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Research article

Investigation on stability and controller design for singular bio-economic systems with stochastic fluctuations

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Abstract: In this paper, the finite-time stability and control of a kind of singular bio-economic systems with stochastic fluctuations are investigated. When economic profit is no longer a constant but a variable, the system shows distinct dynamic behavior. First, a singular system is proposed to describe the bio-economic system with stochastic fluctuation. Then a singular stochastic T-S fuzzy model is established based on T-S fuzzy system theory. Second, a sufficient condition is proposed to satisfy finite-time stochastic stability of bio-economic system. On this basis, a fuzzy state feedback controller is designed which can make corresponding closed-loop singular stochastic bio-economic system admissible in finite-time, and the states of the system can be driven to a bounded range through the management of the open resource. Finally, the validity of the results is verified through the numerical simulation.

Keywords: stochastic singular bio-economic systems; fuzzy systems; stability analysis; controller design

1. Introduction

In recent years, the problems of resource shortage and fragile ecological environment have appeared frequently, threatening the survival and development of posterity to a large extent. Therefore, many people have a keen interest in studying biological systems. Considering the maximization of net economic income, a bio-economic model was proposed in [1] which was based on differential-algebraic equation. Biological and economic stability by adding a popular dynamic was studied in [2–4]. The problem of how to obtain best harvest in the bio-economic model was studied in [5, 6], which provides a theoretical basis for the rational development of biological resources. The optimal cost control problem of Markov jump system was solved in [7, 8]. For the purpose of protecting the environment and maintaining the economic development stably and rapidly, it is very urgent and necessary to research the analysis of the bio-economic system.

Singular systems have a larger application background in bio-economic systems [9]. The singular systems is different from the normal system in that its stability is more complex. It is well-known that a singular system can be stabilized only when it is regular and impulse-free [10, 11]. Therefore, much effort was devoted to singular system and its applications. In the recent years, some singular bio-economic system models with stochastic and bifurcation properties are established, which shows that the research of the singular system is very broad and has good development prospects. A singular biological economy markov jump system is proposed in [12], which takes commodity price as markov chain. The bio-economic singular Markov jumping system was studied and the corresponding control design was proposed in [13].

In natural environment, environmental fluctuation is a very important part of bio-economic system in real life. To a large extent, there are limitations in the application of deterministic methods in mathematical modeling. Therefore, the future dynamics of the system are difficult to accurately predict. In the various dynamic analysis of the system, the stochastic differential equation model is a important part. The ecological population system model was established using randomness in [14]. Some problems of T-S fuzzy system was researched and the applicaton of this type of system in bio-economy was explored in [15, 16]. According to the theory of fishery resource economics, a stochastic singular bio-economic system which based on the T-S fuzzy model was established in [17].

In practice, some systems can maintain asymptotic stability in an infinite time interval but they do not have good transient characteristics. Consequently, it is meaningful to study the transient behavior within a limited time interval. Several sufficient conditions for the continuous-time systems and discrete-time systems to maintain stability in a finite-time are given in [18, 19]. In the singular system, the finite-time stability was redefined which have impulsive effects. Then at this time, sufficient conditions were derived in [20]. The conditions which linear stochastic systems can achieve finite-time stability was studied in [21]. The linear matrix inequality theory was used to obtain a series of properties of linear systems, nonlinear systems and stochastic systems in [22, 23]. However, up to now, there are few studies on stochastic singular systems with parameter uncertainties and external disturbances. These problems are very important in practical application and also the main content of our research.

The purpose of this paper is to research stability in finite-time and achieve control of the singular bio-economic system with stochastic fluctuations. It is undoubtedly challenging to control the density of biological population within a certain range and eliminate the influence of some unfavorable factors. This is also the motivation of this paper. The knowledge of the singular stochastic bio-economic system combined with the application of T-S fuzzy control in [24, 25]. Firstly, the T-S fuzzy control model which based on the T-S fuzzy control method is established. Then, it provides a new sufficient condition for the system to achieve stochastic stability in finite-time. On this basis, a state feedback controller which can control the biological populations in a limited range through open resource management is designed. Finally, the effectiveness of the method is verified through simulation in the feasible region.

Notations: The superscript T of a matrix represents its matrix transpose. A is a positive definite matrix if $A > 0$. $\deg(\cdot)$ represents the degree of the determinant. $\varepsilon(x)$ represents expectation of stochastic variable x. $diag(\cdot)$ denotes a diagonal matrix. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ stand for the largest eigenvalue and the smallest eigenvalue of matrix A.

2. Modeling and problem formulation

A single kind of dynamic model which has stage structure proposed by [26] is introduced as follows:

$$\begin{cases} \dot{x}_1(t) = \alpha x_2(t) - r_1 x_1(t) - \beta x_1(t) - \eta x_1^2(t), \\ \dot{x}_2(t) = \beta x_1(t) - r_2 x_2(t), \end{cases} \tag{2.1}$$

where $x_1(t)$ represents the population density of immature species at time t. $x_2(t)$ represents the population density of mature species at time t. α represents the inherent growth rate of the immature. β denotes the transition rate which from the immature species grow into mature species. r_1 represents the death rate of immature species. r_2 represents the death rate of mature species. $-\eta x_1^2(t)$ reflects restriction on the growth of immature species density.

The economic profit is often affected by tax, season, market demand, capture costs and other factors. By the bio-economic theory, the sustainable economic profit should be the sustainable total revenue minus the sustainable total cost. If the population captured in the model (2.1) and the economic benefits of the young population captured are considered, and the economic profit $m(t)$ changes with time, then the singular bio-economic system model can be established as follows:

$$\begin{cases} \dot{x}_1(t) = \alpha x_2(t) - r_1 x_1(t) - \beta x_1(t) - \eta x_1^2(t), \\ \dot{x}_2(t) = \beta x_1(t) - r_2 x_2(t), \\ \quad 0 = E_1(p(t)x_1(t) - c) - m(t), \end{cases} \tag{2.2}$$

where E_1 represtents harvested effort of the immature species, c denotes the cost coefficient so cE_1 represents the total cost, $p(t)$ and $m(t)$ represent the price coefficient and the economic benefits of each individual at time t, respectively.

We notice that there are many random factors in nature and many other random factors caused by human activities, affecting or interfering the changes of immature population density and mature population density in the real environment. It is assumed that the parameters involved in the deterministic model (2.2) are deterministic and have nothing to do with environmental fluctuations. By considering these factors, we can replace the parameters r_1 and r_2 to introduce randomness into the model. Firstly, it is supposed that the white noise can affects the mortality rate of biological species by $r_1 \to r_1 - \alpha_1 \xi(t)$ and $r_2 \to r_2 - \alpha_2 \xi(t)$. Secondly, it is assumed that the population density will also be directly affected through the external random parameter $\omega(t)$.

Therefore, the stochastic differential-algebraic equations is established as follows:

$$\begin{cases} \dot{x}_1(t) = \alpha x_2(t) - r_1 x_1(t) - \beta x_1(t) - \eta x_1^2(t) - E_1 x_1(t) + \alpha_1 x_1(t)\xi(t) + x_1(t) w(t), \\ \dot{x}_2(t) = \beta x_1(t) - r_2 x_2(t) + \alpha_2 x_2(t)\xi(t), \\ \quad 0 = E_1(p(t)x_1(t) - c) - m(t), \end{cases} \tag{2.3}$$

where α_1, α_2 are used to represent two different intensities of the white noises. It is assumed that $\xi(t)$ and $\omega(t)$ are independent of each other, the mean value is zero and the standard deviation is Gaussian white noises, that is $E[\omega(t)] = 0$, $E[\omega(t)\omega(t + \tau)] = \delta(\tau)$, $\delta(\tau)$ is the Dirac function. In the context of biological systems, all the coefficients in Eq (2.3) are non-negative.

3. T-S fuzzy linearization and preliminaries

There is a positive equilibrium point and a non-positive equilibrium point in system (2.3). In other words, there are two equilibrium points in total, and only the positive balance point is considered in the context of biological. Therefore, this paper only considers the positive equilibrium point under some certain conditions.

For express more clearly, it is supposed that the equilibrium point is $p^* = (x_1^*\ x_2^*\ m^*)$. In order to facilitate further study, the following transformations can be used:

$$\begin{cases} \varsigma_1(t) = x_1(t) - x_1^*, \\ \varsigma_2(t) = x_2(t) - x_2^*, \\ \varsigma_3(t) = m(t) - m^*. \end{cases} \tag{3.1}$$

The system (2.3) can be converted to:

$$\begin{cases} \dot{\varsigma}_1(t) = \alpha(\varsigma_2(t) + x_2^*) - r_1(\varsigma_1(t) + x_1^*) - \beta(\varsigma_1(t) + x_1^*) - \eta(\varsigma_1(t) + x_1^*)^2, \\ \qquad - E_1(\varsigma_1(t) + x_1^*) + \alpha_1(\varsigma_1(t) + x_1^*)\xi(t) + (\varsigma_1(t) + x_1^*)w(t) \\ \dot{\varsigma}_2(t) = \beta(\varsigma_1(t) + x_1^*) - r_2(\varsigma_2(t) + x_2^*) + \alpha_2(\varsigma_2(t) + x_2^*)\xi(t), \\ \quad 0 = E_1\left(p(t)(\varsigma_1(t) + x_1^*) - c\right) - (\varsigma_3(t) + m^*). \end{cases} \tag{3.2}$$

The Eq (3.2) is obviously a nonlinear system. Since species density saturation exists, it can be assumed that $\varsigma_i(t)(i = 1, 2, 3)$ are bounded. Make the following changes to the system (3.2) to make expression more concise:

$$\begin{aligned} E\dot{\varsigma}(t) = &\begin{bmatrix} \Omega_{11} & \alpha & 0 \\ \beta & -r_2 + \alpha_2\xi(t) & 0 \\ E_1 p(t) & 0 & -1 \end{bmatrix} \varsigma(t) \\ &+ \begin{bmatrix} \alpha x_2^* - r_1 x_1^* - \beta x_1^* - \eta x_1^{*2} - E_1 x_1^* + x_1^*\alpha_1\xi(t) + x_1^*\omega(t) + \varsigma_1(t)w(t) \\ \beta x_1^* - r_2 x_2^* + x_2^*\alpha_2\xi(t) \\ E_1 p(t) x_1^* - E_1 c - m^* \end{bmatrix}, \end{aligned} \tag{3.3}$$

where

$$\Omega_{11} = -r_1 - \beta - \eta\varsigma_1(t) - 2\eta x_1^* - E_1 + \alpha_1\xi(t)$$

$$\varsigma(t) = \begin{bmatrix} \varsigma_1(t) \\ \varsigma_2(t) \\ \varsigma_3(t) \end{bmatrix}, E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Let

$$z(t) = -r_1 - \beta - \eta\varsigma_1(t) - 2\eta x_1^* - E_1 + \alpha_1\xi(t),$$

so

$$\begin{aligned} \max z(t) &= -r_1 - \beta - \eta\varsigma_1^{\min}(t) - 2\eta x_1^* - E_1 + \alpha_1\xi(t), \\ \min z(t) &= -r_1 - \beta - \eta\varsigma_1^{\max}(t) - 2\eta x_1^* - E_1 + \alpha_1\xi(t). \end{aligned}$$

$z(t)$ is expressed as follows by the max-min values

$$z(t) = M_{11}(z(t)) \max z(t) + M_{12}(z(t)) \min z(t)$$

where $M_{11} + M_{12} = 1$ and M_{11}, M_{12} denote the membership functions. Given the fuzzy rules as follows:
Model rule 1:
If $z(t)$ is $M_{11}(z_1(t))$, then $E\dot{\varsigma}(t) = A_1\varsigma(t) + B_1\varsigma(t)w(t)$
Model rule 2:
If $z(t)$ is $M_{12}(z_1(t))$, then $E\dot{\varsigma}(t) = A_2\varsigma(t) + B_2\varsigma(t)w(t)$
The system (3.3) can be changed into Eq (3.4) by using the fuzzy rules:

$$Ed\varsigma(t) = A_h\varsigma(t)dt + B\varsigma(t)dw(t) \tag{3.4}$$

where $A_h = \sum_{i=1}^{2} h_i(z(t))A_i \quad h_i(z(t)) \geq 0 \quad \sum_{i=1}^{2} h_i(z(t)) = 1 \quad B = B_1 = B_2$.

Definition 3.1. (i) If there is a constant λ such that $\det(\lambda E - A_h) \neq 0$, then the system (3.4) is regular. (ii) If $rank(E) = \deg(\det(\lambda E - A_h))$, then the system (3.4) is impulse-free.

Definition 3.2. Given a positive definite matrix R, for any two positive numbers l_1, l_2 then satisfy $l_1 \leq l_2$, the system (3.4) for (l_1, l_2, T, R) is stochastic finite-time admissible if

$$\begin{aligned} &\varepsilon\left\{\varsigma^T(0)E^T RE\varsigma(0)\right\} \leq l_1 \\ &\Rightarrow \varepsilon\left\{\varsigma^T(t)E^T RE\varsigma(t)\right\} \leq l_2, \forall t \in [0, T] \end{aligned}$$

Definition 3.3. [27] Given a stochastic Lyapunov function $V(x(t), t)$, where $x(t)$ satisfies the following equation which is a stochastic differential equation in a stochastic system:

$$dx(t) = f(t)dt + g(t)dw(t) \tag{3.5}$$

The definition of the weak infinitesimal operator L in the random process is given as $\{x(t), t > 0\}$:

$$\begin{aligned} &LV(x(t), t) = V(x(t), t)_t + V(x(t), t)_x f(t) \\ &+\tfrac{1}{2}tr\left[g^T(t)V_{xx}(x(t), t)g(t)\right] \end{aligned} \tag{3.6}$$

Lemma 3.4. *([28] Gronwalls inequality) Given a non-negative function $g(t)$, for any two constants m, n and they satisfy $m, n \geq 0$*

$$g(t) \leq m + n\int_0^t g(s)ds, 0 \leq t \leq T \tag{3.7}$$

then

$$g(t) \leq m\ \exp(nt) \tag{3.8}$$

Lemma 3.5. *(Schur's complement) Exist any real matrix A, B, C, among them $B^T = B$ and $C^T = C > 0$, the following three conditions are equivalent:*

$$(i)\quad B + AC^{-1}A^T < 0$$

$$(ii)\quad \begin{pmatrix} B & A \\ A^T & -C \end{pmatrix} < 0$$

$$(iii)\quad \begin{pmatrix} B & -A \\ -A^T & -C \end{pmatrix} < 0$$

Lemma 3.6. *[29](i) For two orthogonal matrices U and V, if rank (E) = r,then E can be decomposed as follows*

$$E = U \begin{bmatrix} \sum_r & 0 \\ * & 0 \end{bmatrix} V^T = U \begin{bmatrix} I_r & 0 \\ * & 0 \end{bmatrix} v^T \tag{3.9}$$

where $\sum_r = diag\{\delta_1, \delta_2, \cdots, \delta_r\}, \delta_k > 0, k = 1, 2, \cdots, r$
Partition $U = \begin{bmatrix} U_1 & U_2 \end{bmatrix}, V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}, v = \begin{bmatrix} V_1 \sum_r & V_2 \end{bmatrix}$ *with* $\sum V_2 = 0, U_2^T E = 0.$
(ii)If P satisfies

$$EP^T = PE^T \tag{3.10}$$

Then

$$\tilde{P} = U^T P v = \begin{bmatrix} p_{11} & p_{12} \\ 0 & p_{22} \end{bmatrix}. \tag{3.11}$$

We can have $P_{11} \geq 0$, $\det(P_{22}) \neq 0$ *if P is nonsingular. Furthermore, P that satisfied Eq (3.10) can be set as follows*

$$P = Ev^{-T} Y v^{-1} + UZV_2^T \tag{3.12}$$

where $Y = diag\{P_{11}, \Phi\}, Z = \begin{bmatrix} P_{11}^T & P_{22}^T \end{bmatrix}^T$*, among this* $\Phi \in \mathbb{R}^{(n-r)\times(n-r)}$ *is an arbitrary parameter matrix.*
(iii) The following equation holds if P is a nonsingular matrix, C and Φ *are two positive definite matrices, Y is a diagonal matrix in Eq (3.12), P and E satisfy Eq (3.13)*

$$P^{-1}E = E^T C^{\frac{1}{2}} S C^{\frac{1}{2}} E \tag{3.13}$$

Then the solution of Eq (3.13) can be expressed by $S = C^{-\frac{1}{2}} U Y^{-1} U^T C^{-\frac{1}{2}}$*, where S is a positive definite matrix.*

4. Main results

4.1. Finite-time stability of the singular stochastic bioeconomic system

In this section, we discuss whether the model (3.4) can be stable in a finite-time.

Theorem 4.1. *If there is a non-singular symmetric positive definite matrix Q makes any given time constant* $T > 0$ *and scalar* $\alpha > 0$ *satisfy*

$$PE^T = EP^T \geq 0 \tag{4.1}$$

$$P^{-1}E = E^T R^{\frac{1}{2}} Q R^{\frac{1}{2}} E \tag{4.2}$$

$$\lambda_{\max}(Q)\, l_1 e^{\alpha T} - l_2 \lambda_{\min}(Q) < 0 \tag{4.3}$$

and satisfy the following matrix inequalities:

$$\begin{bmatrix} A_1 P^T + P A_1^T - \alpha E P^T & \left(EBP^T\right)^T \\ EBP^T & -\tilde{Q} \end{bmatrix} < 0 \tag{4.4}$$

$$\begin{bmatrix} A_2 P^T + P A_2^T - \alpha E P^T & \left(EBP^T\right)^T \\ EBP^T & -\tilde{Q} \end{bmatrix} < 0 \tag{4.5}$$

where $\tilde{Q} = R^{-\frac{1}{2}} Q^{-1} R^{-\frac{1}{2}}$. *Then the system (3.4) is tolerable in a stochastic finite-time for* (l_1, l_2, T, R).

Proof. Introduce the following Lyapunov function

$$V(\varsigma(t), t) = \varsigma^T(t) P^{-1} E \varsigma(t) \tag{4.6}$$

Let L be the infinitesimal generator. We can get the following inequality:

$$LV(\varsigma(t), t) < \alpha V(\varsigma(t), t) \tag{4.7}$$

In the following, we can prove that the three conditions Eqs (4.7), (4.4) and (4.5) are equivalent. Applying *Itô* formula, we can get

$$LV(\varsigma(t), t) = \varsigma^T(t) \left(A_h^T P^{-T} + P^{-1} A_h + B^T P^{-1} E B\right) \varsigma(t) \tag{4.8}$$

then

$$\begin{aligned} &LV(\varsigma(t), t) - \alpha V(\varsigma(t), t) \\ &= \varsigma^T(t) \left(A_h^T P^{-T} + P^{-1} A_h + B^T P^{-1} E B - \alpha P^{-1} E\right) \varsigma(t) \end{aligned} \tag{4.9}$$

Combining condition Eq (4.2) with $\tilde{Q} = R^{-\frac{1}{2}} Q^{-1} R^{-\frac{1}{2}}$, Eq (4.9) can be transformed as

$$\begin{aligned} &LV(\varsigma(t), t) - \alpha V(\varsigma(t), t) \\ &= \varsigma^T(t) \left(A_h^T P^{-T} + P^{-1} A_h + B^T E^T \tilde{Q}^{-1} E B - \alpha P^{-1} E\right) \varsigma(t) \end{aligned} \tag{4.10}$$

Therefore, from Eq (4.10), Eq (4.7) is equivalent to

$$A_h^T P^{-T} + P^{-1} A_h + B^T E^T \tilde{Q}^{-1} E B - \alpha P^{-1} E < 0 \tag{4.11}$$

The left is multiplied by P and on the right multiplied by P^T, we get

$$P A_h^T + A_h P^T + P C^T E^T \tilde{Q}^{-1} E B P^T - \alpha E P^T < 0 \tag{4.12}$$

It can be obtained that Eq (4.12) is equivalent to Eqs (4.4) and (4.5) by using matrix factorization and Schur's complement lemma. Integrating the left and right sides of Eq (4.7) from 0 to t at the same time and then taking the expected value, we can get

$$\varepsilon\left\{V\left(\varsigma(t),t\right)\right\} < V\left(\varsigma(0),0\right) + \alpha\int_0^t \varepsilon\left\{V\left(\varsigma(s),s\right)\right\}ds \tag{4.13}$$

From Lemma 3.4, we have

$$\varepsilon\left\{V\left(\varsigma(t),t\right)\right\} < V\left(\varsigma(0),0\right)e^{\alpha t} \tag{4.14}$$

from inequalities

$$\begin{aligned}&\varepsilon\left\{V\left(\varsigma(t),t\right)\right\} = \varepsilon\left\{\varsigma^T(t)E^TR^{\frac{1}{2}}QR^{\frac{1}{2}}E\varsigma(t)\right\}\\&\geq \lambda_{\min}(Q)\,\varepsilon\left\{\varsigma^T(t)E^TRE\varsigma(t)\right\}\end{aligned} \tag{4.15}$$

and

$$\begin{aligned}&V\left(\varsigma(0),0\right)e^{\alpha t} = \varsigma^T(0)E^TR^{\frac{1}{2}}QR^{\frac{1}{2}}E\varsigma(0)e^{\alpha t}\\&\leq \lambda_{\max}(Q)\,\varsigma^T(0)E^TRE\varsigma(0)e^{\alpha t}\\&\leq \lambda_{\max}(Q)\,l_1e\end{aligned} \tag{4.16}$$

then, we get

$$\varepsilon\left\{\varsigma^T(t)E^TRE\varsigma(t)\right\} < \frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}l_1e^{\alpha T} \tag{4.17}$$

Considering condition Eq (4.3) and inequality (4.17), for the $t \in [0, T]$, have

$$\varepsilon\left\{\varsigma^T(t)E^TRE\varsigma(t)\right\} < l_2 \tag{4.18}$$

4.2. Finite-time control of singular stochastic bioeconomic system

In practice, the bio-economic system is more or less disturbed by the external environment. For example, the growth of a population can be affected by environmental factors such as the intensity of sunlight and the temperature. This section considers this random environment that affects populations as a zero-mean Gauss white noise. In order to achieve effective planning of capture strategies and maintain the sustainable development of market economy, some measures must be taken to stabilize the biological population. Therefore, control is added to the singular stochastic bioeconomic system model (2.3):

$$\begin{cases}\dot{x}_1(t) = \alpha x_2(t) - r_1x_1(t) - \beta x_1(t) - \eta x_1^2(t) - E_1x_1(t) + \alpha_1x_1(t)\xi(t) + u(t) + x_1(t)w(t),\\ \dot{x}_2(t) = \beta x_1(t) - r_2x_2(t) + \alpha_2x_2(t)\xi(t),\\ \quad 0 = E_1\left(p(t)x_1(t) - c\right) - m(t),\end{cases} \tag{4.19}$$

where $u(t)$ represents the management degree of open resources and it is a control variable.
Through the method which is similar to the T-S fuzzy method mentioned in Section 3, we get

$$Ed\varsigma(t) = \sum_{i=1}^{2} h_i(z(t))\left((A_i\varsigma(t) + Cu(t))\,dt + B\varsigma(t)\,dw(t)\right) \tag{4.20}$$

Consider the following state feedback controller:

$$u(t) = \sum_{i=1}^{2} h_i(z(t))\,G_i\varsigma(t) \tag{4.21}$$

By designing the state feedback gain G_i.
Thus, the corresponding closed-loop system can be expressed as

$$Ed\varsigma(t) = \sum_{i=1}^{2} h_i(z(t)) \sum_{j=1}^{2} h_j(z(t))\left[\left(A_i + CG_j\right)\varsigma(t)\,dt + B\varsigma(t)\,dw(t)\right] \tag{4.22}$$

where $C = [1\ 0\ 0]^T$

Next, design parameters of the fuzzy state feedback controller Eq (4.21), a new sufficient condition for the closed-loop singular stochastic bio-economic system to be stable in a finite-time is given.

Theorem 4.2. *The closed-loop system (4.22) is stochastic finite-time admissible for a state feedback controller Eq (4.21) relative to* (l_1, l_2, T, R)*, if* P *is a nonsingular matrix,* Q *is a symmetric positive definite matrix and any matrix* $Y_j, j = 1, 2$ *which can make Eqs (4.1)–(4.3) hold, and the following matrix inequalities can be satisfied:*

$$\begin{bmatrix} \gamma_{ii} & \left(EBP^T\right)^T \\ EBP^T & -\tilde{Q} \end{bmatrix} < 0, i = j, i, j = 1, 2 \tag{4.23}$$

$$\begin{bmatrix} \gamma_{ij} + \gamma_{ji} & \left(EBP^T\right)^T \\ EBP^T & -\tilde{Q} \end{bmatrix} < 0, i < j, i, j = 1, 2 \tag{4.24}$$

where $\gamma_{ij} = PA_i^T + Y_j^T C^T + A_i P^T + CY_j - \alpha EP^T$ *and* $\tilde{Q} = R^{-\frac{1}{2}} Q^{-1} R^{-\frac{1}{2}}$*, Next, we can choose the state feedback* $G_j = Y_j P^{-T}$ *which we need.*

Proof. First, regularity and impulse-free of the system can be proved. Without loss of generality, denoting

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

where $rank(E) = rank(I_r) = r \le n$.
Suppose that there are two non-singular matrices H and K, then

$$\begin{aligned} HEK &= \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \quad H\left(A_i + CG_j\right)K = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \\ HBK &= \begin{bmatrix} B_1 & B_2 \\ 0 & 0 \end{bmatrix} \quad H^{-T}PK^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \end{aligned} \tag{4.25}$$

Now, if $\left(E, A_i + CG_j\right)$ is impulse-free, we can prove the impulselessness of the system (4.22). The system (4.22) is impulse-free if

$$rank\begin{bmatrix} E & A_i + CG_j \\ 0 & E \end{bmatrix} = n + rank\,(E) \tag{4.26}$$

It can be computed from Eq (4.25) that

$$rank\begin{bmatrix} E & A_i + CG_j \\ 0 & E \end{bmatrix} = rank\begin{bmatrix} I_r & 0 & A_{i11} & A_{i12} \\ 0 & 0 & A_{i21} & A_{i22} \\ 0 & 0 & I_r & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 2r + rank\,(A_{i22}) \tag{4.27}$$

This shows Eq (4.26) is equivalent to $n = r + rank\,(A_{i22})$, then $\left(E, A_i + CG_j\right)$ is regular and impulse-free if A_{i22} is non-singular simultaneously.

Considering Theorem 4.1 and the system (4.22), the following matrix inequality can be obtained:

$$\sum_{i=1}^{2}\sum_{j=1}^{2} h_i\,(z\,(t))h_j\,(z\,(t))\,\Delta_{ij} < 0 \tag{4.28}$$

where

$$\Delta_{ij} = P\left(A_i + CG_j\right)^T + \left(A_i + CG_j\right)P^T + PC^T E^T \tilde{Q}^{-1} EBP^T - \alpha EP^T$$

Let

$$Y_j = G_j P^T$$

Then

$$\begin{aligned} &\sum_{i=1}^{2}\sum_{j=1}^{2} h_i\,(z\,(t))h_j\,(z\,(t))\left(PA_i^T + Y_j^T C + A_i P^T + CY_j\right. \\ &\left.+PC^T E^T \tilde{Q}^{-1} ECP^T - \alpha EP^T\right) < 0 \end{aligned} \tag{4.29}$$

Obviously, Eq (4.29) is equivalent to

$$\begin{aligned} &\sum_{i=1}^{2} h_i^2\,(z\,(t))\left(\gamma_{ii} + PC^T E^T \tilde{Q}^{-1} ECP^T\right) \\ &+\sum_{i=1}^{2}\sum_{j=1}^{2} h_i\,(z\,(t))h_j\,(z\,(t))\left[\left(\gamma_{ij} + \gamma_{ji} + PC^T E^T \tilde{Q}^{-1} ECP^T\right)\right] < 0 \end{aligned} \tag{4.30}$$

Using matrix decomposition and Lemma 3.5, the above inequality (4.30) is equivalent to inequalities (4.23) and (4.24). Next, by using Eqs (4.2)–(4.3) and similar proof of Theorem 4.1, the system (4.22) is stochastic finite-time admissible for (l_1, l_2, T, R).

Remark 1. By designing a fuzzy state feedback controller, the government's management of open resource development can be better expressded. The density of biological population can be controlled within a limited range and some unfavorable phenomena can be eliminated. In real life, managers should take some effective measures, such as adjusting taxes, introducing some preferential policies to stimulate the development of fisheries, reducing environmental pollution and so on. Thus, the population density can be strictly controlled and the economic benefits can be maintained steadily.

5. Numerical examples

We use the following special circumstances to prove that the results obtained are true and effective. Selection of ecological parameters based on Nile tilapia data from Lake Tanganyika in Africa.

Consider the finite-time stability of bio-economic model with white noise, we choose the ecological parameters of the appropriate unit:

$$\begin{array}{l} \alpha{=}0.4, r_1 = 0.5, \beta = 0.5, \eta = 0.1 \\ \alpha_1 = 0.1, r_2 = 0.1, \alpha_2 = 0.1, p(t) = 1 \\ E_1 = 0.8, c = 3, \xi(t) = 1, w(t) = 0.1 \end{array}$$

Then, the singular bioeconomic system can be obtained:

$$\begin{cases} \dot{x}_1(t) = 0.4x_2(t) - 0.5x_1(t) - 0.5x_1(t) - 0.1x_1^2(t) - 0.8x_1(t) + 0.1x_1(t)\xi(t) + u(t) + x_1(t)w(t), \\ \dot{x}_2(t) = 0.5x_1(t) - 0.1x_2(t) + 0.1x_2(t)\xi(t), \\ \quad 0 = 0.8\left(p(t)x_1(t) - 3\right) - m(t), \end{cases} \tag{5.1}$$

where

$$x_1(t) \in [0,5], x_2(t) \in [0,10], m(t) \in [0,5]$$

We can get the system (5.1) has an equilibrium point $p^*(5, 15, 1.225)$ when $u(t) = 0$.

For achieveing the transition from the equilibrium point to origin, a following fuzzy models can be constructed by using linear transformation (3.1):

$$\begin{aligned} E\dot{\zeta}(t) = & \begin{bmatrix} \Omega_{11} & 0.4 & 0 \\ 0.5 & -0.1 + 0.1\xi(t) & 0 \\ 0.8p(t) & 0 & -1 \end{bmatrix} \zeta(t) \\ & + \begin{bmatrix} -5.5 + 0.5\xi(t) + 5w(t) + \zeta_1(t)w(t) + u(t) \\ 1 - 1.5\xi(t) \\ 4p(t) - 3.625 \end{bmatrix}, \end{aligned} \tag{5.2}$$

where

$$\zeta_1(t) \in [-5,0], \zeta_2(t) \in [-15,-5], \zeta_3(t) \in [-1.225, 3.775]$$

$$\Omega_{11} = -2.8 - 0.1\zeta_1(t) + 0.1\xi(t)$$

we have

$$\begin{array}{l} \max z(t) = -3.3 + 0.1\xi(t) \\ \min z(t) = -2.8 + 0.1\xi(t) \end{array}$$

By using the fuzzy rules mentioned above, we can execute the fuzzy model as:

$$Ed\varsigma(t) = \sum_{i=1}^{2} h_i(z(t)) \sum_{j=1}^{2} h_j(z(t)) \left[\left(A_i + CG_j \right) \varsigma(t)\, dt + B\varsigma(t)\, dw(t) \right] \tag{5.3}$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -3.2 & 0.4 & 0 \\ 0.5 & 0 & 0 \\ 0.8 & 0 & -1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -2.7 & 0.4 & 0 \\ 0.5 & 0 & 0 \\ 0.8 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -4.5 \\ -0.5 \\ 0.375 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$$

Let

$$\alpha = 0.001, l_1 = 10000, l_2 = 10000000, T = 1000, R = I$$

Then we get

$$\begin{matrix} G_1 = \\ G_2 = \end{matrix} \begin{bmatrix} -8.6579 & 47.2591 & -0.0345 \\ -8.6221 & 47.2785 & -0.0364 \end{bmatrix}$$

Therefore, we can get that $\varepsilon\left\{\zeta^T(t)\, E^T R E\zeta(t)\right\} < 10000000$, for all $t \in [0, 1000]$.

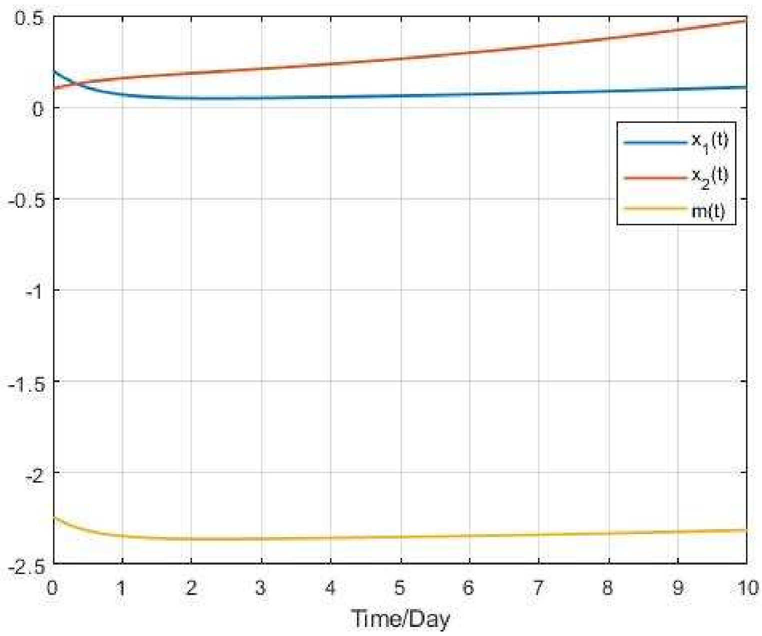

Figure 1. Trajectory of the open-loop stochastic singular system.

From Figure 1, we can see the trajectory of a stochastic singular bio-economic system which is open-loop and considers the white noise. It can be seen that the species density and average price are unstable within a limited time. The economic interests fluctuates randomly in Figure 1, on account of the economic interests is affected by the population unit price and the species density of biological economic model which proposed in this paper in a randomly disturbed environment.

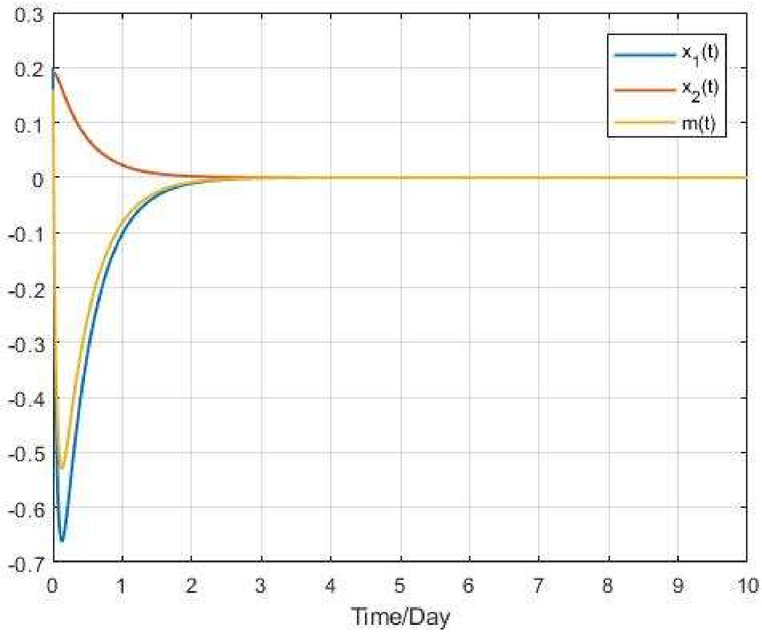

Figure 2. Trajectory of the closed-loop stochastic singular system.

From Figure 2, through the state feedback controller Eq (4.21), we can see the trajectory of a stochastic singular bio-economic system which is closed-loop and considers the white noise. It can be seen from Figure 2 that economic profits tend to be stabilize within a limited time.

6. Conclusions

In this paper, the finite time stability and control of a kind of singular bio-economic system with stochastic fluctuations which based on T-S fuzzy model are studied. Through two theorems, we derived some new sufficient conditions to guarantee the stability of system in finite time. The corresponding controller design method is also given. Finally, the effectiveness of the method is verifies through a numerical simulation. The results are also applicable to other types of systems which is similar to the system of this paper.

From a biological point of view, the biological species density can be controlled within a certain range through the fuzzy state feedback controller which designed in this paper, eliminating influence of some unfavorable factors, It can better control the density of the population and keep the economic profit stable.

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Conflict of interest

The authors declare there is no conflict of interest.

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