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Research article

Interval-valued intuitionistic fuzzy MADM method based on TOPSIS and grey correlation analysis

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Abstract: In this paper, we propose an interval-valued intuitionistic fuzzy Multi-Attribute Decision Making (MADM) method based on improved TOPSIS and Grey Correlation Analysis (GCA), in which the attribute values are interval-valued intuitionistic fuzzy numbers. So that we can deal with imprecise information in fuzzy and rough form in MADM problems by using interval-valued intuitionistic fuzzy numbers Firstly, the concept of interval intuitionistic fuzzy entropy is introduced to calculate the entropy weight of attributes. And the combined weight is calculated by combining the entropy weight with the subjective weight. Secondly, the reverse order phenomenon in the traditional TOPSIS method is eliminated by constructing absolute Positive Ideal Solution (PIS) and absolute Negative Ideal Solution (NIS) in the form of interval-valued intuitionistic fuzzy numbers. Furthermore, the improved TOPSIS method and grey correlation analysis method are combined to describe the degree of closeness for each alternative from the ideal solution, and then the ranking and selection of each alternative are made accordingly to this degree. Finally, the rationality and effectiveness of our method are verified by an example and its sensitivity analysis. The result shows that our method makes the solution of MADM problems more objective and reasonable.

Keywords: multi-attribute decision making (MADM); interval-valued intuitionistic fuzzy numbers; TOPSIS; grey correlation analysis (GCA)

1. Introduction

The Multi-Attribute Decision Making (MADM) problem is an important research content of modern decision making science [1]. It usually means that for the given alternatives, the attribute and the corresponding weight of the attribute are given according to certain rules after expert study and analysis, and then, according to a certain method, the obtained information is statistically arranged to obtain the corresponding comprehensive evaluation value of each alternative. Finally, the best alternative is selected by sorting out the comprehensive evaluation value of all the alternatives. In the traditional MADM problems, the evaluation information of alternatives is generally accurate. However, due to the lack of relevant knowledge of the evaluation or the ambiguity of the evaluation itself, the evaluator cannot give an accurate evaluation value when comparing the alternatives. In this case, it is more accurately expressed by fuzzy number, for example, interval numbers [2], linguistic variable [3], intuitionistic fuzzy numbers [4] Zadeh et al. [5] proposed the concept of Fuzzy Set (FS) in 1965, which laid the theoretical foundation of FS. Fuzzy Linguistic Scale has become a hot topic in MADM research. Atanassov et al. [6] studied multi-attribute group decision making problems with known attribute weights and intuitionistic fuzzy attribute value based on weighted averaging operator. In order to meet the need of practical decision making, Atanassov and Gargov [7] defined the concept of Interval-Valued Intuitionistic Fuzzy Set (IVIFS), and used interval numbers to represent membership degree, non-membership degree and hesitancy degree respectively, which described the information of the research object more completely. Based on this, many scholars have carried out a lot of theoretical research about IVIFS, and used IVIFS as an effective tool to Decision-Making [8–10], Medical Diagnosis [11], Linear Programming [12], Pattern Recognition [13] and Market Forecasting [14]. However, the research on the IVIFS is mainly focused on the fundamental theories, such as the interaction measures, correlation measures, distance measures, similarity measures, and clustering algorithm of IVIFS [15–19].

At present, the MADM methods include Evidential Reasoning (ER) method [20], Data Envelopment Analysis (DEA) method [21], fuzzy evaluation method [22], TOPSIS method [23], ELECTRE method [24], Grey Correlation Analysis (GCA) method [25] et al. TOPSIS is a ranking method of approximate ideal solution, and it is an effective method to solve the MADM problems. However, the Euclidean distance as the evaluation criterion can only reflect the position relation between data series, but not the situation changes of data series in traditional TOPSIS method. In the case of large difference of attribute value, if the distance between the alternative and the ideal solution is close, the result of the alternative is similar. Grey Correlation Analysis (GCA) is a method to measure the similarity of curve shape. However, the traditional grey correlation analysis method uses the absolute value of the difference between two data series to calculate the correlation degree, it only considers the geometric similarity between the data series and neglects the value close degree, so the accuracy cannot be guaranteed. The combination of grey correlation analysis method and TOPSIS method can consider the approach distance and correlation degree between the alternatives and ideal solution at the same time, which makes the solution of MADM problems more objective and reasonable. Zhou et al. [26] used the weighted gray correlation-TOPSIS method to evaluate and compare the quality of Raw Moutan cortex. P. P. Das and S. Chakraborty [27] illustrated the application of grey correlation-based TOPSIS in obtaining the optimal parametric mix for the said process for better surface roughness along with higher micro hardness. Zhou et al. [27] researched the comprehensive evaluation model of TOPSIS based on entropy weight method-gray correlation to reasonably select the best railway route design scheme. The existing research includes the above methods are limited to the application of TOPSIS and grey correlation analysis to the exact numbers, and less to the research of interval-valued intuitionistic fuzzy numbers.

To sum up, the previous research on MADM either neglected the application of interval-valued intuitionistic fuzzy numbers, or neglected the deficiency of single method. Therefore, we propose an interval-valued intuitionistic fuzzy MADM method based on TOPSIS and grey correlation analysis in this paper. Firstly, the concept of interval intuitionistic fuzzy entropy is introduced to calculate the entropy weight of attributes, then combine the entropy weight with the subjective weight calculated by AHP method to calculate the combined weight. Secondly, TOPSIS method is used to calculate the weighted Euclidean distance for the alternatives from the PIS and NIS, and grey correlation analysis method is used to calculate weighted grey correlation degree between the alternatives and PIS and NIS, then combine the weighted Euclidean distance and weighted grey correlation degree to calculate the degree of closeness between the alternatives and the ideal solution, then the alternatives are evaluated and ranked. Finally, the rationality and effectiveness of the method are verified by an example.

2. Preparation knowledge

TOPSIS is a multi- attribute decision-making method, which can rank approach degree between multiple evaluation objects and ideal goals [29]. Preferences or rating of alternatives are often vague or underspecified, and defining them by crisp numbers may be difficult in many real world cases. Chen [30] extended traditional TOPSIS method to the fuzzy environment, in which the fuzzy numbers are used to define preference ratings. Yang et al. [31] used IVIFS for preference rating in fuzzy-TOPSIS, for alternatives selection. In MADM problems, due to the limited statistical data and human factors, many data have no typical distribution law, that is, some information is known and some information is unknown. Under the condition of the poor information, the traditional statistical methods in handling such problems appear insufficient capacity. While the grey correlation analysis method is similarity measure of curve shape, which can well analyze situation changes with less original data and convenient operation by using grey correlation coefficients. It can achieve good results in dealing with the problem of poor information and small samples [32].

L. A. Zadeh proposed Fuzzy Set (FS) in 1965 that only took the membership into account. Therefore, Atanassov [33] extended the traditional FS to the Intuitionistic Fuzzy Set (IFS) in 1986, which considered the information of membership degree, non-membership degree and hesitation degree at the same time. Besides, it is a challenging to express these degrees with single numbers, due to complexity and uncertainties. Thus, these degrees may be defined by interval-valued numbers as IVIFS. The three indexes of IVIFS, membership degree, non-membership degree and hesitation degree can be used to denote respectively three states of support, oppose and neutral. The single membership function of FS can only represent the two states of the positive and negative or support, oppose, so the IVIFS can describe the essential properties of things in a subtler way. At the same time, the IVIFS is an extension of the intuitive set. The range of membership degree and non-membership degree of IFS is on [0,1], while the range of IVIFS is on a subinterval of [0,1]. In the process of describing problems such as uncertainty, inaccuracy, incomplete information, IVIFS has a stronger ability to express uncertainty.

The definition of the IVIFS and the related basic operations are shown as follows.

2.1. Intuitionistic fuzzy set (IFS)

Let $X=\{x_1, x_2, \cdots, x_m\}$ be a non-empty set, then an IFS of X can be expressed as $A=\{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$. Where $\mu_A : X \to [0,1]$ represents the membership degree of x_i

belonging to A, and $\nu_A : X \to [0,1]$ represents the non-membership degree of x_i belonging to A, and for $\forall x_i \in X, \mu_A(x_i), \nu(x_i) \in [0,1]$, there is $\mu_A(x_i) + \nu_A(x_i) \le 1$. $\pi_A(x_i) = 1 - \mu(x_i) - \nu(x_i)$ represents the hesitation degree of x_i belonging to A and satisfies $0 \le \pi_A(x_i) \le 1, \forall x_i \in X$. If $\pi_A(x_i) = 0$, that is $\mu_A(x_i) + \nu_A(x_i) = 1$, then IFS becomes FS [19].

2.2. *Interval-valued intuitionistic fuzzy set (IVIFS)*

Let $X=\{x_1, x_2, \cdots, x_m\}$ be a non-empty set, $\mu_A(x_i) = [\mu_A^L(x_i), \mu_A^U(x_i)]$ and $\nu_A(x_i) = [\nu_A^L(x_i), \nu_A^U(x_i)]$, then the IVIFS can be expressed as follows:

$$A = \{\langle x_i, [\mu_A^L(x_i), \mu_A^U(x_i)], [\nu_A^L(x_i), \nu_A^U(x_i)]\rangle | x_i \in X\} \tag{1}$$

where $\mu_A^L(x) : X \to [0,1]$, $\mu_A^U(x) : X \to [0,1]$, $\nu_A^L(x) : X \to [0,1]$, $\nu_A^U(x) : X \to [0,1]$, and meet the conditions of $\forall x \in X$, $0 \le \mu_A^L(x_i) + \nu_A^L(x_i) \le 1$, $0 \le \mu_A^U(x_i) + \nu_A^U(x_i) \le 1$. $\mu_A^U(x)$ and $\mu_A^L(x)$ means that element x in X belonging to the upper and lower bound of the interval-valued set A, $\nu_A^U(x)$ and $\nu_A^-(x)$ means that element x in X not belonging to the upper and lower bound of the interval-valued set A. Accordingly, the hesitation degree of x_i belonging to A can be expressed in the following form:

$$\pi_A(x_i) = [\pi_A^L(x_i), \pi_A^U(x_i)] = [1 - \mu_A^U(x_i) - \nu_A^U(x_i), 1 - \mu_A^L(x_i) - \nu_A^L(x_i)] \tag{2}$$

Obviously, for $\forall x \in X$, there are $0 \le \pi_A^L(x_i) \le 1$ and $0 \le \pi_A^U(x_i) \le 1$.

The complementary set of IVIFS A can be expressed in the following form:

$$A^c = \{\langle x_i, [\nu_A^L(x_i), \nu_A^U(x_i)], [\mu_A^L(x_i), \mu_A^U(x_i)]\rangle | x_i \in X\} \tag{3}$$

2.3. *Basic operation of IVIFS*

Let $\alpha_1(x_i) = ([\mu_1^L(x_i), \mu_1^U(x_i)], [\nu_1^L(x_i), \nu_1^U(x_i)])$ and $\alpha_2(x_i) = ([\mu_2^L(x_i), \mu_2^U(x_i)], [\nu_2^L(x_i), \nu_2^U(x_i)])$ be any two sets of interval-valued intuitionistic fuzzy numbers of IVIFS A. Then they follow the following operational rules [34–37]:

(1) $$\begin{aligned}\alpha_1(x_i)+\alpha_2(x_i) = (&[\mu_1^L(x_i) + \mu_2^L(x_i) - \mu_1^L(x_i) \cdot \mu_2^L(x_i), \mu_1^U(x_i) + \mu_2^U(x_i) - \mu_1^U(x_i) \cdot \mu_2^U(x_i)],\\ &[\nu_1^L(x_i) \cdot \nu_2^L(x_i), \nu_1^U(x_i) \cdot \nu_2^U(x_i)]);\end{aligned}$$

(2) $$\begin{aligned}\alpha_1(x_i) \cdot \alpha_2(x_i) = (&[\mu_1^L(x_i) \cdot \mu_2^L(x_i), \mu_1^U(x_i) \cdot \mu_2^U(x_i)], [\nu_1^L(x_i) + \nu_2^L(x_i) - \nu_1^L(x_i) \cdot \nu_2^L(x_i),\\ &\nu_1^U(x_i) + \nu_2^U(x_i) - \nu_1^U(x_i) \cdot \nu_2^U(x_i)]);\end{aligned}$$

(3) $\lambda\alpha_1(x_i) = \left(\left[1-(1-\mu_1^L(x_i))^\lambda, 1-(1-\mu_1^U(x_i))^\lambda\right], \left[(\nu_1^L(x_i))^\lambda, (\nu_1^U(x_i))^\lambda\right]\right), \ \lambda > 0.$;

(4) $(\alpha_1(x_i))^\lambda = \left(\left[(\mu_1^L(x_i))^\lambda, (\mu_1^U(x_i))^\lambda\right], \left[1-(1-\nu_1^L(x_i))^\lambda, 1-(1-\nu_1^U(x_i))^\lambda\right]\right), \ \lambda > 0.$;

(5) $\alpha_1(x_i) \subseteq \alpha_2(x_i)$, iff $\forall x_i \in X$, $\mu_{\alpha_1}^L(x_i) \le \mu_{\alpha_2}^L(x_i)$, $\mu_{\alpha_1}^U(x_i) \le \mu_{\alpha_2}^U(x_i)$, $\nu_{\alpha_1}^L(x_i) \ge \nu_{\alpha_2}^L(x_i)$ and $\nu_{\alpha_1}^U(x_i) \ge \nu_{\alpha_2}^U(x_i)$.

2.4. Distance measurement of IVIFS

According to the geometric interpretation of intuitionistic fuzzy set, Szmidt and Kacprzyk [38] defined the distance of intuitionistic fuzzy set, and defined the distance of intuitionistic fuzzy set. The membership degree, non-membership degree and direct hesitation degree are all taken into account. This idea is extended to the distance definition of interval-valued intuitionistic fuzzy set.

Let two sets of IVIFS A and B, where $A = \left\{\left\langle x_i, \left[\mu_A^L(x_i), \mu_A^U(x_i)\right], \left[\nu_A^L(x_i), \nu_A^U(x_i)\right]\right\rangle | x_i \in X\right\}$, $B = \left\{\left\langle x_i, \left[\mu_B^L(x_i), \mu_B^U(x_i)\right], \left[\nu_B^L(x_i), \nu_B^U(x_i)\right]\right\rangle | x_i \in X\right\}$, and their normalized Euclidean distance can be defined as follows:

$$\begin{aligned} d(A,B) = \Bigg\{ \frac{1}{4m} \sum\nolimits_{i=1}^{m} \Big[&\left(\mu_A^L(x_i) - \mu_B^L(x_i)\right)^2 + \left(\mu_A^U(x_i) - \mu_B^U(x_i)\right)^2 \\ &+ \left(\nu_A^L(x_i) - \nu_B^L(x_i)\right)^2 + \left(\nu_A^U(x_i) - \nu_B^U(x_i)\right)^2 \\ &+ \left(\pi_A^L(x_i) - \pi_B^L(x_i)\right)^2 + \left(\pi_A^U(x_i) - \pi_B^U(x_i)\right)^2 \Big] \Bigg\}^{\frac{1}{2}} \end{aligned} \tag{4}$$

The distance measurement satisfies the following basic properties:

(P1) Nonnegative: $d(A,B) \ge 0$.

(P2) Identity: $d(A,B) = 0$, iff $A = B$.

(P3) Symmetry: $d(A,B) = d(B,A)$.

(P4) Triangle Inequality: for any three sets of IVIFS, A, B and C, $d(A,C) \le d(A,B) + d(B,C)$.

Proof.

(P1) According to the square result of any real number is non-negative number, so there must be $d(A,B) \ge 0$.

(P2) if $A = B$, i.e., $\mu_A^L(x_i) = \mu_B^L(x_i)$, $\mu_A^U(x_i) = \mu_B^U(x_i)$, $\nu_A^L(x_i) = \nu_B^L(x_i)$, $\nu_A^U(x_i) = \nu_B^U(x_i)$, $\pi_A^L(x_i) = \pi_B^L(x_i)$, $\pi_A^U(x_i) = \pi_B^U(x_i)$, from Eq (5) we can obtain $d(A,B) = 0$.

On the other hand, if $d(A,B) = 0$, i.e., $\mu_A^L(x_i) - \mu_B^L(x_i) = 0$, $\mu_A^U(x_i) - \mu_B^U(x_i) = 0$, $\nu_A^L(x_i) - \nu_B^L(x_i) = 0$, $\nu_A^U(x_i) - \nu_B^U(x_i) = 0$, $\pi_A^L(x_i) - \pi_B^L(x_i) = 0$, $\pi_A^U(x_i) - \pi_B^U(x_i) = 0$, so $A = B$.

(P3) Because $\left(\mu_A^L(x_i) - \mu_B^L(x_i)\right)^2 = \left(\mu_B^L(x_i) - \mu_A^L(x_i)\right)^2 \ldots \left(\pi_A^U(x_i) - \pi_B^U(x_i)\right)^2 = \left(\pi_B^U(x_i) - \pi_A^U(x_i)\right)^2$, so $d(A,B) = d(B,A)$.

(P4) Firstly, we introduce the Minkowski inequation [39]. Let R^n be n-dimensional Euclidean space, $(a_1, a_2, \cdots, a_n)$, $(b_1, b_2, \cdots, b_n) \in R^n$ and $1 \le p < \infty$, then the following inequation holds:

$$\left(\sum_{k=1}^{n} |a_k + b_k|^p\right)^{\frac{1}{p}} \le \left(\sum_{k=1}^{n} |a_k|^p\right)^{\frac{1}{p}} + \left(\sum_{k=1}^{n} |b_k|^p\right)^{\frac{1}{p}}$$

We can know from Minkowski inequation that:

$$d\left(A,C\right)=\left\{\frac{1}{4m}\sum_{i=1}^{m}\left[\left(\mu_A^L(x_i)-\mu_C^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_C^U(x_i)\right)^2+\left(\nu_A^L(x_i)-\nu_C^L(x_i)\right)^2+\left(\nu_A^U(x_i)-\nu_C^U(x_i)\right)^2\right.\right.$$

$$\left.\left.+\left(\pi_A^L(x_i)-\pi_C^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_C^U(x_i)\right)^2\right]\right\}^{\frac{1}{2}}$$

$$=\left\{\frac{1}{4m}\sum_{i=1}^{m}\left[\left(\mu_A^L(x_i)-\mu_B^L(x_i)+\mu_B^L(x_i)-\mu_C^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_B^U(x_i)+\mu_B^U(x_i)-\mu_C^U(x_i)\right)^2\right.\right.$$

$$+\left(\nu_A^L(x_i)-\nu_B^L(x_i)+\nu_B^L(x_i)-\nu_C^L(x_i)\right)^2+\left(\nu_A^U(x_i)-\nu_B^U(x_i)+\nu_B^U(x_i)-\nu_C^U(x_i)\right)^2$$

$$\left.\left.+\left(\pi_A^L(x_i)-\pi_B^L(x_i)+\pi_B^L(x_i)-\pi_C^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_B^U(x_i)+\pi_B^U(x_i)-\pi_C^U(x_i)\right)^2\right]\right\}^{\frac{1}{2}}$$

$$\leq\left\{\frac{1}{4m}\sum_{i=1}^{m}\left[\left(\mu_A^L(x_i)-\mu_B^L(x_i)\right)^2+\left(\mu_A^U(x_i)-\mu_B^U(x_i)\right)^2+\left(\nu_A^L(x_i)-\nu_B^L(x_i)\right)^2+\left(\nu_A^U(x_i)-\nu_B^U(x_i)\right)^2\right.\right.$$

$$\left.\left.+\left(\pi_A^L(x_i)-\pi_B^L(x_i)\right)^2+\left(\pi_A^U(x_i)-\pi_B^U(x_i)\right)^2\right]\right\}^{\frac{1}{2}}+\left\{\frac{1}{4m}\sum_{i=1}^{m}\left[\left(\mu_B^L(x_i)-\mu_C^L(x_i)\right)^2+\left(\mu_B^U(x_i)-\mu_C^U(x_i)\right)^2\right.\right.$$

$$\left.\left.+\left(\nu_B^L(x_i)-\nu_C^L(x_i)\right)^2+\left(\nu_B^U(x_i)-\nu_C^U(x_i)\right)^2+\left(\pi_B^L(x_i)-\pi_C^L(x_i)\right)^2+\left(\pi_B^U(x_i)-\pi_C^U(x_i)\right)^2\right]\right\}^{\frac{1}{2}}$$

That is $d\left(A,C\right)\leq d\left(A,B\right)+d\left(B,C\right)$.

2.5. Interval-Valued Intuitionistic Fuzzy Entropy

The concept of entropy originated from thermodynamics, and was introduced into information theory by Shannon to represent the size of information. The higher the order degree of a system is, the smaller its entropy is, and the greater the amount of information it contains. On the contrary, the higher the degree of disorder, the greater the entropy, the smaller the amount of information.

Intuitionistic Fuzzy Entropy can objectively reflect the order of IFS. If the lower the entropy value of an attribute, the greater the uncertainty, and then the greater the weight should be given [40]. Reference [41–43] constructed different forms of Intuitionistic Fuzzy Entropy, but did not consider the effect of hesitancy degree on the uncertainty of IFS and thus has some limitations.

In fact, Intuitionistic Fuzzy Entropy contains two kinds of information: the degree of uncertainty and the degree of unknown. The degree of uncertainty is usually expressed by the absolute deviation of membership degree and non-membership degree, and the degree of unknown can be expressed by hesitation degree. Based on this, reference [44] constructed a new Interval-Valued Intuitionistic Fuzzy Entropy based on hesitation degree, which can overcome the shortcoming of only consider uncertainty degree but ignore unknown degree in the former Intuitionistic Fuzzy Entropy.

For any set of IVIFS, $A=\left\{\left\langle x_i,\mu_A\left(x_i\right),\nu_A\left(x_i\right)\right\rangle \middle| x_i\in X,i=1,2,\cdots,m\right\}$, define the Interval-Valued Intuitionistic Fuzzy Entropy as follows:

$$E\left(A\right)=\frac{1}{m}\sum_{i=1}^{m}\frac{\min\left\{\mu_A^L\left(x_i\right),\nu_A^L\left(x_i\right)\right\}+\min\left\{\mu_A^U\left(x_i\right),\nu_A^U\left(x_i\right)\right\}+\pi_A^L\left(x_i\right)+\pi_A^U\left(x_i\right)}{\max\left\{\mu_A^L\left(x_i\right),\nu_A^L\left(x_i\right)\right\}+\max\left\{\mu_A^U\left(x_i\right),\nu_A^U\left(x_i\right)\right\}+\pi_A^L\left(x_i\right)+\pi_A^U\left(x_i\right)}\tag{5}$$

As can be seen from Eq (5), the Interval-Valued Intuitionistic Fuzzy Entropy $E\left(A\right)$ includes not only the information of membership degree $\mu_A\left(x_i\right)$ and non-membership degree $\nu_A\left(x_i\right)$, but also the information of hesitation degree $\pi_A\left(x_i\right)$.

For any two sets of IVIFS, A and B, the above measure of the interval-valued intuitionistic fuzzy entropy satisfies the following axiomatic requirements:

(P1) $E(A)=0$, iff A is a crisp set;

(P2) $E(A)=1$, iff $\mu_A(x_i)=\nu_A(x_i)$, $\forall x_i \in X$;

(P3) if $A \subseteq B$ and $\mu_B^L(x_i) \le \nu_B^L(x_i)$, $\mu_B^U(x_i) \le \nu_B^U(x_i)$, i.e., $\mu_A^L(x_i) \le \mu_B^L(x_i) \le \nu_B^L(x_i) \le \nu_A^L(x_i)$, $\mu_A^U(x_i) \le \mu_B^U(x_i) \le \nu_B^U(x_i) \le \nu_A^U(x_i)$, $\forall x_i \in X$, $E(A) \le E(B)$;

(P4) $E(A)=E(A^c)$.

Proof.

(P1) If A is a crisp set, i.e., $\mu_A(x_i)=[1,1]$, $\nu_A(x_i)=[0,0]$ or $\mu_A(x_i)=[0,0]$, $\nu_A(x_i)=[1,1]$, $\forall x_i \in X$. No matter in which case, we have $E(A)=0$ for Eq (5).

On the other hand, if $E(A)=0$, i.e., $\min\{\mu_A^L(x_i),\nu_A^L(x_i)\}+\min\{\mu_A^U(x_i),\nu_A^U(x_i)\}+\pi_A^L(x_i)+\pi_A^U(x_i)$, $\forall x_i \in X$. So we have:

$\min\{\mu_A^L(x_i),\nu_A^L(x_i)\}=0$, $\min\{\mu_A^U(x_i),\nu_A^U(x_i)\}=0$, $\pi_A^L(x_i)=0=1-\mu_A^L(x_i)-\nu_A^L(x_i)$, $\pi_A^U(x_i)=0=1-\mu_A^U(x_i)-\nu_A^U(x_i)$, i.e., $\mu_A(x_i)=[1,1]$, $\nu_A(x_i)=[0,0]$ or $\mu_A(x_i)=[0,0]$, $\nu_A(x_i)=[1,1]$, $\forall x_i \in X$. Hence, A is a crisp set.

(P2) Let $\mu_A(x_i)=\nu_A(x_i)$ for any $x_i \in X$. From Eq (5) we can obtain $E(A)=1$. On the other hand, if $E(A)=1$, i.e., $\min\{\mu_A^L(x_i),\nu_A^L(x_i)\}+\min\{\mu_A^U(x_i),\nu_A^U(x_i)\}=\max\{\mu_A^L(x_i),\nu_A^L(x_i)\}+\max\{\mu_A^U(x_i),\nu_A^U(x_i)\}$, $\forall x_i \in X$. So we have $\mu_A^L(x_i)=\nu_A^L(x_i), \mu_A^U(x_i)=\nu_A^U(x_i)$, $\forall x_i \in X$. Hence, $\mu_A(x_i)=\nu_A(x_i)$, $\forall x_i \in X$.

(P3) if $A \subseteq B$ and $\mu_B^L(x_i) \le \nu_B^L(x_i)$, $\mu_B^U(x_i) \le \nu_B^U(x_i)$, $\forall x_i \in X$. We can get $\min\{\mu_A^L(x_i),\nu_A^L(x_i)\}=\mu_A^L(x_i)$, $\min\{\mu_A^U(x_i),\nu_A^U(x_i)\}=\mu_A^U(x_i)$, $\max\{\mu_A^L(x_i),\nu_A^L(x_i)\}=\nu_A^L(x_i)$, $\min\{\mu_A^U(x_i),\nu_A^U(x_i)\}=\nu_A^U(x_i)$.

Thus,

$$E(A)=\frac{1}{m}\sum_{i=1}^{m}\frac{\mu_A^L(x_i)+\mu_A^U(x_i)+\pi_A^L(x_i)+\pi_A^U(x_i)}{\nu_A^L(x_i)+\nu_A^U(x_i)+\pi_A^L(x_i)+\pi_A^U(x_i)}=\frac{1}{m}\sum_{i=1}^{m}\frac{2-\left(\nu_A^L(x_i)+\nu_A^U(x_i)\right)}{2-\left(\mu_A^L(x_i)+\mu_A^U(x_i)\right)}.$$

Similarly, $E(B)=\frac{1}{m}\sum_{i=1}^{m}\frac{2-\left(\nu_B^L(x_i)+\nu_B^U(x_i)\right)}{2-\left(\mu_B^L(x_i)+\mu_B^U(x_i)\right)}$.

From the criteria above, we can get $\nu_A^L(x_i)+\nu_A^U(x_i) \ge \nu_B^L(x_i)+\nu_B^U(x_i)$ and $\mu_A^L(x_i)+\mu_A^U(x_i) \le \mu_B^L(x_i)+\mu_B^U(x_i)$. Hence, $E(A) \le E(B)$.

(P4) It is clear that $A^c=\left\{\left\langle x_i,\left[\nu_A^-(x_i),\nu_A^+(x_i)\right],\left[\mu_A^-(x_i),\mu_A^+(x_i)\right]\right\rangle | x_i \in X\right\}$. From Eq (5) we can obtain $E(A)=E(A^c)$.

3. Interval-valued intuitionistic fuzzy TOPSIS-GCA method

The basic idea of TOPSIS method is to construct the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS) of the MADM problems, and to evaluate the alternatives based on the criteria of near PIS and far from NIS. When TOPSIS method is used to solve MADM problems, it is based on the original data sample, and the analysis is based on the distance relationship between data series. The GCA can directly reflect the nonlinear relationship between data series. The closer the curve shape is, the greater the grey correlation degree of the corresponding series is. For MADM problems, if the grey correlation degree between alternatives and ideal solution is higher, the alternatives can be considered to be closer to ideal solution. This hybrid method combines the grey correlation analysis to calculate the weighted grey correlation degree of the alternatives with absolute PIS and absolute NIS, and TOPSIS method to calculate the weighted Euclidean distance for the alternatives from absolute PIS and absolute NIS, for solving interval-valued intuitionistic fuzzy MADM problems. The steps of this proposed method are shown in Figure 1.

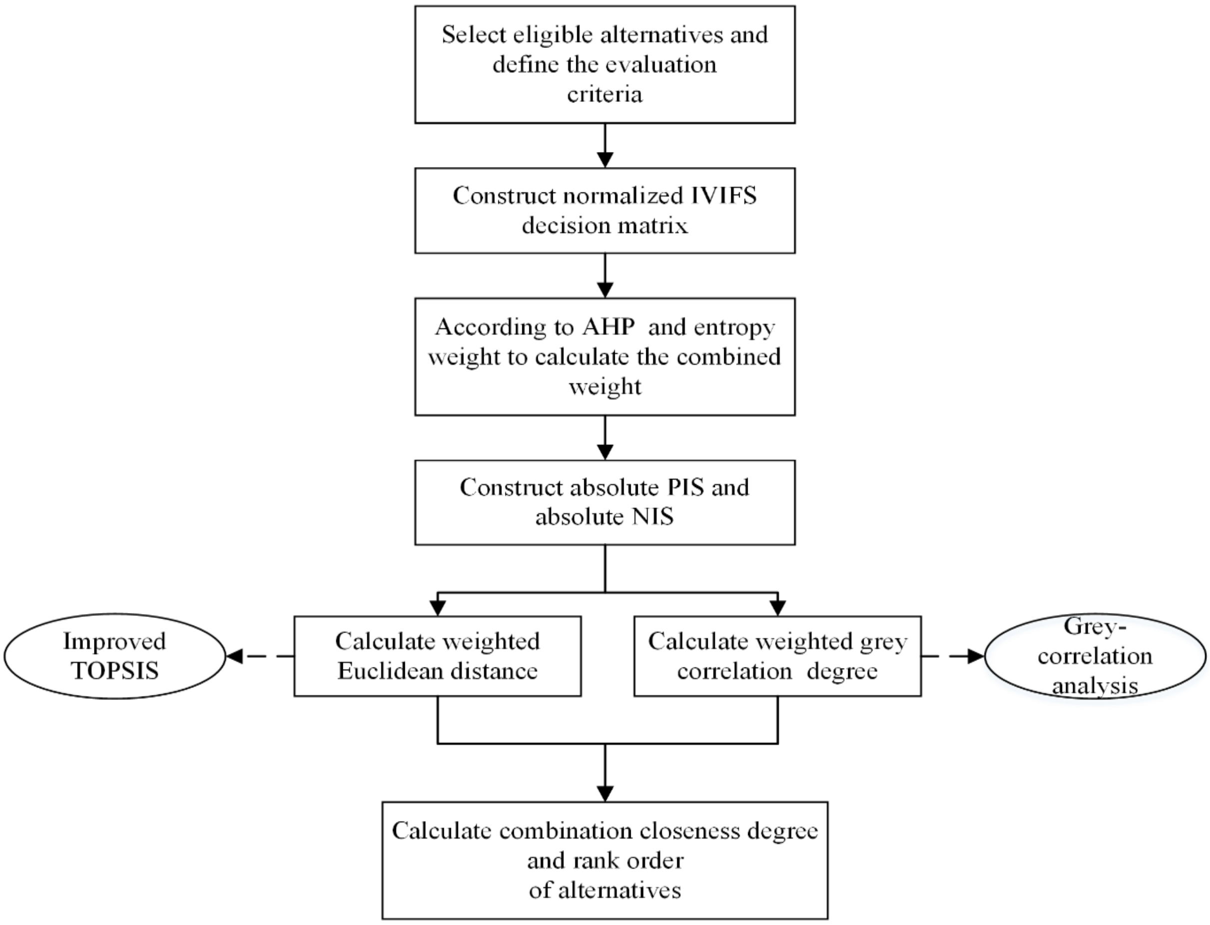

Figure 1. The steps of TOPSIS-GCA method.

The calculation steps of the proposed interval-valued intuitionistic fuzzy TOPSIS-GCA method are as follows:

Step 1: Establish the standard decision matrix in the form of interval-valued intuitionistic fuzzy numbers;

Step 2: Calculate the combined weight of each attribute;

Step 3: Construct absolute PIS and absolute NIS in the form of interval-valued intuitionistic fuzzy numbers;

Step 4: Calculate the weighted Euclidean distances for each alternative from the absolute PIS and the absolute NIS;

Step 5: Calculate the weighted grey correlation degrees for each alternative from the absolute PIS and the absolute NIS;

Step 6: Normalize the weighted Euclidean distances and weighted grey correlation degrees.

Calculate the combination closeness degrees between the alternatives and the ideal solution and rank order of alternatives.

3.1. Establish decision matrix and its standardization

For an interval-valued intuitionistic fuzzy MADM problem with m number of alternatives, $X=\{X_1,X_2,\cdots,X_m\}$ .And each alternative has n number of attributes, $A=\{A_1,A_2,\cdots,A_n\}$. the interval-valued intuitionistic fuzzy decision matrix $H=\left(h_{ij}\right)_{m\times n}$ can be expressed as follows:

$$H=\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix} \tag{6}$$

where h_{ij} represents the j^{th} attribute value with respect to i^{th} alternative, and h_{ij} is in the form of interval-valued intuitionistic fuzzy number.

Consider that there are two main types of attribute categories in the decision-making process:

(1) Benefit attribute: the bigger the attribute value, the better;

(2) Cost attribute: the smaller the attribute value, the better, then it is necessary to convert the value of the cost attribute to the value of the benefit attribute in accordance with Eq (7).

$$r_{ij}=\begin{cases} h_{ij}, & \textit{if } A_n \textit{ is benefit attribute,} \\ h_{ij}^{c}, & \textit{if } A_n \textit{ is cost attribute.} \end{cases} \tag{7}$$

where h_{ij}^{c} is the complementary set of h_{ij} .

The standardized decision matrix $R=\left(r_{ij}\right)_{m\times n}$ is shown in Eq (8).

$$R=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{8}$$

where $r_{ij}=\left(\left[\mu_{ij}^{-},\mu_{ij}^{+}\right],\left[\nu_{ij}^{-},\nu_{ij}^{+}\right]\right)$.

3.2. Calculate the combined weight of each attribute

After the standardized decision matrix is constructed, the attribute weights are determined by the combination of AHP and Interval-Valued Intuitionistic Fuzzy Entropy weighting method. The combination weighting method combines the advantages of subjective weighting method and objective weighting method, and avoids the disadvantage of single weighting method.

The Analytic Hierarchy Process (AHP) quantitatively obtains the judgment matrix according to the relative importance degree of any two attributes, and then obtains the consistency matrix through the consistency check. Finally, the weight vector of each attribute can be obtained by computing the consistency matrix. Reference [45] introduces the detailed calculation process. The subjective weight vector of each attribute is $W_1=\left(\alpha_1,\alpha_2,\cdots,\alpha_n\right)$.

The entropy weight of attribute may be computed as follows:

$$\beta_j = \frac{\left|1 - E(A_j)\right|}{\sum_{j=1}^{n}\left|1 - E(A_j)\right|} \tag{9}$$

where $\beta_j \in [0,1], \sum_{j=1}^{n}\beta_j = 1$, $E(A_j)$ is the Interval-Valued Intuitionistic Fuzzy Entropy in section 2.5 and $0 \le E(A_j) \le 1$.

The entropy weight vector of each attribute obtained by Interval-Valued Intuitionistic Fuzzy Entropy weighting method is $W_2 = (\beta_1, \beta_2, \cdots, \beta_n)$.

The subjective weight and entropy weight are integrated to obtain combined weight by Eq (10).

$$\omega_j = \frac{\sqrt{\alpha_j \beta_j}}{\sum_{j=1}^{n}\sqrt{\alpha_j \beta_j}} \tag{10}$$

The combination weight vector of attributes is $W = (\omega_1, \omega_2, \cdots, \omega_n)$.

3.3. *Construct absolute PIS and absolute NIS*

The Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS) in the traditional TOPSIS are both taken from the actual measurements of the alternatives, so different alternative sets will have different PIS and NIS. The PIS and the NIS must be re-selected and calculated when new alternatives are added or some of the alternatives are removed, and the result of the ranking may change, which is called reverse order phenomenon [46]. In order to solve this problem, this paper put forward the concept of absolute ideal solution, which is to take the limit value of the optimal state and the worst state of each attribute of the alternative as the PIS and NIS. The absolute PIS and the absolute NIS in the form of interval-valued intuitionistic fuzzy numbers are shown as follows:

$$r_j^+ = \left(\left[\mu_j^{+L}, \mu_j^{+U}\right], \left[\nu_j^{+L}, \nu_j^{+U}\right]\right) = ([1,1],[0,0]) \tag{11}$$

$$r_j^- = \left(\left[\mu_j^{-L}, \mu_j^{-U}\right], \left[\nu_j^{-L}, \nu_j^{-U}\right]\right) = ([0,0],[1,1]) \tag{12}$$

The corresponding hesitation degree are $\pi_j^+ = \left[\pi_j^{+L}, \pi_j^{+U}\right] = [0,0]$ and $\pi_j^- = \left[\pi_j^{-L}, \pi_j^{-U}\right] = [0,0]$ respectively.

3.4. *Calculate the weighted Euclidean distances for each alternative from the absolute PIS and absolute NIS*

According to the distance measurement of IVIFS, the weighted Euclidean distances for each alternative from the absolute PIS and the absolute NIS may be calculated as follows:

$$\begin{aligned} d_i^+ = \Bigg\{ \frac{1}{4n}\sum_{j=1}^{n}\omega_j^2 \Big[& \left(\mu_{ij}^L - \mu_j^{+L}\right)^2 + \left(\mu_{ij}^U - \mu_j^{+U}\right)^2 + \left(\nu_{ij}^L - \nu_j^{+L}\right)^2 \\ & + \left(\nu_{ij}^U - \nu_j^{+U}\right)^2 + \left(\pi_{ij}^L - \pi_j^{+L}\right)^2 + \left(\pi_{ij}^U - \pi_j^{+U}\right)^2 \Big] \Bigg\}^{\frac{1}{2}} \end{aligned} \tag{13}$$

$$\begin{aligned} d_i^- = \Bigg\{ \frac{1}{4n}\sum_{j=1}^{n}\omega_j^2 \Big[& \left(\mu_{ij}^L - \mu_j^{-L}\right)^2 + \left(\mu_{ij}^U - \mu_j^{-U}\right)^2 + \left(\nu_{ij}^L - \nu_j^{-L}\right)^2 \\ & + \left(\nu_{ij}^U - \nu_j^{-U}\right)^2 + \left(\pi_{ij}^L - \pi_j^{-L}\right)^2 + \left(\pi_{ij}^U - \pi_j^{-U}\right)^2 \Big] \Bigg\}^{\frac{1}{2}} \end{aligned} \tag{14}$$

where $r_j^+ = \left(\left[\mu_j^{+L}, \mu_j^{+U}\right], \left[v_j^{+L}, v_j^{+U}\right]\right) = ([1,1],[0,0])$, $r_j^- = \left(\left[\mu_j^{-L}, \mu_j^{-U}\right], \left[v_j^{-L}, v_j^{-U}\right]\right) = ([0,0],[1,1])$, $\pi_j^+ = \left[\pi_j^{+L}, \pi_j^{+U}\right] = [0,0]$ and $\pi_j^- = \left[\pi_j^{-L}, \pi_j^{-U}\right] = [0,0]$.

3.5. Calculate the weighted grey correlation degrees for each alternative from the absolute PIS and the absolute NIS

The grey correlation coefficients for each alternative from the absolute PIS and the absolute NIS are shown below:

$$\xi_{ij}^+ = \frac{\min\limits_i \min\limits_j \left|r_{ij} - r_j^+\right| + \rho \max\limits_i \max\limits_j \left|r_{ij} - r_j^+\right|}{\left|r_{ij} - r_j^+\right| + \rho \max\limits_i \max\limits_j \left|r_{ij} - r_j^+\right|} \tag{15}$$

where $\left|r_{ij} - r_j^+\right| = \frac{1}{4}\left(\left|\mu_{ij}^L - \mu_j^{+L}\right| + \left|\mu_{ij}^U - \mu_j^{+U}\right| + \left|v_{ij}^L - v_j^{+L}\right| + \left|v_{ij}^U - v_j^{+U}\right|\right)$.

$$\xi_{ij}^- = \frac{\min\limits_i \min\limits_j \left|r_{ij} - r_j^-\right| + \rho \max\limits_i \max\limits_j \left|r_{ij} - r_j^-\right|}{\left|r_{ij} - r_j^-\right| + \rho \max\limits_i \max\limits_j \left|r_{ij} - r_j^-\right|} \tag{16}$$

where $\left|r_{ij} - r_j^-\right| = \frac{1}{4}\left(\left|\mu_{ij}^L - \mu_j^{-L}\right| + \left|\mu_{ij}^U - \mu_j^{-U}\right| + \left|v_{ij}^L - v_j^{-L}\right| + \left|v_{ij}^U - v_j^{-U}\right|\right)$, ξ_{ij}^+ and ξ_{ij}^- are the grey correlation coefficients of the distances computed for each criterion from the absolute PIS and the absolute NIS, and ρ is the distinguishing coefficient having value between 0 and 1. Its value is usually considered as 0.5.

Then the weighted grey correlation degrees for each alternative from the absolute PIS and the absolute NIS may be calculated as follows:

$$\xi_i^+ = \sum\nolimits_{j=1}^n \xi_{ij}^+ \omega_j, \quad i = 1, 2, \cdots, m. \tag{17}$$

$$\xi_i^- = \sum\nolimits_{j=1}^n \xi_{ij}^- \omega_j, \quad i = 1, 2, \cdots, m. \tag{18}$$

3.6. Normalize the weighted Euclidean distances and weighted grey correlation degrees

The normalized treatment is based on the following generalized equation:

$$\alpha_i = \frac{\beta_i}{\max \beta_i} \tag{19}$$

where $i = 1, 2, \cdots, m$, β_i represents d_i^+ , d_i^- , ξ_i^+ and ξ_i^- , accordingly, α_i represents D_i^+ , D_i^- , E_i^+ and E_i^+.

3.7. Calculate the combination closeness degrees between the alternatives and the ideal solution and rank order of alternatives

According to the improved TOPSIS and grey correlation analysis, if $D_i^+ + E_i^-$ is bigger, the corresponding alternative is closer to the PIS, and if $D_i^- + E_i^+$ is bigger, the corresponding alternative is closer to the NIS. Considering the weighted distance and the weighted grey correlation degree, we can get the close degree coefficient as follows:

$$V_i^+ = \alpha D_i^- + \beta E_i^+, \quad i = 1, 2, \cdots, m. \tag{20}$$

$$V_i^- = \alpha D_i^+ + \beta E_i^-, \quad i = 1, 2, \cdots, m. \tag{21}$$

where α and β are preference coefficients, which reflects the preference degree of decision-maker to the Euclidean distance and the grey correlation degree. They satisfy $\alpha + \beta = 1$ and $\alpha, \beta \in [0,1]$, and the decision maker can determine their values according to preferences. If the decision maker has no preference for TOPSIS and grey correlation analysis, it can make α=0.5, β=0.5. V_i^+ reflects the closeness degree between the alternatives and the ideal solution, and V_i^+ reflects the distance degree between the alternatives and the ideal solution.

The combination closeness degree may be calculated as follows:

$$Z_i = \frac{V_i^+}{V_i^+ + V_i^-}, \quad i = 1, 2, \cdots, m. \tag{22}$$

The combination closeness degree Z_i can be used as the comprehensive evaluation value of the alternatives. It is evident from the equation above that if Z_i is bigger for an alternative, it is near PIS and far from NIS. Thus, a bigger Z_i is rightly connected with a higher rank.

4. Case study

Complex system is a kind of information system with large number and variety of subsystems, and the subsystems have very complicated nonlinear relationship with each other. Reliability evaluation of complex system is to analyze, calculate and evaluate the reliability of simulation system. The purpose of reliability evaluation of complex system includes evaluation, selection and modification of complex system. The quantitative results of complex system reliability can be obtained by reliability evaluation, and then the complex system can be judged and identified. It is difficult to meet the needs of reliability evaluation only by using a single method for complex system. The reliability evaluation of complex system has gradually evolved into a complex multi-attribute comprehensive evaluation problem. Besides, due to the complexity of the system and the limitations of the experts, the evaluation of the complex system is approximate and qualitative. It is suitable for experts use interval-valued intuitionistic fuzzy numbers to evaluate the reliability of simulation system.

4.1. Example application

With the development of information technology and the wide application of information network system, the reliability evaluation of complex information network system becomes more and more important, which is helpful for ensuring the safe operation of information network system. Taking a certain information network system as an example, the evaluation attributes include: Response capability T_1, Transmission capability T_2, Real-time capability T_3, Resource utilization capability T_4, Construction cost T_5 and Anti-Jamming capability T_6. There are three options alternatives of information network system: A_1, A_2 and A_3. The data for each evaluation attribute are shown in Table 1.

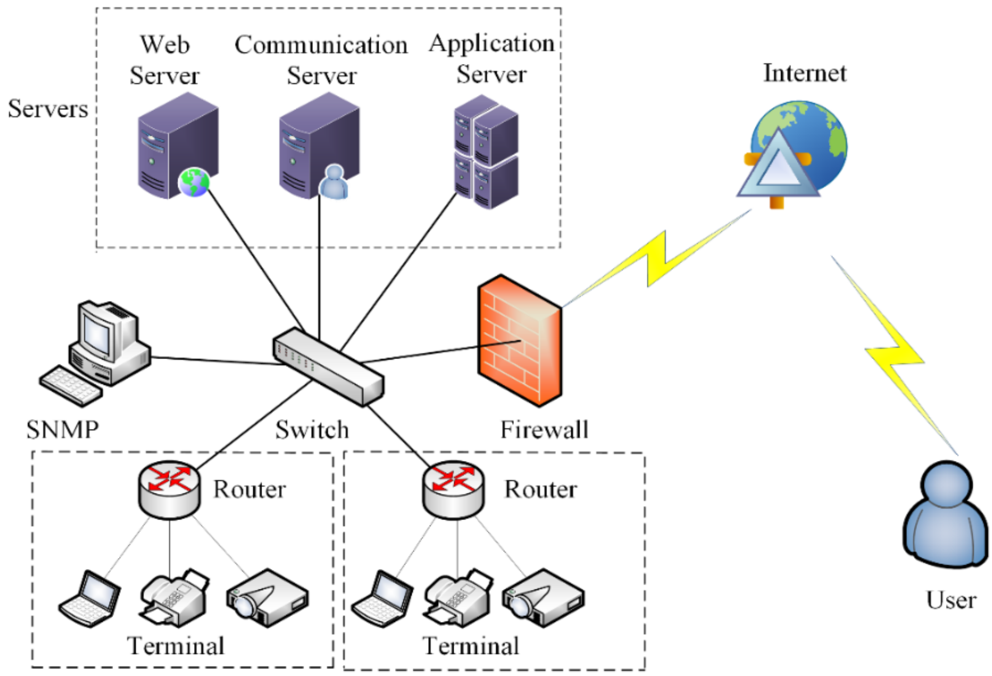

Figure 2. Sample of complex information network system.

Table 1. IVIFS performance matrix.

Alternative attributes	A_1		A_2		A_3	
	μ	ν	μ	ν	μ	ν
T_1	[0.30,0.45]	[0.40,0.45]	[0.60,0.70]	[0.15,0.25]	[0.30,0.40]	[0.45,0.50]
T_2	[0.25,0.45]	[0.35,0.50]	[0.45,0.55]	[0.40,0.45]	[0.20,0.25]	[0.65,0.70]
T_3	[0.60,0.70]	[0.15,0.25]	[0.35,0.40]	[0.45,0.55]	[0.85,0.85]	[0.05,0.10]
T_4	[0.50,0.60]	[0.35,0.40]	[0.15,0.35]	[0.55,0.60]	[0.45,0.65]	[0.25,0.30]
T_5	[0.25,0.40]	[0.35,0.55]	[0.80,0.85]	[0.05,0.10]	[0.60,0.70]	[0.15,0.25]
T_6	[0.65,0.70]	[0.15,0.25]	[0.30,0.40]	[0.35,0.55]	[0.55,0.70]	[0.15,0.20]

4.1.1. Construct standardized decision matrix

Among the evaluation attributes, Response capability T_1, Transmission capability T_2, Resource utilization capability T_4 and Anti-Jamming capability T_6 belong to benefit attribute, Real-time capability T_3 and Construction cost T_5 belong to cost index. The indicator data in Table 1 are processed in accordance with Eq (7) to obtain a standardized decision matrix, as shown in Table 2.

4.1.2. Calculate the combined weight of attributes

The subjective weight of attributes is obtained by AHP method, and the entropy weight is obtained by interval-valued intuitionistic fuzzy entropy method. Finally, the combined weight of attributes is obtained by Eq (10). The results are shown in Table 3.

Table 2. IVIFS performance matrix of standardization.

Alternative attributes	A_1		A_2		A_3	
	μ	ν	μ	ν	μ	ν
T_1	[0.30,0.45]	[0.40,0.45]	[0.60,0.70]	[0.15,0.25]	[0.30,0.40]	[0.45,0.50]
T_2	[0.25,0.45]	[0.35,0.50]	[0.45,0.55]	[0.40,0.45]	[0.20,0.25]	[0.65,0.70]
T_3	[0.15,0.25]	[0.60,0.70]	[0.45,0.55]	[0.35,0.40]	[0.05,0.10]	[0.80,0.85]
T_4	[0.50,0.60]	[0.35,0.40]	[0.15,0.35]	[0.55,0.60]	[0.45,0.65]	[0.25,0.30]
T_5	[0.35,0.55]	[0.25,0.40]	[0.05,0.10]	[0.80,0.85]	[0.15,0.25]	[0.60,0.70]
T_6	[0.65,0.70]	[0.15,0.25]	[0.30,0.40]	[0.35,0.55]	[0.55,0.70]	[0.15,0.20]

Table 3. Weights of attributes.

Attributes weights	Attributes					
	T_1	T_2	T_3	T_4	T_5	T_6
Subjective weight	0.183	0.212	0.145	0.191	0.108	0.161
Entropy weight	0.201	0.200	0.130	0.177	0.134	0.158
Combined weight	0.192	0.206	0.138	0.184	0.121	0.159

The comparative analysis results of weights are shown in Figure 3.

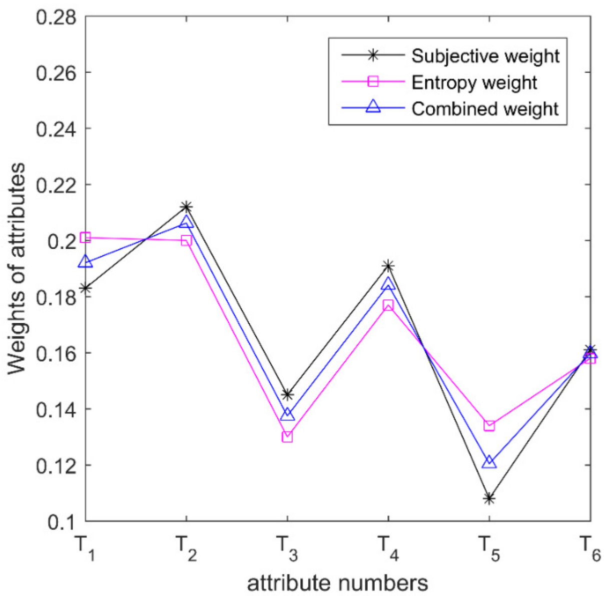

Figure 3. The comparative analysis results of weights.

It can be seen from Figure 3 that among the three weighting methods, the AHP method is greatly affected by the subjective factors of experts, which further affects the accuracy of the weighting results. Because the interval-valued intuitionistic fuzzy entropy method has strong dependence on the original data, the difference of weight distribution results is small and the accuracy is poor. The weights optimized by combination weighting method is between AHP method and the interval-valued intuitionistic fuzzy entropy method Because the subjective and objective factors are comprehensively considered, some random and secondary factors are suppressed, and the influence of expert experience is ignored, so the weighting result is more objective and reasonable.

4.1.3. Calculate the weighted Euclidean distance and the weighted grey correlation degree

According to section 3.3, the absolute PIS and the absolute NIS for this example are shown as follows:

$$A^{+}=\left(([1,1],[0,0]),([1,1],[0,0]),([1,1],[0,0]),([1,1],[0,0]),([1,1],[0,0]),([1,1],[0,0])\right)$$

$$A^{-}=\left(([0,0],[1,1]),([0,0],[1,1]),([0,0],[1,1]),([0,0],[1,1]),([0,0],[1,1]),([0,0],[1,1])\right)$$

According to the Eqs (13) and (14), the weighted Euclidean distances between the alternatives and the absolute PIS and absolute NIS are calculated, and the weighted grey correlation degrees for each alternative from the absolute PIS and absolute NIS are obtained by Eqs (15)–(18). The results are shown in Table 4. The weighted Euclidean distances d_i^+, d_i^- and the weighted grey correlation degrees ξ_i^+, ξ_i^- are normalized according to Eq (19). The results as shown in Table 5.

Table 4. Weighted Euclidean distances and weighted grey correlation degrees.

Alternative	Distance		Grey related degree	
	d_i^+	d_i^-	ξ_i^+	ξ_i^-
A_1	0.090	0.096	0.780	0.571
A_2	0.094	0.092	0.758	0.617
A_3	0.105	0.085	0.729	0.668

Table 5. Normalized treatment to weighted Euclidean distances and weighted grey correlation degrees.

Alternative	Distance		Grey related degree	
	D_i^+	D_i^-	E_i^+	E_i^-
A_1	0.857	1.000	1.000	0.855
A_2	0.895	0.958	0.972	0.924
A_3	1.000	0.885	0.935	1.000

4.1.4. Calculate combination closeness degree

According to Eqs (20)–(22), the combination closeness degrees between the alternatives and the ideal solution are calculated. There are several cases to discuss.

When , $\alpha=0.5, \beta=0.5$ the closeness coefficients V_i^+, V_i^- and the combination closeness degree Z_i between the alternatives and the ideal solution are shown in Table 6.

Table 6. Combination closeness degree and rank.

Alternatives	V_i^+	V_i^-	Z_i	Rank
A_1	1.000	0.856	0.539	1
A_2	0.965	0.910	0.515	2
A_3	0.910	1.000	0.476	3

As can be seen from Table 6, the combination closeness degree of A_1 is 0.539, which is the largest value of three alternatives, so A_1 has the best performance.

4.2. Result analysis

The final ranking result may be changed according to the values of α and β. Taking preference coefficient α as an example (actually, β is limited to α, because $\alpha+\beta=1$.). The final result changes with α as shown in Figure 4.

When $\alpha<0.5$, the decision makers prefer grey correlation analysis. When $\alpha=0.5$, TOPSIS is as important as grey correlation analysis. And when $\alpha>0.5$, TOPSIS is more important. If the decision maker has no preference for TOPSIS and grey correlation analysis, it can make $\alpha=0.5, \beta=0.5$. It can be seen from Figure 4 that in this example, when $\alpha=0.5, \beta=0.5$, TOPSIS method and grey correlation analysis method are both used to calculate the combination closeness degree of each alternative, and the difference in the combination closeness degree Z_i of each alternative is reasonable and accord with the actual situation. At the same time, it can be seen from Figure 4 that no matter what the preference parameter α is, the ranking of the alternatives do not change, which shows the effectiveness of the interval-valued intuitionistic fuzzy TOPSIS-GCA method proposed in this paper.

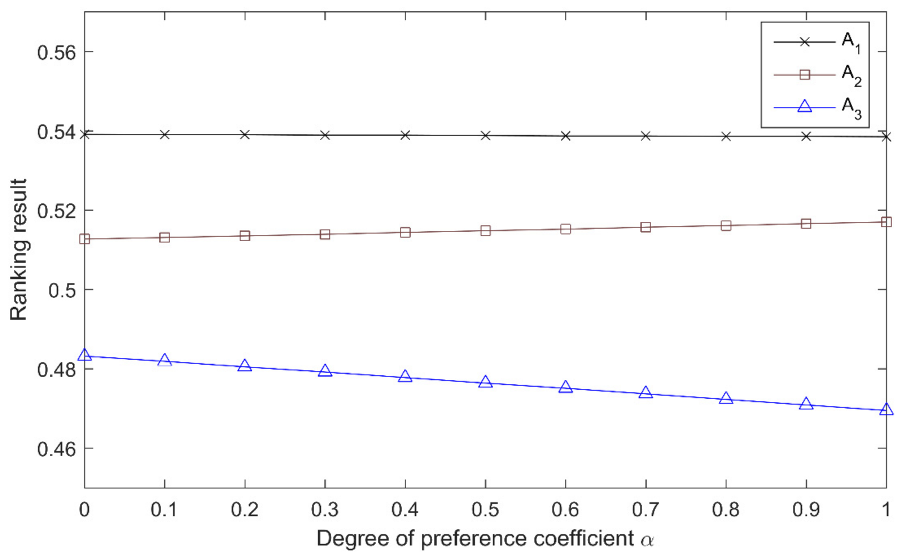

Figure 4. Sensitivity analysis for preference coefficient.

In order to further verify the validity and correctness of the interval-valued intuitionistic fuzzy

TOPSIS-GCA method in this paper, DIF-MADM [47] method and IIHA [48] method are adopted for scheme evaluation based on the examples in this paper. The ranking results of each method are shown in the Table 7.

Table 7. The ranking results of three methods.

Alternative	TOPSIS-GCA		DIF-MADM		IIHA	
	value	rank	value	rank	value	rank
A_1	0.539	**1**	0.520	**1**	0.079	**1**
A_2	0.515	**2**	0.479	**2**	0.020	**2**
A_3	0.476	**3**	0.443	**3**	-0.061	**3**

It can be seen from Table 7 that the ranking results of three methods are all the same, which reflects the correctness of the method in this paper. At the same time, the evaluation result of the interval-valued intuitionistic fuzzy TOPSIS-GCA method has a moderate gradient, which is more reasonable than the other two methods.

5. Conclusions

In this paper, an interval-valued intuitionistic fuzzy TOPSIS-GCA method is proposed by analyzing the interval-valued intuitionistic fuzzy MADM problems. Firstly, the entropy weight of attributes is obtained by introducing the concept of interval-valued intuitionistic fuzzy entropy. And the combined weight is calculated by combining the entropy weight with the subjective weight calculated by AHP method. Then combine weighted Euclidean distance and weighted grey correlation degree for each alternative from the absolute PIS and the absolute NIS to construct the combination closeness degree of each alternative. And the alternatives are ranked and evaluated according to the combination closeness degree. Finally, the effectiveness and feasibility of the proposed method in this paper are verified by an example of Rader performance evaluation. The interval-valued intuitionistic fuzzy TOPSIS-GCA method has great theoretical and practical significance, which can be widely used in the fields of economy, society, management, engineering, etc.

The interval-valued intuitionistic fuzzy TOPSIS-GCA method proposed in this paper combines the advantages of TOPSIS and grey correlation analysis. It not only reflects the degree of similarity between the alternatives and the ideal solution on the scale of physical space distance, but also reflects the degree of geometric similarity between the alternatives and the ideal solution, which makes the solution of MADM problems more objective and reasonable. In addition, the method can also be extended to Intuitionistic Fuzzy Set (IFS), Fuzzy Set (FS) and other application scenarios.

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Conflict of interest

The author declares no conflicts of interest in this paper.

References

1. A. Mardani, E. K. Zavadskas, Z. Khalifah, N. Zakuan, A. Jusoh, K. M. Nor, et al., A review of multi-criteria decision-making applications to solve energy management problems: Two decades from 1995 to 2015, *Renewable Sustainable Energy Rev.*, **71** (2017), 216–256.
2. X. Zeng, L. Shu, S. Yan, Y. Shi, F. He, A novel multivariate grey model for forecasting the sequence of ternary interval numbers, *Appl. Math. Model*, **69** (2019), 273–286.
3. H. Wang, S. He, C. Li, X. Pan, Pythagorean uncertain linguistic variable hamy mean operator and its application to multi-attribute group decision making, *J. Autom. Sin.*, **6** (2019), 194–206.
4. P. Wang, P. Liu, Some Maclaurin symmetric mean aggregation operators based on Schweizer-Sklar operations for intuitionistic fuzzy numbers and their application to decision making, *J. Intell. Fuzzy Syst.*, **36** (2019), 3801–3824.
5. L. A. Zadeh, Fuzzy sets, *Inf. Control*, **8** (1965), 338–353.
6. K. T. Atanassov, G. Pasi, R. R. Yager, Intuitionistic fuzzy interpretations of multi-measurement tool multi-criteria decision making, *Int. J. Syst. Sci.*, **36** (2005), 859–868.
7. K. Atanassov, G. Gargov, Interval valued intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **31** (1989), 343–349.
8. D. K. Joshi, S. Kumar, Entropy of interval-valued intuitionistic hesitant fuzzy set and its application to group decision making problems, *Granular Comput.*, **1** (2018), 1–15.
9. D. Ye, D. Liang, P. Hu, Three-Way decisions with interval-valued intuitionistic fuzzy decision-theoretic rough sets in group decision-making, *Symmetry*, **10** (2018), 281.
10. L. Wu, G. Wei, H. Gao, Y. Wei, Some Interval-Valued Intuitionistic Fuzzy Dombi Hamy Mean Operators and Their Application for Evaluating the Elderly Tourism Service Quality in Tourism Destination, *Mathematics*, **6** (2018), 294.
11. K. Kumar, D. Pandey, Discussion on the switching between type-2 fuzzy sets and intuitionis-tic fuzzy sets: An application in medical diagnosis, *J. Inf. Optim. Sci.*, **39** (2018), 427–444.
12. S. M. Chen, Z. C. Huang, Multiattribute decision making based on interval-valued intuitionistic fuzzy values and linear programming methodology, *Inf. Sci.*, **381** (2017), 341–351.
13. H. C. J. Chao, C. T. Tung, C. H. Chu, Extension theorems for interval-valued intuitionistic fuzzy sets, *J. Discrete Math. Sci. Cryptography*, **3** (2018), 707–712.
14. M. Wei, Q. Dai, S. Sun, S. Ionita, Eva. Volná, A. Gavrilov, et al., A prediction model for traffic emission based on interval-valued intuitionistic fuzzy sets and case-based reasoning theory, *J. Intell. Fuzzy. Syst.*, **31** (2016), 3039–3046.
15. H. Zhao, Z. Xu, Z. Yao, Interval-valued intuitionistic fuzzy derivative and differential operations, *Int. J. Comput. Int. Syst.*, **9** (2016), 36–56.
16. F. Meng, X. Chen, Correlation Coefficient of Interval-Valued Intuitionistic Uncertain Linguistic Sets and Its Application, *J. Cybern.*, **48** (2017), 114–135.
17. Y. Kang, S. Wu, D. Cao, W. Weng, New hesitation-based distance and similarity measures on intuitionistic fuzzy sets and their applications, *Int. J. Syst. Sci.*, **49** (2018), 1–17.
18. D. F. Liu, X. H. Chen, D. Peng, Interval-valued intuitionistic fuzzy ordered weighted cosine similarity measure and its application in investment decision-making, *Complexity*, 2017, 1–11.
19. V. P. Ananthi, P. Balasubramaniam, T. Kalaiselvi, A new fuzzy clustering algorithm for the segmentation of brain tumor, *Soft. Comput.*, **20** (2015), 1–21.

20. Q. Gao, D. L. Xu, An empirical study on the application of the evidential reasoning rule to decision making in financial investment, *Knowl. Based Syst.,* **164** (2019), 226–234.
21. M. Nahangi, Y. Chen, B. Mccabe, Safety-based efficiency evaluation of construction sites using data envelopment analysis (DEA), *Saf. Sci.*, **113** (2019), 382–388.
22. Y. Zhu, X. Wang, S. Deng, M. Zhao, X. Ao, Evaluation of Curtain Grouting Efficiency by Cloud Model - based Fuzzy Comprehensive Evaluation Method, *KSCE. J. Civ. Eng.*, **23** (2019), 2852–2866.
23. A. Kumar, R. N. Rai, Evaluation of Wear Properties of Stir Cast AA7050-10% B4C Ex Situ Composite through Fuzzy-TOPSIS MCDM Method, *Solid State Phenom.*, **291** (2019), 1–12.
24. M. Akram, N. Waseem, P. Liu, Novel approach in decision making with m–Polar fuzzy ELECTRE-I, *Int. J. Fuzzy Syst.*, **21** (2019), 1–13.
25. W. Yunlong, L. Kai, G. Guan, Y. Yanyun, L. Fei, Evaluation method for Green jack-up drilling platform design scheme based on improved grey correlation analysis, *Appl. Ocean. Res.*, **85** (2019), 119–127.
26. S. J. Zhou, B. Liu, J. Meng, Quality evaluation of raw moutan cortex using the AHP and gray correlation-TOPSIS method, *Pharmacognosy. Mag.*, **13** (2017), 528–533.
27. P. P. Das, S. Chakraborty, A grey correlation-based TOPSIS approach for optimization of surface roughness and micro hardness of Nitinol during WEDM operation, *Mater. Today Proceedings,* 2019.
28. Y. Zhou, X. Liu, F. Li, W. Jiang, Railway route selection based on entropy weight method-gray correlation improvement TOPSIS, IOP Conference Series Earth and Environmental Science, 2019. Available from: https://iopscience.iop.org/journal/1755-1315.
29. S. H. Zyoud, D. Fuchs-Hanusch, A bibliometric-based survey on AHP and TOPSIS techniques. *Expert. Syst. Appl*, **78** (2017), 158–181.
30. C. T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy. Set. Syst*, **114** (2000), 1–9.
31. S. S. Yang, N. Nasr, S. K. Ong, A. Y. C. Nee, Designing automotive products for remanufacturing from material selection perspective. *J. Clean. Prod*, **153** (2017), 570–579.
32. S. F. Liu, Y. Lin, *Grey information theory and practical applications*, London: Springer-Verlag, 2011,10–30.
33. K. T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy. Set. Syst*, **20** (1986), 87–96.
34. J. J. Peng, J. Q. Wang, J. Wang, X. H. Chen, Multicriteria decision-making approach with hesitant interval-valued intuitionistic fuzzy sets, *Sci. World Journal*, **2014** (2014), 1–22.
35. Y. Song, X. Wang, L. Lei, A. Xue, Combination of interval-valued belief structures based on intuitionistic fuzzy set, *Knowl. Based. Syst.*, **67** (2014), 61–70.
36. T. Y. Chen, The extended linear assignment method for multiple criteria decision analysis based on interval-valued intuitionistic fuzzy sets, *Appl. Math. Model*, **38** (2014), 2101–2117.
37. Z. Xu, Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making, *Control Decis.*, **22** (2007).
38. E. Szmidt, J. Kacprzyk, Distances between intuitionistic fuzzy sets, *Fuzzy. Set. Syst.*, **114** (2000), 505–518.
39. H. Liao, Z. Xu, X. J. Zeng, Novel correlation coefficients between hesitant fuzzy sets and their application in decision making, *Knowl. Based. Syst.*, **82**(2015), 115–127.

40. P. Burillo, H. Bustince, Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets, *Fuzzy. Set. Syst.*, **78** (1996), 305–316.
41. J. Ye, Two effective measures of intuitionistic fuzzy entropy, *Computing*, **87** (2010), 55–62.
42. W. Zeng, H. Li, Relationship between similarity measure and entropy of interval valued fuzzy sets, *Fuzzy. Set. Syst*, **157** (2006), 1477–1484.
43. Q. S. Zhang, S. Y. Jiang, A note on information entropy measures for vague sets and its applications, *Inf. Sci.*, **178** (2008), 4184–4191.
44. P. Wang, C. P. Wei, Constructing method of interval-valued intuitionistic fuzzy entropy, *Comput. Eng. Appl.*, **47** (2011), 43–45.
45. C. Lin, G. Kou, Y. Peng, F. E. Alsaadi, Aggregation of the nearest consistency matrices with the acceptable consensus in AHP-GDM, *Ann. Oper. Res.* (2020), 1–17.
46. C. Y. Xie, Z. Q. Luo, N. Jia, W. Wang, Goafs' risk discrimination based on improved topsis coupled with ga-bp, *J. Northeast. Univer.*, **3** (2016), 440–445.
47. Z. S. Xu, R. R. Yager, Dynamic intuitionistic fuzzy multi-attribute decison making, *Int. J. Approx. Reason*, **48** (2008), 246–262.
48. Z. S. Xu, J. Chen, Approach to Group Decision Making Based on Interval-Valued Intuitionistic Judgment Matrices, *Syst. Eng. Theory Pract.*, **27** (2007),126–133.

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