Mathematical Biosciences and Engineering

MBE, 18 (6): 7389–7401.
DOI: 10.3934/mbe.2021365
Received: 30 June 2021
Accepted: 11 August 2021
Published: 30 August 2021

http://www.aimspress.com/journal/MBE

*Research article*

# The role of proactive behavior on COVID-19 infordemic in the Chinese Sina-Microblog: a modeling study

**Fulian Yin[1,2], Hongyu Pang[2], Lingyao Zhu[2], Peiqi Liu[2], Xueying Shao[2], Qingyu Liu[3] and Jianhong Wu[4,*]**

[1] State Key Laboratory of Media Convergence and Communication, Communication University of China, Beijing, 100024, China
[2] College of Information and Communication Engineering, Communication University of China, Beijing 100024, China
[3] The third construction CO.LTD of China construction third engineering bureau Beijing, Beijing 100024, China
[4] Fields-CQAM Laboratory of Mathematics for Public Health, Laboratory for Industrial and Applied Mathematics, York University, Toronto M3J1P3, Canada

* **Correspondence:** Email: wujhhida@hotmail.com; Tel: +1 4167365243.

**Abstract:** In order to avoid forming an information cocoon, the information propagation of COVID-19 is usually created through the action of “proactive search”, an important behavior other than “reactive follow”. This behavior has been largely ignored in modeling information dynamics. Here, we propose to fill in this gap by proposing a proactive-reactive susceptible-discussing-immune (PR-SFI) model to describe the patterns of co-propagation on social networks. This model is based on the forwarding quantity and takes into account both proactive search and reactive follow behaviors. The PR-SFI model is parameterized by data fitting using real data of COVID-19 related topics in the Chinese Sina-Microblog, and the model is calibrated and validated using the prediction accuracy of the accumulated forwarding users. Our sensitivity analysis and numerical experiments provide insights about optimal strategies for public health emergency information dissemination.

**Keywords:** Chinese Sina-Microblog; active search-reactive follow; information propagation model

## 1. Introduction

Like many other social network platforms, in Sina-Microblog as the most popular network in China [1], every user can become both an information receipt to browse news and public opinions timely and an information publisher to disseminate the information quickly. For example, COVID-19 pandemic relevant topics increased exponentially on this platform after the initial outbreak and subsequently after the WHO declared a global pandemic. Figure 1 shows two main ways in which users browse and participate in the dissemination of information. One common way is to contact/expose information and then choose to forward it through the follow-ship over the real network [2,3]. Machine interference is another main way that has a profound impact on effective information dissemination. In this way, the Most Searched Hashtags and Hottest Topics are sorted and listed through AI technologies applied to the attention of real-time hot events. Users can read these hot public opinions that interest them by clicking the list of popular search titles or keywords. However, the listed topics are limited, those users having an additional desire for unlisted information usually look for more information using the searching box. This active search behavior has not yet been captured, to our best knowledge, in most modeling studies, so the users' exposure to information is usually underestimated, the description of information propagation dynamics is incomplete, and the use of such modeling framework as a tool to inform early warnings and rapid response strategies about the spread of information through social network is limited.

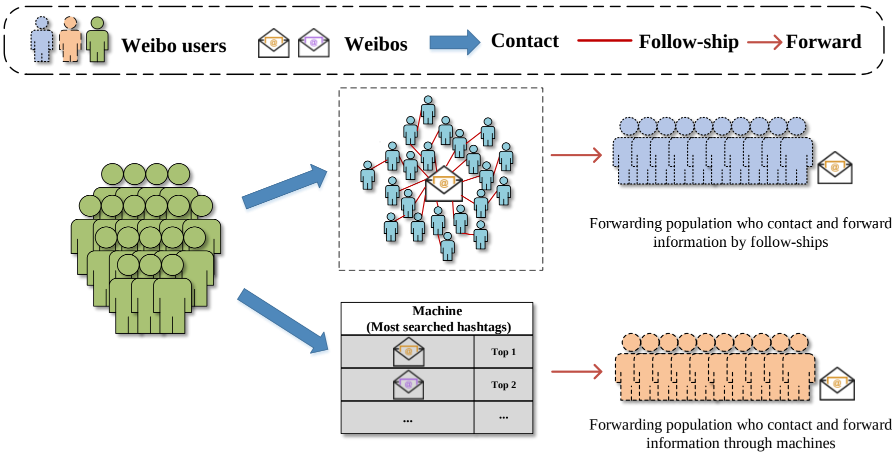


**Figure 1.** An illustration of different ways to passively expose and actively search, followed by information dissemination.

More specifically, based on some similarities between information spreading and infectious disease transmission in terms of dynamic propagation, sub-population proximity in terms of exposure and transmission, and human interventions [4–6], classical epidemic models have been adopted and applied to understand information propagation on social platforms [7–9]. Depending on the disease presentation and disease progression, classical disease transmission models are typically classified by the compartmentalization of the population into the susceptible-infected (SI) model [10,11], the susceptible-infected-susceptible (SIS) [12] model, and the susceptible-infected-recovered (SIR) model [13,14], among others. Almost all of these formulations can find their extensions to information propagation. At present, society gradually emerges new propagation phenomena, so that scholars apply these classical infectious disease models to new application scenarios or establish new models to match the situation [15,16], and describe the process of information dissemination. Wang et al. [17] proposed a model considering microblog propagation behaviors based on the SIS model, which is shown to be able to predict forwarding trends and have strong practicalities in the real network. Liu et al. [18] noticed the super-spreading information propagation on tweet and developed a parameterized model termed SAIR by adding a super-spreader state into the classical SIR model. Wang et al. [19] added the activity of users, the credibility of information, and network weights to the classical SIR model. They analyzed the dissemination rate and immunization rate of information dissemination on Weibo according to the total number of different users' comments, reports, and likes at different times. Zhao et al. [20] extended the SEIR model to investigate the propagation dynamics of the information online, and they used the information propagation data from the Digg website to examine the effectiveness. Zhang et al. [21] extended the traditional SEIR model to a susceptible-exposed-trusted-questioned-recovered (SETQR) model. Sang et al. [22] proposed a new information dissemination model called SEIRD to incorporate heterogeneous characteristics and user awareness. In addition, rumors as a special kind of information are the focus of some published studies. Kumar et al. [23] established a SMIR model and introduced the concept of an anti-rumor spreading agent to investigate how to deal with the propagation of false news or rumors. Chen et al. [24], Zhu et al. [25], Choi et al. [26], and Naim et al. [27] also created further mathematical models for the study of rumor propagation.

In the Chinese Sina-Microblog, users can browse or participate in the propagation in many ways. A previous work [28] proposed a susceptible-forwarding-immune (SFI) model to study the basic information propagation dynamics with consideration of the forwarding behavior. Based on the SFI model, subsequent studies [29,30] considered the cross-transmission of pieces of information and emotional choices of users and respectively, with the developed CT-SFI model and E-SFI model. Active search through keywords is an important way to actively expose and spread information, but this searching behavior has not been captured in the existing modeling studies. Filling in this important gap falls in the scope of our current research. Namely, we seek to make a distinction between active searching and non-active exposure behaviors in modeling the propagation mechanism of COVID-19 related information. In particular, in our proposed proactive susceptible-forwarding-immune dynamics model, we introduce active search factors according to the discussion quantities of COVID-19 topics on the network, with Chinese Sina-Microblog as a case study.

The rest of the paper is organized as follows. In Section 2, we construct a mathematical model of the information spreading on social networks incorporating proactive search and reactive follow behaviors. In Section 3, we fit the model with the real data of typical topics and analyze trends for each state variable over time. In Section 4, we conduct a parameter sensitivity analysis, suggesting effective intervention strategies for optimal information propagation, and comparing the effects of proactive search and passive exposure behavior on information dissemination. In Section 5, we introduce and calculate some key indices to measure public opinion propagation strength at the early stage of public opinion outbreaks. Section 6 summarizes our conclusions.

## 2. Model description

We consider the information transmission on Sina-Microblog, with two supplementary routes for accessing online information simultaneously. According to the propagation process of each information, we classify the relevant Chinese Sina-Microblog users into four distinct states: the proactive susceptible state ($S_P$), in which users are unaware of but susceptible to the Weibo by actively searching on social media; the passive susceptible state ($S_R$), in which users are unaware of but susceptible to the Weibo based on the follow-ship; the forwarding state ($F$), in which users have been forwarding the Weibo actively to influence other users; the immune state ($I$), in which users are further divided into two categories: those who have contacted but have no interest in the content of this Weibo subjectively, and those who have already forwarded the Weibo, but no longer forward the Weibo again, and have no more influence on other users.

Let $N$ denote the total number of users involved in the process of information dissemination, where $N$ is a sufficiently large positive integer. We assume that the susceptible users have at most one forwarding behavior and the users consist of four groups with different states, including the proactive susceptible state $S_P(t)$ ($S_P$-state), the passive susceptible state $S_R(t)$ ($S_R$-state), the forwarding state $F(t)$ ($F$-state) and the immune state $I(t)$ ($I$-state), for any time $t \geq 0$. In particular, the susceptible state (S) is subdivided into the proactive susceptible state ($S_P$) and the passive susceptible state ($S_R$), i.e. $S_P(t) + S_R(t) = S(t)$. We obtain the following susceptible-forwarding-immune model:

$$dS_R(t)/dt = -\beta S_R(t)F(t), \tag{1}$$

$$dS_P(t)/dt = -\theta S_P(t)F(t), \tag{2}$$

$$dF(t)/dt = p\beta S_R(t)F(t) + \theta\gamma S_P(t)F(t) - \alpha F(t), \tag{3}$$

$$dI(t)/dt = (1-p)\beta S_R(t)F(t) + \theta(1-\gamma)S_P(t)F(t) + \alpha F(t). \tag{4}$$

An illustration of this model is given in Figure 2. The definition of parameters is detailed in Table 1.

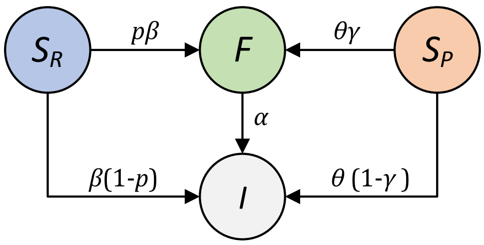


**Figure 2.** The proactive (search)/passive(exposure)-follow susceptible-forwarding-immune propagation model.

**Table 1.** The definition of parameters.

| Parameter | Interpretation |
|---|---|
| $p$ | The probability that the users in the un-initiative susceptible state will forward the Weibo. |
| $\beta$ | The exposure rate that the users in the un-initiative susceptible state will be exposed to the Weibo based on the follow-ship. |
| $\gamma$ | The probability that the users in the initiative susceptible state will forward the Weibo. |
| $\theta$ | The exposure rate that the users in the initiative susceptible state will be exposed to the Weibo by actively searching on the social media. |
| $\alpha$ | The rate at which a user in the $F$-state becomes the immune state. |

In our proposed model, the user's state changes in the forwarding, the direct-immune, and the overtime-immune processes. More specifically, the number of initiative susceptible users is $\theta N$, among which $\theta\gamma N$ will forward the Weibo and $\theta(1-\gamma)N$ will not, where $0<\gamma<1$. The number of un-initiative susceptible users is $\beta N$, among which $p\beta N$ will forward the Weibo and $(1-p)\beta N$ will not, where $0<\beta<1$. The probability of a normal user is a proactive susceptible user or a non-active susceptible is $S_R(t)/N$ or $S_P(t)/N$, and there are $F(t)$ active forwarding users in total, then $dS_R(t)/dt=-\beta S_R(t)F(t)$, and $dS_R(t)/dt=-\beta S_R(t)F(t)$. After a short period (latent), some users in the forwarding state turn into the immune state at a rate of $\alpha$, where $1/\alpha$ is the average duration that an F-user remains active in forwarding.

We denote the number of cumulative forwarding users by $C(t)$ and can compute it according to the proposed model

$$C(t)=\int_0^t\ [\ p\beta S_R(t)F(t)+\gamma\theta S_P(t)F(t)]dt. \tag{5}$$

It follows that

$$dC(t)/dt=\ p\beta S_R(t)F(t)+\gamma\theta S_P(t)F(t). \tag{6}$$

Since one Weibo is usually posted by a single user, we have the following initial conditions for the model, $F_0=C_0=1$, $I_0=0$, $S_{R0}+S_{P0}=N-F_0=N-1$. Eqs 1, 2 and 6 imply that

$$dS(t)/dt=dS_P(t)/dt+dS_R(t)/dt=-\beta S_R(t)F(t)-\theta S_P(t)F(t)<0, \tag{7}$$

and

$$C'(t)=p\beta S_R(t)F(t)+\gamma\theta S_P(t)F(t)>0. \tag{8}$$

Therefore, $S_\infty=\lim_{t\to\infty}S(t)>0$ and $C_\infty=\lim_{t\to\infty}C(t)<N$. Similarly, Eq. (4) implies that $I(t)$ is increasing since $I'(t)=(1-p)\beta S_R(t)F(t)+\theta(1-\gamma)S_P(t)F(t)+\alpha F(t)$ and $I_\infty=N-S_\infty$.

**The reproduction ratio $\mathfrak{R}_0$:** The reproduction ratio is an important indicator to measure whether the disease may break out during the spread of infectious diseases. Similarly, we define the reproduction ratio $\mathfrak{R}_0$ to estimate the propagation tendency of a Weibo.

If the forwarded speed of one Weibo is always less than zero after it was published, i.e.,

$$F'(t)=(p\beta S_{R0}+\theta\gamma S_{P0}-\alpha)F(t)<0. \tag{9}$$

Then this Weibo is unlikely to appear in the top search list. Since F(t) is always positive, we rewrite Eq 9 as

$$\frac{p\beta S_{R0}+\theta\gamma S_{P0}}{\alpha}<1. \tag{10}$$

Based on the definition of epidemic reproduction ratio, we define the Weibo reproduction ratio

$$\mathfrak{R}_0=\frac{p\beta S_{R0}+\theta\gamma S_{P0}}{\alpha}. \tag{11}$$

If $\mathfrak{R}_0<1$, the forwarded number of one message will decrease and this Weibo will get less and less attention. If $\mathfrak{R}_0>1$, the message will be forwarded timely and quickly, leading to the outbreak of information. Therefore, the popularity of one Weibo can be determined by the value of $\Re_0$. The larger $\Re_0$ is, the faster Weibo will be forwarded. Note the role of proactive search, so classical modeling without considering this proactive search normally leads to underestimating the information outbreak potential.

## 3. Numerical simulation

In this section, we verify the effectiveness of the proposed proactive SFI model through numerical experiments. The real data is collected from Weibo's Application Programming Interface (API). Table 2 lists the cumulative forwarding number of users of a particular Weibo at different times between 15:00, March 4, 2020 and 14:00, March 5, 2020. This Weibo is about that a Chinese research team found the mutation of COVID-19 and this observation attracts a lot of attention. This Weibo was forwarded by a fraction of users and its forwarding quantity increased slowly at first, and then more users noticed and forwarded this Weibo quickly after about one to two hours, and the forwarding quantity achieved a huge surge after a steady increase period. Finally, the popularity of this Weibo would fade away or be overwritten by other news.

**Table 2.** The cumulative forwarding numbers of a particular Weibo.

| $t(h)$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| number | 4480 | 8002 | 10847 | 12802 | 14025 | 14978 | 15636 | 16184 |
| $t(h)$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| number | 16607 | 16715 | 16792 | 16847 | 16879 | 16903 | 16909 | 16927 |
| $t(h)$ | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| number | 16957 | 17003 | 17054 | 17103 | 17177 | 17209 | 17233 | 17250 |

Here we use the $LS(Least\ Square)$ method to estimate the parameters of the proactive SFI model and the fitted parameters minimize the error function locally so we have added the parameter ranges that are available based on previous research experience through software called DEDiscover within which the fitting was conducted. The cumulative forwarding number is the available data from Weibo, and we set the auxiliary parameter vector as $\Theta = (\beta, \alpha, p, \theta, \gamma, S_{R0}, S_{P0})$. Let $f_C(t)$ be the numerical value of $C(t)$. To discover the optimal data fitting, $\Theta$ is set to minimize the following formula:

$$LS = \sum_{k=0}^{T} |f_C(t_k, \Theta) - ID_k|^2. \tag{12}$$

where $ID_k$ denotes the real number of cumulative forwarding users, $t_k = k$ is the sampling time, $k = 1,2,\dots,16$.

Figure 3 plots the data fitting and simulation results. Figure 3a is the fitting result between the numerically calculated and the actual numbers of cumulative forwarding, which shows the rapid outbreak of this information and it tends to be stable quickly. From the data fitting, our PR-SFI model achieves accurate estimation, and we can estimate unknown parameters. Based on the findings from Figure 3a, we obtain the changes in the number of users in $S_R$ state, $S_P$ state, $F$ state, and $I$ state as plotted in Figure 3b. We can see from Figure 3b that the number of people in $S_P$ and $S_R$ states have been declining over time, and the number of people in $I$ state has been rising and tends to be stable finally. The observations match the actual situation and validate the feasibility of our model. In addition, we compute the reproduction ratio of this information $\Re_0 = 5.9190 > 1$, and hence we can expect a rapid outbreak of this information at the beginning of the propagation. Fig.3(a) does confirm this inference.

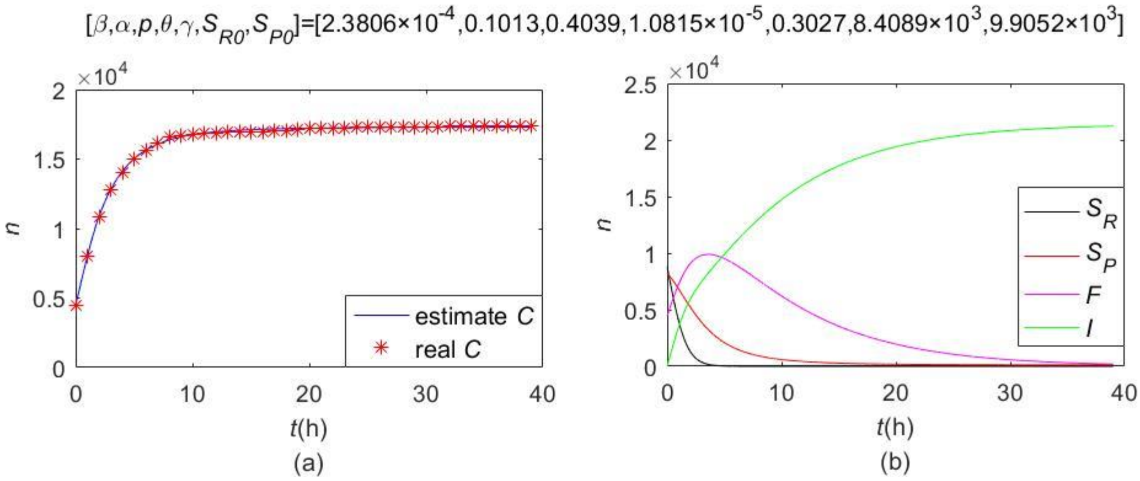


**Figure 3.** Parameter estimation and numerical simulation results (a) Data fitting to the actual number of C population; (b) The number of users in four different states, SR- state, SP- state, F- state, and I- state.

**Table 3.** Parameter results.

| | Estimated Value | Min | Max |
|---|---|---|---|
| $\beta$ | $2.3806\times 10^{-4}$ | 0 | 1 |
| $\alpha$ | 0.1013 | 0 | 2 |
| $p$ | 0.4039 | 0 | 1 |
| $\theta$ | $1.0815\times 10^{-4}$ | 0 | 1 |
| $\gamma$ | 0.3027 | 0 | 1 |
| $S_{R0}$ | $8.4089\times 10^{3}$ | 0 | $1.5\times 10^{4}$ |
| $S_{P0}$ | $9.9052\times 10^{3}$ | 0 | $2.0\times 10^{4}$ |

## 4. Parametric sensitivity analyses

This section analyzes how the parameters in the proactive SFI model influence the information propagation of the COVID-19 topics. To this end, we use the partial rank correlation coefficients (PRCCs) [31] based on 1000 samples under different threshold conditions to evaluate the sensitivity of various input parameters. According to the histogram and scatter diagram of three important indicators $\Re_0$, $F_{max}$ and $C_s$, we find that when the correlation between $F_{max}$ and $C_s$ is positive, the value of the corresponding index will increase with the increase of the value of the parameter. On the contrary, the index will decrease as the parameter decreases.

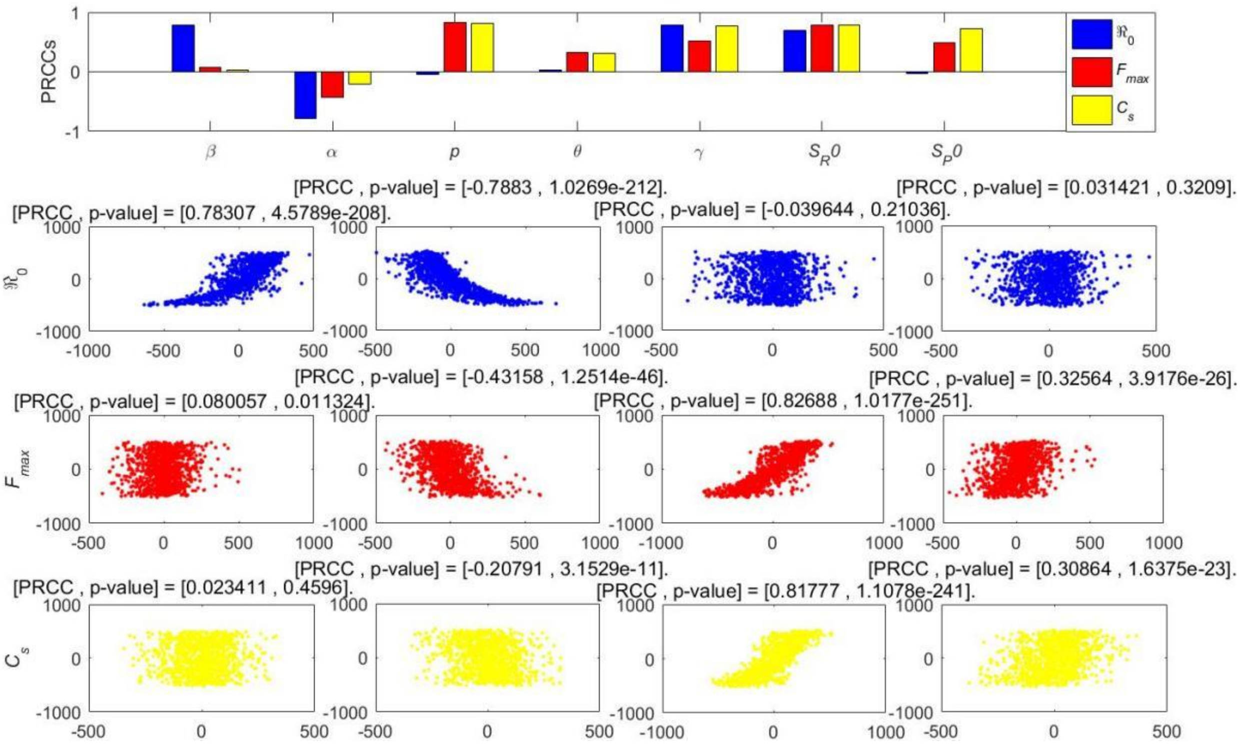


**Figure 4.** PRCCs results of $\Re_0$, $F_{max}$ and $C_s$ under multi-parameter variation.

Figure 4 shows that parameters $\beta$, $p$, $\theta$, $\gamma$, and initial values $S_{R0}$, $S_{P0}$ have positive influences on $\Re_0$, $F_{max}$ and $C_s$, and parameter $\alpha$ has a negative influence on the three indexes. According to the definition of $\Re_0$ in Eq 11, if we want to let the positive topics of the COVID-19 guide the public opinion, we can achieve it by increasing the value of $\beta$, $p$, $\theta$, $\gamma$, $S_{R0}$ and $S_{P0}$ and decreasing the value of $\alpha$. $S_{c0}$ and $S_{s0}$ are the initial values of the susceptible population, and we can increase these two values by taking advantage of the appeal of opinion leaders and persuading them to participate in the information propagation. Since $\beta$ is the average exposure rate for a user to contact the information, the increases of $S_{R0}$ and $S_{P0}$ lead to the increase of $\beta$ as well. Besides, we can make the content richer and more interesting to attract users to forward topics and keep them active in forwarding for a longer time to increase the value of parameter $p$ and decrease the value of $\alpha$. We also can select the information diffusion source with higher network density at the beginning of information release and reduce the network density by truncating key nodes to increase the parameter $\theta$. Correspondingly, if we do not want public opinion to erupt, such as rumor topics of the COVID-19, we can reduce the value of parameter $\beta$ and $S_0$ by requesting the platform to delete the relevant topics of the rumors about the COVID-19. Then the new reading quantity is decreased and the outbreak of public opinion is controlled from the source.

To compare the effects of proactive and non-active behavior on information dissemination, here we focus on the analysis of parameters $\beta$, $\theta$, $p$, and $\gamma$. Figure 5 shows the influence of each parameter on the instantaneous number of forwarding users $F(t)$ and the cumulative number of forwarding users $C(t)$.

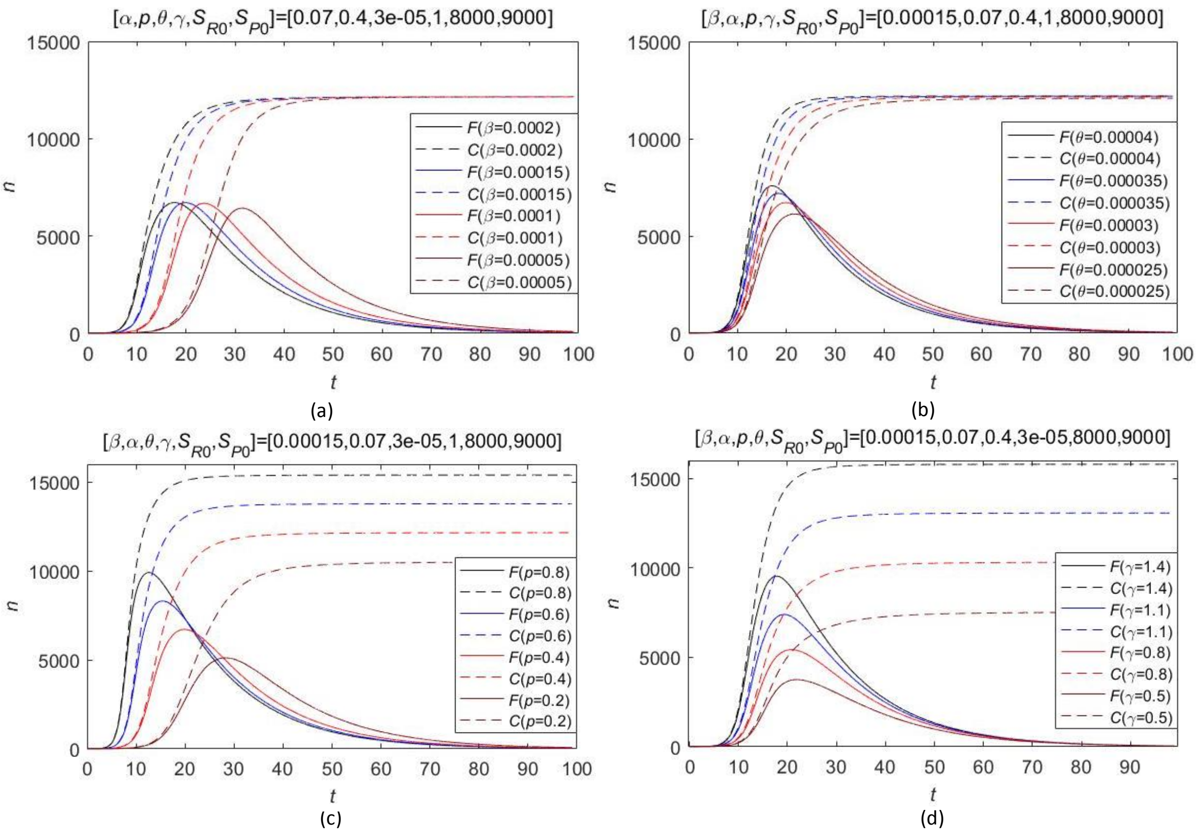


**Figure 5.** The influence of key parameters on $F$ and $C$: (a) only $\beta$ changes; (b) only $\theta$ changes; (c) only $p$ changes; (c) only $\gamma$ changes.

By comparing and analyzing Figure 5a,b, parameters $\beta$, $\theta$, $p$, and $\gamma$ have similar influences on $F(t)$ and $C(t)$. With the increase of these parameters, the instantaneous number of forwarding users $F(t)$ and the cumulative number of forwarding users $C(t)$ shot up. The rate of the outbreak is faster, the peak of the total number of forwarding users $F(t)$ is higher, and the final scale of the cumulative number of forwarding users $C(t)$ is also wider. The difference is that the average contact rate $\beta$ and the average exposure probability $\theta$ are very sensitive, and the experiment is only carried out in a very small range, as shown in Figure 5, although parameters $\beta$ and $\theta$ affect both $F(t)$ and $C(t)$, they have less obvious effects on $C(t)$ than parameters $p$, $\gamma$.

## 5. Early predictive performance of the model

The purpose of public opinion analysis and prediction is to provide early warnings. The proposed SFI dynamic model makes it possible to analyze public opinions about COVID-19 published on the Sina-Microblog, and then we can make predictions on the key communication index of public opinion as early as possible.

Parameter estimations are necessary to determine and apply the proposed model. In particular, $\beta$ and $\theta$ can be determined by the network structure, and $\alpha$ can be obtained from the behavior rules of users, so the unknown parameters include $p$, $\gamma$, $S_{R0}$ and $S_{P0}$.

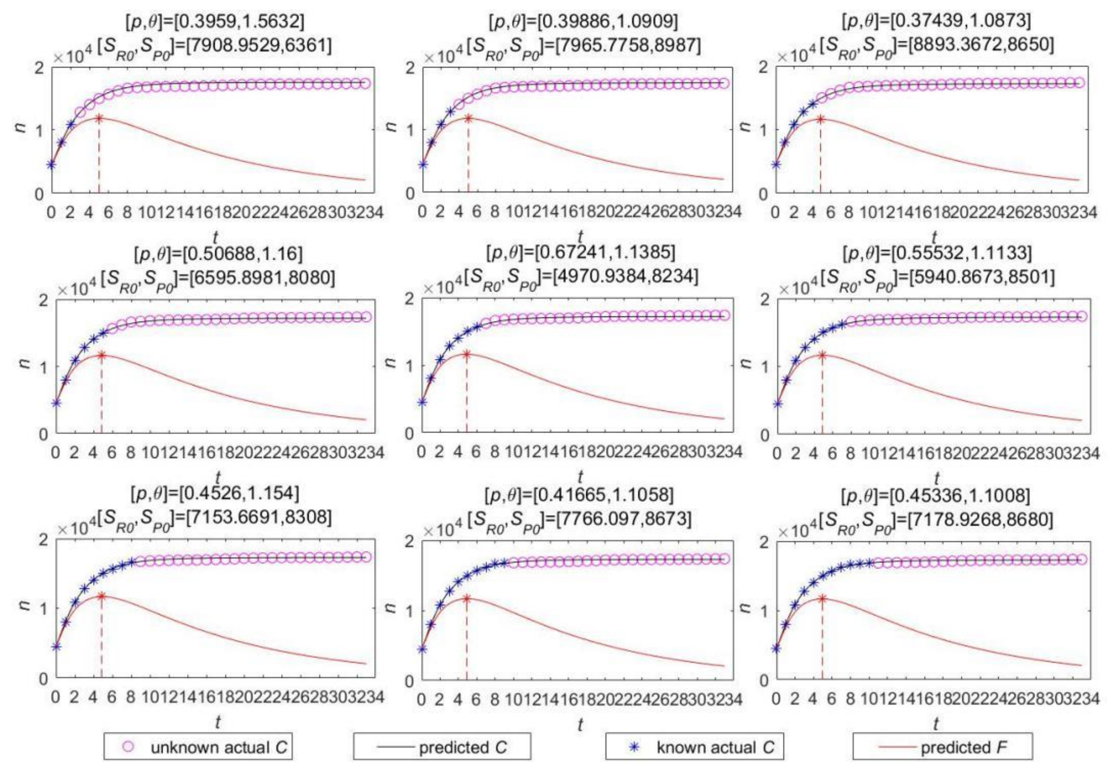

**Figure 6.** Predictive performance of single information propagation proactive SFI dynamic model (only parameters $p$, $\gamma$ and initial values $S_{R0}$ and $S_{P0}$ are estimated).

This section carries on numerical experiments with $\beta = 1.3484 \times 10^{-4}$, $\alpha = 0.1292$, $\theta = 3.1943 \times 10^{-5}$, as shown in Figure 6, where the blue star is used for parameter estimation of the real cumulative number of forwarding users, the pink circle is the unknown real cumulative number of forwarding users, black and red solid lines are respectively the predicted cumulative number of forwarding users obtained from parameter estimation and the total amount of users in forwarding state, and the red star indicates the public opinion peak and the corresponding public opinion climax time.

Figure 6 shows the prediction results of single information transmission about the COVID-19 based on the proactive SFI dynamics model. It only shows the results from the beginning of information release to the third hour of data acquisition, and fewer hours are no longer displayed because the prediction performance is too low and has no practical significance. We can see that our model can predict the whole trend of public opinion of the COVID-19, and the prediction performance of each period is good. The early prediction way is a method to exchange prior knowledge for early warning time and effect. To estimate fewer parameters at an earlier time with better prediction performance of the COVID-19, it is necessary to detect the network structure and individual habits in the network in the early stage.

## 6. Conclusions

In this study, we propose a proactive search-passive exposure-reactive susceptible-discussing-immune (PR-SFI) dynamics model considering both proactive search, passive exposure and reactive follow behaviors of users from the Chinese Sina-Microblog. We parametrize, calibrate, and validate our models using real-life data in the network during the COVID-19 pandemic. Our model examines how these two complementary behaviors, proactive search and passive exposure, influence public opinion and information propagation in different stages of the spread of COVID-19 related information. In our numerical experiments, the results of parameter sensitivity analysis and PRCCs show that both proactive search and reactive follow behaviors have effects on the transmission of information and the influence of the forwarding probability $\gamma$ of proactive search and the exposure rate $\beta$ of reactive follow are more significant than other parameters. Meanwhile, the model can realize early prediction of public opinion. In summary, we believe we have pointed out an important information access route, proactive research, that has been underestimated in the existing literature and we filled in this gap. As a result, we have initiated the discussion towards developing an important technical tool to accurately predict public opinion spread trends and to provide insights on optimal public health communication strategies during a public health crisis, like the ongoing COVID-19 pandemic.

## Acknowledgments

The work was supported by the National Natural Science Foundation of China (Grant numbers: 61801440), the Natural Science and Engineering Research Council of Canada, the Canada Research Chair Program (JWu), the Fundamental Research Funds for the Central Universities, the High-quality and Cutting-edge Disciplines Construction Project for Universities in Beijing (Internet Information, Communication University of China) and the State Key Laboratory of Media Convergence and Communication, Communication University of China.

## Conflict of interest

The authors declare there is no conflict of interest.

## References

1. Q. Gao, F. Abel, G. J. Houben, Y. Yu, A comparative study of users' microblogging behavior on Sina Weibo and Twitter, In *Int. Conf. User Model., Adapt., Pers.*, Springer, Berlin, Heidelberg, (2012), 88–101.
2. X. Li, S. Cheng, W. Chen, F. Jiang, Novel User Influence Measurement based on User Interaction in Microblog, *2013 IEEE/ACM Int. Conf. Adv. Soc. Netw. Anal. Min.*, (2013), 615–619.
3. N. Zhang, Y. Chai, Y. Li, H. Sun, Modeling Micro-blog Network Structure based on Combination of Online Communities, in *Chin. Contr. Decision Conf.*, (2013), 3419–3424.
4. W. Goffman, V. Newill, Generalization of epidemic theory, *Nature*, 1964, **204** (1964), 225–228.
5. K. Dietz, Epidemics and rumours: A survey, *J. R. Stat. Soc. Ser. A (General)*, **130** (1967), 505–528.
6. S. Galam, Modelling rumors: the no plane Pentagon French hoax case, *Physica A*, **320** (2003), 571–580.
7. S. Abdullah, X. Wu, An epidemic model for news spreading on twitter, *2011 IEEE 23rd Int. Conf. Tools Artif. Intell.*, IEEE, 2011, 163–169.
8. B. Zhang, X. Guan, M. J. Khan, Y. Zhou, A time-varying propagation model of hot topic on BBS sites and Blog networks, *Inform. Sci.*, **187** (2012), 15–32.

9. M. Tanaka, Y. Sakumoto, M. Aida, K. Kawashima, Study on the growth and decline of SNSs by using the infectious recovery SIR model, In *2015 10th Asia-Pacific Symp. Inf. and Telecommun. Technol. (APSITT)*. IEEE, 2015, 1–3.
10. F. Chen, A susceptible-infected epidemic model with voluntary vaccinations, *J. Math. Biol.*, **53** (2006), 253–272.
11. Z. Lu, S. Gao, L. Chen, Analysis of an SI epidemic model with nonlinear transmission and stage structure, *Acta Math. Scientia.*, **23** (2003), 440–446.
12. C. Xia, S. Sun, F. Rao, J. Sun, J. Wang, Z. Chen, SIS model of epidemic spreading on dynamical networks with community, *Front. Comput. Sci. China*, **3** (2009), 361–365.
13. W. O. Kermack, A. G. McKendrick, Contributions to the mathematical theory of epidemics--I. 1927, *Bull. Math. Biol.*, **53** (1991), 33–55.
14. L. Stone, B. Shulgin, Z. Agur, Theoretical examination of the pulse vaccination policy in the SIR epidemic model, *Math. Comput. Model.*, **31** (2000), 207–215.
15. X. Feng, L. Liu, S. Tang, X. Huo, Stability and bifurcation analysis of a two-patch SIS model on
16. nosocomial infections, *Appl. Math. Lett.*, **102** (2020), 106097.
17. B. Tang, F. Xia, S. Tang, N. L. Bragazzi, Q. Li, X. Sun, et al., The effectiveness of quarantine and isolation determine the trend of the COVID-19 epidemic in the final phase of the current outbreak in China, *Int. J. Infect. Dis.*, **96** (2020), 636–647.
18. H. Wang, Y. Li, Z. Feng, L. Feng, ReTweeting analysis and prediction in microblogs: An epidemic inspired approach, *China Commun.*, **10** (2013), 13–24.
19. Y. Liu, B. Wang, B. Wu, S. Shang, Y. Zhang, C. Shi, Characterizing super-spreading in microblog: An epidemic-based information propagation model, *Physica A*, **463** (2016), 202–218.
20. B. Wang, J. Zhang, H. Guo, Y. Zhang, X. Qiao, Model study of information dissemination in microblog community networks, *Discrete Dyn. Nat. Soc.*, **2016** (2016), 1–11.
21. D. Zhao, J. Sun, Y. Tan, J. Wu, Y. Dou, An extended SEIR model considering homepage effect for the information propagation of online social networks, *Physica A*, **512** (2018), 1019–1031.
22. Y. Zhang, Y. Feng, R. Yang, Network public opinion propagation model based on the influence of media and interpersonal communication, *Int. J. Mod. Phys. B*, **33** (2019), 1950393.
23. C. Sang, S. Liao, Modeling and simulation of information dissemination model considering user's awareness behavior in mobile social networks, *Physica A*, **537** (2020), 122639.
24. A. Kumar, P. Swarnakar, K. Jaiswal, R. Kurele, SMIR model for controlling the spread of information in social networking sites, *Physica A*, **540** (2020), 122978.
25. X. Chen, N. Wang, Rumor spreading model considering rumor credibility, correlation and crowd classification based on personality, *Sci. Rep.*, **10** (2020), 1–15.
26. L. Zhu, W. Liu, Z. Zhang, Delay differential equations modeling of rumor propagation in both homogeneous and heterogeneous networks with a forced silence function, *Appl. Math. Comput.*, **370** (2020), 124925.
27. S. H. Choi, H. Seo, M. Yoo, A multi-stage SIR model for rumor spreading, *Discrete Contin. Dyn. Syst. Ser. B*, **25** (2020), 2351–2372.
28. M. Naim, F. Lahmidi, A. Namir, Dynamics of a Delayed Rumor Propagation Model with Consideration of Psychological Factors and Forgetting Mechanism, *Appl. Math.*, **14** (2020), 597–604.
29. F. Yin, X. Shao, J. Wu, Nearcasting forwarding behaviors and information propagation in Chinese Sina-Microblog, *Math. Biosci. Eng.*, **16** (2019), 5380–5394.

30. F. Yin, X. Shao, B. Tang, X. Xia, J. Wu, Modeling and analyzing cross-transmission dynamics of related information co-propagation, *Sci. Rep.*, **11** (2021), 1–20.
31. F. Yin, X. Xia, X. Zhang, M. Zhang, J. Lv, J. Wu, Modelling the dynamic emotional information propagation and guiding the public sentiment in the Chinese Sina-microblog, *Appl. Math. Comput.*, **396** (2021), 125884.
32. U. L. Abbas, R. M. Anderson, J. W. Mellors, Potential impact of antiretroviral chemoprophylaxis on HIV-1 transmission in resource-limited settings, *PloS One*, **2** (2007), e875.