



Research article

A new model of dengue fever in terms of fractional derivative

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Abstract: It is eminent that the epidemiological patterns of dengue are threatening for both the global economy and human health. The experts in the field are always in search to have better mathematician models in order to understand the transmission dynamics of epidemics models and to suggest possible control or the minimization of the infection from the community. In this research, we construct a new fractional-order system for dengue infection with carrier and partially immune classes to visualize the intricate dynamics of dengue. By using the basics of fractional theory, we determine the fundamental results of the proposed fractional-order dengue model. We obtain the basic reproduction number R_0 by next generation method and present the results based on it. The stability results are established for the infection-free state of the system. Moreover, sensitivity of R_0 is analyzed through partial rank correlation coefficient (PRCC) method to show the importance of different parameters in R_0 . The influence of different input factors is shown on the output of basic reproduction number R_0 numerically. Our result showed that the threshold parameter R_0 can be decreased by increasing vaccination and treatment in the system. Finally, we illustrate the solution of the suggested dengue system through a numerical scheme to notice the influence of the fractional-order ϑ on the system. We observed that the fractional-order dynamics can explain the complex system of dengue infection more precisely and accurately rather than the integer-order dynamics. In addition, we noticed that the index of memory and biting rate of vector can play a significant part in the prevention of the infection.

Keywords: fractional derivative; dengue disease; mathematical model; sensitivity analysis; numerical simulations

1. Introduction

Dengue fever is a viral disease produced by a pathogenic agent and is spread through female *Aedes aegypti* mosquitoes. Specific temperature areas of the world are affected by dengue infection and are almost spread to 128 countries which brings much more losses in public health and economy [1]. The symptoms include, muscle pain, nausea, red eyes, high fever, joints pain, pain behind eyes, lower back pain, vomiting, severe fatigue and weakness, and mild bleeding etc. The carrier vector becomes infected when they bit an infected host and transmit four different serotypes of dengue to the host through their bites. Rare cases of vertical transmission in dengue infection are reported [2, 3]. Due to the increased prominence of dengue viral infection in the past few decades, much effort has been made recently to develop a dengue vaccine. Yet, there is no fully effective vaccine although the partially effective vaccine is accessible in several states of the world [4, 5]. Reduction of vector bites and environmental domain for vectors are other adopted control policies for dengue infection.

Epidemiological models are of great significance to gain insight into the comprehensive dynamics of infectious disease and to provide better control policies. To be more specific, analysis of these models predicts critical factors that play a central part in the prevention and spread of infection [6, 7]. A number of the dynamical models have been established and approved to visualize the transmission of dengue infection [8, 9, 10, 11]. The basic model of dengue infection was introduced by Lourdes Esteva and analyzed the stability of the model with variable human population [8, 9]. Stability results for the equilibria of dengue infection model were established by in [10, 11]. The role of vaccination on the transmission dynamics of dengue is examined in [12, 13]. In dengue infection, asymptomatic carriers are frequently reported in [14, 15] and are critical in the sense to increase the level of infection in endemic and non-endemic areas of the world.

It is eminent that vector-borne disease posses information about its previous stages and associative learning mechanism, to be more specific memory has a prominent role in the transmission dynamics of vector-borne disease. The memory in the host population associated to the individual awareness which reduces the contact rate between vector and hosts, while the mosquitoes use their prior experience about the human's location, blood selection, color, and the smell of humans sweat [16, 17]. These types of phenomena can easily be captured by a fractional-order system in mathematical modeling of infectious diseases.

Fractional order system posses information about its previous and present stages to provide more realistic information about the dynamics of viral infections. Therefore, the dynamical behavior of epidemic diseases can be conceptualized and explore more accurately through non-integer order derivatives. A number of fractional-order models are introduced in the literature and successfully showed their positive role in different areas of physics, engineering, mathematics, and biology. For instant, a Maxwell model in fractional derivative is studied in [18]. Dengue dynamics in fractional derivative is considered in [19]. Application of fractional calculus to subdiffusion-reaction process is explored in [20]. An SIR model in fractional derivative is considered in [21]. The wave equation in a delay type fractional derivative is considered [22]. Recently, researchers proved that the fractional-order model can effectively handle the crossover behavior and complexity of communicable diseases, and offers a preferable fit for real data of the infection [23, 24]. These properties make fractional-order systems more efficient than integer-order systems. For more related research on fractional order derivative and their applications to science and engineering problems, the authors investigated many practical prob-

lems. For example, the author in [25] studied the fractional derivative and its applications to epidemic models. The corona virus model with lockdown is discussed through a mathematical model in [26]. The CoVID-19 and its impact on HIV and their future observation has been investigated in [27]. The corona virus model with fractional derivative is studied in [28], where the fractional operator is considered the Atangana-Baleanu type. The authors in [29] studied a corona virus model with nonlocal operator and using a power computational method. The M-fractional derivative and its applications to differential equations is discussed in [30]. An engineering problem with a new numerical results is studied in [31]. A fractional Schrodinger-Hirota equation and its optical solution is analyzed in [32]. The computer virus model in fractional derivative is considered in [33]. A review article that describing the harmonic wavelets and its fractal extension is studied in [34]. The solution of heat and diffusion equation with new technology is considered in [35]. A vector-host model with saturated treatment function and its optimal control analysis is discussed in [36]. The solution of Telegraph equation in new fractional operators is considered in [37]. The boundary value problems in fractional derivatives are studied in [38]. A fractional SEIR type model with treatment is studied in [39] while the dynamics of HIV with different transmission rates are analyzed in [40]. Therefore, motivated by these extraordinary properties, we analyze the dynamical behavior of dengue infection in the framework of the fractional-order derivative.

The article is structured as: In the second section, we put forward a brief summary of the basic concepts of fractional calculus. In the third section, we construct a fractional-order model for dengue infection with new assumptions, in addition, we proved basic results for our proposed fractional-order model. We established stability results for the steady-state in section four and carried out sensitivity analysis through the PRCC technique to point out the importance of input parameter on the output of R_0 . We have shown the influence of different input factors on the basic reproduction number R_0 , numerically. The fractional-order system is inspected numerically to observe the influence of ϑ on the dynamics of dengue in Section 5. In the last section of the article, ending remarks and conclusion of the overall analysis is presented.

2. Basics of fractional calculus

Here, we will present a brief summary of the Caputo fractional derivative for further analysis of the dengue model. Caputo fractional derivative has the benefit of dealing with initial value problems and involves integration which posses information about the previous state of the system.

Definition 2.1. Let $g : \mathbb{R}^+ \rightarrow \mathbb{R}$, then the fractional integral of g is given by

$$I_t^\vartheta(g(t)) = \frac{1}{\Gamma(\vartheta)} \int_0^t (t-y)^{\vartheta-1} g(y) dy. \quad (2.1)$$

where ϑ indicates the order of fractional integral and Gamma function is indicated by Γ .

Definition 2.2. The fractional derivative of order ϑ for a given function g in the Caputo form is given by

$${}^C D_t^\vartheta(g(t)) = I^{n-\vartheta} D^n g(t) = \frac{1}{\Gamma(n-\vartheta)} \int_a^t (t-y)^{n-\vartheta-1} g^n(y) dy, \quad (2.2)$$

where $n - 1 < \vartheta < n, n \in \mathbb{N}$.

Let η_1, η_2 be two positive numbers, then the Mittag-Leffler function is given by

$$E_{\eta_1, \eta_2}(s) = \sum_{k=0}^{\infty} \frac{s^k}{\Gamma(\eta_1 k + \eta_2)}. \quad (2.3)$$

Let ${}^C D_t^\vartheta$ be the Caputo fractional derivative (CFD) of order ϑ , then its laplace transform is define by

$$L[{}^C D_t^\vartheta g(t)] = u^\vartheta G(u) - \sum_{k=0}^{n-1} g^{(k)}(0) u^{\vartheta-k-1}, \quad (2.4)$$

further, for the function $t^{\eta_2-1} E_{\eta_1, \eta_2}(\pm \omega t^\vartheta)$, the laplace transform is given by

$$L[t^{\eta_2-1} E_{\eta_1, \eta_2}(\pm \omega t^\vartheta)] = \frac{u^{\eta_1-\eta_2}}{u^{\eta_1} \pm \omega}. \quad (2.5)$$

The following equation hold true by Mittag-Leffler function given in [41]:

$$E_{\eta_1, \eta_2}(s) = s E_{\eta_1, \eta_1 + \eta_2}(s) + \frac{1}{\Gamma(\eta_2)}.$$

Lemma 2.1. [42]. Consider a function $g : R^+ \times R^5 \rightarrow R^5$, which fulfills the conditions:

- The function $g(t, Y(t))$ is Lebesgue measurable in R^+ with respect to t ;
- The function $g(t, Y(t))$ is continuous on R^5 with respect to $Y(t)$;
- The function $\frac{\partial g(t, Y(t))}{\partial Y}$ is continuous on R^5 with respect to $Y(t)$;
- and $\|g(t, Y(t))\| \leq \mathfrak{N} + \mathfrak{h}\|Y\|, \forall t \in R^+, Y \in R^5$, where $\mathfrak{N}, \mathfrak{h}$ are positive constants.

Then the fractional order system

$$\begin{aligned} {}^C D_t^\vartheta Y(t) &= g(t, Y(t)), \\ Y(0) &= Y_0, \end{aligned} \quad (2.6)$$

where $0 < \vartheta \leq 1$, has a unique solution.

Lemma 2.2. [43]. Let a function $g(t) \in C(r, s]$ such that ${}^C D_r^\vartheta g(t) \in C[r, s]$ for $\vartheta \in (0, 1]$, then we have

$$g(t) = g(r) + \frac{1}{\Gamma(\vartheta)} {}^C D_r^\vartheta g(\epsilon)(t-r)^\vartheta, \quad r < \epsilon < t, \forall t \in (r, s].$$

Remark 1. Let a function $g \in C(r, s]$, such that ${}^C D_r^\vartheta g(t) \in C[r, s]$ for $\vartheta \in (0, 1]$. Then by Lemma (2.2) if ${}^C D_r^\vartheta g(t) \leq 0, \forall t \in (r, s)$, then $g(t)$ is non-increasing function for all $t \in [r, s]$, and if ${}^C D_r^\vartheta g(t) \geq 0, \forall t \in (r, s)$, then $g(t)$ is non-decreasing function for all $t \in [r, s]$.

3. Formulation of the model

In construction of the model, we represent the mosquitoes and humans population by N_v and N_h . The total mosquitoes size is divided into susceptible (S_v) and infected (I_v) compartments, while the total humans size is divided into susceptible (S_h), infected (I_h), carrier (I_{hA}), partially immune (P), and recovered (R_h) compartments.

We indicate the natural birth (or death) rate of mosquito and human by μ_v and μ_h , respectively, and assumed that the disease-induced death rate to be negligible. Furthermore, we assumed that a fraction ν of susceptible hosts goes to partially immune class after implementation of vaccination and τ indicates treatment of infected host. The symptomatic fraction of infected class from susceptible and partially immune classes are indicated by ψ and ω , respectively. the parameter ρ denotes the fraction of treated humans which join the partially immune class P while remaining enter to recovered class R_h . Parameter γ_h is used for recovery of symptomatic and asymptomatic humans while ϕ denotes the fraction of recovered humans which join the partially immune class P . The terms $(\frac{b\beta_1}{N_h}I_v)$ and $(\frac{b\beta_2}{N_h}I_v)$ indicate the infection rate per susceptible host and partially immune host, respectively, while $(\frac{b\beta_3}{N_h}I_h)$ denotes the infection rate of per susceptible vector. In these terms b accounts the biting rate, β_1 and β_2 are the rates at which infected vectors transmit disease to susceptible and partially immune humans respectively, whereas β_3 represents the disease transmission rate from infected human to susceptible vector. The dynamics of dengue infection is given by

$$\begin{cases} \frac{dS_h}{dt} = \mu_h N_h - \frac{\beta_1 b}{N_h} S_h I_v - \nu S_h - \mu_h S_h, \\ \frac{dI_h}{dt} = \psi \frac{\beta_1 b}{N_h} S_h I_v + \omega \frac{\beta_2 b}{N_h} P I_v - (\mu_h + \tau + \gamma_h) I_h, \\ \frac{dI_{hA}}{dt} = (1 - \psi) \frac{\beta_1 b}{N_h} S_h I_v + (1 - \omega) \frac{\beta_2 b}{N_h} P I_v - (\mu_h + \gamma_h) I_{hA}, \\ \frac{dP}{dt} = \nu S_h + \rho \tau I_h + \phi \gamma_h (I_h + I_{hA}) - \frac{\beta_2 b}{N_h} P I_v - \mu_h P, \\ \frac{dR_h}{dt} = (1 - \rho) \tau I_h + (1 - \phi) \gamma_h (I_h + I_{hA}) - \mu_h R_h, \\ \frac{dS_v}{dt} = \mu_v N_v - \frac{\beta_3 b}{N_h} S_v (I_h + I_{hA}) - \mu_v S_v, \\ \frac{dI_v}{dt} = \frac{\beta_3 b}{N_h} S_v (I_h + I_{hA}) - \mu_v I_v, \end{cases} \quad (3.1)$$

with the following initial condition

$$S_h(0) \geq 0, I_h(0) \geq 0, I_{hA}(0) \geq 0, P(0) \geq 0, R_h(0) \geq 0, S_v(0) \geq 0, I_v(0) \geq 0.$$

We denote the total human population size by N_h , and vector by N_v so that, $N_h = S_h + I_h + I_{hA} + P + R_h$ and $N_v = S_v + I_v$. As the system (3.1) is independent of the state-variable R_h , thus, the proposed model (3.1) of dengue infection can be re-written in the following form without R_h :

$$\begin{cases} \frac{dS_h}{dt} = \mu_h N_h - \frac{\beta_1 b}{N_h} S_h I_v - \nu S_h - \mu_h S_h, \\ \frac{dI_h}{dt} = \psi \frac{\beta_1 b}{N_h} S_h I_v + \omega \frac{\beta_2 b}{N_h} P I_v - (\mu_h + \tau + \gamma_h) I_h, \\ \frac{dI_{hA}}{dt} = (1 - \psi) \frac{\beta_1 b}{N_h} S_h I_v + (1 - \omega) \frac{\beta_2 b}{N_h} P I_v - (\mu_h + \gamma_h) I_{hA}, \\ \frac{dP}{dt} = \nu S_h + \rho \tau I_h + \phi \gamma_h (I_h + I_{hA}) - \frac{\beta_2 b}{N_h} P I_v - \mu_h P, \\ \frac{dS_v}{dt} = \mu_v N_v - \frac{\beta_3 b}{N_h} S_v (I_h + I_{hA}) - \mu_v S_v, \\ \frac{dI_v}{dt} = \frac{\beta_3 b}{N_h} S_v (I_h + I_{hA}) - \mu_v I_v, \end{cases} \quad (3.2)$$

further, assume that

$$x_1 = \frac{S_h}{N_h}, x_2 = \frac{I_h}{N_h}, x_3 = \frac{I_{hA}}{N_h}, x_4 = \frac{P}{N_h}, x_5 = \frac{I_v}{N_v} \text{ and } x_6 = \frac{S_v}{N_v},$$

then, we have $x_1 + x_2 + x_3 + x_4 = 1$ and $x_5 + x_6 = 1$. After replacing $x_6 = 1 - x_5$ in the last equation of normalized system we obtained the following five dimensional reduced dengue model:

$$\begin{cases} \frac{dx_1}{dt} = \mu_h - \frac{b\beta_1 N_v}{N_h} x_1 x_5 - \nu x_1 - \mu_h x_1, \\ \frac{dx_2}{dt} = \psi \frac{b\beta_1 N_v}{N_h} x_1 x_5 + \omega \frac{b\beta_2 N_v}{N_h} x_4 x_5 - (\mu_h + \tau + \gamma_h) x_2, \\ \frac{dx_3}{dt} = (1 - \psi) \frac{b\beta_1 N_v}{N_h} x_1 x_5 + (1 - \omega) \frac{b\beta_2 N_v}{N_h} x_4 x_5 - (\mu_h + \gamma_h) x_3, \\ \frac{dx_4}{dt} = \nu x_1 + \rho \tau x_2 + \phi \gamma_h (x_2 + x_3) - \frac{b\beta_2 N_v}{N_h} x_4 x_5 - \mu_h x_4, \\ \frac{dx_5}{dt} = b\beta_3 (1 - x_5) (x_2 + x_3) - \mu_v x_5. \end{cases} \quad (3.3)$$

As fractional-order models describe the non-local behavior of biological systems and possess hereditary property, moreover, it provides information about its past and present state for the future, therefore, we represent the dynamical system (3.3) of dengue infection in the framework of fractional order Caputo's derivative to conceptualize the transmission of dengue fever in a more accurate way. Thus, the system consist of fractional derivatives is presented by

$$\begin{cases} {}_0^C D_t^\vartheta x_1 = \mu_h^\vartheta - b^\vartheta a x_1 x_5 - \nu^\vartheta x_1 - \mu_h^\vartheta x_1, \\ {}_0^C D_t^\vartheta x_2 = \psi b^\vartheta a x_1 x_5 + \omega b^\vartheta c x_4 x_5 - (\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta) x_2, \\ {}_0^C D_t^\vartheta x_3 = (1 - \psi) b^\vartheta a x_1 x_5 + (1 - \omega) b^\vartheta c x_4 x_5 - (\mu_h^\vartheta + \gamma_h^\vartheta) x_3, \\ {}_0^C D_t^\vartheta x_4 = \nu^\vartheta x_1 + \rho \tau^\vartheta x_2 + \phi \gamma_h^\vartheta (x_2 + x_3) - b^\vartheta c x_4 x_5 - \mu_h^\vartheta x_4, \\ {}_0^C D_t^\vartheta x_5 = b^\vartheta e (1 - x_5) (x_2 + x_3) - \mu_v^\vartheta x_5, \end{cases} \quad (3.4)$$

where ${}_0^C D_t^\vartheta$ indicates Caputo's fractional derivative of order ϑ , the order ϑ indicates the index of memory in the system. In addition, the values

$$a = \frac{\beta_1 N_v}{N_h}, c = \frac{\beta_2 N_v}{N_h}, \text{ and } e = \beta_3.$$

In Caputo's sense, the derivative of constant is equal to zero which is another advantage to make the system more reliable and flexible for analysis. Next, we will analyze the biologically feasible region of the fractional-order dengue model (3.4).

Theorem 3.1. *The proposed fractional-order dengue infection system of equations (3.4) has a unique solution.*

Proof. For the required result, we first prove that the fractional system (3.4) has a unique solution for all initial conditions in R^5 . Clearly, the first three conditions of Lemma (2.1) are hold by the vector function g of the system (3.4). Next, to prove the last condition of Lemma (2.1), we rewrite system (3.4) as

$${}_0^C D_t^\vartheta x(t) = \mathfrak{N} + A_1 x(t) + A_2 x_1(t)x(t) + A_3 x_2(t)x(t) + A_4 x_3(t)x(t) + A_5 x_4(t)x(t) + A_6 x_5(t)x(t),$$

where

$$\mathfrak{N} = \begin{bmatrix} \mu_h^\vartheta \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, A_1 = \begin{bmatrix} -(\mu_h^\vartheta + \nu^\vartheta) & 0 & 0 & 0 & 0 \\ 0 & -(\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta) & 0 & 0 & 0 \\ 0 & 0 & -(\mu_h^\vartheta + \gamma_h^\vartheta) & 0 & 0 \\ \nu^\vartheta & \rho \tau^\vartheta + \phi \gamma_h^\vartheta & \phi \gamma_h^\vartheta & -\mu_h^\vartheta & 0 \\ 0 & b^\vartheta e & b^\vartheta e & 0 & -\mu_v^\vartheta \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi b^\theta a \\ 0 & 0 & 0 & 0 & (1-\psi)b^\theta a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -b^\theta e \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -b^\theta e \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega b^\theta c \\ 0 & 0 & 0 & 0 & (1-\omega)b^\theta c \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, A_6 = \begin{bmatrix} -b^\theta a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -b^\theta c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Symbolize the above

$$g(t, x(t)) = \mathfrak{N} + A_1 x(t) + A_2 x_1(t)x(t) + A_3 x_2(t)x(t) + A_4 x_3(t)x(t) + A_5 x_4(t)x(t) + A_6 x_5(t)x(t),$$

and taking norm, we get

$$\begin{aligned} & \|g(t, x(t))\| \\ &= \|\mathfrak{N} + A_1 x(t) + A_2 x_1(t)x(t) + A_3 x_2(t)x(t) + A_4 x_3(t)x(t) + A_5 x_4(t)x(t) + A_6 x_5(t)x(t)\| \\ &\leq \|\mathfrak{N}\| + \|A_1\| \|x(t)\| + \|A_2\| \|x_1(t)\| \|x(t)\| + \|A_3\| \|x_2(t)\| \|x(t)\| + \|A_4\| \|x_3(t)\| \|x(t)\| \\ &\quad + \|A_5\| \|x_4(t)\| \|x(t)\| + \|A_6\| \|x_5(t)\| \|x(t)\| \\ &\leq \|\mathfrak{N}\| + \|A_1\| \|x(t)\| + \|A_2\| \|x(t)\| + \|A_3\| \|x(t)\| + \|A_4\| \|x(t)\| + \|A_5\| \|x(t)\| + \|A_6\| \|x(t)\| \\ &= \|\mathfrak{N}\| + \left(\|A_1\| + \|A_2\| + \|A_3\| + \|A_4\| + \|A_5\| + \|A_6\| \right) \|x(t)\| \\ &= \mathfrak{N} + \tilde{h} \|x(t)\|. \end{aligned}$$

Hence, the requirements of Lemma (2.1) are fulfilled, therefore the system of equations (3.4) has a unique solution. \square

Theorem 3.2. *The closed set $\Omega = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}_+^5 : 0 \leq x_1 + x_2 + x_3 + x_4 \leq M_1, 0 \leq x_5 \leq M_2\}$ is a positive invariant set for the proposed fractional order system (3.4).*

Proof. To prove that the system of equations (3.4) has a non-negative solution, the system of equations (3.4) implies

$$\begin{cases} {}_0^C D_t^\theta x_1 |_{x_1=0} = \mu_h^\theta > 0, \\ {}_0^C D_t^\theta x_2 |_{x_2=0} = \psi b^\theta a x_1 x_5 + \omega b^\theta c x_4 x_5 \geq 0, \\ {}_0^C D_t^\theta x_3 |_{x_3=0} = (1-\psi)b^\theta a x_1 x_5 + (1-\omega)b^\theta c x_4 x_5 \geq 0, \\ {}_0^C D_t^\theta x_4 |_{x_4=0} = \nu^\theta x_1 + \rho \tau^\theta x_2 + \phi \gamma_h^\theta (x_2 + x_3) \geq 0, \\ {}_0^C D_t^\theta x_5 |_{x_5=0} = b^\theta e (x_2 + x_3) \geq 0. \end{cases} \quad (3.5)$$

Thus, the fractional system (3.4) has non-negative solutions. In the end, from the first four equations of the fractional system (3.4), we obtain

$$\begin{aligned} {}_0^C D_t^\vartheta(x_1 + x_2 + x_3 + x_4) &\leq \mu_h^\vartheta - \mu_h^\vartheta x_1 - ((1 - \rho)\tau^\vartheta + (1 - \phi)\gamma_h^\vartheta + \mu_h^\vartheta)x_2 - ((1 - \phi)\gamma_h^\vartheta + \mu_h^\vartheta)x_3 - \mu_h^\vartheta x_4, \\ &\leq \mu_h^\vartheta - \mathcal{W}(x_1 + x_2 + x_3 + x_4) \end{aligned} \quad (3.6)$$

where $\mathcal{W} = \min(\mu_h^\vartheta, ((1 - \rho)\tau^\vartheta + (1 - \phi)\gamma_h^\vartheta + \mu_h^\vartheta), ((1 - \phi)\gamma_h^\vartheta + \mu_h^\vartheta), \mu_h^\vartheta)$. Solving the above inequality, we obtain

$$\left(x_1(t) + x_2(t) + x_3(t) + x_4(t)\right) \leq \left(x_1(0) + x_2(0) + x_3(0) + x_4(0) - \frac{\mu_h^\vartheta}{\mathcal{W}}\right) E_\vartheta(-\mathcal{W}t^\vartheta) + \frac{\mu_h^\vartheta}{\mathcal{W}},$$

so by the asymptotic behavior of Mittag-Leffler function [44], we obtain

$$\left(x_1(t) + x_2(t) + x_3(t) + x_4(t)\right) \leq \frac{\mu_h^\vartheta}{\mathcal{W}} \cong M_1,$$

taking the same steps for the last equation of system (3.4), we get $x_5(t) \leq M_2$, where $M_2 = \frac{b^\vartheta e M_1}{b^\vartheta e M_1 + \mu_v^\vartheta}$. Hence, the closed set Ω is a positive invariant region for the fractional-order dengue model (3.4). \square

4. Reproduction number and stability analysis

The biological meaningful equilibria of fractional system (3.4) are DFE and EE, depending on infected classes in both the populations. To obtain the infection-free equilibrium, we set the fractional derivative ${}_0^C D_t^\vartheta x_1, {}_0^C D_t^\vartheta x_2, {}_0^C D_t^\vartheta x_3, {}_0^C D_t^\vartheta x_4$, and ${}_0^C D_t^\vartheta x_5$ to zero of the fractional system (3.4) without infection, and get

$$E_0(x_1^0, x_2^0, x_3^0, x_4^0, x_5^0) = \left(\frac{\mu_h^\vartheta}{\mu_h^\vartheta + \nu^\vartheta}, 0, 0, \frac{\nu^\vartheta}{\mu_h^\vartheta + \nu^\vartheta}, 0\right).$$

The detailed concept of the basic reproduction number is presented in [45], and is normally denoted by R_0 , and is calculated as

$$\mathcal{F} = \begin{bmatrix} \psi b^\vartheta a x_1 x_5 + \omega b^\vartheta c x_4 x_5 \\ (1 - \psi) b^\vartheta a x_1 x_5 + (1 - \omega) b^\vartheta c x_4 x_5 \\ b^\vartheta e(1 - x_5)(x_2 + x_3) \end{bmatrix} \text{ and } \mathcal{V} = \begin{bmatrix} (\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta) x_2 \\ (\mu_h^\vartheta + \gamma_h^\vartheta) x_3 \\ \mu_v^\vartheta x_5 \end{bmatrix},$$

this is because of three infected compartment in the system, i.e., $m = 3$, which provides

$$F = \begin{bmatrix} 0 & 0 & \psi b^\vartheta a x_1^0 + \omega b^\vartheta c x_4^0 \\ 0 & 0 & (1 - \psi) b^\vartheta a x_1^0 + (1 - \omega) b^\vartheta c x_4^0 \\ b^\vartheta e & b^\vartheta e & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} (\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta) & 0 & 0 \\ 0 & (\mu_h^\vartheta + \gamma_h^\vartheta) & 0 \\ 0 & 0 & \mu_v^\vartheta \end{bmatrix},$$

which gives

$$FV^{-1} = \begin{bmatrix} 0 & 0 & \frac{\psi b^\vartheta a x_1^0 + \omega b^\vartheta c x_4^0}{\mu_v^\vartheta} \\ 0 & 0 & \frac{(1 - \psi) b^\vartheta a x_1^0 + (1 - \omega) b^\vartheta c x_4^0}{\mu_v^\vartheta} \\ \frac{b^\vartheta e}{(\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta)} & \frac{b^\vartheta e}{(\mu_h^\vartheta + \gamma_h^\vartheta)} & 0 \end{bmatrix}.$$

The R_0 of the fractional-order dengue model (3.4) is obtained by using the next generation matrix as

$$\begin{aligned}\rho(FV^{-1}) &= \sqrt{\frac{b^\vartheta e}{\mu_v^\vartheta(\mu_h^\vartheta + \nu^\vartheta)} \left[\frac{(1-\psi)b^\vartheta a\mu_h^\vartheta + (1-\omega)b^\vartheta c\nu^\vartheta}{\mu_h^\vartheta + \gamma_h^\vartheta} + \frac{\psi b^\vartheta a\mu_h^\vartheta + \omega b^\vartheta c\nu^\vartheta}{\mu_h^\vartheta + \gamma_h^\vartheta + \tau^\vartheta} \right]}, \\ \Rightarrow R_0 &= \sqrt{\frac{b^\vartheta e}{\mu_v^\vartheta(\mu_h^\vartheta + \nu^\vartheta)} \left[\frac{(1-\psi)b^\vartheta a\mu_h^\vartheta + (1-\omega)b^\vartheta c\nu^\vartheta}{\mu_h^\vartheta + \gamma_h^\vartheta} + \frac{\psi b^\vartheta a\mu_h^\vartheta + \omega b^\vartheta c\nu^\vartheta}{\mu_h^\vartheta + \gamma_h^\vartheta + \tau^\vartheta} \right]}.\end{aligned}$$

Assume a fractional-order linear homogenous system of the following form

$$\begin{aligned}{}^C_0 D_t^\vartheta y(t) &= By(t), \\ y(0) &= y_0,\end{aligned}\tag{4.1}$$

where $B \in M_{m \times m}(\mathbb{R})$ and $0 < \vartheta \leq 1$. The following theorems are on the stability of linear homogenous system (4.1).

Theorem 4.1. [46]. *The origin of the fractional dynamical system (4.1) is asymptotically stable $\Leftrightarrow |\arg(\lambda_i)| > \frac{\vartheta\pi}{2}$ is fulfilled for all eigenvalues λ_i of matrix B .*

Theorem 4.2. [47]. *The steady state of fractional system (2.6) is locally asymptotically stable (LAS) if $|\arg(\lambda_i)| > \frac{\vartheta\pi}{2}$ for all eigenvalues λ_i 's of $g(y)$ at steady state, otherwise unstable.*

In order to prove the LAS of infection-free steady state E_0 it is sufficient to confirm that all of the eigenvalues of the matrix of dynamics given by $J(E_0)$ lie outside the closed angular sector as shown in [46]. We state the following theorem for the desired result.

Theorem 4.3. *The DFE of the fractional-order system (3.4) is LAS if $R_0 < 1$ and $|\arg(\lambda_i)| > \frac{\vartheta\pi}{2}$ for all eigenvalues λ_i 's of the Jacobian matrix $J(E_0)$ at steady state, otherwise it is unstable.*

Proof. To obtain the demanded result for the fractional system (3.4), we take the Jacobian matrix of the system at infection-free equilibrium as

$$J(E_0) = \begin{bmatrix} -(v^\vartheta + \mu_h^\vartheta) & 0 & 0 & 0 & -b^\vartheta ax_1^0 \\ 0 & -(\mu_h^\vartheta + \gamma_h^\vartheta + \tau^\vartheta) & 0 & 0 & \psi b^\vartheta ax_1^0 + \omega b^\vartheta cx_4^0 \\ 0 & 0 & -(\mu_h^\vartheta + \gamma_h^\vartheta) & 0 & (1-\psi)b^\vartheta ax_1^0 + (1-\omega)b^\vartheta cx_4^0 \\ \nu^\vartheta & \rho\tau^\vartheta + \phi\gamma_h^\vartheta & \phi\gamma_h^\vartheta & -\mu_h^\vartheta & -b^\vartheta cx_4^0 \\ 0 & b^\vartheta e & b^\vartheta e & 0 & -\mu_v^\vartheta \end{bmatrix}.$$

The characteristic equation in term of λ of the above matrix J_{E_0} which is given below:

$$(\lambda + \mu_h^\vartheta)(\lambda + (\mu_h^\vartheta + \nu^\vartheta))(\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3) = 0,\tag{4.2}$$

where the associated coefficients are as follow:

$$\begin{aligned}a_1 &= (\mu_h^\vartheta + \gamma_h^\vartheta + \tau^\vartheta) + (\mu_h^\vartheta + \gamma_h^\vartheta) + \mu_v^\vartheta, \\ a_2 &= (\mu_h^\vartheta + \gamma_h^\vartheta + \tau^\vartheta)(\mu_h^\vartheta + \gamma_h^\vartheta) + (\mu_h^\vartheta + \gamma_h^\vartheta + \tau^\vartheta)\mu_v^\vartheta + (\mu_h^\vartheta + \gamma_h^\vartheta)\mu_v^\vartheta - b^\vartheta eB - b^\vartheta eA, \\ a_3 &= (\mu_h^\vartheta + \gamma_h^\vartheta + \tau^\vartheta)(\mu_h^\vartheta + \gamma_h^\vartheta)\mu_v^\vartheta[1 - R_0^2],\end{aligned}$$

, where $A = \psi b^\vartheta ax_1^0 + \omega b^\vartheta cx_4^0$ and $B = (1-\psi)b^\vartheta ax_1^0 + (1-\omega)b^\vartheta cx_4^0$. From (4.2) it is clear the argument of the first two eigenvalues $\lambda = -\mu_h^\vartheta$ and $\lambda = -(\mu_h^\vartheta + \nu^\vartheta)$ satisfy the necessary condition given (i.e.,

$|\arg(\lambda_i)| > \frac{\vartheta\pi}{2}$ for all $\vartheta \in (0, 1)$). Furthermore, if $R_0 < 1$, then all $a_i > 0$, for $i = 1, 2, 3$, and it is easy to show that $a_1 a_2 - a_3 > 0$. Thus, following [46], the arguments of all eigenvalues satisfy the necessary Matignon condition. Consequently, the infection-free steady-state is LAS. \square

Theorem 4.4. *The infection-free steady state of the fractional-order system (3.4) is GAS without vaccination, if $R_0 < 1$.*

Proof. Let $v = 0$, and $(x_1, x_2, x_3, x_4, x_5)$ be the solution of system (3.4) with suitable initial conditions $(x_1(0), x_2(0), x_3(0), x_4(0), x_5(0))$ in Ω . Here, it is clear that the fractional system has only one equilibrium E_0 on the boundary of Ω . Therefore, to achieve the target, it is enough to prove that the solution $(x_1, x_2, x_3, x_4, x_5)$ tends to the infection-free equilibrium as time tends to infinity. Since $x_i \leq x_i^0$, for $i = 1, 4$, then the system implies that

$$\begin{aligned} {}_0^C D_t^\vartheta x_2 &\leq \psi b^\vartheta a x_1^0 x_5 + \omega b^\vartheta c x_4^0 x_5 - (\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta) x_2, \\ {}_0^C D_t^\vartheta x_3 &\leq (1 - \psi) b^\vartheta a x_1^0 x_5 + (1 - \omega) b^\vartheta c x_4^0 x_5 - (\mu_h^\vartheta + \gamma_h^\vartheta) x_3, \\ {}_0^C D_t^\vartheta x_5 &\leq b^\vartheta e(x_2 + x_3) - \mu_v^\vartheta x_5. \end{aligned}$$

Taking the auxiliary system

$$\begin{aligned} {}_0^C D_t^\vartheta y_1 &= \psi b^\vartheta a x_1^0 y_3 + \omega b^\vartheta c x_4^0 y_3 - (\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta) y_1, \\ {}_0^C D_t^\vartheta y_2 &= (1 - \psi) b^\vartheta a x_1^0 y_3 + (1 - \omega) b^\vartheta c x_4^0 y_3 - (\mu_h^\vartheta + \gamma_h^\vartheta) y_2, \\ {}_0^C D_t^\vartheta y_3 &= b^\vartheta e(y_1 + y_2) - \mu_v^\vartheta y_3, \end{aligned}$$

this further implies

$${}_0^C D_t^\vartheta Y = (F - V)Y$$

the coefficient matrix of the above fractional system is $F - V$, and if $R_0 = \rho(FV^{-1}) < 1$, then the eigenvalues of $F - V$ lies in the left half plane. Consequently, each positive solution of the fractional system (4.3) fulfills $\lim_{t \rightarrow \infty} y_1 = 0$, $\lim_{t \rightarrow \infty} y_2 = 0$ and $\lim_{t \rightarrow \infty} y_3 = 0$ by Theorem 4.1. By the comparison theory of fractional differential equations [48], we have $\lim_{t \rightarrow \infty} x_2 = 0$, $\lim_{t \rightarrow \infty} x_3 = 0$ and $\lim_{t \rightarrow \infty} x_5 = 0$. Then from system (3.4), we have

$$\begin{aligned} {}_0^C D_t^\vartheta x_1 &= \mu_h^\vartheta - \mu_h^\vartheta x_1, \\ {}_0^C D_t^\vartheta x_4 &= 0 - \mu_h^\vartheta x_4, \end{aligned}$$

this can be further converted into

$${}_0^C D_t^\vartheta X = \Lambda - BX,$$

having the solution

$$X(t) = t^\vartheta E_{\vartheta, \vartheta+1}(-Bt^\vartheta) \Lambda + E_{\vartheta, 1}(-Bt^\vartheta) X^0,$$

taking the asymptotic behavior of Mittag-Leffler function [44] as

$$E_{\vartheta, 1}(-\omega t^\vartheta) \approx_{t \rightarrow \infty} \frac{t^\vartheta}{\omega \Gamma(1 - \vartheta)}; \quad 0 < \vartheta < 1 \text{ and } \omega > 0,$$

also the real parts of eigenvalues of $-B$ are negative, so it can be observed that $X(t) \rightarrow X^0$ as t tends to infinity. Hence, the infection-free steady-state of the system (3.4) is GAS without vaccination.

Next, we will demonstrate the persistence of infection in the fractional-order system. It describes the level of endemicity of infection in the system. Biologically speaking, the infection persists in the system if the level of infected fraction stays at a higher level for t large enough.

Theorem 4.5. *In the fractional-order system (3.4), the dengue infection is uniformly persistent at $\vartheta = 1$ if $R_0 > 1$.*

□

Proof. The threshold parameter R_0 is of high significance, which measures the level of infection in the system. If R_0 less than one then the asymptomatic stability of infection-free equilibrium (E_0) prevent the system from the persistent of the infection. If R_0 greater than one then the sufficiently close solution of fractional system (3.4) to E_0 moves away from it due to its instability in the interior of Ω by Theorem (4.3). For uniform persistence of our system (3.4) at $\vartheta = 1$, we will prove that it fulfills the criteria of Theorem 4.3 in [49] for $R_0 > 1$. Select $X = R^5$ and $E = \Omega$. Here, the maximal invariant set on the boundary of Ω is the set $\{E_0\}$ and is isolated. So, the criteria of Theorem 4.3 in [49] satisfies for the system. Thus, the system (3.4) is uniformly persistent if E_0 is unstable for $R_0 > 1$. □

4.1. Sensitivity analysis and numerical results

In the section, we present the global sensitivity analysis in order to determine those model parameters which are most influential on the disease dynamics. The main goal of this analysis is to detect and measure the influence of input parameters on the output of the system. To be more specific, it is used to know how the input parameters and initial values contribute to the output of a system. Mostly, when there is a little uncertainty in initial conditions and input parameters partial derivative of output functions are computed with respect to the input factors around the base values. This method is named as the local sensitivity analysis and relies on the variations of parameters close to the base values. This technique is not most suitable for epidemiological models due to the uncertainty in the input of the system. Therefore, global sensitivity analysis is preferred to perform this analysis and to provide more accurate results.

Here, we used the PRCC method [50] for sensitivity analysis to point out the input parameters that highly influence the results of R_0 . It is an effective method and can successfully measure the monotonic, nonlinear relationship between input and output values of the system. PRCC analysis provides PRCC and p values for each factor involve therein, with which we can measure the contribution of each factor. More specifically, the input factors with sizeable PRCC and negligible p-values are considered to be highly effective factors in the system. In our analysis, we investigate all the parameter presented in Table 1 to know their contribution to the outcomes of R_0 and listed all the associated PRCC and p-values provided by the PRCC significance test. Figure 1 and Table 1 demonstrate that the parameter b is highly influential with PRCC value 0.7022. After that, the parameter β_1 , and ϑ are highly influential with PRCC values 0.6427, and -0.5112, respectively. Sensitivity analysis illustrated that b, β_1 , and ϑ are critical parameters in R_0 , which can remarkably influence the infection. Therefore by controlling these factors can greatly decrease and prevent the level of new dengue cases in the community.

Next, we illustrate the influence of vaccination and treatment in on the threshold R_0 of the proposed system 2 (a). We noted that vaccination and treatment decrease the threshold parameter, moreover,

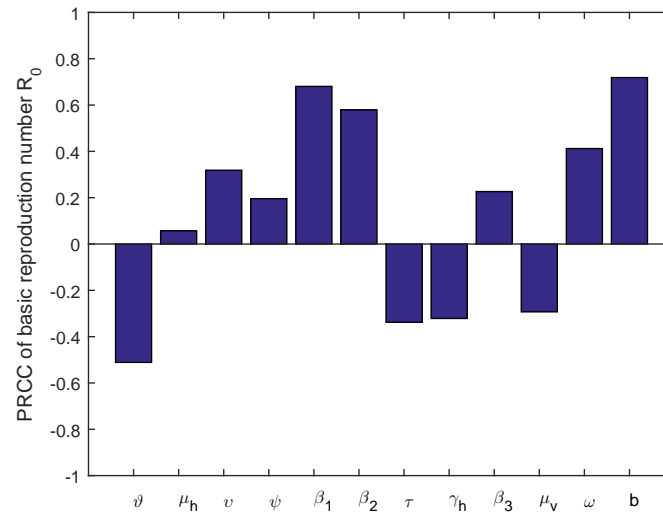


Figure 1. Sensitivity test for input factors of R_0 and PRCC outcomes for R_0 .

Table 1. Sensitivity results of R_0 with PRCC and corresponding p values.

Parameter	Interpretation	PRCC values	p values
ϑ	index of memory or fractional order	-0.5112	0.0000
μ_h	recruitment rate of humans host	0.1095	0.0023
v	vaccinated fraction of susceptible host	+ 0.3133	0.0000
ψ	fraction of incidence rate from S_h to I_h	+ 0.1804	0.0010
β_1	transmission probability from vectors to susceptible host	+ 0.6427	0.0000
β_2	transmission probability from vectors to partially immune host	+ 0.5094	0.0000
τ	treatment fraction of infected host individuals	-0.2110	0.0000
γ_h	recovery rate of host individuals	-0.3361	0.0000
β_3	transmission probability from infected hosts to susceptible vectors	+0.2320	0.0000
μ_v	recruitment rate of mosquitoes vector	-0.3197	0.0000
ω	fraction of incidence rate from P to I_h	+ 0.3596	0.0000
b	bitting rate of vectors	+0.7022	0.0000

we observed that vaccination reduces the susceptibility of host individuals in the system while the treatment reduces the infectivity of host individuals. In Figure 2 (b), we showed the influence of β_3 and γ_h on the basic reproduction number R_0 of the proposed system of dengue fever.

5. Numerical scheme for fractional model

We present in this section the solution of the model (3.4) by using the algorithm presented in [51]. To have a numerical scheme, we write the model (3.4) in the following form:

$$\begin{cases} {}^C D_t^\vartheta g(t) = \mathcal{G}(t, g(t)), \\ g(0) = g_0, \quad 0 < T < \infty, \end{cases} \quad (5.1)$$

where $g = (x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}_+^5$, $\mathcal{G}(t, g(t))$ is used for a continuous real valued vector function, which additionally satisfies the Lipschitz condition and g_0 stands for initial state vector. Taking Caputo

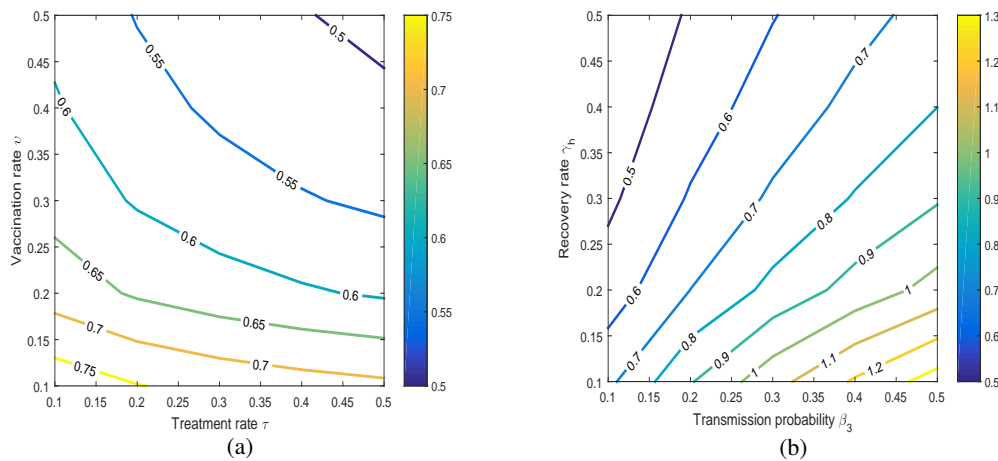


Figure 2. Illustration of basic reproduction number R_0 (a) with the variation of vaccination ν and treatment τ , (b) and with the variation of β_3 and γ_h .

integral on both sides of (5.1) we get

$$g(t) = g_0 + \frac{1}{\Gamma(\vartheta)} \int_0^t (t - \lambda)^{\vartheta-1} \mathcal{G}(\lambda, g(\lambda)) d\lambda. \quad (5.2)$$

In order to formulate an iterative scheme, we consider a uniform grid on $[0, \mathcal{T}]$ with $h = \frac{\mathcal{T}-0}{m}$ is the step size and $m \in \mathbb{N}$. Thus, the equation (5.2) gets the structure as follows after make use of the Euler method [52]

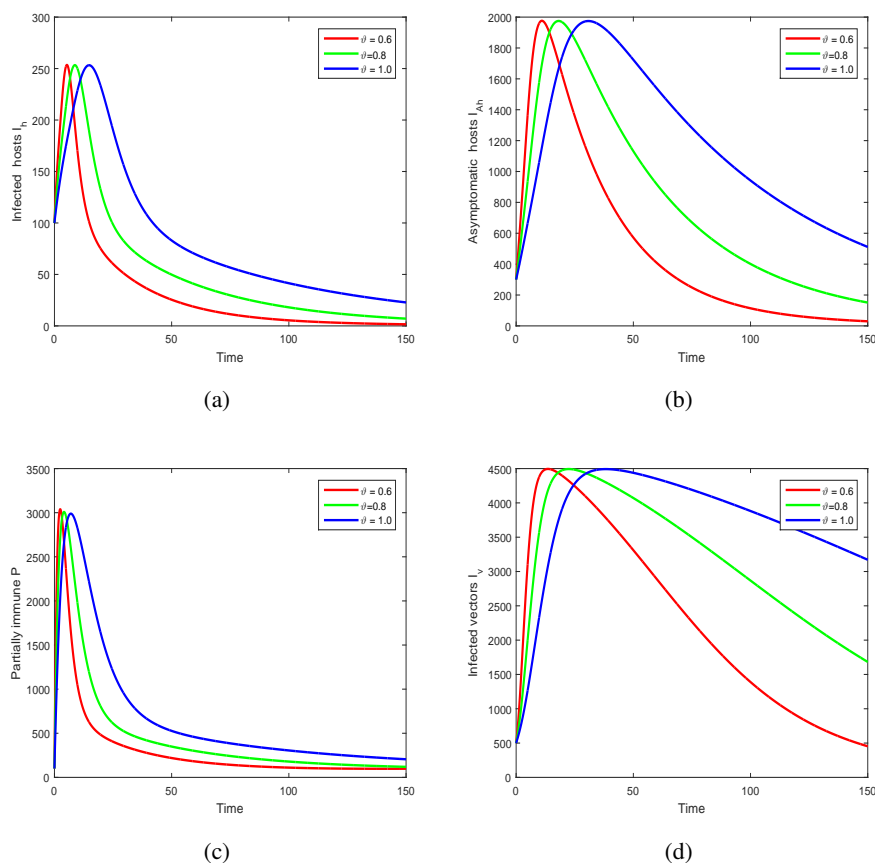
$$\begin{cases} g_{n+1} = g_0 + \frac{h^\vartheta}{\Gamma(\vartheta+1)} \sum_{j=0}^n ((n-j+1)^\vartheta - (n-j)^\vartheta) \mathcal{G}(t_j, g(t_j)), \\ n = 0, 1, 2, \dots, m. \end{cases} \quad (5.3)$$

Thus, utilizing the above scheme (5.3), we deduced the following iterative formulae for the corresponding classes of the model (3.4)

$$\begin{aligned} x_{1_{n+1}} &= x_{1_0} + \frac{h^\vartheta}{\Gamma(\vartheta+1)} \sum_{j=0}^n ((n-j+1)^\vartheta - (n-j)^\vartheta) \left(\mu_h^\vartheta - b^\vartheta a x_{1_j} x_{5_j} - \nu^\vartheta x_{1_j} - \mu_h^\vartheta x_{1_j} \right), \\ x_{2_{n+1}} &= x_{2_0} + \frac{h^\vartheta}{\Gamma(\vartheta+1)} \sum_{j=0}^n ((n-j+1)^\vartheta - (n-j)^\vartheta) \left(\psi b^\vartheta a x_{1_j} x_{5_j} + \omega b^\vartheta c x_{4_j} x_{5_j} - (\mu_h^\vartheta + \tau^\vartheta + \gamma_h^\vartheta) x_{2_j} \right), \\ x_{3_{n+1}} &= x_{3_0} + \frac{h^\vartheta}{\Gamma(\vartheta+1)} \sum_{j=0}^n ((n-j+1)^\vartheta - (n-j)^\vartheta) \left((1-\psi) b^\vartheta a x_{1_j} x_{5_j} + (1-\omega) b^\vartheta c x_{4_j} x_{5_j} - (\mu_h^\vartheta + \gamma_h^\vartheta) x_{3_j} \right), \\ x_{4_{n+1}} &= x_{4_0} + \frac{h^\vartheta}{\Gamma(\vartheta+1)} \sum_{j=0}^n ((n-j+1)^\vartheta - (n-j)^\vartheta) \left(\nu^\vartheta x_{1_j} + \rho \tau^\vartheta x_{2_j} + \phi \gamma_h^\vartheta (x_{2_j} + x_{3_j}) - b^\vartheta c x_{4_j} x_{5_j} - \mu_h^\vartheta x_{4_j} \right), \\ x_{5_{n+1}} &= x_{5_0} + \frac{h^\vartheta}{\Gamma(\vartheta+1)} \sum_{j=0}^n ((n-j+1)^\vartheta - (n-j)^\vartheta) \left(b^\vartheta e (1 - x_{5_j}) (x_{2_j} + x_{3_j}) - \mu_v^\vartheta x_{5_j} \right). \end{aligned} \quad (5.4)$$

Table 2. Interpretation of input parameters with values used in numerical simulations.

Parameter	Interpretation	Values	Reference
μ_v	natural mortality and recruitment rate of vector	0.032300 & 0.029410	[53, 54]
β_1	transmission probability from vectors to susceptible host	0.75	[10]
β_2	transmission probability from vectors to partially immune host	0.375	Assumed
ϑ	index of memory or fractional order	variable	Assumed
b	bitting rate of vectors	0.50	[10]
τ	treatment fraction of infected host	0.40	Assumed
ν	vaccinated fraction of susceptible host	0.20	Assumed
β_3	transmission probability from infected humans to mosquitoes	0.75	[10]
ψ	fraction of incidence rate from S_h to I_h	0.48	Assumed
γ_h	recovery rate of host individuals	0.3288330	[53, 54]
ω	fraction of incidence rate from P to I_h	0.54	Assumed
μ_h	natural mortality and recruitment rate of humans	0.004500 & 0.000046	[53, 54]
ρ	fraction of treated humans enter to P class	0.6	Assumed

**Figure 3.** Illustration of the path moment of (a) Infected host I_h , (b) Carrier host(Asymptomatic) I_{hA} , (c) Partially immune P and (d) Infected host I_v individuals with different values of fractional-order ϑ , i.e., $\vartheta = 0.6, 0.8, 1.0$.

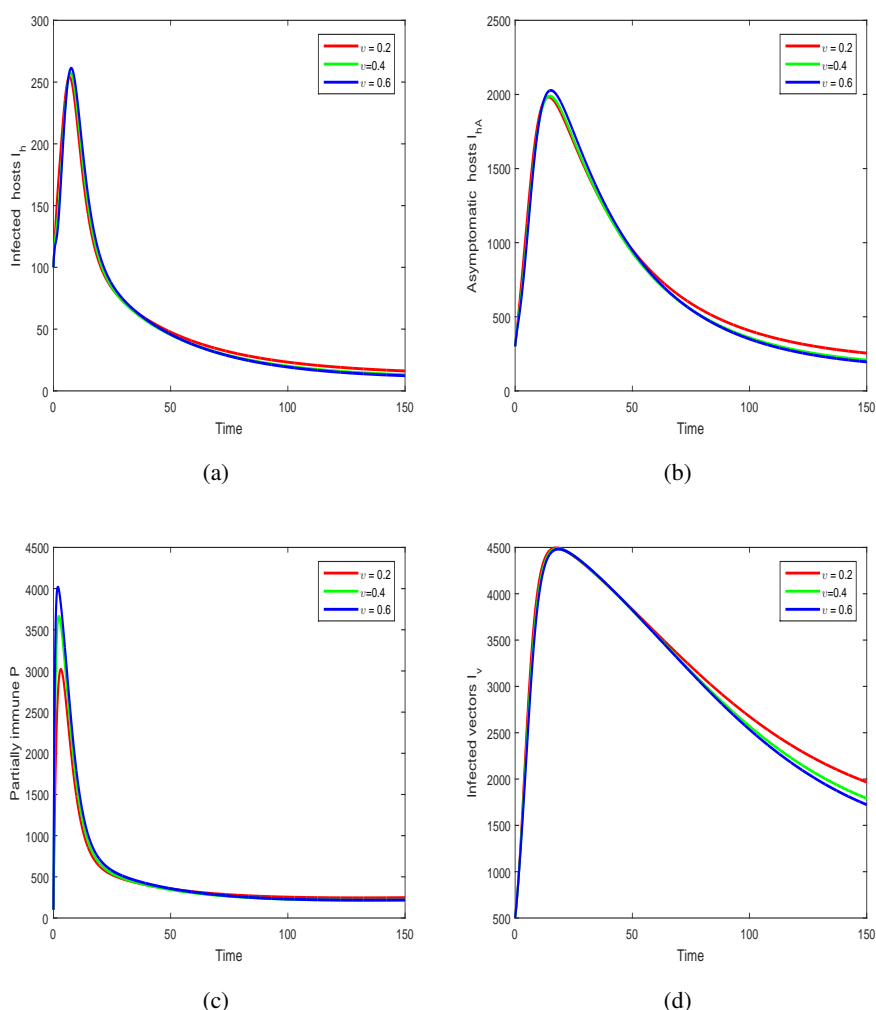


Figure 4. Illustration of the path moment of (a) Infected host I_h , (b) Carrier host(Asymptomatic) I_{hA} , (c) Partially immune P and (d) Infected vector I_v individuals with different vaccination rates ν , i.e., $\nu = 0.2, 0.4, 0.6$.

We used the above approximation for the solution of our fractional system. In Figure (3), we demonstrate the dynamics of infected host (I_h), asymptomatic host (I_{hA}), partially immune (P) host and infected vector (I_v) with the variation of fractional-order ϑ . We noticed that the variation of fractional-order ϑ has a great influence on the infection level of dengue in both the population. In other words, it can highly reduce the level of dengue fever in the community. In Figure (4) and Figure (5), we represent the dynamics of dengue fever with the variation of vaccination and treatment. In these simulations, we observed that vaccination and treatment decrease susceptibility and infectivity of the host population, and have a slight influence on the level of infection. We demonstrated the effect of biting rate b of the mosquitoes on the dynamics of dengue and observed that the peak of infection can be greatly decreased by decreasing the biting rate b in Figure (6). The mosquitoes biting and its generation further can be decreased by spraying or wasting the standing water around home or inside the home, which has great influence on population of mosquitoes that end up biting humans. Using bed-nets, avoid to visit

areas prone to mosquitoes, using mosquito repellent, covering legs and arms by wearing long-sleeves and long pants are useful to prevent himself from the biting of mosquitoes. These scenarios predict that the infection can be controlled and prevented by decreasing the index of memory and biting rate of vectors in the community.

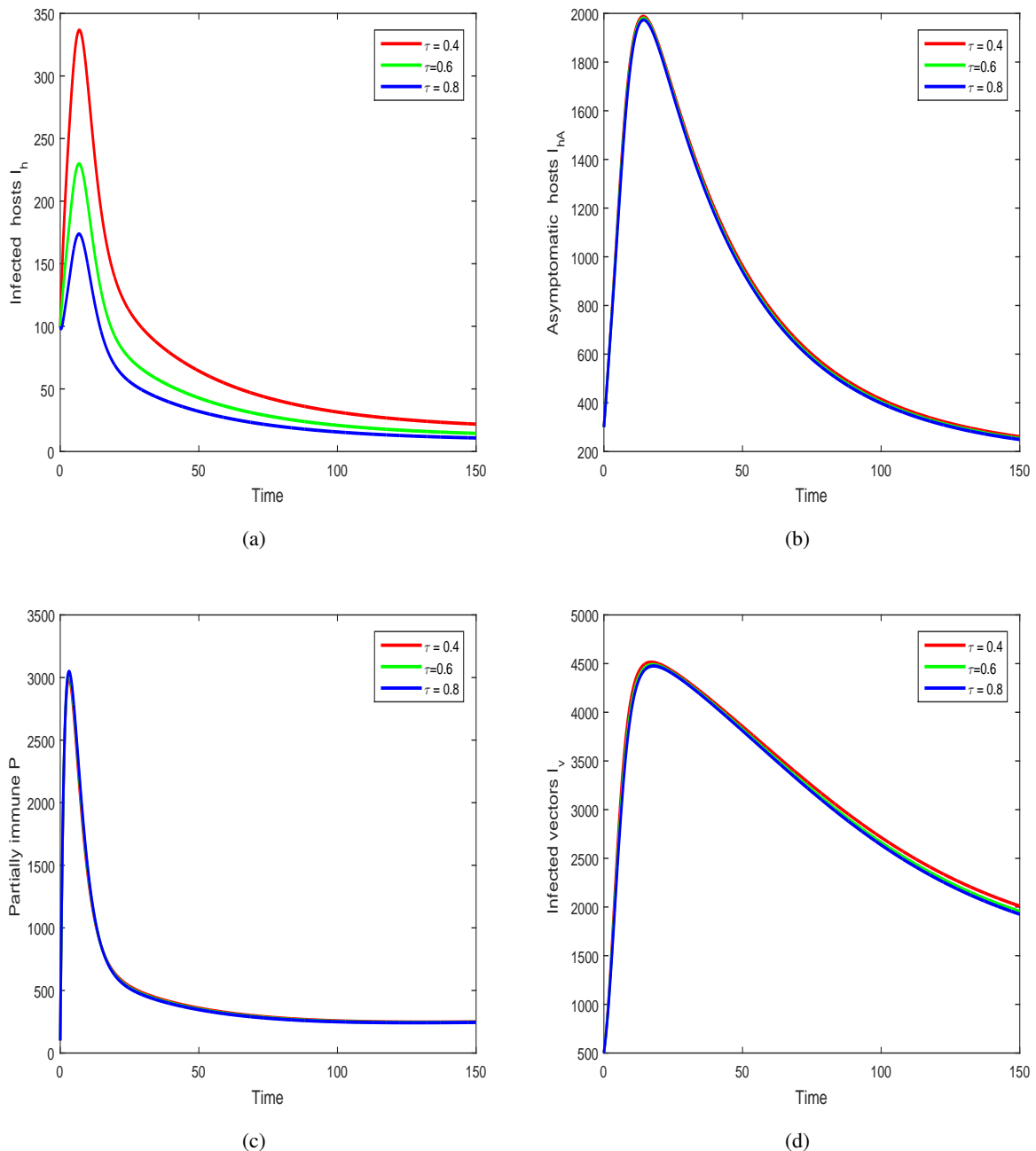


Figure 5. Illustration of the path moment of (a) Infected host I_h , (b) Carrier host(Asymptomatic) I_{hA} , (c) Partially immune P and (d) Infected vector I_v individuals with different treatment rates τ , i.e., $\tau = 0.4, 0.6, 0.8$.

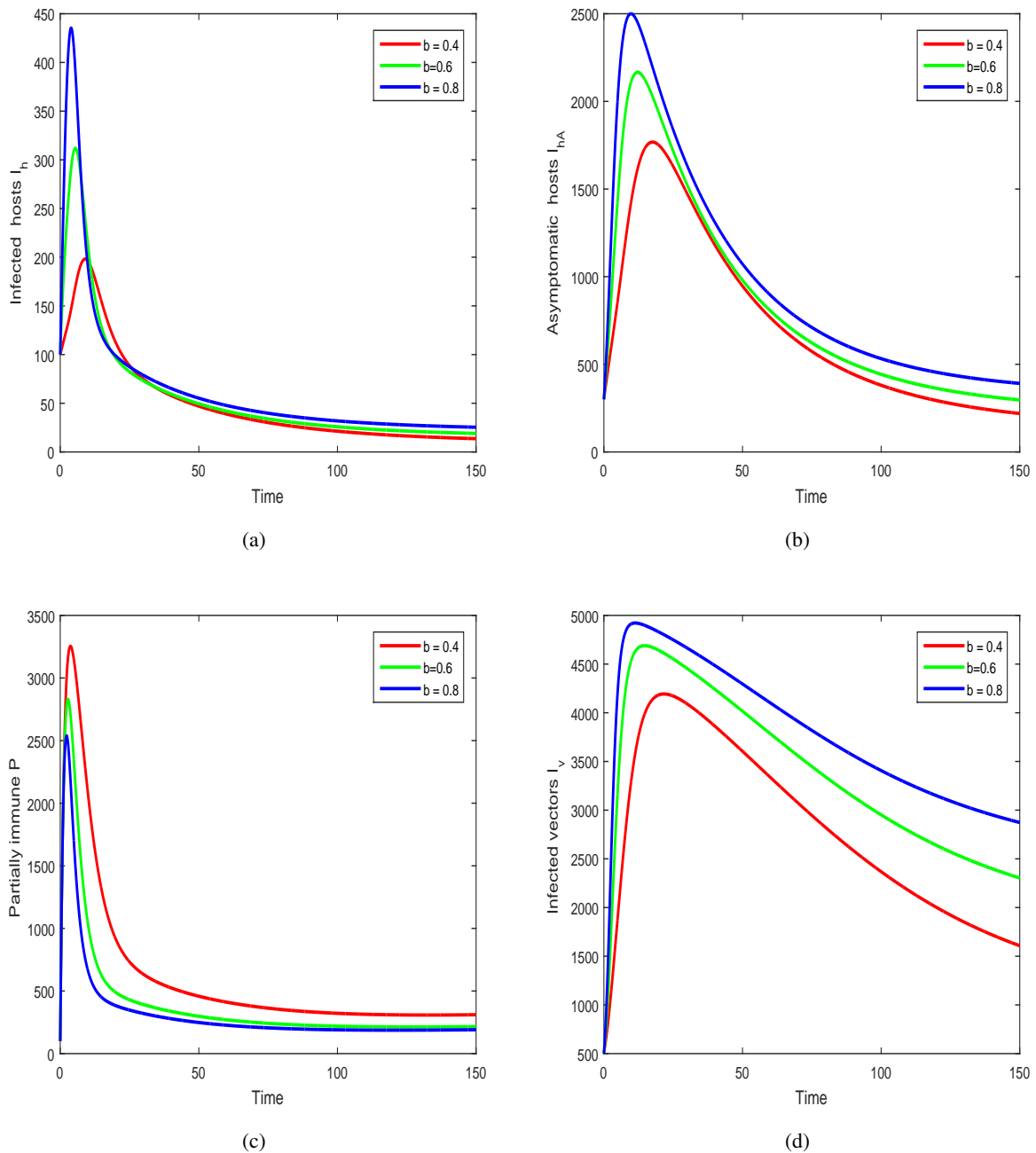


Figure 6. Illustration of the path moment of (a) Infected host I_h , (b) Carrier host(Asymptomatic) I_{hA} , (c) Partially immune P and (d) Infected vector I_v individuals with variation of biting rate of mosquitoes b , i.e., $b = 0.4, 0.6, 0.8$.

6. Conclusion

In this article, we constructed a new epidemic model for the transmission of dengue infection with non-integer derivative. Basic results of the suggested fractional system are investigated through analytic skills. The basic reproduction number of the system is calculated through the next-generation

method, indicated by R_0 . We established stability results of infection-free steady-state of the system. Sensitivity analysis of R_0 is carried out in order to know the contribution of input factors in the results of R_0 and observed that b, μ_v , and β_3 are the most critical parameters that highly contribute in the control and subsequent spread of dengue infection. We showed that the dengue infection is uniformly persistent in the system for $R_0 > 1$. We obtained feasible results for the dynamics of dengue infection with the variation of memory index ϑ and suggested that the index of memory has a dominant influence on the system. We concluded that the fractional-order model can explore more accurately the dengue epidemic disease transmission model rather than the integer-order derivative models. The numerical results for the suggested parameter can be considered useful for the possible eliminations of dengue infection in the community. We suggest that fractional-order(index of memory) ϑ and biting rate b can remarkably control and greatly decrease the level of disease in the society.

Conflict of interest

No conflict of interest exists regarding the publications of this work.

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