(0.294,0.300,0.307)	(0.108,0.162,0.215)	(0.284,0.340,0.397)
a_4	(0.301,0.285,0.277)	(0.261,0.268,0.271)	(0.215,0.269,0.377)	(0.170,0.227,0.340)

Step 1: According to Eqs (5) and (6), the normalized decision matrix $\tilde{\mathbf{D}}'$ was constructed; it is presented in Table 5.

Step 2: On the basis of the regret theory, the distance from the positive ideal point and the distance from the negative ideal point were respectively determined; the results are shown in Table 6 and Table **7**.

Table 6. Distance from positive ideal point.

	c_1	c_2	c_3	c_4
a_1	0.127	0.021	0.000	0.046
a_2	0.000	0.000	0.108	0.113
a_3	0.103	0.002	0.215	0.000
a_4	0.073	0.035	0.093	0.098

Table 7. Distance from negative ideal point.

	c_1	c_2	c_3	c_4
a_1	0.000	0.023	0.215	0.080
a_2	0.127	0.035	0.108	0.000
a_3	0.025	0.034	0.000	0.113
a_4	0.055	0.000	0.128	0.033

Here, let the regret-aversion coefficient be $\gamma = 0.3$; then, the regret value matrix $\mathbf{R}$ and the rejoicing value matrix $\mathbf{G}$ can be established.

$$\mathbf{R} = \begin{bmatrix} -0.039 & -0.006 & 0.000 & -0.014 \\ 0.000 & 0.000 & -0.033 & -0.035 \\ -0.032 & -0.001 & -0.067 & 0.000 \\ -0.220 & -0.011 & -0.028 & -0.030 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 0.000 & 0.007 & 0.063 & 0.024 \\ 0.038 & 0.010 & 0.032 & 0.000 \\ 0.007 & 0.010 & 0.000 & 0.034 \\ 0.016 & 0.000 & 0.038 & 0.010 \end{bmatrix}$$

Using Eqs (7) and (8), the normalized regret value matrix $\mathbf{R}'$ and the normalized rejoicing value matrix $\mathbf{G}'$ can be obtained.

$$\mathbf{R}' = \begin{bmatrix} -1.000 & -0.613 & 0.000 & -0.404 \\ 0.000 & 0.000 & -0.492 & -1.000 \\ -0.807 & -0.054 & -1.000 & 0.000 \\ -0.564 & -1.000 & -0.425 & -0.864 \end{bmatrix}, \quad \mathbf{G}' = \begin{bmatrix} 0.000 & 0.652 & 1.000 & 0.710 \\ 1.000 & 1.000 & 0.508 & 0.000 \\ 0.197 & 0.969 & 0.000 & 1.000 \\ 0.436 & 0.000 & 0.603 & 0.292 \end{bmatrix}$$

Step 3: According to Eq (10), the variable weight of the environmental impact is $w_1 = 0.262$, the variable weight of the expected income is $w_2 = 0.073$, the variable weight of growth is $w_3 = 0.415$ and the variable weight of social benefits is $w_4 = 0.250$. And, it can be seen that the attribute importance is $c_3 \succ c_1 \succ c_4 \succ c_2$.

Step 4: Using the results presented in Table 1, the regret catastrophe progression values $R(a_i)$ and the rejoicing catastrophe progression values $G(a_i)$ of each alternative attribute were obtained.

$$R(a_1) = 2.704,\ R(a_2) = 1.702,\ R(a_3) = 2.489,\ R(a_4) = 3.443;$$

$$G(a_1) = 2.836,\ G(a_2) = 2.712,\ G(a_3) = 2.575,\ G(a_4) = 2.270.$$

According to Eq (11), the ranking values of each alternative are as follows:

$$\Phi(a_1)=\frac{sum(Gx_{1j})}{4}-\frac{sum(Rx_{1j})}{4}=0.033,\quad \Phi(a_2)=\frac{sum(Gx_{2j})}{4}-\frac{sum(Rx_{2j})}{4}=0.252;$$

$$\Phi(a_3)=\frac{sum(Gx_{3j})}{4}-\frac{sum(Rx_{3j})}{4}=0.022,\quad \Phi(a_4)=\frac{sum(Gx_{4j})}{4}-\frac{sum(Rx_{4j})}{4}=-0.293.$$

Step 5: Based on the ranking values of each alternative, the ordering result is $a_2 \succ a_1 \succ a_3 \succ a_4$. Therefore, alternative a_2 is the best suitable investment project.

In order to further illustrate the influence of regret-aversion behavior on decision-making, we conducted a sensitivity analysis by adjusting the regret avoidance coefficient. The results are shown in **Figure 2**. It can be seen in Figure 2 that the first two optimal alternatives remained unchanged; but, as the regret-aversion coefficient increased to about 3.5, the priorities of Alternative a_3 and Alternative a_4 change. This shows that regret-aversion behavior affects the decision results, and that the degree of regret aversion also affects the decision results.

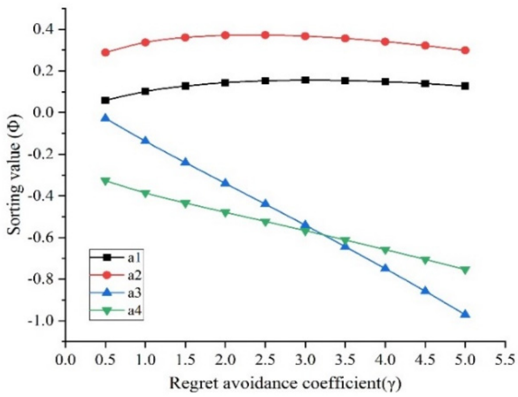

Figure 2. Sensitivity analysis of regret aversion coefficient.

Table 8. Distance from negative ideal point.

Method	Comprehensive prospect value	Ranking order
Traditional catastrophe progression method	$S(a_1)=0.550$, $S(a_2)=0.542$, $S(a_3)=0.518$, $S(a_4)=0.532$	$a_1 \succ a_2 \succ a_4 \succ a_3$
Simple utility theory	$Y(a_1)=0.313$, $Y(a_2)=0.285$, $Y(a_3)=0.241$, $Y(a_4)=0.272$	$a_1 \succ a_2 \succ a_4 \succ a_3$
TOPSIS	$T(a_1)=0.620$, $T(a_2)=0.550$, $T(a_3)=0.350$, $T(a_4)=0.420$	$a_1 \succ a_2 \succ a_4 \succ a_3$

To perform further comparison and analysis of the proposed methods, the example was calculated by using three other methods: the traditional catastrophe progression method without considering the regret-rejoicing action of the DM [47], the TOPSIS method [48] and simple utility theory [49]. This was done so that we could compare the results of the different methods. The ranking order of all of the alternatives, as according to each of the three methods, is showed in Table 8.

As can be ascertained from Table 8, there exist differences in the priority order of the schemes.

The traditional catastrophe progression method and simple utility theory do not take into account the influence of the DM's psychological behavior. However, the regret theory was introduced to the proposed method; thus, the DM's regret psychology regarding the solution is taken as an important reference for solution evaluation. The DM compares each alternative with the optimal one and would regret choosing the worse one. This feeling was quantified into a numerical value. It can be seen from the overall regret value and rejoicing value of each alternative that Alternative a_1 will result in more losses and regrets than Alternative a_2, i.e., $R(a_1) = 2.704 > R(a_2) = 1.702$. But, the rejoicing values of the two alternatives were not very different. Therefore, the comprehensive calculations show that Alternative a_2 is better than Alternative a_1. Therefore, based on regret theory, the DM would prefer to choose the scheme that would not result in regret ($a_2 \succ a_1 \succ a_3 \succ a_4$).

The TOPSIS method is based on the absolute rationality of the DM; it uses distance as the main reference for decision-making and selects the solution in an objective way. Besides, the TOPSIS method considers attribute weights; it is subjective and random when determining weights. For the above two reasons, the solution ranking order obtained via the TOPSIS method ($a_1 \succ a_2 \succ a_4 \succ a_3$) was the same as that obtained via the traditional catastrophe progression method and simple utility theory.

In conclusion, the traditional catastrophe progression method, the TOPSIS method and simple utility theory are based on the presumption that the DM is completely rational. They are not in line with realistic decision-making situations. Furthermore, the method proposed in this paper takes into consideration the psychological behavior of the DM, such as their inclination toward being regret-risk-averse and/or rejoicing-risk-seeking; it also eliminates the subjective influence of traditional methods. Thus, it is more reasonable to choose the proposed method if the DM wants to avoid regret when making decisions and eliminate the influence of subjective weights on attributes.

5. Conclusions

In actuality, the DM's regret-aversion behavior affects their decision-making. Therefore, studying the impact of psychological behavior factors on the decision-making consequences has theoretical and practical significance. As mentioned above, an MADM method with TFNs based on regret theory and catastrophe progression has been developed. First, the regret and rejoicing value matrix is constructed based on the regret theory. Second, the maximizing deviation method is employed to measure the attribute importance degree of the alternative. Finally, using the catastrophe progression method, the comprehensive catastrophe progression value is calculated to rank the alternatives. As compared with other methods, the proposed method has two advantages: 1) it takes the psychological behaviors of the DM into consideration and comprehensively simulates realistic decision-making situations; 2) it eliminates the effect of subjective weighting on decision-making results, and the results are more objective and scientific. In addition, through an illustrative example, it can be seen that the proposed method has excellent operability and practicability. It provides a new way to solve practical MADM problems, such as investment project selection, supply chain management and new product development.

However, there are still some limitations of this study. For example, DMs often evaluate attributes by using textual descriptions, so quantification becomes a problem. Besides, as the MADM problems become more complex, the fuzzy sets must be expanded to better describe the uncertainty of decision-making information. The elements of classical fuzzy sets, such as TFNs, are interpreted based solely on the degree of membership. If the negative attitude of DMs needs to be expressed, fuzzy sets such as an intuitionistic fuzzy set or a Pythagorean fuzzy set should be used because they represent the degree of membership and degree of non-membership. Therefore, in future research, we will focus on the following directions: 1) combining a linguistic term set with the fuzzy set to provide a new way to solve the MADM problems, and 2) expanding the application of the catastrophe progression method under the conditions of a Pythagorean fuzzy environment.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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