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Research article

Modeling the effect of literacy and social media advertisements on the dynamics of infectious diseases

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Abstract: Education empowers humans and gets them ready to face challenges of life efficiently. Literacy and social media campaigns make people aware of the tools and techniques requisite for protection against the emerging diseases. In this paper, we investigate the combined impacts of literacy and social media on the dynamics of infectious diseases spreading through direct contact. Normalized forward sensitivity indices explore the impacts of parameters on basic reproduction number. We perform global sensitivity analysis for the infective population with respect to some controllable epidemiologically important parameters. If the growth rate of broadcasting informations through social media is very high, the system shows limit cycle oscillations. On the other hand, the baseline number of social media advertisements stabilize the system by evacuating persistent oscillations and ultimately settling the system from stable endemic equilibrium to stable disease-free state. The dissemination of awareness among literate people also suppresses the prevalence of limit cycle oscillations and drives the system to disease-free zone. An extension in model is made by assuming the growth rate of social media advertisements as periodic function of time. The simulation results show that the nonautonomous system showcases periodic as well as higher periodic solutions on the increase in the growth rate of advertisements. Our results evoke that media and education play a tremendous role in mounting awareness among the population leading to elimination of disease in the society.

Keywords: epidemic model; social media advertisements; literacy; Hopf bifurcation; periodic solution; global attractivity

1. Introduction

Infectious diseases are major threats for mankind and affect the community at gross root level; posing a huge economic burden due to adverse effects on human health [1, 2]. Mathematical models are considered as one of the most effective tools to predict the transmission mechanism of various infectious diseases. In case a pandemic occurs, at the early stage of progression of infection when health care demands are not sufficient to protect the people against new emerging disease, the first-hand informations propagated through social media, TV, internet, regarding preventive measures, is an easy, fast and less expensive way to suppress the burden of disease [1, 3–6].

In contemporary times, media coverage is identified as an alternate control measure [5, 7–10]. People become alert due to media campaigns and take necessary precautions to avoid their contact with infected individuals [11–13]. The individuals who are substantially aware of transmission mechanisms, adjust their routine work, travel and pay significant attention on isolating themselves and use precautionary measures, and hence reduce their possibility of contracting the infection. Currently, the whole world including government of India is taking unprecedented steps and running various social media advertising campaigns regarding the protection against the coronavirus pandemic. They are employing different modes of propagating informations including social media, TV, radio, internet using various types of slogan such as "Stay home, Stay safe", "Be clean, Be healthy" etc.. The main motive of such social media campaigns is to stimulate people to adopt healthy sanitation practices, frequent hand washing, use of face mask, sanitizer, maintain social distancing, and disseminate awareness among the people regarding the current scenario of disease threats and their control mechanisms [14]. On the other hand, media has some negative effects as well. For example, the negative effects of media at the time of the plague outbreak in India during 1994 largely affected tourism and other businesses [15]. The epicenter of plague was Surat, Gujarat. In total, 52 people died and a large number of people left the city due to media-induced panic, but the plague lasted in two weeks.

Some studies have been conducted to understand the dynamics of infectious diseases and their prevention by considering transmission rate as a decreasing function of infected individuals due to information alerts [16–20]. Authors have also considered media as a dynamic variable and assumed that information is propagated based on the prevalence of disease [4, 21–25]. Awareness programs have the capability to reduce the epidemic threshold and thus control the spread of infection [26–28]. Misra et al. [22] have investigated the impacts of media campaigns and individuals behavioral response towards the disease by assuming that aware individuals are fully immune via protective measures during the infection period whereas Samanta et al. [21] have considered that aware individuals are also vulnerable to infection with lower transmission rate than unaware individuals. Results of these studies reveal that prior knowledge about disease threat through media coverage significantly reduces the burden of disease. To assess the impact of awareness program on cholera dynamics, Yang et al. [29] have investigated two models; in the first one they considered transmission rate as a decreasing function of number of awareness programs while in the second one, they classified the susceptible individuals into two compartments based on their prior knowledge about the disease.

Recently, Misra et al. [4] have investigated the impacts of TV and social media advertisements on the spread of infectious diseases. They considered that the growth rate of information is proportional to disease prevalence, and is a decreasing function of aware individuals. Misra and Rai [5] have assumed equal importance of broadcasting the information via TV and radio, and showed that the information broadcasted through TV, radio and social media have potential to bring behavioral changes among the individuals thereby reduces the disease burden. Misra and Rai [6] have investigated the impacts of TV and radio on the spread of influenza; their findings reveal that media coverage is effective at the early stage of epidemic outbreak. More recently, Chang et al. [3] have assessed the impact of media coverage on the spread of COVID-19 in Hubei Province, China. Continuous propagation of information through media campaigns is found to encourage the people to adopt preventive measures to combat the pandemic together.

Moreover, literacy plays a crucial role to become aware regarding the protection against the disease and change in behavior to adopt a healthy life style. Literate people have the capacity of intellectual thinking and follow all the instructions provided by the government officials to halt the spread of disease. In [30], it is reported that the odds of awareness among highly educated women and men were more than those of uneducated women and men. Besides, both women and men who regularly watch TV were more likely to be aware about AIDS compared to those who never watched TV. During the spread of severe acute respiratory syndrome (SARS) in 2002 and 2004, media coverage and education played important roles in reducing the contact rate of human beings [31]. After the media report and education, people became aware of disease threat and began to reduce their contact with others by stopping some unimportant works such as delayed some meetings, cancelled some dinner parties, schools arranged for their students to teach themselves at home on the internet etc.. These behaviors overall resulted in reduced contact with others. These results motivated us to study the effects of media and education on the dynamics of infectious diseases.

Keeping in mind the importance of media coverage and education on disease control, we extend here the study of Misra et al. [4] by dividing the unaware susceptible in two parts: Literate and illiterate susceptible. In the present study, we investigate the effects of literacy and social media advertisements on the dynamical behavior of the system and try to find out the best strategy to control the spread of the disease. Further, it may be noted that on the progress of an epidemic, government also make expenditure on the treatment of infectives, spraying chemicals for sanitation, etc., and thus reduces the expenditure on advertisements. At the same time, the cost of the advertisement also depends on the time slot and the season. Keeping these facts in mind, it is important to consider the coefficient of growth rate of advertisements as a function of time rather than a constant value. The inclusion of this variation in the parameter captures the fluctuations in the number of advertisements and make the populations of all classes to behave periodically. Positive periodic solution represents an equilibrium situation consistent with the variability of environmental conditions, and ensures that the populations survive.

The remaining portion of this article is organized in the following way: In the next section, a mathematical model is proposed for the effects of literacy and social media advertisements on the spread of infectious diseases. In section 3, we study the dynamics of disease-free and endemic equilibrium. Existence of Hobf-bifurcation is discussed by taking the growth rate of social media advertisements as bifurcation parameter. In section 4, we extend our model by assuming the growth rate of social media advertisements as a periodic function of time. Sufficient conditions are derived for global attractivity of positive periodic solution. We numerically investigate the behaviors of the autonomous and nonautonomous systems in section 5. Finally, the results are compiled and discussed in section 6.

2. The mathematical model

In a region under consideration, let N be the total human population at any instant of time $t > 0$. Depending upon the state of infection and awareness, we divide the total human population into four subpopulations namely; literate susceptible individuals X_l, illiterate susceptible individuals X_u, infected individuals Y, and aware individuals X_a. Let T be the cumulative number of social media advertisements which includes internet informations as well as TV, radio and print media, etc. Descriptions of all the dynamical variables of the considered model are given in Table 1. The urge of social media advertisements is to disseminate accurate and reliable informations about the disease, create awareness about its transmission and prevention, clear existing myths and misconceptions, and induce behavioral changes at the individual level so that the disease prevalence could be minimized. We emphasize that aware individuals are much conscious and take all precautionary measures not to contract the infection and thus are fully protected. However, on loosing the awareness due to forgetfulness or social norms, the individuals of aware class become prone to infection and thus move to susceptible class where they may contract the infection [22].

Table 1. Descriptions of variables used in the system (2.1).

Variables	Descriptions
X_l	Number of literate susceptible individuals
X_u	Number of illiterate susceptible individuals
Y	Number of infected individuals
X_a	Number of aware individuals
T	Cumulative number of social media advertisements

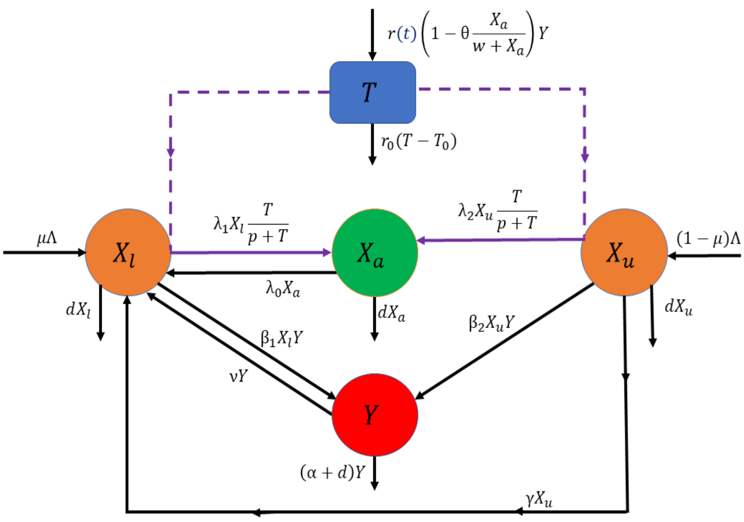

Figure 1. Schematic diagram for the systems (2.1) and (4.1). Here, purple lines represent the impacts of social media advertisements on making literate/illiterate people aware about the disease whereas blue color corresponds to the seasonal variation in the growth rate of social media advertisements.

Our model is based on the following assumptions.

(a) Individuals join the susceptible class at a constant rate Λ, a fraction μ of which join the literate class while the remaining join the illiterate susceptible class.
(b) The population is homogeneously mixed and when literate/illiterate susceptible individuals come in direct contact with infected individuals they become infected and join the infected class, following the simple law of mass action.
(c) Literate susceptible individuals adopt healthy life style during the course of infection, and hence have less possibility to be infected. Also, with the passage of time, illiterate susceptibles become literate through education campaigns at a constant rate γ.
(d) During an endemic, useful informations are broadcasted through different modes of social media. These informations are disseminated frequently to warn people about the epidemic.
(e) Thus, informations broadcasted in public do not remain constant but depend on prevalence of infection, i.e., the growth rate of informations campaigns increases in proportion to the number of infected individuals.
(f) Following [4], we assume that the growth rate of social media advertisements is a decreasing function of aware population in the region. The reason behind such consideration is the involvement of cost in broadcasting the information.
(g) As time passes, informations broadcasted in public lose their impact. Thus, depletion in cumulative number of advertisements is also incorporated in the model. Moreover, a baseline number of social media advertisements is always maintained in the region.
(h) The information broadcasted in public changes individuals' behaviors towards the disease and they (literate and illiterate individuals) avoid their contacts with infected individuals by forming a separate aware class which is fully protected from infection as they adopt all precautionary measures during the infection period [22]. However, individuals in aware class may lose awareness with the passage of time and become susceptible again at a constant rate λ_0.
(i) The informations broadcasted through social media, TV, internet etc., have limited impact on literate and illiterate susceptible individuals. Thus, literate and illiterate susceptible become aware at the rates $\lambda_1 X_l \frac{T}{p+T}$ and $\lambda_2 X_u \frac{T}{p+T}$ with λ_1 and λ_2 as dissemination rates of awareness among literate and illiterate susceptibles, respectively.

With these model assumptions, schematic diagram for the interactions among the considered dynamical variables is depicted in Figure 1, and the proposed model is given as,

$$
\begin{aligned}
\frac{dX_l}{dt} &= \mu\Lambda - \beta_1 X_l Y - \lambda_1 X_l \frac{T}{p+T} + \gamma X_u + \lambda_0 X_a + \nu Y - dX_l,\\
\frac{dX_u}{dt} &= (1-\mu)\Lambda - \beta_2 X_u Y - \lambda_2 X_u \frac{T}{p+T} - \gamma X_u - dX_u,\\
\frac{dY}{dt} &= \beta_1 X_l Y + \beta_2 X_u Y - (\nu + \alpha + d)Y,\\
\frac{dX_a}{dt} &= \lambda_1 X_l \frac{T}{p+T} + \lambda_2 X_u \frac{T}{p+T} - (\lambda_0 + d)X_a,\\
\frac{dT}{dt} &= r\left(1 - \theta\frac{X_a}{w+X_a}\right)Y - r_0(T - T_0).
\end{aligned}
\tag{2.1}
$$

Initial conditions for system (2.1) are chosen as $X_l(0) > 0$, $X_u(0) > 0$, $Y(0) \geq 0$, $X_a(0) > 0$ and $T(0) \geq T_0$. It may be pointed out that $\beta_1 < \beta_2$, $\lambda_1 > \lambda_2$ and $0 < \theta \leq 1$. Note that $\beta_1 < \beta_2$ represents the fact that the literate susceptible individuals have less probability to contract the infection than the illiterate susceptible individuals whereas $\lambda_1 > \lambda_2$ tells that the rate of dissemination of awareness among literate susceptible individuals is more in comparison to that among the illiterate susceptible individuals. All the parameters involved in system (2.1) are assumed to be positive and their descriptions are given in Table 2.

Table 2. Descriptions of parameters involved in the system (2.1) and their values taken within the range prescribed in various previous literature sources [4–6, 26, 27].

Parameters	Descriptions	Units	Values
Λ	Immigration rate in the class of literate/illiterate susceptible population	persons day^{-1}	5
μ	Fraction of newly recruited individuals joining the literate susceptible class	—	0.1
β_1	Contact rate of literate susceptible with infected individuals	person^{-1}day^{-1}	0.0000035
β_2	Contact rate of illiterate susceptible with infected individuals	person^{-1}day^{-1}	0.000004
λ_1	Dissemination rate of awareness among the literate susceptible individuals	day^{-1}	0.014
λ_2	Dissemination rate of awareness among the illiterate susceptible individuals	day^{-1}	0.008
λ_0	Rate of transfer of aware individuals to literate susceptible class	day^{-1}	0.008
p	Half saturation constant	ads.	1200
γ	Rate of transfer of illiterate susceptible individuals to literate susceptible class	day^{-1}	0.005
ν	Recovery rate of human population	day^{-1}	0.2
α	Disease-induced death rate of human population	day^{-1}	0.00001
d	Natural death rate of human population	day^{-1}	0.00004
r	Growth rate of broadcasting the information	ads. person^{-1}day^{-1}	0.01
θ	Decay in advertisements due to increase in number of aware individuals	—	0.0005
w	Half saturation constant	persons	60
r_0	Diminution rate of advertisements due to inefficiency and psychological barriers	day^{-1}	0.005
T_0	Baseline number of social media advertisements	ads.	500

Using the fact that $X_l + X_u + X_a + Y = N$, the system (2.1) reduces to following equivalent system:

$$\frac{dY}{dt} = \beta_1(N - Y - X_u - X_a)Y + \beta_2 X_u Y - (\nu + \alpha + d)Y,$$

$$\begin{aligned}
\frac{dX_u}{dt} &= (1-\mu)\Lambda - \beta_2 X_u Y - \lambda_2 X_u \frac{T}{p+T} - (\gamma + d)X_u, \\
\frac{dX_a}{dt} &= \lambda_1(N - Y - X_u - X_a)\frac{T}{p+T} + \lambda_2 X_u \frac{T}{p+T} - (\lambda_0 + d)X_a, \\
\frac{dN}{dt} &= \Lambda - dN - \alpha Y, \\
\frac{dT}{dt} &= r\left(1 - \theta\frac{X_a}{w + X_a}\right)Y - r_0(T - T_0).
\end{aligned} \tag{2.2}$$

Now onwards, we study the dynamical behavior of system (2.2).

The feasible region for system (2.2) is given in the following lemma [32, 33].

Lemma 2.1. *The region of attraction for all solutions of system (2.2) initiating in the positive orthant is given by*

$$\Omega = \left\{(Y, X_u, X_a, N, T) \in \mathbb{R}^5_+ :\ 0 \le Y, X_u, X_a \le N \le \frac{\Lambda}{d}, 0 \le\ T \le T_0 + \frac{r\Lambda}{r_0 d}\right\},$$

which is compact and invariant with respect to system (2.2). The region Ω is closed and bounded in the positive cone of the five dimensional space. Consequently, the system (2.2) is dissipative and any solution is defined for $t \ge 0$.

Proof. Fourth equation of system (2.2) yields

$$\frac{dN}{dt} = \Lambda - dN - \alpha Y \le \Lambda - dN.$$

Using a standard comparison theorem [34], we have $0 \le N(t) \le \frac{\Lambda}{d} + \left(N(0) - \frac{\Lambda}{d}\right)e^{-dt}$. Thus, as $t \to \infty$, $0 \le N(t) \le \frac{\Lambda}{d}$, we have for any $t > 0$, $0 \le N(t) \le \frac{\Lambda}{d}$. Since $X_l = N - Y - X_u - X_a \ge 0$, so $0 \le Y, X_u, X_a \le \frac{\Lambda}{d}$ for all $t \ge 0$.

Now, from the last equation of system (2.2), we have

$$\frac{dT}{dt} = r\left(1 - \theta\frac{X_a}{w + X_a}\right)Y - r_0(T - T_0) \le rY + r_0T_0 - r_0T \le \frac{r\Lambda}{d} + r_0T_0 - r_0T.$$

Following the above arguments, we can show that for any $t > 0$, $0 \le T(t) \le T_0 + \frac{r\Lambda}{r_0 d}$. Therefore, all feasible solutions of the system (2.2) enter the region Ω implying that the region is an attracting set. □

2.1. System without literacy and awareness

Ignoring the impacts of media and literacy among people, system (2.2) reduces to the following simple subsystem

$$\frac{dY}{dt} = \beta(N - Y)Y - (\nu + \alpha + d)Y,$$

$$\frac{dN}{dt} = \Lambda - dN - \alpha Y, \tag{2.3}$$

which is a simple SIS model with immigration whose dynamics is well studied [35].

System (2.3) has disease-free equilibrium $\overline{E}_0 = (0, \Lambda/d)$ and endemic equilibrium $\overline{E}^* = (\overline{Y}^*, \overline{N}^*)$, where

$$\overline{Y}^* = \frac{\beta\Lambda - d(\nu + \alpha + d)}{\beta(\alpha + d)} \quad \text{and} \quad \overline{N}^* = \frac{\beta\Lambda + \alpha(\nu + \alpha + d)}{\beta(\alpha + d)}.$$

For system (2.3), the expression of basic reproduction number is obtained as,

$$\overline{\mathcal{R}}_0 = \frac{\beta\Lambda}{d(\nu + \alpha + d)}.$$

The disease-free equilibrium $\overline{E}_0$ always exists whereas the endemic equilibrium $\overline{E}^*$ exists if $\overline{\mathcal{R}}_0 > 1$. The equilibrium $\overline{E}_0$ is stable whenever $\overline{\mathcal{R}}_0 < 1$ and unstable for $\overline{\mathcal{R}}_0 > 1$ whereas the equilibrium $\overline{E}^*$ is locally as well as globally asymptotically stable for $\overline{\mathcal{R}}_0 > 1$.

2.2. System without awareness

In the absence of social media advertisements, system (2.2) becomes

$$\begin{aligned} \frac{dY}{dt} &= \beta_1(N - Y - X_u)Y + \beta_2 X_u Y - (\nu + \alpha + d)Y, \\ \frac{dX_u}{dt} &= (1 - \mu)\Lambda - \beta_2 X_u Y - (\gamma + d)X_u, \\ \frac{dN}{dt} &= \Lambda - dN - \alpha Y. \end{aligned} \tag{2.4}$$

System (2.4) has disease-free equilibrium $\widehat{E}_0 = \left(0, \frac{(1-\mu)\Lambda}{\gamma + d}, \frac{\Lambda}{d}\right)$ and endemic equilibrium $\widehat{E}^* = (\widehat{Y}^*, \widehat{X}_u^*, \widehat{N}^*)$. The components of equilibrium $\widehat{E}^*$ are given by $\widehat{N}^* = \frac{\Lambda - \alpha\widehat{Y}^*}{d}$, $\widehat{X}_u^* = \frac{(1-\mu)\Lambda}{\beta_2\widehat{Y}^* + \gamma + d}$ and $\widehat{Y}^*$ is positive root of the following quadratic equation:

$$aY^2 + bY - c = 0, \tag{2.5}$$

where

$$a = \beta_1\beta_2(\alpha + d),\ b = \beta_1(\alpha + d)(\gamma + d) - \beta_2\{\beta_1\Lambda - d(\nu + \alpha + d)\},\ c = d(\gamma + d)(\nu + \alpha + d)(\widehat{\mathcal{R}}_0 - 1)$$

with $\widehat{\mathcal{R}}_0 = \frac{\beta_1\Lambda}{d(\nu + \alpha + d)} + \frac{(\beta_2 - \beta_1)(1-\mu)\Lambda}{(\gamma + d)(\nu + \alpha + d)}$. The quantity $\widehat{\mathcal{R}}_0$ is basic reproduction number for system (2.4). Clearly, Eq (2.5) has exactly one positive root if $\widehat{\mathcal{R}}_0 > 1$.

The disease-free equilibrium $\widehat{E}_0$ is stable whenever $\widehat{\mathcal{R}}_0 < 1$ and unstable for $\widehat{\mathcal{R}}_0 > 1$. If $\widehat{\mathcal{R}}_0 > 1$, the endemic equilibrium $\widehat{E}^*$ exists and always locally asymptotically stable. The equilibrium $\widehat{E}^*$ is globally asymptotically stable if the following condition holds:

$$\beta_2\widehat{X}_u^*(\beta_2 - \beta_1) < \beta_1(\gamma + d). \tag{2.6}$$

2.3. System without literacy

If we ignore literacy among population, then system (2.2) takes the form

$$\begin{aligned}
\frac{dY}{dt} &= \beta(N-Y-X_a)Y-(\nu+\alpha+d)Y,\\
\frac{dX_a}{dt} &= \lambda(N-Y-X_a)\frac{T}{p+T}-(\lambda_0+d)X_a,\\
\frac{dN}{dt} &= \Lambda-dN-\alpha Y,\\
\frac{dT}{dt} &= r\left(1-\theta\frac{X_a}{w+X_a}\right)Y-r_0(T-T_0)
\end{aligned} \tag{2.7}$$

whose dynamics is well studied by Misra et al. [4]. Here, we summarize the dynamics of system (2.7) as follows.

System (2.7) has disease-free equilibrium $\widetilde{E}_0=\left(0,\frac{\lambda\Lambda T_0}{d\{\lambda T_0+(\lambda_0+d)(p+T_0)\}},\frac{\Lambda}{d},T_0\right)$ and endemic equilibrium $\widetilde{E}^*=(\widetilde{Y}^*,\widetilde{X}_a^*,\widetilde{N}^*,\widetilde{T}^*)$. The components of equilibrium $\widetilde{E}^*$ are positive solutions of the equilibrium equations of system (2.7). The expression for basic reproduction number is obtained as

$$\widetilde{\mathcal{R}}_0=\frac{\beta\Lambda}{d(\nu+\alpha+d)}\frac{(\lambda_0+d)(p+T_0)}{\lambda T_0+(\lambda_0+d)(p+T_0)}.$$

The equilibrium $\widetilde{E}_0$ is stable whenever $\widetilde{\mathcal{R}}_0<1$ and unstable for $\widetilde{\mathcal{R}}_0>1$ whereas the equilibrium $\widetilde{E}^*$ exists if $\widetilde{\mathcal{R}}_0>1$ and locally asymptotically stable if $\widetilde{A}_3(\widetilde{A}_1\widetilde{A}_2-\widetilde{A}_3)-\widetilde{A}_1^2\widetilde{A}_4>0$, where

$$\begin{aligned}
\widetilde{A}_1 &= \widetilde{a}_{21}+\lambda_0+2d+r_0+\beta\widetilde{Y}^*,\\
\widetilde{A}_2 &= r_0(\widetilde{a}_{21}+\lambda_0+d)+\widetilde{a}_{24}\frac{r\theta w\widetilde{Y}^*}{(w+\widetilde{X}_a^*)^2}+\beta\widetilde{Y}^*(\lambda_0+r_0+d)+\alpha\beta\widetilde{Y}^*+d(\widetilde{a}_{21}+\lambda_0+r_0+d)+d\beta\widetilde{Y}^*,\\
\widetilde{A}_3 &= \beta\widetilde{Y}^*r_0(\lambda_0+d)+\beta\widetilde{Y}^*\widetilde{a}_{24}\frac{r\theta w\widetilde{Y}^*}{(w+\widetilde{X}_a^*)^2}+\beta\widetilde{Y}^*\widetilde{a}_{24}r\left(1-\theta\frac{\widetilde{X}_a^*}{w+\widetilde{X}_a^*}\right)+\alpha\beta\widetilde{Y}^*(\lambda_0+r_0+d)\\
&\quad +dr_0(\widetilde{a}_{21}+\lambda_0+d)+d\widetilde{a}_{24}\frac{r\theta w\widetilde{Y}^*}{(w+\widetilde{X}_a^*)^2}+\beta d\widetilde{Y}^*(\lambda_0+r_0+d),\\
\widetilde{A}_4 &= \beta d\widetilde{Y}^*r_0(\lambda_0+d)+d\beta\widetilde{Y}^*\widetilde{a}_{24}\frac{r\theta w\widetilde{Y}^*}{(w+\widetilde{X}_a^*)^2}+d\beta\widetilde{Y}^*\widetilde{a}_{24}r\left(1-\theta\frac{\widetilde{X}_a^*}{w+\widetilde{X}_a^*}\right)+\alpha\beta\widetilde{Y}^*r_0(\lambda_0+d)\\
&\quad +\alpha\beta\widetilde{a}_{24}\widetilde{Y}^*\frac{r\theta w\widetilde{Y}^*}{(w+\widetilde{X}_a^*)^2}
\end{aligned}$$

with

$$\widetilde{a}_{21}=\frac{\lambda\widetilde{T}^*}{p+\widetilde{T}^*},\ \widetilde{a}_{24}=\frac{\lambda p(\widetilde{N}^*-\widetilde{Y}^*-\widetilde{X}_a^*)}{(p+\widetilde{T}^*)^2}.$$

It is shown that the increment in growth rate of social media advertisements destabilizes the system and periodic oscillations arise through Hopf-bifurcation. Endemic equilibrium of the system changes its stability from stable to unstable to stable state as the dissemination rate of awareness among susceptible individuals due to popularity of new advertisements increases and the large values of dissemination rate can ceases the feasibility of endemic equilibrium, the system settles to stable disease-free state. The broadcasting of advertisements through TV and social media regarding the spread of infectious diseases are found to have the potential to bring behavioral changes among the people and control the spread of diseases. Further, if the dissemination rate of awareness among the susceptible individuals is fast enough then disease can be effectively controlled in the population.

3. Mathematical analysis of system (2.2)

3.1. Disease-free equilibrium and its stability

For system (2.2), the disease-free equilibrium is $E_0 = \left(0, X_{u_0}, X_{a_0}, \frac{\Lambda}{d}, T_0\right)$, where

$$\begin{aligned} X_{u_0} &= \frac{(1-\mu)\Lambda(p+T_0)}{\lambda_2 T_0 + (\gamma+d)(p+T_0)}, \\ X_{a_0} &= \frac{\lambda_1 T_0 \Lambda\{\lambda_2 T_0 + (\gamma+\mu d)(p+T_0)\} + d(1-\mu)\Lambda\lambda_2 T_0(p+T_0)}{d\{\lambda_1 T_0 + (\lambda_0+d)(p+T_0)\}\{\lambda_2 T_0 + (\gamma+d)(p+T_0)\}}. \end{aligned}$$

The equilibrium E_0 always exists in the system.

3.1.1. Basic reproduction number

The basic reproduction number ($\mathcal{R}_0$), an index worldwide commonly used by public health organizations as a key estimator of the severity of a given epidemic. The new infection terms and transition terms of the system (2.2) are respectively given by

$$\mathscr{F} = \begin{bmatrix} \beta_1(N-Y-X_u-X_a)Y + \beta_2 X_u Y \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathscr{V} = \begin{bmatrix} (\nu+\alpha+d)Y \\ -(1-\mu)\Lambda + \beta_2 X_u Y + \lambda_2 X_u \frac{T}{p+T} + (\gamma+d)X_u \\ -\lambda_1(N-Y-X_u-X_a)\frac{T}{p+T} - \lambda_2 X_u \frac{T}{p+T} + (\lambda_0+d)X_a \\ -\Lambda + dN + \alpha Y \\ -r\left(1 - \frac{\theta X_a}{w+X_a}\right)Y + r_0(T-T_0) \end{bmatrix}.$$

Using next-generation operator method [36], we determine the expression for basic reproduction number. For this, we find the matrices F (of new infection terms) and V (of the transition terms) as

$$F = \begin{bmatrix} F_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} V_{11} & 0 & 0 & 0 & 0 \\ V_{21} & V_{22} & 0 & 0 & V_{25} \\ V_{31} & V_{32} & V_{33} & V_{34} & V_{35} \\ V_{41} & 0 & 0 & V_{44} & 0 \\ V_{51} & 0 & 0 & 0 & V_{55} \end{bmatrix},$$

where

$$F_{11} = \frac{\Lambda}{d}\left[\frac{\beta_1(p+T_0)\{\lambda_2 T_0(\lambda_0+\mu d)+(p+T_0)(\lambda_0+d)(\gamma+\mu d)\}}{\{\lambda_1 T_0+(\lambda_0+d)(p+T_0)\}\{\lambda_2 T_0+(\gamma+d)(p+T_0)\}}+\frac{\beta_2 d T_0(1-\mu)}{\lambda_2 T_0+(\gamma+d)(p+T_0)}\right],$$

$$V_{11} = \nu+\alpha+d,\ V_{21} = \beta_2 X_{u_0},\ V_{22} = \frac{\lambda_2 T_0}{p+T_0}+\gamma+d,\ V_{25} = \frac{\lambda_2 p X_{u_0}}{(p+T_0)^2},\ V_{31} = \frac{\lambda_1 T_0}{p+T_0},$$

$$V_{32} = \frac{(\lambda_1-\lambda_2)T_0}{p+T_0},\ V_{33} = \frac{\lambda_1 T_0}{p+T_0}+\lambda_0+d,\ V_{34} = -\frac{\lambda_1 T_0}{p+T_0},$$

$$V_{35} = -\left[\frac{\lambda_1 p(\Lambda/d - X_{u_0} - X_{a_0})}{(p+T_0)^2}+\frac{\lambda_2 p X_{u_0}}{(p+T_0)^2}\right],\ V_{41} = \alpha,\ V_{44} = d,\ V_{51} = -r\left(1-\frac{\theta X_{a_0}}{w+X_{a_0}}\right),$$

$$V_{55} = r_0.$$

The basic reproduction number is given by $\mathcal{R}_0 = \rho(FV^{-1})$, where ρ is the spectral radius of the next-generation matrix (FV^{-1}). Thus, from the model system (2.2), we obtain the expression for $\mathcal{R}_0$ as

$$\mathcal{R}_0 = \frac{\Lambda}{d(\nu+\alpha+d)}\left[\frac{\beta_1(p+T_0)\{\lambda_2 T_0(\lambda_0+\mu d)+(p+T_0)(\lambda_0+d)(\gamma+\mu d)\}}{\{\lambda_1 T_0+(\lambda_0+d)(p+T_0)\}\{\lambda_2 T_0+(\gamma+d)(p+T_0)\}} + \frac{\beta_2 d T_0(1-\mu)}{\lambda_2 T_0+(\gamma+d)(p+T_0)}\right]. \tag{3.1}$$

Following [36], local stability of the disease-free equilibrium E_0 can be stated as follows.

Theorem 3.1. *For system (2.2), the disease-free equilibrium E_0 is locally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.*

Proof. Jacobian matrix of system (2.2) is given by

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} & 0 \\ J_{21} & J_{22} & 0 & 0 & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\ J_{41} & 0 & 0 & J_{44} & 0 \\ J_{51} & 0 & J_{53} & 0 & J_{55} \end{pmatrix},$$

where

$$J_{11} = \beta_1(N-2Y-X_u-X_a)+\beta_2 X_u-(\nu+\alpha+d),\ J_{12} = (\beta_2-\beta_1)Y,\ J_{13} = -\beta_1 Y,$$

$$J_{14} = \beta_1 Y,\ J_{21} = -\beta_2 X_u,\ J_{22} = -\left[\beta_2 Y+\frac{\lambda_2 T}{p+T}+\gamma+d\right],\ J_{25} = -\frac{\lambda_2 p X_u}{(p+T)^2},$$

$$J_{31} = -\frac{\lambda_1 T}{p+T},\ J_{32} = -\frac{(\lambda_1-\lambda_2)T}{p+T},\ J_{33} = -\left[\frac{\lambda_1 T}{p+T} + \lambda_0 + d\right],\ J_{34} = \frac{\lambda_1 T}{p+T},$$
$$J_{35} = \frac{\lambda_1 p(N-Y-X_u-X_a)}{(p+T)^2} + \frac{\lambda_2 p X_u}{(p+T)^2},\ J_{41} = -\alpha,\ J_{44} = -d,\ J_{51} = r\left(1 - \frac{\theta X_a}{w+X_a}\right),$$
$$J_{53} = -\frac{r\theta w Y}{(w+X_a)^2},\ J_{55} = -r_0.$$

At the equilibrium E_0, the Jacobian matrix becomes

$$J_{E_0} = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & 0 & 0 & a_{44} & 0 \\ a_{51} & 0 & 0 & 0 & a_{55} \end{pmatrix},$$

where

$$a_{11} = (\nu+\alpha+d)(\mathcal{R}_0 - 1),\ a_{21} = -\beta_2 X_{u_0},\ a_{22} = -\left[\frac{\lambda_2 T_0}{p+T_0} + \gamma + d\right],\ a_{25} = -\frac{\lambda_2 p X_{u_0}}{(p+T_0)^2},$$
$$a_{31} = -\frac{\lambda_1 T_0}{p+T_0},\ a_{32} = -\frac{(\lambda_1-\lambda_2)T_0}{p+T_0},\ a_{33} = -\left[\frac{\lambda_1 T_0}{p+T_0} + \lambda_0 + d\right],\ a_{34} = \frac{\lambda_1 T_0}{p+T_0},$$
$$a_{35} = \frac{\lambda_1 p(\Lambda/d - X_{u_0} - X_{a_0})}{(p+T_0)^2} + \frac{\lambda_2 p X_{u_0}}{(p+T_0)^2},\ a_{41} = -\alpha,\ a_{44} = -d,\ a_{51} = r\left(1 - \frac{\theta X_{a_0}}{w+X_{a_0}}\right),$$
$$a_{55} = -r_0.$$

Eigenvalues of the matrix J_{E_0} are obtained as,

$$(\nu+\alpha+d)(\mathcal{R}_0 - 1),\ -\left[\frac{\lambda_2 T_0}{p+T_0} + \gamma + d\right],\ -\left[\frac{\lambda_1 T_0}{p+T_0} + \lambda_0 + d\right],\ -d,\ -r_0.$$

Clearly, the last four eigenvalues are always negative while the first one is negative if $\mathcal{R}_0 < 1$ and positive if $\mathcal{R}_0 > 1$. That is, the equilibrium E_0 is stable if $\mathcal{R}_0 < 1$ and unstable if $\mathcal{R}_0 > 1$.

□

3.2. Endemic equilibrium and its stability

For system (2.2), an endemic equilibrium is $E^*=(Y^*, X_u^*, X_a^*, N^*, T^*)$, whose components are positive solutions of equilibrium equations of the system (2.2).

From the fourth equilibrium equation of system (2.2), we have

$$N^* = \frac{\Lambda - \alpha Y^*}{d}. \tag{3.2}$$

From the fifth equilibrium equation of system (2.2), we have

$$T^* = T_0 + \frac{rY^*}{r_0}\left(1 - \frac{\theta X_a^*}{w+X_a^*}\right) = f_1(Y^*, X_a^*). \tag{3.3}$$

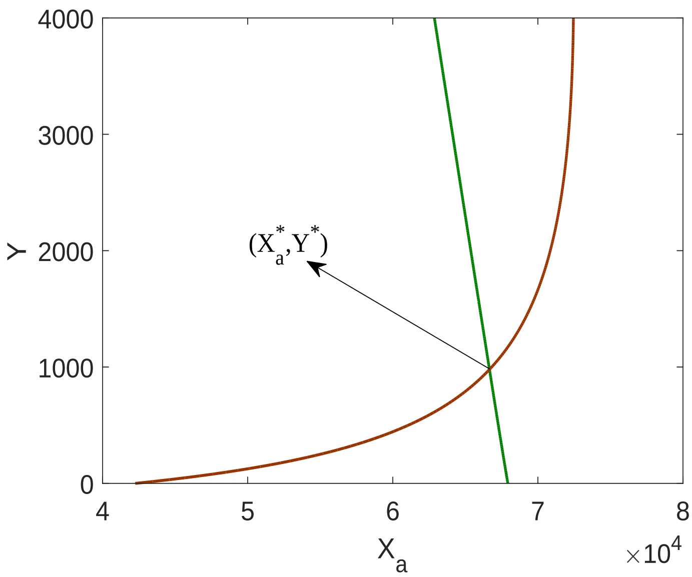

Figure 2. Intersections of isoclines (3.5) and (3.6). Here, green and red colors represent the isoclines (3.5) and (3.6), respectively. Parameters are at the same values as in Table 2.

From the second equilibrium equation of system (2.2), we have

$$X_u^* = \frac{(1-\mu)\Lambda(p+T^*)}{\lambda_2 T^* + (p+T^*)(\beta_2 Y^* + \gamma + d)}. \tag{3.4}$$

From the first equilibrium equation of system (2.2), we have

$$\beta_1 \left[\frac{\Lambda - (\alpha+d)Y}{d} - X_a \right] + \frac{(\beta_2-\beta_1)(1-\mu)\Lambda\{p+f_1(Y,X_a)\}}{\lambda_2 f_1(Y,X_a) + (\beta_2 Y + \gamma + d)\{p+f_1(Y,X_a)\}} - (\nu+\alpha+d) = 0. \tag{3.5}$$

Finally, from the third equilibrium equation of system (2.2), we have

$$\frac{f_1(Y,X_a)}{p+f_1(Y,X_a)} \left[\lambda_1 \left(\frac{\Lambda-(\alpha+d)Y}{d} - X_a \right) - \frac{(\lambda_1-\lambda_2)(1-\mu)\Lambda\{p+f_1(Y,X_a)\}}{\lambda_2 f_1(Y,X_a) + (\beta_2 Y+\gamma+d)\{p+f_1(Y,X_a)\}} \right]$$
$$-(\lambda_0+d)X_a = 0. \tag{3.6}$$

Note that Eqs (3.5) and (3.6) are two isoclines in Y and X_a. It is difficult to analyze the behaviors of isoclines (3.5) and (3.6) mathematically. To visualize numerically, in Figure 2, we plot isoclines (3.5) and (3.6) for the set of parameter values given in Table 2. It is clear from the figure that isoclines (3.5) and (3.6) intersect uniquely in the interior of positive quadrant. Let the point of intersection be (X_a^*, Y^*).

In Table 3, we listed equilibria and basic reproduction for the full system (2.2) and the corresponding subsystems (2.3), (2.4) and (2.7).

Remark 1. The quantity $\overline{\mathcal{R}}_0$ is basic reproduction number when there is no social media advertisements and also literacy is ignored among people, which captures the dynamics of simple SIS model with immigration. In the presence of social media advertisements, this basic reproduction

Table 3. Equilibria and basic reproduction for the systems (2.2)–(2.4) and (2.7).

Models	Equilibria	Basic reproduction number
(2.3)	$\overline{E}_0 = (0, \frac{\Lambda}{d})$ $\overline{E}^* = (\overline{Y}^*, \overline{N}^*)$	$\overline{\mathcal{R}}_0 = \frac{\beta\Lambda}{d(\nu+\alpha+d)}$
(2.4)	$\widehat{E}_0 = \left(0, \frac{(1-\mu)\Lambda}{\gamma+d}, \frac{\Lambda}{d}\right)$ $\widehat{E}^* = (\widehat{Y}^*, \widehat{X}_u^*, \widehat{N}^*)$	$\widehat{\mathcal{R}}_0 = \frac{\beta_1\Lambda}{d(\nu+\alpha+d)} + \frac{(\beta_2-\beta_1)(1-\mu)\Lambda}{(\gamma+d)(\nu+\alpha+d)}$
(2.7)	$\widetilde{E}_0 = \left(0, \frac{\lambda\Lambda T_0}{d\{\lambda T_0+(\lambda_0+d)(p+T_0)\}}, \frac{\Lambda}{d}, T_0\right)$ $\widetilde{E}^* = (\widetilde{Y}^*, \widetilde{X}_a^*, \widetilde{N}^*, \widetilde{T}^*)$	$\widetilde{\mathcal{R}}_0 = \frac{\beta\Lambda}{d(\nu+\alpha+d)} \frac{(\lambda_0+d)(p+T_0)}{\lambda T_0+(\lambda_0+d)(p+T_0)}$
(2.2)	$E_0 = \left(0, X_{u_0}, X_{a_0}, \frac{\Lambda}{d}, T_0\right)$ $E^*=(Y^*, X_u^*, X_a^*, N^*, T^*)$	$\mathcal{R}_0 = \frac{\Lambda}{d(\nu+\alpha+d)}\left[\frac{\beta_1(p+T_0)\{\lambda_2T_0(\lambda_0+\mu d)+(p+T_0)(\lambda_0+d)(\gamma+\mu d)\}}{\{\lambda_1T_0+(\lambda_0+d)(p+T_0)\}\{\lambda_2T_0+(\gamma+d)(p+T_0)\}} + \frac{\beta_2 dT_0(1-\mu)}{\lambda_2T_0+(\gamma+d)(p+T_0)}\right]$

number is modified and becomes $\widetilde{\mathcal{R}}_0$. It can be easily observed that $\widetilde{\mathcal{R}}_0 < \overline{\mathcal{R}}_0$, showing the impact of broadcasting information through social media advertisements on epidemic threshold. The quantity $\widehat{\mathcal{R}}_0$ represents the basic reproduction number in the presence of educational campaigns. From the expression of $\widehat{\mathcal{R}}_0$, it is inferred that the epidemic threshold decreases in the presence of education campaigns indicating the role of education campaigns on controlling the epidemic outbreak. From the expression of $\mathcal{R}_0$, the role of dissemination of awareness among literate people on the disease prevalence is evident. It is to be noted that by increasing the rate of dissemination of awareness among literate people, the value of $\mathcal{R}_0$ decreases. To explore the role of model parameters on $\mathcal{R}_0$, we will perform later sensitivity analysis of $\mathcal{R}_0$ with respect to each parameter.

Regarding local stability of the endemic equilibrium E^*, we have the following theorem.

Theorem 3.2. *The endemic equilibrium E^*, if feasible, is locally asymptotically stable if and only if the following conditions hold:*

$$\begin{aligned} &A_5 > 0,\ A_1A_2 - A_3 > 0,\ A_3(A_1A_2 - A_3) - A_1(A_1A_4 - A_5) > 0, \\ &A_4\{A_3(A_1A_2 - A_3) - A_1(A_1A_4 - A_5)\} - A_5\{A_2(A_1A_2 - A_3) - (A_1A_4 - A_5)\} > 0, \end{aligned} \tag{3.7}$$

where A_i's ($i = 1$–5) are defined in the proof.

Proof. Evaluating the Jacobian matrix J at the endemic equilibrium E^*, we get

$$J_{E^*} = \begin{pmatrix} -\beta_1Y^* & (\beta_2-\beta_1)Y^* & -\beta_1Y^* & \beta_1Y^* & 0 \\ -\beta_2X_u^* & -a_{22} & 0 & 0 & -a_{25} \\ -a_{31} & -(a_{31}-a_{32}) & -(a_{31}+\lambda_0+d) & a_{31} & a_{35} \\ -\alpha & 0 & 0 & -d & 0 \\ a_{51} & 0 & -a_{53} & 0 & -r_0 \end{pmatrix},$$

where

$$a_{22} = \beta_2Y^* + \frac{\lambda_2T^*}{p+T^*} + \gamma + d,\ a_{25} = \frac{\lambda_2pX_u^*}{(p+T^*)^2},\ a_{31} = \frac{\lambda_1T^*}{p+T^*},\ a_{32} = \frac{\lambda_2T^*}{p+T^*},$$

$$a_{35} = \frac{\lambda_1 p(N^* - Y^* - X_u^* - X_a^*)}{(p+T^*)^2} + \frac{\lambda_2 p X_u^*}{(p+T^*)^2},\ a_{51} = r\left(1 - \theta\frac{X_a^*}{w+X_a^*}\right),\ a_{53} = \frac{r\theta w Y^*}{(w+X_a^*)^2}.$$

The associated characteristic equation is obtained as

$$\xi^5 + A_1\xi^4 + A_2\xi^3 + A_3\xi^2 + A_4\xi + A_5 = 0, \tag{3.8}$$

where

$$A_1 = \beta_1 Y^* + a_{22} + a_{31} + \lambda_0 + r_0 + 2d,$$

$$\begin{aligned} A_2 \;=\;& r_0(a_{31} + \lambda_0 + d) + a_{35}a_{53} + a_{22}(a_{31} + \lambda_0 + r_0 + d) + \beta_1 Y^*(a_{22} + \lambda_0 + r_0 + d) \\ &+ (\beta_2 - \beta_1)\beta_2 Y^* X_u^* + d(\beta_1 Y^* + a_{22} + a_{31} + \lambda_0 + r_0 + d) + \alpha\beta_1 Y^*, \end{aligned}$$

$$\begin{aligned} A_3 \;=\;& a_{22}\{r_0(a_{31} + \lambda_0 + d) + a_{35}a_{53}\} + a_{25}a_{53}(a_{31} - a_{32}) \\ &+ \beta_1 Y^*\{r_0(\lambda_0 + d) + a_{35}a_{53} + a_{22}(\lambda_0 + r_0 + d)\} \\ &+ (\beta_2 - \beta_1)Y^*\{\beta_2 X_u^*(a_{31} + \lambda_0 + r_0 + d) + a_{25}a_{51}\} + \beta_1\beta_2 Y^* X_u^*(a_{31} - a_{32}) + \beta_1 Y^* a_{35}a_{51} \\ &+ d\{r_0(a_{31} + \lambda_0 + d) + a_{35}a_{53} + a_{22}(a_{31} + \lambda_0 + r_0 + d) + \beta_1 Y^*(a_{22} + \lambda_0 + r_0 + d) \\ &+ (\beta_2 - \beta_1)\beta_2 Y^* X_u^*\} + \alpha\beta_1 Y^*(\lambda_0 + r_0 + d) + \alpha\beta_1 Y^* a_{22}, \end{aligned}$$

$$\begin{aligned} A_4 \;=\;& \beta_1 Y^*[a_{22}\{r_0(\lambda_0 + d) + a_{35}a_{53}\} + a_{25}a_{53}(a_{31} - a_{32})] \\ &+ (\beta_2 - \beta_1)Y^*[\beta_2 X_u^*\{r_0(a_{31} + \lambda_0 + d) + a_{35}a_{53}\} + a_{25}a_{31}a_{53} + a_{25}a_{51}(a_{31} + \lambda_0 + d)] \\ &+ \beta_1\beta_2 Y^* X_u^* r_0(a_{31} - a_{32}) + \beta_1 Y^*\{a_{22}a_{35}a_{51} + a_{25}a_{51}(a_{31} - a_{32})\} \\ &+ d[a_{22}\{r_0(a_{31} + \lambda_0 + d) + a_{35}a_{53}\} + a_{25}a_{53}(a_{31} - a_{32}) \\ &+ \beta_1 Y^*\{r_0(\lambda_0 + d) + a_{35}a_{53} + a_{22}(\lambda_0 + r_0 + d)\} \\ &+ (\beta_2 - \beta_1)Y^*\{\beta_2 X_u^*(a_{31} + \lambda_0 + r_0 + d) + a_{25}a_{51}\} \\ &+ \beta_1\beta_2 Y^* X_u^*(a_{31} - a_{32}) + \beta_1 Y^* a_{35}a_{51}] \\ &+ \alpha\beta_1 Y^*\{r_0(\lambda_0 + d) + a_{35}a_{53}\} + \alpha a_{22}\beta_1 Y^*(\lambda_0 + r_0 + d), \end{aligned}$$

$$\begin{aligned} A_5 \;=\;& d\beta_1 Y^*[a_{22}\{r_0(\lambda_0 + d) + a_{35}a_{53}\} + a_{25}a_{53}(a_{31} - a_{32})] \\ &+ dY^*(\beta_2 - \beta_1)[\beta_2 X_u^*\{r_0(a_{31} + \lambda_0 + d) + a_{35}a_{53}\} + a_{25}a_{31}a_{53} + a_{25}a_{51}(a_{31} + \lambda_0 + d)] \\ &+ d\beta_1\beta_2 Y^* X_u^* r_0(a_{31} - a_{32}) + d\beta_1 Y^*\{a_{22}a_{35}a_{51} + a_{25}a_{51}(a_{31} - a_{32})\} \\ &+ \alpha\beta_1 Y^* a_{22}\{r_0(\lambda_0 + d) + a_{35}a_{53}\} + \alpha a_{25}a_{53}Y^*\{(\beta_2 - \beta_1)a_{31} + \beta_1(a_{31} - a_{32})\}. \end{aligned}$$

Employing Routh-Hurwitz criterion, roots of the Eq (3.8) are either negative or with negative real parts if and only if the conditions stated in (3.7) are satisfied.

□

3.3. Global stability of endemic equilibrium

Regarding global stability of the endemic equilibrium E^*, we have the following theorem.

Theorem 3.3. *The endemic equilibrium E^*, if feasible, is globally asymptotically stable inside the region of attraction Ω if the following conditions are satisfied:*

$$\left[\frac{\lambda_2 \Lambda r}{d r_0 (p+T^*)}\left(1-\theta\frac{X_a^*}{w+X_a^*}\right)\right]^2 < \frac{\beta_1\beta_2 X_u^*}{6(\beta_2-\beta_1)}\left(\frac{\lambda_2 T^*}{p+T^*}+\gamma+d\right), \tag{3.9}$$

$$\left[\frac{\lambda_1 T^*}{\lambda_1 T^*+(\lambda_0+d)(p+T^*)}\right]^2 \max\left\{\frac{3\beta_1}{2}, \frac{2m_4}{r_0}\left(\frac{r\theta\Lambda}{d(w+X_a^*)}\right)^2\right\} < \frac{1}{9}\min\left\{\frac{2\beta_1}{3}, \frac{\lambda_1^2}{(\lambda_1-\lambda_2)^2}\left(\frac{\lambda_2 T^*}{p+T^*}+\gamma+d\right), \frac{2\beta_1 d}{\alpha}, \frac{m_4 r_0 d^2}{2\Lambda^2}\left(\frac{\lambda_1 T^*}{\lambda_1+\lambda_2}\right)^2\right\}, \tag{3.10}$$

where m_4 is defined in the proof.

Proof. To establish the global stability of the endemic equilibrium E^*, we consider the following positive definite function:

$$\begin{aligned} V &= \left[Y-Y^*-Y^*\ln\left(\frac{Y}{Y^*}\right)\right]+\frac{m_1}{2}(X_u-X_u^*)^2+\frac{m_2}{2}(X_a-X_a^*)^2 \\ &\quad +\frac{m_3}{2}(N-N^*)^2+\frac{m_4}{2}(T-T^*)^2, \end{aligned} \tag{3.11}$$

where m_1, m_2, m_3 and m_4 are positive constants to be chosen appropriately.

Calculating the time derivative of V along the solution of model system (2.2), and choosing $m_1 = \frac{\beta_2-\beta_1}{\beta_2 X_u^*}$ and $m_3 = \frac{\beta_1}{\alpha}$, we get

$$\begin{aligned} \frac{dV}{dt} &= -\beta_1(Y-Y^*)^2-\frac{\beta_2-\beta_1}{\beta_2 X_u^*}\left[\beta_2 Y+\frac{\lambda_2 T^*}{p+T^*}+\gamma+d\right](X_u-X_u^*)^2 \\ &\quad -m_2\left[\frac{\lambda_1 T^*}{p+T^*}+\lambda_0+d\right](X_a-X_a^*)^2-\frac{\beta_1 d}{\alpha}(N-N^*)^2-m_4 r_0(T-T^*)^2 \\ &\quad -\beta_1(Y-Y^*)(X_a-X_a^*)-m_2\frac{\lambda_1 T^*}{p+T^*}(Y-Y^*)(X_a-X_a^*) \\ &\quad +m_4 r\left(1-\theta\frac{X_a^*}{w+X_a^*}\right)(Y-Y^*)(T-T^*) \\ &\quad -m_2\frac{(\lambda_1-\lambda_2)T^*}{p+T^*}(X_u-X_u^*)(X_a-X_a^*)-\frac{\beta_2-\beta_1}{\beta_2 X_u^*}\frac{\lambda_2 p X_u}{(p+T)(p+T^*)}(X_u-X_u^*)(T-T^*) \\ &\quad +m_2\frac{\lambda_1 T^*}{p+T^*}(X_a-X_a^*)(N-N^*)+m_2\frac{\lambda_1 p(N-Y-X_u-X_a)+\lambda_2 p X_u}{(p+T)(p+T^*)}(X_a-X_a^*)(T-T^*) \\ &\quad -m_4\frac{r\theta w Y}{(w+X_a)(w+X_a^*)}(X_a-X_a^*)(T-T^*). \end{aligned}$$

Thus, $\frac{dV}{dt}$ will be negative definite inside the region of attraction Ω provided the following conditions are satisfied:

$$\beta_1 < \frac{2m_2}{9}\left(\frac{\lambda_1 T^*}{p+T^*} + \lambda_0 + d\right), \tag{3.12}$$

$$m_2\left[\frac{\lambda_1 T^*}{p+T^*}\right]^2 < \frac{2\beta_1}{9}\left(\frac{\lambda_1 T^*}{p+T^*} + \lambda_0 + d\right), \tag{3.13}$$

$$m_4 r^2\left[1 - \theta\frac{X_a^*}{w+X_a^*}\right]^2 < \frac{\beta_1 r_0}{3}, \tag{3.14}$$

$$m_2\left[\frac{(\lambda_1-\lambda_2)T^*}{p+T^*}\right]^2 < \frac{1}{3}\left(\frac{\lambda_1 T^*}{p+T^*} + \lambda_0 + d\right)\left(\frac{\lambda_2 T^*}{p+T^*} + \gamma + d\right), \tag{3.15}$$

$$\frac{\beta_2-\beta_1}{\beta_2 X_u^*}\left[\frac{\lambda_2\Lambda}{d(p+T^*)}\right]^2 < \frac{m_4 r_0}{2}\left(\frac{\lambda_2 T^*}{p+T^*} + \gamma + d\right), \tag{3.16}$$

$$m_2\left[\frac{\lambda_1 T^*}{p+T^*}\right]^2 < \frac{2\beta_1 d}{3\alpha}\left(\frac{\lambda_1 T^*}{p+T^*} + \lambda_0 + d\right), \tag{3.17}$$

$$m_2\left[\frac{\Lambda(\lambda_1+\lambda_2)}{d(p+T^*)}\right]^2 < \frac{m_4 r_0}{6}\left(\frac{\lambda_1 T^*}{p+T^*} + \lambda_0 + d\right), \tag{3.18}$$

$$m_4\left[\frac{r\theta\Lambda}{d(w+X_a^*)}\right]^2 < \frac{m_2 r_0}{6}\left(\frac{\lambda_1 T^*}{p+T^*} + \lambda_0 + d\right). \tag{3.19}$$

From inequalities (3.14) and (3.16), we can choose a positive value of m_4 if condition (3.9) holds. Now, from inequalities (3.12), (3.13), (3.15)–(3.19), we can choose a positive value of m_2 provided condition (3.10) holds.

□

Remark 2. Conditions of Theorem 3.3 indicate that the growth rate of social media advertisements may destabilize the system, i.e., on increasing the value of r, conditions stated in (3.7) for local stability of the equilibrium E^* may be violated and the equilibrium E^* may lose its stability. Thus, there is a possibility of occurrence of Hopf bifurcation which leads to existence of limit cycle oscillations around the equilibrium E^* as the parameter r passes through its critical value from below. The stabilizing/destabilizing roles of dissemination rates of awareness among literate/illiterate susceptible individuals and the baseline number of social media advertisements are not clear from the inequalities (3.9) and (3.10). Numerically, we will check which parameters in system (2.2) have capability to alter the stability behavior of the system.

3.4. Existence of Hopf-bifurcation

In this section, we investigate for the possibility of Hopf bifurcation from the endemic equilibrium E^* by taking the growth rate of broadcasting the information through social media, r, as a bifurcation parameter, keeping other parameters fixed. In this regard, we have the following theorem.

Theorem 3.4. *When the growth rate of broadcasting the information through social media, r, exceeds a critical value, r^*, the system (2.2) enters into Hopf bifurcation around the endemic equilibrium E^* if the following necessary and sufficient conditions are satisfied:*

$$(a)\ \psi(r^*) \equiv \{A_3(r^*) - A_1(r^*)A_2(r^*)\}\{A_5(r^*)A_2(r^*) - A_3(r^*)A_4(r^*)\} - \{A_5(r^*) - A_1(r^*)A_4(r^*)\}^2 = 0,$$

$$(b)\ A_1(r^*) > 0,\ A_1(r^*)A_2(r^*) - A_3(r^*) > 0,\ A_3(r^*) - A_1(r^*)\omega_0^* > 0,\quad \omega_0^* = \frac{A_5(r^*) - A_1(r^*)A_4(r^*)}{A_3(r^*) - A_1(r^*)A_2(r^*)} > 0,$$

$$(c)\ \left.\frac{d\psi(r)}{dr}\right|_{r=r^*} \neq 0.$$

Proof. The characteristic polynomial (3.8) has a pair of purely imaginary roots $\xi_{1,2} = \pm i\sqrt{\omega_0}$, $\omega_0 > 0$ if and only if it can be written as

$$p(\xi) = (\xi^2 + \omega_0)g(\xi),\ g(\xi) = \xi^3 + B_1\xi^2 + B_2\xi + B_3. \tag{3.20}$$

Thus, we have

$$p(\xi) = \xi^5 + B_1\xi^4 + (B_2 + \omega_0)\xi^3 + (B_3 + B_1\omega_0)\xi^2 + B_2\omega_0\xi + B_3\omega_0. \tag{3.21}$$

Equating the coefficients of Eqs (3.8) and (3.21), we get

$$A_1 = B_1,\ A_2 = B_2 + \omega_0,\ A_3 = B_3 + B_1\omega_0,\ A_4 = B_2\omega_0,\ A_5 = B_3\omega_0. \tag{3.22}$$

For the consistence of the above relations, we have

$$\omega_0^2 - A_2\omega_0 + A_4 = 0,\ A_1\omega_0^2 - A_3\omega_0 + A_5 = 0. \tag{3.23}$$

The elimination of ω_0^2 gives

$$(A_3 - A_1A_2)\omega_0 = A_5 - A_1A_4. \tag{3.24}$$

Thus, Eq (3.8) can be written as

$$p(\xi) = \xi^5 + A_1\xi^4 + A_2\xi^3 + A_3\xi^2 + \omega_0(A_2 - \omega_0)\xi + \omega_0(A_3 - A_1\omega_0). \tag{3.25}$$

If $(A_3 - A_1A_2)(A_5 - A_1A_4) > 0$, then from Eq (3.24), we have

$$\omega_0 = \omega_0^* = \frac{A_5 - A_1A_4}{A_3 - A_1A_2} > 0. \tag{3.26}$$

Substituting $\omega_0 = \omega_0^*$ in Eq (3.25), we find that Eqs (3.8) and (3.25) are identical if and only if

$$\psi = (A_3 - A_1A_2)(A_5A_2 - A_3A_4) - (A_5 - A_1A_4)^2 = 0. \tag{3.27}$$

Now, the necessary and sufficient condition under which the polynomial

$$g(\xi) = \xi^3 + A_1\xi^2 + (A_2 - \omega_0)\xi + A_3 - A_1\omega_0 = 0 \tag{3.28}$$

does not have zero roots is

$$A_3 - A_1\omega_0 \neq 0. \tag{3.29}$$

The polynomial $g(\xi)$ has all roots with negative real parts if and only if all leading principal minors of the matrix

$$\begin{pmatrix} B_1 & B_3 & 0 \\ 1 & B_2 & 0 \\ 0 & B_1 & B_3 \end{pmatrix} = \begin{pmatrix} A_1 & A_3 - A_1\omega_0 & 0 \\ 1 & A_2 - \omega_0 & 0 \\ 0 & A_1 & A_3 - A_1\omega_0 \end{pmatrix} \tag{3.30}$$

are positive (Routh-Hurwitz conditions for stability [37]). The positivity of the determinants lead to the following conditions

$$A_1 > 0,\ A_1A_2 - A_3 > 0,\ A_3 - A_1\omega_0 > 0. \tag{3.31}$$

To complete the discussion, it remains to verify the transversality condition. The function $\psi(r)$ can be expressed in the form of Orlando's formula as follows:

$$\begin{aligned}\psi(r) = (\xi_1 + \xi_2)(\xi_1 + \xi_3)(\xi_1 + \xi_4)(\xi_1 + \xi_5)(\xi_2 + \xi_3)(\xi_2 + \xi_4)(\xi_2 + \xi_5)\\ (\xi_3 + \xi_4)(\xi_3 + \xi_5)(\xi_4 + \xi_5).\end{aligned} \tag{3.32}$$

As $\psi(r^*)$ is a continuous function of all its roots, there exists an open interval $I_{r^*} = (r^* - \epsilon, r^* + \epsilon)$, where ξ_1 and ξ_2 are complex conjugates for all $r \in I_{r^*}$. Let their general forms in this neighborhood be $\xi_1(r) = \phi_1(r) + i\phi_2(r)$, $\xi_2(r) = \phi_1(r) - i\phi_2(r)$ with $\phi_1(r^*) = 0$, $\phi_2(r^*) = \sqrt{\omega_0} > 0$ while $\mathrm{Re}(\xi_{3,4,5}(r^*)) \neq 0$. Then, we have

$$\begin{aligned}&\psi(r) = 2\phi_1\{(\xi_3 + \phi_1)^2 + \phi_2^2\}\{(\xi_4 + \phi_1)^2 + \phi_2^2\}\{(\xi_5 + \phi_1)^2 + \phi_2^2\}(\xi_3 + \xi_4)(\xi_3 + \xi_5)(\xi_4 + \xi_5),\\ &\psi(r^*) = 0.\end{aligned}$$

Differentiating with respect to r and putting $r = r^*$, we obtain

$$\left[\frac{d\psi(r)}{dr}\right]_{r=r^*} = \left[2(\phi_2^2 + \xi_3^2)(\phi_2^2 + \xi_4^2)(\phi_2^2 + \xi_5^2)(\xi_3 + \xi_4)(\xi_3 + \xi_5)(\xi_4 + \xi_5)\frac{d\phi_1(r)}{dr}\right]_{r=r^*}. \tag{3.33}$$

Since the roots $\xi_{3,4,5}$ have negative real parts at $r = r^*$, therefore

$$\left[\frac{d\phi_1(r)}{dr}\right]_{r=r^*} \neq 0 \iff \left[\frac{d\psi(r)}{dr}\right]_{r=r^*} \neq 0. \tag{3.34}$$

Thus, the transversality condition holds and hence the claim.

□

4. Seasonally forced model

Social media advertisements are increasingly used worldwide to control the prevalence of disease and are viable options to reduce the disease transmission. However, the cost involved in the advertisements through social media varies with time and season. Therefore, it is reasonable to consider the growth rate coefficient of social media advertisements as a function of time, $r(t)$, rather than a constant, r. Incorporating the seasonal variations of the social media advertisements will explore a more realistic and explicit outcome of the social media on disease outbreak. Thus, by assuming the growth rate of social media advertisements as seasonally forced, we extend our autonomous system (2.2) as,

$$\begin{aligned}
\frac{dY}{dt} &= \beta_1(N - Y - X_u - X_a)Y + \beta_2 X_u Y - (\nu + \alpha + d)Y,\\
\frac{dX_u}{dt} &= (1-\mu)\Lambda - \beta_2 X_u Y - \lambda_2 X_u \frac{T}{p+T} - (\gamma + d)X_u,\\
\frac{dX_a}{dt} &= \lambda_1(N - Y - X_u - X_a)\frac{T}{p+T} + \lambda_2 X_u \frac{T}{p+T} - (\lambda_0 + d)X_a,\\
\frac{dN}{dt} &= \Lambda - dN - \alpha Y,\\
\frac{dT}{dt} &= r(t)\left(1 - \theta\frac{X_a}{w + X_a}\right)Y - r_0(T - T_0).
\end{aligned} \tag{4.1}$$

Here, we assume that the rate parameter $r(t)$ is positive, continuous and bounded with positive lower bound. For simplicity, we neglect phase shift and simply incorporate the effect of seasonal changes by considering the periodic rate parameter, $r(t)$, with a period of one year. Denote $r^M = \max_{t>0} r(t)$ and $r^m = \min_{t>0} r(t)$.

Lemma 4.1. *Let κ be a real number and f be a nonnegative function which is integrable and uniformly continuous on $[\kappa, +\infty)$, then $\lim_{t\to+\infty} f(t) = 0$ [38].*

4.1. Global attractivity

Theorem 4.2. *If the system (4.1) has at least one positive periodic solution, the positive periodic solution is unique and globally attractive if there exist $\mu_i > 0$ $(i = 1–5)$ such that the following conditions hold:*

$$\mu_1\beta_1 - \mu_2\beta_2 - \frac{\mu_4\alpha}{e^{\rho_{44}}} - \frac{\mu_3 r^M}{e^{\rho_{55}}} - \frac{\mu_5 r^M \theta e^{\rho_3}}{e^{\rho_{55}}(w + e^{\rho_{33}})} - \frac{\mu_3\lambda_1 e^{\rho_5}}{e^{\rho_{33}}(p + e^{\rho_{55}})} > 0, \tag{4.2}$$

$$\frac{\mu_2(1-\mu)\Lambda}{e^{2\rho_2}} - \mu_1(\beta_1 + \beta_2) - \frac{\mu_3(\lambda_1 + \lambda_2)e^{\rho_5}}{e^{\rho_{33}}(p + e^{\rho_{55}})} > 0, \tag{4.3}$$

$$\frac{\mu_3 e^{\rho_{55}}(\lambda_1 e^{\rho_{44}} + \lambda_2 e^{\rho_{22}})}{e^{2\rho_3}(p + e^{\rho_5})} - \mu_1\beta_1 - \frac{\mu_3\lambda_1 e^{\rho_5}(e^{\rho_1} + e^{\rho_2})}{e^{2\rho_{33}}(p + e^{\rho_{55}})} - \frac{\mu_5 r^M \theta (w e^{\rho_1} + 2e^{\rho_1 + \rho_3})}{e^{\rho_{55}}(w + e^{\rho_{33}})^2} > 0, \tag{4.4}$$

$$\frac{\mu_4\Lambda}{e^{2\rho_4}} - \mu_1\beta_1 - \frac{\mu_4\alpha e^{\rho_1}}{e^{2\rho_{44}}} - \frac{\mu_3\lambda_1 e^{\rho_5}}{e^{\rho_{33}}(p + e^{\rho_{55}})} > 0, \tag{4.5}$$

$$\frac{\mu_2\lambda_2 p}{(p+e^{\rho_5})^2}+\frac{\mu_5(r^m e^{\rho_{11}}+r_0T_0)}{e^{2\rho_5}}-\frac{\mu_5 r^M\theta e^{\rho_1+\rho_3}}{e^{2\rho_{55}}(w+e^{\rho_{33}})}$$
$$-\frac{(p+2e^{\rho_5})\{\mu_3\lambda_1(e^{\rho_1}+e^{\rho_2}+e^{\rho_4})+\mu_3\lambda_2 e^{\rho_2}\}}{e^{\rho_{33}}(p+e^{\rho_{55}})^2}>0. \tag{4.6}$$

Proof. Let system (4.1) has at least one positive periodic solution, $(\overline{Y}(t),\overline{X}_u(t),\overline{X}_a(t),\overline{N}(t),\overline{T}(t))$. Further, let

$$e^{\rho_{11}}\le\overline{Y}(t)\le e^{\rho_1},\ e^{\rho_{22}}\le\overline{X}_u(t)\le e^{\rho_2},\ e^{\rho_{33}}\le\overline{X}_a(t)\le e^{\rho_3},\ e^{\rho_{44}}\le\overline{N}(t)\le e^{\rho_4},\ e^{\rho_{55}}\le\overline{T}(t)\le e^{\rho_5}.$$

Let $(Y(t),X_u(t),X_a(t),N(t),T(t))$ be any positive periodic solution of system (4.1).

Consider the Lyapunov functional,

$$V(t)=\mu_1|\ln Y(t)-\ln\overline{Y}(t)|+\mu_2|\ln X_u(t)-\ln\overline{X}_u(t)|+\mu_3|\ln X_a(t)-\ln\overline{X}_a(t)|$$
$$+\mu_4|\ln N(t)-\ln\overline{N}(t)|+\mu_5|\ln T(t)-\ln\overline{T}(t)|.$$

Calculating the right hand Dini's derivatives, we get

$$D^+V(t)=\mu_1\mathrm{sgn}(Y(t)-\overline{Y}(t))\left[\frac{\dot Y(t)}{Y(t)}-\frac{\dot{\overline{Y}}(t)}{\overline{Y}(t)}\right]+\mu_2\mathrm{sgn}(X_u(t)-\overline{X}_u(t))\left[\frac{\dot X_u(t)}{X_u(t)}-\frac{\dot{\overline{X}}_u(t)}{\overline{X}_u(t)}\right]$$
$$+\mu_3\mathrm{sgn}(X_a(t)-\overline{X}_a(t))\left[\frac{\dot X_a(t)}{X_a(t)}-\frac{\dot{\overline{X}}_a(t)}{\overline{X}_a(t)}\right]+\mu_4\mathrm{sgn}(N(t)-\overline{N}(t))\left[\frac{\dot N(t)}{N(t)}-\frac{\dot{\overline{N}}(t)}{\overline{N}(t)}\right]$$
$$+\mu_5\mathrm{sgn}(T(t)-\overline{T}(t))\left[\frac{\dot T(t)}{T(t)}-\frac{\dot{\overline{T}}(t)}{\overline{T}(t)}\right].$$

Now, we have

$$\mu_1\mathrm{sgn}(Y(t)-\overline{Y}(t))\left[\frac{\dot Y(t)}{Y(t)}-\frac{\dot{\overline{Y}}(t)}{\overline{Y}(t)}\right]$$
$$\le\mu_1[\beta_1\{|N(t)-\overline{N}(t)|+|Y(t)-\overline{Y}(t)|+|X_u(t)-\overline{X}_u(t)|+|X_a(t)-\overline{X}_a(t)|$$
$$+\beta_2|X_u(t)-\overline{X}_u(t)|\}];$$

$$\mu_2\mathrm{sgn}(X_u(t)-\overline{X}_u(t))\left[\frac{\dot X_u(t)}{X_u(t)}-\frac{\dot{\overline{X}}_u(t)}{\overline{X}_u(t)}\right]$$
$$\le\mu_2\left[-\frac{(1-\mu)\Lambda}{X_u(t)\overline{X}_u(t)}|X_u(t)-\overline{X}_u(t)|+\beta_2|Y(t)-\overline{Y}(t)|\right.$$
$$\left.+\frac{\lambda_2 p}{(p+T(t))(p+\overline{T}(t))}|T(t)-\overline{T}(t)|\right];$$

$$\mu_3\mathrm{sgn}(X_a(t)-\overline{X}_a(t))\left[\frac{\dot X_a(t)}{X_a(t)}-\frac{\dot{\overline{X}}_a(t)}{\overline{X}_a(t)}\right]$$

$$
\begin{aligned}
&\leq \mu_3 \Bigg[\lambda_1 \Bigg\{ \frac{T(t)}{\overline{X}_a(t)(p+T(t))} |Y(t) - \overline{Y}(t)| + \frac{T(t)}{\overline{X}_a(t)(p+T(t))} |X_u(t) - \overline{X}_u(t)| \\
&\quad - \frac{T(t)(N(t)-Y(t)-X_u(t))}{X_a(t)\overline{X}_a(t)(p+T(t))} |X_a(t) - \overline{X}_a(t)| \\
&\quad + \frac{T(t)}{\overline{X}_a(t)(p+T(t))} |N(t) - \overline{N}(t)| \\
&\quad + \frac{X_a(t)\{p(\overline{N}(t)+\overline{Y}(t)+\overline{X}_u(t)) + 2T(t)(N(t)+Y(t)+X_u(t))\}}{X_a(t)\overline{X}_a(t)(p+T(t))(p+\overline{T}(t))} |T(t) - \overline{T}(t)| \Bigg\} \\
&\quad + \frac{\lambda_1 p}{(p+T(t))(p+\overline{T}(t))} |T(t) - \overline{T}(t)| \\
&\quad + \lambda_2 \Bigg\{ - \frac{X_u(t)T(t)}{X_a(t)\overline{X}_a(t)(p+T(t))} |X_a(t) - \overline{X}_a(t)| + \frac{T(t)}{\overline{X}_a(t)(p+T(t))} |X_u(t) - \overline{X}_u(t)| \\
&\quad + \frac{X_a(t)(p\overline{X}_u(t) + 2X_u(t)T(t))}{X_a(t)\overline{X}_a(t)(p+T(t))(p+\overline{T}(t))} |T(t) - \overline{T}(t)| \Bigg\} \Bigg];
\end{aligned}
$$

$$
\begin{aligned}
&\mu_4 \mathrm{sgn}(N(t) - \overline{N}(t)) \left[\frac{\dot{N}(t)}{N(t)} - \frac{\dot{\overline{N}}(t)}{\overline{N}(t)} \right] \\
&\quad \leq \mu_4 \left[- \frac{\Lambda}{N(t)\overline{N}(t)} |N(t) - \overline{N}(t)| + \alpha \left\{ \frac{Y(t)}{N(t)\overline{N}(t)} |N(t) - \overline{N}(t)| + \frac{1}{\overline{N}(t)} |Y(t) - \overline{Y}(t)| \right\} \right];
\end{aligned}
$$

$$
\begin{aligned}
&\mu_5 \mathrm{sgn}(T(t) - \overline{T}(t)) \left[\frac{\dot{T}(t)}{T(t)} - \frac{\dot{\overline{T}}(t)}{\overline{T}(t)} \right] \\
&\quad \leq \mu_5 \Bigg[r(t) \left\{ \frac{1}{T(t)} |Y(t) - \overline{Y}(t)| - \frac{Y(t)}{T(t)\overline{T}(t)} |T(t) - \overline{T}(t)| \right\} - \frac{r_0 T_0}{T(t)\overline{T}(t)} |T(t) - \overline{T}(t)| \\
&\quad + r(t)\theta \Bigg\{ \frac{T(t)(w\overline{Y}(t) + X_a(t)\overline{Y}(t) + X_a(t)Y(t))}{T(t)\overline{T}(t)(w+X_a(t))(w+\overline{X}_a(t))} |X_a(t) - \overline{X}_a(t)| \\
&\quad + \frac{X_a(t)}{\overline{T}(t)(w+\overline{X}_a(t))} |Y(t) - \overline{Y}(t)| + \frac{X_a(t)Y(t)}{T(t)\overline{T}(t)(w+X_a(t))} |T(t) - \overline{T}(t)| \Bigg\} \Bigg].
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
D^+V(t) \quad \leq \quad & - \left[\mu_1\beta_1 - \mu_2\beta_2 - \frac{\mu_4\alpha}{N(t)} - \frac{\mu_5 r(t)}{T(t)} - \frac{\mu_5 r(t)\theta X_a(t)}{\overline{T}(t)(w+\overline{X}_a(t))} - \frac{\mu_3\lambda_1 T(t)}{\overline{X}_a(t)(p+T(t))} \right] |Y(t) - \overline{Y}(t)| \\
& - \left[-\mu_1(\beta_1+\beta_2) + \frac{\mu_2(1-\mu)\Lambda}{X_u(t)\overline{X}_u(t)} - \frac{\mu_3(\lambda_1+\lambda_2)T(t)}{\overline{X}_a(t)(p+T(t))} \right] |X_u(t) - \overline{X}_u(t)| \\
& - \Bigg[-\mu_1\beta_1 - \frac{\mu_5 r(t)\theta(w\overline{Y}(t) + X_a(t)\overline{Y}(t) + X_a(t)Y(t))}{\overline{T}(t)(w+X_a(t))(w+\overline{X}_a(t))} \\
& + \mu_3 \frac{T(t)\{\lambda_1(N(t)-Y(t)-X_u(t)) + \lambda_2 X_u(t)\}}{X_a(t)\overline{X}_a(t)(p+T(t))} \Bigg] |X_a(t) - \overline{X}_a(t)|
\end{aligned}
$$

$$-\left[-\mu_1\beta_1+\frac{\mu_4(\Lambda-\alpha Y(t))}{N(t)\overline{N}(t)}-\frac{\mu_3\lambda_1 T(t)}{\overline{X}_a(t)(p+T(t))}\right]|N(t)-\overline{N}(t)|$$
$$-\left[\frac{\mu_2\lambda_2 p}{(p+T(t))(p+\overline{T}(t))}+\frac{\mu_5(r(t)Y(t)+r_0T_0)}{T(t)\overline{T}(t)}-\frac{\mu_5 r(t)\theta X_a(t)Y(t)}{T(t)\overline{T}(t)(w+X_a(t))}\right.$$
$$-\mu_3\lambda_1\frac{X_a(t)\{p(\overline{N}(t)+\overline{Y}(t)+\overline{X}_u(t))+2T(t)(N(t)+Y(t)+X_u(t))\}}{X_a(t)\overline{X}_a(t)(p+T(t))(p+\overline{T}(t))}$$
$$\left.+\mu_3\lambda_2\frac{p\overline{X}_u(t)+2X_u(t)T(t)}{X_a(t)\overline{X}_a(t)(p+T(t))(p+\overline{T}(t))}\right]|T(t)-\overline{T}(t)|.$$

Therefore,

$$D^+V(t)\le -k_1|Y(t)-\overline{Y}(t)|-k_2|X_u(t)-\overline{X}_u(t)|-k_3|X_a(t)-\overline{X}_a(t)|-k_4|N(t)-\overline{N}(t)|$$
$$-k_5|T(t)-\overline{T}(t)|, \tag{4.7}$$

where

$$k_1=\mu_1\beta_1-\mu_2\beta_2-\frac{\mu_4\alpha}{e^{\rho_{44}}}-\frac{\mu_3 r^M}{e^{\rho_{55}}}-\frac{\mu_5 r^M\theta e^{\rho_3}}{e^{\rho_{55}}(w+e^{\rho_{33}})}-\frac{\mu_3\lambda_1 e^{\rho_5}}{e^{\rho_{33}}(p+e^{\rho_{55}})},$$
$$k_2=\frac{\mu_2(1-\mu)\Lambda}{e^{2\rho_2}}-\mu_1(\beta_1+\beta_2)-\frac{\mu_3(\lambda_1+\lambda_2)e^{\rho_5}}{e^{\rho_{33}}(p+e^{\rho_{55}})},$$
$$k_3=\frac{\mu_3 e^{\rho_{55}}(\lambda_1 e^{\rho_{44}}+\lambda_2 e^{\rho_{22}})}{e^{2\rho_3}(p+e^{\rho_5})}-\mu_1\beta_1-\frac{\mu_3\lambda_1 e^{\rho_5}(e^{\rho_1}+e^{\rho_2})}{e^{2\rho_{33}}(p+e^{\rho_{55}})}-\frac{\mu_5 r^M\theta(we^{\rho_1}+2e^{\rho_1+\rho_3})}{e^{\rho_{55}}(w+e^{\rho_{33}})^2},$$
$$k_4=\frac{\mu_4\Lambda}{e^{2\rho_4}}-\mu_1\beta_1-\frac{\mu_4\alpha e^{\rho_1}}{e^{2\rho_{44}}}-\frac{\mu_3\lambda_1 e^{\rho_5}}{e^{\rho_{33}}(p+e^{\rho_{55}})},$$
$$k_5=\frac{\mu_2\lambda_2 p}{(p+e^{\rho_5})^2}+\frac{\mu_5(r^m e^{\rho_{11}}+r_0T_0)}{e^{2\rho_5}}-\frac{\mu_5 r^M\theta e^{\rho_1+\rho_3}}{e^{2\rho_{55}}(w+e^{\rho_{33}})}$$
$$-\frac{(p+2e^{\rho_5})\{\mu_3\lambda_1(e^{\rho_1}+e^{\rho_2}+e^{\rho_4})+\mu_3\lambda_2 e^{\rho_2}\}}{e^{\rho_{33}}(p+e^{\rho_{55}})^2}.$$

If the conditions (4.2)–(4.6) hold, then $V(t)$ is monotonic decreasing on $[0,\infty)$. Now integrating inequality (4.7) over $[0,t]$, we have

$$V(t)+\int_0^t[k_1|Y(t)-\overline{Y}(t)|+k_2|X_u(t)-\overline{X}_u(t)|+k_3|X_a(t)-\overline{X}_a(t)|+k_4|N(t)-\overline{N}(t)|$$
$$+k_5|T(t)-\overline{T}(t)|]dt\le V(0)<\infty,\ \forall\ t\ge 0.$$

Hence, by Lemma 4.1, we have

$$\lim_{t\to\infty}|Y(t)-\overline{Y}(t)|=0,\ \lim_{t\to\infty}|X_u(t)-\overline{X}_u(t)|=0,\ \lim_{t\to\infty}|X_a(t)-\overline{X}_a(t)|=0,$$
$$\lim_{t\to\infty}|N(t)-\overline{N}(t)|=0,\ \lim_{t\to\infty}|T(t)-\overline{T}(t)|=0.$$

Therefore, the positive periodic solution $(\overline{Y}(t),\overline{X}_u(t),\overline{X}_a(t),\overline{N}(t),\overline{T}(t))$ is globally attractive.

To prove that the globally attractive periodic solution $(\overline{Y}(t),\overline{X}_u(t),\overline{X}_a(t),\overline{N}(t),\overline{T}(t))$ is unique, we assume that $(\overline{Y}_1(t),\overline{X}_{u_1}(t),\overline{X}_{a_1}(t),\overline{N}_1(t),\overline{T}_1(t))$ is another globally attractive periodic solution of system (4.1) with period 1. If this solution is different from the solution $(\overline{Y}(t), \overline{X}_u(t), \overline{X}_a(t), \overline{N}(t), \overline{T}(t))$, then there exists at least one $\kappa \in [0, 1]$ such that $\overline{Y}(\kappa) \neq \overline{Y}_1(\kappa)$, which means $|\overline{Y}(\kappa) - \overline{Y}_1(\kappa)| = \epsilon_{11} > 0$. Thus,

$$\begin{aligned}\epsilon_{11} &= \lim_{n\to\infty} |\overline{Y}(\kappa + n) - \overline{Y}_1(\kappa + n)| \\ &= \lim_{t\to\infty} |\overline{Y}(t) - \overline{Y}_1(t)| > 0,\end{aligned}$$

which contradicts the fact that the periodic solution $(\overline{Y}, \overline{X}_u, \overline{X}_a, \overline{N}, \overline{T})$ is globally attractive. Therefore, $\overline{Y}(t) = \overline{Y}_1(t)$ for all $t \in [0, 1]$. Similar arguments can be used for other components $\overline{X}_u$, $\overline{X}_a$, $\overline{N}$ and $\overline{T}$ also. Hence, the system (4.1) has unique positive 1-periodic solution, which is globally attractive.

□

5. Numerical simulations

Here, we report the simulations to investigate the behaviors of systems (2.2) and (4.1). The set of parameter values are chosen within the range prescribed in various previous literature sources [4–6, 26, 27], and are given in Table 2. Unless it is mentioned, the values of parameters used for numerical simulations are the same as in Table 2. For the set of parameter values in Table 2, the components of endemic equilibrium E^* are obtained as:

$$\begin{aligned}&Y^* = 979.2622032,\ X_u^* = 313.9752,\ X_a^* = 66663.63299,\ N^* = 124755.1844,\\ &T^* = 2457.546025.\end{aligned}$$

The eigenvalues of the Jacobian matrix at the equilibrium E^* are given by,

$$-0.02480,\ -0.00056 \pm 0.01025i,\ -0.01429,\ -0.00004. \tag{5.1}$$

Note that three eigenvalues are negative whereas two have negative real parts showing that the equilibrium E^* is locally asymptotically stable. Further, for the parameters in Table 2, the value of basic reproduction number is found to be $\mathcal{R}_0 = 1.440734831$. As $\mathcal{R}_0 > 1$, the disease always persists in the system for this set of parameter values. Thus, we look for the controllable parameters which can drive the value of $\mathcal{R}_0$ below unity, and hence enhance the possibility of disease eradication.

5.1. Sensitivity analysis

To see how the model parameters influence the basic reproduction number $\mathcal{R}_0$, we find the normalized forward sensitivity indices of $\mathcal{R}_0$ to each parameter involved in the expression of $\mathcal{R}_0$ [39]. We choose $\lambda_2 = 0.012$, $\alpha = 0.02$, $p = 500$ and $\lambda_0 = 0.00005$, and keep rest of the parameters at the same values as in Table 2. The normalized forward sensitivity index of a variable to a parameter is a ratio of the relative change in the variable to the relative change in the parameter. The sensitivity indices of $\mathcal{R}_0$ with respect to the parameters of interest are plotted in Figure 3. The figure shows that when the parameters Λ, β_1, β_2, γ and λ_0 increase, keeping the other parameters constant, the value of $\mathcal{R}_0$ increases as they have positive indices. Instead, increase in the values of parameters ν, α, d, λ_1, λ_2 and T_0 cause decrement in the values of $\mathcal{R}_0$ as they have negative indices. It is noted that the

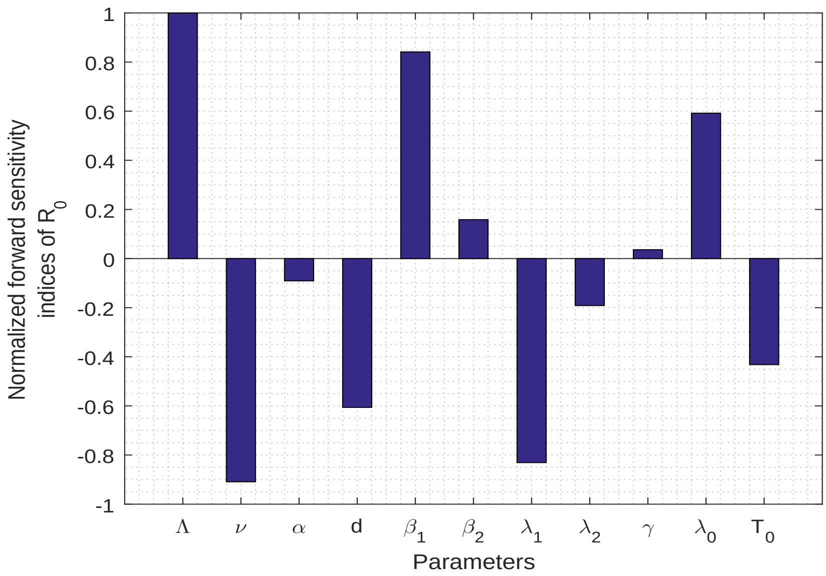

Figure 3. Normalized forward sensitivity indices of $\mathcal{R}_0$ with respect to Λ, ν, α, d, β_1, β_2, λ_1, λ_2, γ, λ_0 and T_0. Parameters are at the same values as in Table 2 except $\lambda_2 = 0.012$, $\alpha = 0.02$, $p = 500$ and $\lambda_0 = 0.00005$.

sensitivity index of $\mathcal{R}_0$ is 1 for the parameter Λ. It means that 1% increase in Λ, keeping other parameters fixed, will produce 1% increase in $\mathcal{R}_0$. A lower value of $\mathcal{R}_0$ is preferable because it increases the possibility of disease eradication in the region. Therefore, it is imperative to prevent an increase in the parameters $\Lambda, \beta_1, \beta_2$, γ and λ_0, while increasing ν, α, d, λ_1, λ_2 and T_0 should instead be fostered. Thus, any external measure aiming at reducing the former parameters and enhancing the latter should therefore be taken into serious consideration.

Next, we select some controllable parameters: Λ, β_1, β_2, λ_1, λ_2 and T_0 as input parameters and infective population, Y, as response function. We employed the approach of [40, 41] to perform global sensitivity analysis. The procedure consists of two statistical techniques: Latin Hypercube Sampling (LHS) and Partial Rank Correlation Coefficients (PRCCs). LHS allows us to vary several parameters simultaneously in an efficient way while PRCCs correlate the model output and the input parameters. PRCCs assign values between -1 and 1; the sign indicates the type of correlation while the value its strength. We observed nonlinear and monotone relationships for the infective population with the input parameters, a prerequisite condition before computing PRCCs. We consider a uniform distribution for each parameter and run 500 simulations per LHS. We choose baseline values of parameters as

$$\begin{aligned} &\Lambda = 50,\ \mu = 0.05,\ \beta_1 = 0.032,\ \beta_2 = 0.035,\ \lambda_1 = 0.9,\ \lambda_2 = 0.85,\ \lambda_0 = 0.05,\ p = 5, \\ &\gamma = 0.8,\ \nu = 0.2,\ \alpha = 0.02,\ d = 0.4,\ r = 0.5,\ \theta = 0.05,\ w = 6,\ r_0 = 0.3,\ T_0 = 150 \end{aligned} \tag{5.2}$$

and allow them to deviate $\pm 25\%$ from these values. The PRCCs values are depicted in Figure 4. It can be noted from the figure that the parameters Λ, β_1 and β_2 have positive correlation with infective population whereas the parameters having negative impact on infective population are λ_1, λ_2 and T_0.

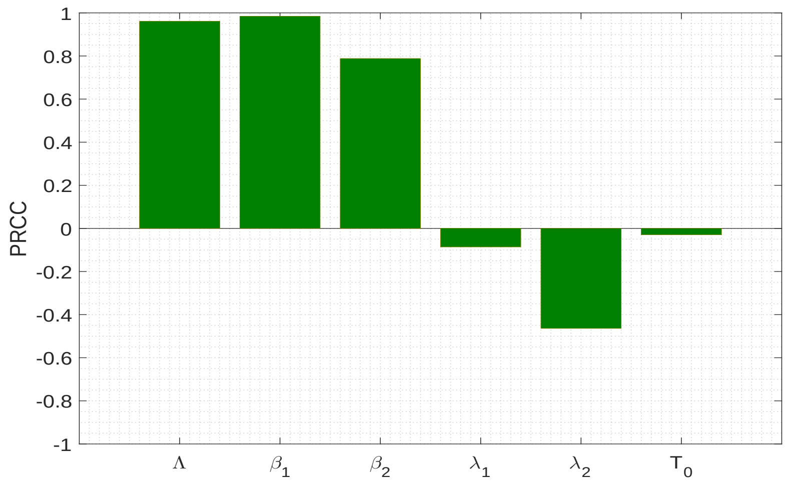

Figure 4. Effect of uncertainty of the model (2.2) on infected population. Baseline values of parameters are given in (5.2).

Of these, the parameters having significant correlations with the infective population are $\Lambda, \beta_1, \beta_2$ and λ_2 (p-value$<$ 0.05). Identification of parameters with positive or negative correlation with infective population is crucial information for the formulation of effective control strategy necessary for controlling the burden of disease.

5.2. Effects of some key parameters on disease control

To better represent the combined actions of three important model parameters−the dissemination rate of awareness among literate people (λ_1) and illiterate people (λ_2), and the baseline number of social media advertisements (T_0)−on the basic reproduction number ($\mathcal{R}_0$), we plot $\mathcal{R}_0$ with respect to T_0 and λ_1 (Figure 5a), and T_0 and λ_2 (Figure 5b). It is apparent from the graphs that, if the dissemination rates of awareness among literate/illiterate people and the baseline number of social media advertisements increase, the basic reproduction number decreases and can be less than unity for certain ranges of λ_1, λ_2 and T_0. Thus, it can be inferred that the prevalence of disease can be controlled by increasing the dissemination rates of awareness among literate/illiterate people and the baseline number of social media advertisements in the region. Next, we plot the infective population by varying two parameters at a time viz. (r, r_0), (λ_1, p), (λ_2, p) and (T_0, λ_0) (Figure 6). We find that the infective population decreases with increments in the parameters r, λ_1, λ_2 and T_0 whereas increase in the parameters r_0, p and λ_0 lead to rise in the infected population. It can be easily noted that the most effective parameters are p, λ_1 and T_0. Diminution of advertisements due to psychological barrier causes significant rise in infectives. Thus, the active advertisements should be maintained in the society.

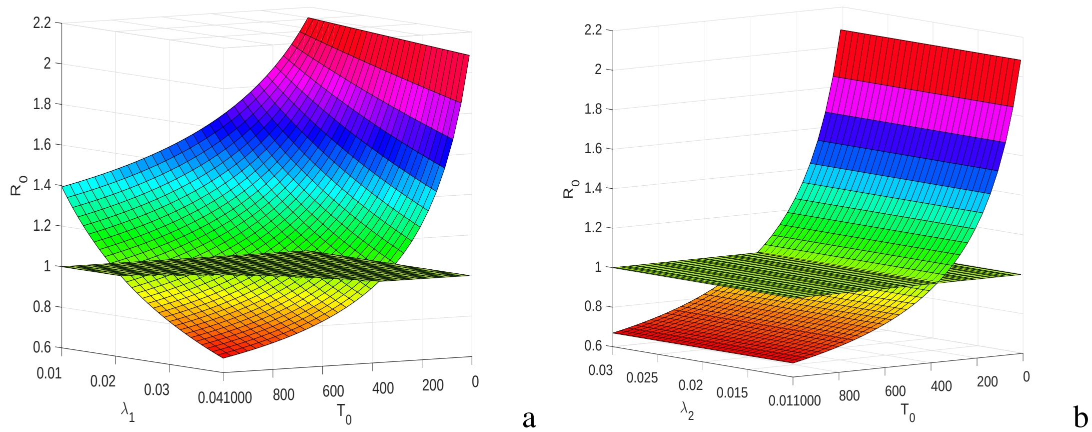

Figure 5. Plot of $\mathcal{R}_0$ with respect to (a) T_0 and λ_1, and (b) T_0 and λ_2. Rest of the parameters are at the same values as in Table 2 except $\lambda_1 = 0.04$ in (b).

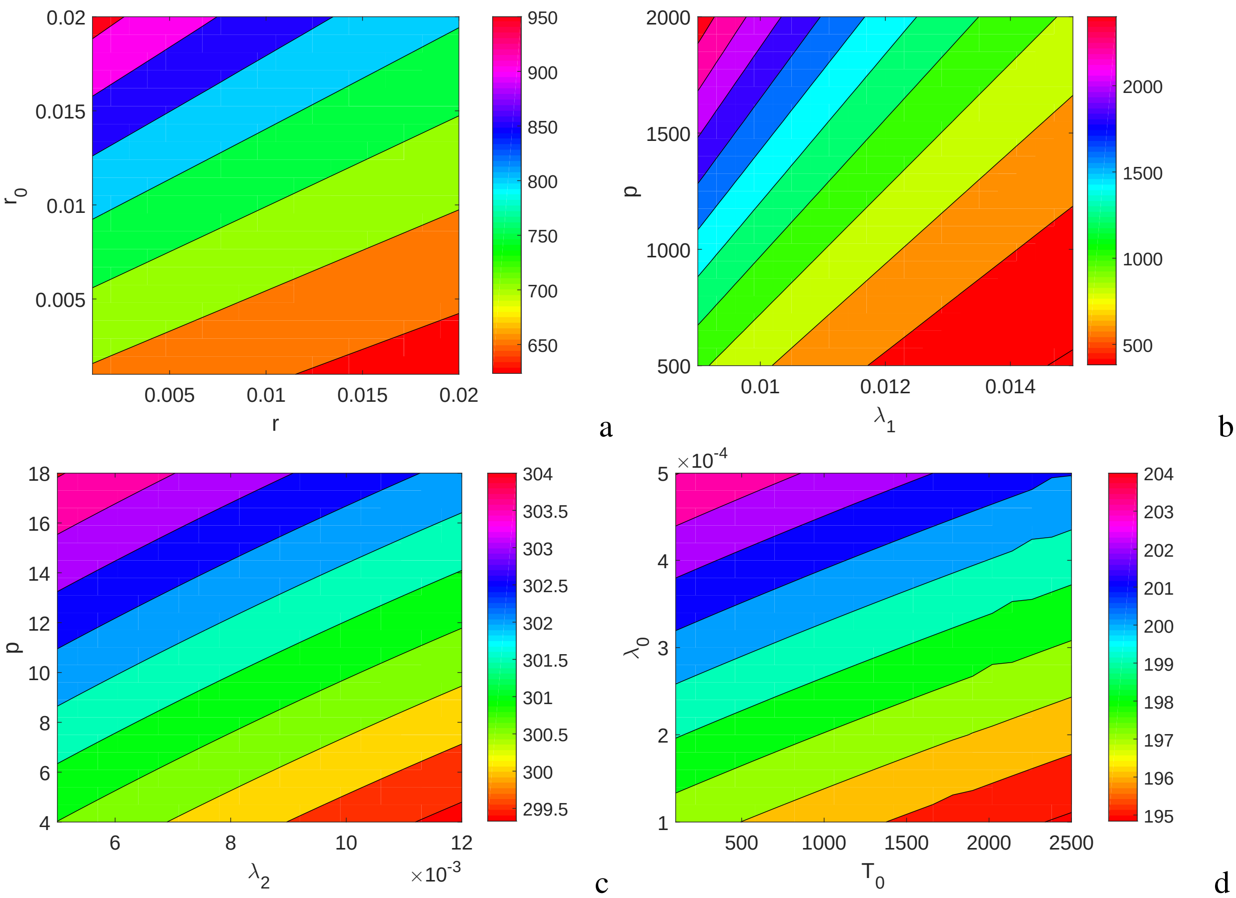

Figure 6. Contour line representing the equilibrium value of infected population, $Y(t)$, as functions of (a) r and r_0, (b) λ_1 and p, (c) λ_2 and p, and (d) T_0 and λ_0. Rest of the parameters are at the same values as in Table 2.

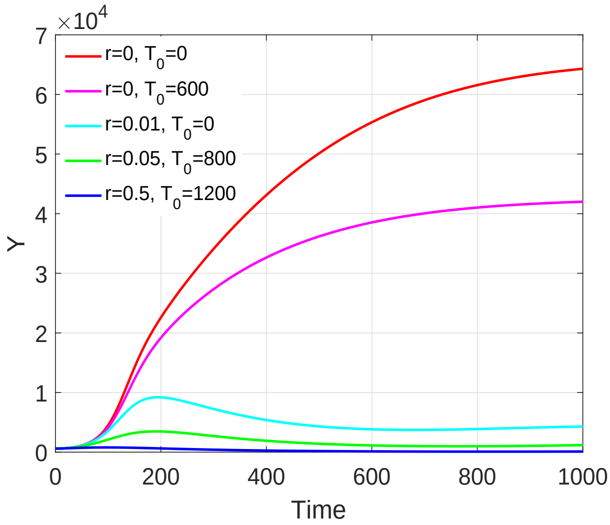

Figure 7. Effects of social media advertisements on the equilibrium level of infective population. Parameters are at the same values as in Table 2 except $\lambda_1 = 0.01$.

Further, we see the equilibrium level of infective population for different values of r and T_0, Figure 7. We fix $\lambda_1 = 0.01$ and keep rest of the parameters at the same values as in Table 2. First, we choose $r = T_0 = 0$ i.e., there is no advertisements in the region; we find that the equilibrium level of infective is very high is such case. Next, we assume there is baseline number of advertisements but no growth in number of advertisements, the number of infective is found to attain a lower equilibrium level than in the previous case. Sudden decrease in the infective is seen if there is continuous growth in the number of advertisements although the baseline number of advertisements is zero. Further, we consider that there is always baseline number of advertisements and the number of advertisements also increases; due to this combined efforts of social media, the equilibrium level of infective decreases to a low value. Finally, we see that the equilibrium value of infective becomes negligible if the baseline number of advertisements is sufficiently large and also there is continuous growth in the number of advertisements.

5.3. Bifurcation results

At first, we see how the growth rate of broadcasting the informations, r, affects the dynamics of system (2.2). For this, we vary the parameter r in the interval [0.01, 0.04] and draw the bifurcation diagram of the system (2.2), Figure 8. This figure demonstrates the rising of periodic solution from stationary solution on increasing the value of r in $Y{-}X_a$ plane. We note that for lower values of r, the system (2.2) showcases the steady state dynamics while on increasing r, we get a critical value, namely $r^* \approx 0.02789$, at which the system's behavior changes drastically. At $r = r^*$, the system enters into limit cycle oscillations from stable equilibrium via supercritical Hopf bifurcation. Thus, broadcasting the information through social media may induce instability in the system. Next, we observe the impact of dissemination rate of awareness among the literate susceptible individuals on system's stability behavior. In Figure 9, we plot bifurcation diagram of system (2.2) with respect to the parameter λ_1. We find that for lower ranges of λ_1, system (2.2) exhibits stable dynamics at the

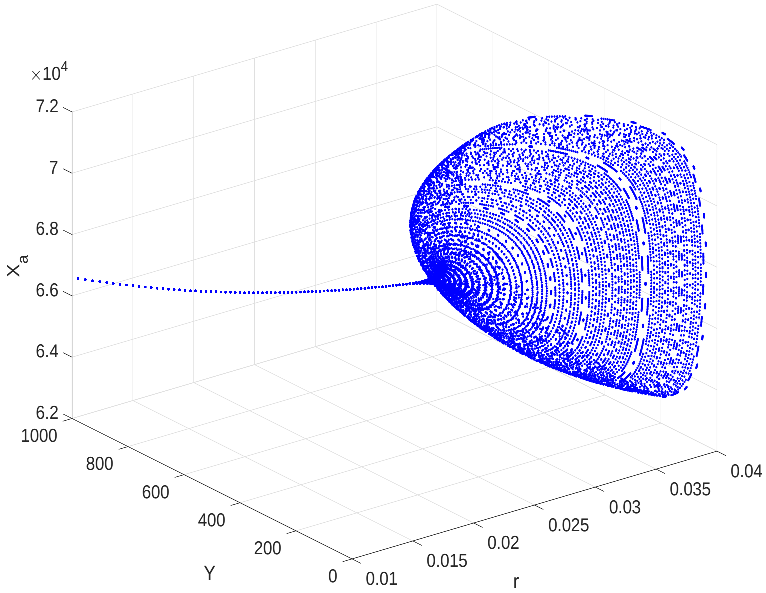

Figure 8. Bifurcation diagram of system (2.2) with respect to r. Limit cycles arising in $Y-X_a$ plane on increasing the value of r. Rest of the parameters are at the same values as in Table 2.

endemic equilibrium. On increasing the values of λ_1, we obtain two threshold values of λ_1: $\lambda_1^{h_1} \approx 0.01342$ and $\lambda_1^{h_2} \approx 0.0247$, at which the stability behavior of the system changes. At the former, the system enters into limit cycle oscillations from stable dynamics via supercritical Hopf bifurcation whereas at the latter, the oscillations are killed out and the system settles to stable disease-free steady state. Therefore, awareness among literate people due to popularity of social media advertisements are helpful to eradicate the disease in the region. Further, we explore the effect of baseline number of social media advertisements (T_0) on the dynamics of system (2.1), Figure 10. The dynamics of system is oscillatory upto certain level of baseline number of advertisements and stability is achieved after the critical value $T_0^c = 462$; the disease dies out and the system settles to stable disease-free equilibrium for $T_0 > 614$. Thus, we find that whenever the baseline number of social media advertisements is above a threshold value, the disease can not persist in the system. Therefore, one should focus on the augmentation in baseline number of advertisements together with dissemination of awareness among literate people in order to have disease-free system.

5.4. Simulation results of nonautonomous system (4.1)

We simulate system (4.1) by considering the growth rate of broadcasting the information through social media, $r(t)$, as a sinusoidal function: $r(t) = r + r_{11}\sin(\omega t)$ with period of 365 days. The solution trajectories of the nonautonomous system (4.1) are plotted in Figure 11. We observe that there exists a positive periodic solution at $r = 0.015$. Thus, for the same set of the parameter values, the nonautonomous system (4.1) exhibits positive periodic solution whereas the corresponding autonomous system shows stable dynamics. Next, we show global stability of the endemic equilibrium E^* of system (2.2) and positive periodic solution of system (4.1), Figure 12. We note that

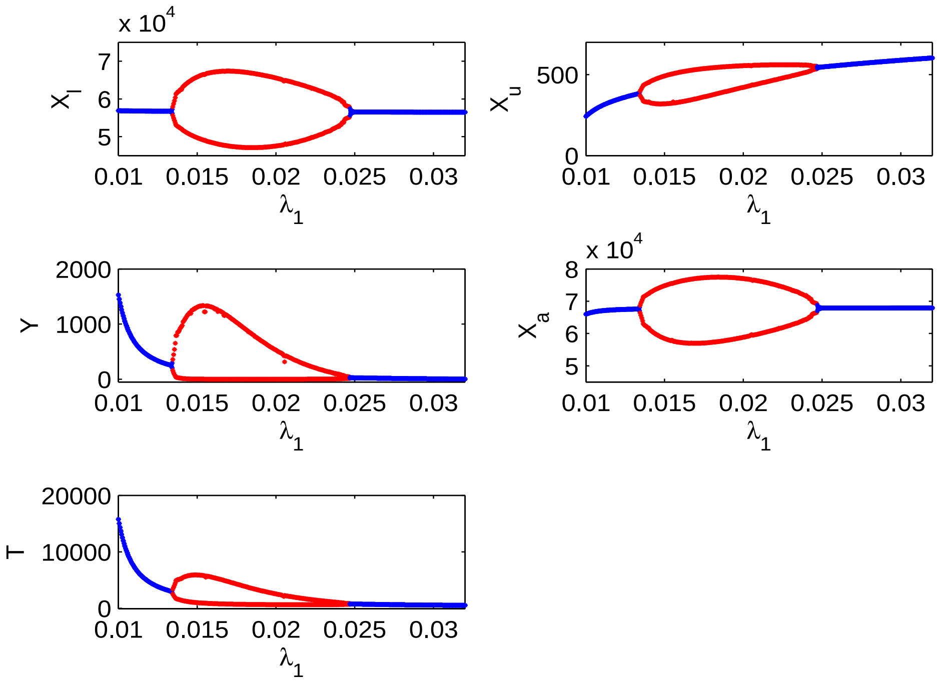

Figure 9. Bifurcation diagram of system (2.2) with respect to λ_1. Rest of the parameters are at the same values as in Table 2 except $r = 0.05$.

the conditions (3.9) and (3.10) for the global stability of the endemic equilibrium E^* are satisfied for $\lambda_1 = 0.0003$ and $\lambda_2 = 0.00002$ while the remaining parameters are at the same values as in Table 2. In Figure 12a, we have shown the global stability of the endemic equilibrium E^* inside the region of attraction Ω in Y−X_a−T space. It is evident from the figure that all the solution trajectories that originate inside the region of attraction approach the point (Y^*, X_a^*, T^*). Using this approach, the global asymptotic stability of the endemic equilibrium E^* in other spaces can also be shown. Thus, the statement of Theorem 3.3 is numerically verified. Further, we see the global stability of the positive periodic solution of the nonautonomous system (4.1). For better understanding, we choose two values of r from below and above the threshold value r^*. We fix the value of r at $r = 0.015$ and plot the solution trajectories of the system (4.1) for the infected population only by choosing three different initial conditions, Figure 12b. It is apparent from the figure that all the periodic solutions initiating from three different initial values converge to a single periodic solution. It is noted that for other variables of the system, different solutions trajectories coincide to unique positive periodic solution. Thus, the positive periodic solution is globally attractive. Hence, the results of Theorem 4.2 is numerically validated. Next, we choose $r = 0.03$ ($r > r^*$) and plot the solution trajectories of the nonautonomous system for the parameter values given in Table 2, Figure 13. It is evident from the figure that the system shows higher periodic solutions for $r = 0.03$. Therefore, the global stability of the nonautonomous system is affected due to enhancement in broadcasting the information whereas the persistence of the system is not affected by broadcasting of the information through social media.

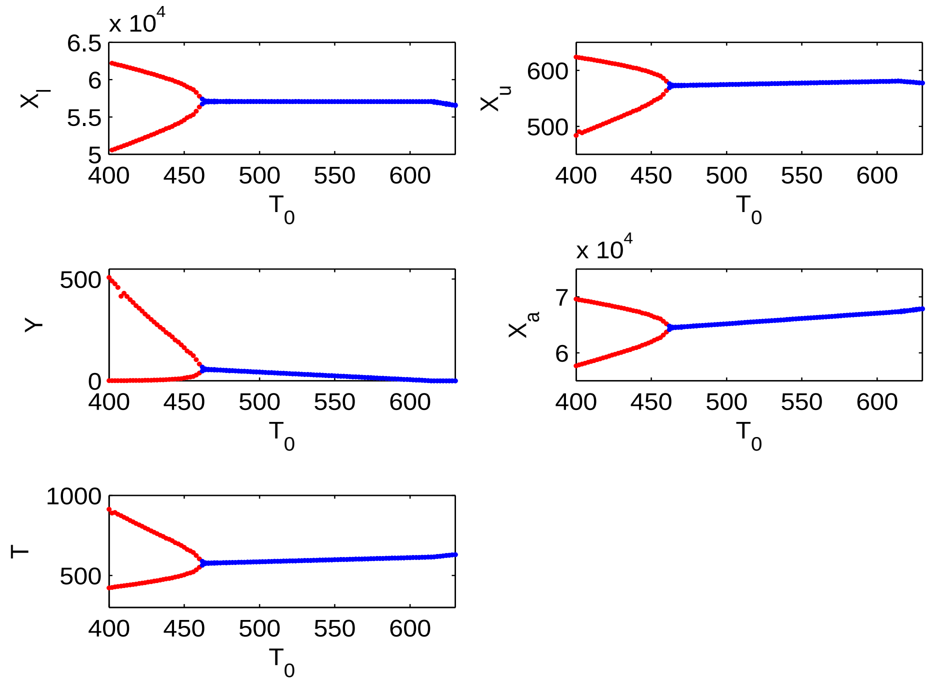

Figure 10. Bifurcation diagram of system (2.2) with respect to T_0. Rest of the parameters are at the same values as in Table 2 except $\lambda_0 = 0.004$ and $\alpha = 0.002$.

6. Results and discussions

Here, we have investigated the roles of social media advertisements and literacy on the prevalence of infectious diseases. Social media makes people aware about the disease and brings behavioral changes among the individuals regarding the risk of infection whereas literacy plays a very crucial role to guide people towards awareness. Our model comprises five dynamical variables: Literate and illiterate population, aware population, infective population and cumulative number of social media advertisements. We find that literacy, awareness and baseline number of social media advertisements have crucial impacts on basic reproduction number and can push back the epidemic threshold below unity. Our sensitivity results suggest to device a strategy of intervention which aimed to reduce the immigration of population, contact rates of literate and illiterate susceptibles with infectives; simultaneously one should focus on the augmentation in dissemination rate of awareness due to popularity of new advertisements among the people and baseline number of awareness programs in the endemic regions.

We have found that in absence of awareness or literacy and awareness, the system is locally as well as globally stable at the endemic state. Our numerical results show destabilizing role of growth rate of advertisements; the system enters into limit cycle oscillations from stable endemic state as the growth rate of social media advertisements surpasses a critical value. In contrast, the baseline number of advertisements play an important role in the system dynamics; it stabilizes an otherwise unstable endemic equilibrium and drive the system to disease-free environment after crossing a threshold value. Previous studies have only shown reduction in infected populations with enhancement in

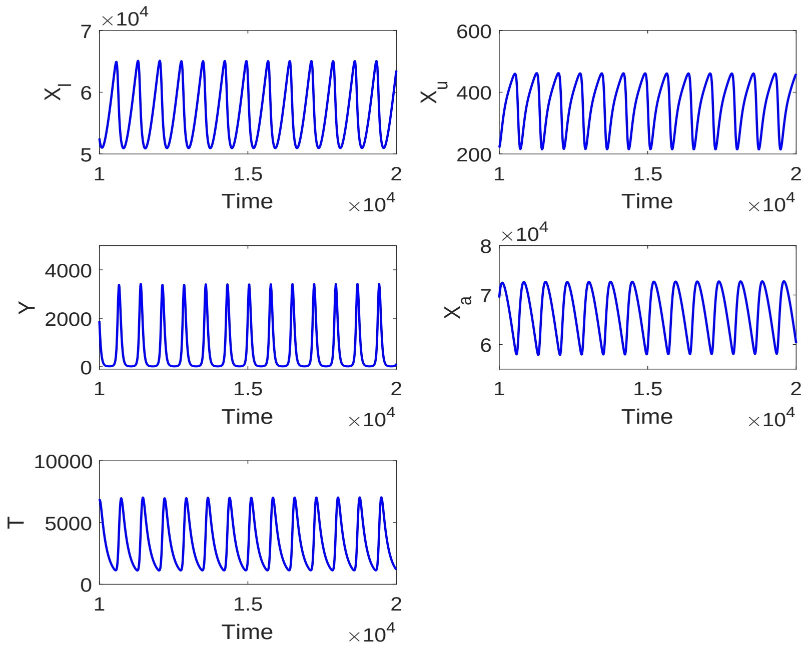

Figure 11. System (4.1) exhibits periodic solution at $r = 0.015$. Rest of the parameters are at the same values as in Table 2, $r_{11} = 0.01$ and $\omega = 2\pi/365$.

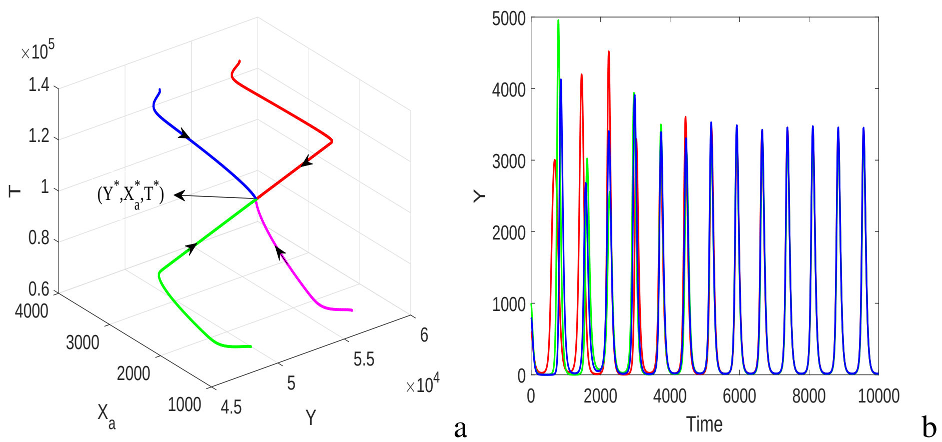

Figure 12. Global stability of (a) equilibrium E^* of systems (2.2) and (b) positive periodic solution of system (4.1). Parameters are at the same values as in Table 2 except in (a) $\lambda_1 = 0.0003$ and $\lambda_2 = 0.00002$, and (b) $r = 0.015$, $r_{11} = 0.01$ and $\omega = 2\pi/365$. Figures show that solution trajectories starting from different initial points ultimately converge to the equilibrium E^* in (a) and a unique positive periodic solution in (b).

baseline number of media campaigns [4–6, 22]. The impact of dissemination rate of awareness among literate susceptible population shows interesting dynamics. The system is stable for low and high rate of disseminating awareness among literate susceptible people but for moderate values of this

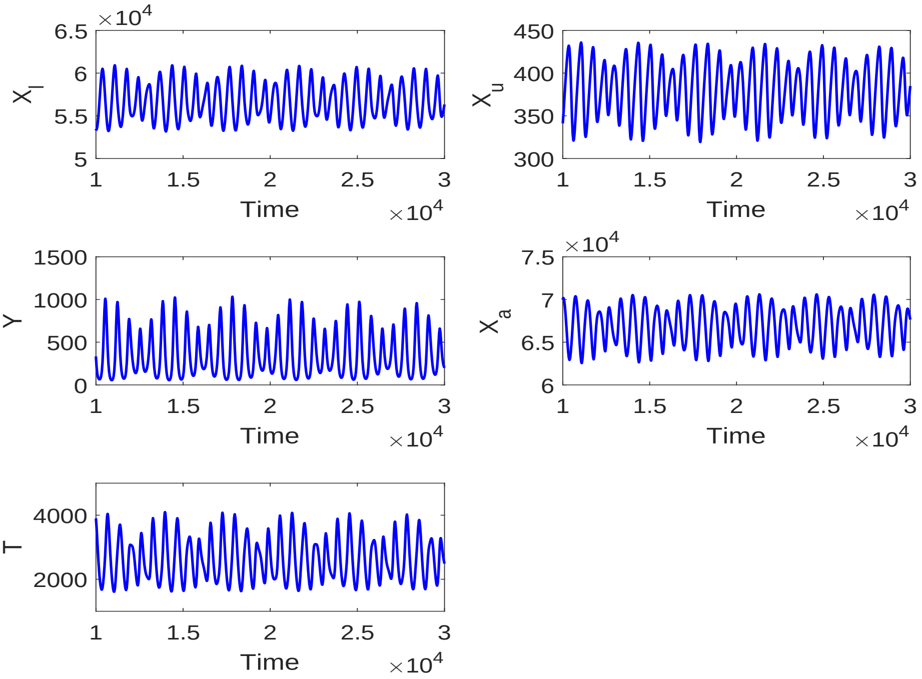

Figure 13. System (4.1) exhibits higher periodic solution at $r = 0.03$. Rest of the parameters are at the same values as in Table 2, $r_{11} = 0.01$ and $\omega = 2\pi/365$.

parameter, the system coexist in oscillatory state. However, if the dissemination rate of awareness among literate susceptible population is very high, then the endemic equilibrium disappears and stable disease-free steady state emerges in the system. In epidemiology, it can be interpreted that the augmentation in dissemination rate of awareness among the literate susceptible individuals due to popularity of new advertisements increases the number of aware individuals leading to decrease in the number of social media advertisements, and hence increases the infected individuals in the region. Now, as the infectives increased, the number of social media advertisements again rise, causing increments in the aware individuals. Such interplay between the number of infected individuals and dissemination of awareness through social media advertisements give the birth to oscillation in the system. However, if the infectives are too low and aware individuals reach a plateau, persistent oscillations disappeared and system gained stable coexistence. Moreover, if the dissemination rate of awareness among literate susceptibles is too large, the disease is completely eradicated from the region. Recall that Misra et al. [4] have also shown the similar effects of growth rate of social media advertisements and dissemination rate of awareness among susceptible individuals. Note that in their study, effect of literacy was not considered. Our study shows an additional and excitable result: The stabilizing role of baseline number of social media advertisements, higher values of which results in complete eradication of the disease.

We have also seen the effect of seasonally varying rate of broadcasting advertisements through social media. We have considered the growth rate of social media advertisements as a periodic function of time with a period of one year. The numerical results revealed that the nonautonomous system exhibited positive periodic solution which is globally attractive whenever the corresponding autonomous system shows stable dynamics. The existence of periodic solutions should be viewed as a condition allowing for the survival of the populations under consideration. We observed that the global attractivity of the periodic solution is altered if the social media advertisements are broadcasted at a high rate. For the larger values of rate of advertisements, the nonautonomous system shows the occurrence of higher periodic solutions. Thus, the seasonal variations of the rate of broadcasting social media advertisements impose synergistic effects for inducing higher periodic oscillations in the system. The outcomes of the present investigation suggest that the dissemination of awareness due to popularity of new advertisements among literate people and the baseline number of social media advertisements act as effective control parameters by altering the prevalence of limit cycle oscillations to order, and ultimately settling the system to disease-free region. Moreover, the dissemination of awareness among illiterate people through social media advertisements greatly reduces the infective cases. Recall that in the case of autonomous system, the disease can be eradicated for higher dissemination rate of awareness among literate susceptible individuals and also by maintaining a large number of social media advertisements in the endemic region. However, for the nonautonomous system, existence of unique positive periodic solution is observed. It is worthy to note that the existence of positive periodic solution represents an equilibrium situation consistent with the variability of social media advertisements, and hence persistence of disease in the society. That is to say, in the former case there is possibility of disease termination while in the latter case, the population density of infective behaves periodically.

The results reported in this paper indicate that the broadcasting of information through social media advertisements is a crucial factor in the prevention of disease transmission and may be used as a potential strategy in controlling the disease. The effects of awareness should be increased in both literate and illiterate susceptible populations. Efforts should focus on radical behavioral changes among literate and illiterate susceptible populations, in particular among those deemed at-risk, in order to completely eradicate the disease. In this context, the public-health authorities and policy makers have a major contribution. They should monitor the situation to ensure that intervention strategies are being implemented properly. For example, Guinea-worm disease is eradicated only through proper program-implementation strategies [42–44]. Similarly, the burden of other diseases such as HIV [10], coronavirus [3] etc., can be reduced by disseminating awareness among literate and illiterate susceptible individuals, and increasing the baseline number of social media advertisements in the endemic zones. Due to absence of any proper vaccine or therapeutics, different non-pharmaceutical interventions are imposed to impede COVID-19 transmission. Several countries are focusing on media advertising campaigns for stimulating people to adopt healthy sanitation practices, frequent hand washing, use of face mask, sanitizer, maintain social distancing etc. In this regard, different modes of media such as social media, TV, radio, internet etc., are playing tremendous role to propagate information among people. These social media campaigns are helpful to disseminate awareness among the people about menace of the pandemic and their non-pharmaceutical prevention practices [14]. Disseminations of information through social media advertisements have definitely encouraged the people to adopt preventive measures to combat the coronavirus pandemic. After the media reporting about COVID-19, people became aware of the disease threat and began to reduce their contact with others. In almost all the affected countries, government imposed complete lockdown to reduce social distancing, educational institutions arranged online classes, webinars etc. These behaviors overall resulted in reduced contact with others and delayed disease spread, and consequently suppress the burden of disease.

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Conflict of interest

The authors declare that there is no conflict of interest in this paper.

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